Characteristics of Final Particles in Multiple Compton Backscattering Process

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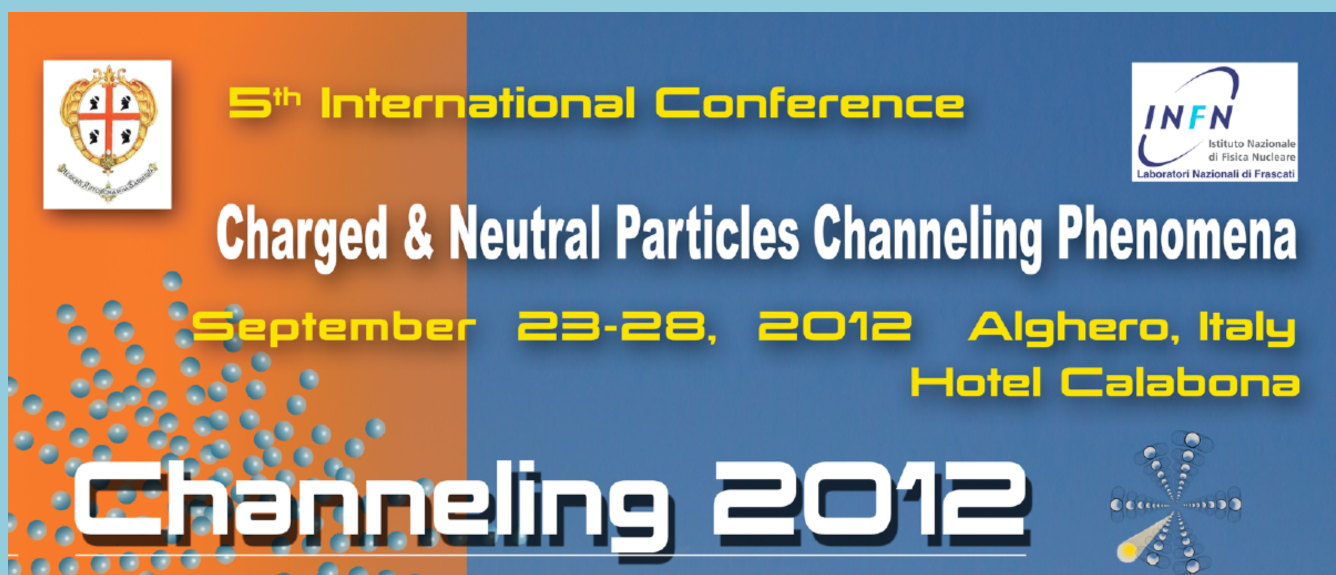
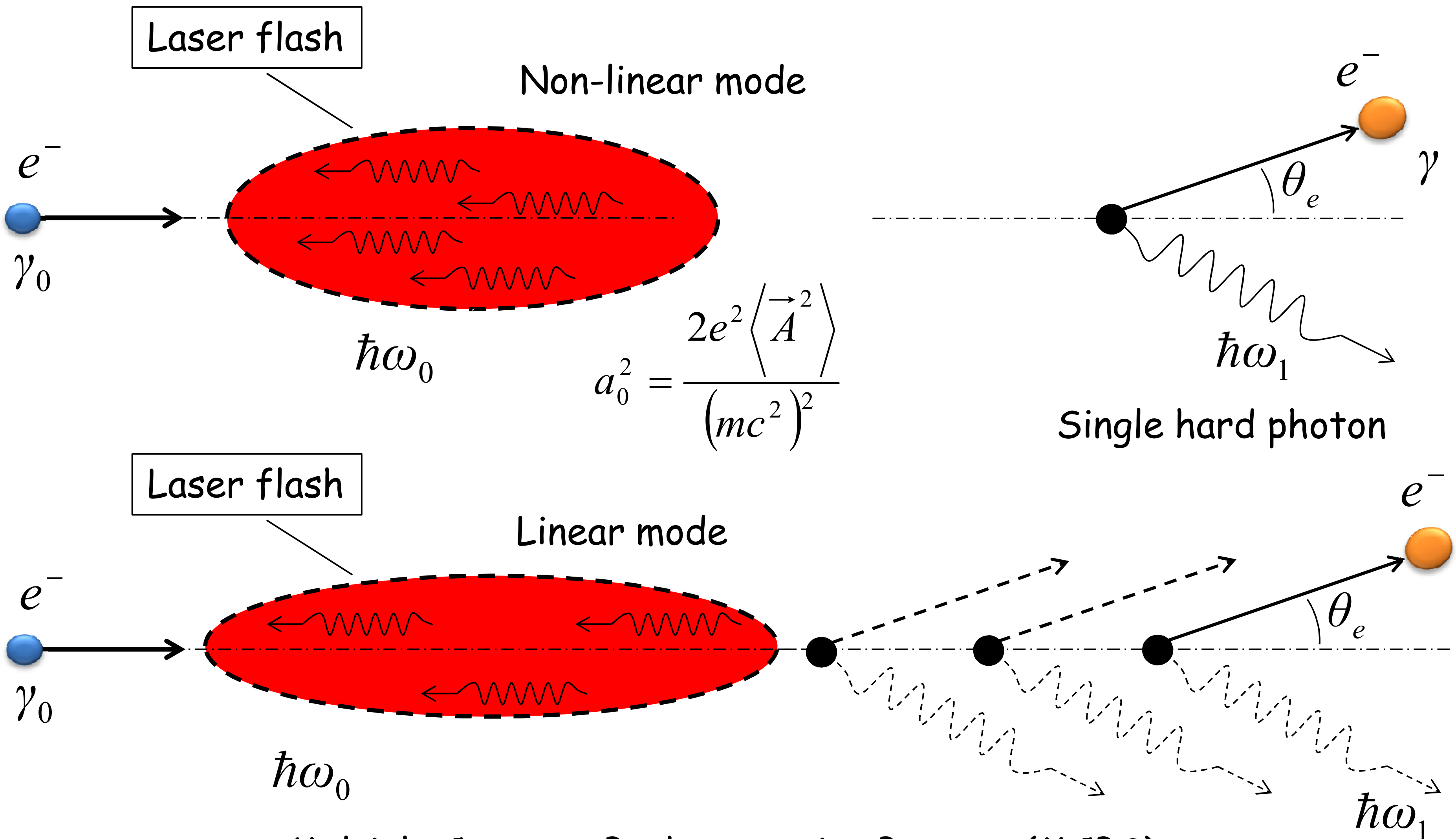


Compton backscattering



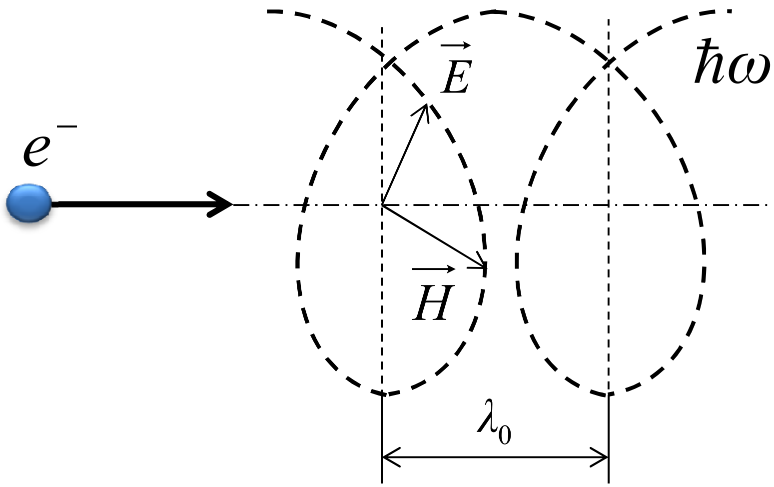
Multiple Compton Backscattering Process (MCBS)

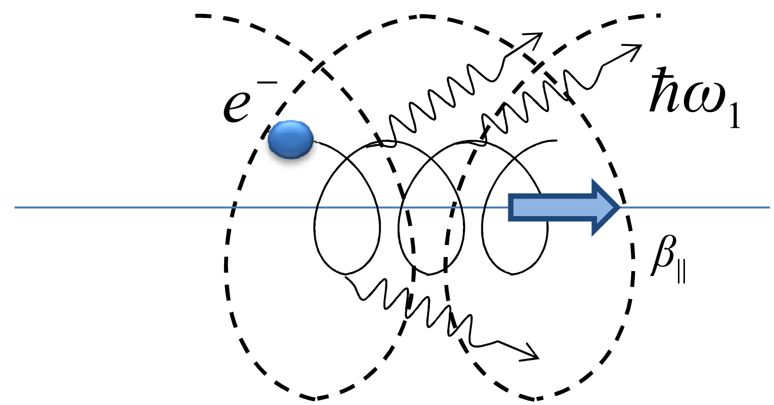
$a_0^2 << 1$

$\theta_e \sim \frac{\hbar\omega_0}{mc^2} \leq 10^5$

Plane – wave approximation

Circularly-polarized wave



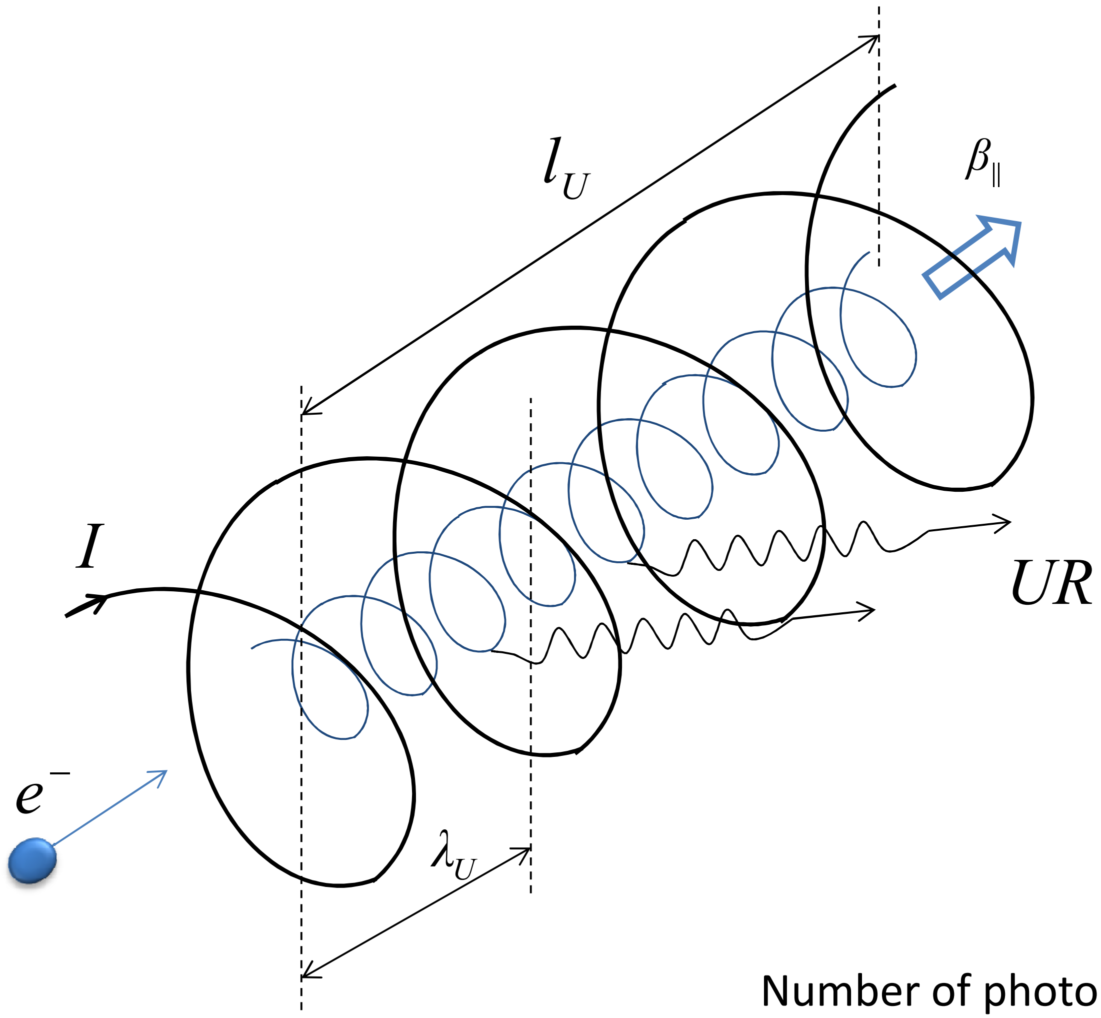


Radius of electron trajectory

$$r_\perp \sim a_0 \lambda_0$$

MCBS may be treated as undulator radiation in a field of intense plane wave ("light undulator")

Undulator radiation



$$K = \frac{eH\lambda_U}{2\pi mc^2}$$

Number of photons per electron in undulator

$$\bar{k}_{UR} = Rl_U\,,$$

Rate of emitted photons $R = \frac{2}{3}\pi\alpha K^2 \frac{1}{\lambda_U},\quad \left(K^2 << 1\right)$

$\bar{k}_{UR}$ Is a random value

MCBS process ("light undulator")

$$K^2 \to a_0^2, \quad \lambda_U \to \frac{\lambda_0}{2}$$

$$\bar{k}_{MCBS} = \frac{2}{3}\pi\alpha a_0^2 l_L, \quad l_L -$$ length of a laser pulse, $l_L = N_0\lambda_0$

$$a_0^2 = 4\pi\alpha\lambda\!\!\!^{-}{}_e^2\lambda_0 n_0,$$

$\lambda\!\!\!^{-}{}_e = \frac{\hbar}{mc}, \quad n_0 -$ mean concentration of laser photons.

$$\bar{k}_{MCBS} = 2\cdot\frac{8\pi}{3}r_e^2 n_0 l_L = 2\sigma_T n_0 l_L$$

where $\sigma_T = \frac{8\pi}{3}r_e^2 -$ Thomson cross-section

The strict number of MCBS photons is defined from luminosity

$$L = c(1+\beta_0)\sigma N_e N_L \int dVdtF_e(x,z,y,t)F_L(x,y,z,t)$$

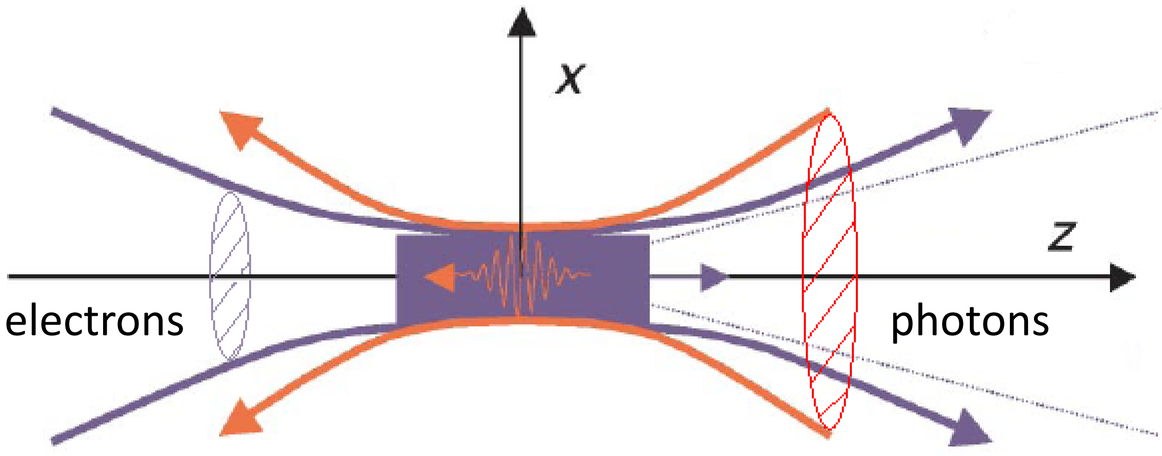
$N_{e(L)} -$ number of electrons (photons) in bunches

$F_{e(L)}(x,y,z,t) -$ distribution of each bunches in the space and time

Mean number of emitted photons (number of collisions) per electron

$$\bar{k}_{MCBS} = L/N_e$$

Mean collision number per each initial electron. (number of photons per particle)



$$f_L \sim \exp\left(-\frac{\rho^2}{\rho_L^2}\right),$$

$$f_e \sim \exp\left(-\frac{\rho^2}{\rho_e^2}\right)$$

F. Hartemann et al. PRST – AB, (2005)1007 02

$$\rho_L^2 = \rho_0^2\left(1+\frac{z^2}{z_R^2}\right), \quad \rho_e^2 = \rho_b^2\left(1+\frac{z^2}{\beta_f^2}\right).$$

$\rho_0(\rho_b)$ is the minimum radius of photon (electron) beam. z_R is the Rayleigh length, β_f is the beta function.

Longitudinad distributions of beams were approximated by Gaussians with parameters l_L and l_e.

In this model the mean number of scattered photons is determined by the flollowing expression :

$$\bar{k} = \frac{16\sqrt{\pi}}{3}\frac{r_e^2}{\rho_0^2}N_L f(l_e,l_L,r), \text{ where}$$

$$f(l_e,l_L,r) = \exp[(1+r^2)/(\mu^2+r^2\eta^2)]/[(\mu^2+r^2\eta^2)(1+r^2)]^{1/2} \times \left\{1-\Phi\left[(1+r^2)^{1/2}/(\mu^2+r^2\eta^2)^{1/2}\right]\right\},$$

$$\mu = \frac{l_L}{2\sqrt{2}z_R}, \quad \eta = \frac{l_e}{2\sqrt{2}\beta_f}, \quad r = \frac{\rho_b}{\rho_0}, \quad \Phi(x) \text{ is the error function.}$$

Simplest case $\rho_{\mathrm{L}} = const$, $\rho_{\mathrm{e}} = const$

In the limit $z_R \to \infty, \beta_f \to \infty \; (\mu, \eta \to 0)$

$$1 - \Phi(x) \approx \frac{1}{\sqrt{\pi}} \frac{\exp(-x^2)}{x},$$

$$\bar{k} \approx \frac{16}{3} N_L \frac{r_e^2}{\rho_0^2 + \rho_e^2}$$

or using luminosity L : $\bar{k} = \frac{L\sigma}{N_e}$

Introducing a mean photon concentration n_0 and effective length l_L of a laser flash :

$$n_0 = \frac{N_L}{V_{eff}} = \frac{W_L}{\hbar\omega_0} \frac{1}{\pi \rho_{eff}^2 l_L} :$$

$$\bar{k} \approx l_L 2\sigma_T (1 - x_0) n_0 \approx \frac{l_L}{l_T} \;\; for \;\; x_0 << 1, \quad x_0 \approx \frac{4\gamma_0 \hbar\omega_0}{mc^2}$$

Where $l_T = \frac{1}{2 n_0 \sigma_T}$ – so - called Thomson free path electron length in a "light target".

Scheme for production of polarized positrons

[T. Omori et al. NIMA 500(2003)232]

The mean number of collisions $\overline{k}$ may be estimated for parameters:

$\rho_0 \approx 20\ \mu m,\ z_R \approx 220\ \mu m,\ l_L \approx 3\ mm$

$\rho_b \approx 20\ \mu m,\ \beta_f \approx 3600\ mm,\ le \approx 1\ mm$

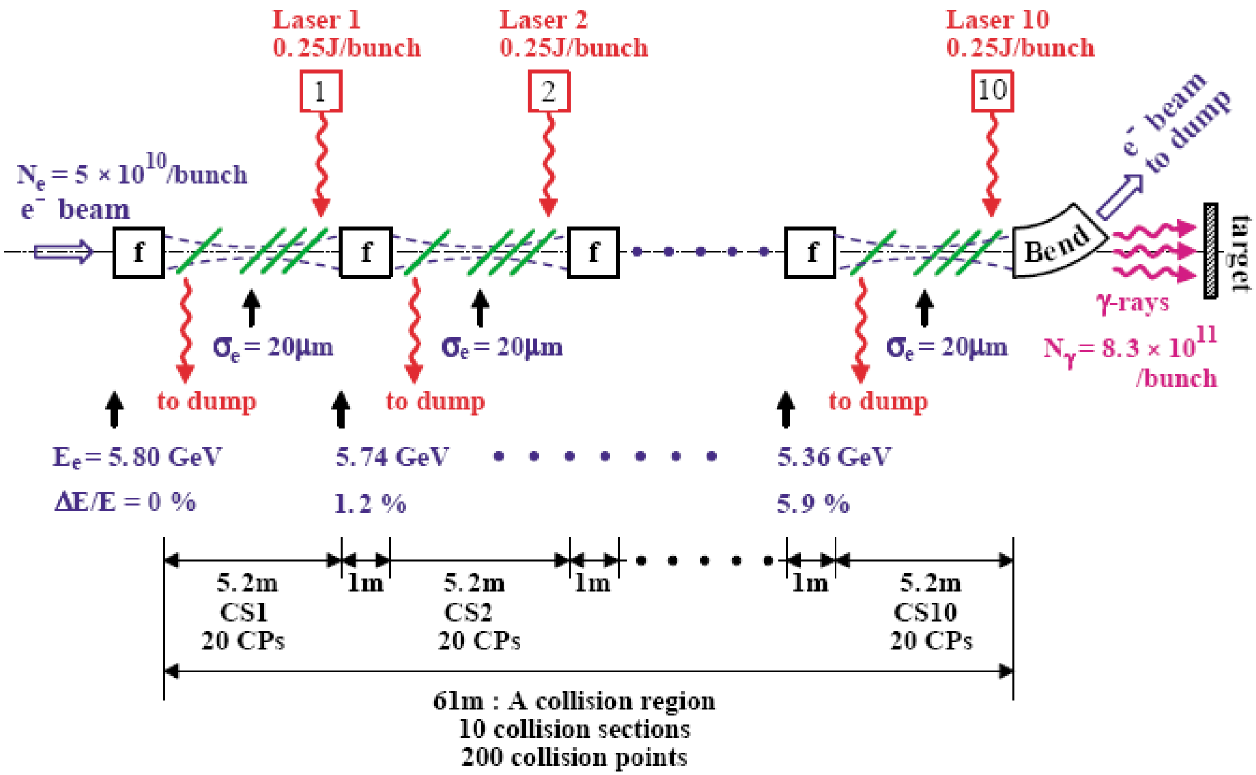
$\overline{k_0} \approx 0.2$

and for 200 collision points $\overline{k} \approx 200\overline{k_0} \approx 40$

Due to non-gaussian laser beam authors have got the value:

$\overline{k} = N_\gamma / N_e = 16.6$

$\left(N_\gamma = 8.3 \cdot 10^{11}\ photons/bunch,\ N_e = 5 \cdot 10^{10}\ e^-/bunch\right)$



Number of photons per electron:

$\overline{k} = 2.4$ CLICHE [D. Asher et al. Eur. Phys. J. C., 28 (2003) 27]

$\overline{k} = 1.2$ SAPPHiRE [S. A. Bogacz et. al. arXiv: 1208.2827]

Analytic description of MCBS process

The cross-section of the linear MCBS process

$$\frac{d\sigma}{dy} = \frac{2\pi r_e^2}{x_0}\left[\frac{1}{1-y} + 1 - y - \frac{4y}{x_0(1-y)}\left(1 - \frac{y}{x_0(1-y)}\right)\right],$$

$$x_0 = \frac{2p_0k_0}{(mc^2)^2} \approx \frac{4\gamma_0\hbar\omega_0}{mc^2}, \qquad y = 1 - \frac{p_0k}{p_0k_0} \approx \frac{\hbar\omega}{\gamma_0 mc^2}$$

MCBS is the stochastic process where $\bar{k}$ and electron energy losses $\Delta\varepsilon = \varepsilon_0 - \varepsilon$

along a path l are described by the probability distribution $P(\varepsilon, \varepsilon_0, l)$

$$\frac{\partial}{\partial l}P(\varepsilon,\varepsilon_0,l) + \Sigma(\varepsilon_0)P(\varepsilon,\varepsilon_0,l) \int_0^{\hbar\omega_{\max}} \Sigma(\hbar\omega;\varepsilon_0)P(\varepsilon,\varepsilon_0-\hbar\omega,l)\,d\hbar\omega = 0$$

with the boundary condition

$$P(\varepsilon,\varepsilon_0,l=0) = \delta(\varepsilon-\varepsilon_0).$$

Here

$\Sigma(\hbar\omega,\varepsilon_0) = 2\bar{n}_0 \dfrac{d\sigma}{d\hbar\omega}$ is the differential macroscopic cross-section of the Compton scattering process and

$\Sigma(\varepsilon_0) = \displaystyle\int_0^{\hbar\omega_{\max}} \Sigma(\hbar\omega;\varepsilon_0)\,d\hbar\omega = 0$ is the total macroscopic cross-section.

In the paper [A.Kolchuzhkin, A. Potylitsyn, S. Strokov et al., Nucl. Instr. and Meth. B 201 (2003) 307] we have derived analytical formulae for the main parameters

$$\bar{k}(\varepsilon_0, l_L) = 2\sigma_T \overline{n_0} l_L - 2\log\left(1 + {g_1(\varepsilon_0) l_L}/{\varepsilon_0}\right),$$

$$\bar{\varepsilon}(\varepsilon_0, l_L) = \frac{\varepsilon_0}{1 + {g_1(\varepsilon_0) l_L}/{\varepsilon_0}},$$

$$\Delta(\varepsilon_0, l_L) = \frac{g_2(\varepsilon_0) l_L}{\left[1 + {g_1(\varepsilon_0) l_L}/{\varepsilon_0}\right]^4}.$$

where coefficients $g_{1,2}(\varepsilon_0)$ are defined as

$$g_1(\varepsilon_0) = \int_0^{\hbar\omega_{\max}} \hbar\omega \Sigma(\hbar\omega; \varepsilon_0) d\hbar\omega,$$

$$g_2(\varepsilon_0) = \int_0^{\hbar\omega_{\max}} (\hbar\omega)^2 \Sigma(\hbar\omega; \varepsilon_0) d\hbar\omega.$$

In the case $x_0 << 1$

$$\bar{\varepsilon}(\varepsilon_0, l_L) \approx \frac{\varepsilon_0}{1 + \frac{1}{2}\bar{k} x_0 \left(1 - \frac{21}{10} x_0\right)},$$

$$\Delta(\varepsilon_0, l_L) \approx \frac{{7}/{20}\, \bar{k} \varepsilon_0^2 x_0^2}{\left(1 + \frac{1}{2}\bar{k} x_0\right)^4}.$$

Or, substituting n_0 and $\hbar\omega_{max}$ instead $\overline{k}$

$$\overline{\varepsilon} = \varepsilon_0(1 - x_0\sigma_T n_0 l_L) = \varepsilon_0 - \sigma_T n_0 \hbar\omega_{max} l_L,$$

$$\Delta^2 = \frac{7}{20} 2\sigma_T n_0 l_L \varepsilon_0^2 x_0^2 = \frac{7}{10} \sigma_T n_0 (\hbar\omega_{max})^2 l_L.$$

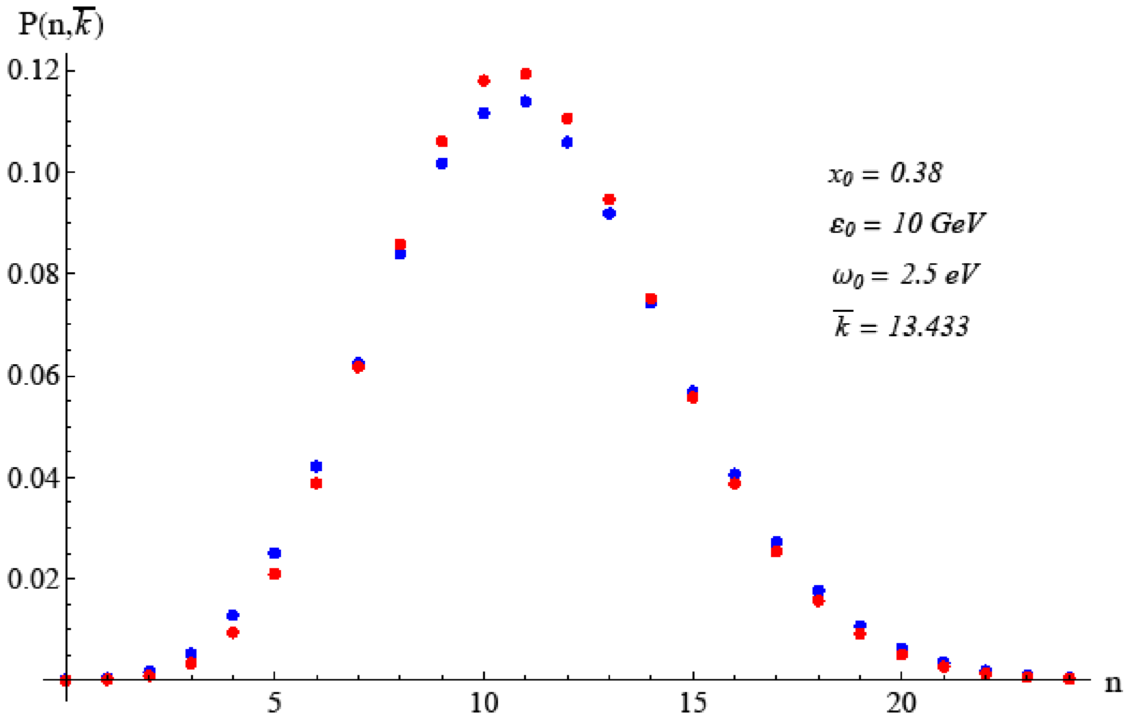
it is possible to compare expressions obtained with results:

[Robb G.R.M., Bonifacio R., Europhys. Lett., 94 (2011) 34002].

The difference is $^7/_{10}$ instead 1 in the second expression.

Monte - Carlo simulation

Distribution over the collision number *k* (number of emitted photons)

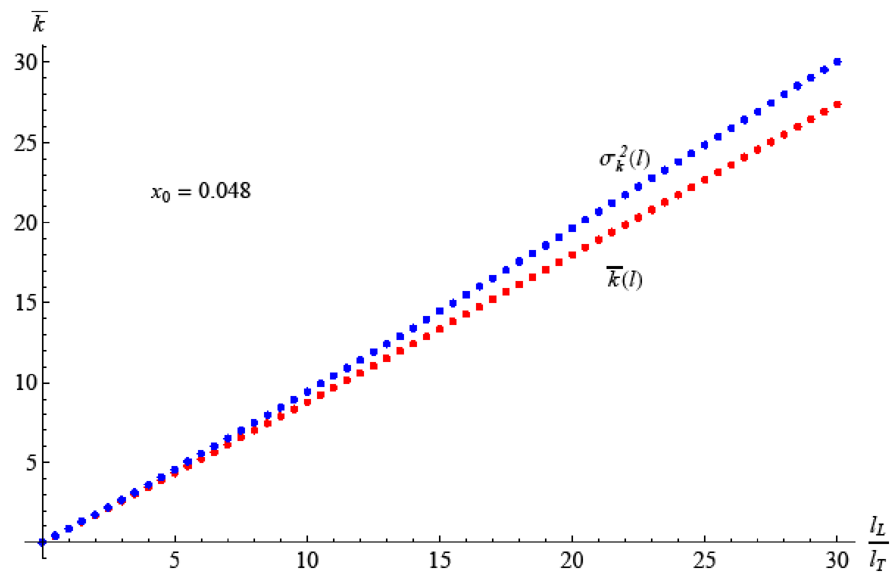


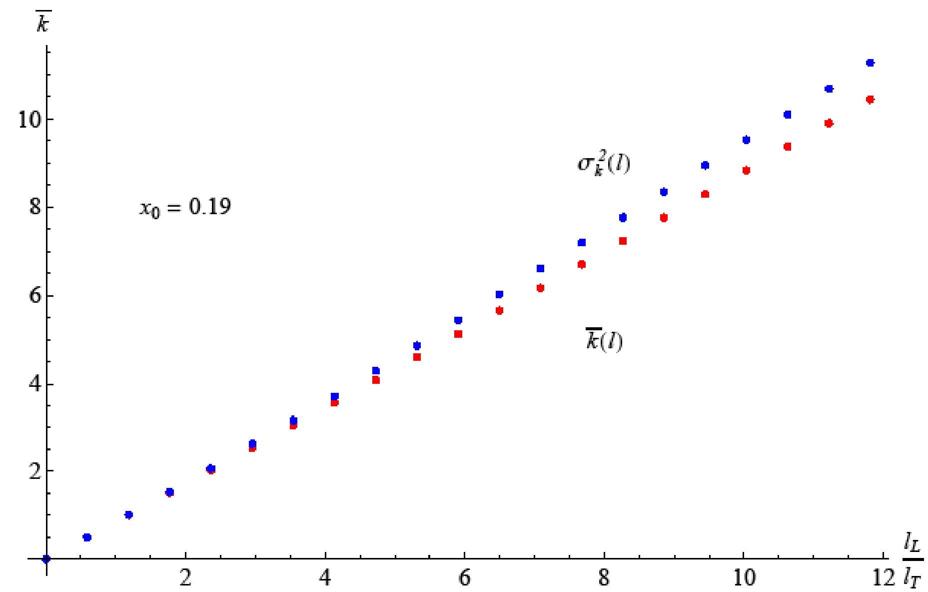
Blue – Poisson law, red – simulation rezults

$$\begin{cases} \sigma_k^2(l) = \bar{k}(l) \text{ - main characteristics of the Poisson law} \\ P(n,\bar{k}) = \bar{k}^2 \exp(-\bar{k})/n! \\ \text{For any value of } \bar{k} \text{ there is a part of electrons passing through a light target without interaction } (n=0): \\ P(0,\bar{k}) = \exp(-\bar{k}) \\ \text{For } \bar{k} = 1, \exp(-\bar{k}) = 0.37 = 37\,\% \end{cases}$$

A random path length and a random energy loss were successively simulated in each collision

Comparison of $\bar{k}$ and σ_k^2



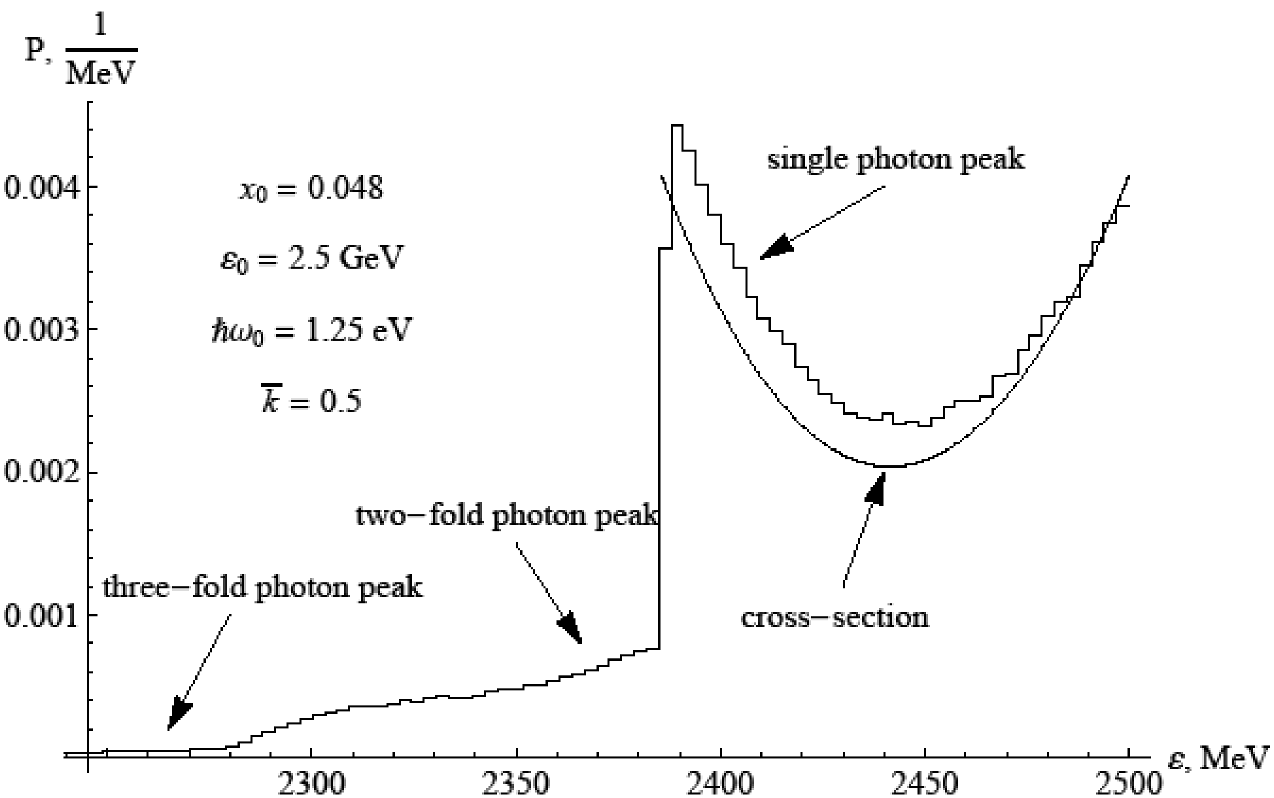


Dependencies $\bar{k}$ and σ_k^2 on a light target thickness.

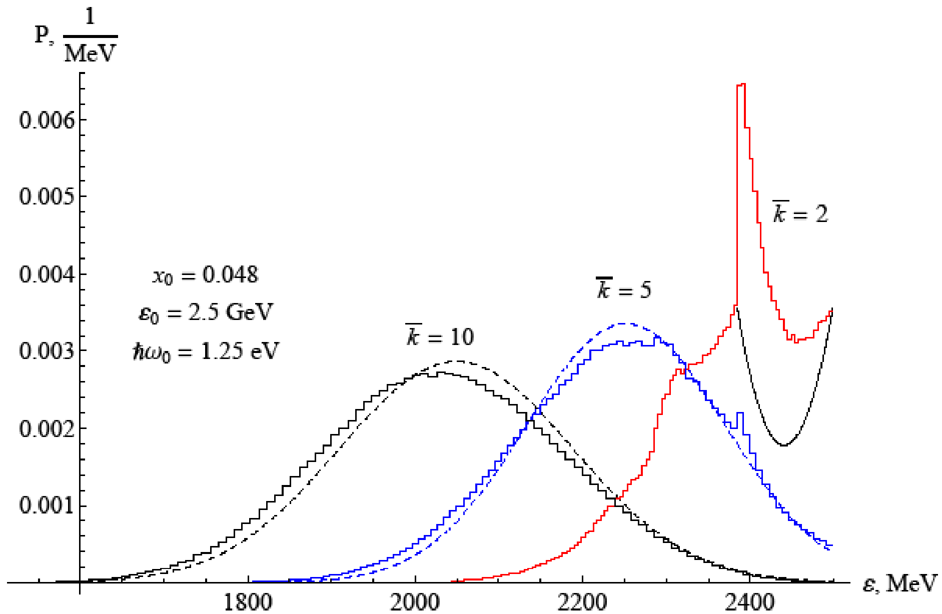
$\bar{k} = \sigma_k^2$ for small x_0 and small thickness - typical characteristic of Poisson law

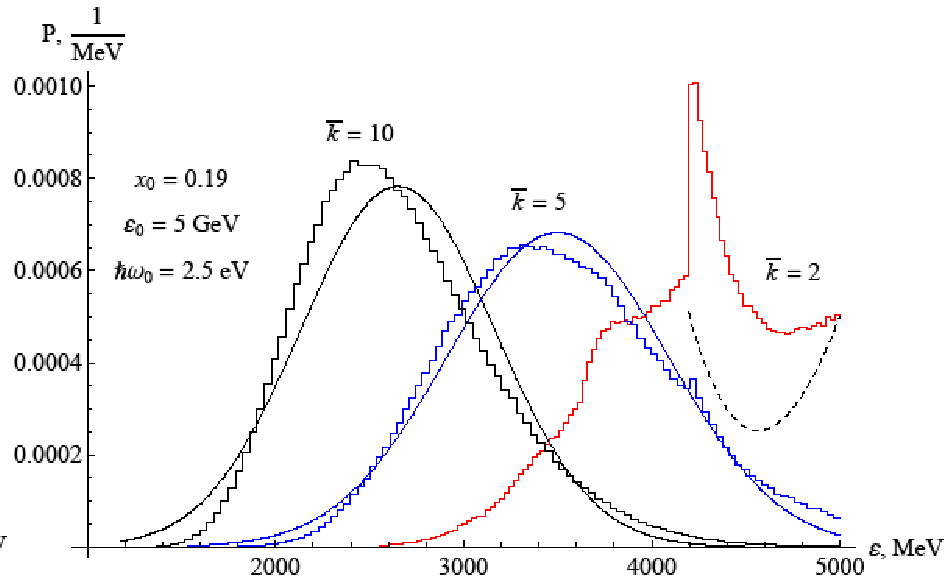
Energy distribution of recoil electrons

for small collision number $\left(\overline{k} = 0.5\right)$



Energy distribution of beam passed through a light target



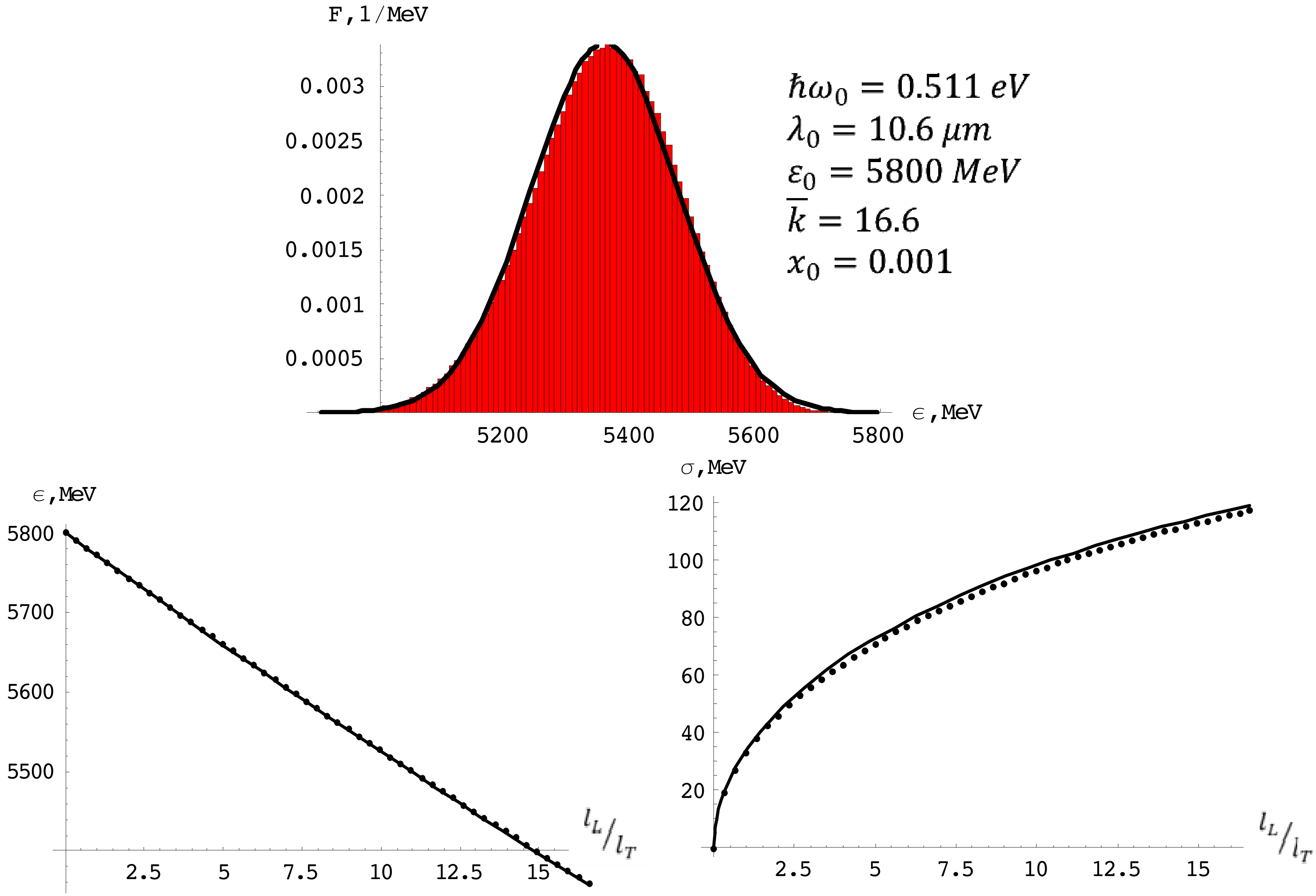


Curves – gaussian distribution with parameters

$$\bar{\varepsilon}(\varepsilon_0, l_L) \approx \frac{\varepsilon_0}{1 + \frac{1}{2}\bar{k}x_0\left(1 - \frac{21}{10}x_0\right)}, \quad \Delta(\varepsilon_0, l_L) \approx \frac{7}{20}\frac{\bar{k}\varepsilon_0^2 x_0^2}{\left(1 + \frac{1}{2}\bar{k}x_0\right)^4}$$

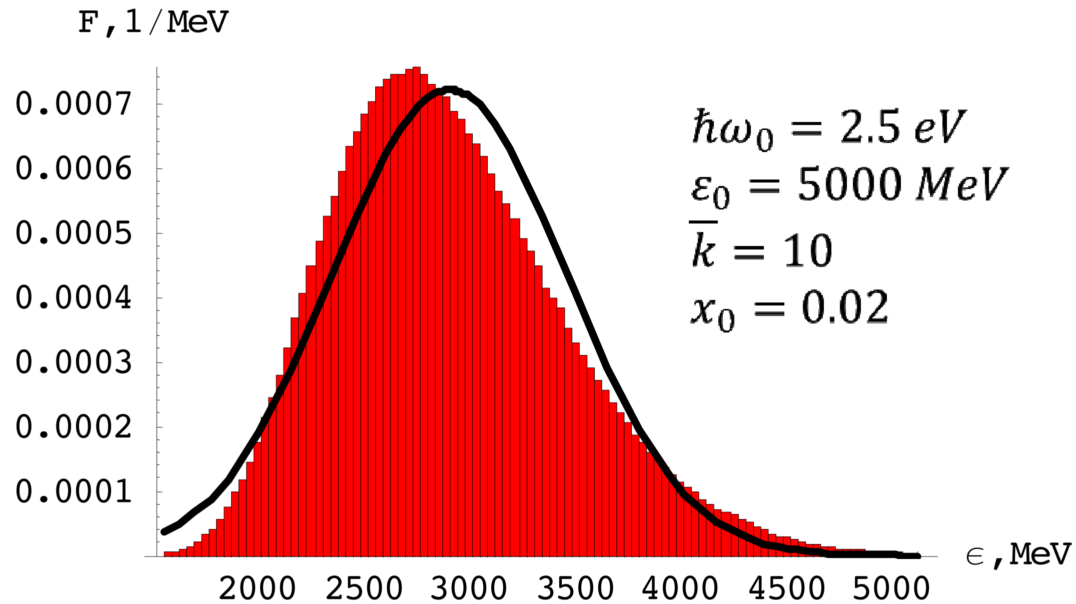
Analitical solution are valid for $x_0 << 1$ (see Kolchuzhkin et al. NIMB 201 (2003) 307)

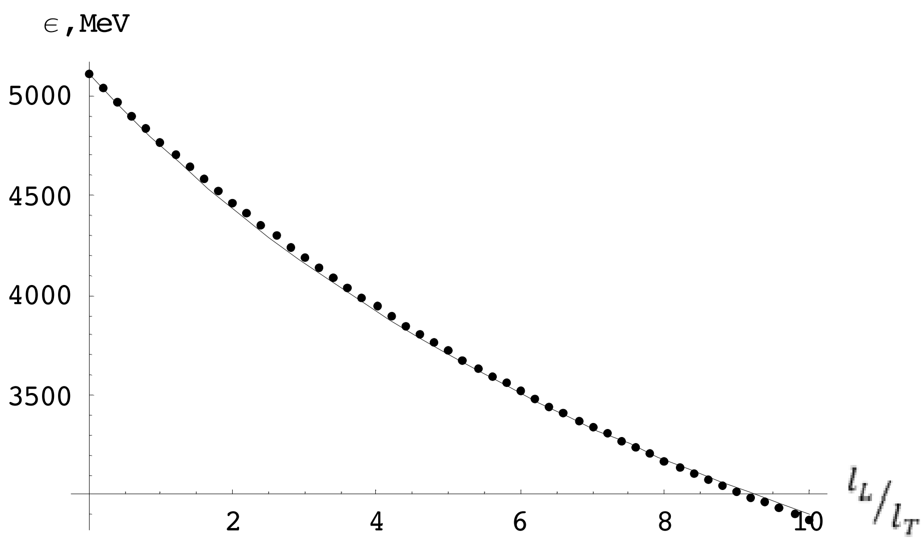
Energy distribution of electron beam passed through a light target

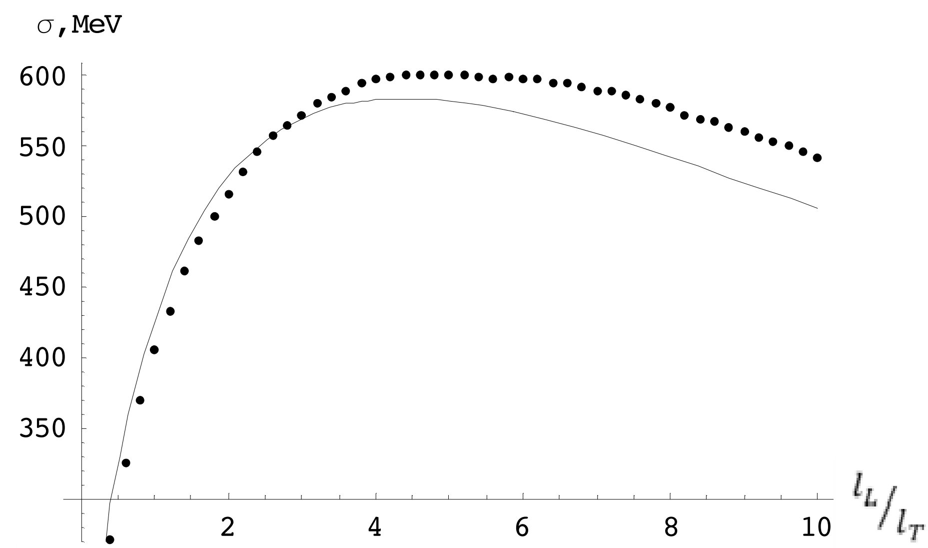


Points – M-C simulation, curves – analytical dependences

Energy distribution of electron beam passed through a light target

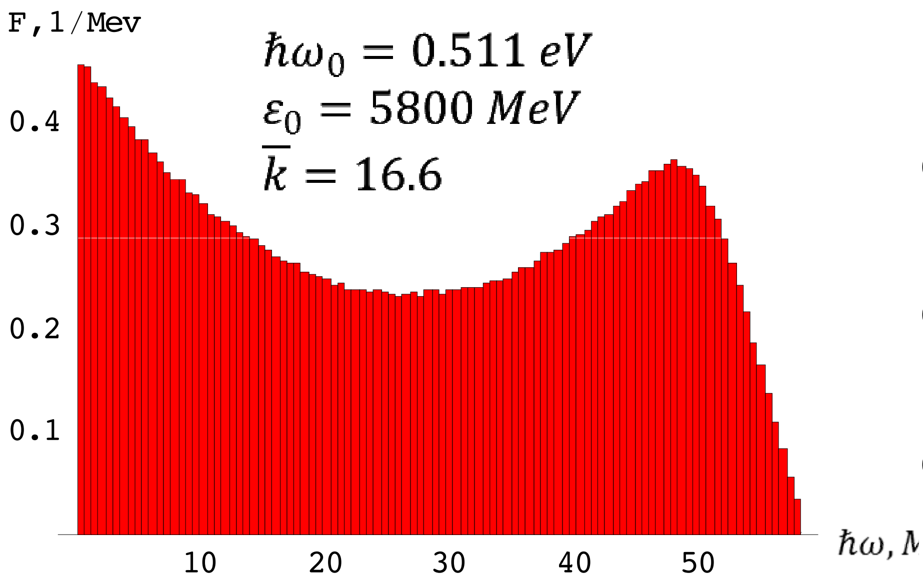


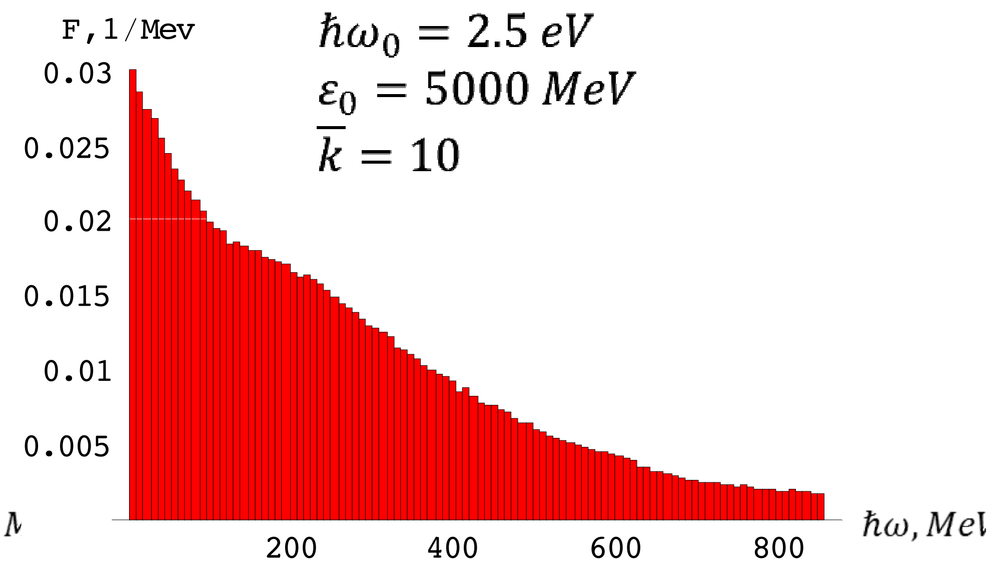




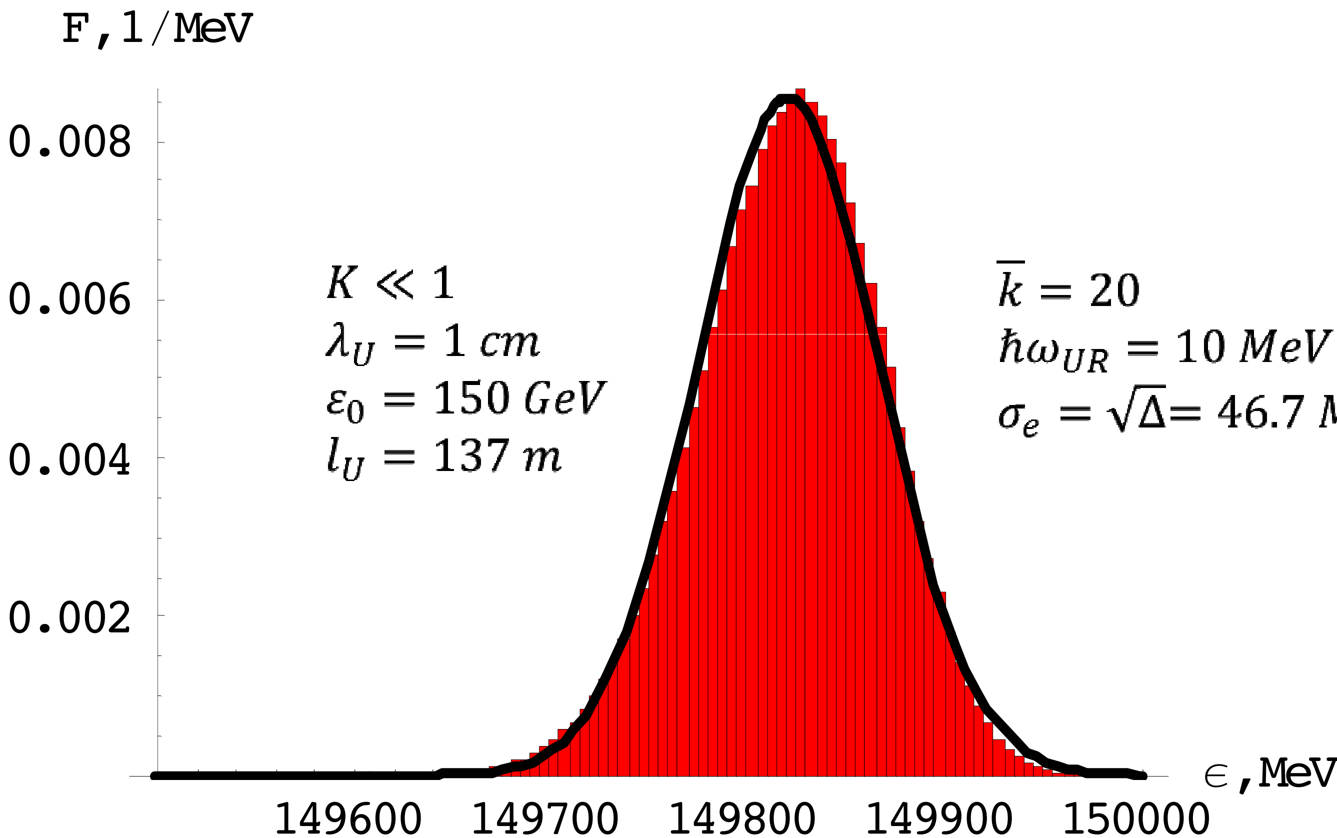
Points – M-C simulation, curves – analytical dependences

Photon spectra as a result of MCBS process





Energy distribution of electron beam at the undulator exit



Spin-dependent cross section

Unpolarized e^+ beam may be considered as a sum of two polarized components with a half of the initial intensity and polarized in opposite directions (indice + means parallel orientation of positron spin and momentum, + - antiparallel one)

In this case for $P_c = +1, \left|\xi_z\right| = \left|\xi_{z0}\right| = 1$

$$\frac{d\sigma_+}{dy} = \frac{d\sigma_{++}}{dy} + \frac{d\sigma_{+-}}{dy} = 2\left(\frac{d\sigma_0}{dy} + \frac{d\sigma_1}{dy}\right)$$

$$\frac{d\sigma_-}{dy} = \frac{d\sigma_{--}}{dy} + \frac{d\sigma_{-+}}{dy} = 2\left(\frac{d\sigma_0}{dy} - \frac{d\sigma_1}{dy}\right)$$

where

$$\frac{d\sigma_{++}}{dy} = \frac{d\sigma_0}{dy} + \frac{d\sigma_1}{dy} + \frac{d\sigma_2}{dy} + \frac{d\sigma_3}{dy}; \quad \text{- non spin - flip term}$$

$$\frac{d\sigma_{+-}}{dy} = \frac{d\sigma_0}{dy} + \frac{d\sigma_1}{dy} - \frac{d\sigma_2}{dy} - \frac{d\sigma_3}{dy}; \quad \text{- spin - flip term}$$

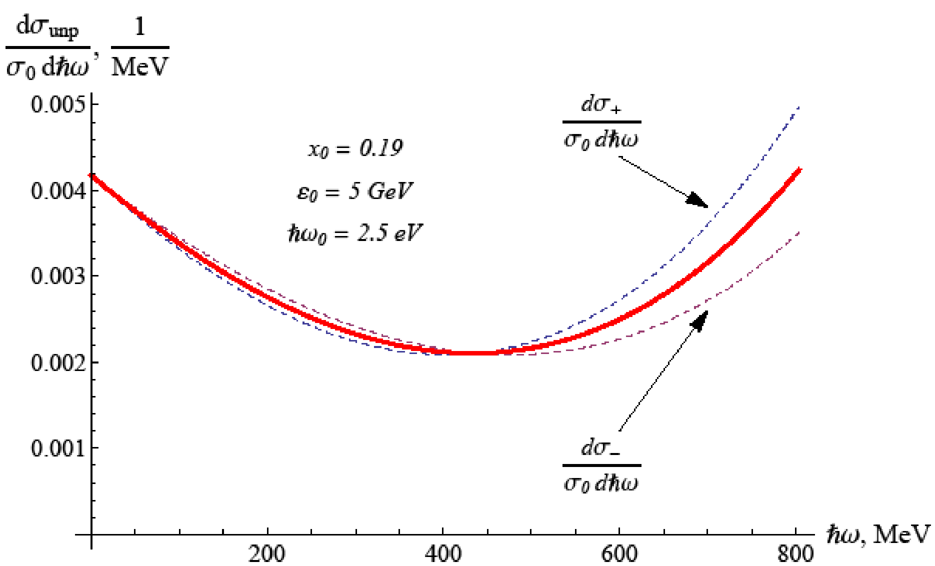
$$\frac{d\sigma_{--}}{dy} = \frac{d\sigma_0}{dy} - \frac{d\sigma_1}{dy} - \frac{d\sigma_2}{dy} + \frac{d\sigma_3}{dy}; \quad \text{- non spin - flip term}$$

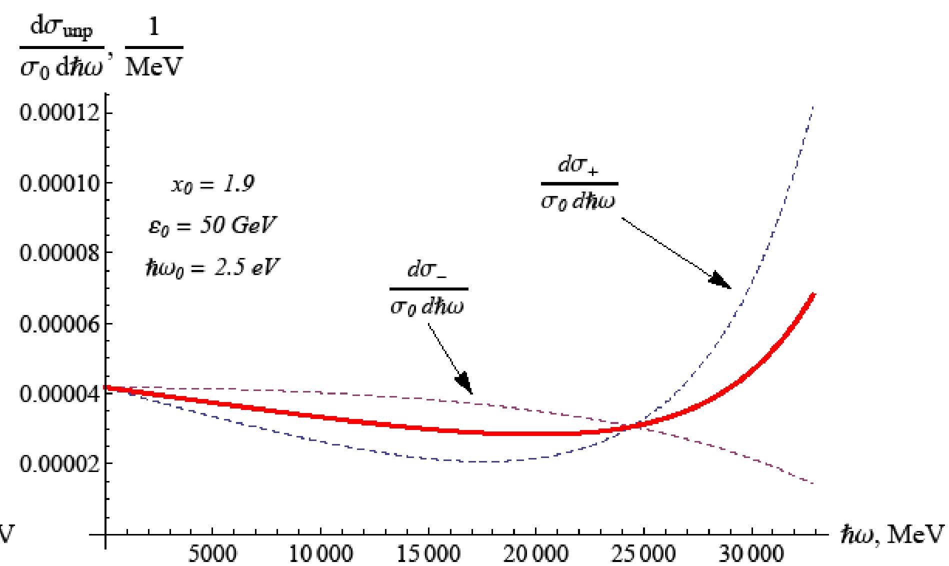
$$\frac{d\sigma_{-+}}{dy} = \frac{d\sigma_0}{dy} - \frac{d\sigma_1}{dy} + \frac{d\sigma_2}{dy} - \frac{d\sigma_3}{dy}; \quad \text{- spin - flip term}$$

For unpolarized beam (after averaging over initial polarization states).

$$\frac{d\sigma_{unp}}{dy} = 2\frac{d\sigma_0}{dy} \approx \frac{\pi r_0^2}{x_0}\left\{\frac{1}{1-y} + 1 - y - s^2\right\}$$

$$\hbar\omega_{\max} = \gamma_0 mc^2 \frac{x_0}{1+x_0}$$

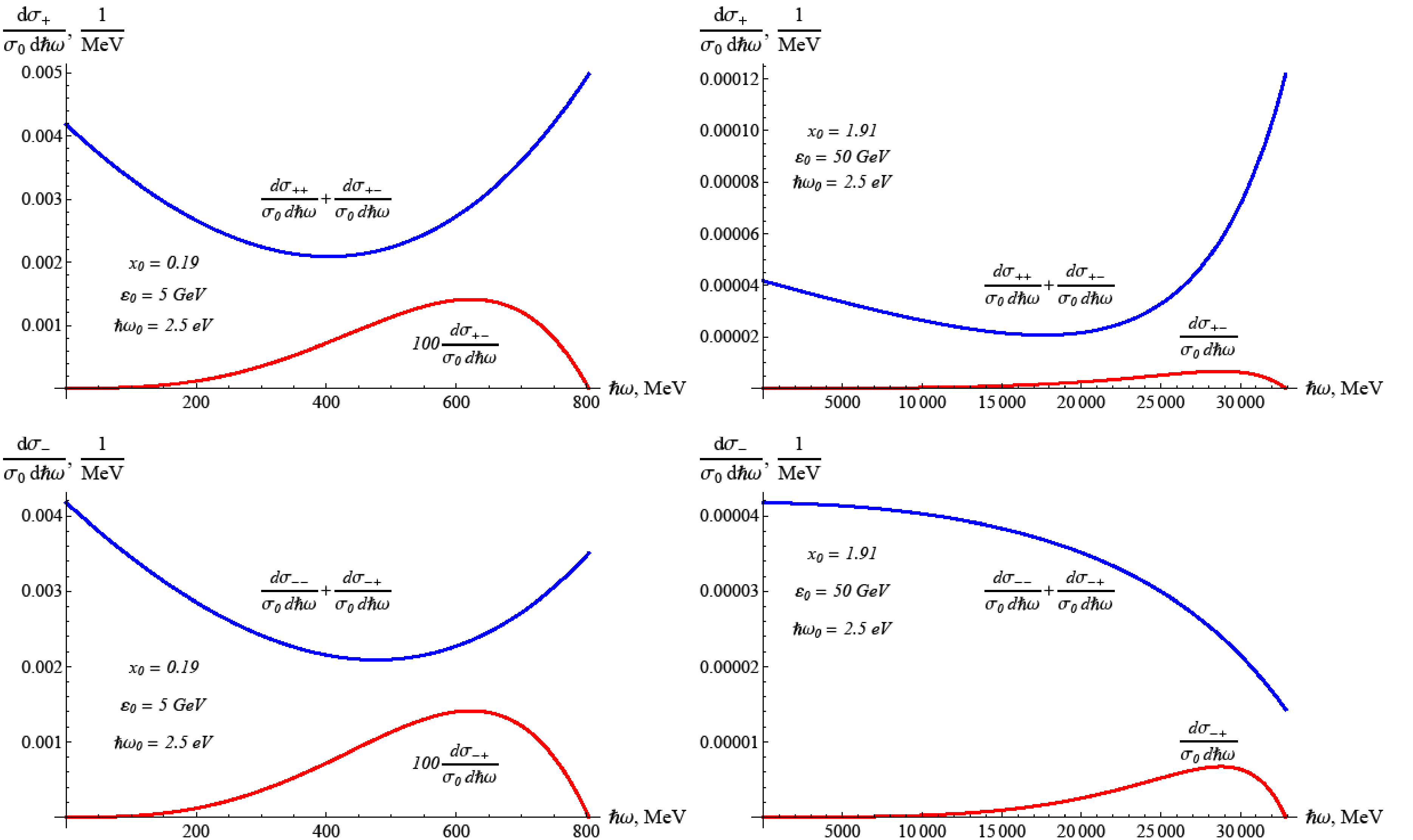




$\hbar\omega_{\max} = 803.546$ MeV

$\hbar\omega_{\max} = 32846.3$ MeV

Comparison of both parts of cross-section for different electron energies



It is evidently for any electron energy

$$\frac{d\sigma_{+-}}{dy} = \frac{d\sigma_{-+}}{dy}$$

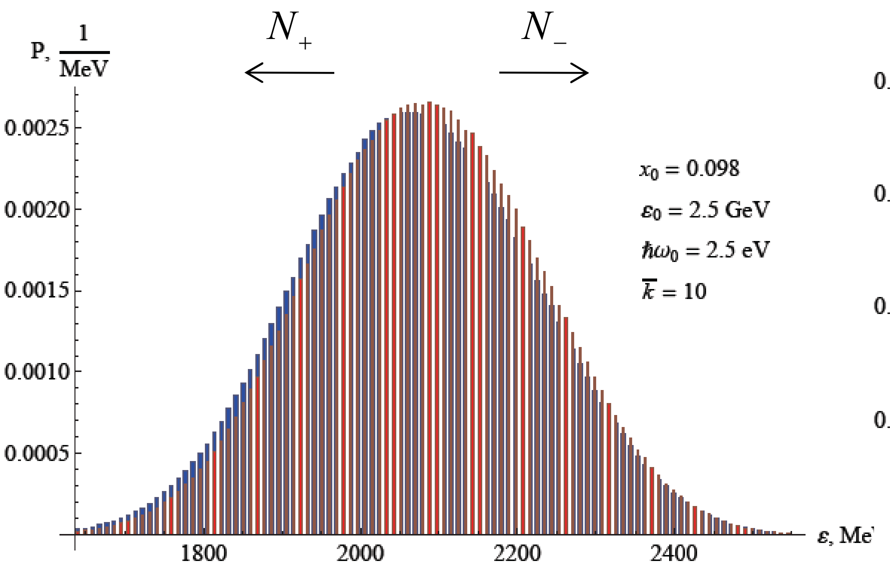
within an accuracy γ_0^{-1}.

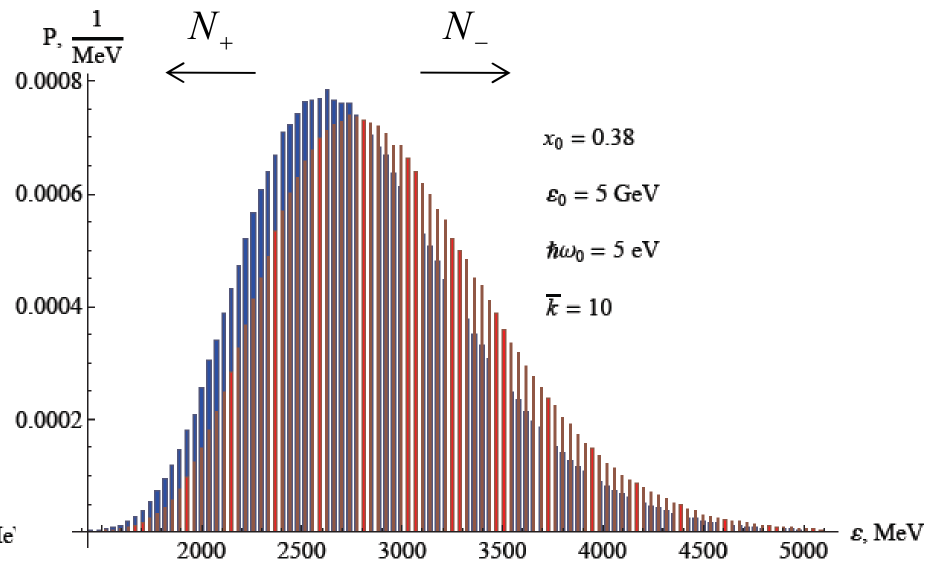
It means there is no polarisation of the final beam as whole.

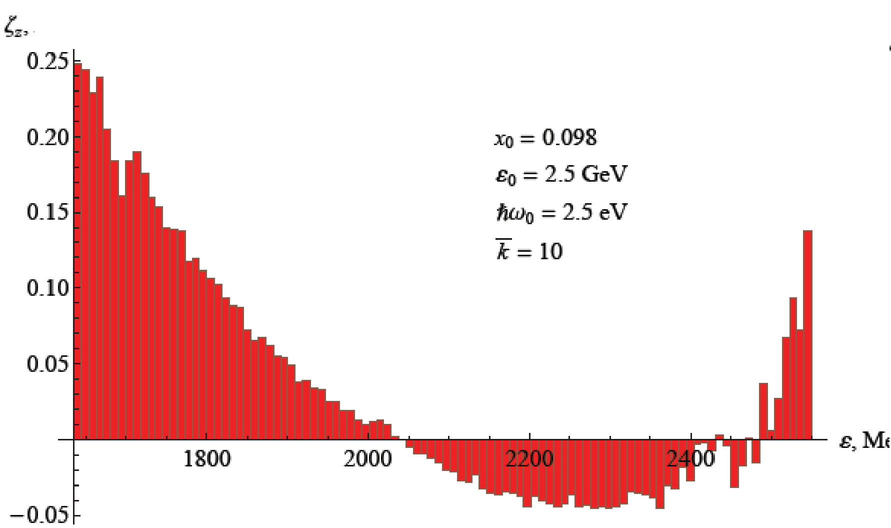
But for the case when each positron in a beam will interact with *CP* laser photons a few times (multiple Compton backscattering proccess) the final positron beam will have the non - zeroth polarization for a part of beam due to difference in cross - sections :

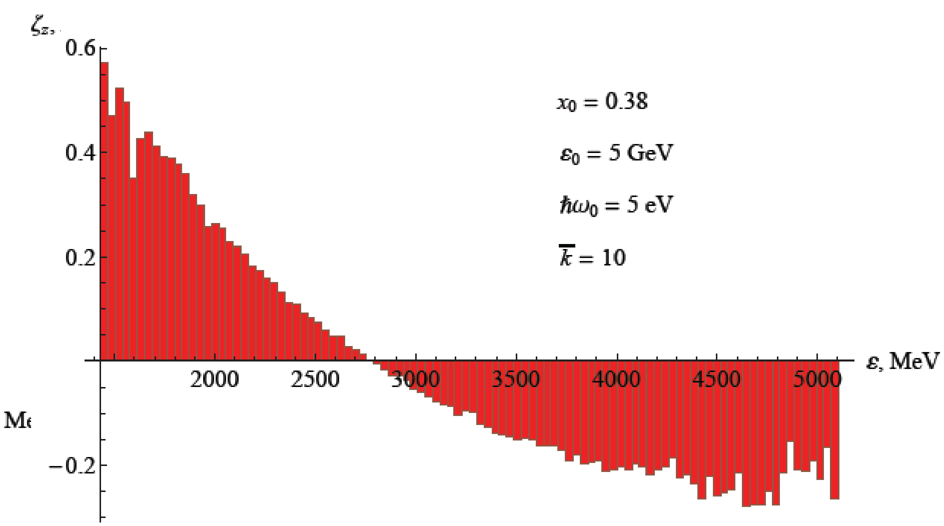
$$\boxed{\frac{d\sigma_{+}}{dy} \neq \frac{d\sigma_{-}}{dy}}$$

Polarization of a final beam

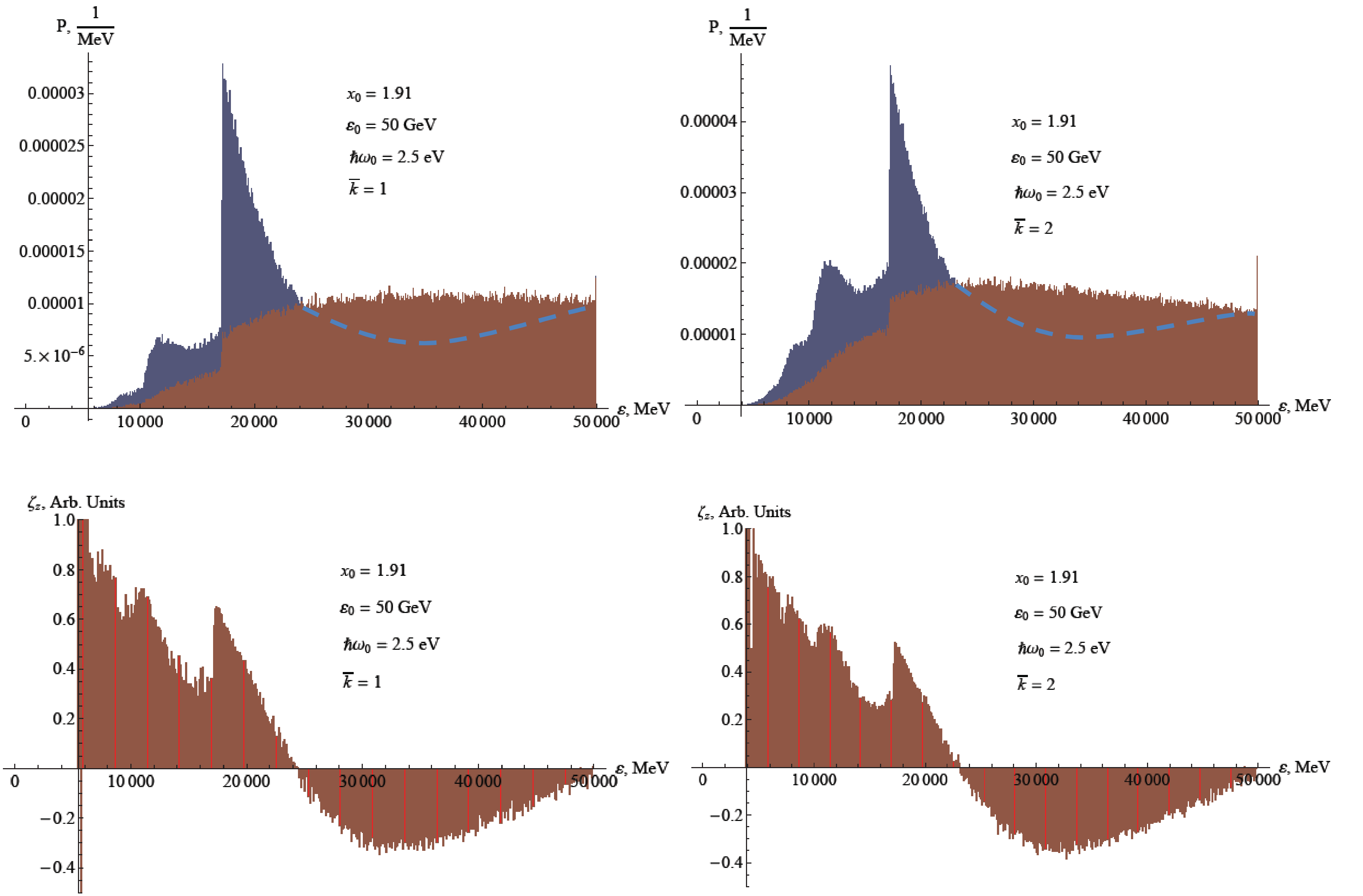








Initial electron energy 50 GeV



Summary

- Distribution over number of emitted photons describes by Poisson law with a good accuracy
- In the MCBS process (and for ordinary UR also but not for FEL) the final distribution of electrons is continuos, which may be described by gaussian one.

Thanks for your attention!

- Even for small mean number of collisions (<k>~1) there is significant contribution of events with k = 2,3… photons from each electron/positron
- The ordinary multiple Compton – backscattering process (plane – wave approximation) may provide polarization of a part of unpolarized beam
- For the case $x_0 \geq 0.1$ a resulting photon spectrum is distorted significantly if $\overline{k}_{MCBS} \gg 1$ in comparison with a single - photon spectrum