

Coherent X-Ray Radiation Produced by Microbunched Beams in Amorphous and Crystalline Radiators

(CXBR, CXTR, CXRTR, CXDR, CXCUR, CXUR, CXChR and CXPXR)

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Abstract

A review on the coherent X-ray bremsstrahlung (CXBR), X-ray transition (CXTR), resonance transition (CXRT), diffraction (CXDR), channeling (CXCHR), parametric (CXPXR) and crystalline undulator (CXCUR) radiation produced by microbunched beams passing through crystalline radiators without the accompanying SASE beams of X-ray FELs is given. Formula for the spectral and angular distributions as well as for the total number of photons of these radiations are derived and numerically studied. It is discussed the possibility of observing of these types of radiation and their application for the study of the parameters of the electron beam microbunching which is important for the effectiveness of XFELs and for production of additional beams of intense monochromatic X-ray beams.

1.Introduction

Using one of the methods proposed in [1] the first X-ray transition radiation (XTR) detector has been constructed [2]. In [3] the first study of the AD and SD of the optical transition radiation (OTR) of relativistic particles has been carried out and proposed to use OTR for HEP particle beam diagnostics. “Initiated” by [3], OTR was used [4] for beam parameters measurements. Conference sections on XTR, OTR and TRD (see Yerevan 1977, 1981, RREPS1991-2010, TRDs 2005-2011) took place. Recently following the proposal [5] microwave TR has been used [6,7] for the study of microbunching (MB) of beams of FELs (see Table 1 for XFELs).

1. A.I. Alikhanian, F.R. Arutiunian, K.A. Ispirian, M.L. Ter-Mikaelian, “On a Possibility of Identifying of High Energy Particles”, Zh. Teor. Eksp. Fiz. 41. 2002, 1961.

2. F.R.Arutiunian, K.A.Ispirian and A.G.Oganesian, Yad. Fiz., 1, 842, 1965

3. A.I. Alikhanian, K.A. Ispirian, A.G. Oganesian, “Experimental study of the (Opt.) Transition Radiation and Its Possible Application For the measurement of Energy of High Energy Particles” Zh. Teor. Eksp. Fiz. 57. 1696, 1969.

4. R. Wartski et al, EEEE Trans. Nucl. Sci. NS, 20,44, 1973; J. Appl. Phys. 46,3644, 1975.

5. J. Rosenzweig, G. Travish, A. Treimane, Nucl. Instr. And Meth. A 365, (1955) 255.

6. Y.Liu et al, Phys. Rev. Lett. 80 (1998) 44.

7. A. Treimane et al, Phys. Rev. Lett. 81 (1988) 5816.

Table 1. Some parameters of MB Electron beams at LCLS and projected for other X FELS [8-10].

Parameter	Notations	Units	LCLS (USA) [8]2009	SCSS* (Japan) 2011	EurXFEL(Germany) [9] 2015?	SwissFEL (Switzerland) [10]?
Electron energy ^{μ}	E	GeV	13.6	8	17.5	5.8
Bunch charge	Q	nC	0.25			
Repetition rate		Hz	30	60	10	100-400
Bunch length (rms)	2.36 σ_z	mkm	6-8			
Projected emittance	$\gamma\epsilon_{x,y}$	mkm	0.5-1.6			
Photon wavelength	λ	nm	0.15	0.1	0.1	0.1

~50 articles have been published in Nature , PRL based on works at LCLS (the first atomic X Laser(N.Rohringer et al Nature 2012).

8. P. Emma et al, a) Proc. FEL2009,p 397; b) Nature Photonics, 4, 641, 2010; c) LCLS, Design Study Report, 1998 SLAC-R-521.

9. TESLA Technical Design Report 2001 DESY 011.

10. B.D. Patterson et al, New Journal of Physics, 12, 035012, 2010.

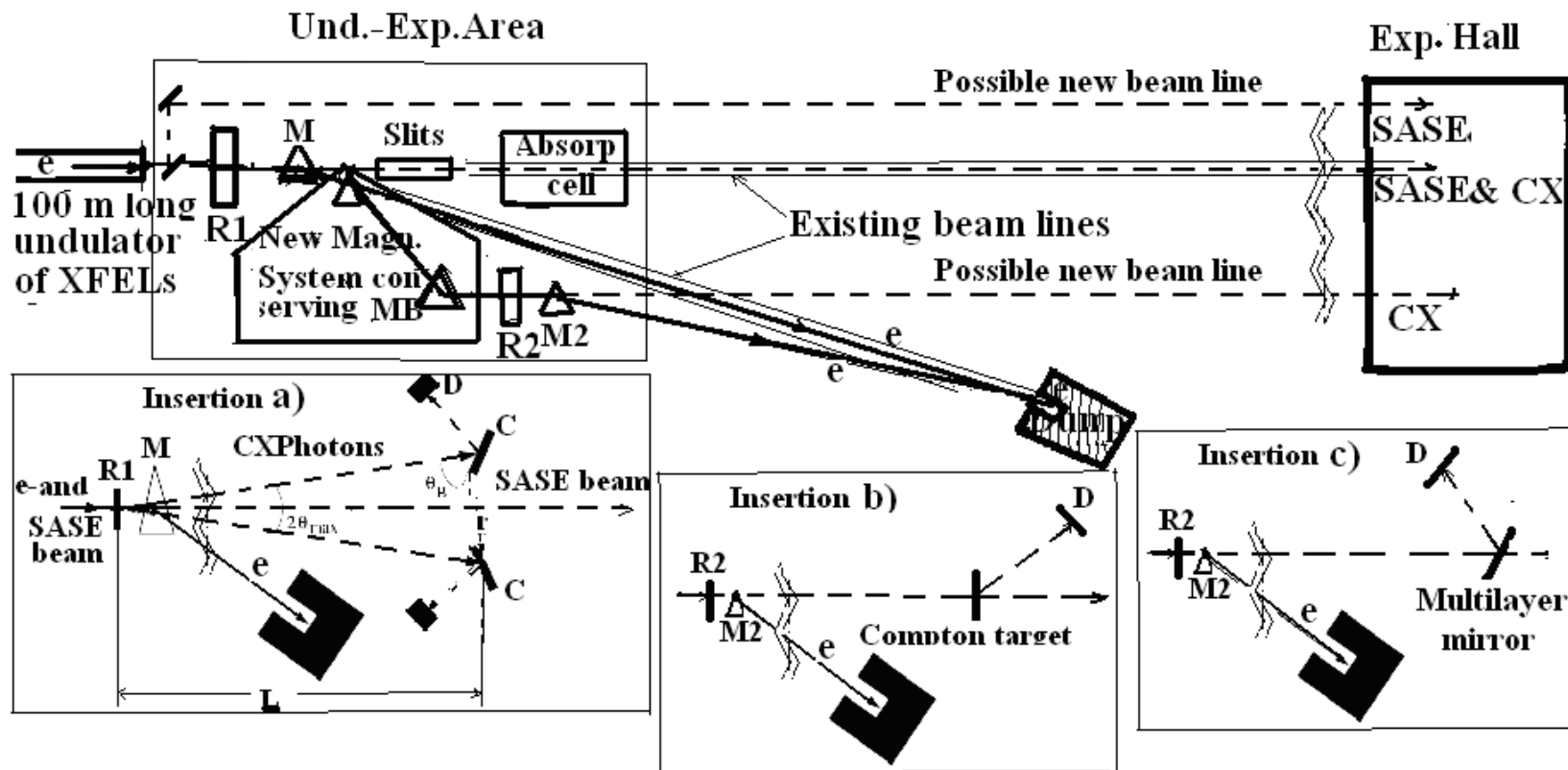


Fig.1. Existing and new arrangements at LCLS for the study and use of the new coherent radiation photons (CXBR, etc) with and without deflection of microbunched beam conserving the microbunching [11,12] and corresponding photon detections (Insertion a),b) and c)).

11. Y.Li, W. Decking, B. Faatz and J.Pflueger, Phys. Rev. STAB, 13, 080705, 2010.

12. G. Geloni, V. Kocharyan, E. Saldin, Arxiv 1106.1776, 2011.

The el. beams after long XFEL undulators are microbunched with period

$$\lambda_r = 2\pi c / \omega_r = \lambda_{und} (1 + K^2 / 2) / 2\gamma^2$$

and density distribution

$$f(r, z) = \frac{N_b \exp(-r^2 / 2\sigma_r^2) \exp(-z^2 / 2\sigma_z^2)}{2\pi\sigma_r^2 (2\pi)^{1/2} \sigma_z} [1 + b_1 \cos(k_r z)] \quad (1)$$

where $N_b = Mn_b$ and n_b are the number of electrons in the macropulse and M microbunches, $k_r = 2\pi / \lambda_r$, $0 \leq b_1 \leq 1$ is the most important microbunching parameter or modulation depth, which is measured at soft FELs and not measured for XFELs, $\sigma_{r,z}$ are ...

In [13] we have developed theory of CXTR produced by these microbunched beams and proposed to measure b_1 at XFELs. There are a proposal [14] and even preparatory works [15] how one can measure and use CXTR at SLAC. Nevertheless, there is no MB measurement at LCLS. In the work [16] it has been studied the properties of the bunch coherence of CXPXR, produced under larger angles.

13. E.D. Gazazian, K.A. Ispirian, R.K. Ispirian, M.I. Ivanian, Pisma Zh. v Eksp. Teor. Fiz. 70, 664, 1999; Nucl. Instr. And Meth. B173 (2001) 160.

14. A.H. Lumkin, W.M. Fawley, D.W. Rule, Proc. FEL2004, p.515.

15. A.H. Lumkin, J.B. Hastings, D.W. Rule, SLAC-PUB-12451, 200??

16. X. Artru, K.A. Ispirian, Proc. NATO Workshop Electron-Photon Interaction in Dense Media Nor-Hamberd, Yerevan, Armenia 25-29 June, 2001, n. 191, ArXive nh/0208018, 2002.

The aim of this work is: to use the MB el. Beams (now sent to dumps) for production of intense X-ray beams of CXTR, CXPXR etc, giving for the measurement of b_1 since their intensity is $\sim N_b^2$.

2. Method of Calculations and Physics

a) The method In [17] it is given a review and derivation of the formula:

$$\frac{d^2 N_{CXi}}{d\omega d\theta} \cong N_b^2 F(\omega, \theta) \frac{d^2 N_{Xi}}{d\omega d\theta}. \quad \mathbf{i = XTR \dots PXR} \quad (2)$$

The el. bunch with density distribution (1) has the form-factor

$$F(\omega, \theta) = \exp\left(- (k\sigma_r \sin \theta)^2\right) \left[\exp\left(\frac{-\omega^2 \sigma_z^2}{2v^2}\right) + b_1 \exp\left(- \left(\frac{\omega}{v} - k_r\right)^2 \frac{\sigma_z^2}{2}\right) + b_1 \exp\left(- \left(\frac{\omega}{v} + k_r\right)^2 \frac{\sigma_z^2}{2}\right) \right]^2 \quad (3)$$

Always our considerations will be limited in a region near to ω_r (i.e we shall neglect the first and third terms in (3)). Then using the dimensionless frequency and angles $\zeta = \omega / \Omega \gamma^2$, $u = \gamma \theta$ where $\Omega = 2\pi V / L$ and L is the period of various types of motion. 7

17. N.A. Korkhmazian, L.A. Gevorgian, M.L. Petrosian, Zh. Tekh. Fiz. 47, 1583, 1977.

(2) and (3) will be written in the forms

$$\frac{d^2 N_{CXi}}{d\zeta du} = N_b^2 F(\zeta, u) \frac{d^2 N_{Xi}}{d\zeta du} \quad (2')$$

$$F(\zeta, u) = F_{\parallel}(\zeta) F_{\perp}(\zeta, u) = b_1^2 \exp\left\{-[A(\zeta - \zeta_r)^2 + B\zeta^2 u^2]\right\} \quad (3')$$

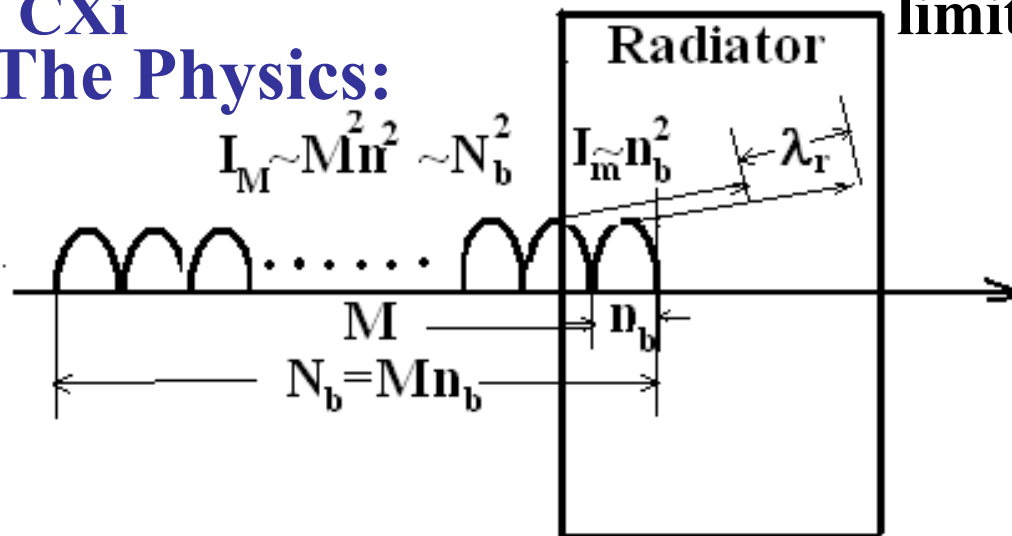
where

$$A = \left(\frac{2\pi\gamma^2 \sigma_z}{L}\right)^2, \quad B = \left(\frac{2\pi\gamma\sigma_r}{L}\right)^2 \quad (4)$$

Our receipt always is the same: 1) To find suitable $d^2 N_{Xi} / dx du$,
 2) Substitute it and (3') into (2') to derive $d^2 N_{CXi} / dx du$ and then
 3) after integrations dN_{CXi} / du and dN_{CXi} / dx

b) Main properties: Monochromatic around ζ_r and diffraction limited $I \rightarrow 0$ if B or σ_r is great.

c) The Physics:



The trajectories inside the radiator can be ⁸ sinusoidal

Attention

Since the results on CXBR, CXTR, CXRTR and CXDR (amorphous radiators) could not be presented at CHANNELING 2010 and RREPS2011 for financial reasons, but part of them have been or will be published in [18-20] here we shall omit the details and to save time will present only numerical results.

18. K.A. Ispirian, To be published in Proc. of Megri 2011 conference.

19. M.A. Aginian, K.A. Ispirian, M.K. Ispiryanyan, Izvestia NAS of Armenia, N2 , p.83, 2012.

20. K.A. Ispirian, To be published.

Numerical Results on CXTR, CXRTR, CXBR and CXDR

The below given numerical calculations [18-20] are for the following parameters of LCLS [8]; $E=13.6$ GeV, $\hbar\omega = 8.3$ keV, $\sigma_z = 9 \cdot 10^{-4}$ cm, $\sigma_r = 6.12 \cdot 10^{-4}$ cm, $N_b = 1.56 \times 10^9$, $T=75$ fs, $L=3$ cm, $K=q=3.5$, $L=112$ m. Radiators for 1) CXTR and 3)CXBR- Ti, $a = 0.0035$ cm; for 2) CXRTR -268 foil of Be with $a=0.005$ cm, $b=0.2$ cm; For CXDR - $a = 0.0035$ cm, $b=0.2$ cm, $H=0.0003$ cm. We assume $b_1=1$..

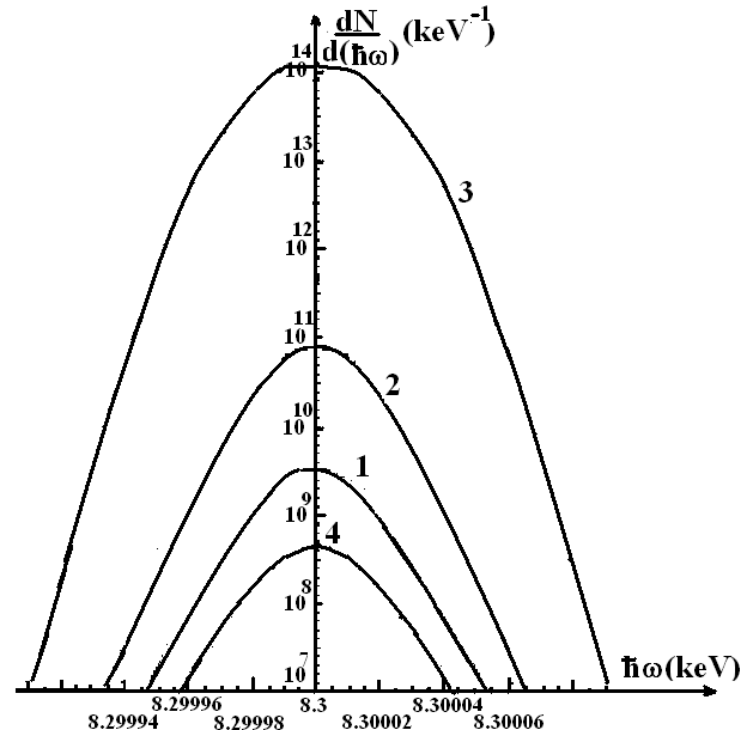


Fig. 1 the spectral distributions of CXBR, CXTR, CXRTR and CXDR (curves 1,2,3 and 4, respectively) 11

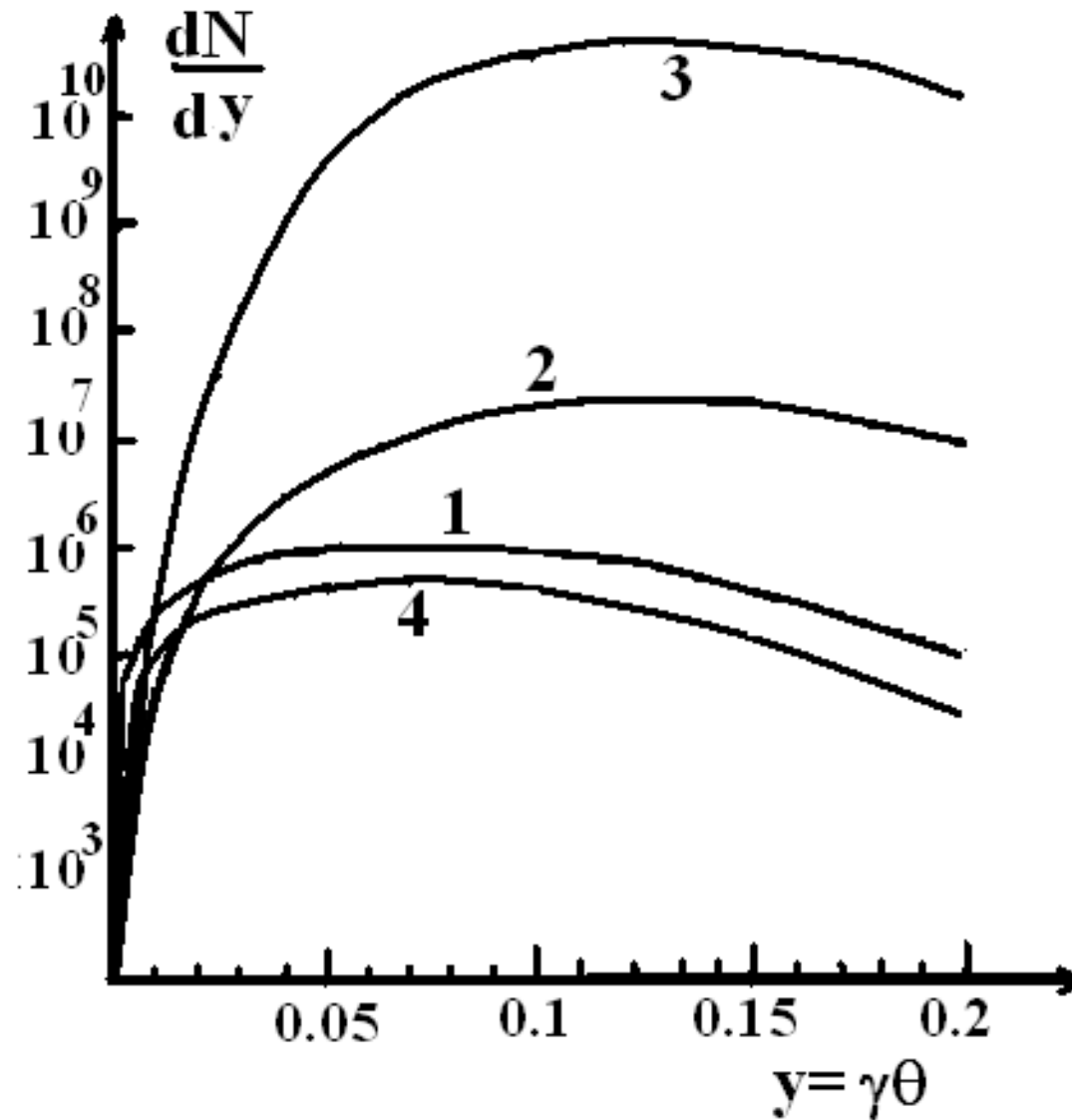


Fig. 2. The angular distributions of CXBR, CXTR, CXRTR and CXDR (curves 1, 2, 3 and 4, respectively) for the same parameters as in Fig. 1.

$$N_{CXBR} = 2.8 \times 10^5$$

$$N_{CXTR} = 6.7 \times 10^6$$

$$N_{CXRTTR} = 2.0 \times 10^9$$

$$N_{CXDR} = 3.4 \times 10^4$$

3. CXCUR Taking into Account the Medium Polarization

There is no time to review the first and important works on X-ray UR, CU and CUF

18. N.A. Korkhmazian, Izvestiya Akad. Nauk. Arm SSR, Fizika, 5,287, 1970; 5, 418, 1970.

19. A.I. Alikhanian, S.K. Esin, K.A. Ispirian et al, Pisma Zh. Eksper. Teor. Fiz. 15, 142, 1972.

20. K.A. Ispirian, A.G. Oganesian, Lectures on VII Intern. School on Exp. and Theor Physics, Yerevan, 1971, Preprint YerPhI, EFI-ME-4(71) 1971.

21. D.F. Alferov, Yu. A. Bashmakov, E.G. Bessonov, Proc. FIAN, Ser. 80, Ed. N.G. Basov, 1976, p.97.

22. R.H. Pantel et al, Nucl. Instr. and Meth. A 250, 312, 1986.

23. L.A. Gevorgian and N.A. Korkhmazian, Zh. Eksp. Teor. Fiz. 76, 1226, 1979.

24. V.V. Kaplin, S.V. Plotnikov and S. A. Vorobev, Zh. Tekh. Fiz. 50, 1079, 1980.

25. A.A. Korol, A.V. Solovev, W. Greiner, J. Phys. G. 24, L.45, 1998.

26. R.O. Avakian, L.A. Gevorgian, K.A. Ispirian, R.K. Ispirian, Pisma JETP. 68, 437, 1998; NIM. B 173, 112, 2001.

27. U. Mikkelsen and E. Uggerhoy, Nucl. Instr. and Meth. A, 483, 455, 2002.

28. R.O. Avakian, K.A. Avetian, K.A. Ispirian, E.M. Melikian, NIM. A 492, 11, 2002; A508, 496, 2003;

29. S. Bellucci et al, Phys. Rev. Lett. 90. 03801, 2003

31. S. Bellucci et al, Phys. Rev. STAB, 7, 023501, 2004.

32. B.T. Baranov et al, Pisma Zh. Eksp. Teor. Fiz. 82, 638, 2005.

33. H. Backe et al, Proc. of Intern. Conf. CHANNELING 2008, p.281.

34. H. Backe et al, Proc. Of Intern. Conf. CHANNELING 2010, p.157.

In brief, the achieved results are: 1) Si CU have (by scratching) $L=(50-100) \mu m$, $A=(20-100) \text{ \AA}$ and $N \sim 10$. 2) $\text{Ge}_x\text{Si}_{1-x}$ CU (epitaxial) $L=(9.9-50) \mu m$, $A=(4-90) \text{ \AA}$.and $N \sim 4$.

Last note before beginning: Since $L_{Dech}^e(\text{GeV}) \approx 13 \mu m / \text{GeV} \times E_e = 170 \mu m$ or $\sim 90 \mu m$ (Biryukov, arXiv0712.3904, CATH) the el. CUs are not serious, and further we consider CXCUR for positrons though

Using the formula (32) of the work [20] for CUR produced by a single particle in a unit length CU having amplitude A and period L one has

$$\frac{d^2 N_{CUR}}{d\zeta du} = \frac{\pi \alpha \eta^2}{r^2 L} u \left[1 + (1 - u^2 \zeta)^2 \right] \delta(u^2 - \phi(\zeta)) \quad (5)$$

In (5) the following notations [21] are used:

$$\alpha = e^2 / \hbar c, \quad \Omega = \frac{2\pi V}{L}, \quad \eta = \sqrt{2} \pi A / \lambda_p, \quad \phi(\zeta) = Q \frac{(\zeta - \zeta_1)(\zeta_2 - \zeta)}{\zeta^2}, \quad \zeta_{1,2} = \frac{1 \mp \sqrt{1 - r^2 Q}}{Q} \quad (6)$$

$$\gamma_0 = \frac{\omega_p}{\Omega} = \frac{L}{\lambda_p}, \quad r = \frac{\gamma_0}{\gamma}, \quad \lambda_p = 2\pi c / \omega_p, \quad \omega_p = \sqrt{4\pi n e^2 / m}, \quad q = \frac{2\pi A \gamma}{L}, \quad Q = 1 + \frac{q^2}{2} = 1 + \frac{\eta^2}{r^2}$$

20. L.A. Gevorgian and N.A. Korkhmazian, Zh. Eksp. Teor. Fiz. 76, 1226, 1979.

21. R.O. Avakian, L.A. Gevorgian, K.A. Ispirian, R.K. Ispirian, Pisma Zh. Eksp. Teor. Fiz. 68, 437, 1998; Nucl. Instr. and Meth. B 173, 112, 2001.

In (6) $\varsigma_{1,2}$ are the edges of the narrowed spectra of CUR [21].

By an appropriate choice of the experimental parameters (E_e, A, L) one can have $\varsigma_r = \omega_r / \Omega\gamma^2 \geq \varsigma_1$. Then substituting (3') and (5) into (2') one obtains

$$\frac{d^2 N_{CXCUR}}{d\varsigma du} = \frac{\pi\alpha\eta^2}{r^2 L} N_B^2 b_1^2 u [1 + (1 - \varsigma u^2)^2] \exp\left\{-\left[A(\varsigma - \varsigma_r)^2 + B\varsigma^2 u^2\right]\right\} \delta(u^2 - \phi(\varsigma)) \quad (7)$$

After appropriate integrations for $\varsigma_r \approx \varsigma_1$ one derives (see [22])

$$\frac{dN_{CXCUR}}{d\varsigma} = \frac{\pi\alpha}{L} \left(\frac{N_b b_1 \eta}{r}\right)^2 \exp\left(-\frac{B^2}{A}\right) \exp[-A(\varsigma - \varsigma_1)^2] \quad (8)$$

$$\frac{dN_{CXCUR}}{du} = \frac{2\pi\alpha}{L} \left(\varsigma_1 \frac{N_b b_1 \eta}{r}\right)^2 u \exp\left(-\frac{B^2}{A}\right) \exp\left(-\frac{A\varsigma_1^4 u^4}{4}\right) \quad (9)$$

$$N_{CXCUR} = \frac{\pi^{3/2}\alpha}{L\sqrt{A}} \left(\frac{N_b b_1 \eta}{r}\right)^2 \exp\left(-\frac{B^2}{A}\right) \quad (10)$$

3'. CXUR (Radiation in Magnetic Undulators as by-Product)

It is clear that taking in (8-10) $\omega_P = \eta = 0$ one derives for CXUR without any filling in a unit length magnetic undulator gaps:

$$\frac{dN_{CXUR}}{d\zeta} = \frac{\pi\alpha}{2L} (N_B b_1 q)^2 \exp\left(\frac{B^2}{A}\right) \exp[-A(\zeta - \zeta_R)^2] \quad (11)$$

$$\frac{dN_{CXUR}}{du} = \frac{4\pi\alpha}{L} (Nbq)^2 u \frac{1}{Q^2} \exp\left[-\frac{4Au^4}{Q^4}\right] \quad (12)$$

$$N_{CXUR} = \frac{\pi^{3/2}\alpha}{2L\sqrt{A}} (N_b b_1 q)^2 \exp\left(\frac{B^2}{A}\right) \quad (13)$$

Let us note that our simple results differ from other results (G.N. Kulipanov et al Nucl. Instr. and Meth. A 375,576, 1996; E.G. Bessonov et al, Proc. of the conference CHANNELING2010, p. 93) by the fact that in our case there are no accompanying SASE photons.

4. CXChR Taking into Account the Medium Polarization

Theory of ChR of planarly channeled single electrons has been developed in [23] without taking into account medium polarization. The theory of ChR of planarly channeled positrons taking into account the medium polarization has been considered recently in [24,25]. Since in spite of positrons, the electrons have small and their oscillation periods are different for various entrance coordinate we shall consider CXChR for e^+ .

If the particle enters the crystal with $\theta_{entr} = 0$, $s = 2y_0 / d_P$ (where d_P is the distance between two neighboring crystallographic planes) according to [24,25] for a unit length crystal one has

$$\frac{d^2 N_{ChR}}{d\zeta du} = \frac{2\pi\alpha q^2(s)}{L_{Ch}} u \left[1 + (1 - \zeta u^2)^2 \right] \delta(u^2 - \phi(\zeta)) \quad (14)$$

23. M.K. Khokonov, *Rad. Effects*, **80**, 93, 1984.

24. L. Gevorgian and L. Hovsepyan, *Proc. Intern. Conf. on Charged and Neutral Particles Channeling Phenomena, Channeling 2006, Proc. of SPIE, Vol. 6634, 663408, 2007.*

25. L. Gevorgyan, *Proc. Intern. Conf. on Charged and Neutral Particles Channeling Phenomena, Channeling 2008, World Scientific, New Jersey, 2010, p. 370.*

In (14)

$$\zeta = \frac{\omega}{\Omega_0 \gamma^{3/2}} \quad \Omega_0 = \Omega_{Ch} \sqrt{\gamma} = \frac{2c\sqrt{2\nu}}{d_p}, \quad \nu \equiv U_0 / mc^2, \quad L_{Ch} = L_0 \sqrt{\gamma} \quad L_0 = \pi d_p / \sqrt{2\nu}$$

$$q(s) = s\sqrt{2\nu\gamma} \quad \phi(\zeta) = \frac{2}{\zeta} - Q(s) - \frac{r_{Ch}^2}{\zeta^2} \quad Q(s) = 1 + q^2(s)/2$$

$$r_{Ch} = L_{Ch} / (\gamma \lambda_p) \quad \zeta_{1,2} = \frac{1 \mp \sqrt{1 - r_{Ch}^2 Q(s)}}{Q(s)} \quad \zeta_1 \sim r_{Ch}^2 / 2 \quad (15)$$

U_0 is the crystal potential depth.

According to our receipt substituting (14) and (3') into (2') one obtain

$$\frac{d^2 N_{CXChR}}{d\zeta du} = \frac{2\pi\alpha\nu\sqrt{\gamma}s^2}{L_0} N_b^2 b_1^2 u \exp\left\{-\left[A(\zeta - \zeta_r)^2 + B\zeta^2 u^2\right]\right\} \delta(u^2 - \phi(\zeta)) \quad (16)$$

The second term of the arg. of exp. strongly reduces the CXChR intensity. As it is shown in [26] one can overcome this providing

$\zeta_r \approx \zeta_1$ by an appropriate choice of only available parameter $\gamma = \gamma_r$

26. M.A. Aginian, L.A. Gevorgian, K.A. Ispirian, to be publ. in proc. of this conference

$$\gamma_r = \left(\frac{L_0 L_{und} (1 + K^2 / 2)}{4\lambda_P^2} \right)^{2/3} \quad (17)$$

For this chosen particle energy

$$\omega_r = \frac{4\pi c \gamma^2}{L(1 + K^2 / 2)} \quad (18)$$

Then, averaging (16) over s, after corresponding integrations one derives [26]

$$\frac{dN_{CXChR}}{d\zeta} = \frac{4\pi\alpha}{3L_0} v \sqrt{\gamma} N_b^2 b_1^2 \exp\left[-\frac{B^2}{A}\right] \exp[-A(\zeta - \zeta_1)^2] \quad (19)$$

$$\frac{dN_{CXChR}}{du} = \frac{8\pi\alpha}{3L_0} v \sqrt{\gamma} N_b^2 b_1^2 \zeta_1^2 u \exp\left[-\frac{B^2}{A}\right] \exp\left[-\frac{A\zeta_1^4 u^4}{4}\right] \quad (20)$$

$$N_{CXChR} = \frac{4\pi^{3/2}\alpha}{3L_0 \sqrt{A}} v \sqrt{\gamma} N_b^2 b_1^2 \exp\left[-\frac{B^2}{A}\right] \quad (21)$$

5. Taking into Account the Absorption and Multiple Scattering (Dechanneling)

All the above formulae are for unit length radiator without taking into account absorption and multiple scattering. As in 1961 with RTR [1] we shall show that for this purpose

It is necessary to multiply by some L_{eff} . Indeed, if the calculated number of the homogeneously produced photons per unit length is $N(1)$ then the number of photons coming out from the radiator with account of only absorption [1]

or
$$N(L_{\text{Rad}}) = N(1)L_{\text{abs}} [1 - \exp(-L_{\text{Rad}} / L_{\text{abs}})] \quad (22)$$

$$L_{\text{eff}} = \frac{N(1)}{N(L_{\text{Rad}})} = L_{\text{abs}} [1 - \exp(-L / L_{\text{abs}})] \quad (23)$$

These formulae can be generalized taking into account L_{Dech} , see Fig.4

When n_0 particles enters the radiator the number of photons dN produced in the layer dx at distance x from the entrance is

$$dN = N_0(x)N(1)dx - N\mu dx \quad (24)$$

Since due to dechanneling the number of particles at x is $n_0(x)=n_0\exp(-x/L_{Dech})$, and $\mu = 1/L_{abs}$ one obtains the diff. equation

$$\frac{dN}{dx} + \frac{N}{L_{abs}} = n_0 N(1) \exp\left(-\frac{x}{L_D}\right) \quad (25)$$

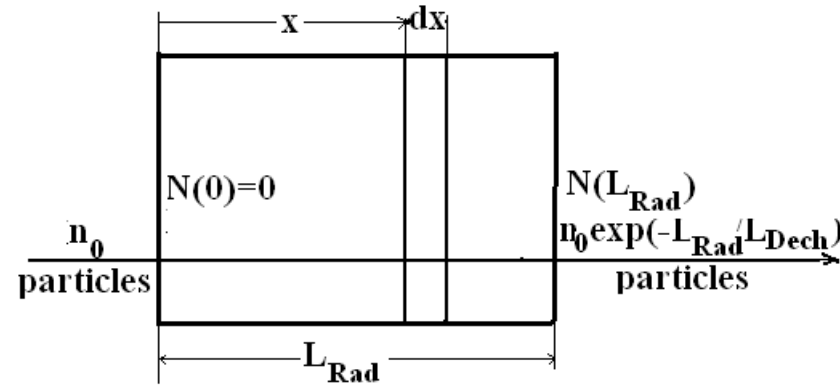


Fig. 4

The solution of (25) has the form

$$N(L_{Rad}) = \frac{n_0 N(1)}{1/L_{abs} - 1/L_{Deck}} [\exp(-L_{Rad} / L_{Deck}) - \exp(-L_{Rad} / L_{abs})] \quad (26)$$

Or for $n_0=1$

$$L_{eff} = \frac{1}{1/L_{abs} - 1/L_{Deck}} [\exp(-L_{Rad} / L_{Deck}) - \exp(-L_{Rad} / L_{abs})] \quad (27)$$

As it is seen when $L_{\text{Dech}} \gg L_{\text{abs}}$ and $L_{\text{Dech}} \gg L_{\text{Rad}}$ (26) and (27) give (22) and (23). In the below given numerical calculations for positron it has been used L_{eff} given by (23) since $L_{\text{Dech}} \gg L_{\text{abs}}$. It is very difficult to use (27) and make predictions about electrons because a) as it has been mentioned above the periods of electron oscillations depend on s and there is no theory taking into account medium polarization, b) No correct data on L_{Dech}^{e-} . Nevertheless taking the above sparse $L_{\text{Dech}}^{e-} \sim 140$ mkm and $L_{\text{abs}}^{\text{Si}} = 77$ mkm according to (27) one obtains $L_{\text{eff}}^{e-} = 0.0015$ mkm, when for positrons $L_{\text{eff}}^{e+} = 0.0077$ mkm. Nevertheless, the developed theory is not valid for electrons because of a), and new approach is necessary.

6. Calculation of CXPXR and Estimates of CXDTR

Since it is impossible to find the necessary $d^2 N_{PXR} / d\omega d\theta$ in the literature taking into account that the spectrum of PXR is very narrow (δ -function) we write such an $d^2 N_{PXR} / d\omega d\Omega$ which after integration over ω gives the well known kinematic FI $dN_{PXR} / d\Omega$ [27-30]

$$\frac{d^2 N_{PXR}}{d\omega d\Omega} = \frac{\alpha}{4\pi c} \frac{\omega}{\sin^2(\theta_B)} L_{eff}(\omega) J(\theta, \varphi) |\chi_\tau(\omega)|^2 \delta(\omega - \omega_{PXR}) \quad , (28)$$

Where θ_B is the Bragg angle, $\chi_\tau(\omega) = S(\tau) \exp(-W) \frac{F(\tau)\omega_P^2}{Z\omega^2} S \dots$, (29),

$$J(\theta, \varphi) = \frac{\theta^2 [\cos^2(\varphi) \cos^2(2\theta_B) + \sin^2(\varphi)]}{(\theta^2 + \gamma^{-2} + \omega_P^2 / \omega^2)^2} \quad , (30) \text{ and}$$

$$\hbar\omega_{PXR} = \frac{2\pi\hbar c}{d} \frac{\beta \sin \theta_B}{1 - \beta\sqrt{\epsilon} \cos \theta} \quad , (31)$$

27. A.M. Afanasev, M.A. Aginian, Proc. of Intern. Symp. on Transition Radiation of High Energy Particles, Yerevan, 1977, p.193; Zh. Eksp. Teor. Fiz. 74, 570, 1978.

28. I.D. Feranchuk, A.V. Ivashin, J. Physique, 46, 1981, 1985.

29. A.P. Potilitsin, Electromagnetic Radiation of Electrons in Periodic Structures, Springer-Heidelberg, 2011.

30. M.A. Aginian, K.A. Ispirian, M.K. Ispiryan to be publ. in proc. of this conference.

From (31) at $\gamma \gg 1$ $\omega_{PXR} = \omega_B$ and, indeed, as a control after integration of (28) over ω with the help of δ -function one obtains the well known formula of FI.

Now integrating (28) over φ one obtains the necessary for receipt:

$$\frac{d^2 N_{PXR}}{d\omega d\theta} = A1 J(\theta) \omega \delta(\omega - \omega_{PXR}) \quad , \quad (32)$$

where

$$A1 = \frac{\alpha |\chi(\omega)|^2 L_{eff}}{4c \sin^2 \theta_B}; \quad J(\theta) = \frac{\theta^3 [1 + \cos^2(2\theta_B)]}{(\theta^2 + \gamma^{-2} + \omega_P^2 / \omega_B^2)^2} \quad (33)$$

Rewrite (32) using $u = \gamma\theta$, $\zeta = \omega / \Omega\gamma^2$, $\Omega = 2\pi V \sin \theta_B / d_P$

$$\frac{d^2 N_{PXR}}{d\zeta du} = K \frac{\zeta^5 u^3 \delta(\zeta - \zeta_B)}{[(u^2 + 1)\zeta^2 + r^2]^2} \quad (34)$$

where

$$K = \frac{\pi\alpha\beta\gamma^2 L_{eff} |\chi(\omega)|^2}{2d_P \sin \theta_B} [1 + \cos^2(2\theta_B)] \quad (35) \quad \text{and} \quad r = \frac{\omega_P}{\Omega\gamma} \quad (36)$$

Substituting (32) and F-F (3) or (3') into receipt (2) or (2') one obtains

$$\frac{d^2 N_{CXPXR}}{d\zeta du} = N_b^2 b_1^2 K \frac{\zeta^5 u^3 \delta(\zeta - \zeta_B)}{[(u^2 + 1)\zeta^2 + r^2]} \exp\{-[A(\zeta - \zeta_B) + B\zeta^2 u^2]\} \quad (37)$$

To have maximal yield of CXPXR it is necessary to choose θ_B so ω_B (or ζ_B) becomes equal to ω_r (or ζ_R), namely, to choose $\theta_B = \theta_r = \arcsin(\pi\hbar c / d_p \hbar \omega_r)$ and then integrating (37) over ζ with the help of δ -function, i.e. substituting $\zeta = \zeta_B = \zeta_r$, one obtains CXPXR AD (As we assumed SD is δ -function).

$$\frac{dN_{CXPXR}}{du} = N_b^2 b_1^2 K \frac{\zeta_R^5 u^3}{[(u^2 + 1)\zeta_R^2 + r^2]} \exp[-(B\zeta_R^2 u^2)] \quad (38)$$

Finally integrating (38) over u one derives the total photon number

$$N_{CXPXR} = \frac{N_b^2 b_1^2 K \zeta_R}{2} [\exp(D)(1 + D)E_1(D) - 1] \quad (39)$$

Where $E_1(z) = \int_z^\infty \frac{\exp(-t)}{t} dt$ is the exponential integral function and

$$D = B\zeta_R^2 \left(1 + \frac{r^2}{\zeta_R^2} \right) \quad (40)$$

Since for real conditions $D \gg 1$ one can write (14) in the form

$$N_{CXPXR} = \frac{N_b^2 b_1^2 K \zeta_R^5}{2D^2} \quad (41)$$

Roughly one can estimate the number of CXDTR photons as equal to

$$N_{CXDTR} = R \times N_{CXTR} \quad (42)$$

(R is the reflection or diffraction coefficient for CXTR photons)

7. Numerical Results on CXCUR, CXUR, CXChR,

Results for CXPXR and CXDTR, Discussion

1) CXCUR (formulae (8), (9), (10))

a) The parameters of LCLS [8]; $E=13.6$ GeV, $\hbar\omega_r=8.3$ keV, $\sigma_x=9.10^{-4}$ cm, $\sigma_z=6.12.10^{-4}$ cm, $N_b=1.56 \times 10^9$, $T=75$ fs which further will be called “Standard parameters of LCLS beams”.

b) The parameters of CU; Si (110), $L=20.7$ mkm, $A=3$ nm, $L_{abs}=77.5$ mkm, $L_{CU}=15$.

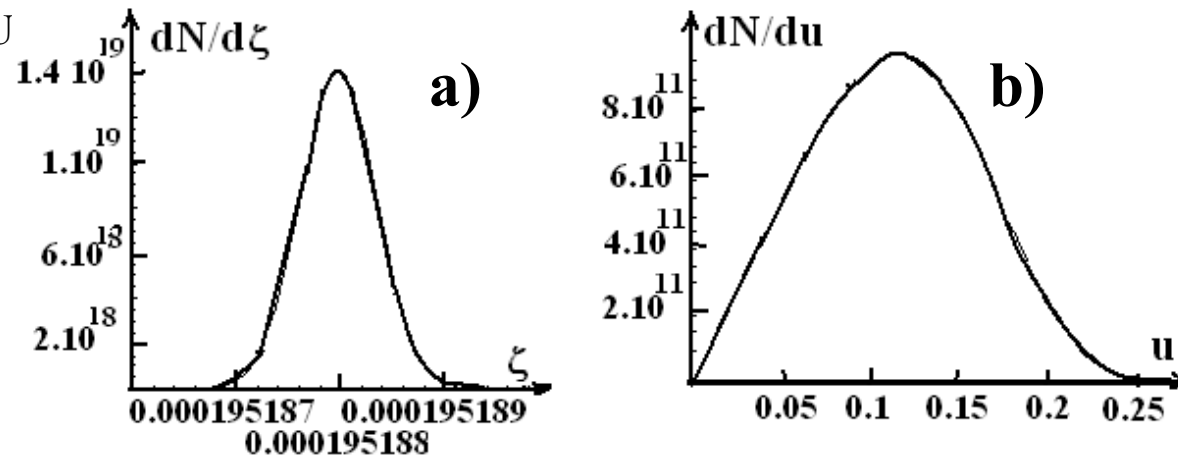


Fig.5 Spectral (a)) and angular (b)) distributions of CXCUR. $\zeta=0.000195188$ corresponds $\hbar\omega=8.3$ keV.

$$N_{CXCUR} = 1.2 \times 10^{10}$$

2) CXUR (formulae (11), (12), (13))

a) Standard parameters of LCLS;

b) The parameters of the magnetic undulator, a section of LCLS long undulator [8] ; $L=3$ cm, $K=q=3.5$, $L_U=L_{eff}= 340$ cm, $N_{osc}=113$.

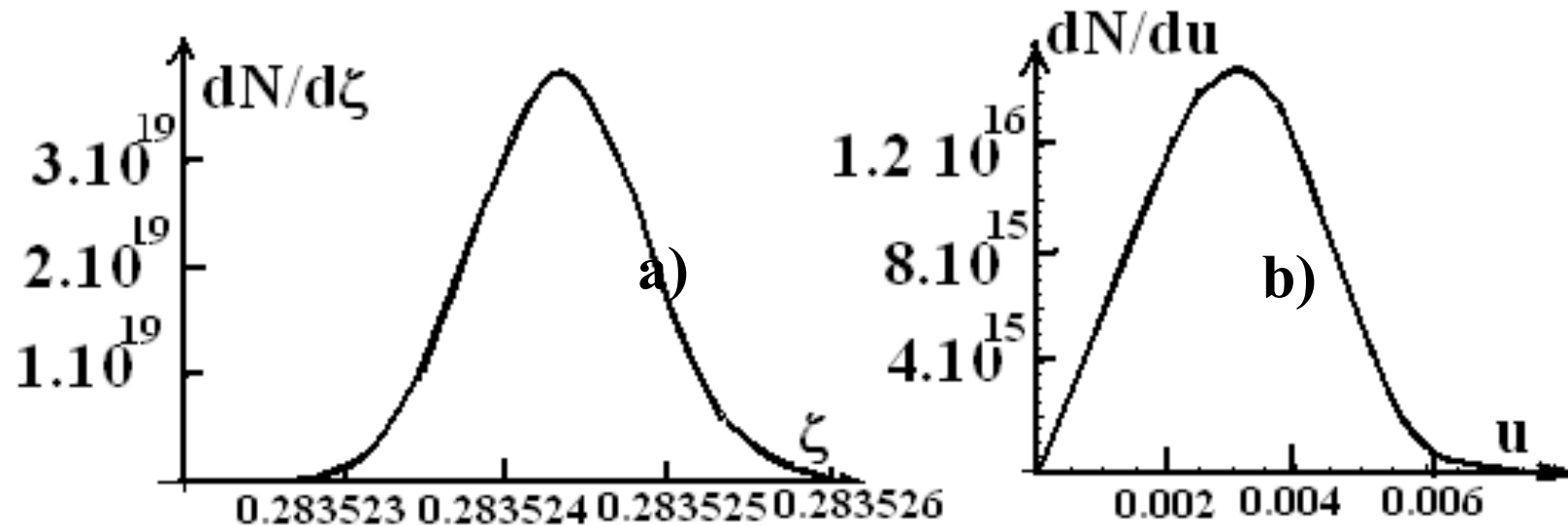


Fig. 6 The spectral (a)) and angular (b)) distributions of CXUR

$$N_{CXUR} = 5.1 \times 10^{13}$$

This number is greater than SASE per pulse $N_{SASE}=(1-2.3) \times 10^{12}$.

However, $\sim b_1^2$, long section length on which b_1 can be reduced, etc.

3) CXChR (formulae (17), (18), (19), (20), (21))

- a) The parameters of LCLS: The LCLS particle and photon energies according to (17) and (18) are: $E=9.27$ GeV and $\hbar\omega_r=3.82$ keV.
- b) The parameters of the crystal: Since the photon energy is very low Si($Z=14$) cannot be used because L_{abs} is small. Diamond($Z=6$) cannot be used because due to small d_p the values of B^2/A and exponential reduction in the formulae (19)-(20) are large. The calculations of CXChR have been carried out [27] for LiH($Z=2$) (100) having large $L_{\text{abs}}(3.82 \text{ keV})=4700$ mkm. It has been taken a realistic LiH thickness $L_{\text{cr}}=1000$ mkm giving $L_{\text{eff}}=900$ mkm, $N_{\text{osc}}=34$.

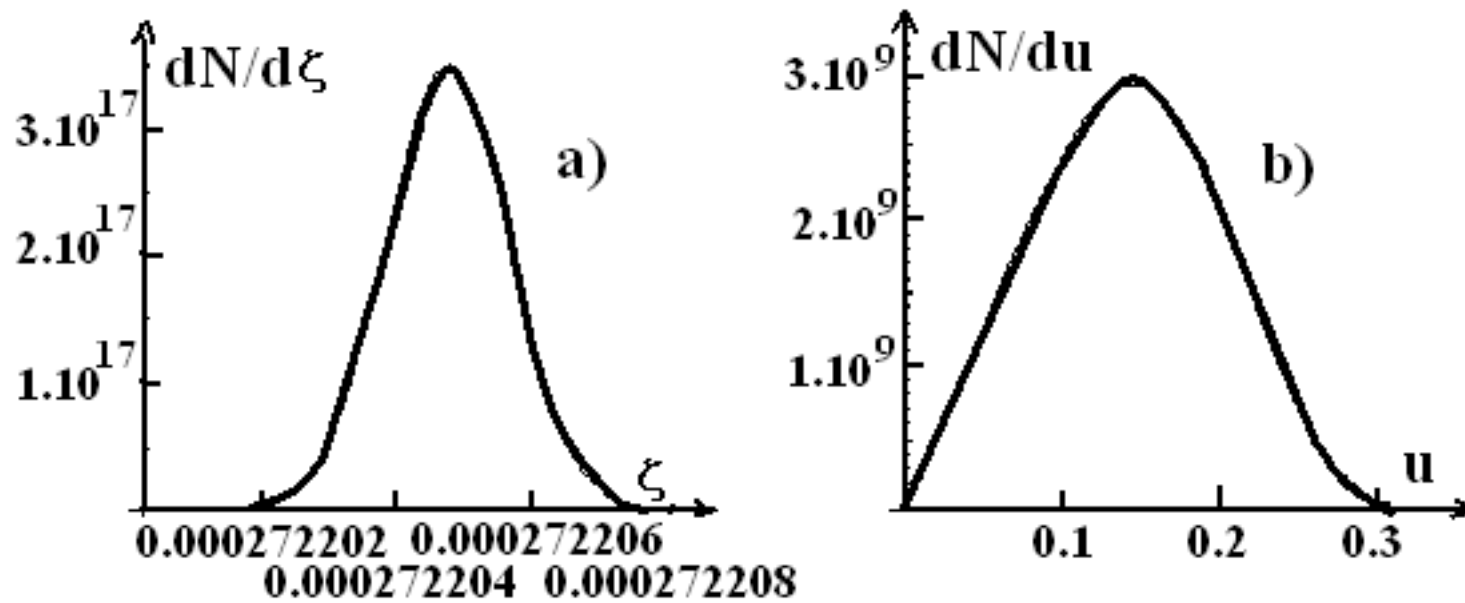


Fig. 7 The spectral (a)) and angular (b)) distributions of CXChR

$$N_{CXChR} = 9.6 \times 10^8$$

4a) CXPXR (formulae (38), (41))

a) Standard parameters of LCLS

b) The parameters of crystal; Si (220), $L_{\text{abs}}=77.5$ mkm, $L_{\text{Rad}}=4L_{\text{abs}}=310$ mkm, $L_{\text{eff}}=L_{\text{abs}}=77.5$ mkm,

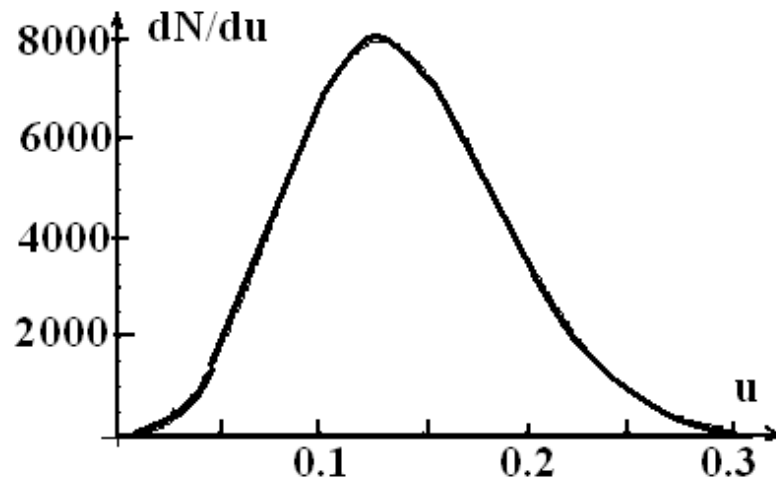


Fig. 7 The angular distribution of CXPXR

$$N_{CXPXR} = 1.0 \times 10^3$$

This number is unexpectedly very small (see the below explanation), and we plan to carry out calculations i) with the help of the formulas of the works [28] taking into account angular spreads and of [29], as well as ii) for X-ray multilayer [30] radiators which can be used for deflecting the X-ray beams with (50-80)% reflectivity at (5-50) keV .

28. R.B. Fiorito et al, Phys. Rev. Lett. 71, 704, 1993.

29. X.Artru and P. Rullhusen, Nucl. Instr. and Meth. B145, 1, 1998; B173, 16, 2001. 31

30. <http://www-cxro.lbl.gov>.

The especial interest to CXPXR is conditioned by the fact that CXPXR is produced under not under “microangles” $\sim 1/\gamma$, but under “macroangles” $\sim 2\theta_B$. Why despite to expectations N_{CXPXR} is small, of the order of the number N_{PXR} of PXR photons produced by N_b electrons $N_{\text{PXR}} \sim 10^{-6} \times 1.6 \cdot 10^9 = 1.6 \cdot 10^3$? The following can serve as a possible explanation.

The enhancement occurs for a narrow band determined by the electron beam form-factor where the number of photons of usual XTR, BR, DR, UR, CUR and ChR is not $\sim 10^{-3}$, but much less $\sim 10^{-10}$. Because the width of these types of radiation have spectra is much wider all the electrons in the microbunch give coherence. In the case of PXR the spectra are much narrower than the form-factors. Therefore, only a small fraction of microbunched electrons satisfy the coherence condition. May be, it is necessary to take into consideration that the electron beam has certain angular distribution and the photons are detected in large angular acceptance, so that the all the electrons could work.

Another evidence for smallness of N_{CXPXR} can serve the below estimated number of coherent DTR number

4b) CXDTR (estimates (42))

It is well known that increasing γ after $\sim \hbar\omega / \hbar\omega_p \sim 10^3$ when $N_{\text{PXR}} \sim N_{\text{DTR}}$, N_{PXR} is saturated, while N_{DTR} increases logarithmically. Therefore, at our energies one can expect that N_{CXDTR} is ~ 1 order greater than N_{CXPXR} .

Roughly, $N_{\text{CXDTR}} \sim R \times N_{\text{CXTR}}$. Since the widths of AD and SD of N_{CXTR} (see Fig.1 and 2) are narrower than the corresponding Darwin widths one can expect that $R \sim (0.1-0.5) \sim 0.3$. As it was shown above $N_{\text{CXTR}} \sim 6 \times 10^6$

$$N_{\text{CXDTR}} = 0.3 \times 6 \times 10^6 \sim 1.8 \times 10^6$$

This is a sufficient number of photons emitted under large, $\sim 2\theta_B$ angle.

SUMMARY TABLE , $N_{\text{SASE}} = (1-3) \times 10^{12}$

CXBR	CXTR	CXRTR	CXDR	CXCUR	CXUR	CXChR	CXPXR	CXDXR
2.8×10^5	6.7×10^6	2.0×10^9	3.4×10^4	1.2×10^{10}	5.1×10^{13}	9.6×10^8	1.0×10^3	1.8×10^6

RESUME

- 1) Though many factors were not taken (Nonstabilities, $\Delta\omega_r$, etc). Nevertheless, all these types of radiation can serve for the study of MB in XFELS for measurement of the MB parameter b_1 measure the ratio $R=N_{\text{Coh}}/N_{\text{Incoh}}$ of the numbers of detected photons N_{Coh} produced per microbunched and not microbunched N_{Incoh} pulse. Since $N_{\text{Coh}} \sim N_B^2 b^2$ while $N_{\text{Incoh}} \sim N_B$ and $b_1=(R/N_B)1/2$.
- 2) The CXBR background is small in the case of massive radiators.
- 3) CXDR (as well as XDR can be observed for the first time since there is no bremsstrahlung background and $\sim N_b^2$ enhancement.
- 4) These numbers are sufficiently high (Even $N_{\text{CXUR}}=5 \cdot 10^{13} > N_{\text{SASE}}=3 \cdot 10^{12}$, because $b_1=1$, $L_u=3\text{m}$) . So that these beams having less than SASE ($\sim 0.2\%$ [8]) can find other applications too.
- 5) The simplest and highest intensity under large angle is achieved Using CXRTR with multilayer mirrors.
- 6) CXDTR also can provide high intensity under large angle, but new calculations and complications seem unavoidable.

THANK YOU