Time Oscillations of the Intensity of Parametric and Diffracted Channeling X-Ray Radiation

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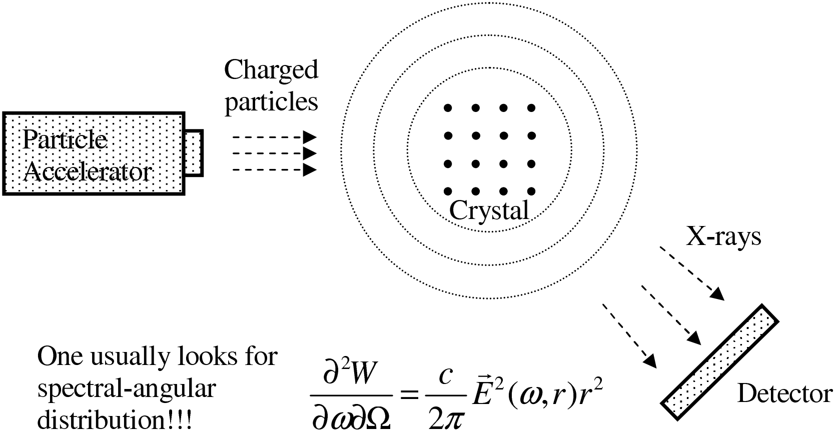
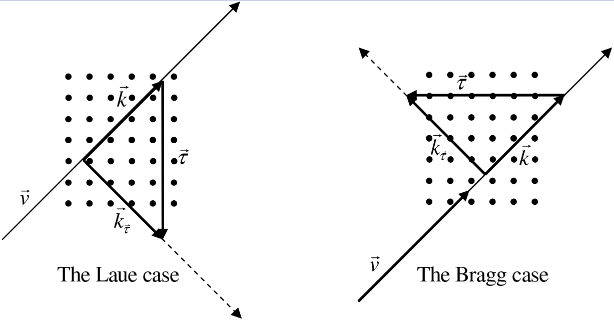
Scattering process



One usually looks for spectral-angular distribution!!!

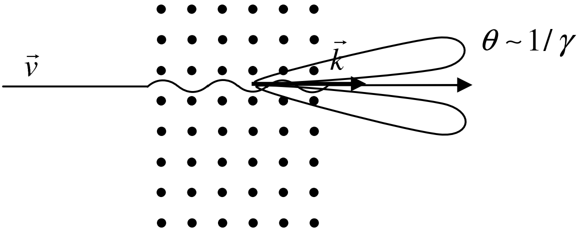
$$\frac{\partial^2 W}{\partial \omega \partial \Omega} = \frac{c}{2\pi} \vec{E}^2(\omega, r) r^2$$

Parametric radiation[1]



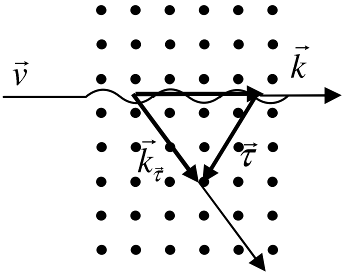
[1] P. Rullhusen, X. Artru and P. Dhez, Novel radiation sources using relativistic electrons: from infrared to x-rays, World Scientific Publishing, Singapore, 1998; Baryshevsky V. G., Feranchuk I. D., Ulyanenkov A. P., *Parametric X-Ray Radiation in Crystals: Theory, Experiment and Applications* (Series: Springer Tracts in Modern Physics, Vol. 213 2005); Baryshevsky V. G. *High-Energy Nuclear Optics of Polarized Media*. World Scientific Pub. 2012.

Channeling radiation[2]

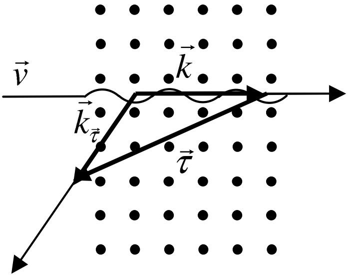


[2]X. Artru and P. Dhez, Novel radiation sources using relativistic electrons: from infrared to x-rays, World Scientific Publishing, Singapore, 1998; Baryshevsky V.G. *Channeling, Radiation and Reactions in Crystals at High Energy*. Minsk: BSU, 1982; Baryshevsky V. G. *High-Energy Nuclear Optics of Polarized Media*. World Scientific Pub. 2012.

Diffracted channeling radiation[3]



The Laue case



The Bragg case

[3] V.G. Baryshevsky, I.Ya. Dubovskaya, DAN USSR. V. 231. P. 1335–1338 (1976); V.G. Baryshevsky, I.Ya. Dubovskaya, J. Phys. C16 (1983) 3663–3672; Baryshevsky V. G. *High-Energy Nuclear Optics of Polarized Media*. World Scientific Pub. 2012.

- One usually looks for spectral-angular distribution of the emitted radiation.
- But! It is also possible to investigate X-ray radiation (parametric radiation, diffracted channeling radiation, etc) in time domaine.

Time-dependent particle radiation in crystals[4]

- Intensity

$$I(t) = \frac{c}{4\pi} |\vec{E}(\vec{r}, t)|^2 r^2$$

- Electric field

$$\vec{E}(\vec{r}, t) = \frac{1}{2\pi} \int \vec{E}(\vec{r}, \omega) e^{-i\omega t} d\omega$$

- Fourier component

$$E_i(\vec{r}, \omega) = \frac{e^{ikr}}{r} \frac{i\omega}{c^2} \sum_s e_i^s \int \vec{E}_{\vec{k}}^{(-)s*}(\vec{r}', \omega) \vec{j}(\vec{r}', \omega) d^3 r'$$

[4] V. G. Baryshevsky, A. A. Gurinovich // arXiv:1101.4162v1 (2011); Baryshevsky V. G. *High-Energy Nuclear Optics of Polarized Media*. World Scientific Pub. 2012.

Solutions of homogeneous Maxwell's equations

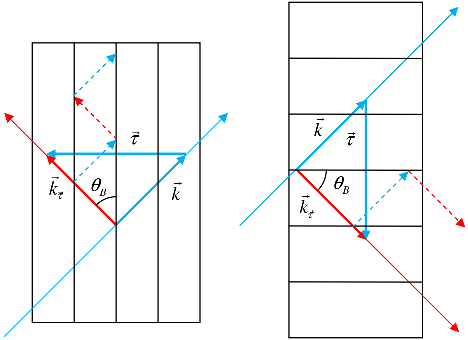
- $\vec{E}^{(-)s}_{\vec{k}}(\vec{r},\omega)$ contains at infinity the incoming spherical waves

$$\vec{E}^{(-)s}_{\vec{k}}(\vec{r},\omega) = \vec{e}^{s}\exp(i\vec{k}\vec{r}) + \text{const}\,\frac{\exp(-ikr)}{r}$$

- $\vec{E}^{(-)s*}_{\vec{k}}(\vec{r},\omega) = \vec{E}^{(+)s}_{-\vec{k}}(\vec{r},\omega)$,
  where $\vec{E}^{(+)s}_{\vec{k}}(\vec{r},\omega)$ describes scattering of electromagnetic waves by a target

$$\vec{E}^{(+)s}_{\vec{k}}(\vec{r},\omega) = \vec{e}^{s}\exp(i\vec{k}\vec{r}) + \text{const}\,\frac{\exp(ikr)}{r}$$

Diffraction geometries[5]



Bragg case (left), Laue case (right).

[5] Z.G. Pinsker, *Dynamical Scattering of X-rays in Crystals*, Springer, Berlin, 1988; Chang Shih-Lin, *Multiple Diffraction of X-Rays in Crystals*, Springer-Verlag Berlin Heidelberg New-York Tokyo, 1984; J. M. Cowley, *Diffraction Physics*. North-Holland Pub. 1975.

Electromagnetic field in crystals

- Maxwell's equations

$$\left[-curl\ curl\ \vec{E}_{\vec{k}}(\vec{r},\omega) + \frac{\omega^2}{c^2}\vec{E}_{\vec{k}}(\vec{r},\omega)\right]_i + \chi_{ij}(\vec{r},\omega)\ E_{\vec{k},j}(\vec{r},\omega) = 0,$$

- Two-wave approximation

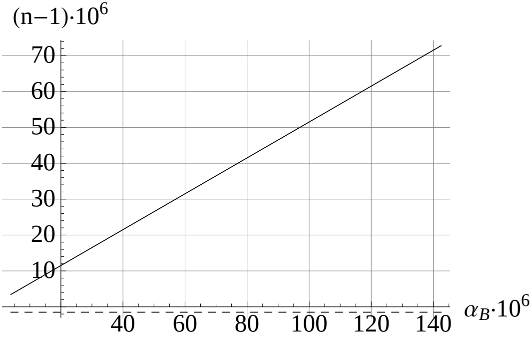
$$\left(\frac{k^2}{\omega^2} - 1 - \chi_0\right)\vec{E}^s_{\vec{k}} - c_s\chi_{-\vec{\tau}}\vec{E}^s_{\vec{k}_{\vec{\tau}}} = 0$$

$$\left(\frac{k^2_{\vec{\tau}}}{\omega^2} - 1 - \chi_0\right)\vec{E}^s_{\vec{k}_{\vec{\tau}}} - c_s\chi_{\vec{\tau}}\vec{E}^s_{\vec{k}} = 0$$

- General solution

$$\vec{E}^s_{\vec{k}}(\vec{r}) = \sum_{\mu=1}^{2}\left[\vec{e}^{\ s}\Phi^s_{\vec{k}\mu}\exp(i\vec{k}_{\mu s}\vec{r}) + \vec{e}^{\ s}_{\vec{\tau}}\Phi^s_{\vec{k}_\tau\mu}\exp(i\vec{k}_{\mu s\vec{\tau}}\vec{r})\right]$$

Index of refraction (the Bragg case)



Slow wave (solid line), fast wave (dashed line)

Bragg parameter $\alpha \approx -\frac{\tau^2}{k_B^2}\frac{(\omega-\omega_B)}{\omega_B}$

Group velocity

- General expression

$$\vec{v}_{gr} = \frac{\partial \omega}{\partial \vec{k}}$$

- Without diffraction

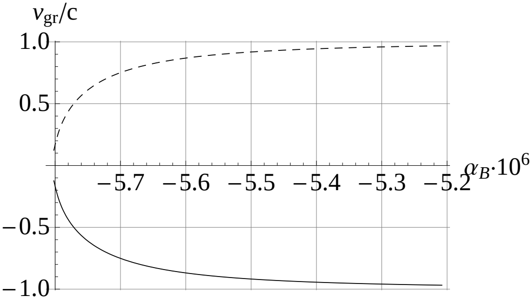
$$v_{gr} = 1 - \frac{\omega_L^2}{2\omega^2}$$

- In crystals [6]

$$\frac{v_{gr}^{(1,2)s}}{c} = \left(\varepsilon_s^{(1,2)} + \frac{(\alpha_B - 2\chi_0)(\chi_0 - 1) + 4C_s^2 \chi_{\vec{\tau}} \chi_{-\vec{\tau}}}{4\varepsilon_s^{(1,2)} - \alpha_B}\right)^{-1}.$$

[6] V. G. Baryshevsky // Izvestia AN BSSR. Ser. Phiz.-mat. 1989. No. 5. P. 109.

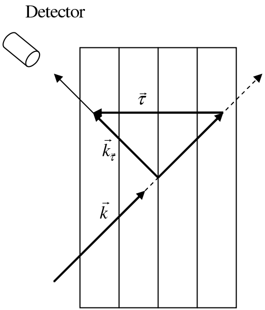
Group velocity [7]



Time delay $\Delta T \approx \frac{L}{v_{gr}} - \frac{L}{v}$ is much greater than a particle time of flight $\frac{L}{v}$ through a crystal target!!!

[7] V. G. Baryshevsky // Izvestia AN BSSR. Ser. Phiz.-mat. 1989. No. 5. P. 109.

Wave-packet diffraction



$E_{diffracted} = \int B_s(t - t_1) E_{incident}(t_1) dt_1,$
where B_s — response function

Diffracted pulse

- General expression

$$B_s(t) = \frac{\omega \chi_{\vec{\tau}} k_B^2}{\tau^2} \times \Bigg(2\frac{J_1(a_s t)}{a_s t} + \sum_{n=1}^{\infty} (-1)^n \Theta(t - 2nt_x) B_n^{n-1}$$

$$\times \left[J_{2n-2}(a_s t_n) + 2B_n J_{2n}(a_s t_n) + B_n^2 J_{2n+2}(a_s t_n) \right] \Bigg) e^{-i(\omega_b + \Delta\omega_B)t},$$

where

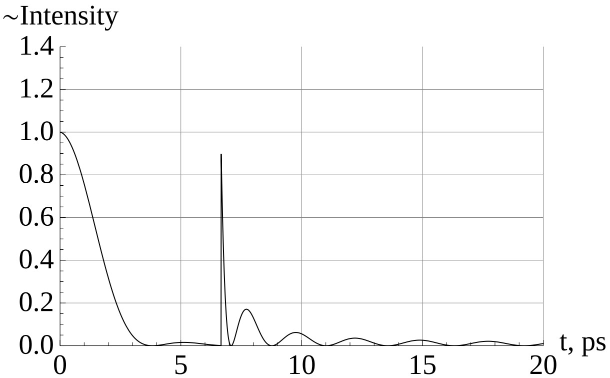
$$t_x = \frac{L}{c} \frac{2\sin^2\theta_B}{|\gamma_1|}, \qquad t_n = \sqrt{t^2 - 4n^2 t_x^2}, \qquad B_n = \frac{t - 2nt_x}{t + 2nt_x},$$

$$a_s = \frac{2\sqrt{C_s \chi_{\vec{\tau}} \chi_{-\vec{\tau}} \omega_B}}{\sqrt{|\beta_1| \frac{\vec{\tau}^2}{k_B^2}}}, \qquad \Delta\omega_B = -\frac{\chi_0 (1 + |\beta_1|) \omega_B k_B^2}{|\beta_1| \, \vec{\tau}^2}.$$

- Simplified formula

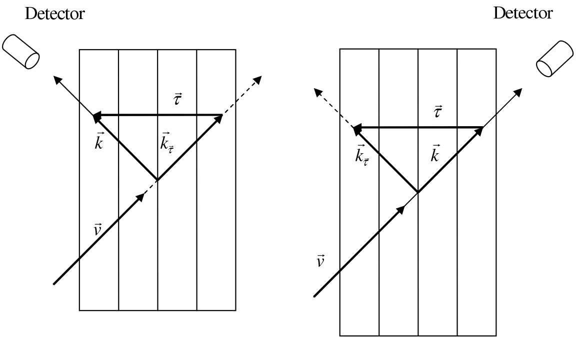
$$B_s(t) = -\frac{\omega \chi_{\vec{\tau}} k_B^2}{\tau^2} \left(2\frac{J_1(a_s t)}{a_s t} - \Theta(t - 2t_x) J_0\left(a_s \sqrt{t^2 - 4t_x^2}\right) \right) e^{-i(\omega_B + \Delta\omega_B)t}.$$

Intensity of a diffracted pulse [8]



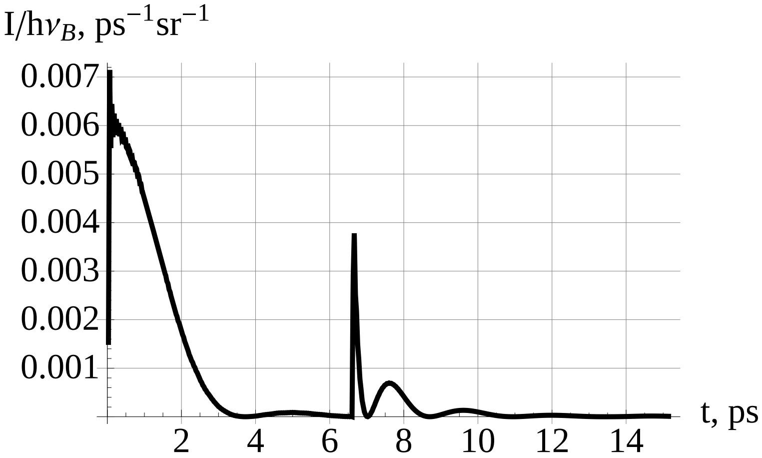
[8] V. G. Baryshevsky // Izvestia AN BSSR. Ser. Phiz.-mat. 1989. No. 5. P. 109; V. G. Baryshevsky, S. A. Maximenko // Optika i spectroskopiya. 1991. Vol. 71. No. 4. P. 659; S. A. Maximenko // J. Mod. Opt. 1994. Vol. 41. No. 10. P. 1975; F. N. Chukhovskii, E. Forster // 1995. Acta. Cryst. A51. P. 668.

Parametric radiation: the Bragg geometry



Diffracted maximum (left), forward maximum (right).

The Bragg geometry (large angles)



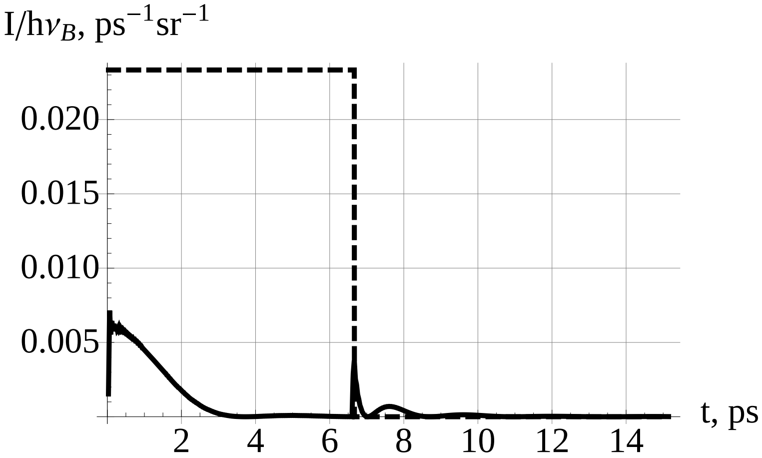
Parameters: LiF crystal, diffraction plane (5,5,5); $\theta_b = 81.5^o$, $\gamma = 2 \cdot 10^4$, $L = 1$ mm, $d = 0.0465$ nm

Parametric radiation (the kinematic approximation)

$$\frac{I}{\hbar\omega} = \frac{\omega_B}{2\pi}\frac{e^2}{\hbar c}|\chi_\tau|^2\frac{\theta^2}{(\theta^2+1/\gamma^2)^2}\left(\Theta(t)-\Theta(t-2T)\right)$$

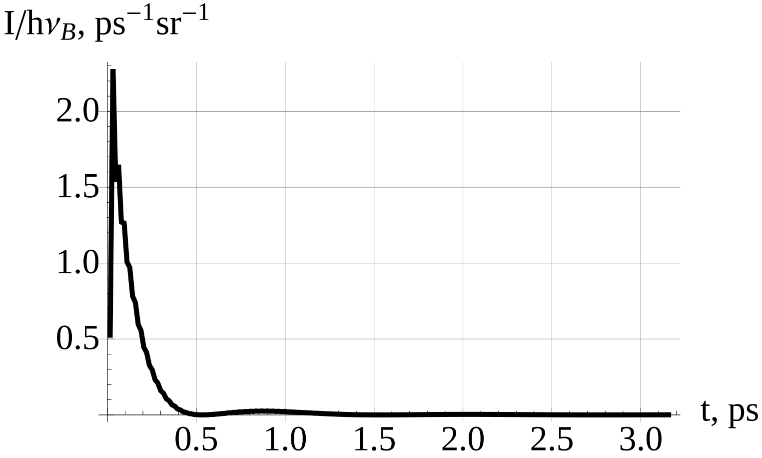
- Here, $\Theta(t)$ - Heaviside function

Bragg geometry (the kinematic approximation)



Parameters: LiF crystal, diffraction plane (5,5,5); $\theta_b = 90^o$, $\gamma = 2 \cdot 10^4$, $L = 1$ mm, $d = 0.0465$ nm

Bragg geometry (small angles, forward radiation)



Parameters: $\theta_b = 81.5^o$, $\gamma = 2 \cdot 10^4$, $L = 1$ mm, $d = 0.0465$ nm

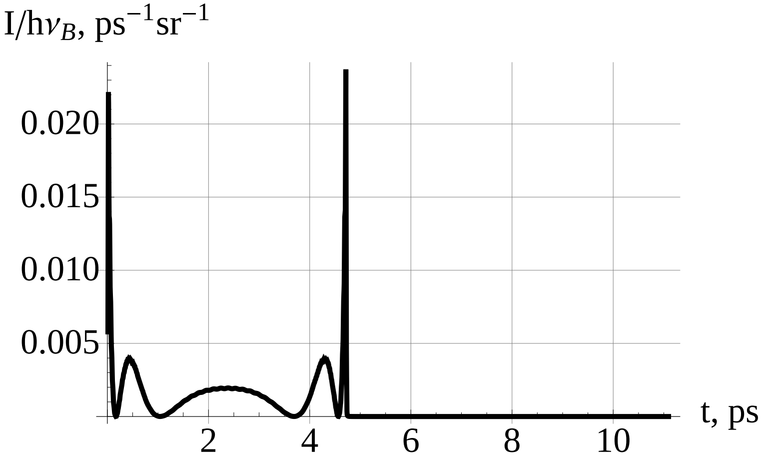
Transition radiation

$$I = \frac{e^2\omega_L^2}{2\pi c}\frac{\theta^2}{(\theta^2 + 1/\gamma^2)^2}\exp\left(-\frac{2\omega_L t}{\sqrt{\theta^2 + 1/\gamma^2}}\right)$$

Here,

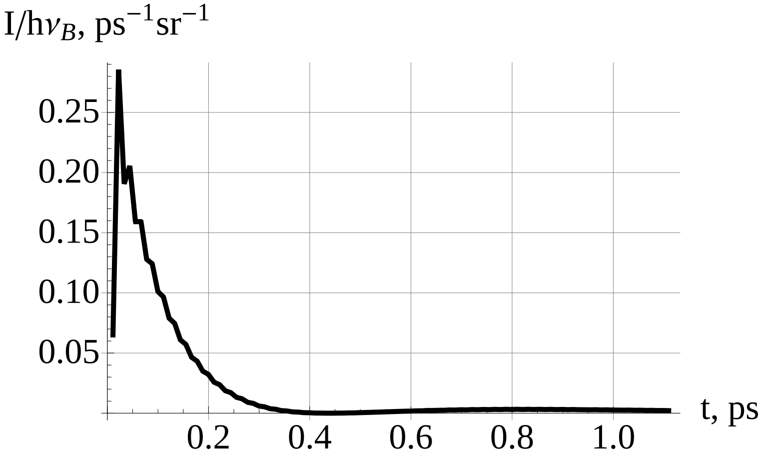
- γ - Lorenz-factor
- ω_L — Langmuir frequency
- $(\gamma\omega_L)^{-1} \sim 10^{-18}$ sec — typical radiation time

Laue geometry (large angles)



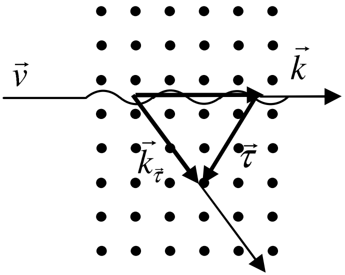
Parameters: $\theta_b = 45^o$, $\gamma = 2 \cdot 10^4$, $L = 1$ mm, $d = 0.0465$ nm

Laue geometry (small angles, forward radiation)

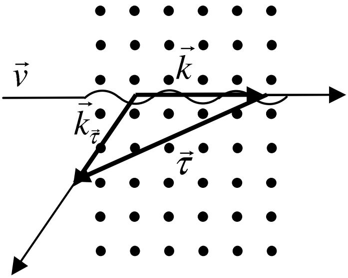


Parameters: $\theta_b = 45^o$, $\gamma = 2 \cdot 10^4$, $L = 1$ mm, $d = 0.0465$ nm

Diffracted channeling radiation

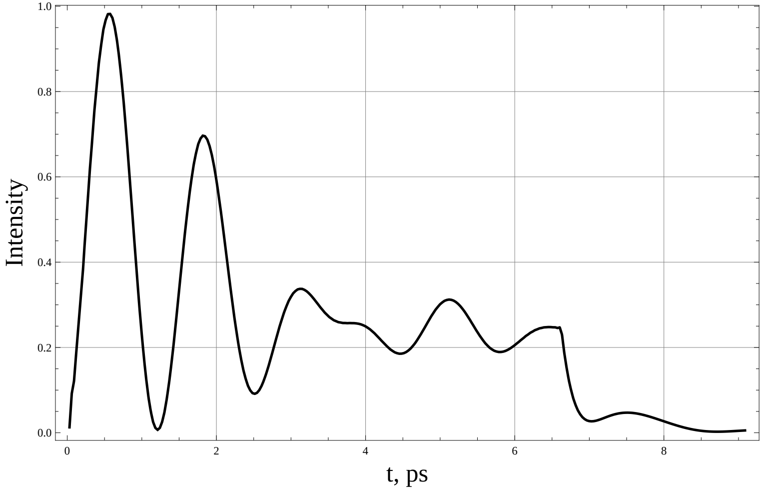


The Laue case



The Bragg case

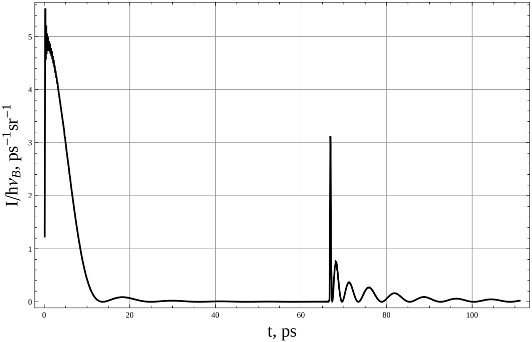
Diffracted channeling radiation (the Bragg case)



Parameters: $\theta_b = 81^o$, $\gamma = 2 \cdot 10^4$, $L = 1$ mm, $d = 0.0465$ nm

Optical range

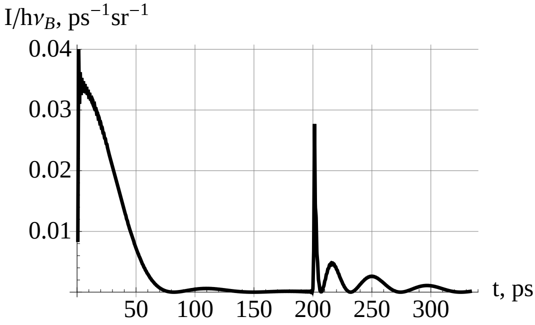
- Constraints on the diameter of a hole: $d < \frac{\lambda\gamma}{4\pi}$
- for $\lambda \sim 300$nm (optical range) and $\gamma \sim 10^3$: we have $d < 3\mu$m !!!



$\lambda = 500$ nm, $\chi_\tau = 3 \cdot 10^{-4}$, $L = 1$ cm, $\theta_b = 80^o$, $\gamma = 2 \cdot 10^3$.

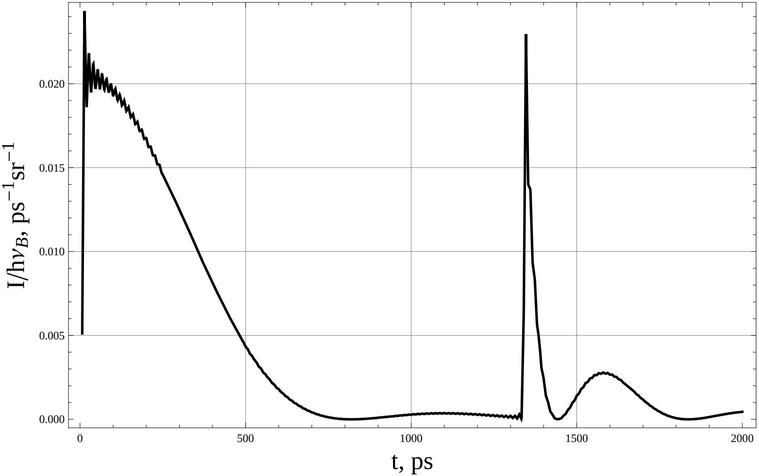
Parametric radiation in photonic crystals (terahertz range)

- Constraints on the diameter of a hole: $d < \frac{\lambda\gamma}{4\pi}$
- for $\lambda \sim 0.3$ mm (terahertz range) and $\gamma \sim 10^3$: we have $d < 3$ mm!!!



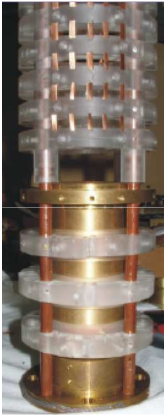
$\lambda = 0.3$ mm, $\chi_\tau = 5 \cdot 10^{-3}$, $L = 3$ cm, $\theta_b = 80^o$, $\gamma = 2 \cdot 10^3$.

Microwave range

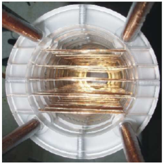


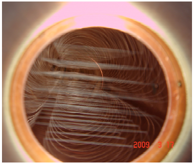
$\lambda = 3$ mm, $\chi_\tau = 3 \cdot 10^{-2}$, $L = 20$ cm, $\theta_b = 80^o$, $\gamma = 2 \cdot 10^3$.

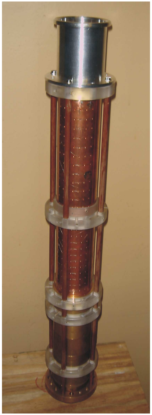
Volume free electron lasers (VFEL)



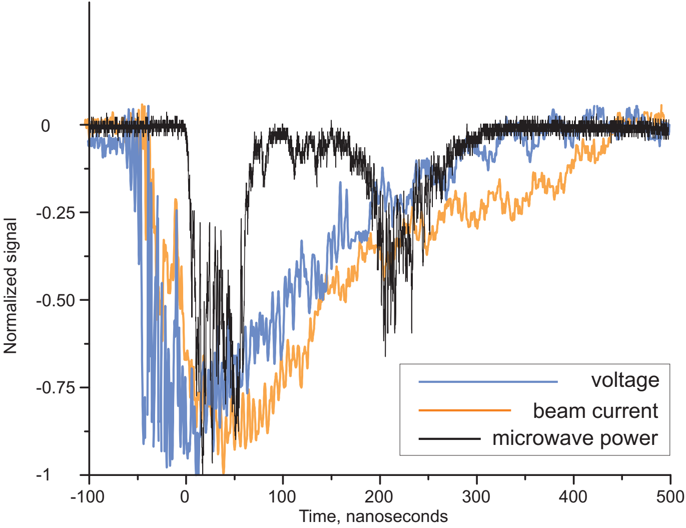
Photonic crystals for
VFEL lasing



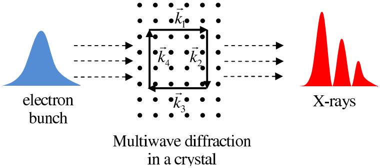




Radiation in the microwave range



VFEL - the way from microwave to optical and X-ray[9]



Multiwave distributed feedback results in a new type of radiative instability of relativistic beams passing through the crystal and enables achieving the generation threshold for induced X-ray radiation even at the beam current density $j \sim 10^8$ A/cm^2.

[9] V.G.Baryshevsky and I.D.Feranchuk. Phys. Lett. A102 (1984), 141; V.G.Baryshevsky, K.G.Batrakov, I.Ya.Dubovskaya. Journ. Phys. D: Appl. Phys. 24 (1991), 1250; V.G.Baryshevsky Docklady Academy of Science of the USSR 299 N6, (1988),1363; V.G. Baryshevsky, http://arxiv.org/abs/1101.0783 (2011).

Conclusions

- The long duration of parametric radiation and diffracted channeling radiation makes possible the detailed experimental investigation of the complicated time structure of the radiation pulses, generated by electron bunches, which are available with modern acceleration facilities.
- A crystal acts as a high quality resonator: photons undergo multiple rescattering by diffraction planes. The period of the oscillations and the distance between peaks may exceed time of the particle flight through the crystal sufficiantly.
- In the case of multiwave diffraction, the delay time of photon exit from a crystal increases.
- Considered phenomena could be observed in all spectral ranges (X-ray, optical, terahertz, microwave).

The end

Thank you for your attention!