Undulator Radiation Inside a Dielectric Waveguide

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Content

- Motivation
- On features of the radiation from an electron moving along a helix around a cylindrical waveguide
- Electromagnetic fields Inside a cylinder
- Radiation fields inside a dielectric cylinder
- Conclusion

Motivation

- The motion of charged particles along a helical orbit is used in helical undulators for generating electromagnetic radiation

- The Unique Characteristics of undulator radiation are the following:
  - Broad Spectrum (from radio or millimeter waves to hard X-rays
  - High Flux
  - High Brilliance: highly collimated photon beam generated by a small divergence and small size source (spatial coherence)
  - High Stability: submicron source stability
  - Polarization: both linear and circular
  - Pulsed Time Structure: pulsed length down to tens of picoseconds allows the resolution of process on the same time scale

Experimental Techniques and Usage

- Synchrotron light is an ideal tool for many types of research and also has industrial applications

- Some of the experimental techniques in synchrotron beamlines are:

  - Structural analysis of crystalline and amorphous materials
  - Powder diffraction analysis
  - X-ray crystallography of proteins and other macromolecules
  - Tomography
  - Photolithography for MEMS structures as part of the LIGA process.
  - Photoemission spectroscopy and Angle resolved photoemission spectroscopy
  - High energy X-rays which can penetrate matter and interact with atoms
  - High concentration, tunability and polarization thus ensuring focusing accuracy for even the smallest of targets

Radiation processes in medium

- Wide applications of synchrotron radiation motivate the importance of investigations for various mechanisms of controlling the radiation parameters

- From this point of view, it is of interest to consider the influence of a medium on the spectral and angular distributions of the radiation

- It is well known that the presence of medium gives rise to new types of radiation processes:
  - Cherenkov radiation
  - Transition radiation
  - Diffraction radiation

- High energy electromagnetic processes essentially change their characteristics when boundaries are present

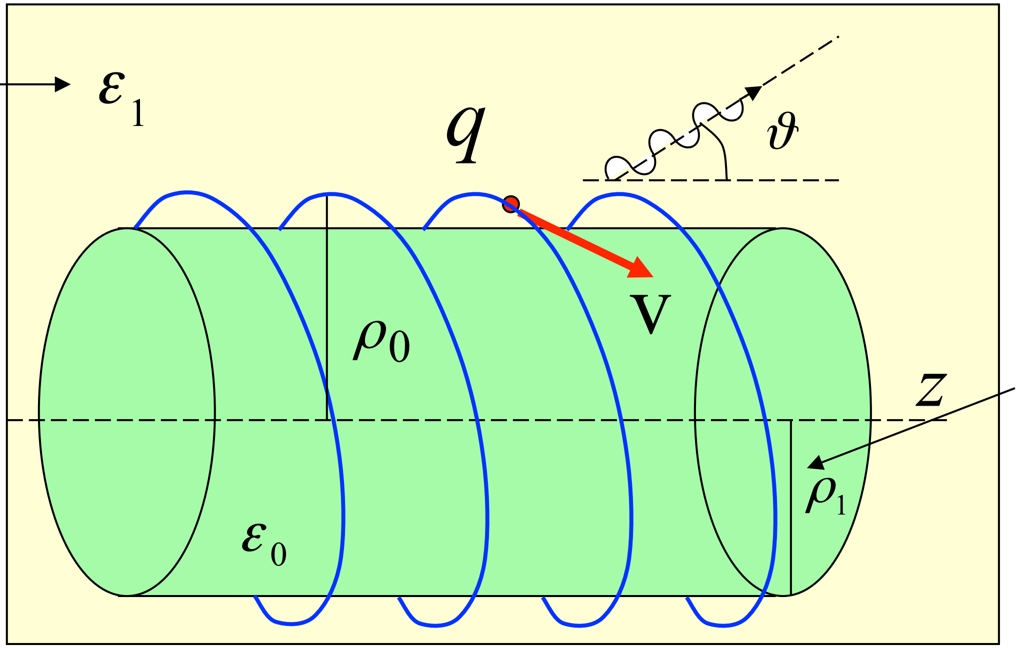
- We consider combined effects of medium and boundaries

Geometry of the problem

Dielectric permittivity for surrounding medium

$\omega_0 = v_\perp / \rho_0$

Rotation frequency



Dielectric cylinder

Notations used: $\beta_{j\perp} = \frac{v_\perp}{c}\sqrt{\varepsilon_j}\,,\beta_{j\|} = \frac{v_\|}{c}\sqrt{\varepsilon_j}, \; j = 0, 1$

The components of the charge velocity along the axis of the cylinder (drift velocity) and on the perpendicular plane we will denote by v_P and $v_\perp$ respectively

Radiation intensity in the exterior medium

- Under the Cherenkov condition, $\beta_{1\|} > 1$, the total radiation intensity at large distances from the charge trajectory is presented in the form

$$I = I_0 + I_{m\neq 0}$$

- I_0 describes the radiation with a continuous spectrum propagating along the Cherenkov cone of the external medium

$$\vartheta = \vartheta_0 \equiv \arccos\left(\beta_{1\|}^{-1}\right)$$

- $$I_{m\neq 0} = \sum_{m=1}^{\infty} \int d\Omega \frac{dI_m}{d\Omega}, \quad d\Omega = \sin\vartheta \, d\vartheta \, d\phi,$$

describes the radiation, which, for a given angle ϑ, has a discrete spectrum determined by

$$\omega_m = \frac{m\omega_0}{|1 - \beta_{1\|}\cos\vartheta|}, \qquad m = 1, 2, \ldots, \quad \omega_0 = v_\perp/\rho_0$$

- Normal Doppler effect $\beta_{1\|} < 1$ and $\beta_{1\|} > 1, \vartheta > \vartheta_0,$
- Anomalous Doppler effect $\vartheta < \vartheta_0$, in the case $\beta_{1\|} > 1$

Features of the radiation

The asymptotical cases

- Non-relativistic motion, $\beta_{1\perp}, \beta_{1\|} \ll 1$,

$$\frac{dI_m}{d\Omega} \approx \frac{2q^2 c(m\beta_{1\perp}/2)^{2(m+1)}}{\pi \rho_0^2 \varepsilon_1^{3/2}(m!)^2} \left[1 + \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_0 + \varepsilon_1} \left(\frac{\rho_1}{\rho_0} \right)^{2m} \right]^2 (1 + \cos^2\vartheta) \sin^{2(m-1)}\vartheta,$$

Induced by the presence of the cylinder

Contribution of the harmonics with $m > 1$ is small compared to that in the fundamental one, $m = 1$

- In the limit $\rho_1 \to 0$, when the radius of cylinder is small, the difference between the radiation intensities in the cases when the cylinder is present and absent, $dI_m/d\Omega - dI_m^{(0)}/d\Omega \propto \rho_1^{2m}$ for $m \geqslant 1$

$$\frac{dI_m^{(0)}}{d\Omega} = \frac{q^2\omega_0^2 m^2}{2\pi c\sqrt{\varepsilon_1}|1 - \beta_{1\|}\cos\vartheta|^3} \left[\beta_{1\perp}^2 J_m'^{\,2}(\lambda_1\rho_0) + \left(\frac{\cos\vartheta - \beta_{1\|}}{\sin\vartheta} \right)^2 J_m^2(\lambda_1\rho_0) \right]$$

- In the same limit and for the radiation corresponding to $m = 0$, the part induced by the cylinder vanishes like ρ_1^2

Features of the radiation

- Behavior of the radiation intensitv near the Cherenkov angle when

$$\frac{\mathrm{d}I_m}{\mathrm{d}\Omega} \propto |1-\beta_{1\|}\cos\vartheta|^{-2}, \quad |1-\beta_{1\|}\cos\vartheta| \ll 1$$

- Near the Cherenkov cone the frequencies of the radiated photons are large and the dispersion of the dielectric permittivity ε_1 should be taken into account

- For the radiation intensity in a homogeneous medium with dielectric permittivity ε_1 we have the same behavior

- For the charge helical motion inside the dielectric cylinder $(\rho_0 < \rho_1)$ the behavior of the radiation intensity near the Cherenkov cone is radically different for

$$\beta_{0\|} > 1 \qquad \frac{\mathrm{d}I_m}{\mathrm{d}\Omega} \propto |1-\beta_{1\|}\cos\vartheta|^{-4}$$

$$\beta_{0\|} < 1 \qquad \frac{\mathrm{d}I_m}{\mathrm{d}\Omega} \propto |1-\beta_{1\|}\cos\vartheta|^{-4}\exp\left[-2(\omega_m/v_\|)(\rho_1-\rho_0)\sqrt{1-\beta_{0\|}^2}\right]$$

Strong peaks in the radiation intensity

- Strong narrow peaks are present in the angular distribution for the radiation intensity at a given harmonic *m*
- The condition for the appearance of the peaks is obtained from the equation determining the eigenmodes for the dielectric cylinder by the replacement $H_m \to Y_m$

H_m: Hankel function of the first kind

Y_m: Neumann function

J: Bessel function

$$\sum_{l=\pm 1}\left[\frac{\lambda_1}{\lambda_0}\frac{J_{m+l}(\lambda_0\rho_1)Y_m(\lambda_1\rho_1)}{J_m(\lambda_0\rho_1)Y_{m+l}(\lambda_1\rho_1)}-1\right]^{-1}=\frac{2\varepsilon_0}{\varepsilon_1-\varepsilon_0}$$

$$\lambda_j^2=\frac{\omega_m^2(k_z)}{c^2}\varepsilon_j-k_z^2, \qquad j=0,1,\ \ \omega_m(k_z)=m\omega_0+k_z v_\parallel$$

This equation has no solutions for the case $\varepsilon_0<\varepsilon_1$

Strong peaks in the radiation intensity

- As necessary conditions for the presence of the strong narrow peaks in the angular distribution for the radiation intensity one has

$$\frac{\omega_0 \rho_0}{c}\sqrt{\varepsilon_1}\sin\vartheta < |1-\beta_{1\|}\cos\vartheta| < \frac{\omega_0 \rho_1}{c}\sqrt{\varepsilon_0 - \varepsilon_1\cos^2\vartheta}$$

- These conditions can be satisfied only if we have

$$\varepsilon_0 > \varepsilon_1, \qquad \tilde{v}\sqrt{\varepsilon_0}/c > 1, \text{ where } \tilde{v} = \sqrt{v_\|^2 + \omega_0^2\rho_1^2}$$

velocity of the charge image on the cylinder surface

Analytic estimates for the heights and widths of peaks

- Angular dependence of the radiation intensity near the peak

$$\frac{dI_m}{d\Omega} \propto \frac{1}{(\vartheta-\vartheta_p)^2/b_p^2+1}\left(\frac{dI_m}{d\Omega}\right)_{\vartheta=\vartheta_p}, \qquad b_p \propto \exp[-2m\zeta(\lambda_1\rho_1/m)]$$

- Angular widths of the peaks: $\Delta\vartheta \propto \exp[-2m\zeta(\lambda_1\rho_1/m)]$

$$\zeta(z) = \ln\frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2}, \qquad \lambda_1 = \frac{m\omega_0}{c}\frac{\sqrt{\varepsilon_1}\sin\vartheta}{1-\beta_{1\|}\cos\vartheta}$$

Numerical examples

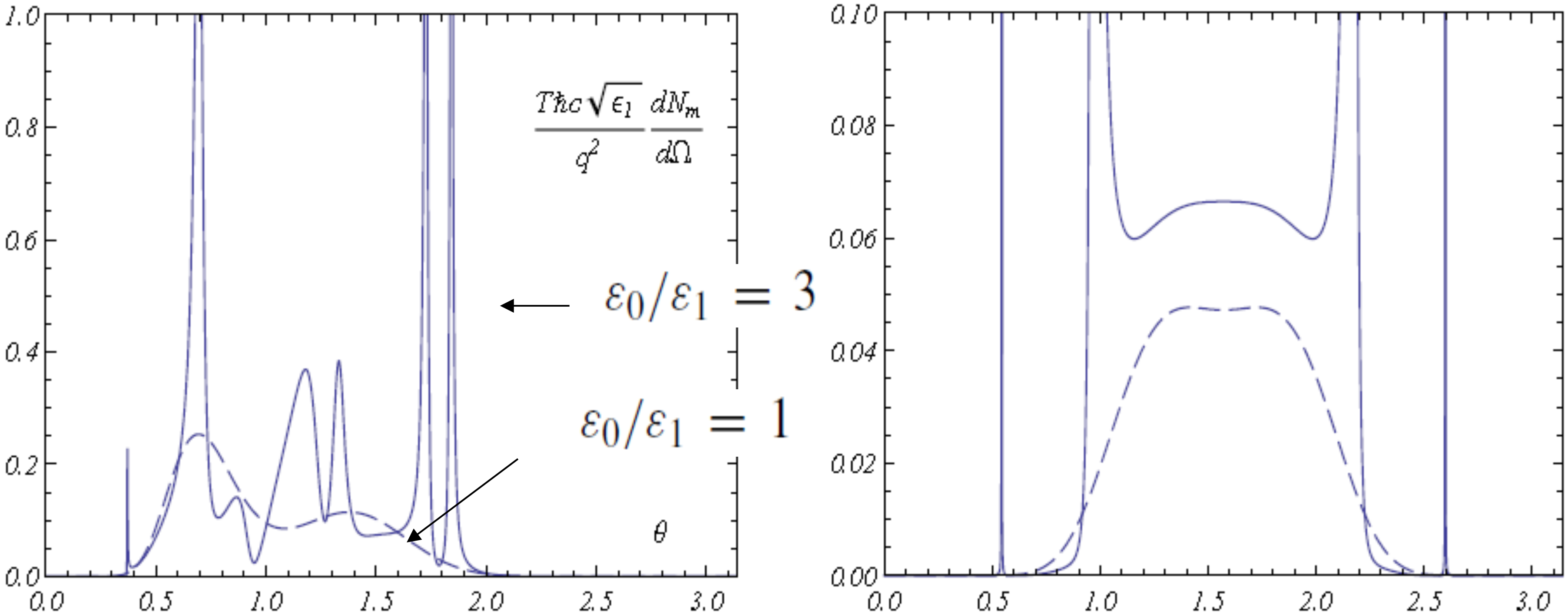
Angular density for the Number of the Radiated Quanta

$$\frac{dN_m}{d\Omega} = \frac{1}{\hbar\omega_m}\frac{dI_m}{d\Omega}$$

$$\beta_{1\perp} = 0.9,\ \rho_1/\rho_0 = 0.95,\ m = 10$$

$$\beta_{1\parallel} = 0.5$$

$$\frac{T\hbar c\sqrt{\epsilon_1}}{q^2}\frac{dN_m}{d\Omega},\quad T = 2\pi/\omega_0 \quad \beta_{1\parallel} = 0$$



Radiation Inside a Dielectric Waveguide

- Vector potential is presented as the Fourier expansion

$$A_l(\mathbf{r},t) = \sum_{m=-\infty}^{\infty} e^{im(\phi-\omega_0 t)} \int_{-\infty}^{\infty} dk_z e^{ik_z(z-v_\parallel t)} A_{ml}(m,k_z,\rho)$$

- Fourier coefficients for the fields determined have poles corresponding to the zeros of the function

$$\alpha_m = \frac{\varepsilon_0}{\varepsilon_1-\varepsilon_0} - \frac{\lambda_0 J_m(\lambda_0\rho_1)}{2} \sum_{l=\pm 1} l \frac{H_{m+l}(\lambda_1\rho_1)}{V_{m+l}^H}$$

Notation: $V_m^F = J_m(\lambda_0\rho_1)\frac{\partial F_m(\lambda_1\rho_1)}{\partial\rho_1} - F_m(\lambda_1\rho_1)\frac{\partial J_m(\lambda_0\rho_1)}{\partial\rho_1},\ F = J, H$

$$\lambda_j^2 = \frac{\omega_m^2(k_z)}{c^2}\varepsilon_j - k_z^2, \qquad j = 0, 1,$$

- Function α_m has zeros only under the conditions $\lambda_1^2 < 0 < \lambda_0^2 \Rightarrow \varepsilon_1 < \varepsilon_0$
- Corresponding modes in the exterior region are exponentially damped with the distance from the cylinder surface
- These modes are the eigenmodes of the dielectric cylinder and they correspond to the waves propagating inside the cylinder

Poles and the integration contour

- We denote by $\lambda_0\rho_1 = \lambda_{m,s},\ s = 1, 2, \ldots,$ the solutions to equation $\alpha_m = 0$
- Corresponding modes for $k_z = k_{m,s}^{(\pm)}$ $\qquad \sigma = +(-)$ for $z - v_\| t > 0$ $(z - v_\| t < 0)$

$$k_{m,s}^{(\pm)} = \frac{m\omega_0\sqrt{\varepsilon_0}}{c\left(1-\beta_{0\|}^2\right)}\left[\beta_{0\|} \pm \sqrt{1 + b_{m,s}^2\left(\beta_{0\|}^2 - 1\right)}\right], \quad b_{m,s} = \frac{c\lambda_{m,s}}{m\omega_0\rho_1\sqrt{\varepsilon_0}}$$

- For $\beta_{0\|} < 1$, the condition for $k_{m,s}^{(\pm)}$ to be real defines the maximum value for *s*:

$$\lambda_{m,s_m} < \frac{m\omega_0\rho_1\sqrt{\varepsilon_0}}{c\sqrt{1-\beta_{0\|}^2}} < \lambda_{m,s_m+1}$$

- If the Cherenkov condition $\beta_{0\|} > 1$ is satisfied, the upper limit *s* is determined by the dispersion law for the dielectric permittivity via the condition $\varepsilon_0(\omega_m) > c^2/v_\|^2$
- Integration contour in the complex plane k_z should be specified: By taking into account the imaginary part of the dielectric permittivity it can be seen:
  - For $\beta_{0\|} < 1$ in the integral over k_z the contour avoids the poles $k_{m,s}^{(-)}$ from above and the poles $k_{m,s}^{(+)}$ from below
  - For $\beta_{0\|} > 1$ poles $k_{m,s}^{(\pm)}$ should be avoided from above

Electromagnetic fields inside the cylinder

- Closing the integration contour by the large semicircle, under the condition $\beta_{0\|} < 1$ one finds

$$F_l^{(\mathrm{rad})}(\mathbf{r}, t) = \sigma F_l^{(\sigma)}(\mathbf{r}, t) = \sigma 4\pi \,\mathrm{Re}\left[\mathrm{i} \sum_{m=1}^{\infty} \mathrm{e}^{im(\phi-\omega_0 t)} \sum_{s=1}^{s_m} \operatorname*{Res}_{k_z=k_{m,s}^{(\sigma)}} \mathrm{e}^{ik_z(z-v_\| t)} F_{ml}(k_z, \rho) \right]$$

$$\sigma = +(-) \text{ for } z - v_\| t > 0 \ (z - v_\| t < 0)$$

$$b_{m,s} = \frac{c\lambda_{m,s}}{m\omega_0 \rho_1 \sqrt{\varepsilon_0}}$$

- Describes waves propagating
  - Along the positive direction of the axis *z* for $\sigma = +$ and for $\sigma = -$, $b_{m,s} < 1$
  - Along the negative direction to the axis *z* for $\sigma = -$, $1 < b_{m,s} < 1/\sqrt{1-\beta_{0\|}^2}$
- Radiation fields under the condition $\beta_{0\|} > 1$

$$F_l^{(\mathrm{rad})}(\mathbf{r}, t) = -\sum_{\sigma=\pm} F_l^{(\sigma)}(\mathbf{r}, t)\theta(v_\| t - z)$$

Heaviside unit step function

- Describes waves propagating
  - Along the positive direction of the axis *z* for $\sigma = +$ and for $\sigma = -$, $b_{m,s} > 1$
  - Along the negative direction to the axis *z* for $\sigma = -$, $b_{m,s} < 1$
- For $b_{m,s} > 1$ there are no waves propagating along the negative direction of the axis *z*

All these features are for the laboratory system

Radiation energy flux inside the cylinder

- Energy flux through the cross section of the cylinder

$$I = \frac{c}{4\pi}\int_0^{\rho_1} d\rho \int_0^{2\pi} d\phi\, \rho (E_\rho H_\phi - E_\phi H_\rho)$$

- For simplicity consider the case $v_{\|} = 0$

$$I = \frac{q^2 v^2}{8\varepsilon_0}(\varepsilon_1 - \varepsilon_0)^2 \sum_{m=1}^{\infty}\sum_{s=1}^{s_m} \frac{J_m^{-2}(\lambda_{m,s})}{K_m^2(\lambda_{m,s}^{(1)})} \frac{\lambda_{m,s}^2}{U_m'^2(k_{m,s})} \frac{k_{m,s}}{m\omega_0} \left(\sum_{l=\pm 1} \frac{l K_{m+l}(\lambda_{m,s}^{(1)}\rho_{01})}{V_m - lmu} \right)^2$$
$$\times \sum_{p=\pm 1}\left[\left(\frac{m^2\omega_0^2}{c^2}\rho_1^2\varepsilon_0 + k_{m,s}^2\rho_1^2 \right)(V_m + pmu)^2 - \lambda_{m,s}^2 \left(V_m^2 - m^2 u^2 \right) \right]$$
$$\times \left[J_{m+p}'^2(\lambda_{m,s}) + J_{m+p}^2(\lambda_{m,s}) \left(1 - \frac{(m+p)^2}{\lambda_{m,s}^2} \right) \right].$$

- Notations

$$\lambda_{m,s} = \rho_1 \sqrt{\left(\frac{m\omega_0}{c}\right)^2 \varepsilon_0 - k_{m,s}^2}, \quad \lambda_{m,s}^{(1)} = \rho_1 \sqrt{k_{m,s}^2 - \left(\frac{m\omega_0}{c}\right)^2 \varepsilon_1}$$

$$V_m = \lambda_{m,s}^{(1)} \frac{J_m'(\lambda_{m,s})}{J_m(\lambda_{m,s})} + \lambda_{m,s} \frac{K_m'(\lambda_{m,s}^{(1)})}{K_m(\lambda_{m,s}^{(1)})}, \quad u = \frac{\lambda_{m,s}}{\lambda_{m,s}^{(1)}} + \frac{\lambda_{m,s}^{(1)}}{\lambda_{m,s}},$$

Radiation energy flux inside the cylinder (continued)

- Notations

$$U_m = V_m \left(\varepsilon_0 \lambda_{m,s}^{(1)} \frac{J'_m(\lambda_{m,s})}{J_m(\lambda_{m,s})} + \varepsilon_1 \lambda_{m,s} \frac{K'_m(\lambda_{m,s}^{(1)})}{K_m(\lambda_{m,s}^{(1)})} \right) - m^2 \frac{\lambda_{m,s}^2 + \lambda_{m,s}^{(1)2}}{\lambda_{m,s}^2 \lambda_{m,s}^{(1)2}} \left(\varepsilon_1 \lambda_{m,s}^2 + \varepsilon_0^2 \lambda_{m,s}^{(1)2} \right)$$

- Eigenmodes of the cylinder are solutions of the equation

$$U_m = 0$$

- Number of the radiated quanta at a given harmonic *m* per period of the charge rotation

$$N_m = 2 \frac{T I_m}{\hbar m \omega_0}, \quad T = \frac{2\pi}{\omega_0}$$

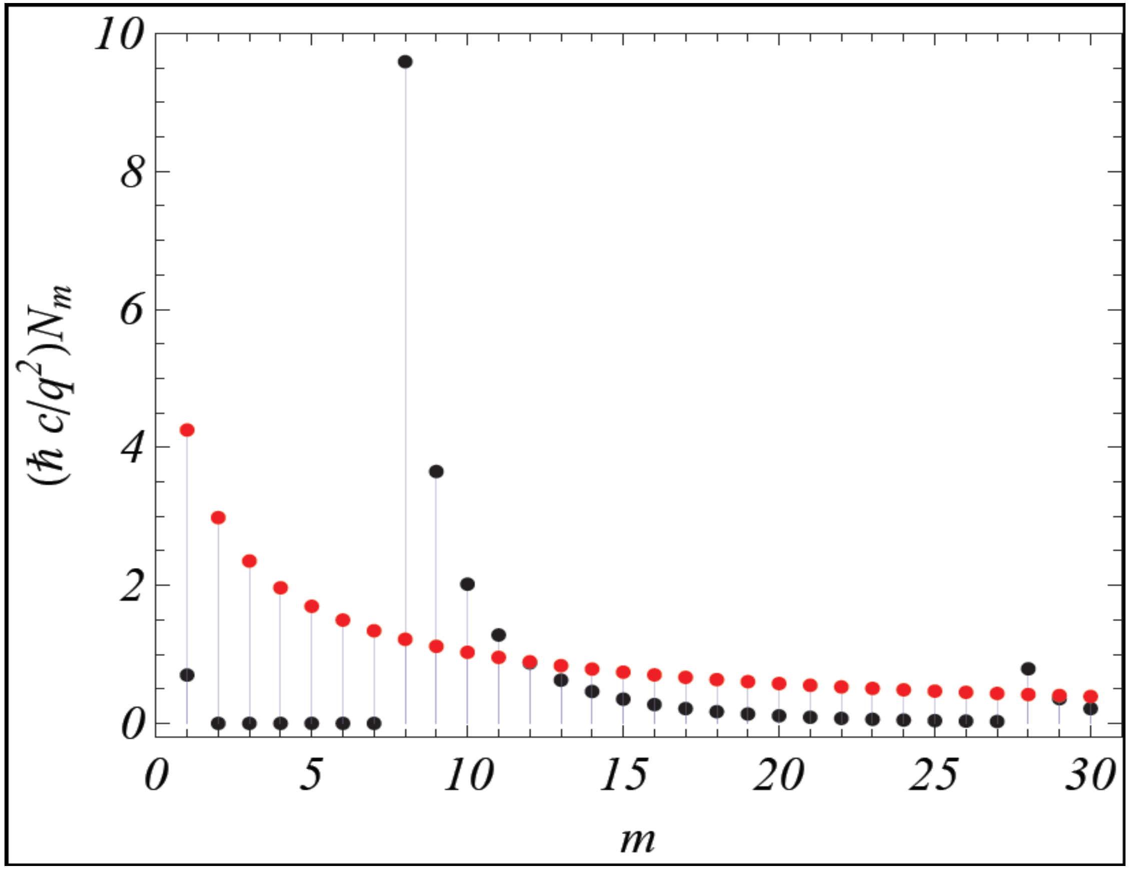
Corresponds to that we have the same amount of the energy flux in the regions $z > 0$ and $z < 0$

- In addition to the radiation propagating inside the dielectric cylinder, in the exterior region we have also radiation fields localized near the surface of the cylinder
- These fields are the tails of the eigenmodes for the dielectric cylinder

Numerical example

- Values of the parameters: Electron energy=2MeV, $\varepsilon_1 = 1, \ \varepsilon_0 = 3, \rho_1/\rho_0 = 0.95$

  Red points -> Synchrotron radiation in the absence of the cylinder

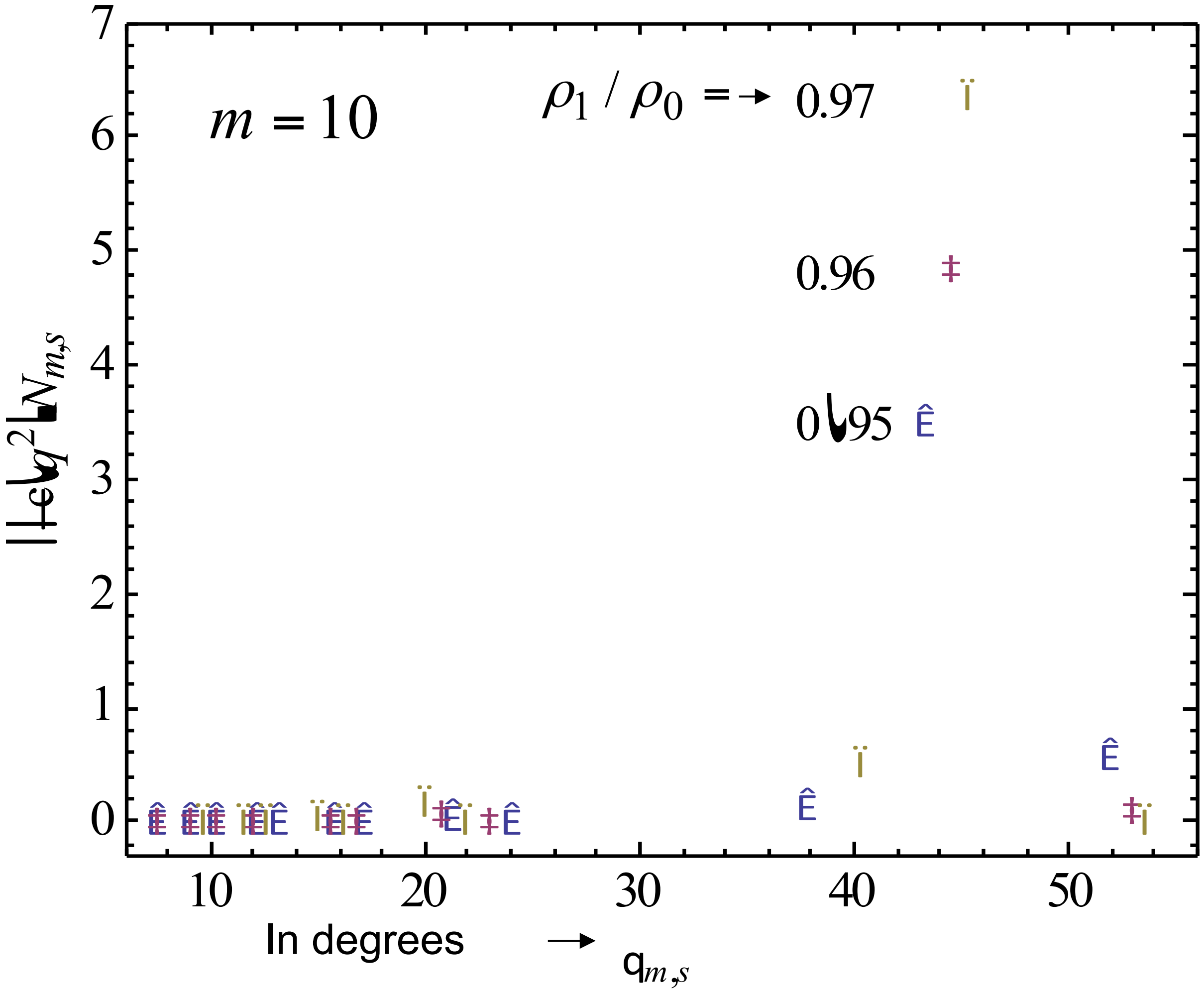


- At m=8 and m=28 new modes of the dielectric cylinder appear

Numerical example: Helical motion

- Values of the parameters: $v_\perp = 0.85c,\ \ v_\parallel = 0.5c,\ \ \varepsilon_0 = 3,\ \varepsilon_1 = 1$

Number of radiated quanta



Total number of radiated quanta at a given m

Radiation frequency

$$\omega_{m,s} = \frac{m\omega_0}{1 - v_\parallel \sqrt{\varepsilon_0}\cos\theta_{m,s}/c}$$

$$k_{z,s} = \frac{\omega_{m,s}}{c}\sqrt{\varepsilon_0}\cos\theta_{m,s}$$

In the numerical example

$$k_{z,s} > 0$$

There is no radiation inside the cylinder propagating along the negative direction of the z axis (in the laboratory frame)

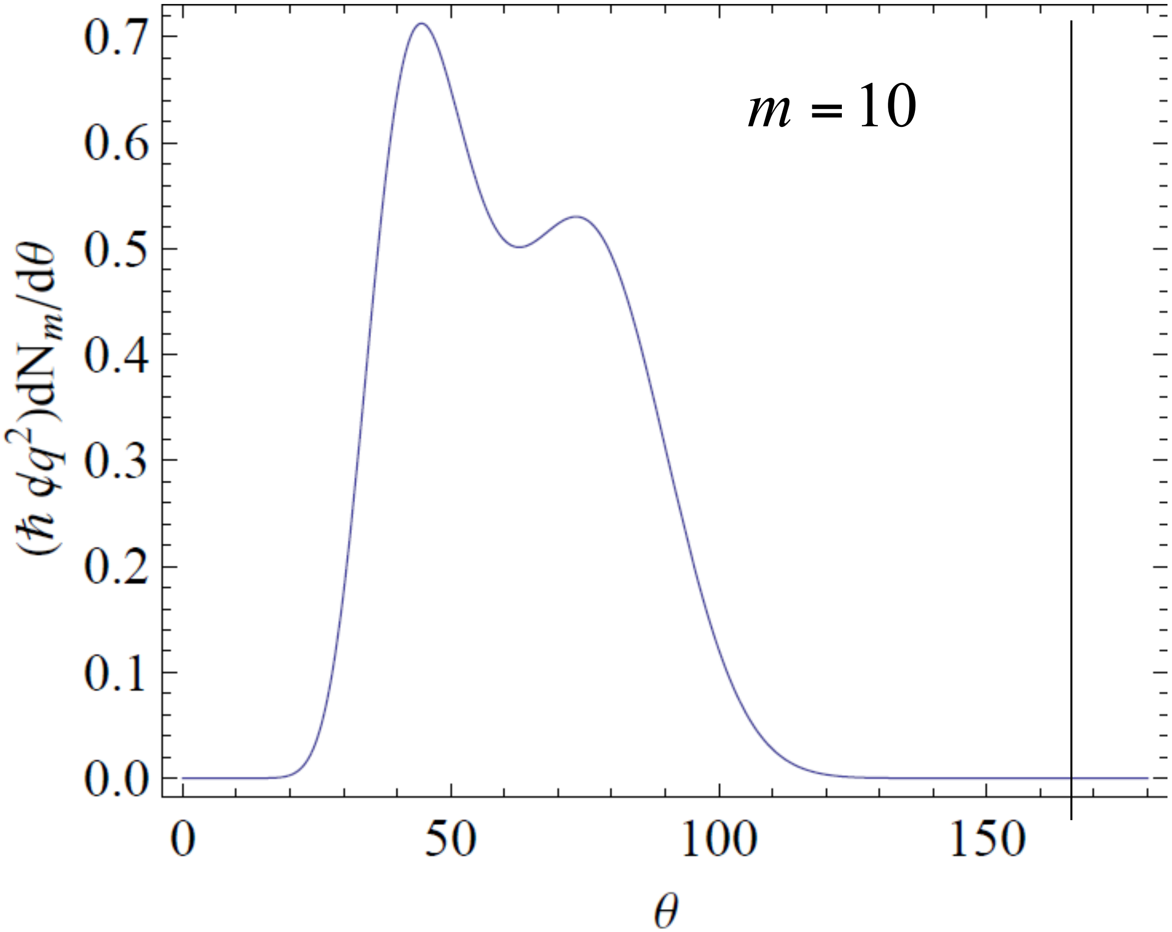
$$\sum_s N_{m,s} = \frac{q^2}{\hbar c}(4.2,\ 4.98,\ 7.02)$$

$\rho_1/\rho_0 \rightarrow$ 0.95 0.96 0.97

Helical motion: Radiation in homogeneous medium

- Values of the parameters: $v_\perp = 0.85c, \quad v_\parallel = 0.5\,\mathrm{c}$

Angular density for the number of radiated quanta



Total number of radiated quanta at a given m

$$\int N_{m,s}\,d\theta = 0.6\,\frac{q^2}{\hbar c}$$

Conclusions

- We have investigated the properties of the radiation from a charged particle moving along a helical orbit around a dielectric cylinder
- Under certain conditions on the parameters strong narrow peaks appear in the angular distribution of the radiation intensity in the exterior medium
- We have specified the conditions for the appearance of the peaks and analytically estimated their heights and widths
- Peaks are present only when dielectric permittivity of the cylinder is greater than the permittivity for the surrounding medium and the Cherenkov condition is satisfied for the velocity of the charge image on the cylinder surface and the dielectric permittivity of the cylinder
- Presence of the cylinder provides a possibility for an essential enhancement of the radiated power as compared to the radiation in a homogeneous medium

Conclusions (continued)

- Radiated energy inside the cylinder is redistributed among the cylinder modes and, as a result, the corresponding spectrum differs significantly from the homogeneous medium or free-space results

- Radiation emitted on the waveguide modes propagates inside the cylinder and the waveguide serves as a natural collector for the radiation

- This eliminates the necessity for focusing to achieve a high-power spectral intensity