Experimental and Theoretical Study of PXRC (Parametric X-Radiation at Channeling) from 255 MeV Electrons in Si

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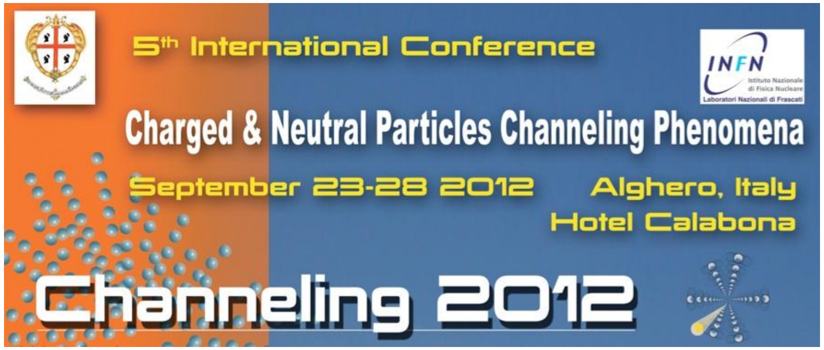


Radiation types from channeled particle in a crystal

Terminology (definitions):

PXR = Parametric X-Radiation = diffraction of relativistic electron electromagnetic field (virtual photons) on the crystal = system of crystallographic planes = well known since first experimental observation (1985, Tomsk, electron synchrotron "Sirius")

DCR = Diffracted Channeling Radiation (Nitta et al, 2002-2008; Bogdanov-Korotchenko-Pivovarov et al, JETP Letters, 2007) … . Diffraction of (virtual) channeling radiation.

PXRC = Parametric X-Radiation from channeled electrons (H.Nitta et al , 1996) limiting case of DCR. Diffraction of channeled relativistic electron electromagnetic field (virtual photons) on the crystal. Electron is bound with channeling plane.

PXR vs PXRC: plane waves vs "squeezed states" (quantum states) bound to channeling planes → **quantum** difference ?

Radiation types from channeled particle in a crystal

H.Nitta: Channeling 2008 → CLASSIFICATION

Diffracted Channeling Radiation (DCR)

Radiation process and matrix elements

Fermi's golden rule:

$$w_{IF} = \frac{2\pi}{\hbar} |< F | H_{int} | I >|^2 \rho_F$$

The interaction Hamiltonian:

$$H_{int} = -\frac{e}{\gamma mc} \mathbf{A} \cdot \hat{\mathbf{p}}$$

Bloch wave "photon":

$$\mathbf{A}(\mathbf{r}) = \sum_{\mathbf{k}} \sum_{\mathbf{g}} \mathbf{A}_{\mathbf{g}} \exp[i(\mathbf{k}+\mathbf{g})\cdot\mathbf{r}] + c.c..$$

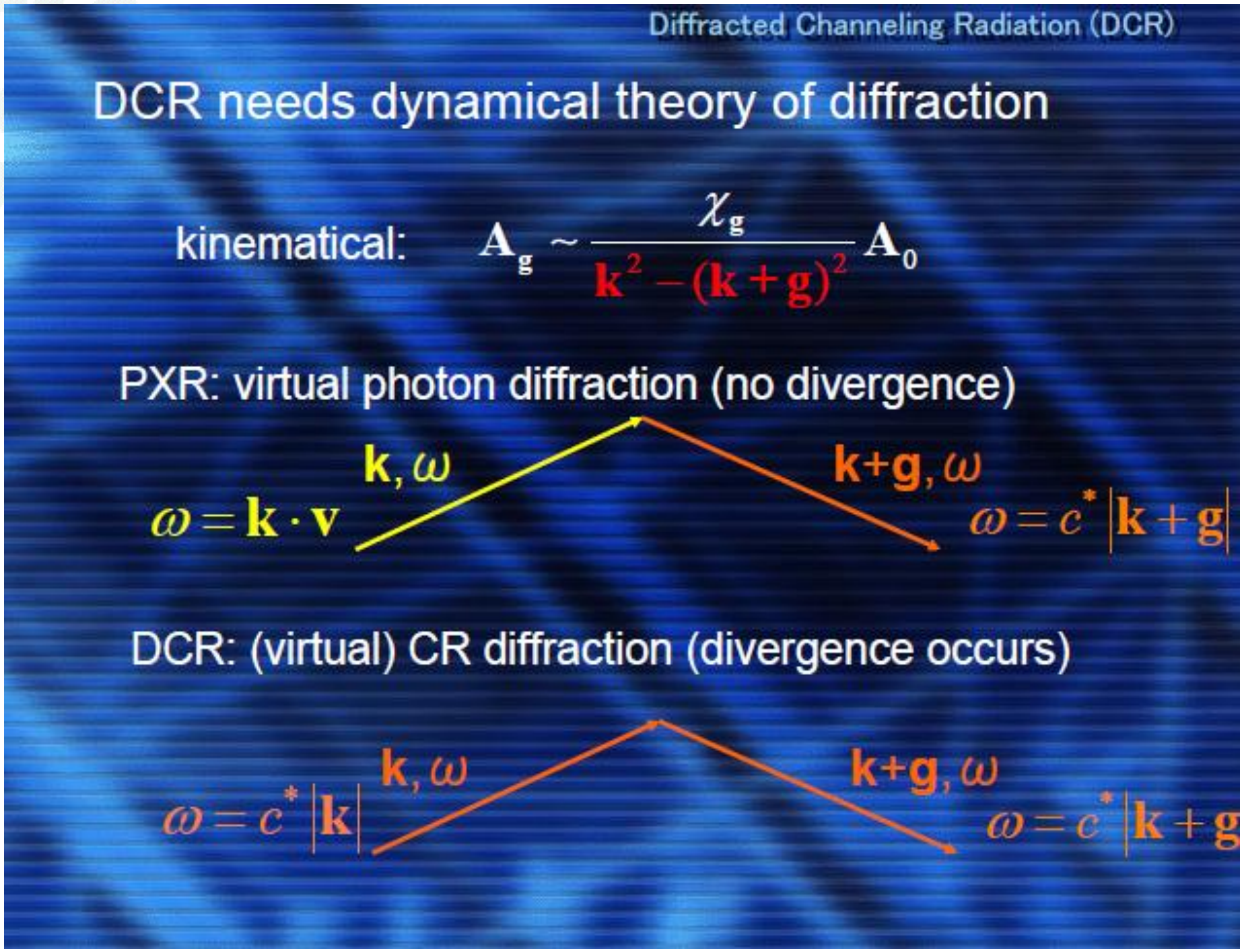
quantum states of channeled electron:

$$\psi^{(s)}(\mathbf{r}) = \frac{1}{\sqrt{L_x L_z}} \varphi_n(y) e^{i\mathbf{p}_{\parallel}\cdot\mathbf{r}_{\parallel}/\hbar} \qquad \varphi_n(y) = \frac{1}{\sqrt{L_y}} \sum_G C_G^{(n)}(p_y) \exp[i(p_y/\hbar + G)$$

$$\left[\frac{\hat{\mathbf{p}}_{\perp}^2}{2\gamma m} + V(y)\right] \varphi_n(y) = E_{\perp,n} \varphi_n(y)$$

Radiation types from channeled particle in a crystal

H.Nitta: Channeling 2008 → CLASSIFICATION



Radiation types from channeled particle in a crystal

H.Nitta: Channeling 2008 → CLASSIFICATION

Diffracted Channeling Radiation (DCR)

Matrix elements

$$< F | H_{int} | I > =$$

$$- < \varphi_f, \mathbf{p}_{\parallel} | \frac{e}{\gamma mc} \sum_{\mathbf{g}(\neq 0)} \left[\mathbf{A}_{\mathbf{g}}^{*} \exp(-i(\mathbf{k}+\mathbf{g})\cdot\mathbf{r}) \right] \cdot \hat{\mathbf{p}} | \varphi_i, \mathbf{p}_{\parallel} >$$

$$- < \varphi_f, \mathbf{p}_{\parallel} | \frac{e}{\gamma mc} \left[\mathbf{A}_{0}^{*} \exp(-i\mathbf{k}\cdot\mathbf{r}) \right] \cdot \hat{\mathbf{p}} | \varphi_i, \mathbf{p}_{\parallel} >$$

$$\equiv M_0^{(if)} + \sum_{\mathbf{g}(\neq 0)} M_{\mathbf{g}}^{(if)}.$$

$$M_{-\mathbf{g}}^{(if)} = -\left(\frac{e}{c}\right)$$

$$\times \left[(\mathbf{A}_{-\mathbf{g}}^{*} \cdot \mathbf{v}_{\parallel}) < \varphi_f | e^{-i(\mathbf{k}-\mathbf{g})_y y} | \varphi_i > + \frac{1}{\gamma m} (\mathbf{A}_{-\mathbf{g}}^{*})_y < \varphi_f | e^{-i(\mathbf{k}-\mathbf{g})_y y} \hat{p}_y | \varphi_i > \right]$$

$$\times \delta(\mathbf{p}_{\parallel} - \mathbf{p}_{\parallel} | \hbar \mathbf{k}_{-\mathbf{g}\parallel})$$

Radiation types from channeled particle in a crystal

H.Nitta: Channeling 2008 → CLASSIFICATION

Diffracted Channeling Radiation (DCR)

intraband transition: i=f

$$M_{-\mathbf{g}}^{(ii)} = -\left(\frac{e}{c}\right)(\mathbf{A}^{*}_{-\mathbf{g}} \cdot \mathbf{v}_{\parallel}) F_{ii}((\mathbf{k}-\mathbf{g})_y)\,\delta(\mathbf{p}_{\parallel} - \mathbf{p}_{\parallel} \mid \hbar \mathbf{k}_{-\mathbf{g}\parallel})$$

$$F_{ii}(q) = <\varphi_i \mid e^{-iqy} \mid \varphi_i>$$ form facto

$$\left(\frac{dN}{d\theta_x d\theta_y dz}\right)_{PXR} = \frac{\alpha\omega_B}{4\pi c \sin^2\theta_B}\left(\frac{\theta_x^2}{4(1+W_{v\parallel}^2)} + \frac{\theta_y^2}{4(1+W_{v\perp}^2)}\right)$$

$$W_{v\sigma} = \frac{1}{2|\chi_g| P_\sigma}\left[\theta_x^2 + \theta_y^2 + \theta_{kin}^2 - \frac{|\chi_g|^2 P_\sigma^2}{\theta_x^2 + \theta_y^2 + \theta_{kin}^2}\right], (\sigma = \parallel, \perp)$$

$$\theta_{kin}^2 = \gamma^{-2} + |\chi_0|$$

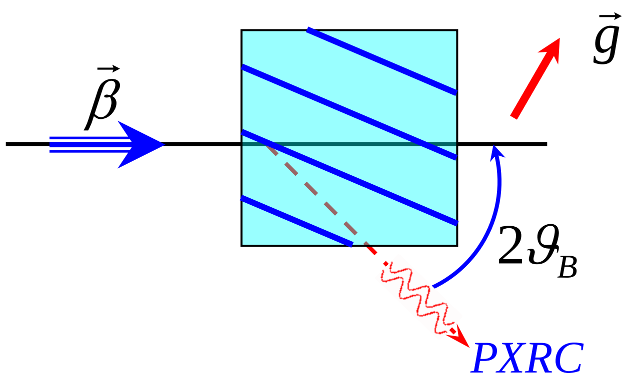
Radiation types from channeled particle in a crystal

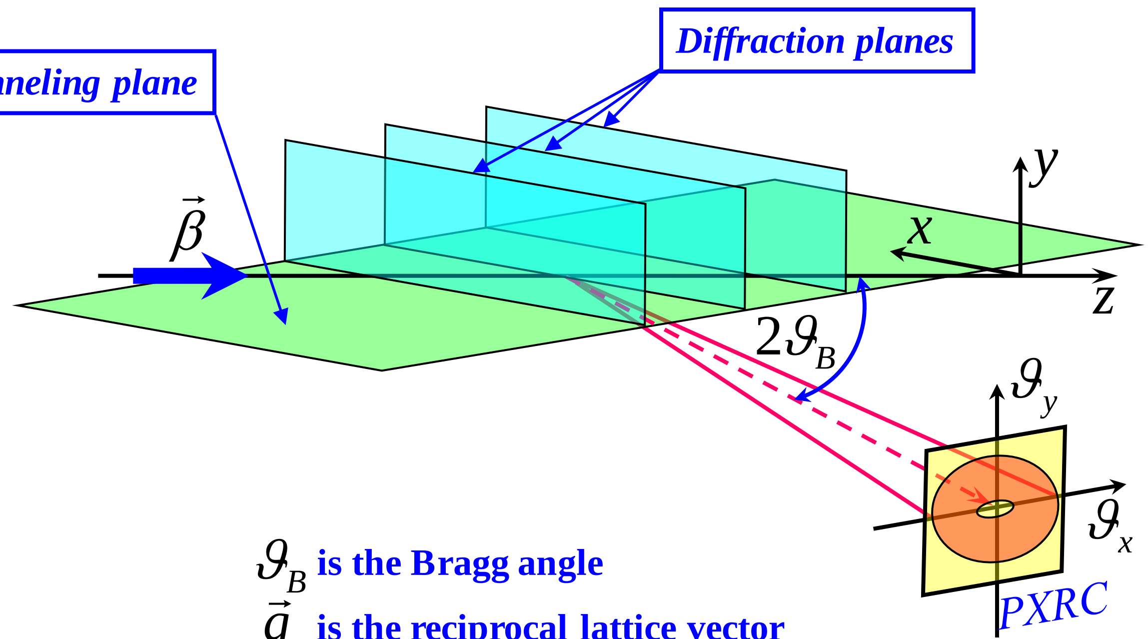
- Interband transitions (i=f) correspond to “DCR” = Diffracted Channeling Radiation
- Intraband transitions (i = f) correspond to “PXRC” = Parametric X-radiation at channeling
- Nitta suggested the form factors of channeled states to be equal approximately to 1
- In this case, angular distribution of PXRC does not differ from that of PXR
- Experiment: SAGA-LS (JETP Letters, 2012) → a difference exists !
- Motivation to re-calculate PXRC angular distribution → a subject of theoretical part of this presentation

PXRC at planar channeling

The PXRC appears when an electron passes through a crystal in the channeling regime. The channeling means that the electron is in a bound state with the crystal plane (axis)

The scheme of observing the angular distribution of PXRC





ϑ_B **is the Bragg angle**

$\vec{g}$ **is the reciprocal lattice vector**

$\vec{v} = c\vec{\beta}$ **is the electron velocity**

PXRC: first experiment

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Quantum Effects for Parametric X-ray Radiation during Channeling: Theory and First Experimental Observation¶

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The theory of X-ray radiation from relativistic channeled electrons at the Bragg angles—parametric X-ray radiation (PXR) during channeling (PXRC)—is developed while accounting for two quantum effects: the initial population of bound states of transverse motion and the transverse "form-factor" of channeled electrons. An experiment was conducted using a 255 MeV electron beam from a linac at the SAGA Light Source. We have identified a difference in the angular distributions of PXR and PXRC and obtained a fairly good agreement between the theoretical and experimental results.

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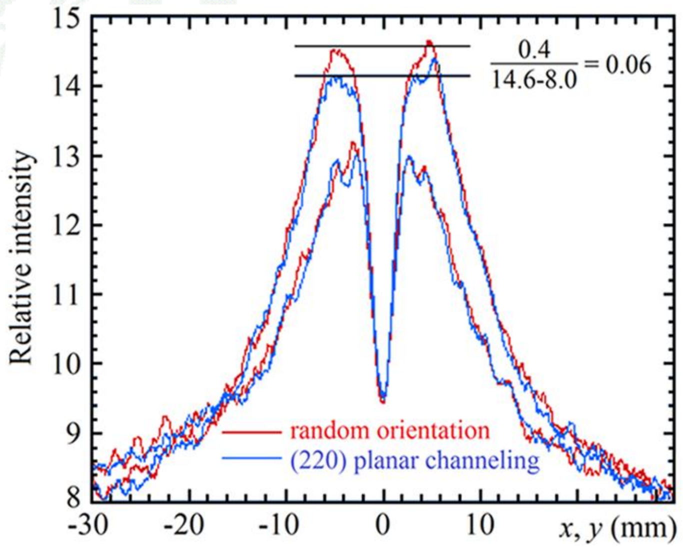
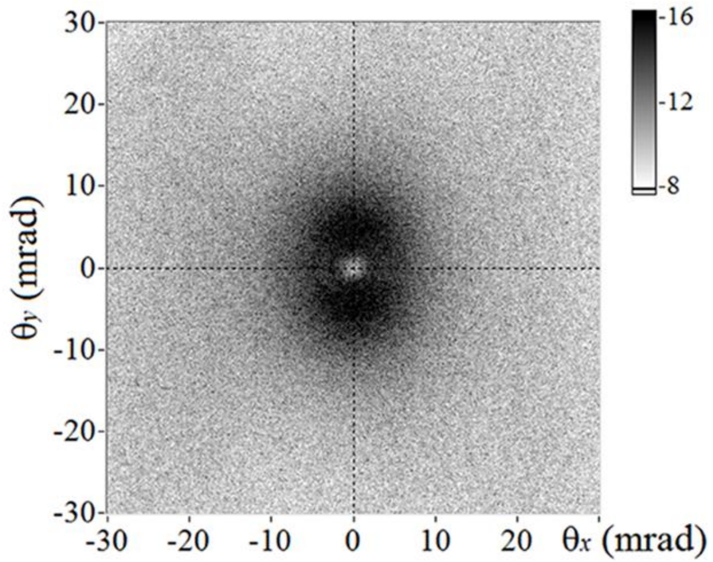
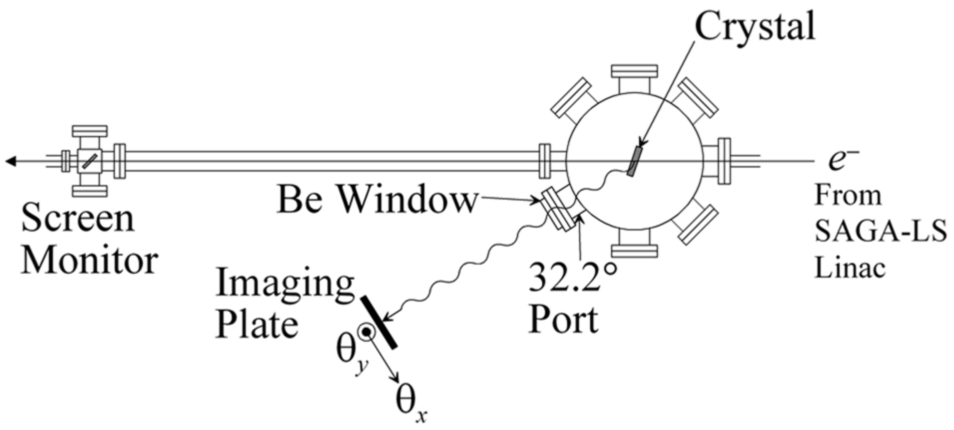
PXRC: first experiment







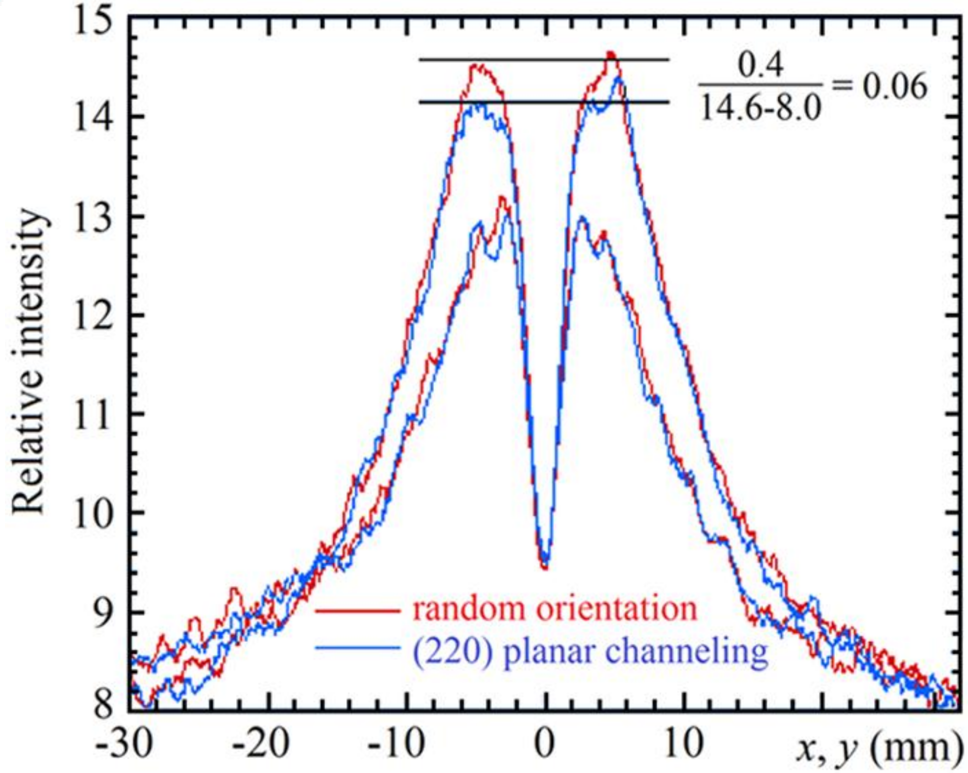
PXRC: experimental details

JETP Letters 2012

- Dechanneling in a 20 micrometers Si: H.Backe (Channeling 2010) – negligible ?
- Initial angular spread of 255 electron beam at SAGA-LS: $\theta_o = \theta_L/3 \rightarrow$ could be better
- Imaging plate: all reflection orders are detected ? Improvements ?
- Bragg direction: could be even better ?
- Axial channeling ?
- Another crystal (e.g. diamond) ?

PXRC: experimental results

Angular distribution of the PXRC and PXR from electron beam
with energy 255 MeV at (220) Si channeling (section along $\theta_x = 0$ and $\theta_y = 0$)



$\hbar\omega_B$ = 7.133 keV

θ_B = 16.1°

PXRC from planar channeled electrons (theory)

Angular distribution of PXRC from channeled electron being in a quantum state “n” of transverse motion

$$dN^{n}_{\mathrm{PXRC}} = \frac{d^3 N_{nn}}{d\theta_x d\theta_y dz} = dN_{\mathrm{PXR}} \, | F_{nn} |^2$$

$$dN_{\mathrm{PXR}} = \frac{\alpha\omega_B}{16\pi c \sin^2\theta_B}\left[\frac{\theta_x^2}{1+W_\pi^2} + \frac{\theta_y^2}{1+W_\sigma^2}\right]$$

$$| F_{nn} |^2 = \left| \int_{-d/2}^{d/2} \phi_n^*(y) \exp(-i\omega_B \theta_y y/c) \phi_n(y) dy \right|^2$$

$$W_\tau = \frac{1}{2|\chi_g| P_\tau}\left(R - \frac{|\chi_g|^2 P_\tau^2}{R}\right), \quad \tau = (\pi, \sigma),$$

$$R = \left[\theta_x - \frac{\Omega_{if}}{\omega_B}\cos\theta_B\right]^2 + \theta_y^2 + R_\circ,$$

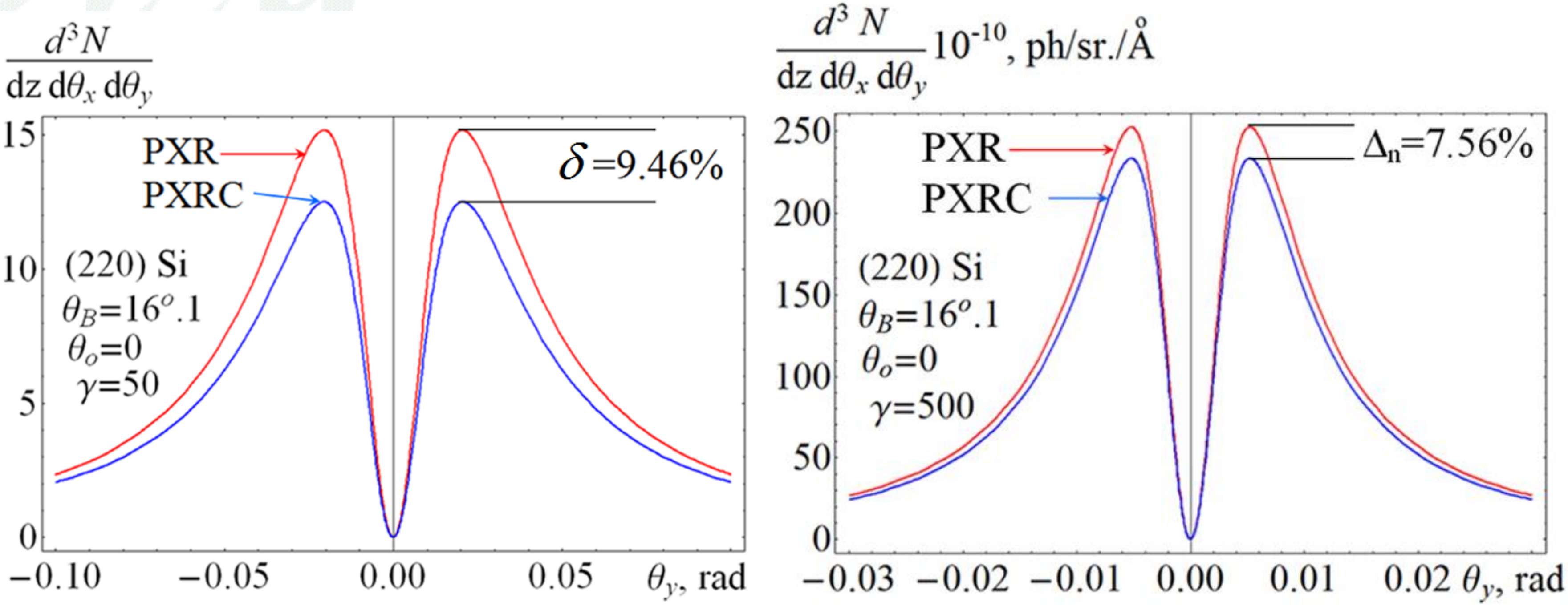
$$R_\circ = \theta_{kin}^2 - 2\frac{\Omega_{if}}{\omega_B}, \quad \theta_{kin}^2 = \gamma^{-2} + |\chi_0|,$$

$\phi_n(y)$ is the wave function for planar channeled electrons and d is the distance between the channeling planes, $\alpha = e^2/c\hbar$

Key question: what we are looking for comparing PXR and PXRC ?
Answer: recent experiment at SAGA-LS

Calculated angular distributions of PXRC and PXR

(sections along $\theta_x = 0$)



Electron beam energy: 25 MeV and 255 MeV

Planar channeling in (220) Si

Theory: relative difference of PXR and PXRC

The PXRC angular distribution = sum over populated quantum states

$$\frac{d^3N_{\mathrm{PXRC}}}{d\theta_x d\theta_y dz} = dN_{\mathrm{PXR}} \sum_n P_n(\theta_\circ)\,|F_{nn}|^2$$

Initial population of the n-th energy level

$$P_n(\theta_\circ) = \frac{1}{d}\left|\int_{-d/2}^{d/2} \exp(ik_y\theta_\circ y)\phi_n(y)dy\right|^2$$

Relative difference in PXR and PXRC

$$\delta = \frac{dN_{\mathrm{PXR}} - \sum_n dN^n_{\mathrm{PXRC}}}{dN_{\mathrm{PXR}}} = 1 - \sum_n P_n(\theta_\circ)\,|F_{nn}|^2$$

Calculation of wave functions, populations & form-factors

- Single potential well: Poeschl-Teller $U(y) = -U_\circ \cosh^{-2} \lambda y.$
- Levels number $E_{n\perp} = -U_\circ (1+n-s)^2/s(s-1), \quad n < s-1$
- Levels parity
- Levels population

$$\phi_e(y) = \cosh^s \lambda y \, {}_2F_1\left(\frac{1}{2}+n, -\frac{1}{2}-n+s; \frac{1}{2}; -\sinh^2 \lambda y\right),$$

$$\phi_o(y) = \cosh^s \lambda y \sinh \lambda y \, {}_2F_1\left(-\frac{1}{2}-n+s, \frac{3}{2}+n; \frac{3}{2}; -\sinh^2 \lambda y\right).$$

$$U_\circ = \frac{\lambda^2 \hbar^2}{2\gamma m} s(s-1), \quad E_{n\perp} = -\frac{\lambda^2 \hbar^2}{2\gamma m} \kappa^2, \quad s = \left(1+\sqrt{8\gamma m U_\circ / \hbar^2 \lambda^2}\right)/2,$$

Calculation of wave functions, populations & form-factors

- Single potential well: Poeschl-Teller
- Level number n=1 (ground state)
- γ - dependence of formfactor F_{nn}
- γ - dependence of level population $P_n(\theta_0)$
- Beyond single potential well: periodic (100) Si channeling potential → band structure of transverse energy levels → more complicated calculations (e.g., in : BKP JETP Letters 2007)

Summary: form-factors & populations change significantly.

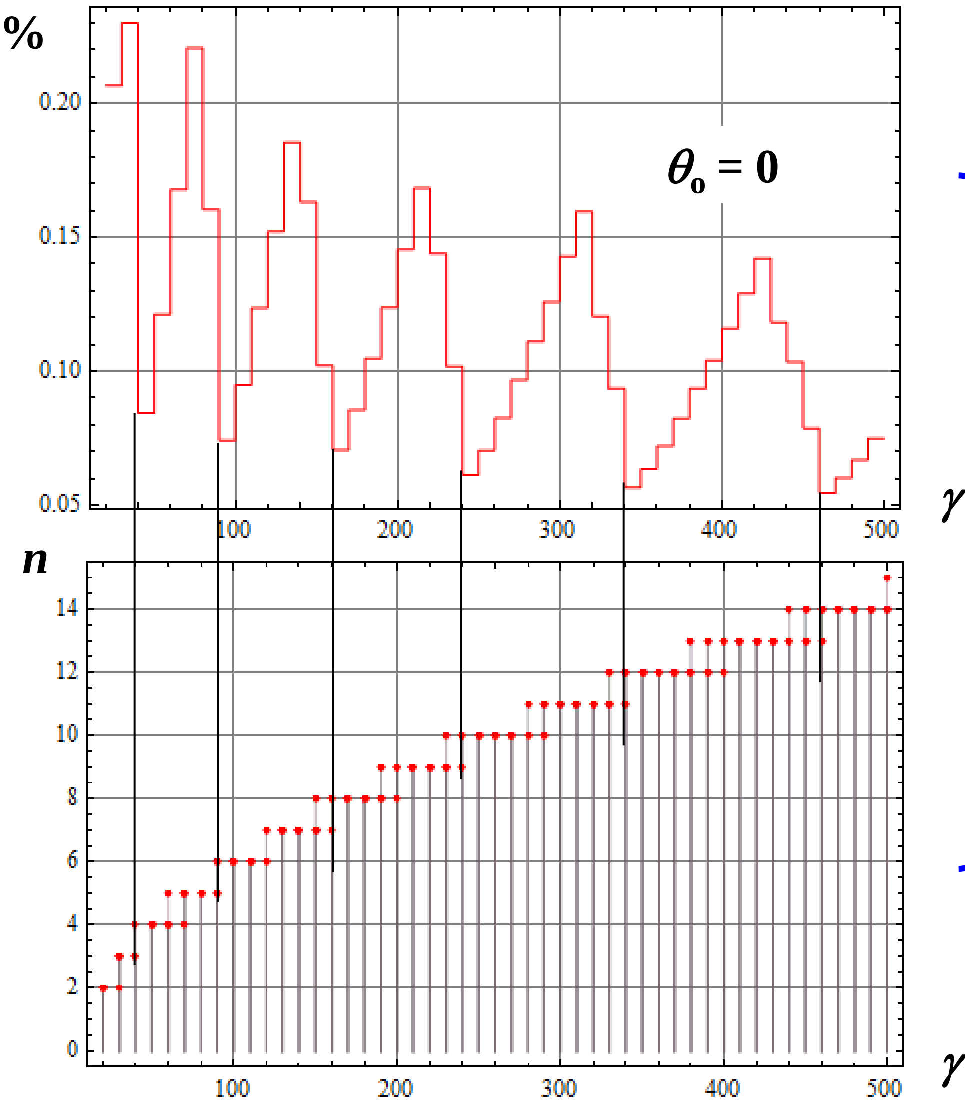
PXRC: Key parameters

- Relativistic factor γ = number of quantum states (bands)
- Relativistic factor γ = form-factors of quantum states (bands)
- Relativistic factor γ = population of quantum states (bands)
- Angle of incidence θ_0 = population of quantum states
- Emission angle (θ_y enters into form-factor)
- Reflection order → PXRC photon energy (enters form-factor)

What is the best conditions for manifestation of "asymmetry" ? Answer: let study the δ - dependence on γ and θ_0

γ - dependence of relative difference δ

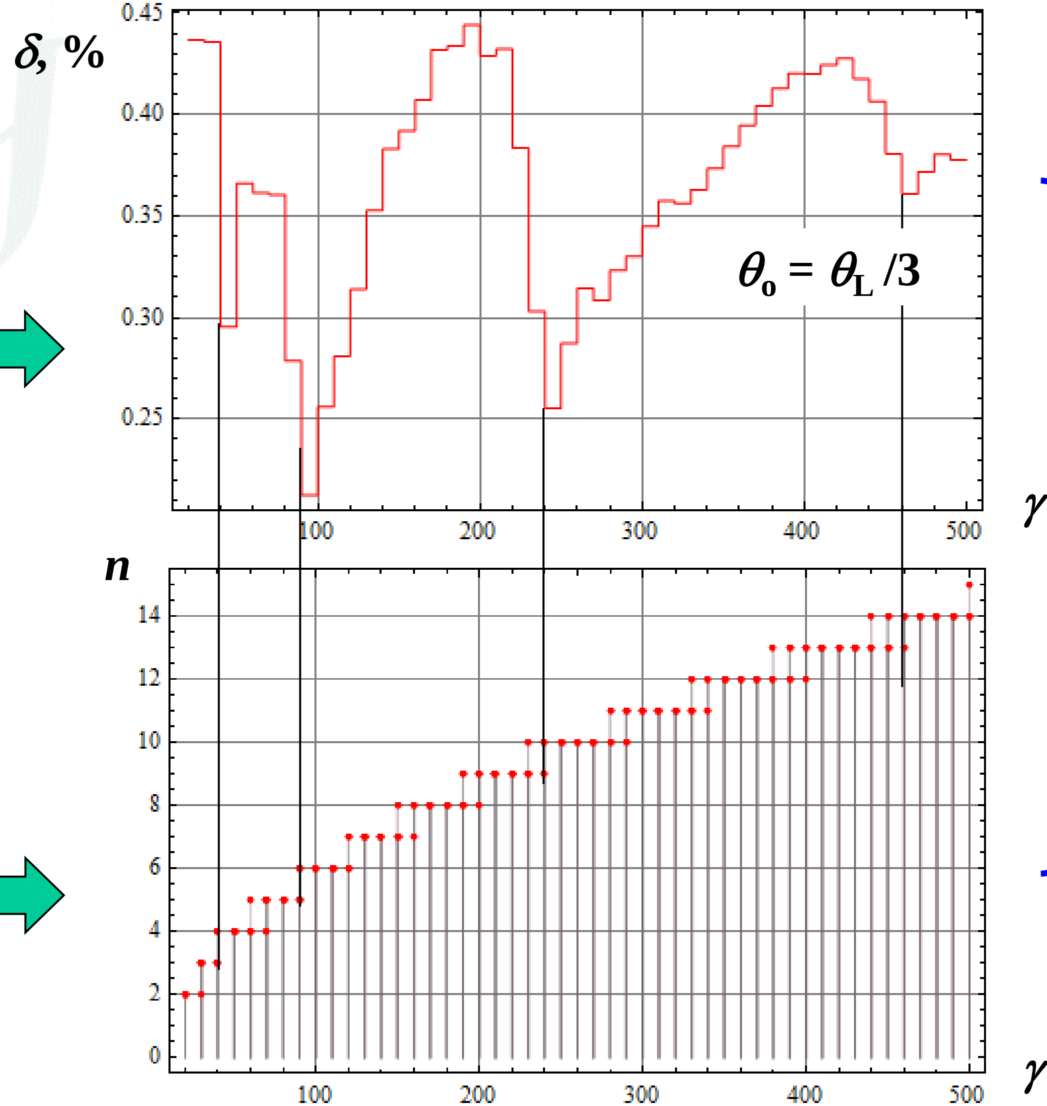
γ - dependence of number of channeled states



Correlation !

γ - dependence of relative difference δ

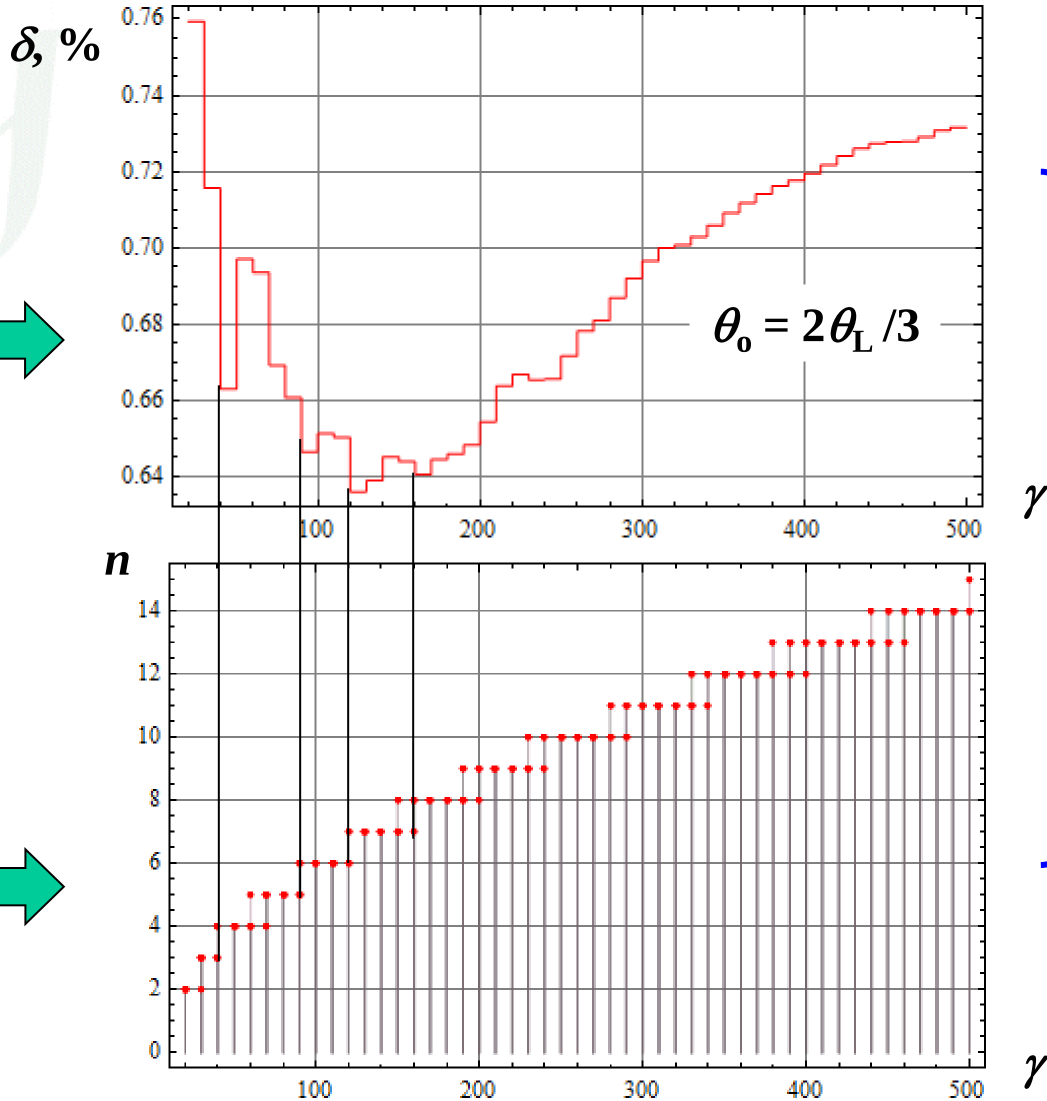
γ - dependence of number of channeled states



Correlation !

γ - dependence of relative difference δ

γ - dependence of number of channeled states



Correlation !

Conclusions

- Recent experimental results (SAGA-LS Linac, JETP Letters 2012) on angular distributions of PXRC from channeling electrons with energy 255 MeV show deviations from ordinary PXR angular distribution
- Observed "asymmetry" is a quantum effect connected with "transverse" form-factors of channeled electrons
- Theory: strong correlation between number of quantum states in a potential well of crystallographic plane and maximums in asymmetry parameter delta
- Experiment was performed (unfortunately) at a gamma value when δ = minimal
- Further experiments at SAGA-LS using thinner (dechanneling !) crystal and changing electron beam energy (exploring gamma-dependence) will bring new information

Thank you for attention !