Parametric gamma-radiation: parametric x-rays from relativistic electrons passing through a Mossbauer crystal

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MOTIVATION

Mössbauer spectroscopy is a powerful and well established method employed for conducting research in various fields such as physical, chemical, biological, earth, and fundamental physical sciences.

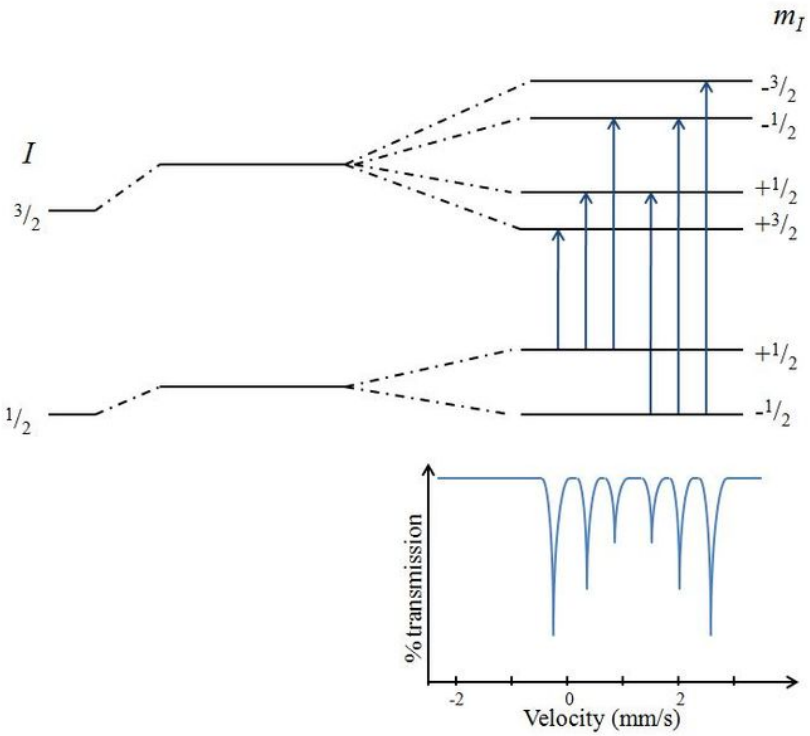
Mössbauer spectroscopy provides element-specific information on surrounding electronic states and magnetism, which is required in modern materials science and in complex systems such as biological substances.

Mössbauer spectroscopy probes tiny changes in the energy levels of an atomic nucleus in response to its environment. Typically, three types of nuclear interaction may be observed: an isomer (chemical) shift; quadrupole splitting; and magnetic or hyperfine splitting (Zeeman effect).

Due to the high energy and extremely narrow line widths of gamma rays, Mössbauer spectroscopy is one of the most sensitive techniques in terms of energy (and hence frequency) resolution, capable of detecting change in just a **few parts per 10^{11}**.

Mössbauer effect

In 1957, Rudolf Mössbauer discovered that, in some circumstances, if the nucleus is bound in a crystal lattice, the whole crystal recoils rather than the individual nucleus. Due to the much greater mass involved in recoil the energy of the emitted ray is very close to that of the difference in energy between the nuclear energy levels, and resonant absorption is possible.



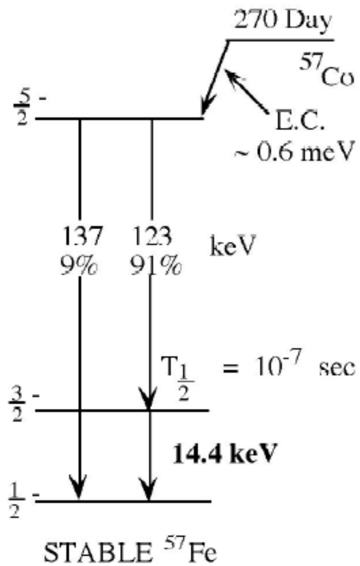
Magnetic splitting of the nuclear energy levels and the corresponding Mössbauer spectrum

Mössbauer spectroscopy is limited by the need for a suitable gamma-ray source. And samples must be solid (frozen)

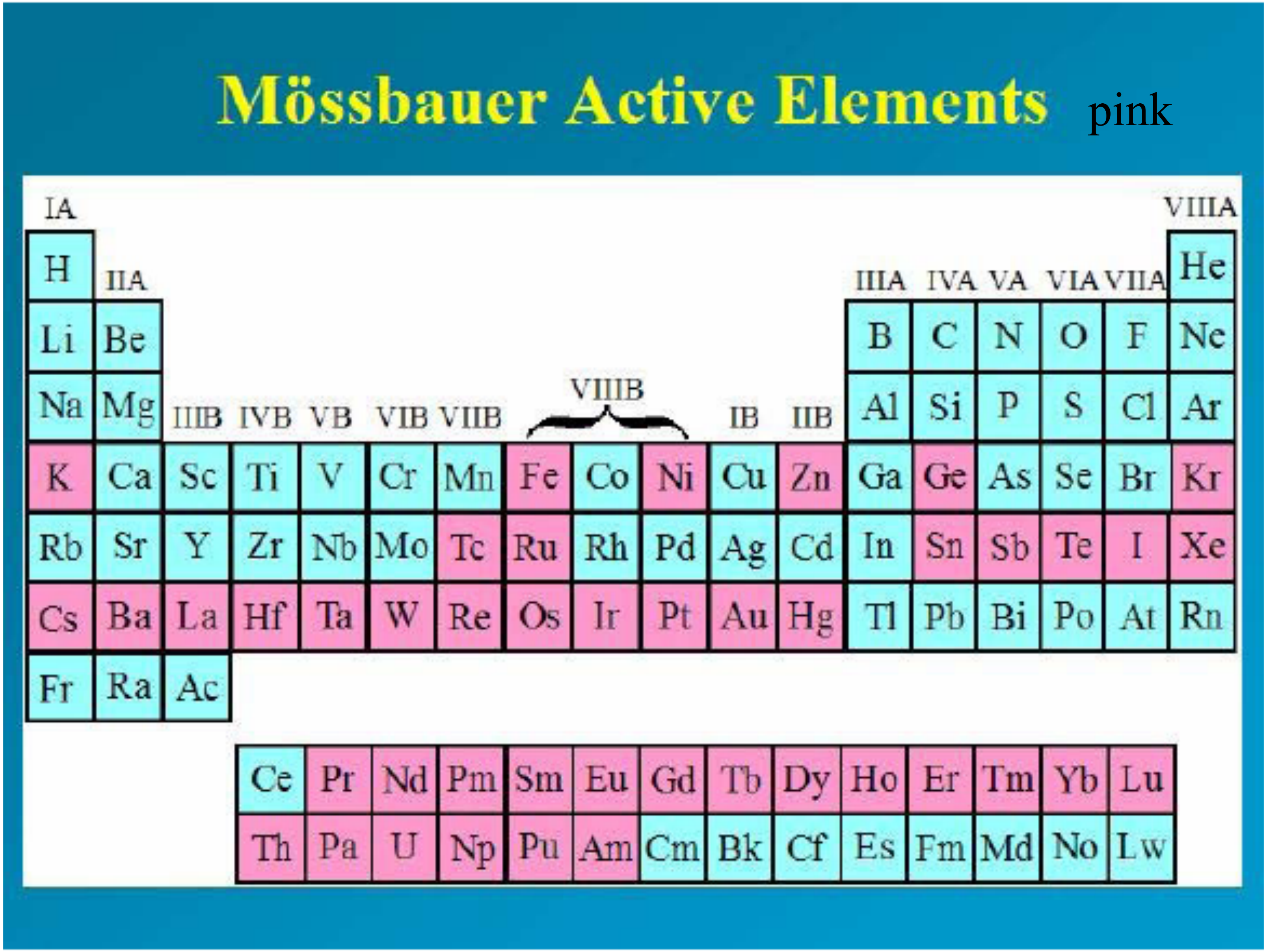
PROPERTIES OF ^{57}Fe $^{\alpha}$

	Ground state	First excited state
Energy (keV)	0	14.36
Spin and parity	$\frac{1}{2}$ -	$\frac{3}{2}$ -
Magnetic moment (nm)	0.0903	-0.153
Quadrupole moment (barns)	0	0.29
Mean life (sec)	Stable	1.4 x 10 ^{7}

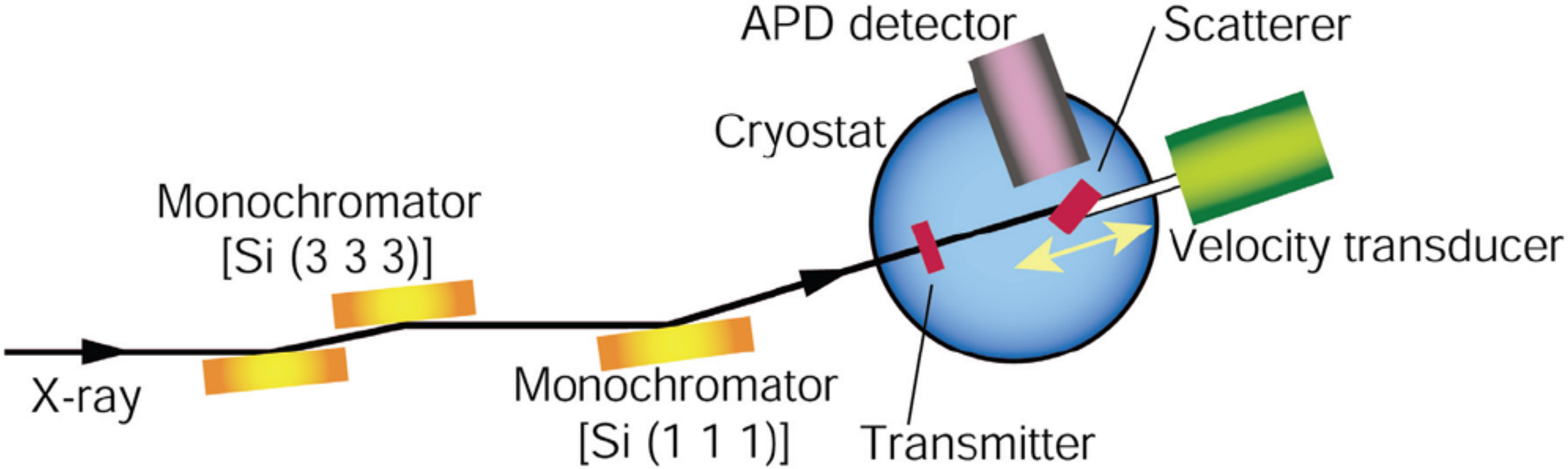
α Internal conversion coefficient: 9.7 ± 0.2.



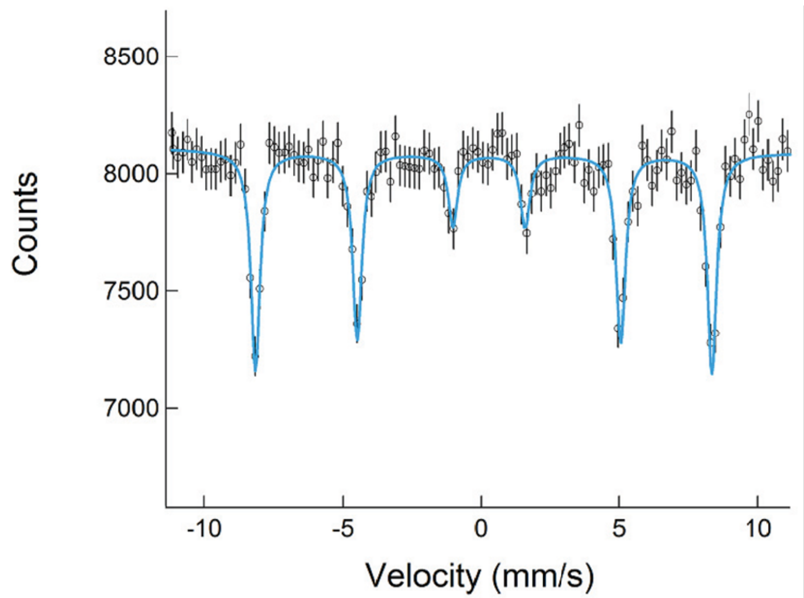
Mössbauer effect



Synchrotron-Radiation-Based Mossbauer Spectroscopy



The applicability
of SR-based Mossbauer
spectroscopy increasing.



M. Seto at al. PRL 102, 217602 (2009)

Synchrotron-Radiation-Based Mossbauer Spectroscopy

TABLE I. Comparison between SR-based Mössbauer spectroscopy and Mössbauer spectroscopy using radioactive sources.

	Synchrotron radiation [proposed energy domain method (E) and time domain NFS method (T)]	Radioactive sources
Surface, interfaces, multilayers	Clean, unclean, and nanostructured samples (E); limited to clean surfaces (T)	Clean, unclean, and nanostructured samples
Multiple extreme conditions [a], imaging	Relatively easy, sub-μm resolution is possible (E,T)	Usually not easy, limited to μm resolution
Applicable isotopes	Short- and extremely long- lived and high-energy isomers are possible (E); efficient for low-energy and sufficiently long-lived isomers (T)	Isotopes for which appropriate sources are available

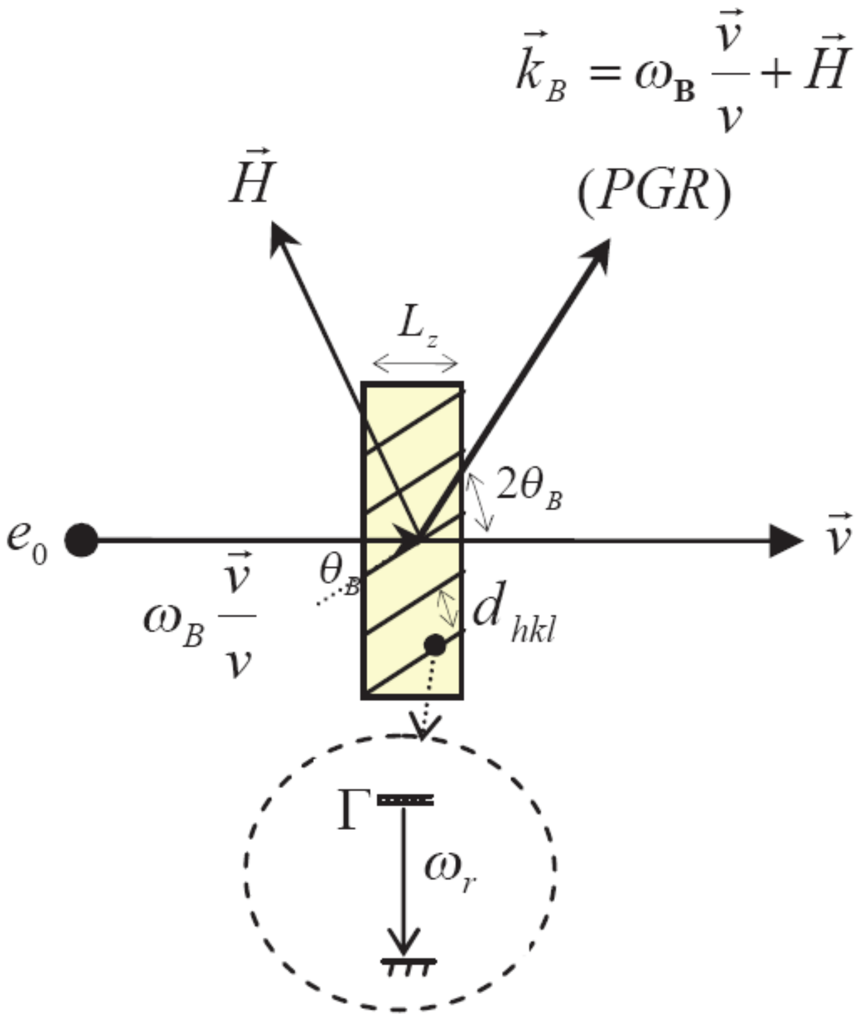
[a]High pressure (~200 GPa), applied magnetic field (~10 T), high (~1000 K) or low (~ mK) temperature.

The spectral density of PXR in the peaks exceeds the analogous value for the SR to one electron. It is also important that the PXR frequencies are defined solely by the crystal parameters and do not depend on the electron energy. It allows generate hard X-rays from electrons with essentially less energy than in the case of SR.

CONTENTS

1. Characteristics of the parametric γ-radiation (PGR) from relativistic electrons in Mössbauer crystals (as source of gamma-radiation for the Mössbauer experiments) are analyzed in the framework of the kinematical diffraction theory taking into account the interference between electron and nuclear resonant parts of the crystal X-ray polarizability.
2. Optimization of the crystal parameters, electron energy and the diffraction geometry is carried out in order to increase brightness of the resonant γ-radiation source based on PGR.
3. Possible experimental setup for observation of RGR is discussed

FORMATION OF PGR



Main parameters and vectors for formation of PGR

$$\omega_B = \frac{|\vec{H}|}{2\sin\theta_B} \approx \omega_r;$$

$$\sin\theta_r \approx \sin\theta_B = \frac{|\vec{H}|}{2\omega_r}$$

$$\frac{\Delta\omega}{\omega_r} \approx \frac{1}{\omega_r L_z}$$

PGR SPECTRAL DENSITY

$$\frac{\partial N_r}{\partial \omega} \approx \frac{e_0^2 \cos 2\theta_r}{4 \sin 2\theta_r} \gamma L_z |\chi(\mathbf{H}, \omega)|^2;$$

$$\chi(\mathbf{H}, \omega) = \chi_e(\mathbf{H}, \omega_r) + \chi_n(\omega),$$

$$\gamma = \frac{E}{m};$$

Electronic part of the polarisability

$$\chi_e(\mathbf{H}, \omega_r) = \frac{4\pi S(\mathbf{H})}{\Omega_0 \omega_r^2} f_e(\mathbf{H}), \quad f_e(\mathbf{H}) = -\frac{e_0^2}{m} F_a(\mathbf{H})$$

Nuclear part of the polarisability

$$\chi_n(\omega) = \frac{4\pi S(\mathbf{H})}{\Omega_0 \omega_r^2} \eta f_n(\omega),$$

$$f_n(\omega) = -\frac{1}{\omega_r(1+\alpha_c)} \frac{\Gamma/2}{\omega - \omega_r + i\Gamma/2}.$$

OPTIMIZATION OF PGR PARAMETERS

PGR is most effective if the following conditions are fulfilled

$$\left|\frac{f_n}{f_e}\right| \approx \frac{\eta m}{\omega_r e_0^2 Z_a} > 1$$

$$\frac{\partial N_r}{\partial \omega} \propto \frac{1}{\theta_B}$$

$$|\mathbf{H}| \ll \omega_r; \quad d_{hkl} \gg \lambda_r$$

PARAMETERS OF SOME CRYSTALS

$$^{183}_{74}W \qquad (110)$$

$$\omega_r = 46.48\ keV;\ \ \Gamma = 4.07\ 10^{-10}\ keV; \alpha_c = 8.2.$$

$$|\chi_e(\mathbf{H}, \omega_r)| \approx 2.40\ 10^{-6}; \theta_{B0} \approx 3.42^o;\ \ F(\mathbf{H}) \approx 68.$$

$$^{57}_{26}Fe \qquad (110)$$

$$\omega_r = 14.41\ keV;\ \ \Gamma = 4.66\ 10^{-12}\ keV; \alpha_c = 9$$

$$|\chi_e(\mathbf{H}, \omega_r)| \approx 1.03\ 10^{-5}; \theta_{B0} \approx 12.26^o;\ \ F(\mathbf{H}) \approx 23;$$

CALCULATION OF THE PGR SOURCE

$$\frac{\partial^3 N_{hkl}}{\partial \vartheta_x \partial \vartheta_y \partial \omega} = \frac{e_0^2}{4\pi^2} \frac{\vartheta_x^2 \cos^2 2\theta_B + \vartheta_y^2}{(\vartheta_x^2 + \vartheta_y^2 + \gamma^{-2})^2} \times$$

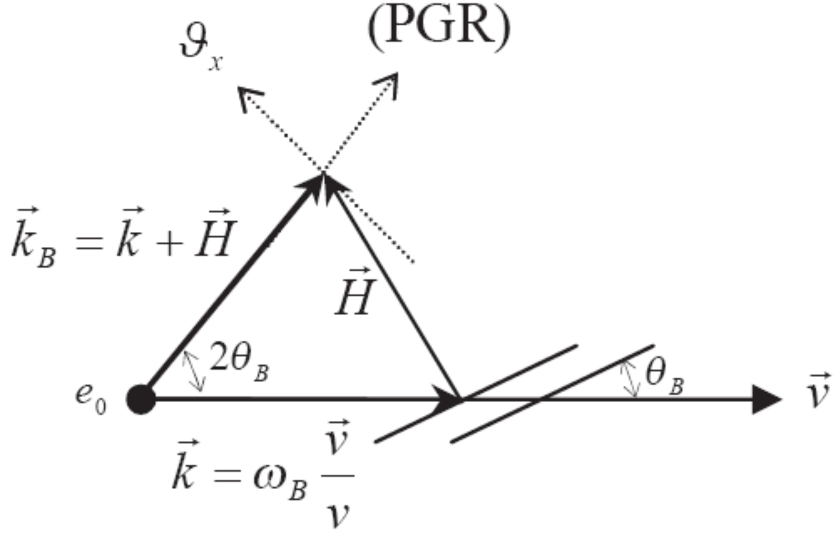
Formula is applicable if the crystal length is less than the extinction length

$$\times |\chi(\mathbf{H}, \omega)|^2 \frac{\sin^2 \nu \omega_B L_z}{\omega_B \nu^2};$$

$$\nu = \frac{1}{2}[\frac{2\sin^2\theta_B}{\cos 2\theta_B} \frac{(\omega - \omega_B)}{\omega_B} - \vartheta_x \tan 2\theta_B].$$

$$\vartheta_x \approx \vartheta_{xr} = \tan\theta_B \frac{(\omega_r - \omega_B)}{\omega_B}$$

$$\Delta\vartheta_x \approx \frac{1}{\omega_B L_z} \approx |\chi(\mathbf{H}, \omega)| \sim 10^{-6}$$



Formula for the analysis is based on the kinematic approximation for the parametric radiation from a relativistic particle

MAXIMUM OF THE SPECTRAL DENSITY

$$\Phi'(u_0) = 0; u_0 = \pm\sqrt{\frac{2\cos^2 2\theta_B - 1}{(1+\cos^2 2\theta_B)}};$$

$$\Phi(u_0) \approx \frac{4\sqrt{2}}{3\sqrt{3}};\ \gamma \tan\theta_B \frac{(\omega_r - \omega_B)}{\omega_B} = u_0 \approx \pm\frac{1}{\sqrt{2}};$$

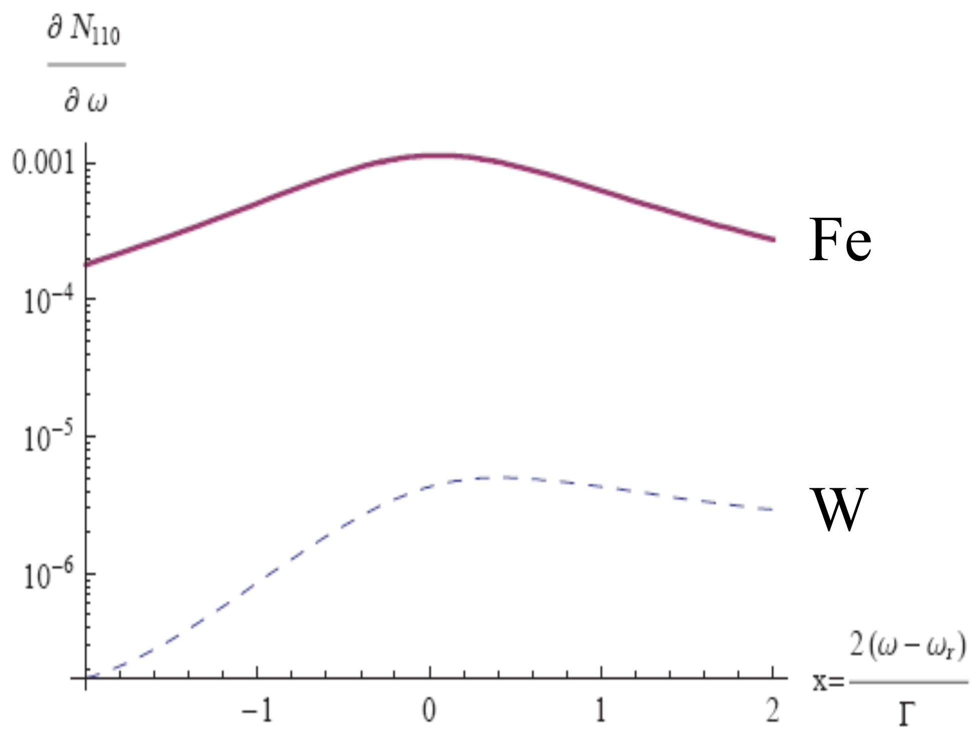
$$\theta_{B0} \approx \frac{|\mathbf{H}|}{4\omega_r}[1 + \sqrt{1 \pm \frac{4\omega_r\sqrt{2}}{\gamma|\mathbf{H}|}}]$$

$$\frac{\partial N_{hkl}}{\partial\omega} = \frac{e_0^2\sqrt{2}}{3\sqrt{3}\omega_r \tan 2\theta_{B0}}\gamma|\chi_e(\mathbf{H},\omega_r)|\Psi(\omega);$$

$$\Psi(\omega) = |1 + \frac{A}{x+i}|^2;$$

$$A = \eta\frac{m}{e_0^2 F(\mathbf{H})\omega_r(1+\alpha_c)};\ x = \frac{2(\omega - \omega_r)}{\Gamma}.$$

CHARACTERISTICS OF PGR FROM ONE ELECTRON



$$N_{hkl} = \frac{e_0^2\sqrt{2}}{3\sqrt{3}\tan 2\theta_{B0}}\gamma|\chi_e(\mathbf{H},\omega_r)|\frac{\Gamma}{\omega_r}(1+\frac{\pi}{4}A^2),$$

Integral number of quanta emitted by one electron in the spectral interval $\Delta\omega = \Gamma$

COMPARISON OF BRIGHTNESS FOR PGR AND SYNCHROTRON SOURCES

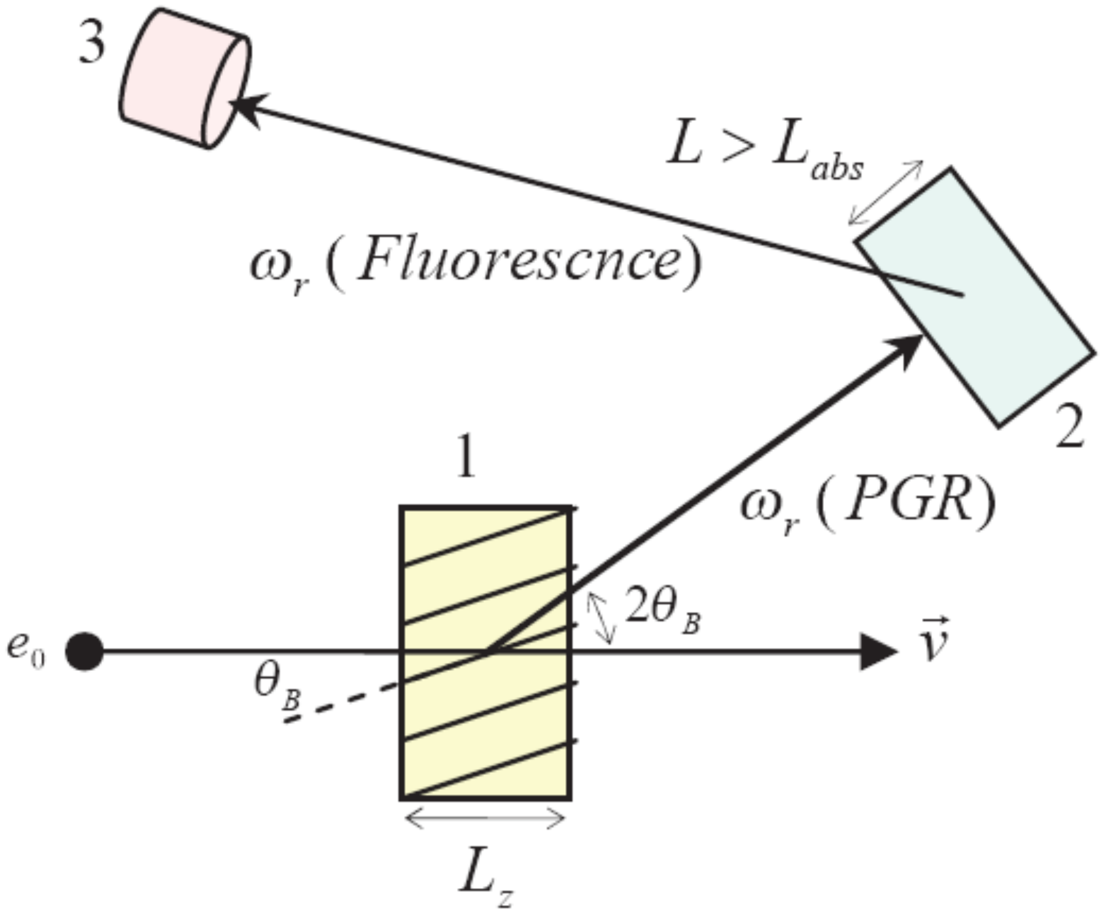
$$\Upsilon_{SR}\ [\mathrm{photons}\cdot \mathrm{s}^{-1}\cdot \mathrm{mrad}^{-2}\cdot (0.1\%\mathrm{BW})^{-1}] \approx$$

$$1.327\times 10^{13}\ E^2\ [\mathrm{GeV}]\ J\ [\mathrm{A}],$$

$$\Upsilon_{PGR} \approx 3\ 10^{16} E^2 [GeV] J[A],\ for\, {}^{57}_{26}Fe\ and$$

$$\Upsilon_{PGR} \approx 1.9\ 10^{15} E^2 [GeV] J[A],\ for\, {}^{183}_{74}W.$$

EXPERIMENTAL SETUP FOR PGR OBSERVATION



Sketch of the experimental setup for the observation of PGR:
1) Mossbauer crystal-radiator; 2) Mossbauer absorber;
3) detector-counter of the fluorescence photons from the absorber

COUNT RATE OF THE PGR SOURCE

$$\dot{N}_\gamma[s^{-1}] \approx N_{hkl}\frac{J}{e_0}\frac{\Delta\Omega}{4\pi} = 0.63\ 10^{19} N_{hkl} J[A]\frac{\Delta\Omega}{4\pi}$$

$$\dot{N}_\gamma \approx 2.6\ 10^4 J\frac{\Delta\Omega}{4\pi} \text{ for } {}^{57}_{26}Fe$$

$$\dot{N}_\gamma \approx 1.02\ 10^4 J\frac{\Delta\Omega}{4\pi} \text{ for } {}^{183}_{74}W.$$

Effect can be observable!

Thank you for attention