Channeling-2012

High-energy wave packets in processes of bremsstrahlung, coherent and transition radiation

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- Spreading of high energy wave packets
- Coherent and transition radiation
- Ionization energy losses by half-bare electron
- LPM-effect and TSF-effect

HIGH-ENERGY WAVE PACKETS IN WKB APPROXIMATION

$$\left[\left(p_\mu - eA_\mu\right)^2 - m^2\right]\phi = 0$$

$$\phi^{WKB}(\vec{r},t) = \left[\frac{1}{\varepsilon - eA_0}\int d^3 r_0 \delta(\vec{r} - \vec{r}(t,\vec{r}_0,\vec{p}))\right]^{1/2} e^{\frac{i}{\hbar}S(\vec{r},\vec{p},t)} =$$

$$= \sqrt{\left|\frac{\partial^2 S}{\partial\vec{r}\partial\vec{p}}\right|}\; e^{\frac{i}{\hbar}S}$$

$$\left|\frac{\partial^2 S}{\partial\vec{r}\partial\vec{p}}\right|$$ – Van Vleck determinant

W.H. Miller. Adv. Cham. Phys. 25 (1974) 69.

A.I. Akhiezer, N.F. Shul’ga. *High-Energy Electrodynamics in Matter.* Gordon and Breach Pub., Amsterdam, 1996.

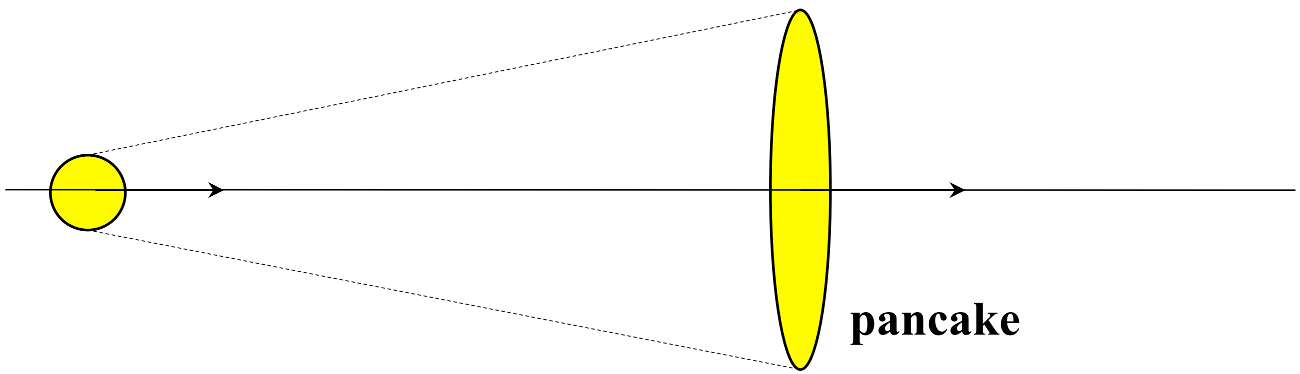
SPREADING OF RELATIVISTIC WAVE PACKETS

(D.I. Blokhintsev, 1967)

$$\left(\partial_t^2 - \nabla^2 - m^2\right)\varphi(\vec{r},t) = 0$$

$$\phi(\vec{r},t) = e^{i(\vec{p}\vec{r}-\varepsilon t)} A(t)\exp\left\{ i\alpha(\vec{r},t) - \frac{(z-t)^2}{2\Delta_{\parallel}^2} - \frac{\rho^2}{2\Delta_{\perp}^2} \right\}$$

$$\Delta_{\parallel}^2(t) = a_{\parallel}^2 + \left(t / a_{\parallel}\varepsilon\gamma^2\right)^2 \qquad \Delta_{\perp}^2(t) = a_{\perp}^2 + \left(t / a_{\perp}\varepsilon\right)^2$$

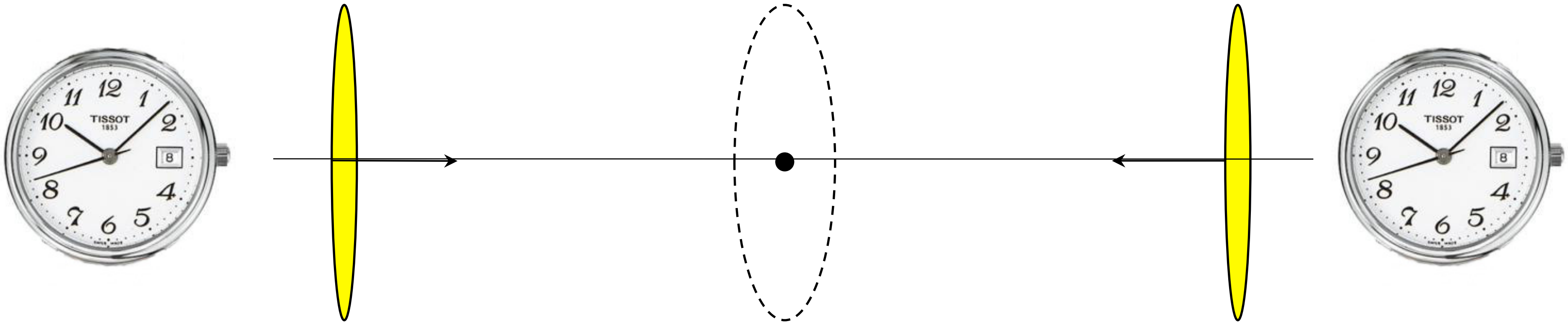


For $\varepsilon >> m$

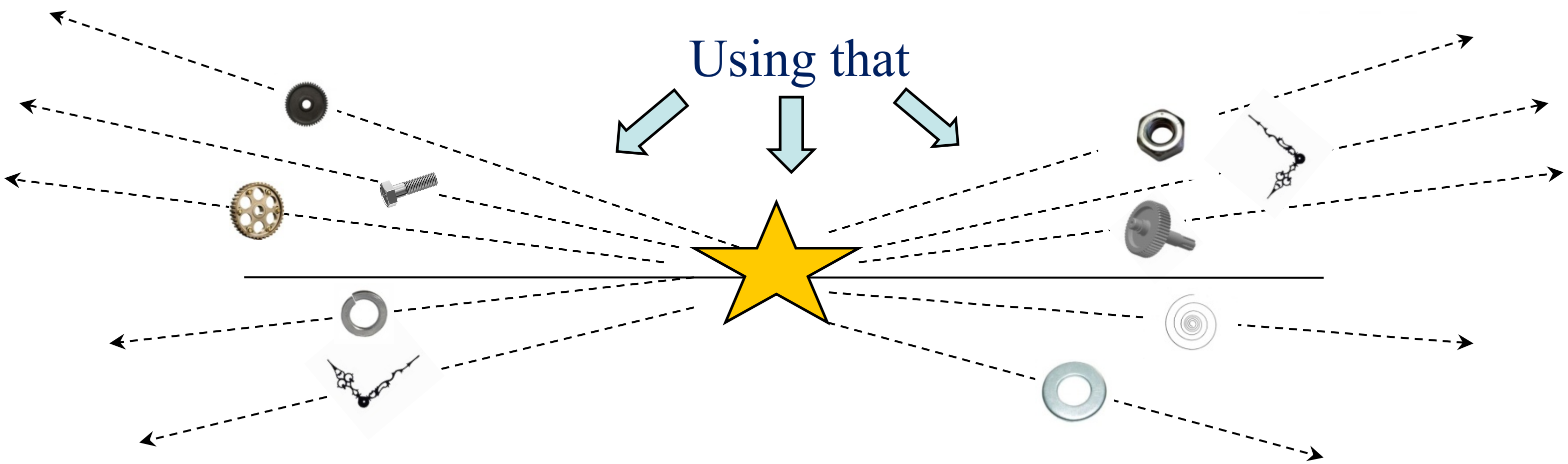
1. Strong stabilizing effect
2. Dispersion mostly in transverse direction

R. Feynman: What are high-energy particle physicists doing (in CERN)?

Let us consider the collision of two watches:



Using that



they want to understand how the watches work

SPREADING OF HIGH-ENERGY ELECTROMAGNETIC PACKETS

N.F. Shul'ga, S.V. Trofymenko, *in the book* *"Electromagnetic Waves"*, InTech, 2012

$$\phi(\vec{r},t) = e^{i(\vec{k}\vec{r}-\omega t)} A(t) \exp\left\{ i\alpha(\vec{r},t) - \frac{(z-t)^2}{2\Delta_{\parallel}^2} - \frac{\rho^2}{2\Delta_{\perp}^2} \right\}$$

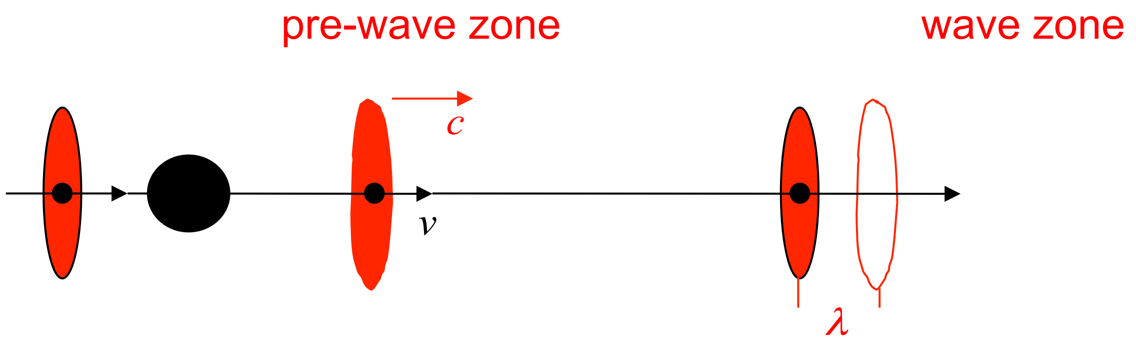
$$\Delta_{\parallel}^2(t) = a_{\parallel}^2 \qquad \Delta_{\perp}^2(t) = a_{\perp}^2 + \left(t / a_{\perp}\omega\right)^2$$

$$A(t)\exp\{i\alpha(\vec{r},t)\} \rightarrow \frac{1}{r} e^{i\omega r} \qquad \text{for } t \approx z \rightarrow \infty$$

1. The equivalent photon method
2. Bremsstrahlung, coherent and transition radiation etc.
3. Ionization energy losses

ULTRATRAHIGH FORMATION (COHERENT) LENGTHS

The excitation is small



$v_{rel} = c - v$

$$l_{coh} = 2\gamma^2\lambda >> \lambda$$

$\lambda = (c - v)\Delta t_c \rightarrow \Delta t_c \approx 2\gamma^2\lambda$

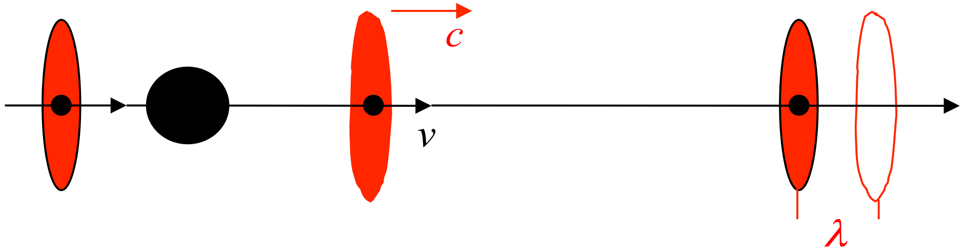
$E \sim 100\, GeV$ $\quad\omega \sim 500\, MeV$ $\quad l_c \sim 10^{-3}\, cm$

$E \sim 50\, MeV$ $\quad\lambda \sim 0.1 cm$ $\quad l_c \sim 20\, m$

EFFECTIVE CONSTANT OF INTERACTION FOR LARGE COHERENCE LENGTH

The excitation is small

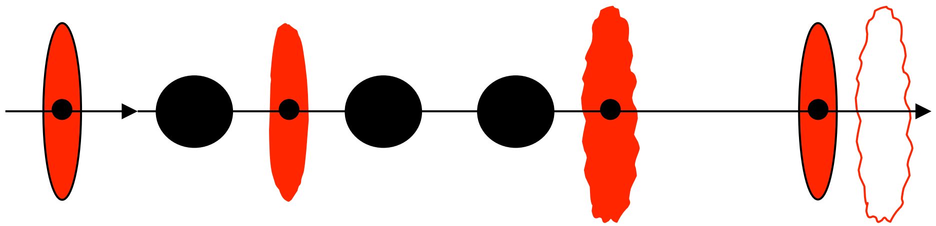
$\gamma\theta_{scat} << 1$



$$\alpha_{eff} = \frac{Ze^2}{hc}$$

The excitation is increased

$\gamma\theta_{scat} \approx 1$

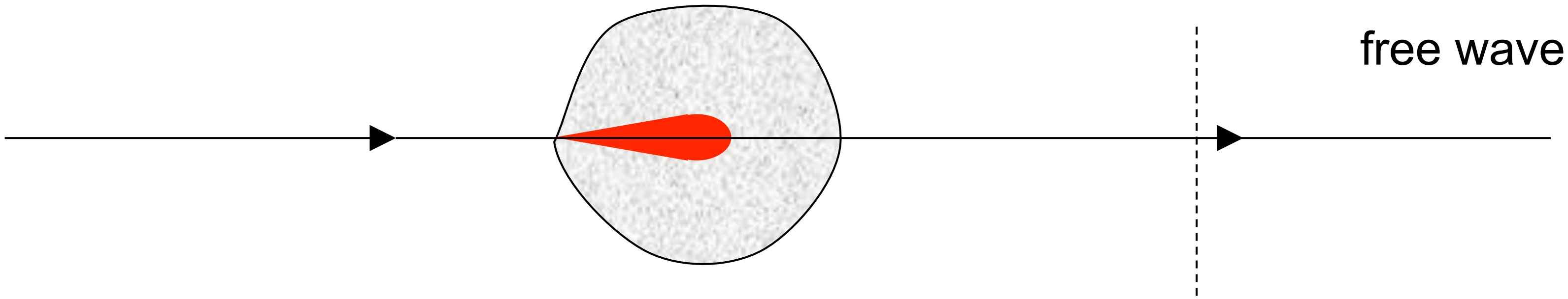


$$\alpha_{eff} = N_{coh}\frac{Ze^2}{hc} = \frac{l_{coh}}{a}\frac{Ze^2}{hc}$$

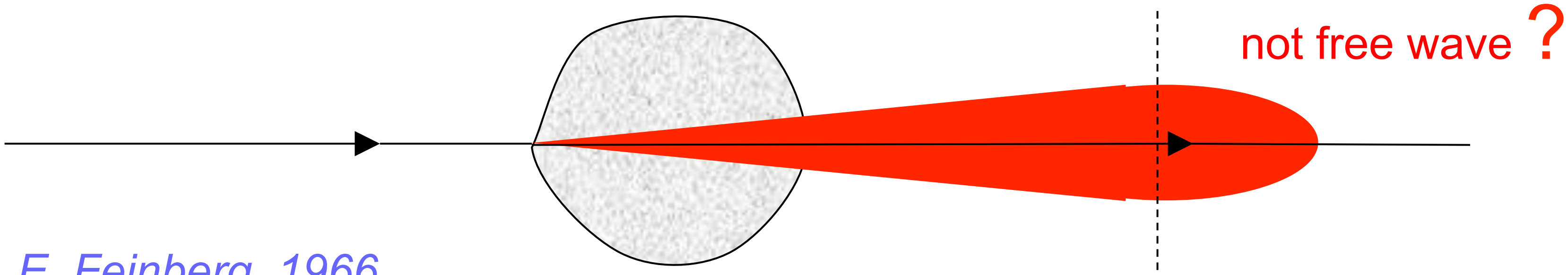
Problems

- **Methods of description of radiation at $\alpha_{eff} > 1$ (eikonal, semiclassical, operator semiclassical, classical electrodynamics, …)**
- **Evolution in space and time**
- **Medium influence on radiation**
- **S-matrix and boundary conditions**

S-matrix and Boundary Conditions for Ultra-High Coherent Lengths



$$S = T\exp\left\{ie^2\int_{-\infty}^{\infty}dt\int d^3r\,J_\mu A_\mu\right\}$$

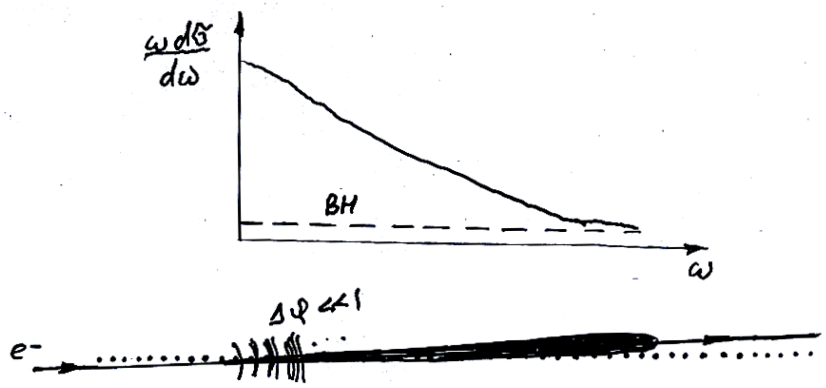


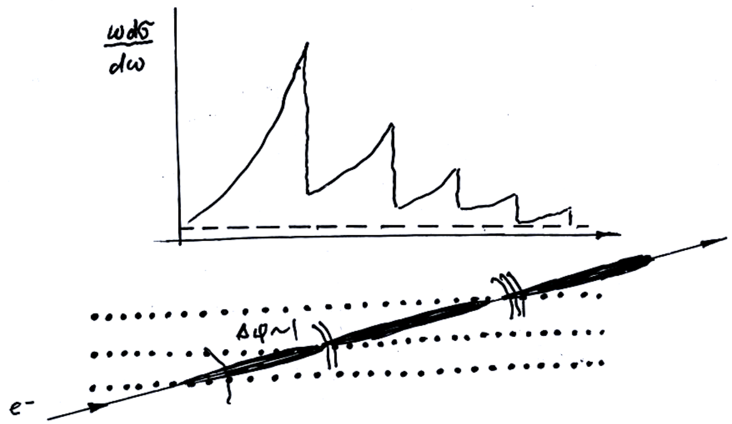
E. Feinberg, 1966

$$S = T\exp\left\{ie^2\int_{-\infty}^{T}dt\int d^3r\,J_\mu A_\mu\right\}$$???

COHERENT RADIATION IN CRYSTAL

Ter-Mikaelian 1953





Coherent effect

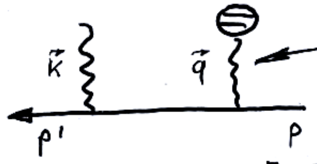
Coherence + Interference

$$\theta_c^2 << \theta^2$$

$$N_c Ze^2 / \hbar c < 1, \quad \text{where} \quad N_c = \frac{l_c}{a}$$

Coherent Bremsstrahlung Theory

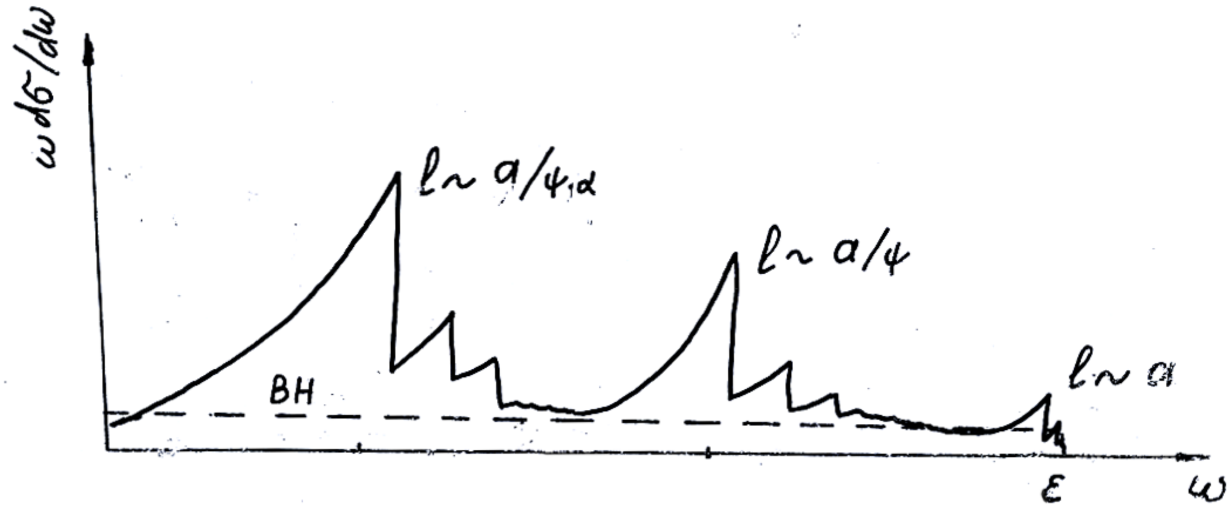
(Ferretti 1950, Ter-Mikaelian 1952, Überall 1956, 1960)



$$U(\vec{r}) = \sum_n u(\vec{r} - \vec{r}_n)$$

$$\vec{q} = \vec{p} - \vec{p}' - \vec{k}$$

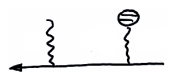
$$d\sigma_{atom} \sim \dots \int d^3q \dots \longrightarrow d\sigma_{crystal} \sim \dots \sum_{\vec{g}} \dots$$



Akhiezer, Shul'ga 1982

The main Akhiezer's idea (1969)

- For coherent bremsstrahlung

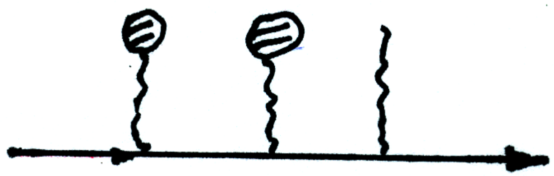


$$d\sigma_{coh} >> d\sigma_{BH}$$

- The idea: relative contribution of higher Born approximation can also be large!!!

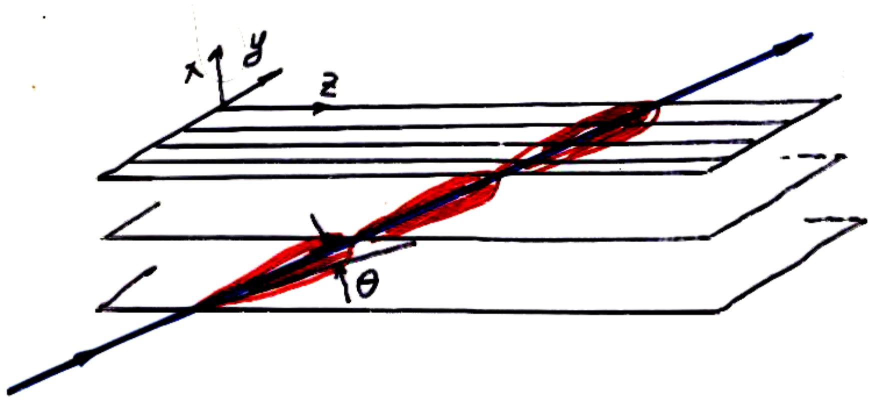
Second Born approximation in CB theory

A.Akhiezer, P.Fomin, N.Shul'ga (1970)



$$d\sigma_c = d\sigma^{Born}_{coh} \cdot \left(1 \pm \eta \frac{\theta_c^{\,2}}{\theta^2}\right), \qquad \hbar\omega = \varepsilon$$

$\eta : 1$ θ_c – *crytical channelling angle*



Higher Born Approximation in the CB Theory

A.Akhiezer, N.Shul'ga (1975)

$$N_{coh} : \min\left(\frac{l_{coh}}{a}, \quad \frac{R}{\psi_a}\right)$$

$$\frac{Ze^2}{\mathrm{h}c} = 1 \quad \rightarrow \quad N_{coh}\frac{Ze^2}{\mathrm{h}c} : \frac{R}{\psi a}\frac{Ze^2}{\mathrm{h}c} = 1$$

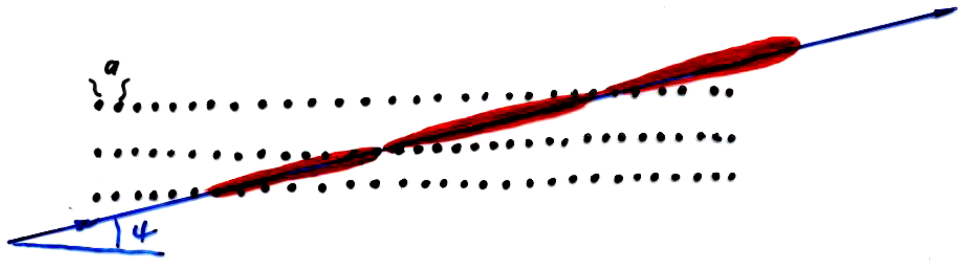
Quickly destroys for $\psi \rightarrow 0$

PARADOX

This condition did not fulfill practically for experiments (1960-1970) on verification of F – T – Ü theoretical results.

But the experiments were in good agreement with this theory !!!
Why ???

Eikonal, Semiclassical, Classical CB Theory



Semiclassical approximation

$$\boxed{\frac{N_c Ze^2}{\text{h}c} = \frac{R}{\psi a}\,\frac{Ze^2}{\text{h}c} \;?\; 1}$$

!!!

Classical Electrodynamics

$$N_c \frac{Ze^2}{\text{h}c} \;?\; 1, \qquad \text{h}\omega = \varepsilon$$

$$d\sigma^{(WKB)} = d\sigma\left\{\vec{r}_{cl}(t)\right\}$$

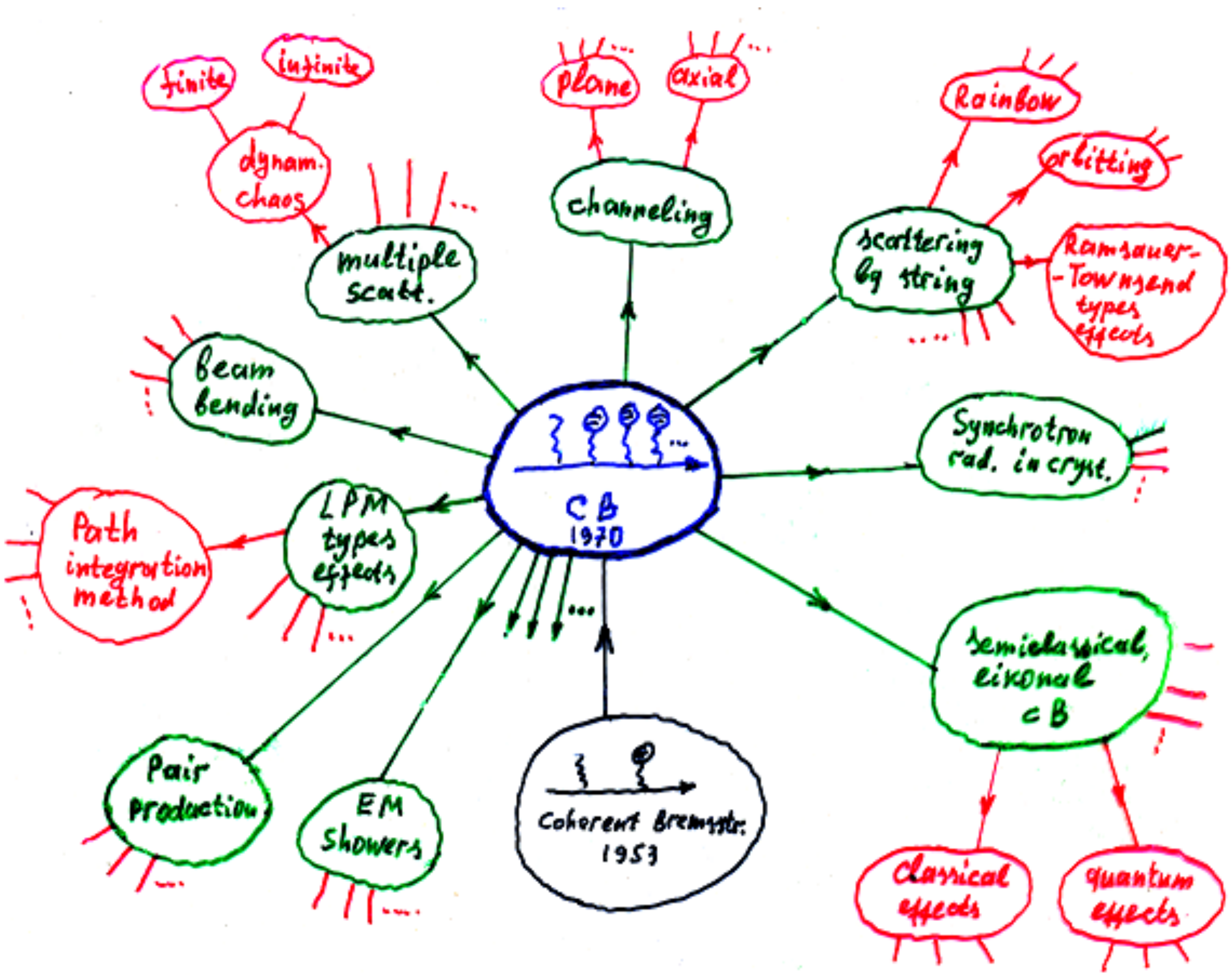
- Radiation is determined by the classical trajectory !!!
- It is necessary to know the types of particles' motion in crystal
- Same methods for description of CB and LPM effects !!!

New direction of research

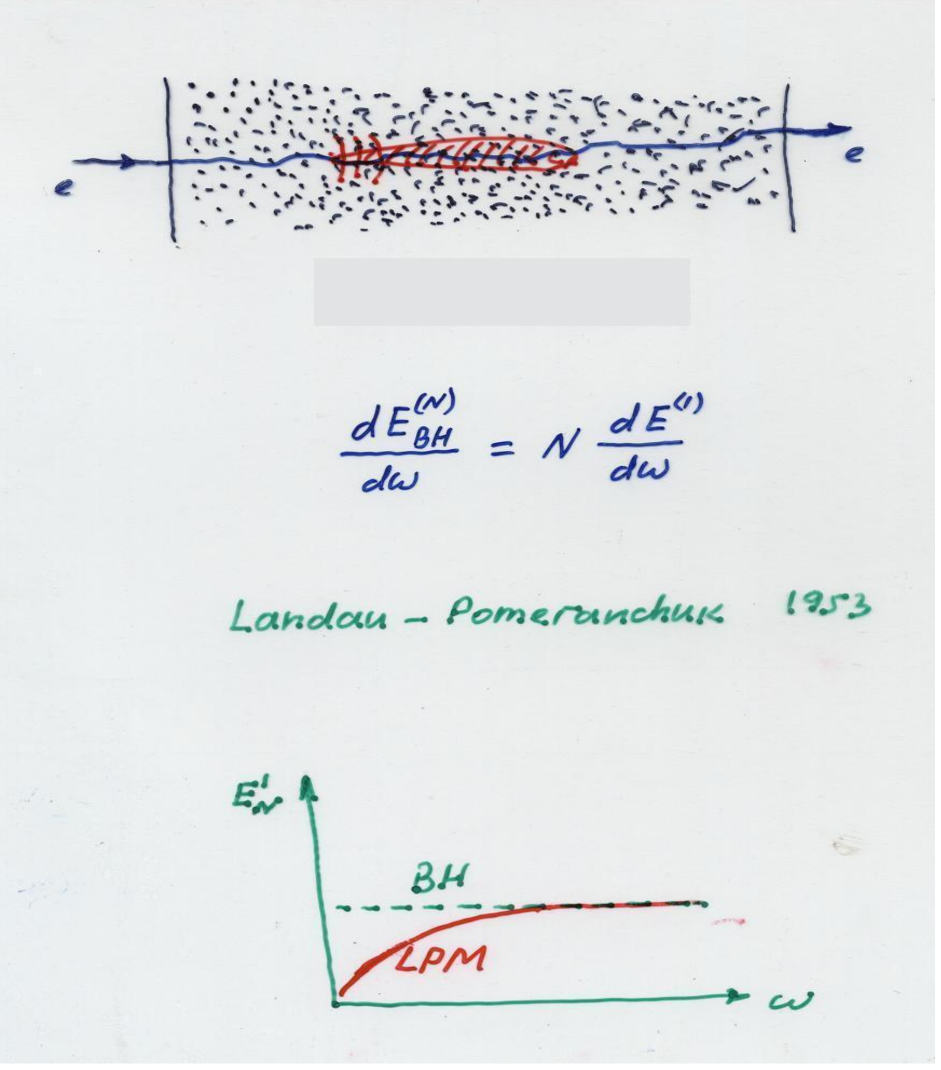
The interaction of high-energy particles with matter in conditions of effectively strong interaction of the particle with atoms of media (semiclassical, classical approximations)

$$N_c \frac{Ze^2}{\mathrm{h}c} \ ? \ 1$$

Проблемы, порожденные теорией когерентного излучения в кристаллах



LPM – effect (1953)



$$\frac{dE_{BH}}{d\omega} = \frac{4}{3}\frac{L}{X_0}\frac{E'}{E}\left(1+\frac{\omega^2}{EE'}\right)$$

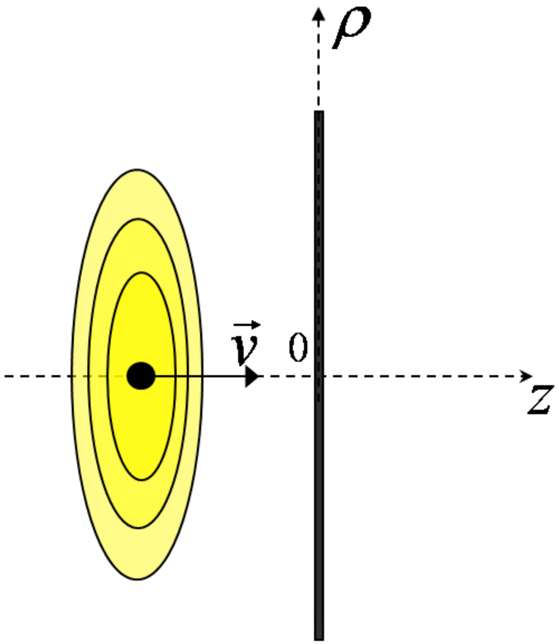
$$X_0^{-1} = \frac{4Z^2e^6n}{m^2}\ln(mR)$$

$$\frac{dE^{(N)}_{BH}}{d\omega} = N\frac{dE^{(1)}}{d\omega}$$

$$\frac{dE_{LP}}{d\omega} \approx \frac{L}{X_0}\sqrt{\frac{2\pi}{3}\frac{\omega E_0}{E}}$$

Development: LPM, TSF-effects etc. see report S.Fomin et al. at Channeling 2012

TRANSITION RADIATION BY ELECTRON WITH EQUILIBRIUM FIELD



Total field:

$$\varphi = \varphi^{C} + \varphi^{f}$$

Boundary condition:

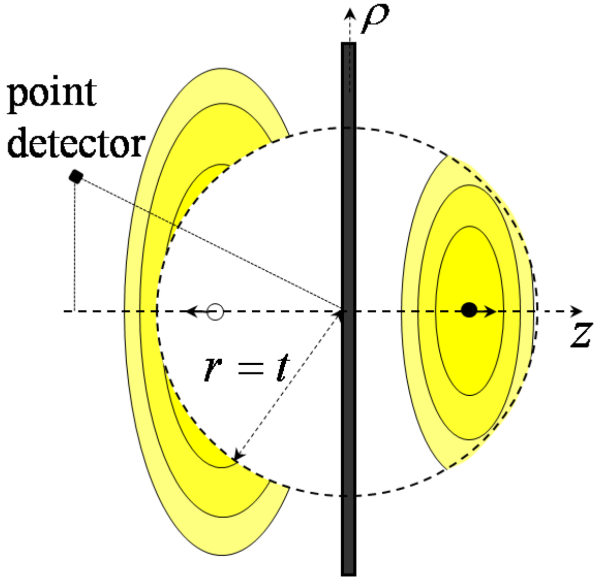
$$\vec{E}_{\perp}^{C}(\vec{\rho}, z = 0, t) + \vec{E}_{\perp}^{f}(\vec{\rho}, z = 0, t) = 0$$

Fourier integral for radiation field:

$$\varphi^{f}(\vec{r}, t) = -\frac{e}{2\pi^{2} v}\int d^{2}k_{\perp}\int_{-\infty}^{\infty} d\omega \frac{1}{k_{\perp}^{2} + \omega^{2}/p^{2}} e^{i\left(z\omega\sqrt{1-k_{\perp}^{2}/\omega^{2}} - \omega\, t + \vec{k}_{\perp}\vec{\rho}\right)}$$

STRUCTURE OF TR ELECTROMAGNETIC FIELD

N.Shul'ga, S. Trofymenko, V. Syshchenko, Nuovo Cimento (2011)



$$E = 50 Mev \qquad \lambda \approx 0.1 cm$$

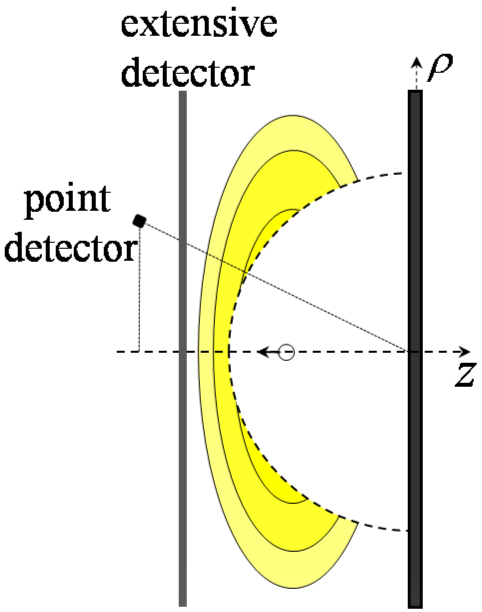
$$l_C \approx 2\gamma^2\lambda \approx 20m \qquad l_T \approx \gamma\lambda \approx 10cm$$

For $t > 0$ **:**

$$\mathbf{z > 0}: \quad \varphi(\vec{r},t) = \left[\frac{e}{\sqrt{\rho^2\gamma^{-2} + (z - vt)^2}} - \frac{e}{\sqrt{\rho^2\gamma^{-2} + (z + vt)^2}}\right]\theta(t - r)$$

$$\mathbf{z < 0}: \quad \varphi(\vec{r},t) = \left[-\frac{e}{\sqrt{\rho^2\gamma^{-2} + (|z| - vt)^2}} + \frac{e}{\sqrt{\rho^2\gamma^{-2} + (z - vt)^2}}\right]\theta(r - t)$$

The Problem of TR Measurement



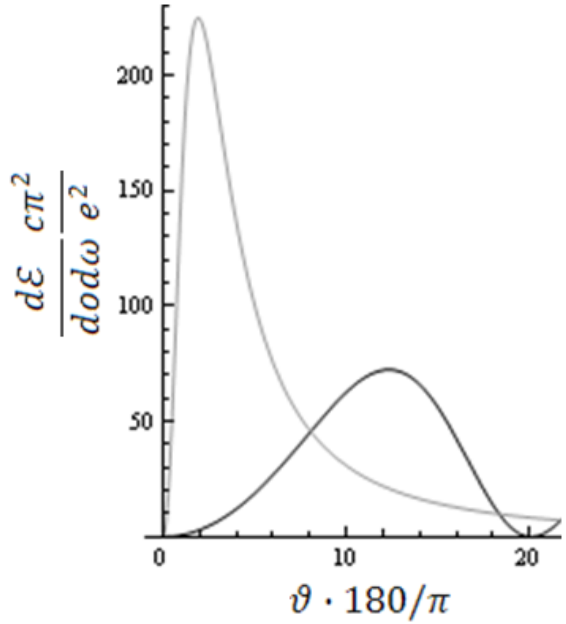
Point detector $\Delta\rho << \gamma / \omega$**:**

$|z| >> l_c$:

$$\frac{d\mathcal{E}}{d\omega do} = \frac{e^2}{\pi^2} \frac{\vartheta^2}{\left(\gamma^{-2} + \vartheta^2\right)^2}$$

$|z| << l_c$:

$$\frac{d\mathcal{E}}{d\omega do} = \frac{4e^2}{\pi^2} \frac{1}{\vartheta^2} \sin^2\left(\frac{\omega\,|z|\vartheta^2}{4}\right)$$



- **Verzilov V. // Phys. Lett. A. , 2000**
- **Dobrovolsky S., Shul'ga N. // NIM B, 2003**

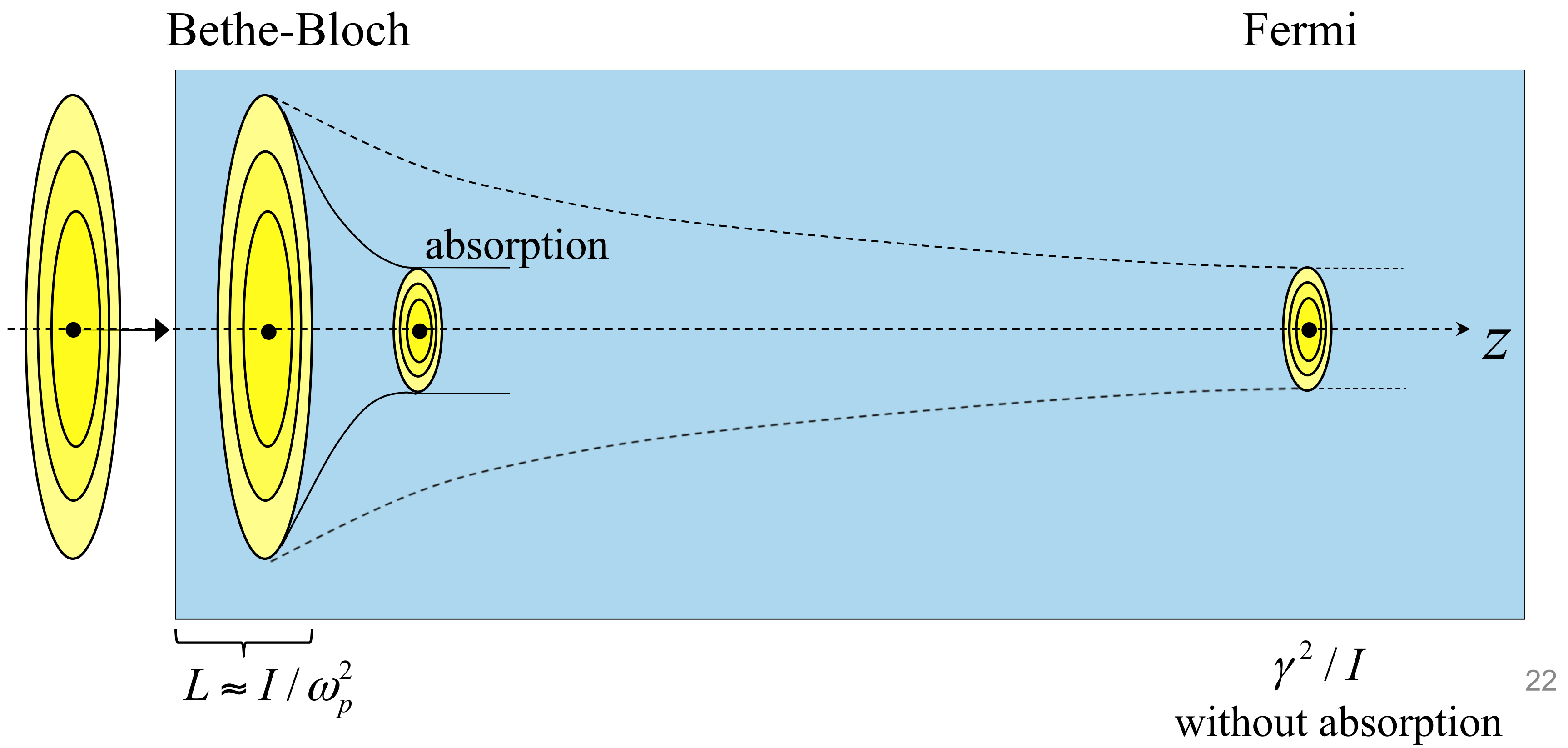
Extensive detector (infinite plate) $\Delta\rho >> \gamma / \omega$ **:**

arbitrary z **:**

$$\frac{d\mathcal{E}}{d\omega do} = \frac{e^2}{\pi^2} \frac{\vartheta^2}{\left(\gamma^{-2} + \vartheta^2\right)^2}$$

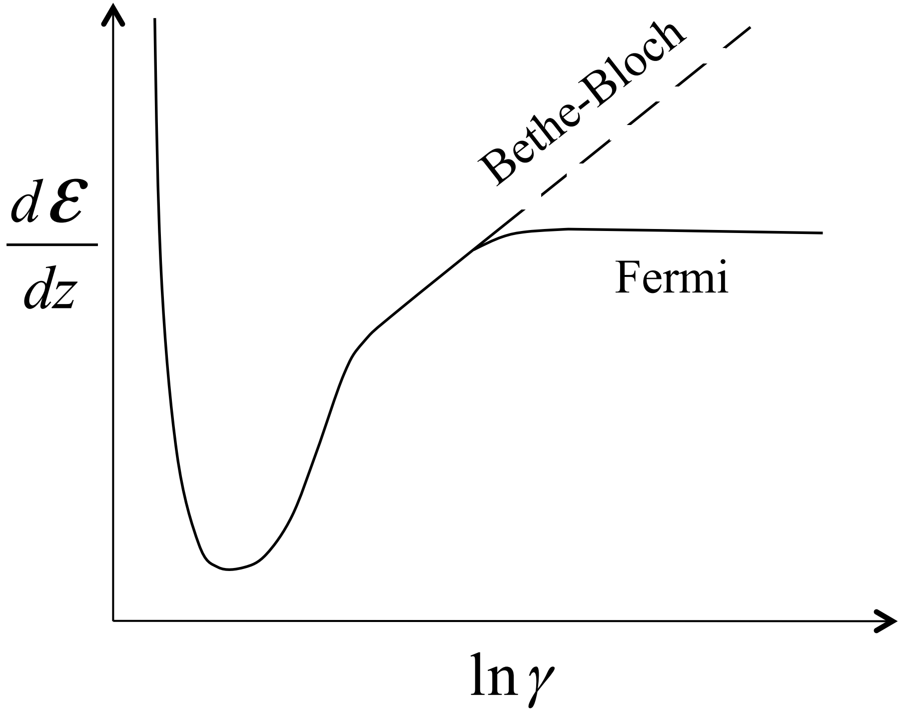
IONIZATION ENERGY LOSSES

N. Shul'ga, S. Trofymenko, 2012



G. Garibian, 1959

IONIZATION ENERGY LOSSES IN THIN AND THICK TARGETS

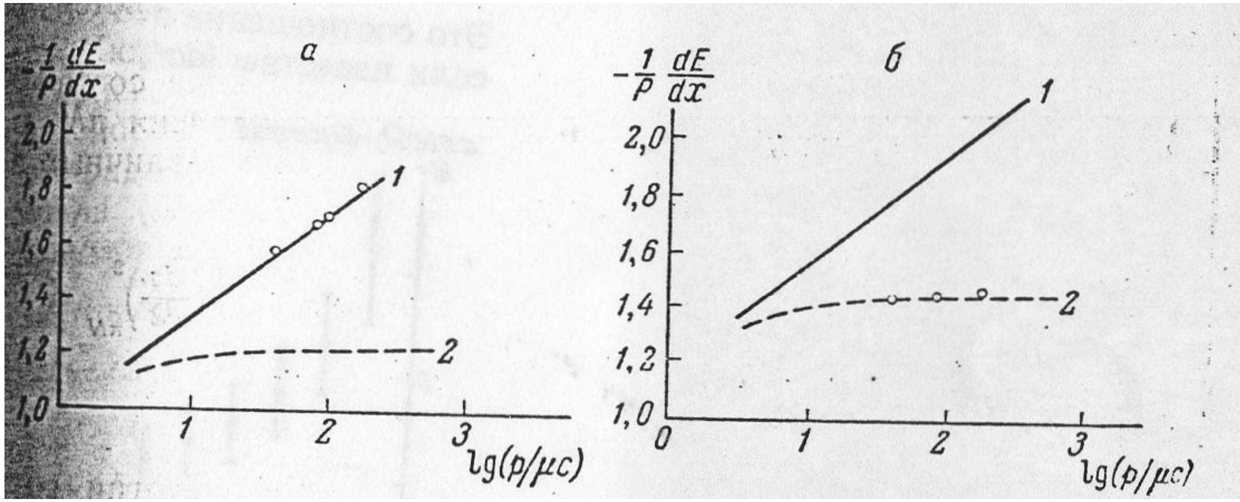


$L < I/\omega_p^2$ ***(G. Garibian, 1959)***

$L >> I/\omega_p^2$

FIRST EXPERIMENT (Kharkov, 1963)

A.I. Alikhanian, G.M. Garibian, M.P. Lorikian, A.K. Walter,
I.A. Grishaiev, V.A. Petrenko, G.L. Fursov



Electron energy losses in thin films of polystyrene of thicknesses $10^{-6}cm$ (a) and $2\times10^{-3}cm$ (б)

1 – theoretical curve without density effect

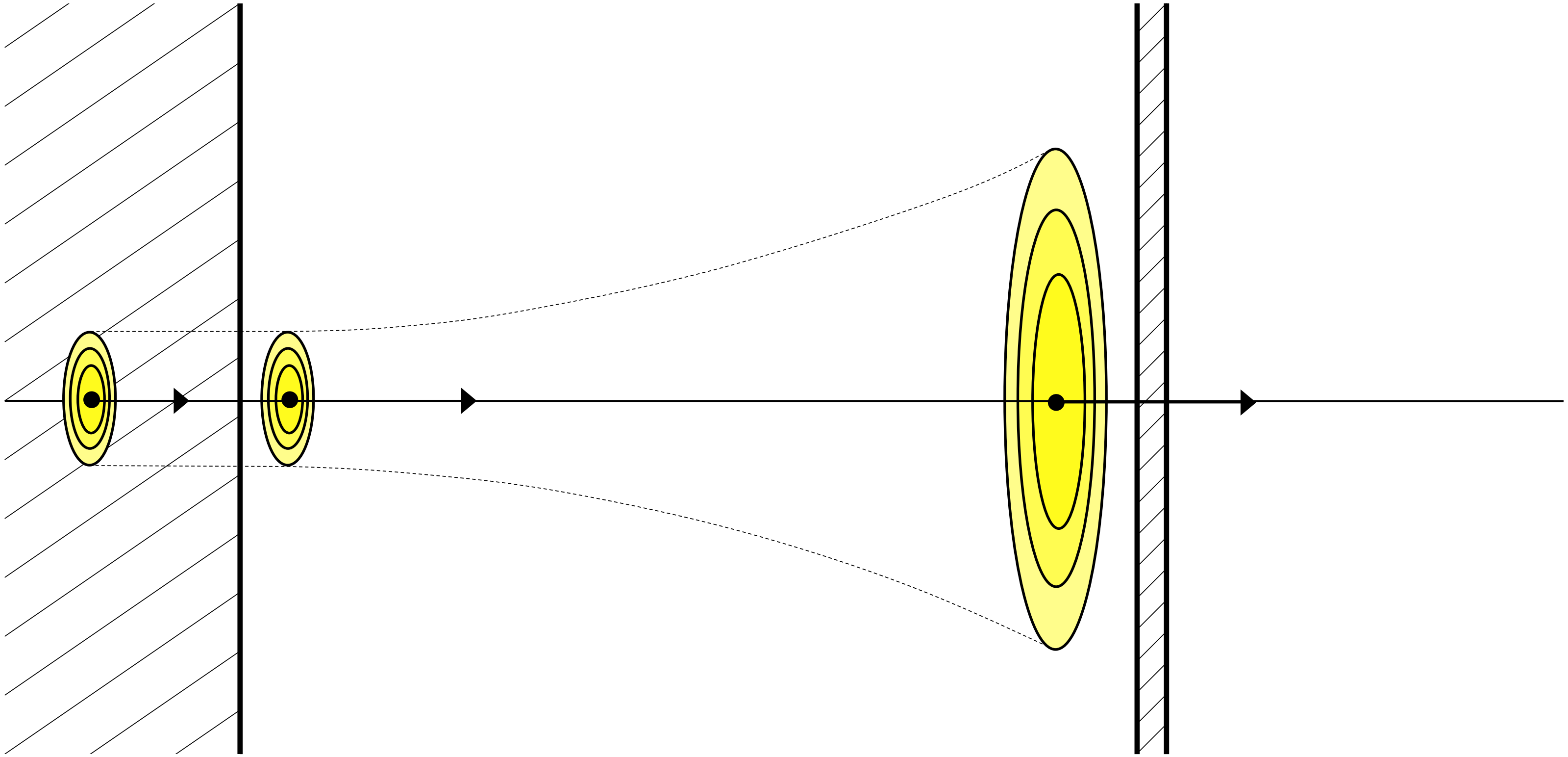
2 – theoretical curve with density effect

circles show the measurement results

20 MeV < ε < 100 MeV

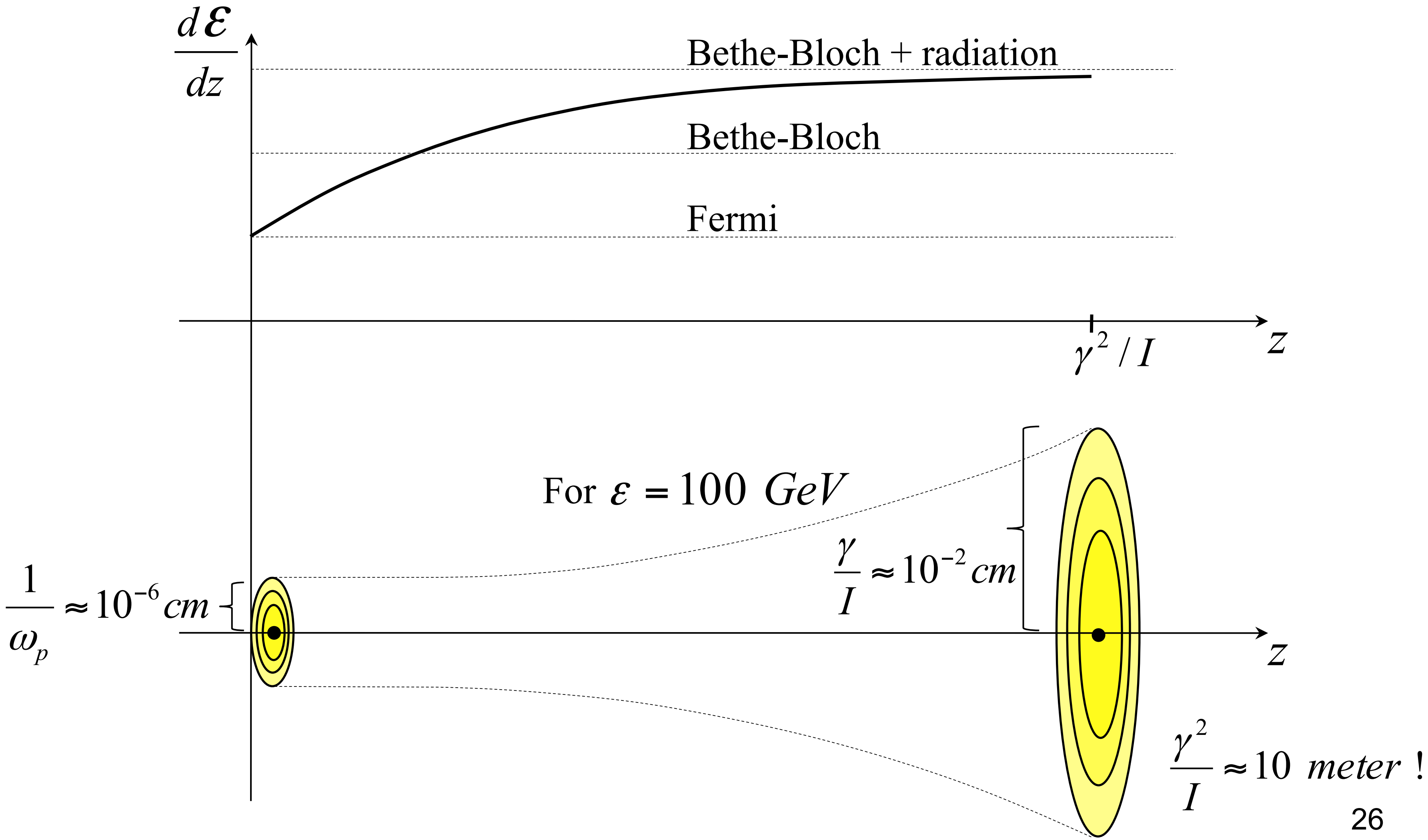
IONIZATION ENERGY LOSSES BY HALF-BARE ELECTRON

N. Shul'ga, S. Trofymenko, 2012



This is not the same as in G.Garibian and K.Ispirian JETP Lett, 1972

IONIZATION ENERGY LOSSES BY HALF-BARE ELECTRON (from Fermi to Bethe-Bloch formula)



More in report S.Trofymenko et al. at Channeling-2012

CONCLUSIONS

- stabilizing effect for high-energy wave packets
- large and ultra large formation lengths for electromagnetic packets
- half-bare electron in bremsstrahlung and transition radiation
- half-bare electron in ionization energy losses (transition from Fermi to Bethe-Bloch formula)
- analogies in solid state physics (polaron),

  in QCD (quark-gluon plasma, …)

THANK YOU FOR ATTENTION!