Method of Induced Currents and its applications to the different problems of Polarization Radiation

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Types of radiation

Electromagnetic radiation by particles can be divided into two categories:

- **Bremsstrahlung** - radiation of hard photons due to acceleration (synchrotron radiation, undulator rad., channeling rad., etc.);
- **Polarization radiation** - radiation of soft photons; acceleration is **not** required (Vavilov-Cherenkov rad., transition rad., etc.);

Why only two types?

- Typical Bremsstrahlung diagram is **infra-red divergent** because an electron is able to radiate infinitely large number of soft photons.
- Polarization Radiation diagram is **ultra-violet divergent** because hard photons break the condition of small radiation losses.

Their sum if always finite: divergences cancel each other!

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Polarization Radiation (PR)

Microscopically, PR arises as a result of the dynamical polarization of the atomic electron shells by the particle's field. PR may dominate over the "ordinary" bremsstrahlung (*Amusia, et al., 1976*) especially in relativistic case and for heavy particles or ions.

Macroscopic treatment of PR began from works on

- Vavilov-Cherenkov radiation, VChR (Cherenkov, 1934; Tamm, Frank, 1937),
- Transition radiation, TR (Ginzburg, Frank, 1945),
- Diffraction radiation, DR (Bobrinev, Braginsky, 1958; Dnestrovsky, Kostomarov, 1959),
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and this is not the end of the list...

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Nowaday status of PR

Today PR has a wide range of applications: from detectors in high-energy physics, proposals of the beam diagnostics for accelerators to the new tunable radiation sources for industry, medicine and biology.

The modern applications of PR require
the adequate methods of calculations!

What the reality-based models should describe?

- Finite permittivity $\varepsilon(\omega) = \varepsilon' + i\varepsilon''$ of a target. Such a model should describe metals as well as dielectrics, photonic crystals, etc. As a result, it should be applicable from the very long waves to the X-rays.
- Real geometrical sizes of a target (screen, grating, cylinder, etc.)
- Real distance to the detector: not only wave zone, but the pre-wave zone and the near-field zone as well.
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Method of Induced Currents

Is it possible to develop such a model?

If we treat VChR, TR, DR, SPR, etc. as the only one PR, its source would be $\mathbf{j}_{pol}$ induced in substance by the particle's field $\mathbf{E}^0$ (Durand, 1975; Ryazanov, et al., 1976):

$$\mathbf{j}_{pol} = \mathbf{j}_{pol}[\mathbf{E}], \;\; \mathbf{E} = \mathbf{E}^0 + \mathbf{E}^{pol}.$$

However, the problem is that this is an integral equation.

There are two ways how to overcome this difficulty:

Perturbative approach: $\mathbf{j}_{pol} = \mathbf{j}_{pol}^{(0)} + \mathbf{j}_{pol}^{(1)} + ...$ with an expansion parameter $\varepsilon(\omega) - 1 \ll 1$ (Durand, 1975; Alikhanian, Chechin, 1979) - mainly for X-rays. By choosing the appropriate $\mathbf{j}_{pol}^{(0)}$, it is possible to make the region of applicability wider: $\varepsilon(\omega) - 1 \lesssim 1$ (Shul'ga, Syshchenko, 2004).

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Non-perturbative approach, which begins with a shift of the photon propagator's pole (Ryazanov, 1957):

$$\frac{1}{\mathbf{k}^2 - \omega^2/c^2} \to \frac{1}{\mathbf{k}^2 - \varepsilon(\omega)\omega^2/c^2}$$

Physically, such a shift, $\omega^2/c^2 \to \varepsilon\omega^2/c^2$, fixes the "bare" current:

$$\mathbf{j}_{pol}^{(0)}(\mathbf{r},\omega) = \sigma(\omega)\mathbf{E}^0(\mathbf{r},\omega),$$

but the key difference of this approach from the 0-th order of perturbation theory is that the emitted photons become "dressed". Here, the complex conductivity is $\sigma(\omega) = i\omega(1-\varepsilon(\omega))/4\pi$.

Shall we obtain an exact result just integrating the current $\mathbf{j}_{pol}^{(0)}$ over the target's volume?

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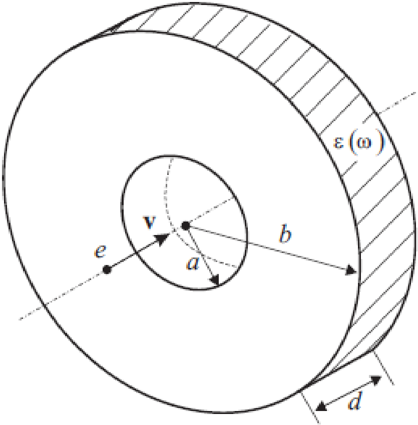
Methodical example: PR from a cylindrical screen

Consider the problem of PR from a cylindrical screen of a finite width d and a hole. We expect two channels: DR (TR) and VChR.



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As long as we integrate $\mathbf{j}_{pol}^{(0)}$ over the target's volume, we only neglect those waves of PR that are reflected back into target. As a result, the exact field of PR inside the target is

$$\mathbf{E}_{pol} = \mathbf{E}_{pol}^{(0)} + \mathbf{E}_{pol}^{(1)} + \mathbf{E}_{pol}^{(2)} + ...,$$

where $\mathbf{E}_{pol}^{(1)}$ is the field once reflected from the interface, $\mathbf{E}_{pol}^{(2)}$ is the one twice re-reflected, etc.

We've again obtained the infinite series, but this time it converges very fast. As we shall see, the contribution of multiple re-reflections is almost negligible even for transparent media.

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The crucial (and a bit unexpected) point here is that this series can be summed exactly.

Note: inside the target, the summation of the far-fields $\mathbf{E}_{pol}$ is not quite correct procedure.

The correct answer can be obtained by solving the inverse problem and applying the reciprocity theorem:

$$(\mathbf{E}^{R(vac)}, \mathbf{d}^{(vac)}) = (\mathbf{E}^{R(m)}, \mathbf{d}^{(m)})$$

where $\mathbf{d}$ is the effective dipole moment of the induced current.

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As soon as we find the field refracted into the target when falling from vacuum, the radiated energy becomes

$$\frac{d^2W}{d\omega d\Omega} = \frac{cr^2}{|\varepsilon|^2}\left(|f_H^{(d)}|^2|H^R_{\perp(F)}+R_H^{(d)}H^R_{\perp(B)}|^2+|\sqrt{\varepsilon}f_E^{(d)}|^2|H^R_{\parallel(F)}+R_E^{(d)}H^R_{\parallel(B)}|^2\right).$$

with $f_E^{(d)}, f_H^{(d)}, R_E^{(d)}, R_H^{(d)}$ being the Fresnel and reflection coefficients, respectively, in which all the multiple re-reflection are taken into account.

All the effects of the target's width d are hidden in $f_E^{(d)}, f_H^{(d)}, R_E^{(d)}, R_H^{(d)}$!

In order to understand whether the multiple re-reflections are important or not we should compare $dW(f^{(d)}, R^{(d)})$ with the "one-reflection" expression $dW(f, 0)$:

$$\frac{d^2W(f,0)}{d\omega d\Omega} = \frac{cr^2}{|\varepsilon|^2}\left(|f_H|^2|H^R_{\perp(F)}|^2+|\sqrt{\varepsilon}f_E|^2|H^R_{\parallel(F)}|^2\right).$$

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All the re-reflections are taken into account in the coefficients:

$$f_H^{(d)} = 2\varepsilon \cos\theta \times \frac{(\sqrt{\varepsilon - \sin^2\theta} + \varepsilon\cos\theta) e^{-id\frac{\omega}{c}\sqrt{\varepsilon - \sin^2\theta}}}{(\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta})^2 e^{-id\frac{\omega}{c}\sqrt{\varepsilon-\sin^2\theta}} - (\varepsilon\cos\theta - \sqrt{\varepsilon - \sin^2\theta})^2 e^{id\frac{\omega}{c}\sqrt{\varepsilon - \sin^2\theta}}},$$

$$f_E^{(d)} = 2\cos\theta \times \frac{(\sqrt{\varepsilon - \sin^2\theta} + \cos\theta) e^{-id\frac{\omega}{c}\sqrt{\varepsilon - \sin^2\theta}}}{(\cos\theta + \sqrt{\varepsilon - \sin^2\theta})^2 e^{-id\frac{\omega}{c}\sqrt{\varepsilon-\sin^2\theta}} - (\cos\theta - \sqrt{\varepsilon - \sin^2\theta})^2 e^{id\frac{\omega}{c}\sqrt{\varepsilon - \sin^2\theta}}},$$

$$R_H^{(d)} = \frac{\sqrt{\varepsilon - \sin^2\theta} - \varepsilon\cos\theta}{\sqrt{\varepsilon - \sin^2\theta} + \varepsilon\cos\theta} e^{i2d\frac{\omega}{c}\sqrt{\varepsilon - \sin^2\theta}},$$

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They explicitly depend on the width d of the plate.

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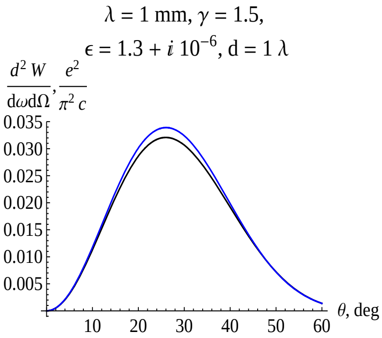

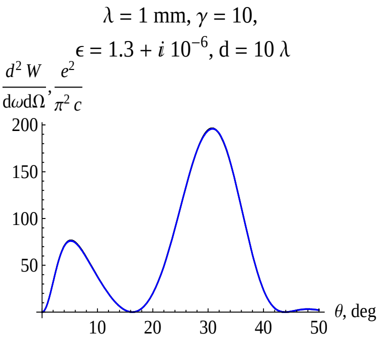

The black curve: without re-reflections; the blue curve: with re-reflections (the exact Pafomov's formula)

The difference is always negligible!

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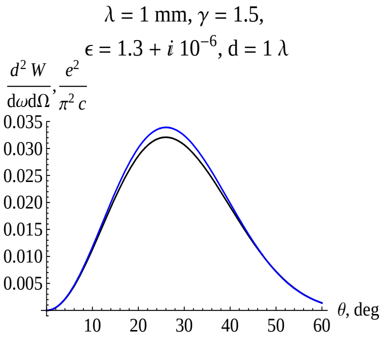

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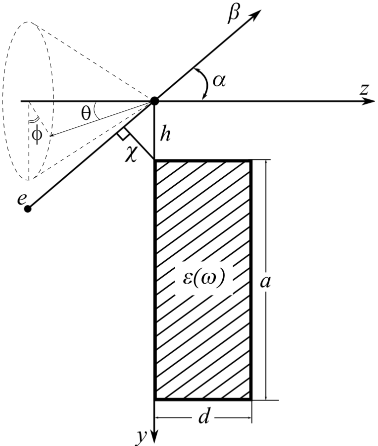
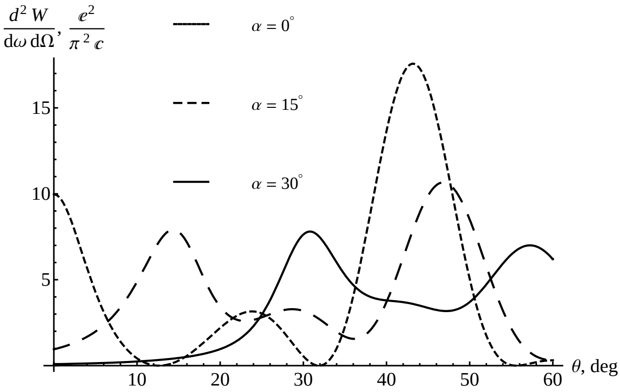
Example 1: rectangular screen

Let us consider the real example: PR from a rectangular screen.
Here we expect two channels: DR and VChR.



In a general $\alpha \neq 0$ case the VChR condition becomes:

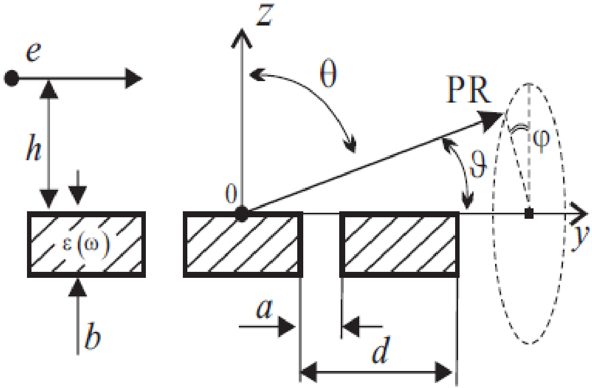
$$\mathrm{Re}\left\{ \cos\alpha - \beta\sqrt{\varepsilon - \sin^2\theta} + i\gamma^{-1}\sin\alpha\sqrt{1+(\beta\gamma\sin\theta\sin\phi)^2} \right\} \to 0$$



$\lambda = 1\text{mm}, \gamma = 10, d = 10\lambda, \varepsilon = 1.5 + i10^{-6}, a = \infty, \phi = 0, \chi = 1\lambda$

Example 2: rectangular grating

Let us consider another example: PR from a rectangular grating with $b \ll d$. One can expect two channels: SPR and VChR.



Example 2: rectangular grating

The general structure of the final formula is as follows:

$$\frac{d^2W}{d\omega d\Omega} = \frac{d^2W}{d\omega d\Omega}\Big|_{screen} F_{strip} F_N,$$

with

$$F_{strip} = \frac{\sin^2\left(\frac{d-a}{2}\frac{\omega}{c}(\beta^{-1}-\cos\vartheta)\right)}{(1-\beta\cos\vartheta)^2} \propto \delta(\beta^{-1}-\cos\vartheta)$$

when $d \to \infty$.

The total internal reflection: no VChR!

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Do our results coincide with those available in literature for DR and SPR?

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A common method for DR and SPR is to describe radiation as a far-field of the surface current $\mathbf{j}_s$.

The key question: How to obtain the surface current?

There were several methods: to solve integral equation (Kazantsev, Surdutovich, 1963; Kesar, 2005), to apply method of images (Bolotovsky, Voskresensky, 1968; Brownell, et al., 1998), etc.

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We got the following vectors in the TR problem:

- a normal to the screen $\mathbf{n}$,
- a wave vector $\mathbf{e} = \mathbf{k}/k$ (which is equivalent to ∇ in a sense),
- a particle's velocity β,
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The possible variants for the current with two tangential components are:

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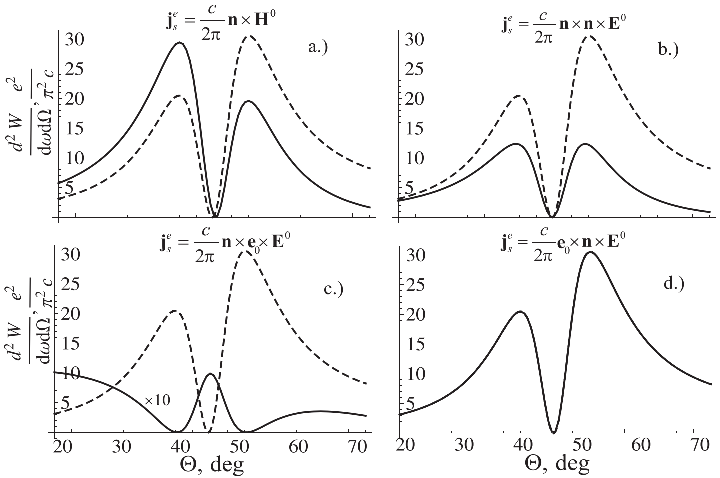

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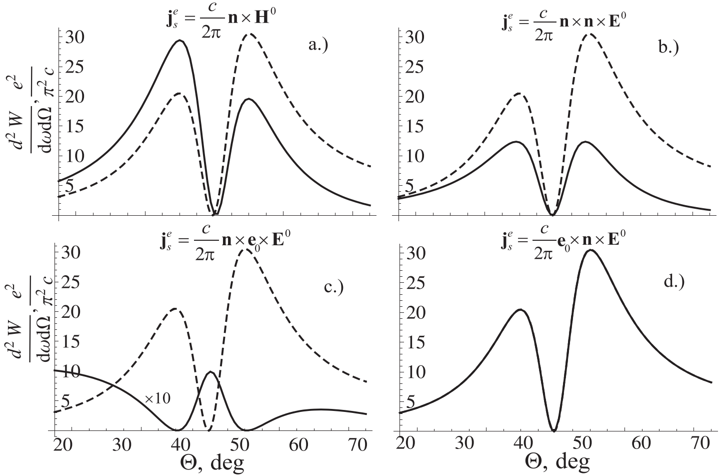

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Thank you for your attention!

Based on:
PLA **373**, 1988 (2009) [arXiv:0908.2331];
JETP Lett. **90**, 326 (2009) [arXiv:1002.1522];
JETP **113**, 27 (2011) [arXiv:1012.5001];
Russ. Phys. J. **55**, 9 (2012).

What we didn't discuss

- Limitation 1: When using Reciprocity, we reduce the initial problem of PR to the one of refraction of a plane wave. In general case, this problem may have no analytical solution, because the target's shape may be rather complicated. So, usually we require targets to be thin: $d \ll b - a$ (cylinder), $d \ll a$ (screen), etc.
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- Justification 2: For pre-wave zone, one can use the more general variant of reciprocity theorem (non-dipole approximation).
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