DIFFRACTION RADIATION FROM PERIODICAL STRUCTURES AS A SOURCE OF X-RAYS

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UV AND X-RAY DR: FEATURES AND PARAMETERS

Spectral-angular distribution of Diffraction Radiation (DR)

$$\frac{dW(\mathbf{n},\omega)}{d\Omega d\omega} \propto \exp\left\{-\frac{2\omega h}{c\beta\gamma}\right\} \qquad 2\omega h < c\beta\gamma \quad \rightarrow \quad 4\pi h < \gamma\beta\lambda$$

$$\left|\varepsilon(\omega)-1\right| < 1, \quad \omega > \omega_p$$

$$\begin{matrix} \hbar\omega_p = 30\,eV \;\leftrightarrow\; \lambda = 50\,nm \\ h = 10\,\mu m \end{matrix} \Rightarrow \begin{cases} \lambda = 50\,nm \rightarrow E > 1.3\ GeV \\ \lambda = 7\,nm \rightarrow E > 10\ GeV \\ \lambda = 3.5\,nm \rightarrow E > 20\ GeV \end{cases}$$

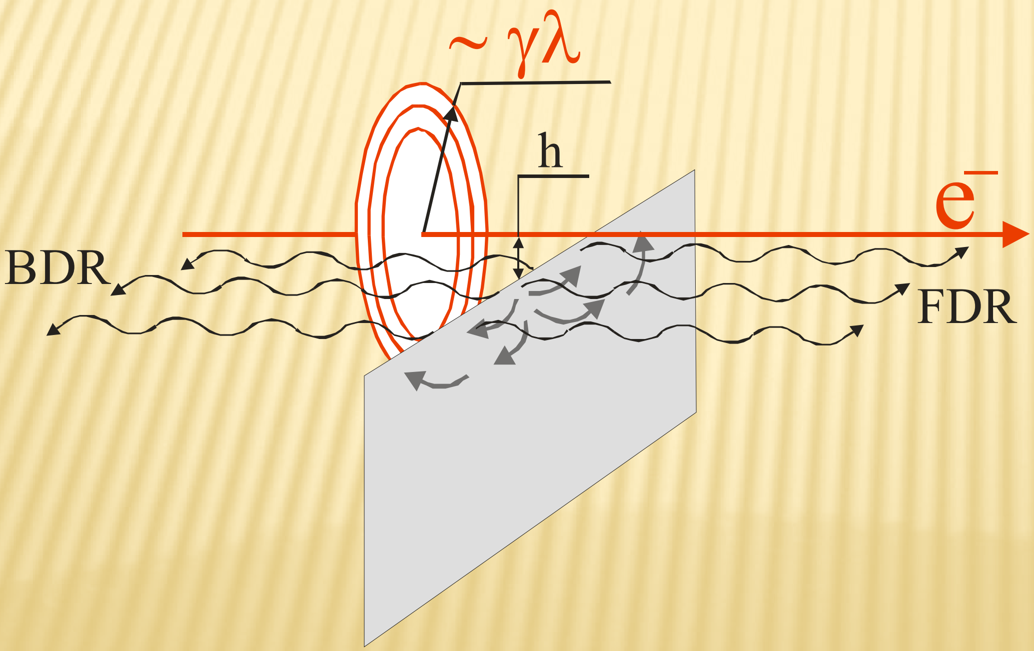
“Water window” frequencies domain ?!
(most interest in the soft X-rays for practical applications in physics, biology and medicine

$$\hbar\omega_p = 284 - 543\ eV \quad \leftrightarrow \quad \lambda = 4.47 - 2.36\ \text{nm}$$

GENERAL OF DIFFRACTION RADIATION

Scattering of the Coulomb field of a moving charged particle – diffraction of the moving particle Coulomb field – diffraction radiation

Dynamical polarization of a target material by the moving particle field – polarization radiation



I.M. Frank,

S.J. Smith and H.M. Purcell,

V.L. Ginzburg,

M.L. Ter-Mikhaelyan

Vavilov-Cherenkov radiation,

Transition radiation,

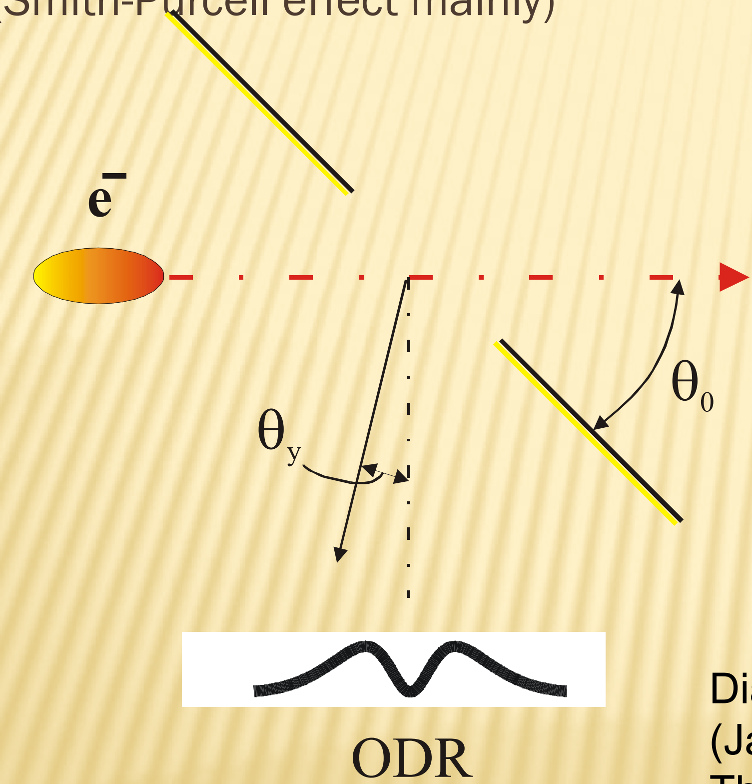
X-ray parametric radiation,

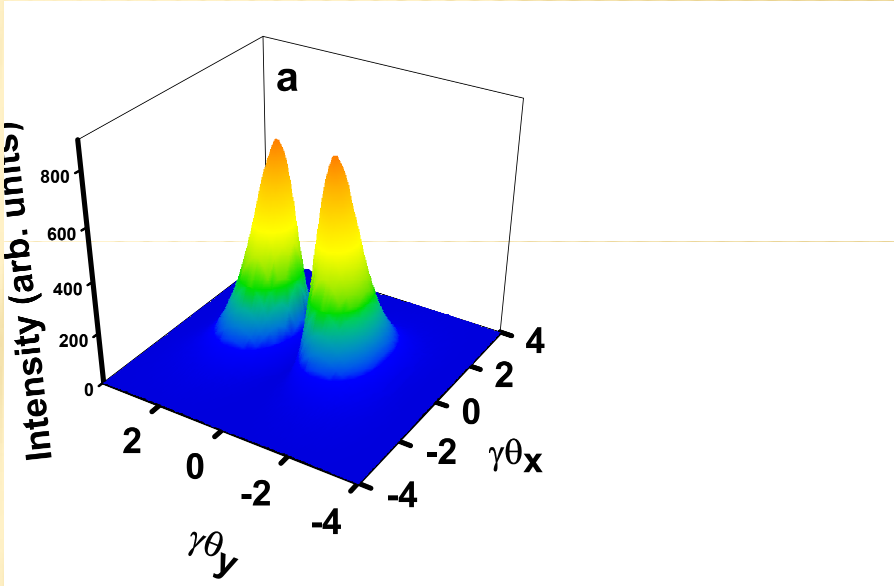
Smith-Purcell radiation

GENERAL OF DIFFRACTION RADIATION

DR is used for:

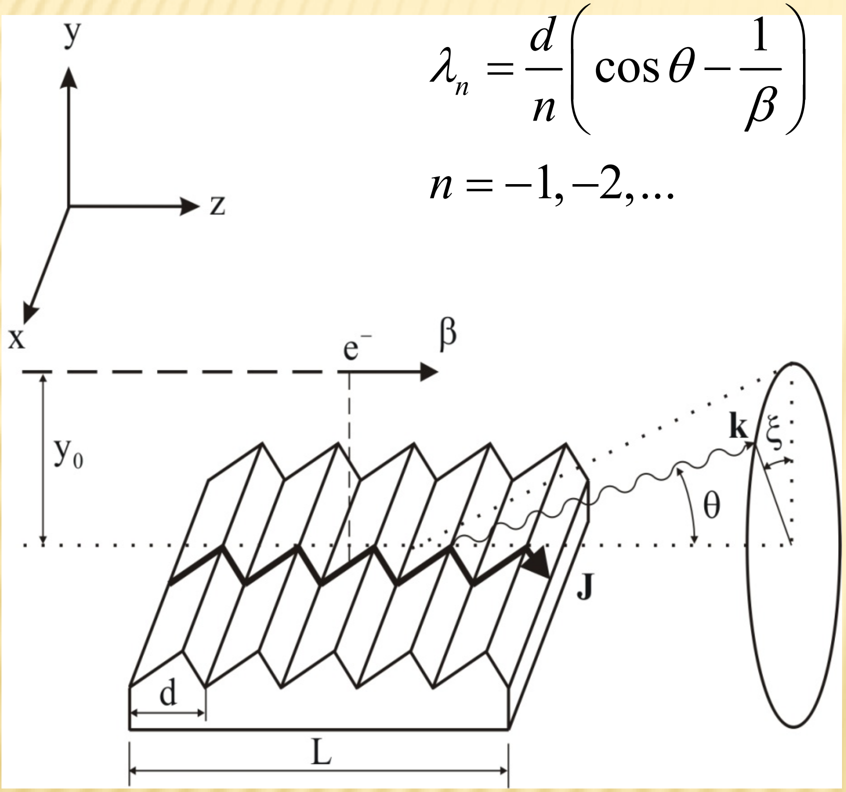
- Nondestructive diagnostics of charged particles bunches
- Source of radiation of electromagnetic waves in different ranges of frequencies (Smith-Purcell effect mainly)

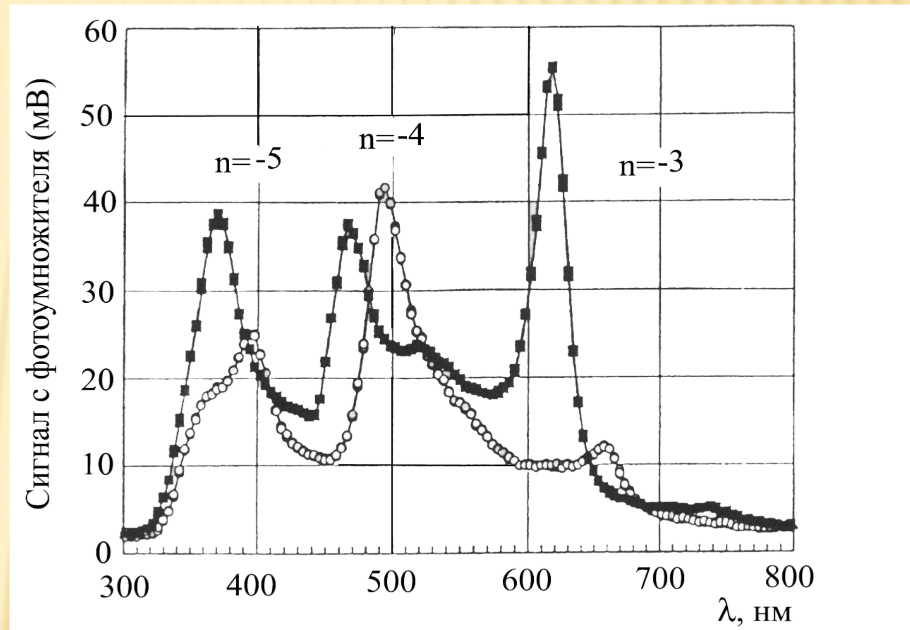




Diagnostic with DR is used today at KEK ATF (Japan), FLASH (Germany), Cornell (USA); The machine on DR diagnostics is being made at SLS (Switzerland); at SLAC (USA) used diagnostics on Smith-Purcell radiation.

SMITH-PURCELL RADIATION





Spectrum of SPR from the electrons with
20KeV (light circles) and
22,5 кэВ (black rectangles).

SPECIAL FEATURES OF X-RAY DIFFRACTION RADIATION

Optical schemes of diagnostics on TR and DR are restricted: wavelength is of the order of transversal bunch size.

For submicron diagnostics one need submicron (UV and X-ray) DR and SPR!

CLIC (CERN, Compact Multi-TeV Linear Collider), others?

UV and X-ray DR: the key theory problems is developed in

A.A. Tishchenko, A.P. Potylitsyn, M.N. Strikhanov,
Phys. Rev. E **70**, 066501 (2004);
NIMB **227**, 63 (2005).
Phys. Lett. A **359**, 509 (2006).

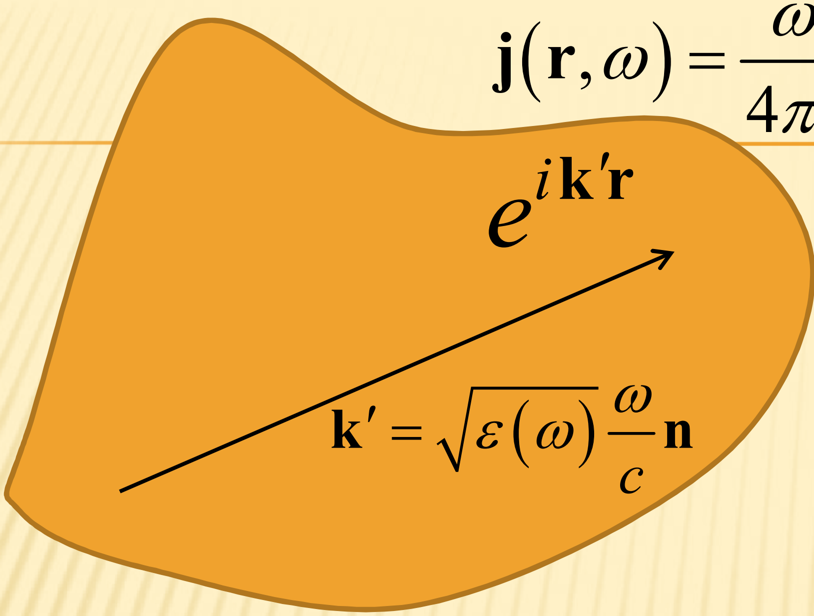
A.P. Potylitsyn, M.I. Ryazanov, M.N. Strikhanov, A.A. Tishchenko,
Diffraction Radiation from Relativistic Particles, Springer, 2010.

experiment?

P. Karataev, T. Lefevre et al,
UV/X-ray Diffraction Radiation for Non-intercepting Micron-Scale Beam Size Measurement,
RREPS-2011, London

L.Bobb, M.Billing, N. Chritin, P. Karataev and T. Lefevre,
UV/X-ray Diffraction Radiation for Non-intercepting Micron-Scale Beam Size Measurement,
Channeling-2012, Alghero - ?

$$\mathbf{j}(\mathbf{r},\omega)=\frac{\omega}{4\pi i}\left(\varepsilon(\omega)-1\right)\mathbf{E}^{act}(\mathbf{r},\omega)$$



$$\mathbf{E}^{0}_{vac}(\mathbf{q},\omega)=-\frac{ie}{2\pi^2}\frac{\mathbf{q}-\mathbf{v}\omega/c^2}{q^2-\omega^2/c^2}\delta(\omega-\mathbf{q}\mathbf{v})$$

L. Durand, Phys. Rev. D 11, 89 (1975).

$$E^{r}_{i}(\mathbf{r},\omega)=\frac{\exp\{ikr\}}{r}\frac{i\omega}{c^2}\left(\delta_{is}-n_i n_s\right)\times$$
$$\times\int_V d^3r\exp\left\{-i\sqrt{\varepsilon(\omega)}\mathbf{k}\mathbf{r}\right\}j_s(\mathbf{r},\omega)$$

+ reciprocity theorem - D. Karlovets, JETP 140, 36 (2011).

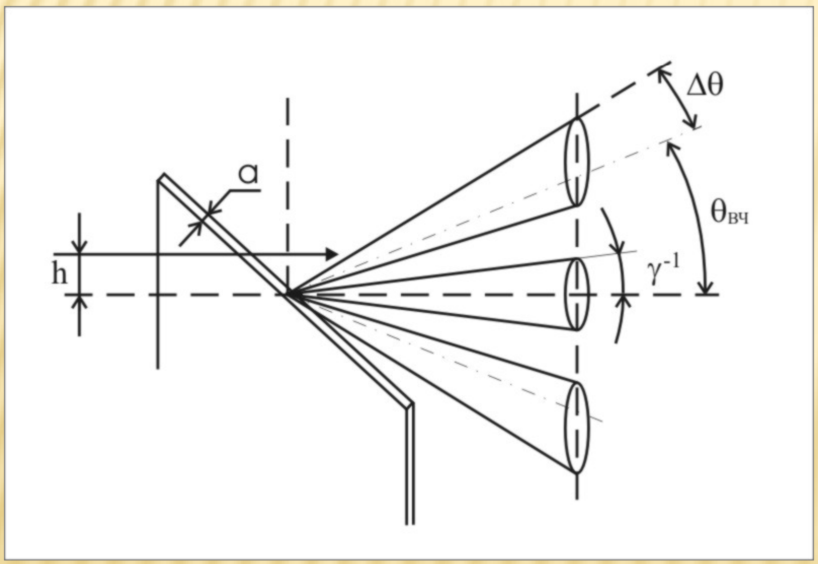
X-ray DR (... + absorption)

$$\frac{d^2W(\mathbf{n},\omega)}{d\Omega d(\hbar\omega)} = \alpha\frac{|\chi|^2}{\pi^2}\frac{1+2\theta^2\gamma^2\sin^2\phi}{1+\theta^2\gamma^2\sin^2\phi}\exp\left(-\frac{2h\omega}{c\gamma}\sqrt{1+\gamma^2\theta^2\sin^2\phi}\right)\times$$

$$\times\frac{\left|1-\exp\left(-i\frac{a\omega}{2c}\left(\gamma^{-2}+\theta^2-\chi'\right)\right)\exp\left(-\frac{a\omega}{2c}\chi''\right)\right|^2}{\left(\gamma^{-2}+\theta^2\right)\left[\left(\gamma^{-2}+\theta^2-\chi'\right)^2+\chi''^2\right]}$$

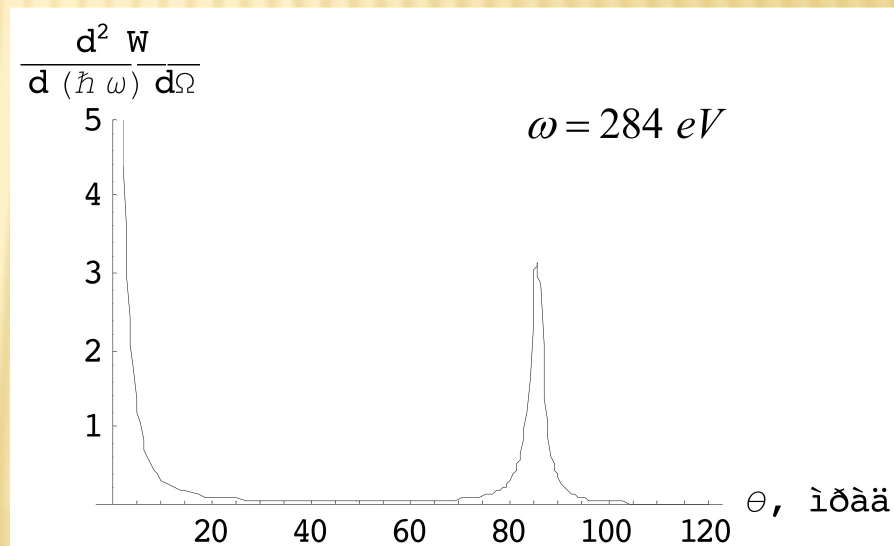
$$\varepsilon(\omega) = 1+\chi'(\omega)+i\chi''(\omega)$$

$$\chi'(\omega) = -\omega_p^2/\omega^2 < 0 \quad \chi'(\omega) \square \chi''(\omega)$$



$$\theta_{ДИ} = 0, \qquad \Delta\theta = \gamma^{-1}$$

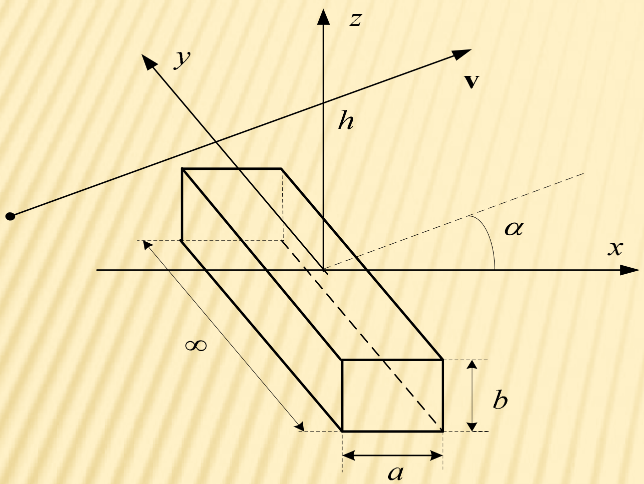
$$\theta_{ВЧ} = \sqrt{\chi'-\gamma^{-2}},\ \Delta\theta = \chi''/\sqrt{\chi'-\gamma^{-2}}$$



Angular distribution of DR for electrons with energy 5 GeV for carbon target

X-ray DR from periodical targets

Parametric DR и PXR, crystal target



$$\mathbf{E}^r_{DR+PDR}(\mathbf{r},\omega)=\mathbf{E}^r_{DR}(\mathbf{r},\omega)+\mathbf{E}^r_{PDR}(\mathbf{r},\omega)$$

$$\varphi_{\mathbf{g}}=\frac{\omega-\mathbf{v}\cdot\mathbf{k}'+\mathbf{g}\cdot\mathbf{v}}{v_x},\quad \rho_1^2=\left[\frac{\omega-v_y\left(k'_y-g_y\right)}{v_x}\right]^2+\left(k'_y-g_y\right)^2-\varepsilon\,\omega^2/c^2$$

$$\mathbf{A}_{\mathbf{g}}=\frac{\omega-v_y\left(k'_y-g_y\right)}{v_x}\mathbf{e}_x+\left(k'_y-g_y\right)\mathbf{e}_y-\mathbf{v}\omega/v^2$$

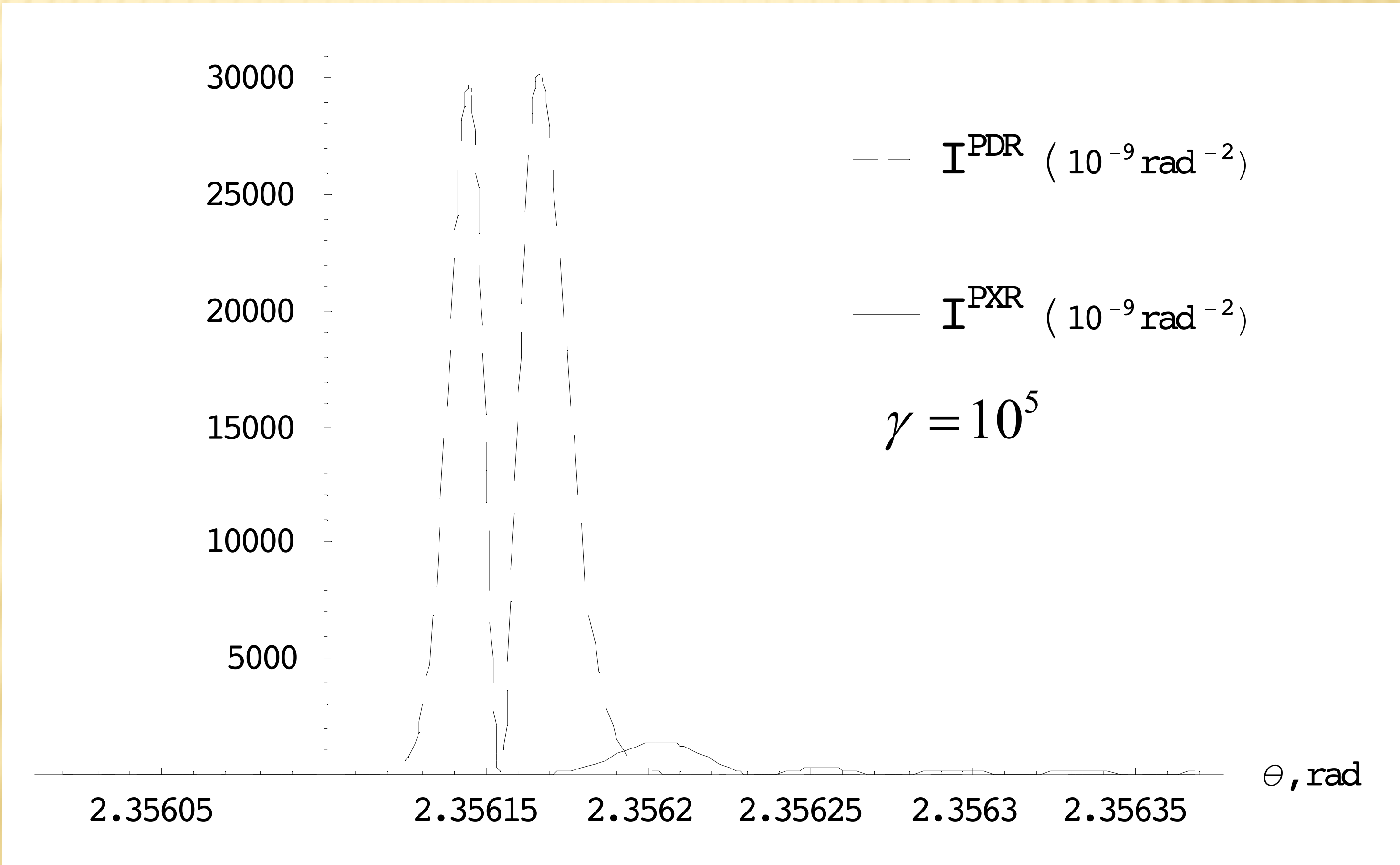
$$\mathbf{E}^r_{DR}(\mathbf{r},\omega)=-\frac{e^{ikr}}{r}\frac{ie}{2\pi}\frac{1}{v_x}\frac{\omega_p^2}{\omega^2}\frac{\sin\left(a\varphi_0/2\right)}{\varphi_0}\exp\left(-h\rho_0\right)\mathbf{k}\times\mathbf{k}\times\left(\frac{\mathbf{A}_0}{\rho_0}-i\mathbf{e}_z\right)\frac{1-\exp\left(-b\rho_0\right)\exp\left(ibk'_z\right)}{\rho_0-ik'_z}$$

$$\mathbf{E}^r_{PDR}(\mathbf{r},\omega)=-\frac{e^{ikr}}{r}\frac{ie}{2\pi}\frac{1}{v_x}\sum_{\mathbf{g}\neq0}\chi_{\mathbf{g}}\frac{\sin\left(a\varphi_{\mathbf{g}}/2\right)}{\varphi_{\mathbf{g}}}\times$$

$$\times\exp\left(-h\rho_2\right)\mathbf{k}\times\mathbf{k}\times\left(\frac{\mathbf{A}_{\mathbf{g}}}{\rho_2}-i\mathbf{e}_z\right)\frac{1-\exp\left(-b\rho_1\right)\exp\left(ib\left(k'_z-g_z\right)\right)}{\rho_1-i\left(k'_z-g_z\right)}$$

X-ray DR from periodical targets

Parametric DR и PXR, crystal target



UV and X-ray Smith-Purcell radiation

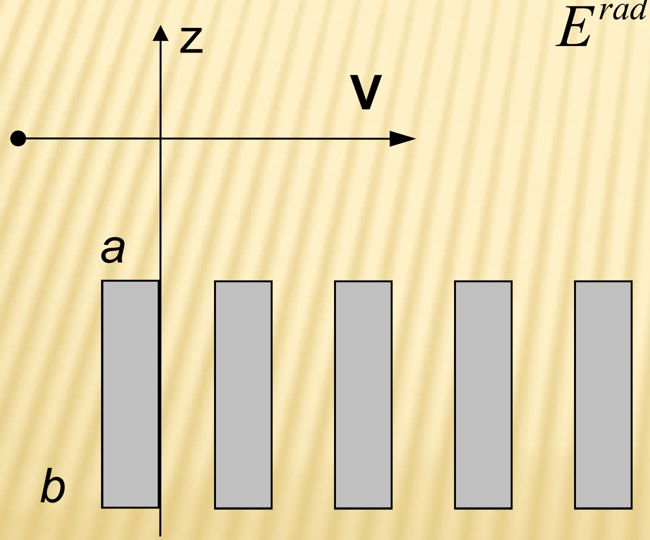
M.J. Moran,
X-ray generation by the Smith-Purcell effect,
Phys. Rev. Lett. 69, 2523 (1992).

opaque screen in X-rays ??

J.C. McDaniel, D.B. Chang, J.E. Drummond, W.W. Salisbury,
Smith-Purcell radiation in the high conductivity and plasma frequency limits,
Applied Optics 28, 4924 (1989).

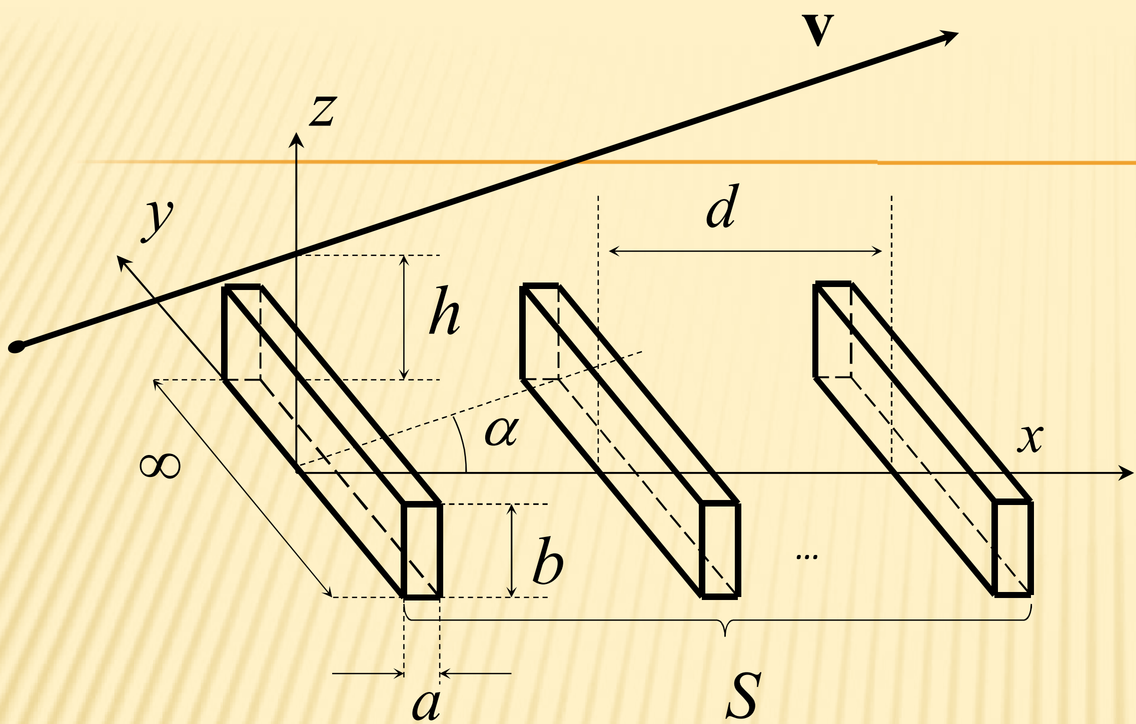
$\beta << 1$ but

$$E^{rad} \propto \exp\left\{-\frac{h\omega}{c\beta\gamma}\right\}, \quad at\ \beta << 1 \ => h = \lambda\ ??$$



UV and X-ray DR: the key theory problems is developed in

A.P. Potylitsyn, M.I. Ryazanov, M.N. Strikhanov, A.A. Tishchenko,
Diffraction Radiation from Relativistic Particles, Springer, 2010



$$\rho_0^2 = \left(\frac{\omega - v_y k'_y}{v_x} \right)^2 + k'^2_y - \omega^2 / c^2$$

$$\varphi = \frac{\omega - \mathbf{v}\mathbf{k}'}{v_x},$$

$$\mathbf{A} = \frac{\omega - v_y k'_y}{v_x} \mathbf{e}_x + k'_y \mathbf{e}_y - \mathbf{v}\omega / v^2$$

$$I^{SPR}(\mathbf{n},\omega) = \frac{e^2}{c\hbar} \left(\frac{\omega_p^2}{2\pi\omega^2} \right)^2 F_b F_S \frac{\sin^2(\varphi a/2) \exp(-2h\rho)}{(c/\omega)^2 (\rho^2 + k'^2_z)} \frac{1 - n_z^2 + \left[A^2 - (\mathbf{A}\mathbf{n})^2 \right] / \rho^2}{\left(1 - \mathbf{n}\mathbf{v}\sqrt{\varepsilon}/c\right)^2}$$

$$F_b(b,\omega) = 1 - 2\exp(-b\rho)\cos(bk'_z) + \exp(-2b\rho)$$

$$F_S = \frac{\sin^2\left(\frac{Sd}{2} \frac{\omega - \mathbf{v}\mathbf{k}}{v_x} \right)}{\sin^2\left(\frac{d}{2} \frac{\omega - \mathbf{v}\mathbf{k}}{v_x} \right)} \xrightarrow{S \gg 1} 2\pi S \sum_m \delta\left(d \frac{\omega - \mathbf{v}\mathbf{k}}{v_x} - 2\pi m \right)$$

UV and X-ray Smith-Purcell Radiation

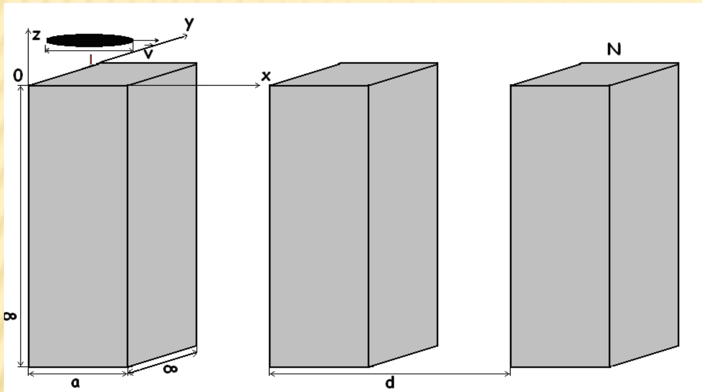
$$\frac{d^2E(\mathbf{n},\omega)}{d\Omega d(\hbar\omega)} \propto \frac{1}{\left(1-\sqrt{\varepsilon}\beta\cos\theta\right)^2}\exp\left\{-\frac{2h\omega}{c\gamma}\right\}\sum_m \delta\left(\frac{d\omega}{v}(1-\beta\cos\theta)-2\pi m\right)$$

$$\beta^{-1}-\cos\theta=\frac{\lambda m}{d}$$

There are two effective frequencies

$$\omega_c = c\gamma/h \qquad \omega_c = \gamma\omega_p$$

Coherent and incoherent X-ray DR



$$\frac{dW_S(\mathbf{n},\omega)}{d\Omega d\omega} = \frac{dW_1(\mathbf{n},\omega)}{d\Omega d\omega}\langle F_S\rangle$$

$$\frac{dW_1(\mathbf{n},\omega)}{d\Omega d\omega} = \left(\frac{\varepsilon(\omega)-1}{v\varphi}\right)^2 \frac{ce^2}{(4\pi)^2} \frac{\sin^2\left(\frac{Sd\varphi}{2}\right)}{\sin^2\left(\frac{d\varphi}{2}\right)} \left(2-2\cos(a\varphi)\right) \frac{\varepsilon k^2\cos^2\theta\sin^2\theta}{\left(\frac{1}{\gamma^2\beta^2}+\varepsilon\sin^2\theta\right)^2} e^{-\frac{2\omega}{c\beta\gamma}h}$$

$$\langle F_N\rangle = N\frac{c\beta\gamma}{\omega r_0} I_1\left(\frac{2\omega}{c\beta\gamma}r_0\right) + N(N-1)\frac{16}{r_0^2 l^2}\frac{v^4\gamma^2}{\omega^4}\sin^2\left(\frac{\omega}{2v}l\right) I_1^2\left(\frac{\omega}{c\beta\gamma}r_0\right)$$

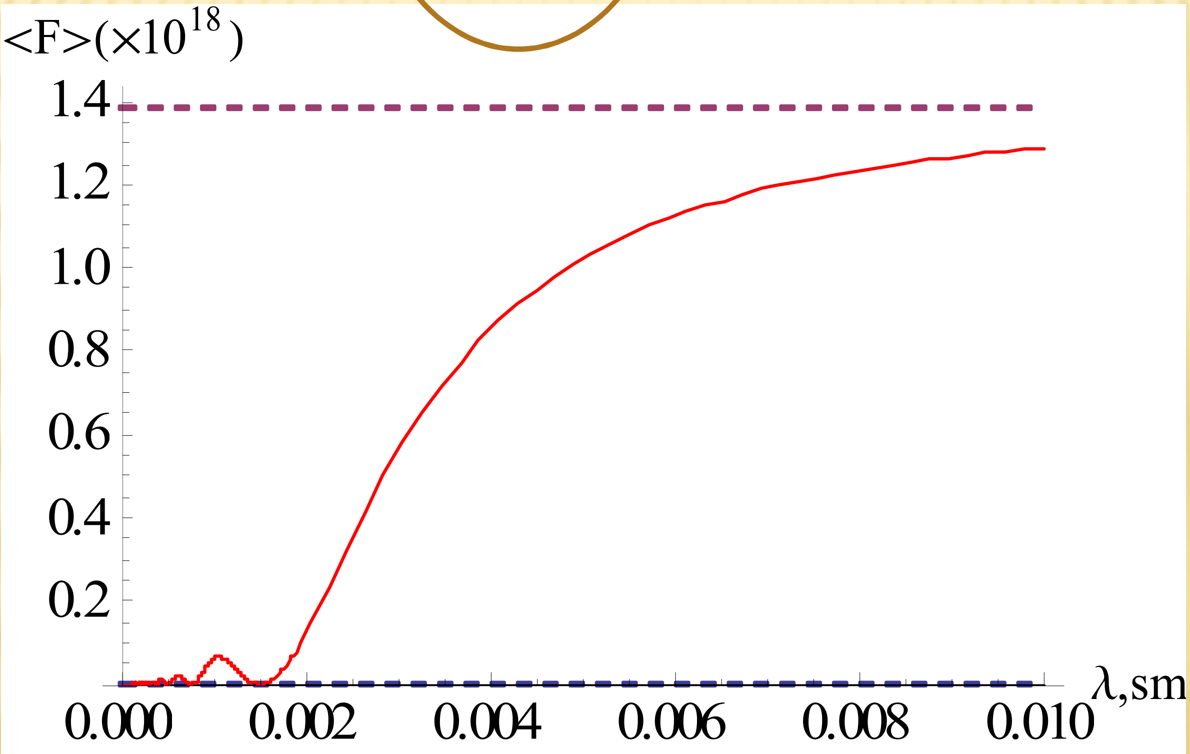
where

$$\xi = \frac{\omega}{v} \qquad \mathbf{k}' = \mathbf{k}\sqrt{\varepsilon(\omega)} = \frac{\omega}{c}\sqrt{\varepsilon(\omega)}\mathbf{n} \qquad \varphi = \xi - k\sqrt{\varepsilon(\omega)}\cos\theta \qquad \rho^2 = \frac{\omega^2}{v^2}\left(1-\frac{v^2}{c^2}\right) = \left(\frac{\omega}{c\gamma\beta}\right)^2$$

Coherent and incoherent X-ray DR

Form-factor (incoherent radiation + coherent)

$$\langle F\rangle = N\frac{2I_1\left(\frac{2\omega}{c\beta\gamma}r_0\right)}{\left(\frac{2\omega}{c\beta\gamma}r_0\right)} + 4N(N-1)\frac{\sin^2\left(\frac{\omega}{2v}l\right)}{\left(\frac{\omega}{2v}l\right)^2}\frac{I_1^2\left(\frac{\omega}{c\beta\gamma}r_0\right)}{\left(\frac{\omega}{c\beta\gamma}r_0\right)^2}$$

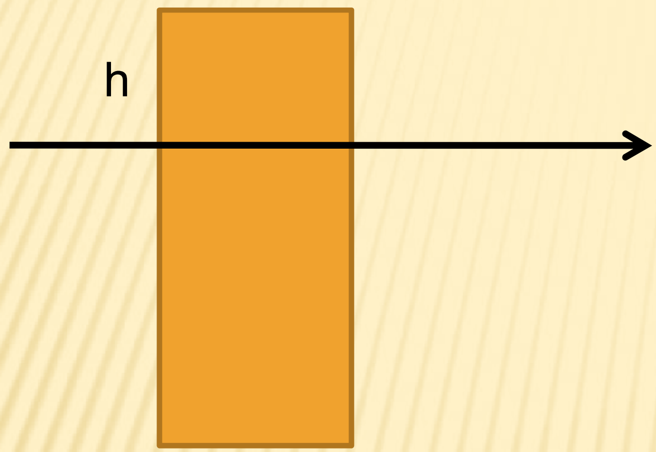


$$\gamma = 10^5$$

$$r_0 = 5\cdot 10^{-4}\ sm$$

$$l = 100 r_0$$

Coherent and incoherent X-ray polarization radiation



Bunch moves
under
the target surface

$$\left\langle \frac{dW(\mathbf{n},\omega)}{d\Omega d\omega} \right\rangle = \frac{c^2(\varepsilon-1)^2}{v^2\omega^2\left(\frac{1}{\beta}-\sqrt{\varepsilon-\sin^2\Theta}\right)^2}\frac{ce^2}{(4\pi)^2}\left(2-2\cos(a\varphi')\right)\langle F\rangle$$

$$\varphi' = \frac{\omega}{c}\left(\frac{1}{\beta}-\sqrt{\varepsilon-\sin^2\Theta}\right)$$

$$\langle F\rangle=\frac{1}{\frac{1}{\beta^4}\left(1-\beta^2\cos^2\Theta\right)^2}\left[\begin{array}{l} N\left(4Q_1'+2e^{-\frac{2\omega h}{c\beta\gamma}}Q_2'\frac{I_1\left(\frac{2\omega r_0}{c\beta\gamma}\right)}{\frac{2\omega r_0}{c\beta\gamma}}-\right.\\ -4e^{-h\frac{\omega}{c\beta\gamma}}\left(\frac{I_1\left(Z_+'\right)}{Z_+'}+\frac{I_1\left(Z_-'\right)}{Z_-'}\right)\left[Q_1'\cos\left(kh\sin\Theta\right)-Q_4'\sin\left(kh\sin\Theta\right)\right]-\\ \left.-4e^{-h\frac{\omega}{c\beta\gamma}}\left(\frac{I_1\left(Z_-'\right)}{Z_-'}-\frac{I_1\left(Z_+'\right)}{Z_+'}\right)\left[Q_4'\cos\left(kh\sin\Theta\right)+Q_1\sin\left(kh\sin\Theta\right)\right]\right)+\\ +4N\left(N-1\right)\frac{\sin^2\left(\frac{\omega l}{2v}\right)}{\left(\frac{\omega l}{2v}\right)^2}\times\left(4Q_1'\frac{J_1^2\left(kr_0\sin\Theta\right)}{k^2r_0^2\sin^2\Theta}+Q_2'e^{-2\frac{\omega h}{c\beta\gamma}}\frac{I_1^2\left(\frac{\omega r_0}{c\beta\gamma}\right)}{\left(\frac{\omega r_0}{c\beta\gamma}\right)^2}-\right.\\ \left.-4e^{-h\rho}\left(Q_1'\cos\left(kh\sin\Theta\right)+Q_4'\sin\left(kh\sin\Theta\right)\right)\frac{I_1\left(\frac{\omega r_0}{c\beta\gamma}\right)}{\left(\frac{\omega r_0}{c\beta\gamma}\right)}\frac{J_1\left(kr_0\sin\Theta\right)}{kr_0\sin\Theta}\right) \end{array}\right]$$

$$Q_1' = \frac{\sin^2\Theta}{\varepsilon}\frac{\omega^2}{c^2}\frac{1}{\beta^2}\left(1-\beta^2-\beta\sqrt{\varepsilon-\sin^2\Theta}\right)^2$$

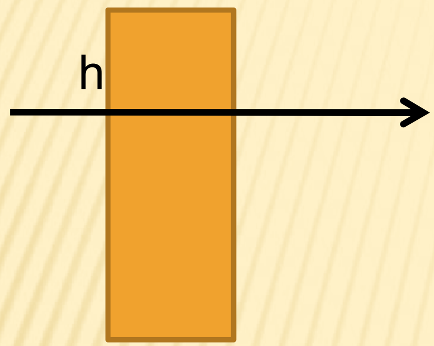
$$Q_2' = \frac{1}{\varepsilon}\frac{\omega^2}{c^2}\left(\varepsilon-\beta^2\sin^2\Theta\right)\left(1-\beta^2+\sin^2\Theta\right)$$

$$Q_4' = \frac{1}{\varepsilon}\frac{1}{\beta^2}\sin\Theta\frac{\omega^2}{c^2}\frac{1}{\gamma}\left(\sqrt{\varepsilon-\sin^2\Theta}+\beta\sin^2\Theta\right)\left(\beta\sqrt{\varepsilon-\sin^2\Theta}-1+\beta^2\right)$$

$$Z_+' = \frac{\omega r_0}{c\beta\gamma}+ikr_0\sin\Theta$$

$$Z_- = \frac{\omega r_0}{c\beta\gamma}-ikr_0\sin\Theta$$

Coherent and incoherent X-ray polarization radiation



$$h \to +\infty$$

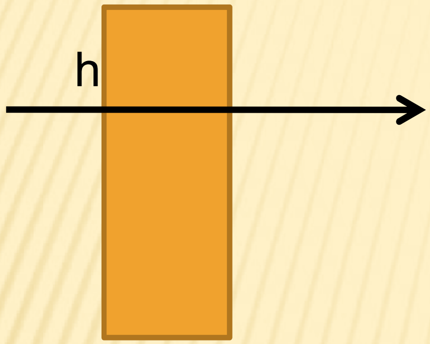
X-ray Transition Radiation

$$\left\langle \frac{dW(\mathbf{n},\omega)}{d\Omega d\omega} \right\rangle = \frac{\left(\varepsilon(\omega)-1\right)^2}{c\left(1-\beta\sqrt{\varepsilon-\sin^2\Theta}\right)^2} \frac{e^2\beta^2}{4\pi^2} \frac{\sin^2\Theta\left(1-\beta^2-\beta\sqrt{\varepsilon-\sin^2\Theta}\right)^2}{\left(1-\beta^2+\beta^2\sin^2\Theta\right)^2} \times$$

$$\times \left[N + 4N(N-1) \frac{\sin^2\left(\dfrac{\omega l}{2v}\right)}{\left(\dfrac{\omega l}{2v}\right)^2} \frac{J_1^2(kr_0\sin\Theta)}{k^2 r_0^2 \sin^2\Theta} \right]$$

G.M. Garibian and Y. Shi,
X-ray Transition Radiation,
1983

Coherent and incoherent X-ray polarization radiation



$$N = 1, \; r_0 \to 0$$

One particle, TR near edge of the target

$$\left\langle \frac{dW(\mathbf{n},\omega)}{d\Omega d\omega} \right\rangle = \frac{c^2(\varepsilon-1)^2}{v^2\omega^2\left(\frac{1}{\beta}-\sqrt{\varepsilon-\sin^2\Theta}\right)^2}\frac{ce^2}{(4\pi)^2}\left(2-2\cos(a\varphi')\right)\langle F\rangle$$

$$\langle F\rangle = \frac{k^4}{\left(\rho^2+\varepsilon k^2\sin^2\theta\right)^2}\left[4Q_1 + e^{-\frac{2\omega h}{c\beta\gamma}}Q_2 - e^{-h\frac{\omega}{c\beta\gamma}}\left[Q_1\cos\left(\sqrt{\varepsilon}kh\sin\theta\right)-Q_4\sin\left(\sqrt{\varepsilon}kh\sin\theta\right)\right]\right]$$

$$Q_1 = Q_3 = A^2\sin^2\theta + \varepsilon k^2\sin^2\theta\cos^2\theta - 2Ak\sqrt{\varepsilon}\sin^2\theta\cos\theta$$

$$Q_2 = A^2\sin^2\theta + \rho^2\cos^2\theta + \varepsilon k^2\sin^4\theta\frac{A^2}{\rho^2} + \varepsilon k^2\sin^2\theta\cos^2\theta$$

$$Q_4 = \sqrt{\varepsilon}k\rho\sin\theta\cos^2\theta - \sqrt{\varepsilon}k\frac{A^2}{\rho}\sin^3\theta + \varepsilon k^2\frac{A}{\rho}\sin^3\theta\cos\theta - \rho A\sin\theta\cos\theta$$

A.A. Tishchenko, M.N. Strikhanov, A.P. Potylitsyn, NIMB **227**, 63 (2005).

Discussion

$$\frac{dW_S(\mathbf{n},\omega)}{d\Omega d\omega} = \frac{dW_1(\mathbf{n},\omega)}{d\Omega d\omega}\langle F_N \rangle$$

$$\langle F_N \rangle = NG_{incoh} + N(N-1)G_{coh}$$

Incoherent form-factor in X-ray polarization radiation?!

Yes, when near the target edge (diffraction radiation).

Usually

$$\langle F_N \rangle = N + N(N-1)G$$

$$G = \iint d^3r d^3r' f_2(\mathbf{r},\mathbf{r}') \exp(i\boldsymbol{\xi}\mathbf{r})$$

$$\mathbf{E}_N = \mathbf{E}_1 \sum_{n=1}^{N} \exp\left(i\psi_n\right)$$

$$\left|\mathbf{E}\right|^2 = \sum_i E_i E_i^* + \sum_i \sum_{j\neq i} E_i E_j^*$$

$$\left\langle F_N \right\rangle = NG_{incoh} + N\left(N-1\right)G_{coh}$$

$$if\ \psi_n\ is\ real,\ \left|\exp\left(i\psi_n\right)\right| = 1 \quad \Rightarrow \quad G_{incoh} = 1$$

$$Is\ \psi_n\ always\ real?\quad No!$$

$$\mathbf{E}(\mathbf{r},\omega) = -\frac{e\left(\varepsilon(\omega)-1\right)}{4\pi}\frac{1}{v_x}\frac{e^{ikr}}{r}\left(\frac{e^{ia\varphi}-1}{\varphi}\right)\left(\frac{1}{\rho - ik_z}\right)\frac{\omega^2}{c^2}\left[\mathbf{n}\left[\mathbf{n}\left(\frac{\mathbf{A}}{\rho} - i\mathbf{e}_z\right)\right]\right]\sum_{m=1}^{M} e^{-i\xi x_m} e^{-ik_y y_m} e^{-\rho z_m}$$

$$\rho^2 = \left(\xi\right)^2 + \left(k_y\right)^2 - \left(\frac{\omega}{c}\right)^2 \qquad \xi = \frac{\omega - k_y v_y}{v_x}$$

Coulomb field, decreasing + target edge

Thank you
for your attention!