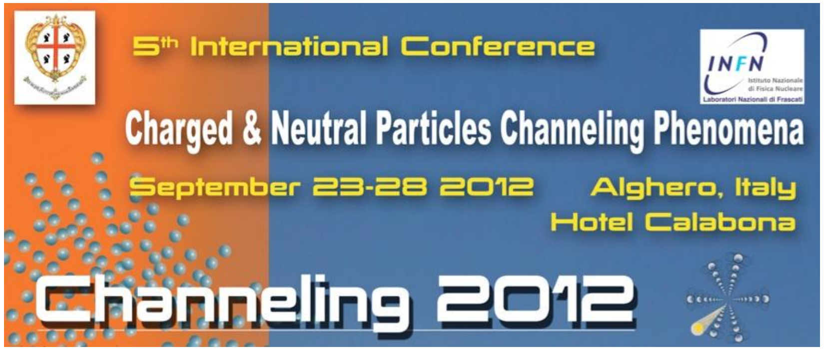
High-Order Diffraction Reflection and DCR

K.B. Korotchenko[1], Yu.P. Kunashenko[1,2]

[1]Tomsk Polytechnic University,Tomsk, Russia

[2]Tomsk State Pedagogical University,Tomsk, Russia



$\omega_{nB} > \omega_B$

Diffracted Channeling Radiation (DCR) is a new kind of x-ray radiation [1,2]. DCR arises due to spontaneous transitions between the transverse energy levels of channeled electrons with photon emission under the Bragg diffraction conditions.

As a rule one use first-order diffraction reflection i.e., the Bragg frequency

$$\omega_{nB} = \frac{n\pi c}{d_p \sin\theta_B}, \quad n = 1$$

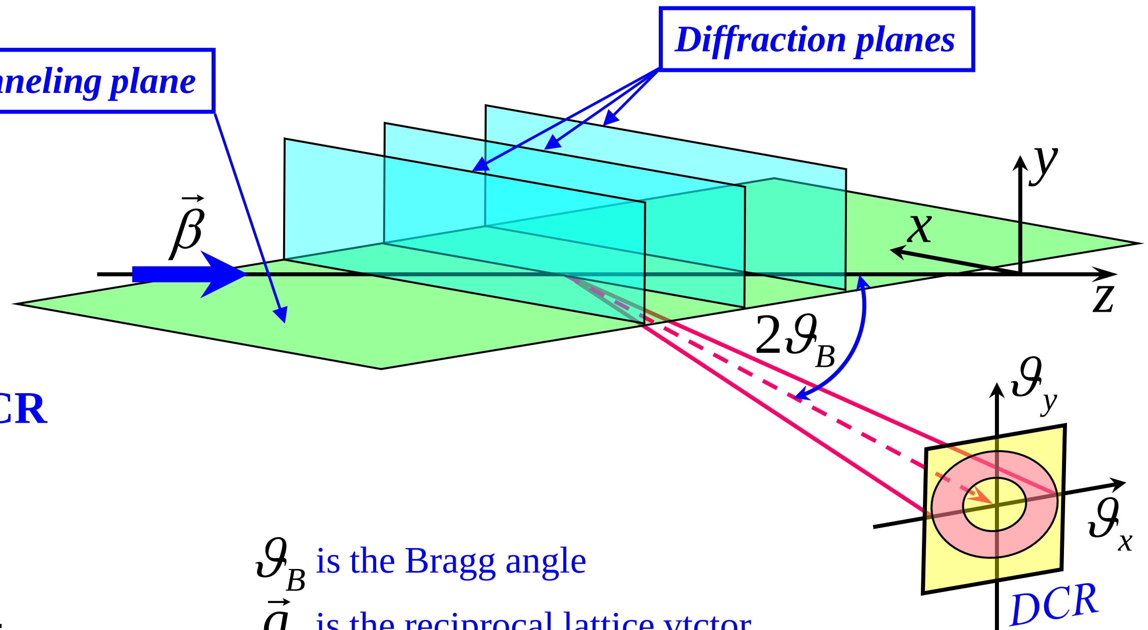
In this case the dipole approximation satisfactory describes the DCR angular distribution [1,2]. But for high-order diffraction reflection we have $\omega_{nB} > \omega_{1B}$ and condition of dipole approximation is broken.

Therefore we have derived the exact (without dipole approximation) formulas for angular distribution of DCR for planar channeling. The goal of our repot is compare the results obtained by exact formula of DCR angular distribution and ones in dipole approximation.

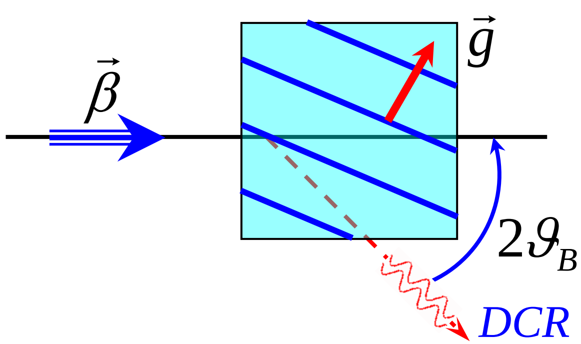
1. Yabuki R, Nitta H, T. Ikeda T, Ohtsuki Y H 2001 Phys. Rev. B **63** 174112
2. Bogdanov O V, Korotchenko K B, Pivovarov Yu L 2007 *JETP Lett*. ***85*** No 11 555–559

Scheme of the DCR formation at planar channeling

The DCR appears when an electron passes through a crystal in the channeling regime. The channeling means that the electron is in a bound state with the crystal plane (axis).



The scheme of observing the **angular distribution** of **DCR**



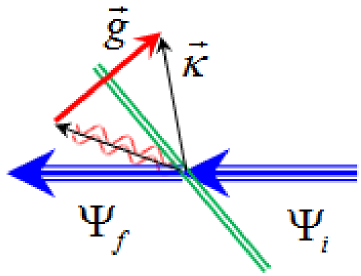
ϑ_B is the Bragg angle

$\vec{g}$ is the reciprocal lattice vtctor

$\vec{v} = c\vec{\beta}$ is the electron velocity

Probability of DCR

The DCR appears when an electron passes through a crystal in the channeling regime. The channeling means that the electron is in a bound state with the crystal plane (axis). Therefore the DCR cannot be described by ordinary perturbation theory of QED. The modied Feynman rules for quantum electrodynamics in a crystal were discussed in Refs. [1]. The radiation from channeled particle according to Ref. [1] is described by first order perturbation theory.



Probability of DCR

$$d\sigma = \frac{2\pi}{\hbar}\left|M_{if}\right|^2 \delta\left(\hbar\omega - (E_i - E_f)\right)\frac{L}{2\pi} d^3\kappa$$

$$M_{if} = -e\left\langle \Psi_f(\vec{r})\right|\vec{A}^*(\vec{r})\vec{\gamma}\left|\Psi_i(\vec{r})\right\rangle \quad \vec{A}(\vec{r}) = \sum_n \left(\vec{A}_{0\kappa}\exp(\mathbf{i}\vec{\kappa}\vec{r}) + \sum_{g\neq 0}\vec{A}_{g\kappa}\exp[\mathbf{i}(\vec{\kappa}+\vec{g})\vec{r}]\right)$$

$$M_{if} = -e\left\langle \Psi_f(\vec{r})\right|\vec{\gamma}\vec{A}^*_{0\kappa}\exp(-\mathbf{i}\vec{\kappa}\vec{r}) + \sum_{g\neq 0}\sum_{\tau}\vec{\gamma}\vec{\varepsilon}_{g\tau}A^{\tau *}_{g\kappa}\exp[-\mathbf{i}(\vec{\kappa}+\vec{g})\vec{r}]\left|\Psi_i(\vec{r})\right\rangle$$

$\vec{\gamma}$ are gamma matrices

$\vec{\varepsilon}_{g\tau}$ is the polarization vector

$$M_{if}^{(-g)\tau} = -e\vec{\varepsilon}_{g\tau}A^{\tau *}_{g\kappa}\left\langle \Psi_i(\vec{r})\right|\vec{\gamma}\exp[-\mathbf{i}(\vec{\kappa}+\vec{g})\vec{r}]\left|\Psi_i(\vec{r})\right\rangle$$

$$\left|\Psi_i(\vec{r})\right\rangle = \sqrt{E_{i\parallel} + mc^2/2E_{i\parallel}}\left|u_i\psi_i(\vec{r})\right\rangle$$

$$\psi_i(\vec{r}) = L^{-N/2}\phi_i(\vec{r}_\perp)\exp(\mathbf{i}p_{i\parallel}\vec{r}_\parallel/\hbar)$$

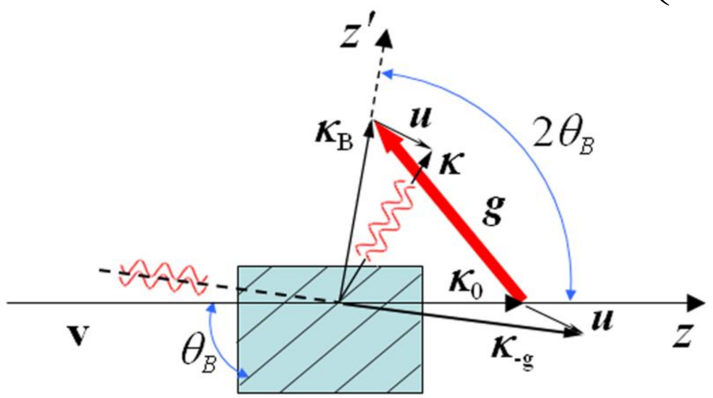
1. Kimball J. C. and Cue N. 1985 Physics Reports (Review Section of Phys.Lett.) 125 No 2 69 – 101

DCR matrix element

The transverse motion of the channeled electron can be described by the Schrödinger like equation with relativistic mass

$$\hat{H}\phi_i(\vec{r}_\perp) = \left(\frac{\hat{\vec{p}}_\perp^{\,2}}{2m\gamma_i} + U(\vec{r}_\perp)\right)\phi_i(\vec{r}_\perp) = E_{i\perp}\phi_i(\vec{r}_\perp), \quad \gamma_i = E_{i\|}/mc^2$$

$$i\hbar\hat{\vec{p}}_\perp^{\,2} = m\gamma_i[\hat{\vec{r}}_\perp, \hat{H}]$$



$$\vec{r}_{\perp if} = \langle\phi_f^*(\vec{r}_\perp)\vec{r}_\perp \exp(-\mathbf{i}\vec{\kappa}_{-g\perp}\vec{r}_\perp)\phi_i(\vec{r}_\perp)\rangle_\perp,$$

$$\mathbf{i}F_{if} = -\langle\phi_f^*(\vec{r}_\perp)\exp(-\mathbf{i}\vec{\kappa}_{-g\perp}\vec{r}_\perp)\phi_i(\vec{r}_\perp)\rangle_\perp,$$

DCR matrix element

$$M_{if}^{(-g)\tau} = \mathbf{i}\frac{e}{mc\gamma}\frac{A_{\circ\kappa}^{\tau *}}{\sqrt{(1+W_\tau^2)}}\delta\left(\frac{\Delta\vec{p}_{if\|}}{\hbar}\,|\,\vec{\kappa}_{-g\|}\right)\times$$
$$\times\ (\vec{\varepsilon}_{g\tau\|}\vec{p}_\| F_{if} + m\gamma\Omega_{if}\vec{\varepsilon}_{g\tau\perp}\vec{r}_{\perp if}).$$

$$\Omega_{if} = (E_{i\perp} - E_{f\perp})/\hbar.$$

Exact formula (without dipole approximation)
for angular distribution of the DCR at planar channeling

Angular distribution DCR

$$\frac{d^3N_{if}}{d\theta_x d\theta_y dz} = \frac{\alpha\omega_B}{16\pi c\sin^2\theta_B}\left(\frac{|Q_{\pi x}|^2}{1+W_\pi^2} + \frac{|Q_{\sigma y}|^2}{1+W_\sigma^2}\right)P_{\circ i}$$

$$Q_{\pi x} = F_{if}\theta_x,\ \ Q_{\sigma y} = F_{if}\theta_y - P_\sigma y_{if}(\Omega_{if}/c)$$

$$F_{if} = \langle\phi_f^* \exp(-\mathbf{i} y\theta_y\omega_B/c)\phi_i\rangle_y,$$

$$y_{if} = \langle\phi_f^* y \exp(-\mathbf{i} y\theta_y\omega_B/c)\phi_i\rangle_y.$$

$$W_\tau = \frac{1}{2|\chi_g|P_\tau}\left(R - \frac{|\chi_g|^2 P_\tau^2}{R}\right),$$

$$\tau = (\pi,\sigma),$$

$$R = [\theta_x - \frac{\Omega_{if}}{\omega_B}\cos\theta_B]^2 + \theta_y^2 + R_\circ,$$

$$R_\circ = \theta_{kin}^2 - 2\frac{\Omega_{if}}{\omega_B},\ \ \theta_{kin}^2 = \gamma^{-2} + |\chi_0|,$$

$$\theta_x^{\max} = X_\circ \pm R_\pm,$$

$$R_\pm^2 = -\gamma^{-2} + Y_o + (\chi_o \pm \chi_g),$$

$$X_o = \Omega_{if}\cos\theta_B d_p/\pi c,\ Y_o = \Omega_{if}\sin\theta_B d_p/\pi c.$$

$$(\theta_x^{\max} - X_\circ)^2 + (\theta_y^{\max})^2 = R_\pm^2,$$

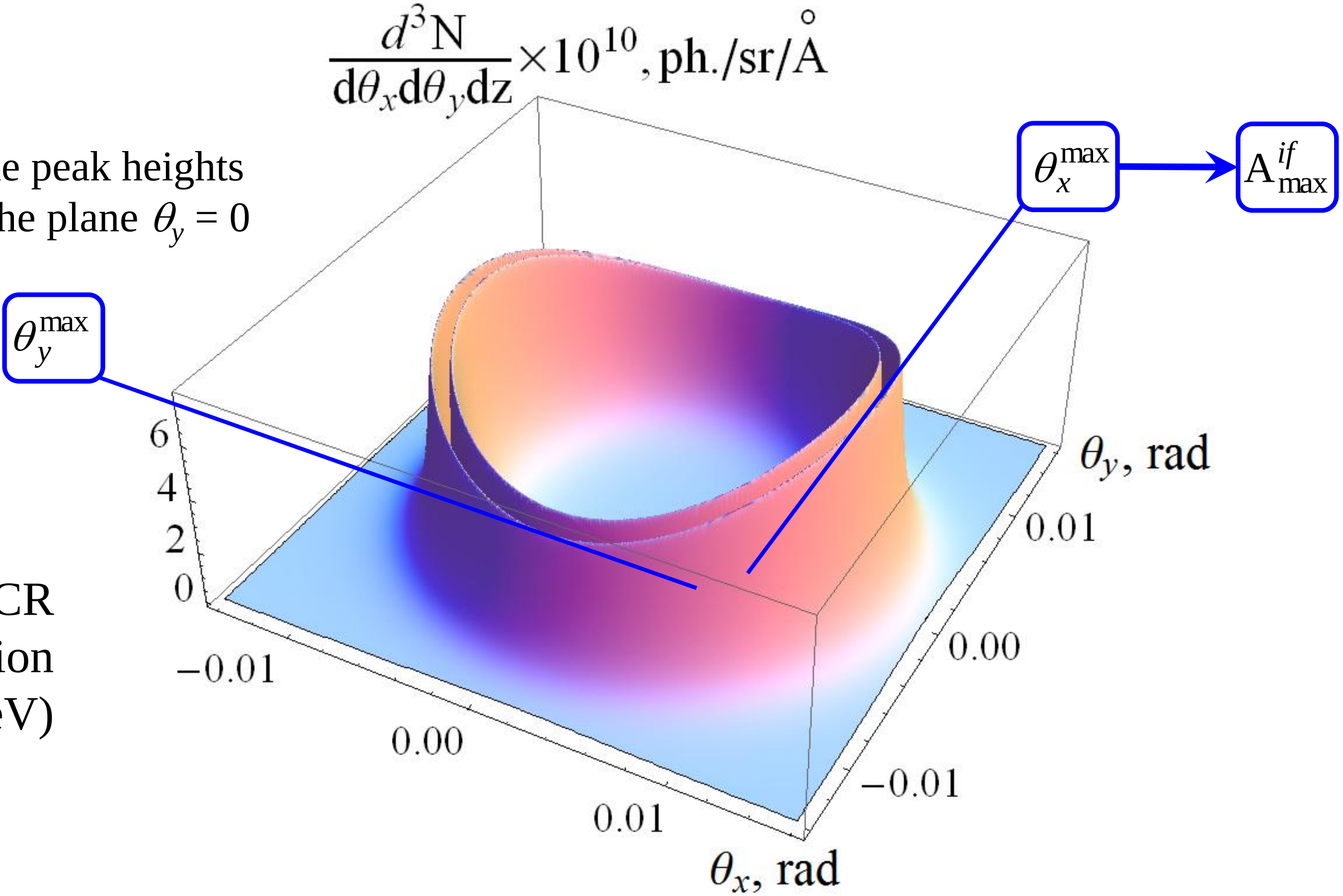
χ_o and χ_g are Fourier components of the electric susceptibility, $\Omega_{if} = (E_{i\perp} - E_{f\perp})/\hbar$.

Angular distribution of the DCR
(electron beam with energy 15 MeV at (220) Si channeling)

A^{if}_{max} is the maxima of the peak heights of the DCR along the plane $\theta_y = 0$

Contribution to the DCR gives only one transition $2 \rightarrow 1$ (Ω_{21}=4 eV)

$\hbar\omega_B = 7.133$ keV



DCR from planar channeled electrons

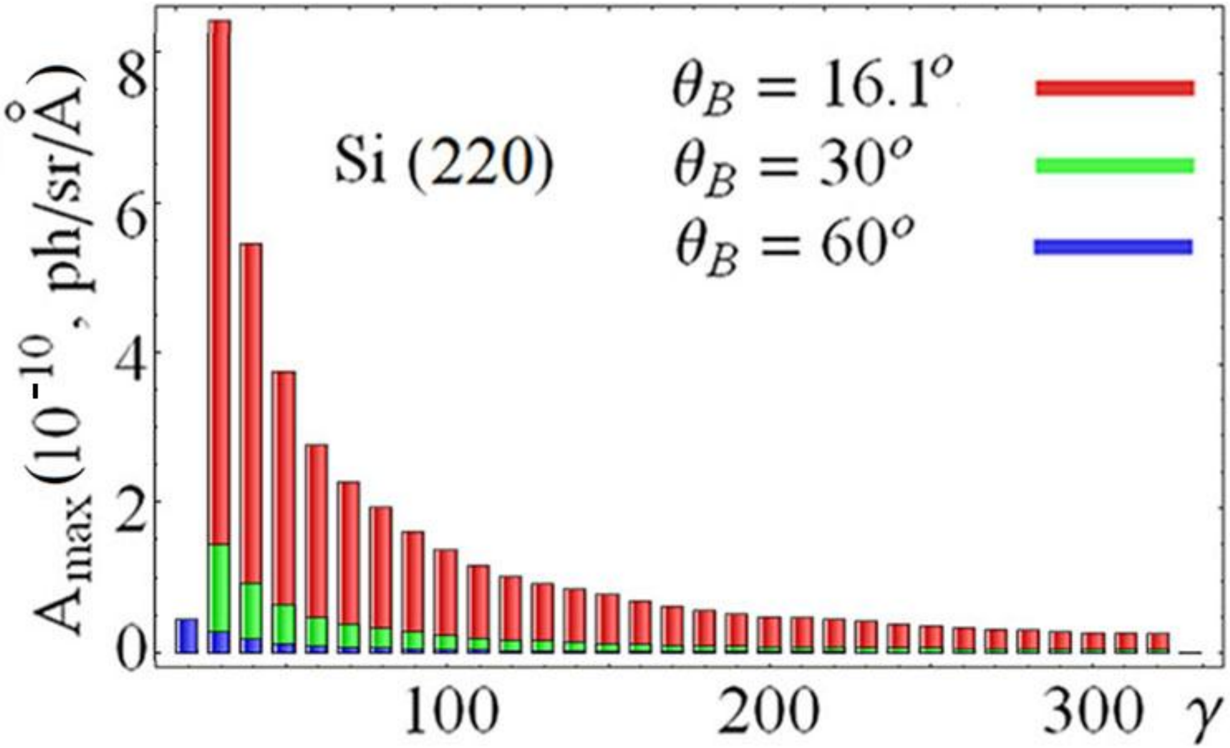
$$A^{if}_{max} = \frac{\alpha\omega_B}{16\pi c \sin^2\theta_B} V_{if} P_{\circ i} \, ,$$

$$V_{if} = \left| F_{if}\theta_y - y_{if}\frac{\Omega_{if}}{c} \right|^2 + f_{xy} \, | F_{if} |^2 ,$$

$$f_{xy} = \frac{8(1+\cos 4\theta_B)}{(3+\cos 4\theta_B)^2}\theta_x^2 \, .$$

The maxima of the peak heights of the DCR

$$A_{max} = \frac{\alpha}{16 d_p c^2} \frac{(y_{max}\Omega_{max})^2}{\sin^3\theta_B} P_{max} \, .$$



DCR from planar channeled electrons
(relative difference ε)

ε is the relative difference between the heights of DCR peaks calculated along the plane $\theta_x = 0$ by **exact** formulae and by one in the **dipole approximation**

$\theta_y = 0$

$$\Delta A^{if}_{\max|x} = A^{if}_{\max|x} - \tilde{A}^{if}_{\max|x} \equiv 0.$$

$\theta_x = 0$

$$\Delta A^{if}_{\max|y} = \frac{\alpha\omega_B}{16\pi c \sin^2\theta_B}\Delta U_{if} P_{\circ i}.$$

$$\Delta U_{if} = U_{if} - \tilde{U}_{if},$$

$$U_{if} = \left| F_{if}\theta_y^{\max} - y_{if}\frac{\Omega_{if}}{c}\right|^2 =$$

$$= \left|\frac{\omega_B}{c} y_{if}\right|^2 \left|\frac{F_{if}c}{y_{if}\omega_B}\theta_y^{\max} - \frac{\Omega_{if}}{\omega_B}\right|^2,$$

$$\tilde{U}_{if} = \left|\frac{\omega_B}{c}\tilde{y}_{if}\right|^2 \left((\theta_y^{\max})^2 - \frac{\Omega_{if}}{\omega_B}\right)^2,$$

$$\Delta A^{if}_{\max|y} > 0$$

$$\frac{F_{if}c}{y_{if}\omega_B} \simeq \theta_y^{\max}$$

For electron beam with energy 255 MeV at (220) Si channeling

$$\varepsilon = \frac{\Delta A^{if}_{\max|y}}{A^{if}_{\max|y}} = \frac{\Delta U_{if}}{U_{if}} \simeq \frac{|y_{if} - \tilde{y}_{if}|^2}{|y_{if}|^2} < 0.1\%.$$

The dipole approximation correctly describes the DCR angular distribution (for the first order diffraction reflection)

$A^{if}_{\max|x(y)}$ is the maxima of the peak heights of the DCR along the plane $\theta_y = 0$ ($\theta_x = 0$).

$\tilde{A}^{if}_{\max|x(y)}$ is one, but calculated by formulae in **dipole approximation**

Angular distribution of the DCR

for higher-order diffraction reflection

$$\omega_{Bn} = n\frac{\pi c}{d_p \sin\theta_B}$$

For electron beam with energy 150 MeV at (220) Si channeling

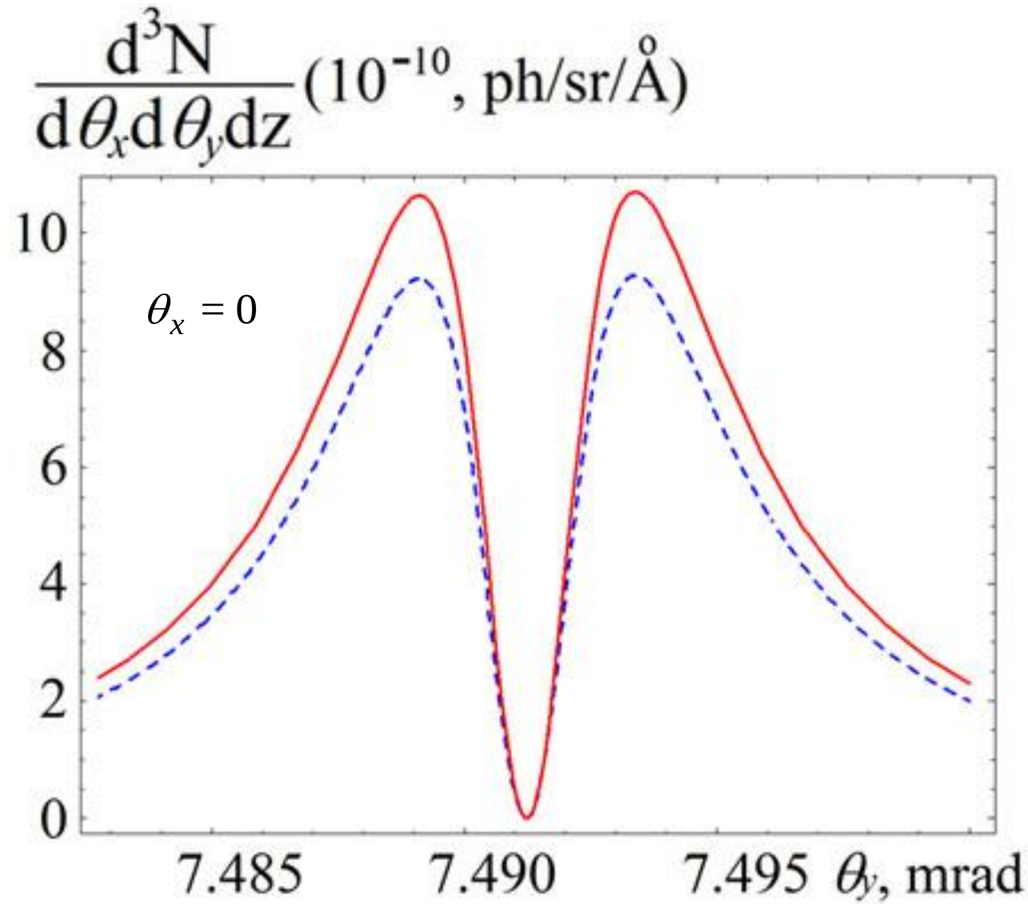
$$\varepsilon = \frac{\Delta A^{if}_{\max|y}}{A^{if}_{\max|y}} \simeq 13.1\%.$$

$$2\theta_B = 10^\circ$$

$$n = 4$$

$$\hbar\omega_B = 90.8 \quad \text{keV}$$

Contribution to the DCR gives transition $6 \rightarrow 5$



$$\Delta\theta_y \approx 10\,\mu\text{rad}$$

Conclusions

- We have derived the exact (without dipole approximation) formulas for angular distribution of DCR as for planar so for axial channeling.
- The dipole approximation satisfactory describes the first DCR diffraction reflection at planar channeling.
- For the higher-order diffraction reflections the relative difference ε significantly increases - the applicability of the dipole approximation is broken.

Thank you for attention