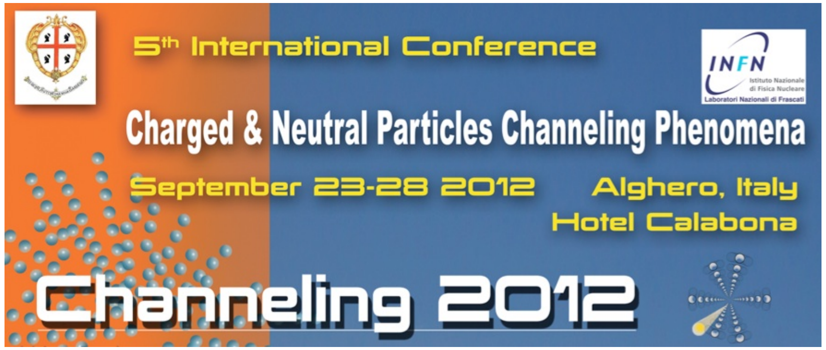

Influence of Slowing Down in the Radiator on the Cherenkov Radiation Angular Distributions from Relativistic Heavy Ions at FAIR, SPS and LHC Energies

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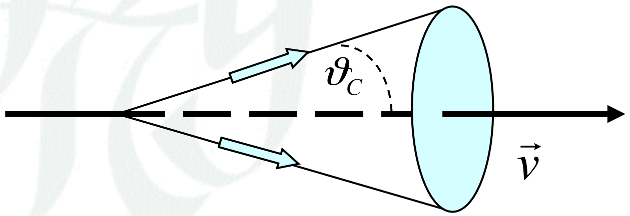
- Introduction: calculation of Cherenkov radiation (ChR)
- Slowing down of relativistic heavy ions (RHI)
- Dependence of ChR angular distribution width on RHI slowing down
- Calculation of RHI slowing down
- Lindhard-Sorensen theory
- Influence of RHI slowing down on ChR angular distribution
- Conclusion

Introduction: calculation of ChR



$$\vartheta_C = \arccos \frac{c}{n\,\mathrm{v}} = \arccos \frac{1}{\beta n}$$

The spectral-angular distribution of the ChR emitted by RHI moving rectilinearly along the trajectory **x(t)** in a radiator with a decreasing velocity:

$$\frac{\mathrm{d}I}{\mathrm{d}\omega\,\mathrm{d}(\cos\theta)} = \left(\frac{ze\sin\theta}{c}\right)^2 \frac{\omega^2 n}{2\pi c} \left| \int_0^T v(t) \exp\left[i\omega \left\{ t - \frac{n}{c} x(t)\cos\theta \right\} \right] \mathrm{d}t \right|^2$$

Here:

c - speed of light in vacuum, ze – ion charge, ω - wave vector and frequency of the Cherenkov photon,

$T = \int_0^L \mathrm{d}x / v(x)$ - RHI time-of-flight through radiator

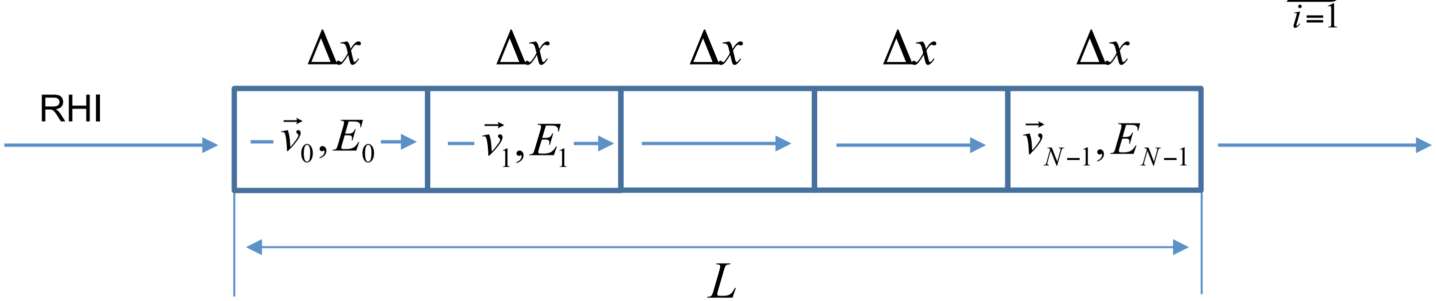
$v(t) = \mathrm{d}x(t)/\mathrm{d}t$ - <u>velocity that decreases because of RHI slowing down</u>

L and n – thickness and refractive index of the radiator

RHI slowing down

To calculate $$I = \int_0^T v(t)\exp\left[i\omega\left\{t - \frac{n}{c}x(t)\cos\theta\right\}\right]\mathrm{d}t$$

we divide a radiator of thickness L into N >>1 thin slices: $$L = \sum_{i=1}^{N}\Delta x$$



The entrance energy (velocity) into every next slice is calculated according to RHI energy loss in previous slice:

$$E_i = E_{i-1} - \frac{\mathrm{d}E_{i-1}}{\mathrm{d}x}\Delta x$$

Here, $\mathrm{d}E/\mathrm{d}x$ - stopping power of the radiator

After passage through every slice of the radiator, the energy (velocity) of the RHI changes, that's why the emission angle $\vartheta_C = \arccos\frac{c}{n\mathrm{v}}$ of ChR also changes.

Dependence of ChR angular distribution width on RHI slowing down

ChR angular distribution width [1-2] may be expressed as:

$$\Delta\theta_S = \frac{1}{\gamma_0^2\beta_0^2 E_0} L \frac{1}{\sqrt{(n(\lambda)\beta_0)^2 - 1}} S(E_0)$$

$$S(E_0) = -\left.\frac{\mathrm{d}E}{\mathrm{d}x}\right|_{x=0}$$

stopping power of the radiator

The slowing down of RHI in a radiator can drastically change the ChR angular distributions at RHI energies of order of 1 GeV/u, as shown earlier in [1-3].

[1] O.V. Bogdanov, E.I. Fiks, Yu.L. Pivovarov, Journal of Physics. V. **357** (2012) 012002 pp.1-9

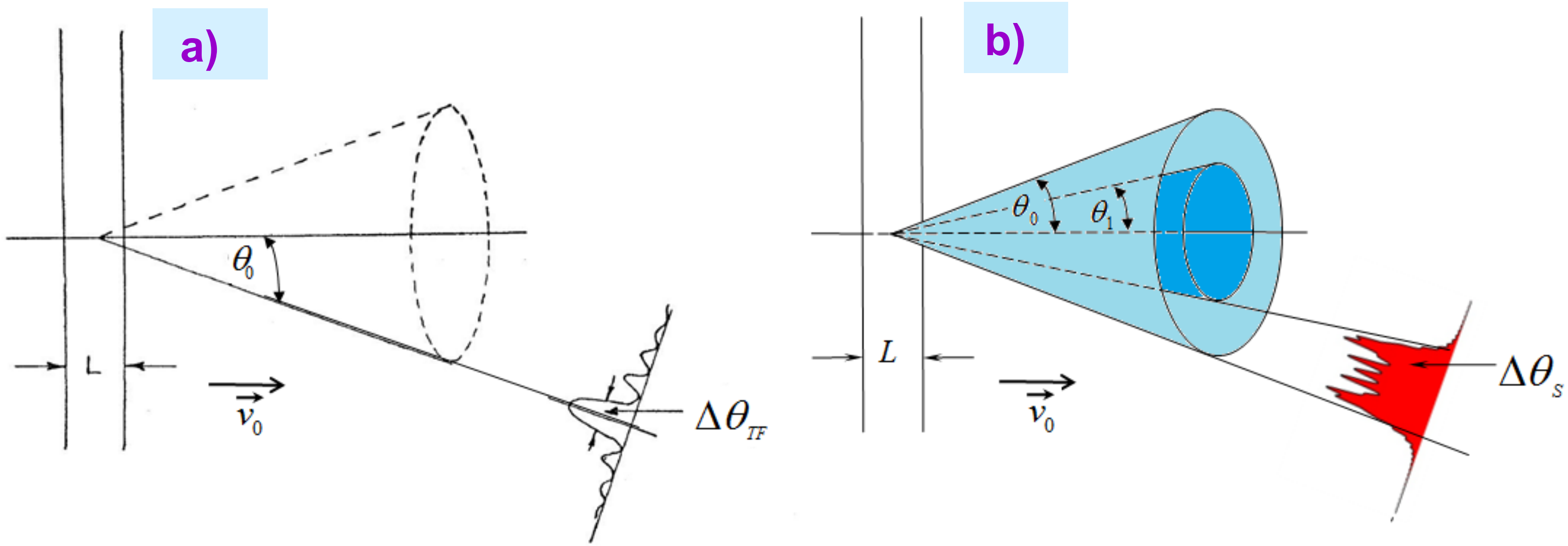
[2] O.V. Bogdanov, E.I. Fiks, Yu.L. Pivovarov, Zh. Exp. Teor. Fiz. V. **142** № 2 (8) (2012) pp. 1-11

[3] V.R. Altapova, O.V. Bogdanov, Yu.L. Pivovarov, Nucl. Instr. and Meth. B **256** (2009) pp. 109-113.

Dependence of ChR angular distribution width on RHI slowing down

At RHI energies of order of 1 GeV/u ChR emission angles $\vartheta_C = \arccos \frac{c}{n\,\mathrm{v}}$

& angular distribution significantly change because of RHI slowing down:



a): Fine structure of ChR angular distribution near θ_C, Tamm-Frank theory

b): New structure and broadening due to slowing down in radiator

Calculation of RHI slowing down

The standard formula for RHI energy-loss reads:

$$-\frac{dE}{dx} = 4\pi z^2 N_A r_e^2 m_e c^2 \frac{Z}{A}\frac{1}{\beta^2} L_{stopping}$$

Here $L_{stopping} = L_0 + \Delta L$ - the stopping logarithm:

ΔL - relativistic corrections

Since Bethe-Bloch and Fermi, L_0 for heavy ions is expressed as:

$$L_0 = \ln\left(\frac{2m_e c^2 \beta^2 \gamma^2}{I}\right) - \beta^2 - \frac{\delta}{2}$$

$m_e c^2$ - electron rest energy, r_e - electron radius, N_A - Avogadro number;

$\gamma = 1/\sqrt{1-\beta^2}$ - relativistic factor

Z, A and I - charge, mass number and mean ionization potential of the target:

δ - correction to take account of the “density effect”

Estimation of RHI slowing down

Bethe-Bloch formula [1]: $$L_{stopping} = L_0 = \ln\left(\frac{2m_e c^2 \beta^2 \gamma^2}{I}\right) - \beta^2$$

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = 2\pi z^2 N_A r_e^2 m_e c^2 \frac{Z}{A}\frac{1}{\beta^2}\left(\ln\left(\frac{2m_e c^2 \beta^2 \gamma^2}{I}\right) - \beta^2\right)$$

For high energies: Bethe-Bloch with density effect:

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = 2\pi z^2 N_A r_e^2 m_e c^2 \frac{Z}{A}\frac{1}{\beta^2}\left(\ln\left(\frac{2m_e c^2 \beta^2 \gamma^2}{I}\right) - \beta^2 - \frac{\delta}{2}\right)$$

Recent LS theory [2] is more correct, is consistent with experimental data on RHI energy loss obtained in GSI and CERN. LS theory: correction ΔL_{LS} to the Bethe-Bloch stopping logarithm, especially important for RHI with high Z (takes into account finite size of projectile nucleus).

[1] H. Bethe, Z. Phys. 76 (1932) 293

[2] J. Lindhard and A.H. Sorensen, Phys. Rev. V. **53**, N.4 (1996) pp. 2443-2456.

Lindhard-Sorensen theory [1]

$$L_{stopping} = L_0 + \Delta L_{LS}$$

$$\Delta L_{LS} = \sum_{k=1}^{\infty}\left[\frac{k}{\eta^2}\frac{k-1}{2k-1}\sin^2(\delta_k - \delta_{k-1}) + \frac{k}{\eta^2}\frac{k+1}{2k+1}\sin^2(\delta_{-k} - \delta_{-k-1}) + \frac{4k}{4k^2-1}\frac{1}{\gamma^2 k^2 + \eta^2} - \frac{1}{k}\right] + \frac{\beta^2}{2}$$

δ_k - relativistic Coulomb phase shift in scattering of e- on finite-size projectile nucleus, $\eta = (Z/137)\beta$

LS theory is implemented into **ATIMA** [2]:

ATIMA is a program developed in GSI which calculates various physical quantities characterizing the slowing-down of protons and heavy ions in matter for specific kinetic energies ranging from **1 keV/u** to **450 GeV/u**:

Ultra relativistic case : $\gamma >> 1$

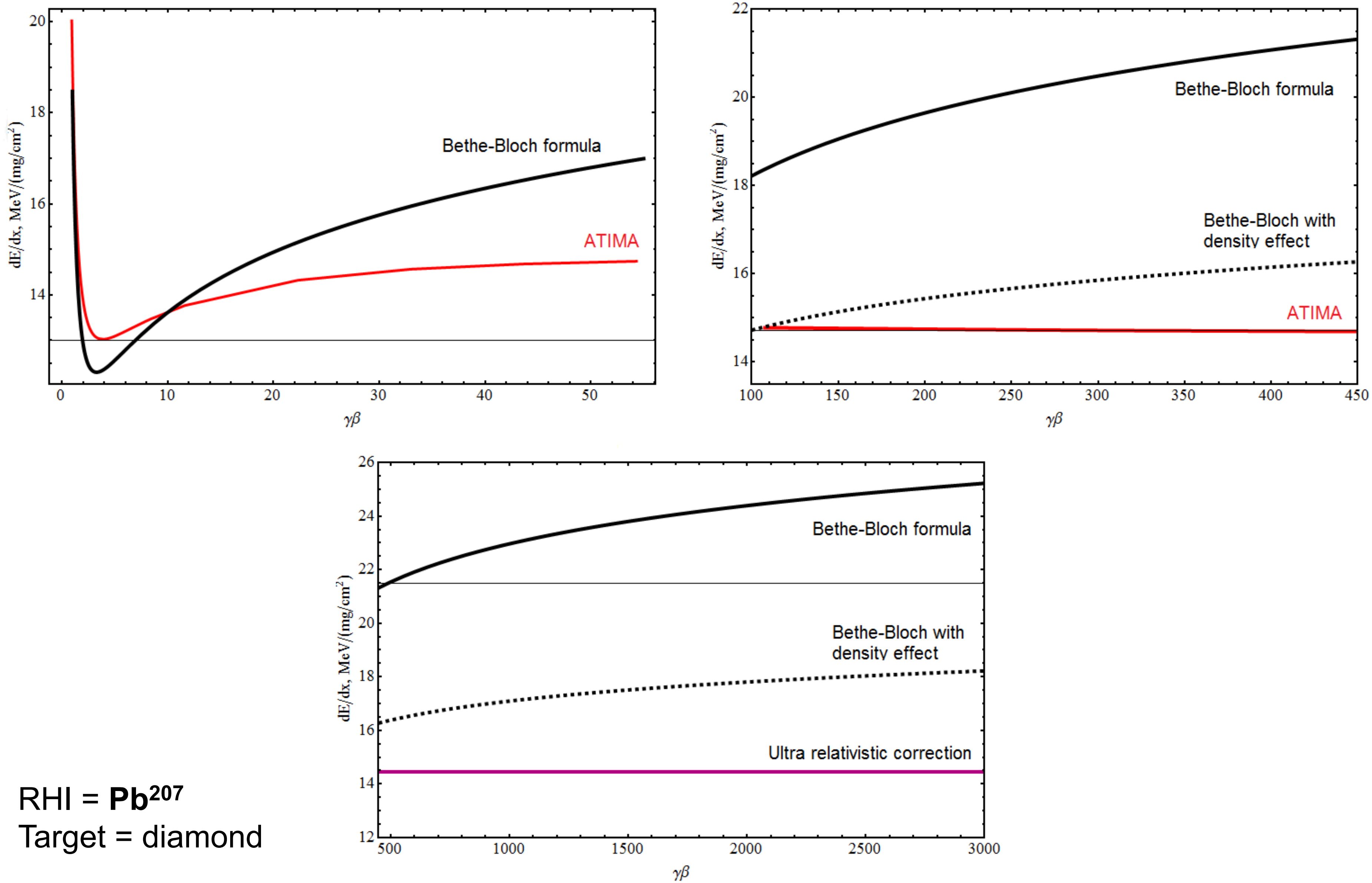
$$L_{stopping} = L_0 + \Delta L = \ln\left(\frac{2c}{R\omega_p}\right) - 0.2$$

$R = 1.18A^{1/3}$ fm - is projectile nuclear radius: ω_p - plasma frequency of the radiator material

[1] J. Lindhard and A.H. Sorensen, Phys. Rev. V. **53**, N.4 (1996) pp. 2443-2456.

[2] http://www-linux.gsi.de/~weick/atima/

Dependence of RHI energy loss $\mathrm{d}E/\mathrm{d}x$ on the RHI beam parameter $\gamma\beta$:

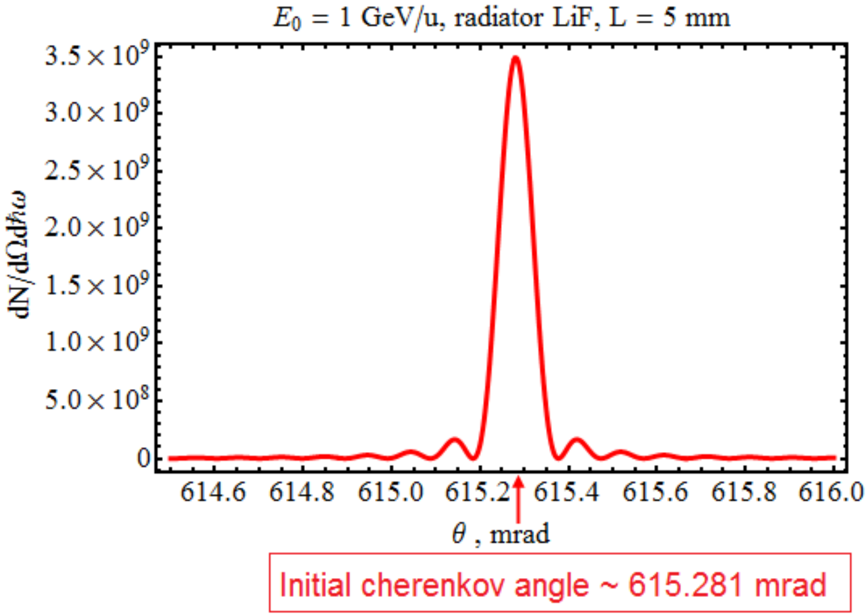


RHI = **Pb**^{207}
Target = diamond

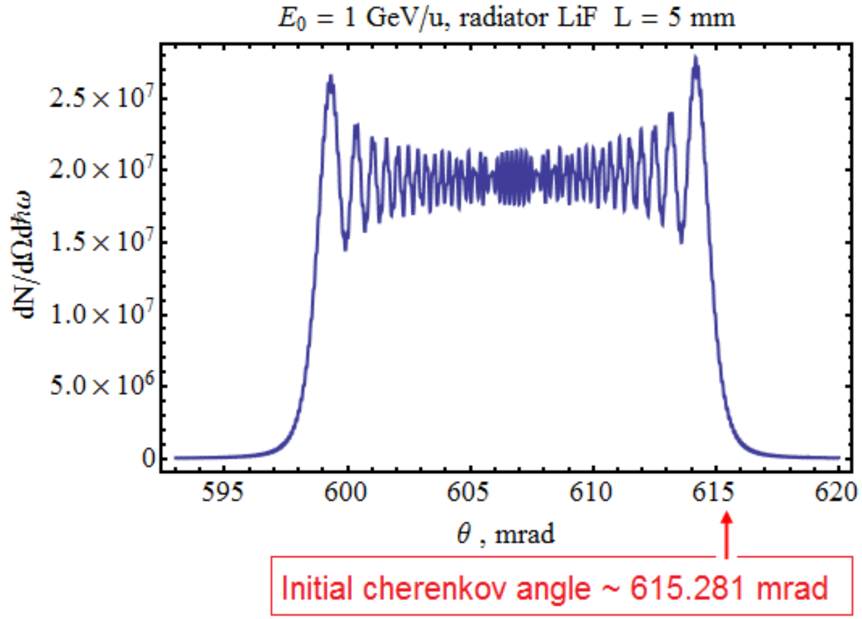
Influence of RHI slowing down on ChR angular distribution

Angular distribution of ChR, initial energy of Au^{197} RHI = 1 GeV/u:

Tamm-Frank theory



Bethe-Bloch formula*



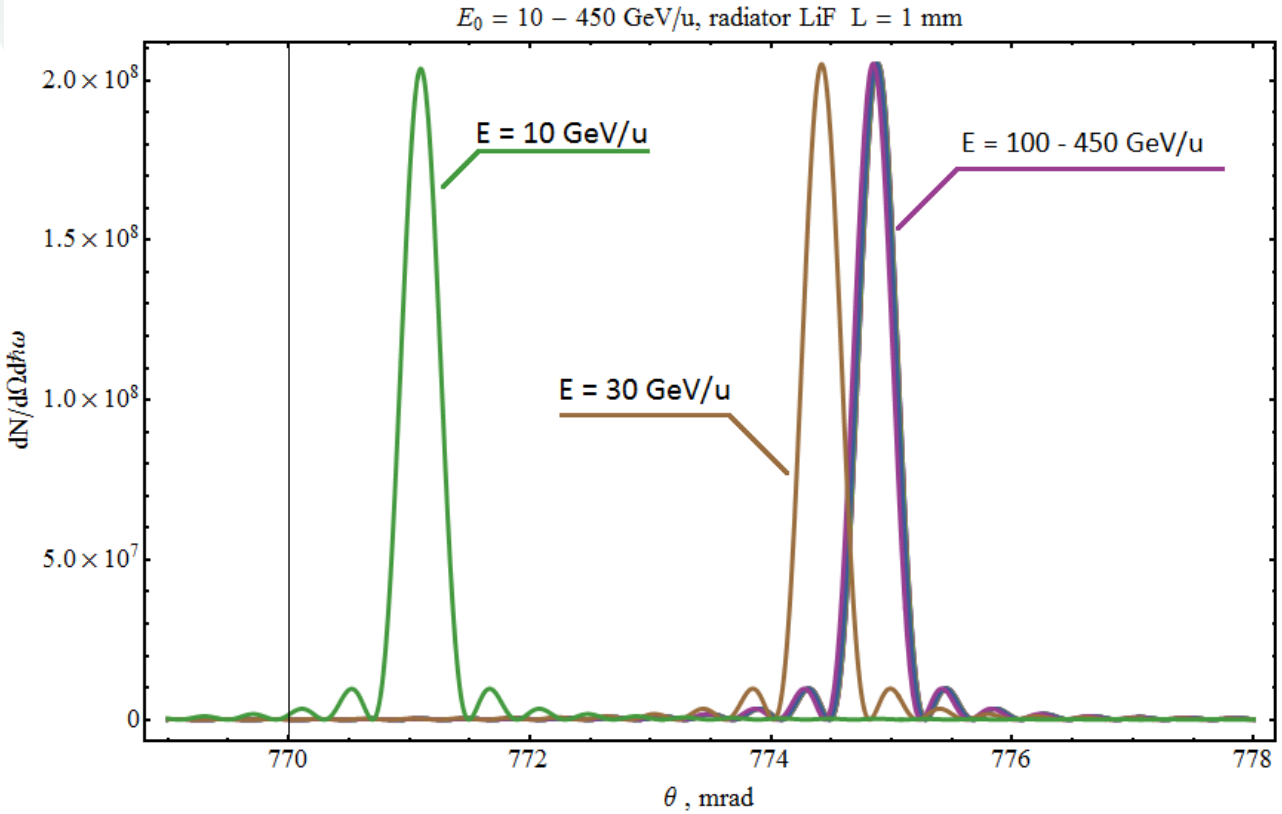
Diffraction-like structure is the result of interference of waves emitted from slices of trajectory at different angles and at decreasing velocity

* Calculations were made using theory developed in [1-2]

[1] O.V. Bogdanov, E.I. Fiks, Yu.L. Pivovarov, Journal of Physics. V. **357** (2012) 012002 pp.1-9
[2] O.V. Bogdanov, E.I. Fiks, Yu.L. Pivovarov, Zh. Exp. Teor. Fiz. V. **142** № 2 (8) (2012) pp. 1-11

Influence of RHI slowing down on ChR angular distribution

Angular distribution of ChR for energy range 10-450 GeV/u (FAIR & CERN)



RHI = **Au^{197}**
Target = **LiF**,
Thickness **L = 1 mm**.
Radiation wave length = **390 nm**

Calculations are performed using ATIMA to calculate energy loss

Influence of RHI slowing down on ChR angular distribution

High energies, FAIR & SPS, 30 - 450 GeV/u:

The form of ChR angular distribution is very close to Tamm-Frank distribution, because at very high energy a change in RHI velocity (for reasonable radiator thickness) is so small that the value of β is still very close to 1. As a sequence, the diffraction-like structure in ChR angular distribution disappears.

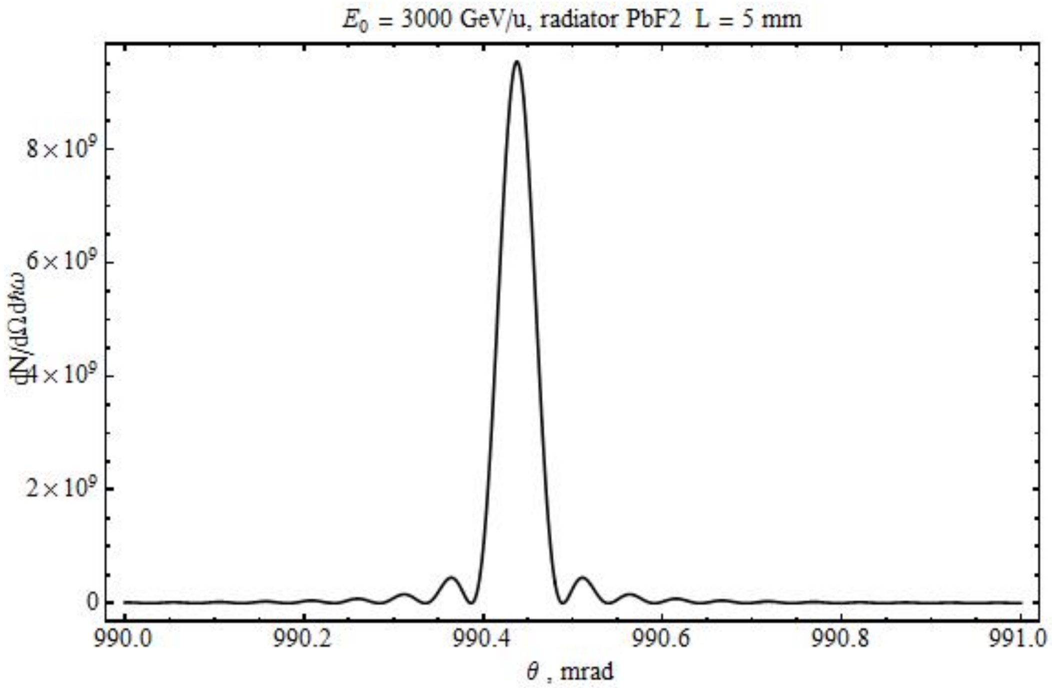
Some values:

L = 1 mm

Initial energy E_0, GeV/u	Final energy E_1, GeV/u	$\frac{dE}{dx}$, MeV/(mg/cm^2)	β_0	β_1	θ_0, mrad	θ_1, mrad
450	449.980	15.025084	0.9999978	0.9999978	774.888	774.888
170	169.980	14.844629	0.999985	0.999985	774.875	774.875
30	29.981	13.86329	0.99954	0.99954	774.42	774.419
20	19.982037	13.428483	0.9990093	0.9990076	773.862	773.855
1	0.983	12.320654	0.876	0.873	615.281	611.519

Influence of RHI slowing down on ChR angular distribution

Ultra relativistic case, LHC, energy ~ 3000 GeV/u:



RHI = **Au^{197}**
Target = **PbF_2**,
Thickness **L = 5 mm**.
Radiation wave length = **390 nm**

Conclusions

- At RHI energies up to several GeV/u (GSI) the RHI slowing down in a radiator results in specific diffraction-like structure of ChR angular distribution. The width of this distribution brings an information on slowing down (stopping), thus on the charge (mass) of RHI.
- To compare, TF theory predicts only z^2 dependence of ChR total intensity.
- At RHI energies from 5 to 10 GeV/u (FAIR) the same effect may appear only in a very heavy solid radiators (PbF_2, $PbWO_4$).
- At RHI high energies (FAIR, SPS and LHC) the ChR angular distribution (the reasonable radiator thickness) is very close to the Tamm-Frank distribution and practically doesn't depend on RHI slowing down in a radiator (until new mechanism of RHI energy-loss – electron-positron pairs production will enter into consideration).

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Thank you for attention !