

**ON THE INFLUENCE OF A PARTICLE'S FIELD  
EVOLUTION ON ITS IONIZATION ENERGY  
LOSSES IN THIN LAYERS OF SUBSTANCE**

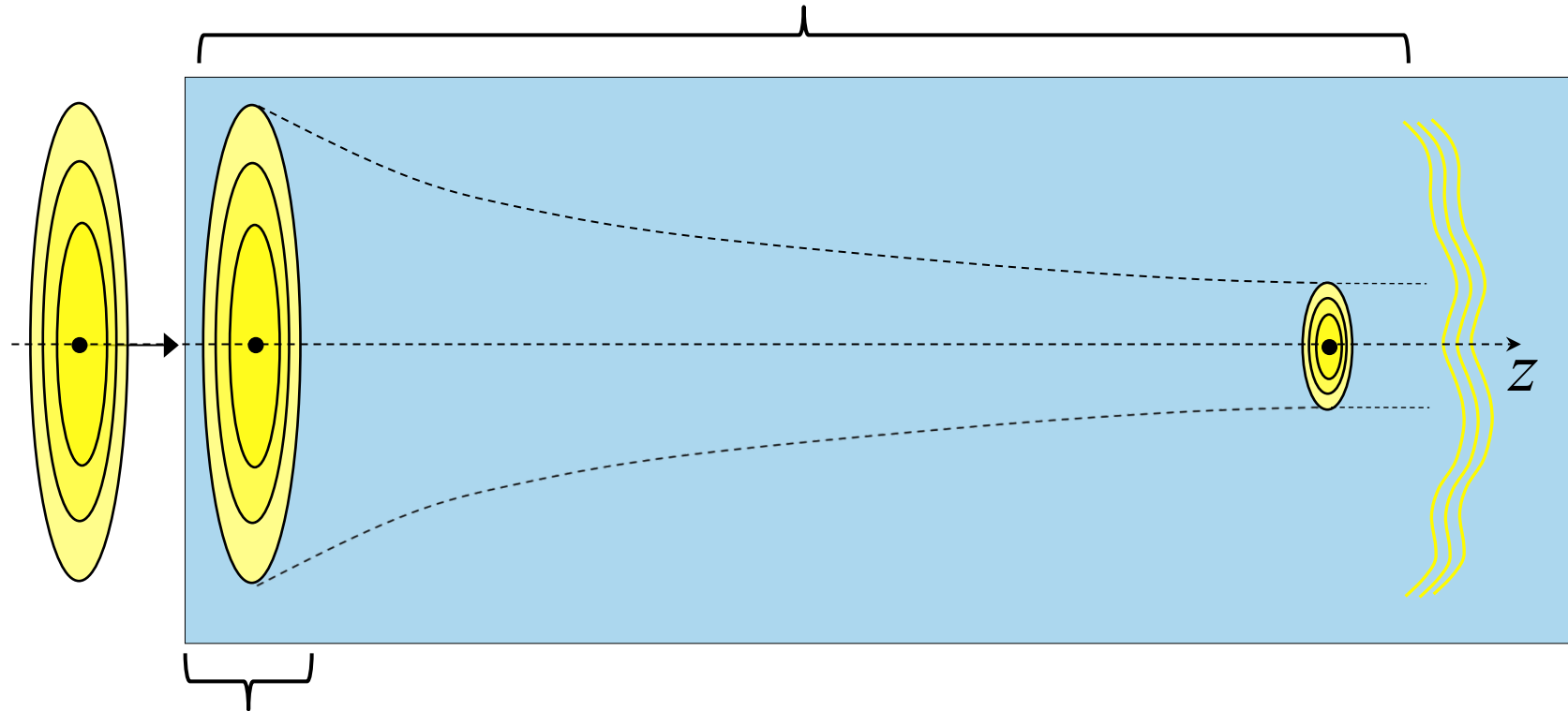
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- Energy losses in infinite medium (Bethe-Bloch, Fermi)
- Energy losses in thin plates (Garibian)
- Field structure after electron emergence from substance
- Energy losses of half-bare electron in thin plate

# RECONSTRUCTION OF THE FIELD AND IONIZATION ENERGY LOSSES

Field reconstruction  $L \approx \gamma^2 / I$



Ionization losses  
reconstruction

$$L \approx I / \omega_p^2 \approx \text{absorption length}$$

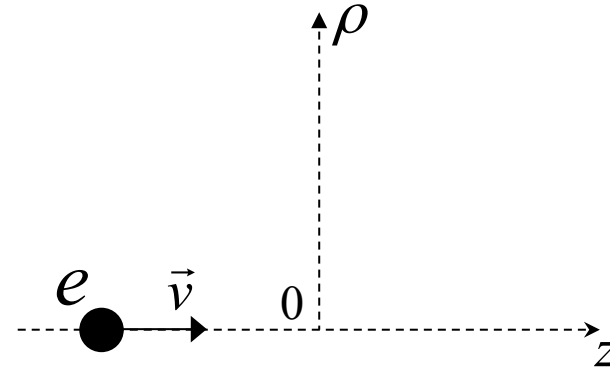
*G. Garibian, 1959*

$I$  – mean ionization potential

# BETHE-BLOCH FORMULA

## Infinite substance

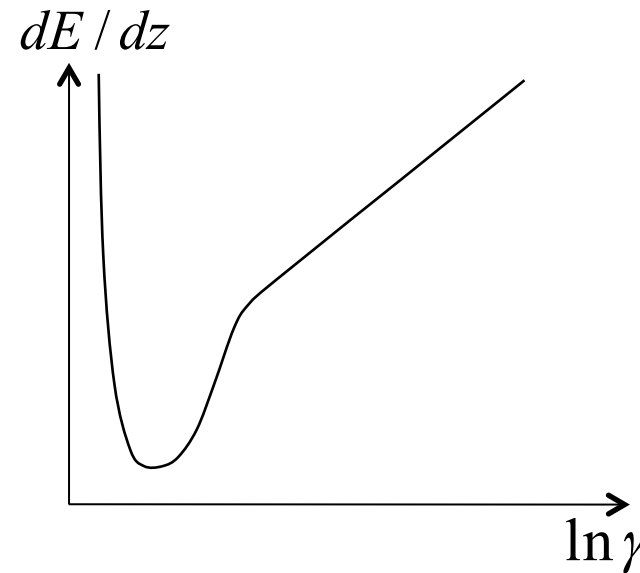
We consider ionization losses with the momentum transfer less than  $q_0$  (the collisions with impact parameter  $\rho > b = 1/q_0$ )



For  $1 \leq \gamma \leq I/\omega_p$ :

$$\frac{dE}{dz} = \frac{\omega_p^2 e^2}{v^2} \ln \frac{\gamma}{bI}$$

$$\varphi_{Coul} = e / \sqrt{\rho^2 \gamma^{-2} + (z - vt)^2}$$

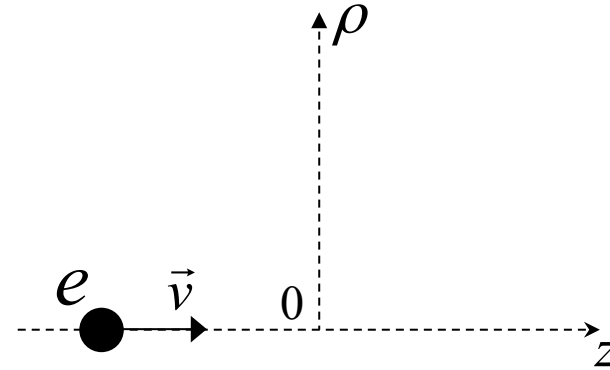


# FERMI FORMULA

For  $\gamma > I / \omega_p$

The influence of medium polarization:

$$\varphi(\vec{r}, t) = \varphi_{Coul} e^{-\omega_p \sqrt{\rho^2 + \gamma^2 (z-vt)^2}}$$

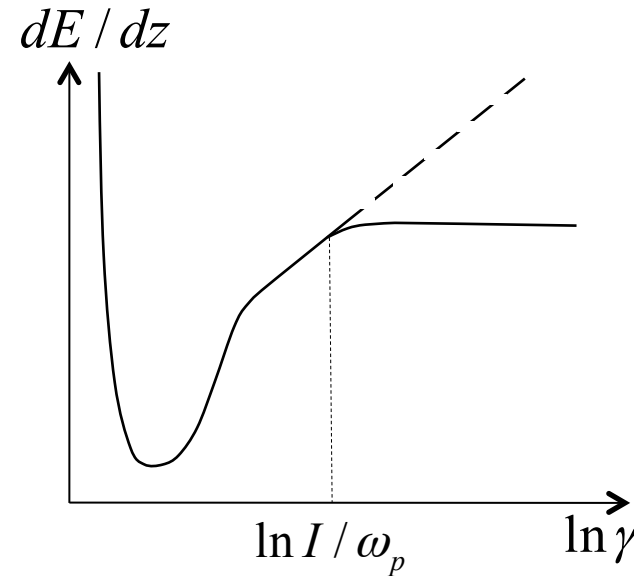


Screening of the field for:

$$\rho > 1 / \omega_p$$

Ionization energy losses:

$$\frac{dE}{dz} = \frac{\omega_p^2 e^2}{v^2} \ln \frac{v}{b \omega_p}$$



# THIN LAYER OF SUBSTANCE

**Bethe-Bloch and Fermi formulae are valid in boundless homogeneous substance**

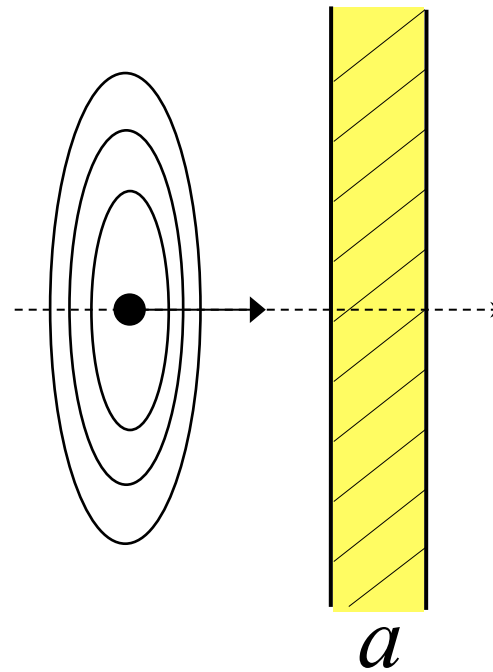
*Garibian G.M.// JETP, 1959*

**Total absence of the density effect in thin plates:**

$$a \leq I / \omega_p^2$$

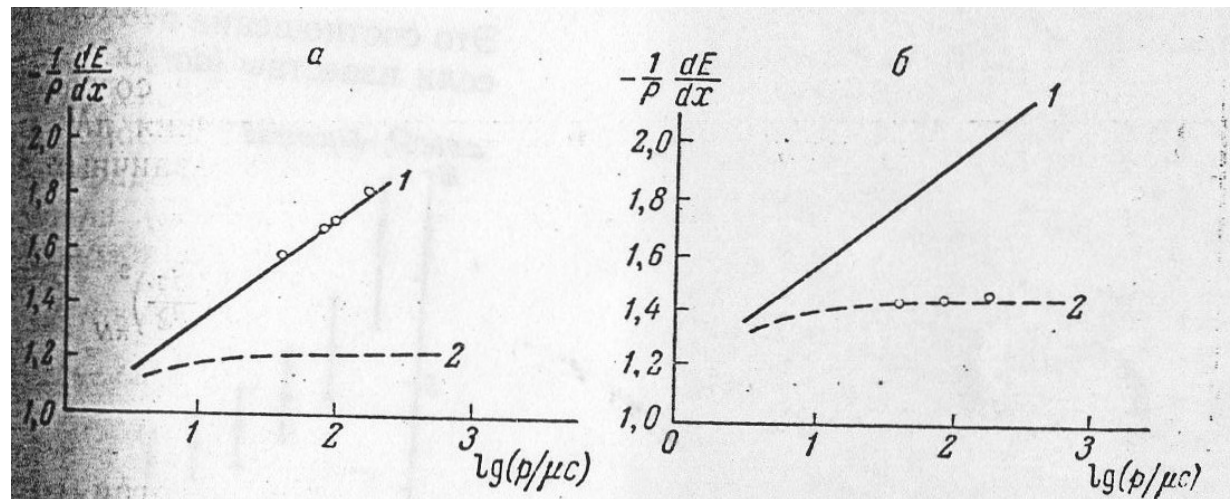
**Particle energy loss:**

$$\Delta E = \frac{\omega_p^2 e^2}{v^2} a \ln \frac{\gamma}{bI} \begin{cases} 1 \leq \gamma \leq I / \omega_p \\ \gamma > I / \omega_p \end{cases}$$



# FIRST EXPERIMENT (Kharkov, 1963)

A.I. Alikhanian, G.M. Garibian, M.P. Lorikian, A.K. Walter,  
I.A. Grishaiev, V.A. Petrenko, G.L. Fursov // JETP, 1963



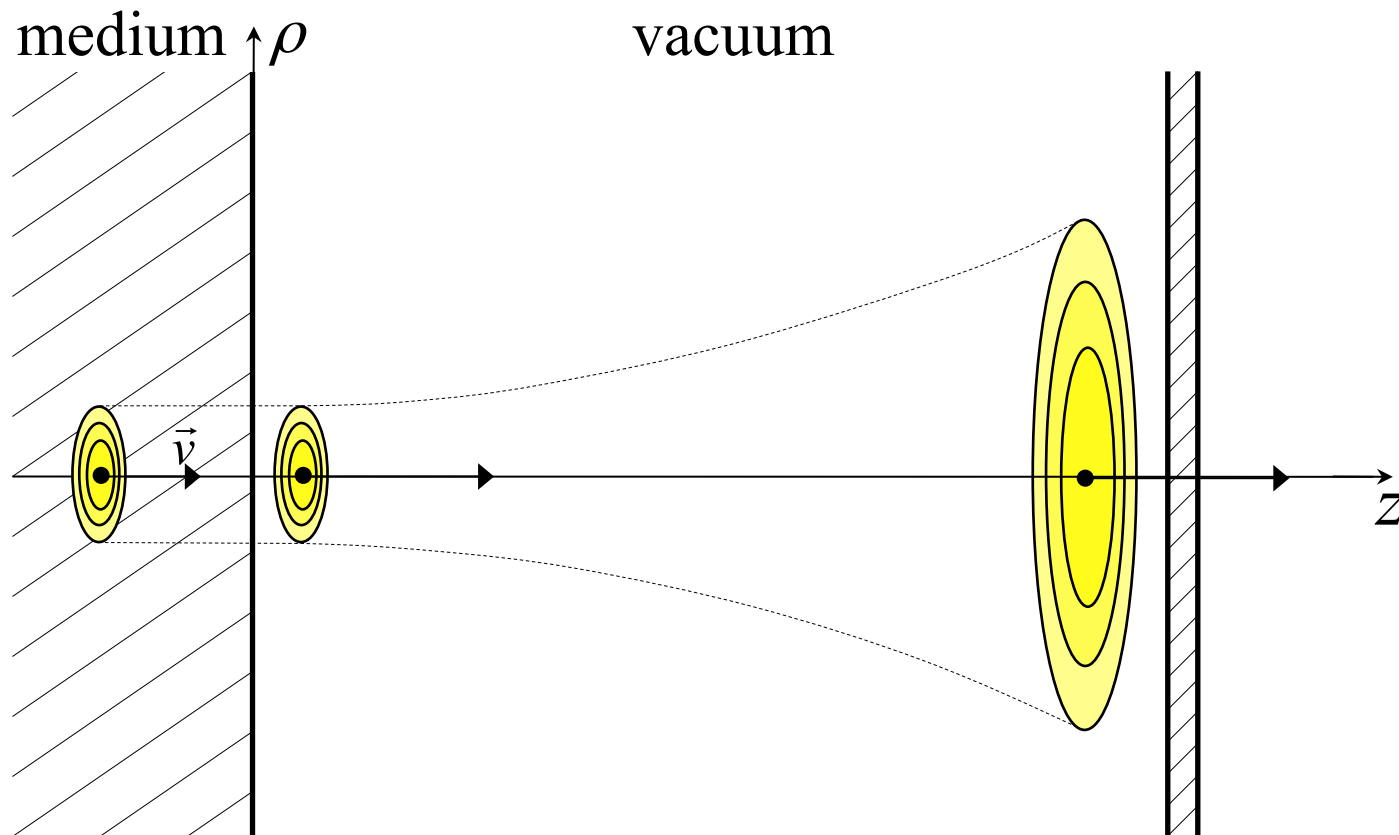
Electron energy losses in thin films of polystyrene of thicknesses  $10^{-6} \text{ cm}$  (a) and  $2 \times 10^{-3} \text{ cm}$  (б)

1 – theoretical curve without density effect

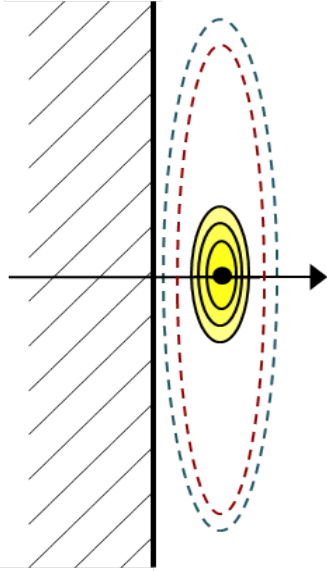
2 – theoretical curve with density effect

circles show the measurement results

# EVOLUTION OF THE FIELD AROUND THE ELECTRON IN VACUUM AT ULTRA HIGH ENERGIES



# EVOLUTION OF THE FIELD AROUND THE ELECTRON IN VACUUM AT ULTRA HIGH ENERGIES



**Total field:**

$$\vec{E} = \vec{E}^C + \vec{E}^F$$

**Boundary conditions:**

$$\left. \begin{aligned} (E_\rho^C + E_\rho^F)^{diel} \Big|_{z=0} &= (E_\rho^C + E_\rho^F)^{vac} \Big|_{z=0} \\ (E_z^C + E_z^F)^{diel} \Big|_{z=0} &= \hat{\epsilon} (E_z^C + E_z^F)^{vac} \Big|_{z=0} \end{aligned} \right\} + \operatorname{div} \vec{E}^F = 0$$

**Fourier expansion of the free field:**

$$E_\rho(\vec{r}, t) = \frac{e}{\pi v} \int_0^\infty dq q^2 J_1(q\rho) \times \int_{-\infty}^{+\infty} d\omega \frac{\sqrt{\omega^2 - q^2}}{\sqrt{\omega^2 \epsilon - q^2} + \epsilon \sqrt{\omega^2 - q^2}} \left[ \frac{1 + \frac{v}{\omega} \sqrt{\omega^2 \epsilon - q^2}}{q^2 + \frac{\omega^2}{v^2} - \epsilon \omega^2} - \frac{\epsilon + \frac{v}{\omega} \sqrt{\omega^2 \epsilon - q^2}}{q^2 + \frac{\omega^2}{v^2} - \omega^2} \right] e^{i\sqrt{\omega^2 - q^2} z - i\omega t}$$

$\vec{q}$  – component of vector  $\vec{k}$  orthogonal to  $z$  axis



# THE STRUCTURE OF THE FIELD IN VACUUM

$$\varepsilon(\omega) = 1 - \omega_p^2 / \omega^2 \quad \omega_p - \text{plasma frequency of the first substance}$$

For  $\gamma \gg 1$  characteristic values:  $\omega_p / \omega \ll 1$  and  $q / \omega \ll 1$

**Free field:**

$$E_\rho^F(\vec{r}, t) = \frac{e}{\pi v} \int_0^\infty dq q^2 J_1(q\rho) \int_{-\infty}^{+\infty} d\omega \left[ \frac{1}{q^2 + \omega_p^2 + \frac{\omega^2}{v^2 \gamma^2}} - \frac{1}{q^2 + \frac{\omega^2}{v^2 \gamma^2}} \right] \exp \left\{ i\omega(z - t) - i \frac{q^2}{2\omega} z \right\}$$

**Total field:**

$$E_\rho = E_\rho^C + E_\rho^F$$

**Particle's own Coulomb field:**

$$E_\rho^C(\vec{r}, t) = -\frac{\partial}{\partial \rho} \frac{e}{\sqrt{\rho^2 \gamma^{-2} + (z - vt)^2}} = \frac{e}{\pi v} \int_{-\infty}^{\infty} d\omega \int_0^{\infty} dq \frac{q^2 J_1(q\rho)}{q^2 + \frac{\omega^2}{v^2 \gamma^2}} e^{i \frac{\omega}{v} z}$$

# IONIZATION OF SUBSTANCE BY EXTERNAL FIELD

$$\omega_0 = I$$

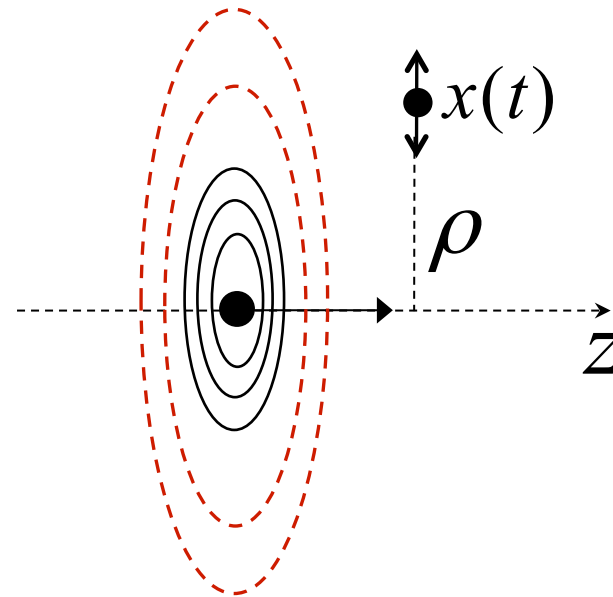
$$\ddot{x} + \beta\dot{x} + \omega_0^2 x = \frac{e}{m} (E_\rho^C + E_\rho^F)$$

**Energy transfer to a harmonic oscillator by external field:**

$$\Delta \mathcal{E} = \frac{e^2}{2m} \left| E_{\omega_0}(\vec{r}) \right|^2$$

$$E_{\omega_0}(\vec{r}) = \int_{-\infty}^{+\infty} E(t) e^{-i\omega_0 t} dt$$

$\omega_0$  – oscillator's own frequency



*J.D. Jackson // Classical electrodynamics, 1999*

# IONIZATION ENERGY LOSSES BY HALF-BARE ELECTRON

$$\omega = \omega_0$$

**Total field Fourier-component:**

$$E_{\omega}^{\rho}(\vec{r}) = 2 \frac{e}{v} \int_0^{\infty} dq q^2 J_1(q\rho) \left\{ \left[ \frac{1}{q^2 + \omega_p^2 + \frac{\omega^2}{v^2 \gamma^2}} - \frac{1}{q^2 + \frac{\omega^2}{v^2 \gamma^2}} \right] e^{i\omega z - \frac{q^2 z}{2\omega}} + \frac{e^{\frac{i\omega}{v} z}}{q^2 + \frac{\omega^2}{v^2 \gamma^2}} \right\}$$

**Total energy loss per unit path:**

$$\frac{d\mathcal{E}}{dz} = n \frac{e^2}{2m} \int_0^{\infty} d\rho 2\pi\rho \left| E_{\omega_0}^{\rho}(\vec{r}) \right|^2$$

# IONIZATION ENERGY LOSSES BY HALF-BARE ELECTRON

$$\omega = \omega_0$$

After integration over  $\rho$  :

$$\frac{d\mathcal{E}}{dz} = \frac{\Omega_p^2 e^2 q_0}{v^2} \int_0^{q_0} dq q^3 \left\{ \left( \frac{1}{q^2 + \frac{\omega^2}{v^2 \gamma^2}} \right)^2 + \left( \frac{1}{q^2 + \omega_p^2 + \frac{\omega^2}{v^2 \gamma^2}} - \frac{1}{q^2 + \frac{\omega^2}{v^2 \gamma^2}} \right)^2 + \right. \\ \left. + \left( \frac{1}{q^2 + \omega_p^2 + \frac{\omega^2}{v^2 \gamma^2}} - \frac{1}{q^2 + \frac{\omega^2}{v^2 \gamma^2}} \right) \frac{1}{q^2 + \frac{\omega^2}{v^2 \gamma^2}} 2 \cos \left( \frac{\omega z}{2v^2 \gamma^2} + \frac{q^2 z}{2\omega} \right) \right\}$$

$\Omega_p$  – plasma frequency of the plate

# IONIZATION ENERGY LOSSES BY HALF-BARE ELECTRON

**Assumption:**  $q_0 \gg \omega_p \gg I/\gamma$

**For**  $z \rightarrow 0$

$$\frac{d\mathcal{E}}{dz} = \frac{\Omega_p^2 e^2}{v^2} \ln \frac{q_0}{\omega_p}$$

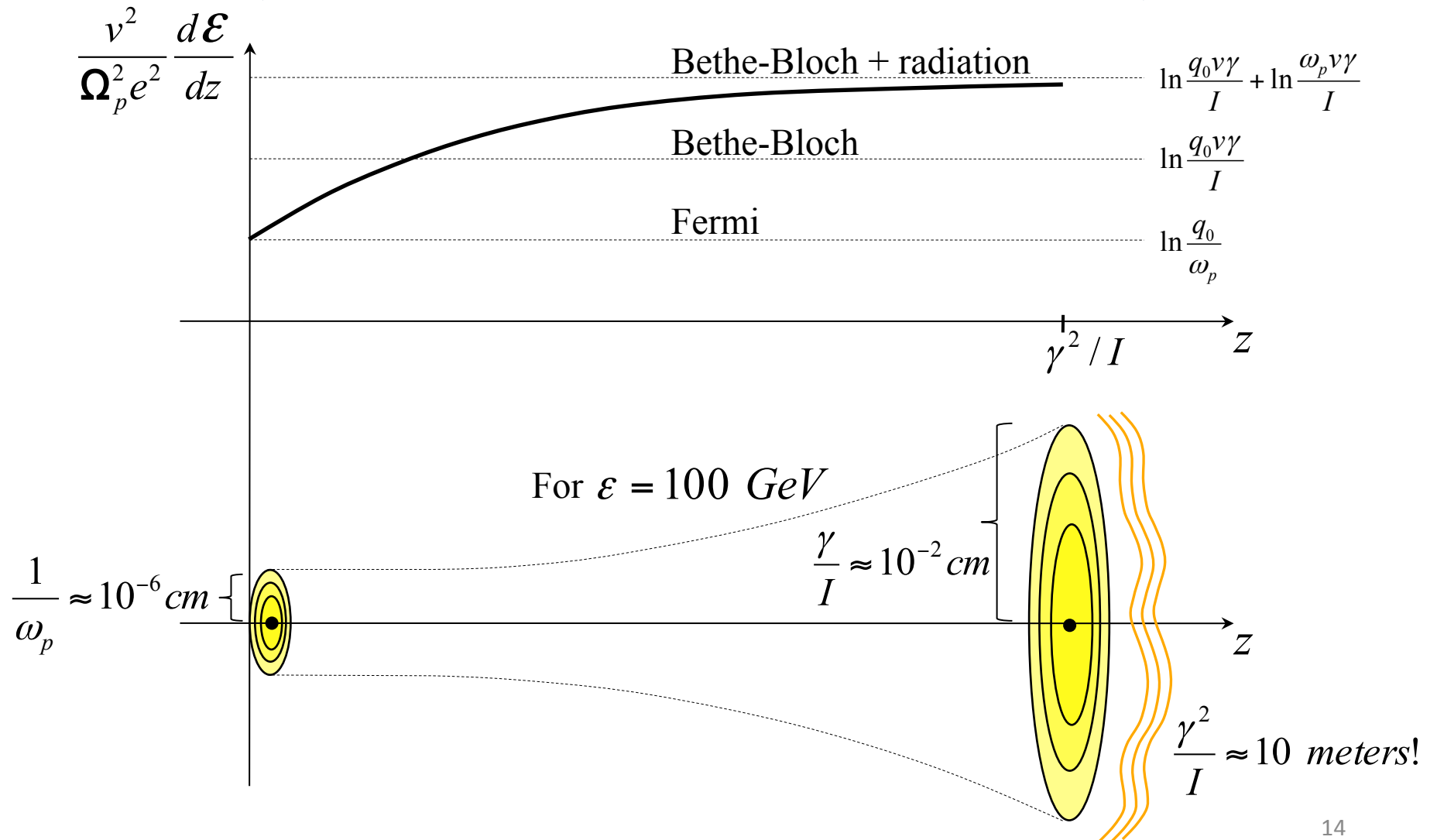
**For**  $I/\omega_p^2 \ll z \ll 2\gamma^2/I$

$$\frac{d\mathcal{E}}{dz} = \frac{\Omega_p^2 e^2}{v^2} \ln \frac{q_0 z \omega_p}{2I}$$

**For**  $z \geq 2\gamma^2/I$

$$\frac{d\mathcal{E}}{dz} = \frac{\Omega_p^2 e^2}{v^2} \left\{ \ln \frac{q_0 v \gamma}{I} + \ln \frac{\omega_p v \gamma}{I} \right\}$$

# IONIZATION ENERGY LOSSES BY HALF-BARE ELECTRON (from Fermi to Bethe-Bloch formula)



# CONCLUSIONS

- ❖ The existence of a transition process in which the ionization energy losses of a particle are defined by the mechanism of reconstruction of the field around it into radiation (not by the absorption of the free field)
- ❖ Gradual change of particle ionization energy losses from Fermi to Bethe-Bloch mode (if taking into account residual ionization by transition radiation) in the considered process

**THANK YOU!**