Twisted electron in a strong laser wave

Dmitry Karlovets

Tomsk Polytechnic University, Tomsk, Russia

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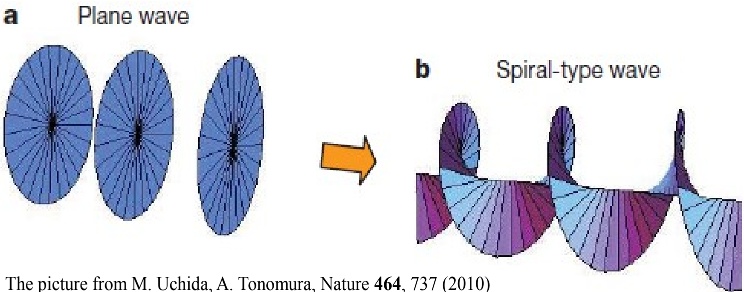
Twisted photons

As was predicted theoretically and demonstrated experimentally in 1990s, the light can carry orbital angular momentum (OAM).

This is a fundamentally new degree of freedom of photons!

They are no longer the plane waves, since their Pointing vector looks like a corkscrew:



The picture from M. Uchida, A. Tonomura, Nature **464**, 737 (2010)

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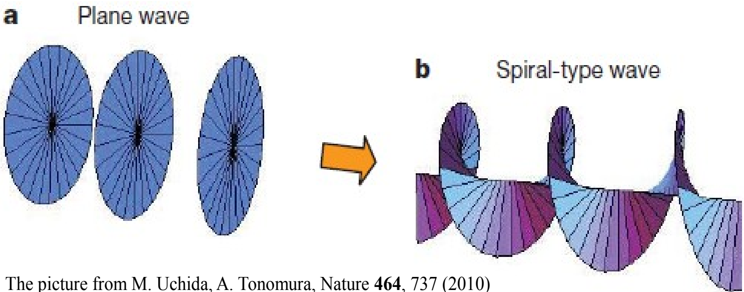

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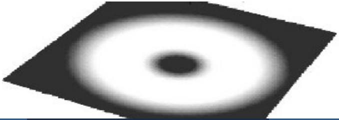
Some interesting phenomena with the twisted photons

- Micro-particles get a torque and start to rotate after absorbing or scattering these photons (e.g. M.E.J. Friese, et al. PRA **54**, 1593 (1996)).
- They are used as the optical tweezers allowing one to trap and move different micro- and nano-objects relevant for biology, condensed matter physics, quantum information, etc. (e.g. D.G. Grier, Nature **424**, 810 (2003)).
- They have many applications in modern astrophysics and astronomy.
- Focused laser beams (in particular, Lagguere-Gaussian ones) were shown to carry OAM, as well as the photons in waveguides and optical fibers (e.g. F.L. Kien, et al., PRA **73**, 053823 (2006)). Such a laser beam has a typical "doughnut" structure:



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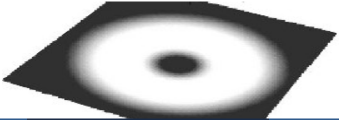

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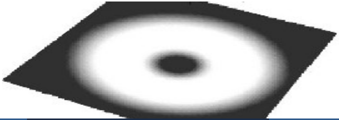

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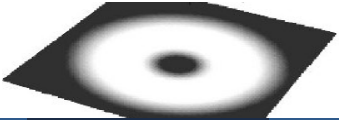

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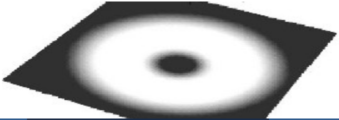

Twisted electrons

Can the massive particles carry OAM as well?

Usually, we describe the particles via the plane-wave states obeying the Dirac or Klein-Gordon equations:

$$\psi(x) \propto u(p)e^{-ipx},\ \hat{p}^{\mu}\psi(x) = p^{\mu}\psi(x),\ \hat{p}^{\mu} = i\frac{\partial}{\partial x_{\mu}}$$

These states have a well-definite 4-momentum p^{μ} and spin (e.g. helicity λ).

The twisted states are not described with the plane-waves, since the wavefront of the de Broglie wave represents a spiral.

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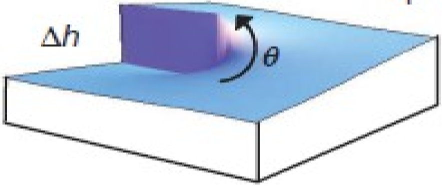
Twisted electrons

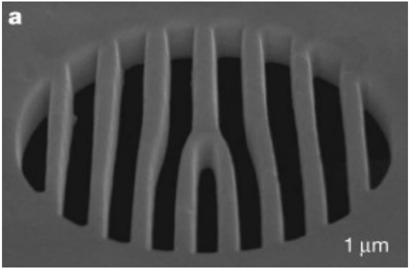
In order to obtain the electrons with OAM, one should imprint the phase singularity on their wavefront. This could be done by using the spiral phase plates or diffraction gratings with the edge dislocations:

Spiral phase plate



M. Uchida, A. Tonomura, Nature **464**, 737 (2010)

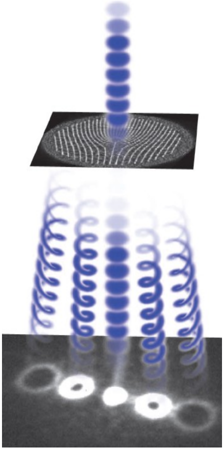
Diffraction grating with an edge dislocation made of Pt foil



J. Verbeeck, et al., Nature **467**, 301 (2010)

The first creation of 300 KeV twisted electrons

1. M. Uchida, A. Tonomura, Nature **464**, 737 (2010);
2. J. Verbeeck, et al., Nature **467**, 301 (2010);
3. B.J. McMorran, et. al., Science **331**, 192 (2011).



The value of observed OAM was as high as $m \sim 100\hbar$!

Motivation

Why these twisted particles (electrons, photons, ions, etc.)
may be of interest for radiation physics?

The values of OAM can be, in principle, arbitrary large!
For instance, the "ordinary" gyromagnetic ratio
for a plane-wave electron is:

$$\boldsymbol{\mu} = \mu_B 2\mathbf{s}$$

with $\mu_B =$ being the Bohr's magneton.

However if the OAM is non-zero, the last is to be added to the spin
(K. Bliokh, et al., PRL **107**, 174802 (2011)):

$$\boldsymbol{\mu} \propto \mu_B(\mathbf{l} + 2\mathbf{s}) \text{ (!!!)}$$

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If the OAM is large, $l_z \equiv m \gg \hbar$, then

$$\mu \gg \mu_B!$$

For electrons with $l_z \equiv m \sim 100\hbar$ the magnetic moment is rudely 10^2 **times larger than the ordinary** μ_B**!**

The radiation intensity of such a magnetic moment **increases in** $\sim (10^2)^2 = 10^4$ **times!!!**

- This could make the radiation effects of magnetic moments of particles (electrons, protons, ions, etc.) much more significant!
- Whereas the spin magnetic moment's radiation is usually too tiny to be measured, radiation of this OAM's magnetic moment can, in principal, be detected much easier.

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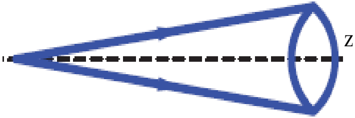
Free twisted state

Generally, a particle can be described with the different quantum numbers:

- The (linear) momentum and energy $p^\mu = \{\varepsilon, \mathbf{p}\}$;
- The angular momentum $\mathbf{j} = \mathbf{l} + \mathbf{s}$ consisting of OAM $\mathbf{l}$ and spin $\mathbf{s}$.
- The absolute value of the momentum $\mathbf{l}^2$, etc.

1. If a particle has zero expectation values of $\mathbf{l}^2$ and $l_z \equiv m$, this is called a plane-wave state.
2. If the state is an eigenfunction of operators $\hat{\mathbf{l}}^2$ and $\hat{l}_z$, it is a spherical wave.
3. The twisted state represents in a sense the intermediate case: they have the well-definite $l_z \equiv m$ only: $|\varepsilon, p_z, \kappa, m, \lambda\rangle$



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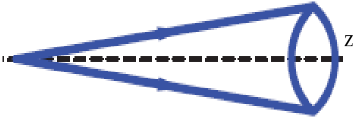

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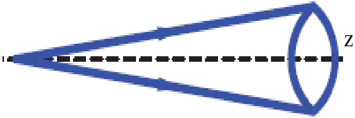

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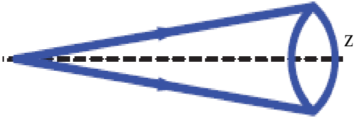

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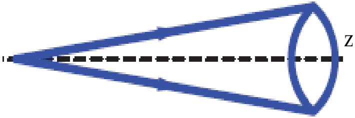

Free twisted scalar

Let us begin with a scalar massive particle in a state $|\psi\rangle$:

$$|\psi\rangle = \int \frac{d^4p}{(2\pi)^4}\psi(p)|p\rangle$$

with $|p\rangle \propto e^{-ipr}$, $p^2 = m^2$. Then, we choose:

$$\psi(p) = (2\pi)^3(-i)^m\delta(p_0-\varepsilon)\delta(p_z-p_{||})\delta(p_\perp-\kappa)\frac{e^{im\phi_p}}{p_\perp},$$

As a result, we get:

$$|\psi\rangle \equiv |\varepsilon, p_{||}, \kappa, m\rangle = \text{const}\ J_m(\rho\kappa)e^{-i\varepsilon t+ip_{||}z+im\phi_r}.$$

Here, ϕ_r is the azimuthal angle. This state is an eigenfunction of the OAM's z-projection operator:

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How OAM can co-exist with spin?

For electrons, we are seeking the solution by analogy with the scalar case:

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Considering the helicity states, we obtain finally:

$$\psi(r) = \text{const}\ e^{-i\varepsilon t + ip_{\|} z} \Big(u^{(1/2)} J_{m+\lambda-1/2}(\rho\kappa) e^{i\phi_r (m+\lambda-1/2)} + u^{(-1/2)} J_{m+\lambda+1/2}(\rho\kappa) e^{i\phi_r (m+\lambda+1/2)} \Big)$$

The bispinors $u^{(\pm 1/2)}$ are the eigenfunctions of the spin's z-projection operator with the eigenvalues $s_z = \pm 1/2$:

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$$|\varepsilon, p_{\|}, \kappa, j_z = l_z + s_z = m + \Lambda\rangle = |...l_z = m + \Lambda - 1/2, s_z = +1/2\rangle + \\ + |...l_z = m + \Lambda + 1/2, s_z = -1/2\rangle,$$

We call this the spin-orbital connection.

These states represent the complete and orthogonal set of functions:

$$\int d^3r \psi^*_{\varepsilon, p'_{\|}, \kappa', m', \lambda'}(r) \psi_{\varepsilon, p_{\|}, \kappa, m, \lambda}(r) = \\ = N^2 (2\pi)^2 2\varepsilon \delta(p_{\|} - p'_{\|}) \frac{\delta(\kappa - \kappa')}{\kappa} \delta_{\lambda\lambda'} \delta_{mm'},$$

$$\sum_m \int \frac{d^3p}{(2\pi)^3} \psi^*_{\varepsilon, p_{\|}, \kappa, m, \lambda}(r') \psi_{\varepsilon, p_{\|}, \kappa, m, \lambda}(r) = N^2 2\varepsilon \delta(r - r').$$

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Consider an electron in a plane circularly polarized e-m wave with $A^\mu = a\{0, \cos\varphi, \sin\varphi, 0\},\ A^2 = -a^2$. It moves oppositely to the z-axis (so that $k = \{\omega, 0, 0, -\omega\}$), head-on to the electron.

Usually, for describing electron in a plane-wave field the Volkov states are used (see e.g. *V.B. Berestetskii, et al., Quantum electrodynamics, Oxford, Pergamon, 1982*). They may be called the "plane-wave" states.

We seek the "non-plane wave" solution of the Dirac equation as usually

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As a result, we get the following twisted state:

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Its general difference from the free electron twisted state is that the momentum p is exchanged for the quasi-momentum q

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$$\langle \hat{l}_z \rangle = \int d^3\mathcal{R}\psi^*(r)\hat{l}_z\psi(r) \Big/ \int d^3\mathcal{R}|\psi(r)|^2 = m.$$

It means that at least for a scalar particle the OAM is conserved in a plane-wave field.

For an electron, the spin-orbital coupling breaks this simple property, but this is not important when $m \gg \hbar$.

OAM appears as an effective integral of motion for an electron in a e-m plane wave (V. Bagrov, D. Gitman, Ann. Phys. (Leipzig) **14**, 467 (2005))!

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Strong field regime

There is a "classical" parameter of the plane wave's intensity:

$$\eta^2 = -\frac{e^2 \bar{A}^2}{m^2 c^4}$$

- When it is small, $\eta \ll 1$, we get the regime of non-linear Compton effect.
- The opposite situation, $\eta \gtrsim 1$, is called the strong field regime.

The modern lasers such as Vulcan and the future facilities (such as Extreme Light Infrastructure) provide the experimental values of $\eta \gtrsim 1$ and even $\eta \gg 1$. The first non-linear quantum effects were observed at the SLAC (C. Bamber, et al., Phys. Rev. D **60**, 092004 (1999)).

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Spin's shift in a strong laser wave

Consider the "ordinary" plane-wave electron with $\kappa = m = 0, j_z = s_z = \lambda$.

$$\langle \hat{s}_z \rangle = \int d^3 r \psi_V^*(r) \frac{1}{2} \Sigma_z \psi_V(r) = \lambda \frac{2\varepsilon - \omega \frac{e^2 a^2}{(pk)}}{2\varepsilon + \omega \frac{e^2 a^2}{(pk)}}$$

which is $\langle \hat{s}_z \rangle < \lambda$.

The electron become spontaneously depolarized, $\langle \hat{s}_z \rangle \to 0$, when

$$\varepsilon = \omega \frac{e^2 a_0^2}{2(pk)} = \eta_0^2 \frac{m^2}{2(p+\varepsilon)} \Rightarrow \eta_0^2 = 2\gamma^2(1+\beta),$$

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This shift can be observed for non-relativistic electrons!

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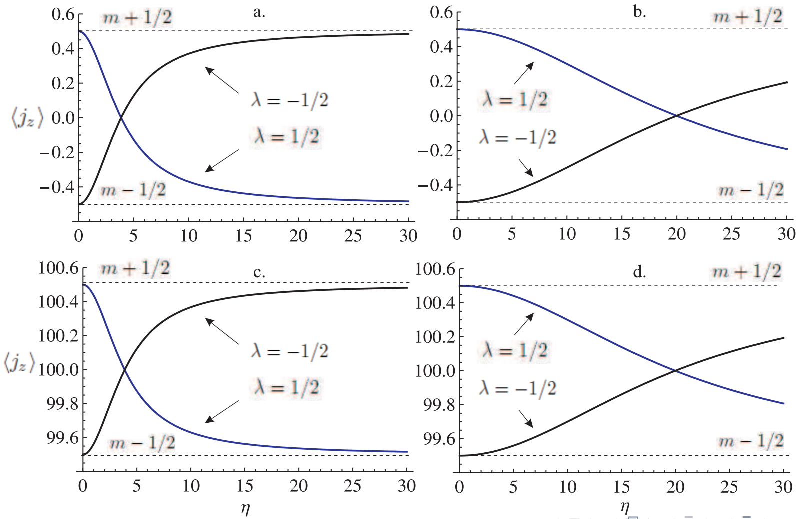
For a twisted electron: $\langle \hat{j}_z \rangle \neq m + \lambda$ due to the spin's shift!



Discussion

- The OAM of particles is a fundamentally new degree of freedom, which role can be much more important than that of the spin.
- Whereas the 300-KeV twisted electrons have already been produced, the key question is how to obtain such electrons (positrons) in GeV-energy range.
- In order to accelerate non-relativistic twisted electrons, it is necessary to understand whether they lose their OAM or not.
- In a plane e-m wave the electron's OAM is conserved. It is enough to conclude that it is also conserved for a more general combination of fields: $E_z + H_z + A$ with k_z (V. Bagrov, D. Gitman, 2005).

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We see (at least) two possibilities for accelerating the twisted electrons:

- One can use the E_z together with H_z or A (with k_z) in order to trap and focus the electrons.
- One can also use the focused laser beams, whose TM-modes have E_z. At least for a weakly focused beam, the electron OAM's z-projection stays to be an integral of motion (Narozhny, Fofanov, 2000).

Some possible applications of the twisted particles

- Along with an increase of the magnetic moment's radiation effects, the development of new sources for OAM-polarized relativistic particles (positrons, protons, etc) seems to be essential.
- Production of twisted electron beams may be also done by analogy with the spin-polarized electrons by using the laser-driven photocathode when the laser photons are twisted. Such beams can be used for next generation colliders (ILC, CLIC) instead of the spin-polarized beams.

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Thank you for your attention!

Based on arXiv:1206.6622

Problems to be solved

In order to understand what effects the OAM can lead to, we should study in more detail the following problems:

- a free twisted boson and fermion;
- a twisted boson and fermion in some external electromagnetic field.
- In particular, we are to understand in which fields the OAM stays stable when accelerating.
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