QFT with generalized statistics and QG phenomenology

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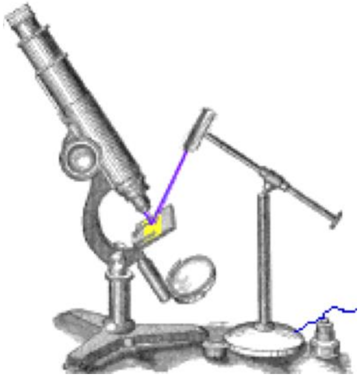
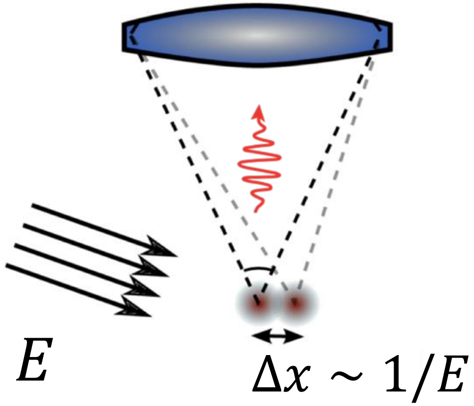
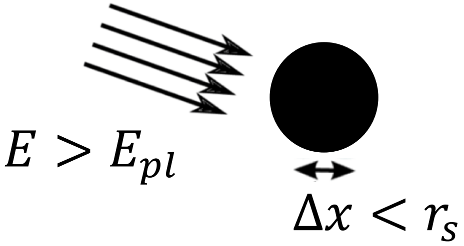
LNF 24/06/25 – Fundamental Physics with Exotic Atoms

Quantum mechanics





+ General Relativity



Infinite localization implies black hole creation

Quantum spacetime

$$\Delta x_{min} \sim l_{pl}$$

$$\Delta t_{min} \sim t_{pl}$$

$$[x^{\mu}, x^{\nu}] = i\theta^{\mu\nu}$$

Doplicher, Fredenhagen, Roberts 1994

String theory

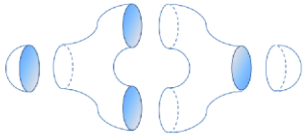
Open strings with background B-fields experience $[x^\mu, x^\nu] = i\theta^{\mu\nu}$

$$\theta^{\mu\nu} = 2\pi\alpha' \left(\frac{1}{g + 2\pi\alpha' B}\right)_A^{\mu\nu}$$

Seiberg & Witten 1999

Loop quantum gravity

κ-Minkowski NCT may emerge as a mesoscopic limit of the spin-foam



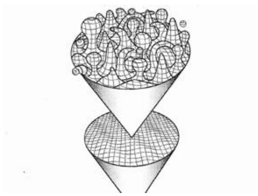
3D quantum gravity

Effective dynamics of matter is descrived by **κ-Minkowski NCST** and **GUP**

Outlook

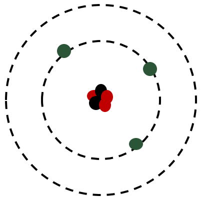
❖ Noncommutative QFT

- Covariance requires deformation of statistics
- Implications



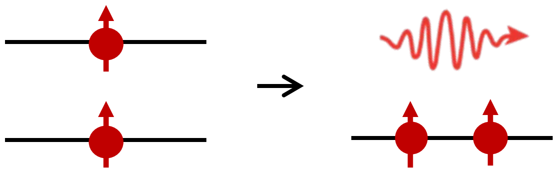
❖ Multi-particle states

- Relativistic generalization of quon model
- Superselection rules



❖ Pauli-forbidden atomic transitions

- Theoretical predictions
- Phenomenological bounds



❖ Final remarks

Covariant construction of NCQFT

- **Deform Poincaré (co-)algebra** to make $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$ invariant (*Oekl '00, Chaichian et al. '04, Aschieri et al. '05*)

$$\Delta_\theta(\Lambda)|p\rangle \otimes |q\rangle = \mathcal{F}^{-1}\Delta_0(\Lambda)\mathcal{F}|p\rangle \otimes |q\rangle = f_\theta^{-1}(p',q')f_\theta(p,q)|p'\rangle \otimes |q'\rangle$$

$$f_\theta(p,q) = \exp\left(-\frac{i}{2}p_\mu \theta^{\mu\nu} q_\nu\right) \text{ Moyal phase}$$

- **Deform permutations** to restore covariance of particle exchange: $\tau_\theta|p\rangle \otimes |q\rangle = \mathcal{Q}(p,q)|q\rangle \otimes |p\rangle$ (*Oekl '00, Balachandran et al. '06*)

$$|p\rangle \otimes |q\rangle \xrightarrow{\Lambda} f_\theta^{-1}(p',q')f_\theta(p,q)|p'\rangle \otimes |q'\rangle \xrightarrow{\tau_\theta} f_\theta^{-1}(p',q')f_\theta(p,q)\mathcal{Q}(p',q')|q'\rangle \otimes |p'\rangle$$

$$\tau_\theta \downarrow$$

$$\mathcal{Q}(p,q)|q\rangle \otimes |p\rangle$$

$$\Lambda \downarrow$$

$$f_\theta^{-1}(q',p')f_\theta(q,p)\mathcal{Q}(p,q)|q'\rangle \otimes |p'\rangle = f_\theta(p',q')f_\theta^{-1}(p,q)\mathcal{Q}(p,q)|q'\rangle \otimes |p'\rangle$$

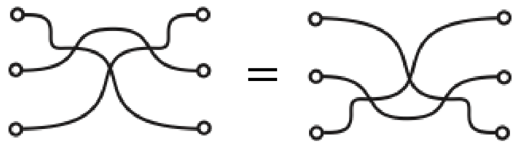
$$\mathcal{Q}(p,q) = \eta(p,q) f_\theta^2(p,q)$$

with $\eta(p,q)$ Lorentz-invariant

τ_θ^i generate braid statistics

$$\tau_\theta^i \tau_\theta^j = \tau_\theta^j \tau_\theta^i \qquad |i-j| \geq 2$$

$$\tau_\theta^i \tau_\theta^{i+1} \tau_\theta^i = \tau_\theta^{i+1} \tau_\theta^i \tau_\theta^{i+1} \qquad 1 \leq i \leq n-2$$



Creation and annihilation operator algebra

Define **$\mathcal{Q}$-mutators**: $[\phi^+(x),\phi^+(y)]_{\mathcal{Q}} \coloneqq \int d\mu(p)d\mu(q)\, e^{-ip\cdot x}e^{-ip\cdot y}[a_p,a_q]_{\mathcal{Q}}$, $[a_p,a_q]_{\mathcal{Q}} \coloneqq a_p a_q - \mathcal{Q}(p,q)a_q a_p$

General case: $\mathcal{Q}(p,q) = \eta(p,q)f_\theta^2(p,q)$

$$[a_p,a_q^\dagger]_{\mathcal{Q}} = \delta_{pq}$$

$\eta \in \mathbb{R}, |\eta| \leq 1$ ensure consistency of QFT

No other relations needed to compute observables

For real constant $\mathcal{Q} = q$ this reduces to quon model

All previous analyses fixed $\mathcal{Q}(p,q) = \pm f_\theta^2(p,q)$

Then $\left(\tau_\theta^i\right)^2 = 1$ and additional relations hold

$$[a_p,a_q^\dagger]_{\mathcal{Q}} = \delta_{pq} \quad + \quad [a_p^\#,a_p^\#]_{\mathcal{Q}^*} = 0$$

$a_p^\# \coloneqq a_p, a_p^\dagger$

More similar to ordinary QFT

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Results

- Pauli-violating transitions heavily depends on the form of η
- $\eta = \pm 1$ predict wrong results
- η ensures UV regularization

Theoretical consistency

A satisfactory quantization scheme has to guarantee $i\partial_t\psi(x) = [\psi(x), H]$. Free fields / interaction representation:

$\psi(x) = \int d\mu(p) \sum_s [e^{-ip\cdot x} u_s(p) a_s(p) + e^{ip\cdot x} v_s(p) b_s^\dagger(p)]$ ⟹ $[a_s^\#(p), H] = \pm E_p a_s^\#(p),\ [b_s^\#(p), H] = \pm E_p b_s^\#(p)$

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Para-fields (H. S. Green 1953)

Assume bilinear mode expansion for number operators

$$H = \int d^3p\, E_p \sum_s \left([a_s^\dagger(p), a_s(p)]_\pm + [b_s^\dagger(p), b_s(p)]_\pm \right)$$

Then derive relations for creation and annihil. operators

$$\left[a_s^\#(p), [a_r^\dagger(q), a_r(q)]_\pm\right] = \pm\delta_{sr}\delta(p-q) a_s^\#(p)$$

$$\left[b_s^\#(p), [b_r^\dagger(q), b_r(q)]_\pm\right] = \pm\delta_{sr}\delta(p-q) b_s^\#(p)$$

Bose and Fermi fields are the trivial solutions

Para-Bose and para-Fermi fields yield consistent local QFT

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Q-deformed fields

$$\begin{cases} H = \int d^3x \psi^\dagger(x) h_D \psi(x) \\ h_D \psi^+(x) = [\psi^+(x), H] + \int d^3y\, \psi^{-\dagger}(y) h_D^y [\psi^+(y), \psi^+(x)]_{Q^*} \end{cases}$$

- If $\boldsymbol{\eta = \pm 1}$: $[\psi(x), \psi(y)]_{Q^*} = 0 \Rightarrow \boldsymbol{N = a_s^\dagger(p) a_s(p)}$
- If $|\boldsymbol{\eta}| < \mathbf{1}$: $\begin{cases} \boldsymbol{H = \int d^3p E_p (N(p) + N^c(p))} \\ \boldsymbol{N, N^c} \textbf{ admit infinite expansion in } \boldsymbol{a^\#} \textbf{ and } \boldsymbol{b^\#} \end{cases}$

$$N(p) = \sum_s a_s^\dagger(p) a_s(p) + \int d^3p \frac{1}{1-\eta^2(p,q)} \sum_{sr} \left[a_r^\dagger(q), a_s^\dagger(p)\right]_{Q^*} [a_s(p), a_r(q)]_{Q^*} + \cdots$$

For $Q = const$ it gives Stanciu's expression for quon-model

Correlation functions

- Two-particle propagators similar to ordinary QFT

$$\left[\phi^{+}(x), \phi^{-\dagger}(y)\right]_{\mathcal{Q}} = \left[\phi^{-}(x), \phi^{+\dagger}(y)\right]_{\mathcal{Q}} = \Delta^{+}(x-y) \text{ with } \Delta^{+}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{e^{-ip\cdot x}}{2E_p}$$

$$G^{(2)}(x_1, x_2) = \langle 0|T\{\phi(x)\phi^{\dagger}(y)\}|0\rangle = \Delta(x-y) \text{ with } \Delta(x) = \int \frac{d^4p}{(2\pi)^4} \frac{ie^{-ip\cdot x}}{(p-m+i\epsilon)^2}$$

- The generic n-point Green function $G^{(n)}(x_1, \ldots, x_n) = \langle 0|T\{\phi(x_1) \ldots \phi^{\dagger}(x_n)\}|0\rangle$ given in terms of $\Delta(x-y)$

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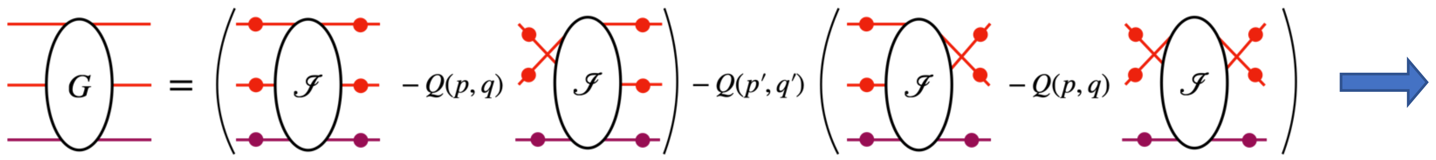
- Examples



diagrams with particles exchanged weighted by $Q(p,q) = \eta(p,q) f_{\theta}^{2}(p,q)$

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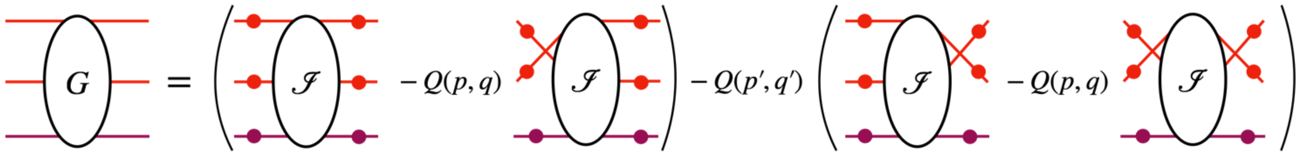
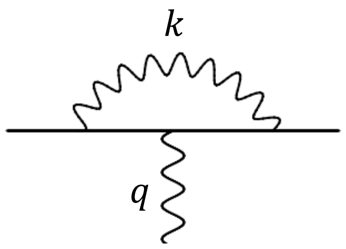


diagrams with particles exchanged weighted by $\mathcal{Q}(p,q) = \eta(p,q) f_{\theta}^{2}(p,q)$



$$\sim \int \frac{d^4k}{(2\pi)^4}\eta(p,q) I_0(k)$$

η may provide UV regolarization. E.g. $\eta(p,q) = \exp\left(-\frac{p\cdot q}{\Lambda_{\theta}^{2}}\right)$

Two-particle Hilbert space

Need new inner product compatible with θ-Poincaré symmetry: $\langle\psi|\phi\rangle_\theta \coloneqq \int d^3p d^3q \psi^\dagger(\boldsymbol{p},\boldsymbol{q})[\phi(\boldsymbol{p},\boldsymbol{q}) + \mathcal{Q}(\boldsymbol{p},\boldsymbol{q})\phi(\boldsymbol{q},\boldsymbol{p})]$

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Untwisted-states

$$\begin{cases} \psi(\boldsymbol{p},\boldsymbol{q}) = \frac{1}{2N}\psi_s(\boldsymbol{p},\boldsymbol{q}) + \frac{1}{2N}\psi_a(\boldsymbol{p},\boldsymbol{q}) \\ \psi_{s,a}(\boldsymbol{p},\boldsymbol{q}) = \psi(\boldsymbol{p},\boldsymbol{q}) \pm \psi(\boldsymbol{p},\boldsymbol{q}) \end{cases}$$

- Completely symmetric states allowed

$$N_s^2 = \int d^3p d^3q \left(1 + \eta(p,q)\cos p_\mu \theta^{\mu\nu} q_\nu\right)|\psi_s(\boldsymbol{p},\boldsymbol{q})|^2$$

- $\psi_{s,a}$ no more superselected: $\langle\psi_s|\mathcal{O}|\psi_a\rangle \neq 0$

Transitions between different symmetric components

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Transitions between different symmetric components

Twisted-states

$$\psi_{s_\theta,a_\theta}(\boldsymbol{p},\boldsymbol{q}) = \frac{\psi(\boldsymbol{p},\boldsymbol{q}) \pm f_\theta^{-2}(\boldsymbol{p},\boldsymbol{q})\psi(\boldsymbol{p},\boldsymbol{q})}{N_{s_\theta,a_\theta}} = \pm f_\theta^2(\boldsymbol{p},\boldsymbol{q})\psi_{s_\theta,a_\theta}(\boldsymbol{p},\boldsymbol{q})$$

$$N_{s_\theta,a_\theta}^2 = \int d^3p d^3q \left(1 \pm \eta(p,q)\right)|\psi_{s_\theta,a_\theta}(\boldsymbol{p},\boldsymbol{q})|^2$$

$$\psi(\boldsymbol{p},\boldsymbol{q}) = c_{s_\theta}\psi_{s_\theta}(\boldsymbol{p},\boldsymbol{q}) + c_{a_\theta}\psi_{a_\theta}(\boldsymbol{p},\boldsymbol{q}), \quad |c_{s_\theta,a_\theta}| = \frac{N_{s_\theta,a_\theta}}{\sqrt{N_{a_\theta}^2 + N_{s_\theta}^2}}$$

η interpolates smootlhy between symmetric/antisymmetric states

- ψ_{s_θ,a_θ} are superselected: $\langle\psi_{s_\theta}|\mathcal{O}|\psi_{a_\theta}\rangle = 0$ for all observables

$$\rho^{(2)} = |c_{a_\theta}|^2|\psi_{a_\theta}\rangle\langle\psi_{a_\theta}| + |c_{s_\theta}|^2|\psi_{s_\theta}\rangle\langle\psi_{s_\theta}|$$

Electrons are indistinguishable

Fock space

Given a permutation $\pi = \pi_{k_1} \dots \pi_{k_m}$, define the representations $\tau_\theta^\pi \coloneqq \tau_\theta^{k_1} \dots \tau_\theta^{k_m}$ and $\sigma_\theta^\pi \coloneqq \sigma_\theta^{k_1} \dots \sigma_\theta^{k_m}$ in $\mathcal{H}^{\otimes n}$ with

$\tau_\theta^k |p_1, \dots, p_n\rangle = \eta(p_k, p_{k+1}) f_\theta^2(p_k, p_{k+1}) |p_1, \dots, p_{k+1}, p_k, \dots, p_n\rangle$ and $\sigma_\theta^k |p_1, \dots, p_n\rangle = f_\theta^2(p_k, p_{k+1}) |p_1, \dots, p_{k+1}, p_k, \dots, p_n\rangle$

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- Inner product of $|\psi\rangle, |\phi\rangle \in \mathcal{H}^{\otimes n}$: $\langle\psi|\phi\rangle_\theta := \langle\psi| \sum_{\pi \in S_n} \tau_\pi |\phi\rangle$

Positive for $|\eta(p,q)| \leq 1$
(*Bozejko, Lytvynov and Wysoczanski 2017*)

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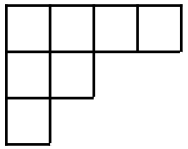
- A generic n-electron state decomposes into all irreducible representations of the symmetric group S_n.

  Each component is given by the Young symmetrizer associated to a given Young tableau λ:

λ: , $\mathcal{P}_\theta^{(n)}(\lambda) := \frac{1}{n!} \sum_{\substack{\pi_r \in R_\lambda \\ \pi_c \in C_\lambda}} \mathrm{sgn}(\pi_c)\, \sigma_\theta^{\pi_r \pi_c}$ ⇒

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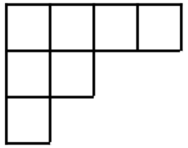
λ:  , $\mathcal{P}_\theta^{(n)}(\lambda) := \frac{1}{n!} \sum_{\substack{\pi_r \in R_\lambda \\ \pi_c \in C_\lambda}} \mathrm{sgn}(\pi_c)\, \sigma_\theta^{\pi_r \pi_c}$ ⇒

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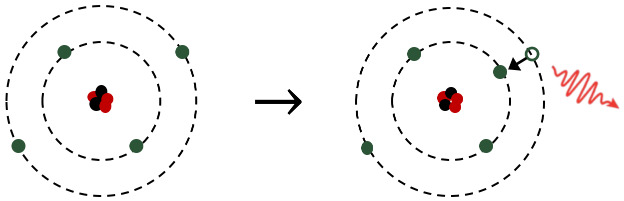
$|\psi_{\lambda_\theta}\rangle = \mathcal{P}_\theta^{(n)}(\lambda)|\psi\rangle, \quad |\psi\rangle = \Sigma_\lambda |\psi_{\lambda_\theta}\rangle$

- $\mathcal{F}_\theta(\mathcal{H})$ defined as the completion of $\mathcal{F}(\mathcal{H}) = \bigoplus_{n=0}^{\infty} \mathcal{H}^{\otimes n}$ with respect to the inner product $\langle \cdot | \cdot \rangle_\theta$

> - For $\eta = \pm 1$: $\langle \cdot | \cdot \rangle_\theta$ automatically projects onto $\mathcal{F}_\theta^\pm(\mathcal{H})$ and remain only twisted-bosons or twisted-fermions
> - For $-1 \leq \eta \leq 1$: **smooth interpolation between twisted-Bose and twisted-Fermi statistics**
> - **Relativistic generalization of the quon-model. Consistency ensured by twisted Poincaré symmetry**

PEP-violating transition rates

- Transition rates are given by standard time-dependent perturbation theory



$|\psi\rangle$ $\quad$ $|\psi'\rangle|\boldsymbol{k},\lambda\rangle$

$|\psi\rangle = \frac{1}{2N}(|\psi_a\rangle + |\psi_s\rangle), |\psi'\rangle = \frac{1}{N_s'}|\psi_s'\rangle$ solutions of $H_0\psi = E\psi$

$$\delta H = \sum_i \mathbb{I}^{\otimes i-1} \otimes V \otimes \mathbb{I}^{\otimes n-i}, \qquad V(t) = e\boldsymbol{\alpha} \cdot \sum_{\boldsymbol{k},\lambda} \boldsymbol{\varepsilon}_\lambda\, e^{ik\cdot\hat{\boldsymbol{x}}} e^{-i\omega t} c_\lambda(\boldsymbol{k}) F_\theta(\boldsymbol{k},\hat{\boldsymbol{p}}) + h.c.$$

$F_\theta(\boldsymbol{k},\hat{\boldsymbol{p}})$ combines corrections arising from noncommutative QED and deformed statistics and $F_\theta(\boldsymbol{k},\hat{\boldsymbol{p}}) \to 1$ as $\theta \to 0$

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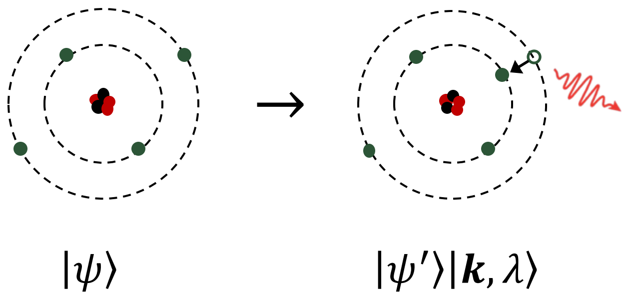

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$$d\Gamma_{PEPV} = 2\pi\delta(E' - E + \omega)\left|\delta H_{if}\right|^2 \frac{d^3k}{(2\pi)^3}$$

$$\delta H_{if} = \frac{e}{\sqrt{2\omega}} \int d^3p\, d^3q\, F_\theta(\boldsymbol{k},\boldsymbol{p}) \psi_s'(\boldsymbol{p},\boldsymbol{q}) \boldsymbol{\alpha}_1 \cdot \boldsymbol{\varepsilon}_\lambda^* \left[\left(\frac{1 - \eta(\boldsymbol{p},\boldsymbol{q}) \cos p\theta q}{N N_s'} \right) \psi_s(\boldsymbol{p}+\boldsymbol{k},\boldsymbol{q}) + \left(\frac{i\eta(\boldsymbol{p},\boldsymbol{q}) \sin p\theta q}{N N_s'} \right) \psi_a(\boldsymbol{p}+\boldsymbol{k},\boldsymbol{q}) \right]$$

At leading order in θ: $F_\theta(\boldsymbol{k},\boldsymbol{p}) \to 1, N \to 1, \psi_{s,a}, \psi_s' \to$ QED wavefunctions:

➢ **δH_{if} reduce to QED matrix elements modified solely by the non-trivial norm N_s' and the exchange factor $Q = \eta f_\theta^2$**

$\boldsymbol{\theta}$-expansion

Convenient writing $\theta^{\mu\nu} = \dfrac{c^{\mu\nu}}{\Lambda_\theta^2}$ where $c^{\mu\nu} \sim \mathcal{O}(1)$ and $\boldsymbol{\Lambda_\theta}$ **energy scale of noncommutativity.** Expand in powers of Λ_θ^{-1}:

$\eta(\boldsymbol{p}, \boldsymbol{q})$:

- must be a function of the invariants $m^2, p \cdot q, (p+q)^2$ and $(p-q)^2$
- depends on Λ_θ as $\eta \to 1$ for $\Lambda_\theta \to \infty$

$$\begin{cases} \eta(\boldsymbol{p}, \boldsymbol{q}) \sim \pm\left[1 - \left(\dfrac{\sigma(p,q)}{\Lambda_\theta^2}\right)^{\frac{\kappa}{2}}\right], & \kappa \geq 0 \\ \sigma_{nr}(\boldsymbol{p}, \boldsymbol{q}) = m^{2-a}|\boldsymbol{p} - \boldsymbol{q}|^2, & a \leq 2 \end{cases}$$

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Leading order of PEPV matrix elements heavily depends on the expansion of $\boldsymbol{\eta}$

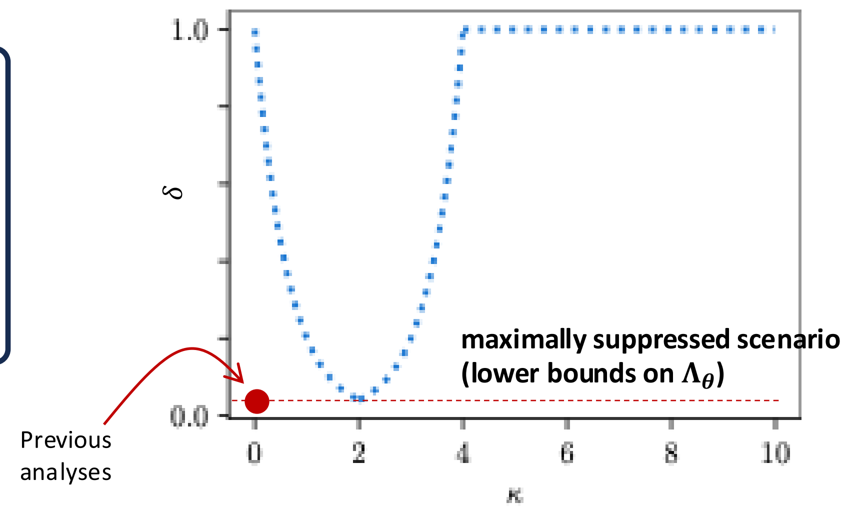
$$N_s' = \left[\iint d^3p d^3q \, (1 - \eta(p,q) \cos p\theta q)|\psi_s'(\boldsymbol{p}, \boldsymbol{q})|^2\right]^{\frac{1}{2}} = \frac{\mathcal{N}_\kappa}{\Lambda_\theta^{h(\kappa)}}, \quad \text{where } h(\kappa) = \begin{cases} 2 & \kappa = 0, \kappa \geq 4 \\ \kappa & 0 < \kappa < 4 \end{cases}$$

Then the matrix elements are proportional to

$$\begin{cases} \frac{1 - \eta(\boldsymbol{p}, \boldsymbol{q}) \cos p\theta q}{N_s'} \sim \frac{\left(\sigma/\Lambda_\theta^2\right)^{\frac{\kappa}{2}}}{\mathcal{N}_\kappa/\Lambda_\theta^{h(\kappa)}} \\ \frac{\eta(\boldsymbol{p}, \boldsymbol{q}) \sin p\theta q}{N_s'} \sim \frac{pcq/\Lambda_\theta^2}{\mathcal{N}_\kappa/\Lambda_\theta^{h(\kappa)}} \end{cases}$$

$$\frac{d\Gamma_{PEPV}}{d\Omega} = \delta\left(\frac{E}{\Lambda_\theta}\right)\frac{d\Gamma_0}{d\Omega}$$

δ depends on $\Lambda_\theta, m, E_0 = Z^2\, 13.6$ eV and the expansion parameters κ, a



- **$\kappa = 0$ and $\kappa \geq 4$ - Analogous to parastatistics:**

Previous analyses: $\delta \sim \mathcal{O}(\Lambda_\theta^{-2})$. We found that:

Λ_θ cancels in the matrix elements: no suppresion as $\Lambda_\theta \to \infty$!

This agrees with a well known but result:

E.g. using $\psi_s \approx \varphi_{nS}\varphi_{nS}$ then

$$\langle p, q|\psi_s\rangle \to \phi(\boldsymbol{p},\boldsymbol{q}) := i\frac{\hat{\boldsymbol{c}}\cdot(\boldsymbol{p}\times\boldsymbol{q})}{mE_0}\psi_s^{(0)}(\boldsymbol{p},\boldsymbol{q}) \in \mathcal{H}_-^{\otimes 2}$$

where $\hat{\boldsymbol{c}} = \frac{\boldsymbol{c}}{|\boldsymbol{c}|}, c^i = \frac{1}{2}\epsilon^{ijk}c^{jk}$

- **$0 < \kappa < 4$ - Analogous to quon model:**

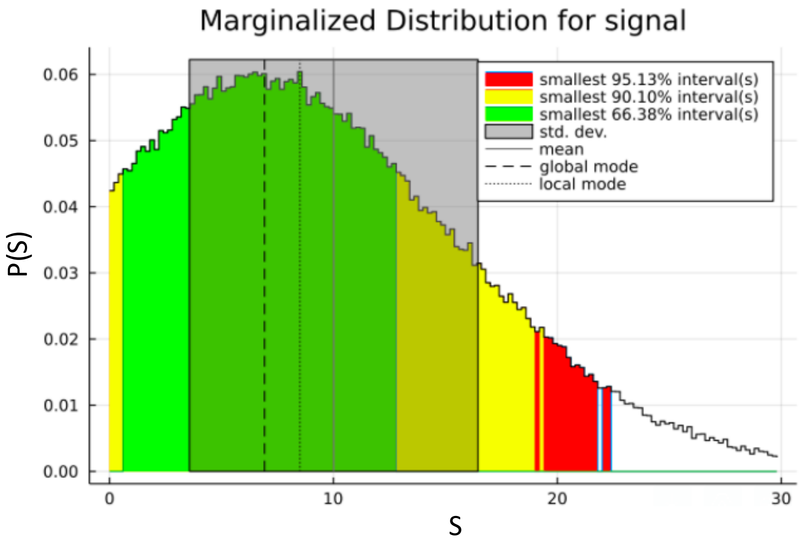
$$\delta\left(\frac{E}{\Lambda_\theta}\right) = \begin{cases} \gamma_{a\kappa}\left(\frac{m^{1-a/4}E_0^{a/4}}{\Lambda_\theta}\right)^{\kappa} & \kappa \leq 2 \\ \gamma_{a\kappa}\left(\frac{\sqrt{mE_0}}{\Lambda_\theta}\right)^{4-\kappa} & 2 < \kappa < 4 \end{cases}$$

$\gamma_{a\kappa} \sim \mathcal{O}(1)$. Strongest suppressions $\delta \sim \mathcal{O}(\Lambda_\theta^{-2})$

Atoms with $\frac{2E_0}{m} \sim \mathcal{O}(1)$ (e.g. Pb, Ta): δ's same O.o.m for all $a \sim 1$

VIP experiments search for anomalous X-ray emissions in the low-background environment of the underground Gran Sasso National Laboratory of INFN

- Extract upper limit $\bar{S}$ on the number of PEPV counts S



- Convert into a lower limit on Λ_θ: $\Lambda_\theta \geq \left(\frac{\bar{S}}{\aleph}\right)^{\frac{1}{h(\kappa)}}$

Wait Kristian's talk for more details…

We obtain the strongest lower bounds on Λ_θ

No evidence for QG up to the Planck energy

Final remarks

Motivations

- Violations of locality expected/predicted to emerge in the UV regime from **QG effects**
- Possibility to implement covariance provided by deformation of the Poincaré (Hopf-)algebra and statistics

Results:

- Generalized statistics yield perfectly **consistent QFT** even for strict braidings ($-1 < \eta < 1$). **Promising UV behaviour**…
- We provide the first **relativistic formulation of quon-model** for $-1 < \eta < 1$
- First detailed derivation of PEP-violating transition rates, showing much **richer phenomenology**
- Although based on too semplicistic analyses, **previous bounds effectively hold**, corresponding to maximal suppressions $\Gamma_{PEPV} \sim \mathcal{O}\left(\Lambda_\theta^{-2}\right)$
- Refined analysis yields stronger lower bounds: **no evidence of QG effects up to the Planck energy**

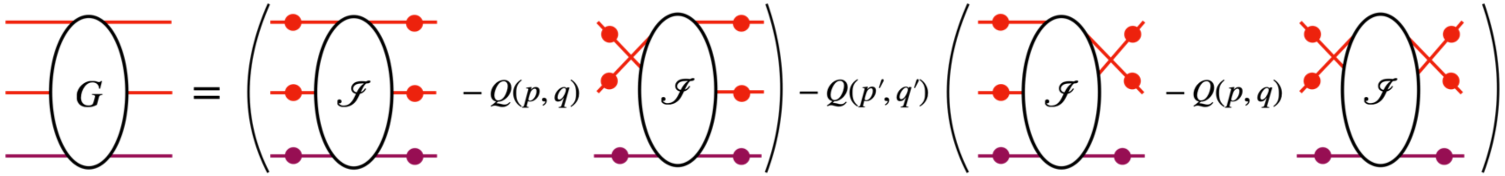
Thank you!

Relativistic wave equation for helium

QFT provides a rigorous relativistic treatment for bound states in terms of Green functions

 nucleus dressed propagator

 electron dressed propagator



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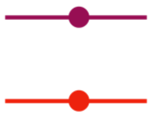
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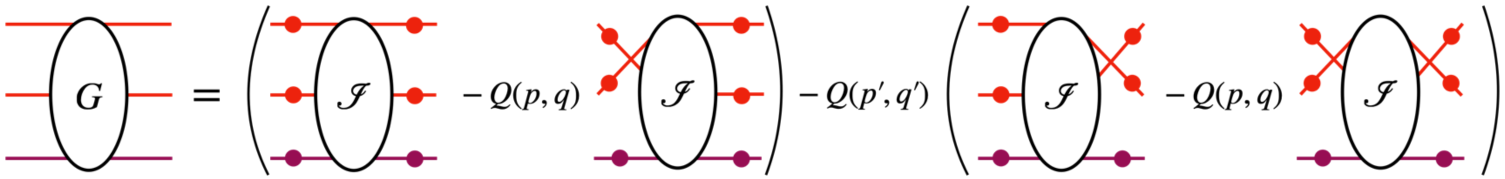
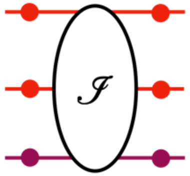


Poles of G contain bound states



$$\sim \frac{\widetilde{\Phi}(p,q)\widetilde{\Phi}^{\dagger}(p',q')}{2E_P(P^0 - E_P + i\epsilon)}$$

Deformed Bethe-Salpeter amplitude

$$\Phi_{\mathrm{BS}}(p,q) = \widetilde{\Phi}(p,q) - \mathcal{Q}(p,q)\widetilde{\Phi}(p,q)$$

Multi-time $\widetilde{\Phi}$ can be expressed by single-time Ψ

$$\widetilde{\Phi}(\{p_i\}) = [(S_1 + S_2)G_{12} + (23) + (13)]\Psi(\{\boldsymbol{p}_i\})$$

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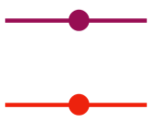

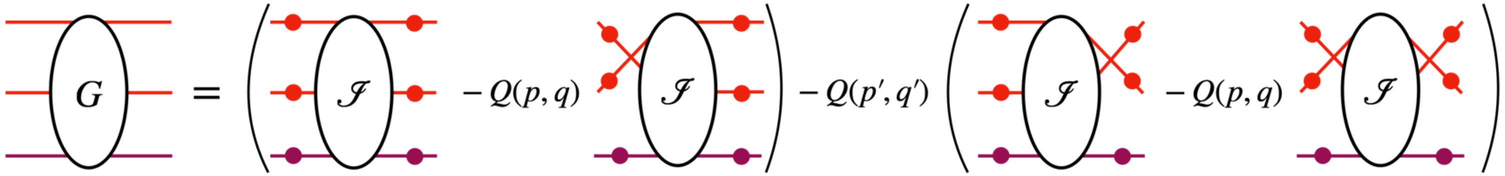

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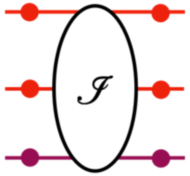

$$\sim \frac{\tilde{\Phi}(p,q)\tilde{\Phi}^{\dagger}(p',q')}{2E_P(P^0 - E_P + i\epsilon)}$$

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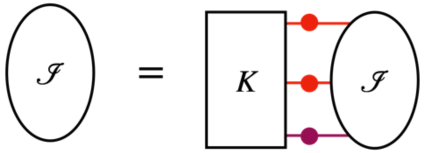


yields B-S equation

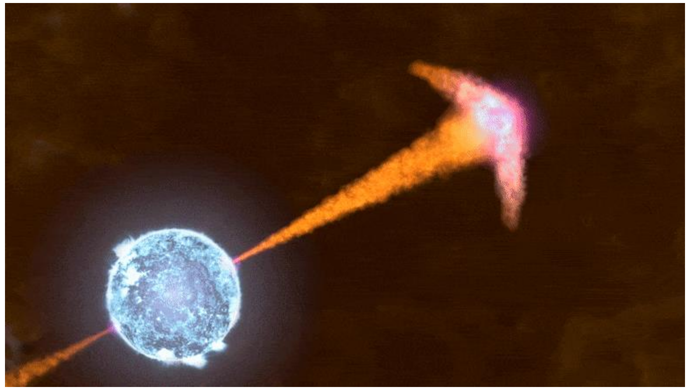
$$S_e(p_1)S_e(p_2)S_n(p_n)\tilde{\Phi}(\{p_i\}) = \left(K \cdot \tilde{\Phi}\right)(\{p_i\})$$

$$\tilde{\Phi}(p_1, p_2, p_n) \to \Psi(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_n) \to \psi(\boldsymbol{p}_1, \boldsymbol{p}_2)$$

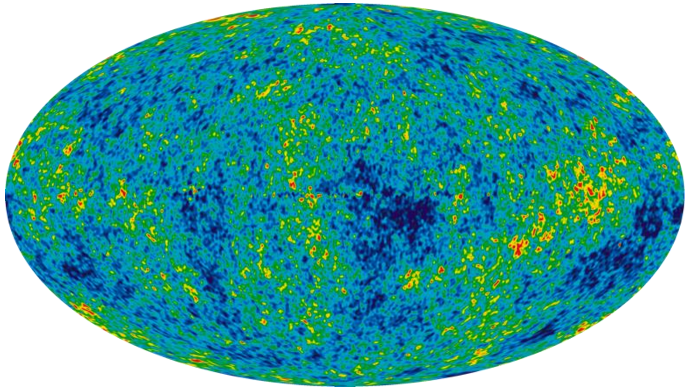
Salpeter 3D equation for two electrons in central field

$$\left[h_D^{(1)} + h_D^{(2)} + \mathcal{V}_1^{+} + \mathcal{V}_2^{+} + \mathcal{V}_{12}^{+}\right]\psi = E\psi$$

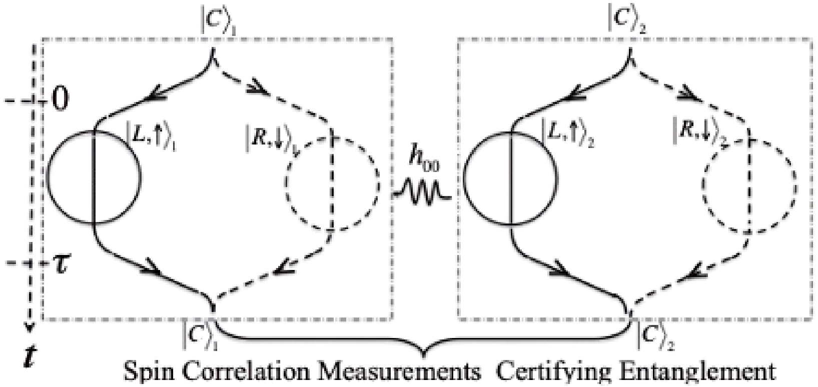
$\mathcal{V} = \Lambda_+^{(1)}\Lambda_+^{(2)} V \Lambda_+^{(1)}\Lambda_+^{(2)}$, $\Lambda_+^{(i)}$ positive energy projectors



20 years research of in vacuo dispersion effects in GRBs



No detection of tensor B-modes in CMB (primordial gravitons)



In future table-top tests of gravity's quantumness

In a **4-dim spacetime** the spin-(para)statistics theorem follows directly from **locality** and **Poincaré invariance**

Guido & Longo Commun. Math. Phys 1995

↓

PEP violations provide smoking guns for QG