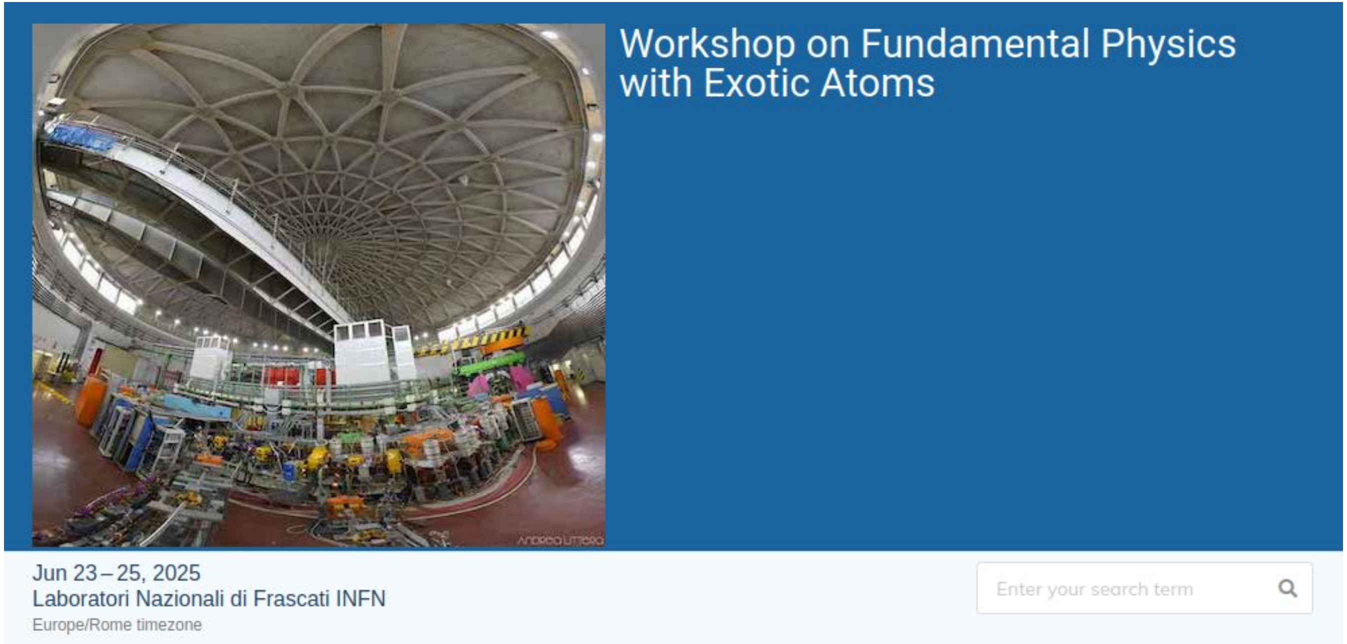


Kaonic Neon Spectroscopy: Testing Bound-State QED, Extracting the Kaon Mass and Probing Beyond Standard Model Physics

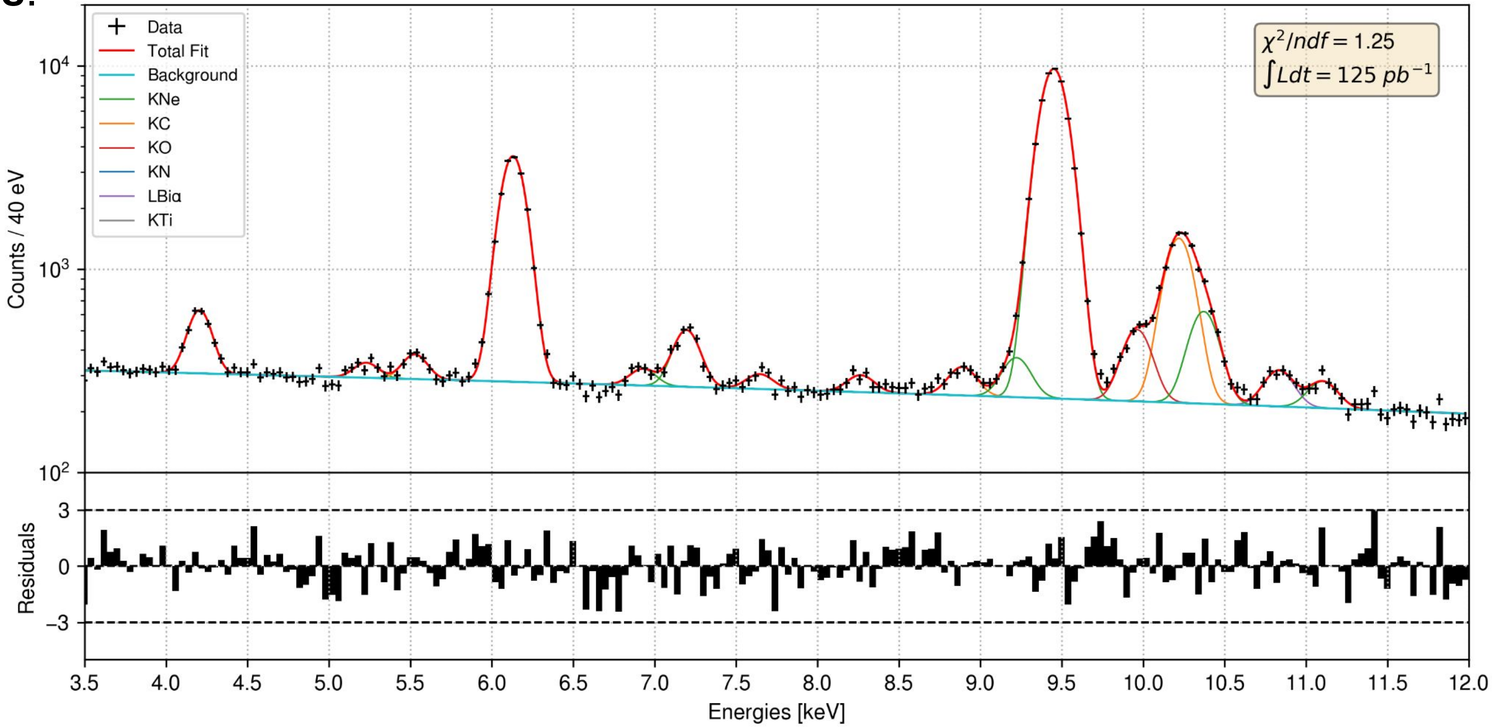
Simone Manti

Outline

- **Kaonic Neon Spectroscopy:**
  - Testing Bound-State QED
  - Extracting the Kaon Mass
  - Probing Beyond Standard Model Physics

High Precision X-ray Spectroscopy in KNe

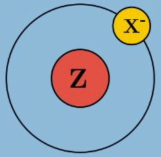
The **Kaonic Neon (KNe)** spectrum exhibits several transitions with **Intense Yields** resulting from the **Radiative De-Excitation** in the X-ray range of the kaonic atom during its **Cascade Process**.



The MCDFGME code

- We run **MCDF code** for Kaonic Atom Calculations, including Relativistic and QED effects, in collaboration with **Prof. Paul Indelicato (Paris–CNRS).**

Ab Initio Input



- **Ab Initio Approach:** based only on **Physical Laws** and **Fundamental Constants.**
- Supports **Normal, Exotic,** or **Mixed Atomic Systems.**
- Supported **Exotic Particles:**
  - K^- : Kaons
  - $\bar{\mu}$: Muons
  - π^- : Pions
  - Σ^- : Sigmas
  - $\bar{p}^-$: Antiprotons
  - X^- : Test Particles

MCDF Framework

Basis Wavefunctions:

- Dirac Equation (spin-½ particles)

$$\left[c\boldsymbol{\alpha}\cdot\mathbf{p}+\beta mc^2+V(\mathbf{r})\right]\psi_i(\mathbf{r})=\varepsilon_i\psi_i(\mathbf{r})$$

- Klein-Gordon Equation (spin-0 particles)

$$\left[-\hbar^2c^2\nabla^2+m^2c^4\right]\phi(\mathbf{r})=\left[E-V(\mathbf{r})\right]^2\phi(\mathbf{r})$$

Multi-Configuration Dirac-Fock:

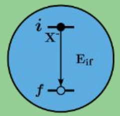
$$\Psi=\sum_{v=1}^{NCF}W_v\phi^v(1,2,\ldots,N;J,J_z)$$

$$\mathrm{E_{tot}}=\sum_{\nu\mu}W_\nu W_\mu\langle\phi^\nu|H|\phi^\mu\rangle/\sum_\nu W_\nu^2$$

Total Energies Contributions:

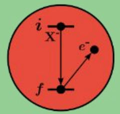
- QED
- Breit Interaction
- Recoil Effect
- Nuclear Finite Size
- Screening Effect

Output Quantities

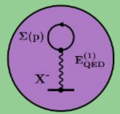


Energy Levels & Transitions

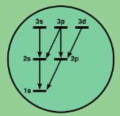
Radiative Transition Probabilities



Auger Decay Rates



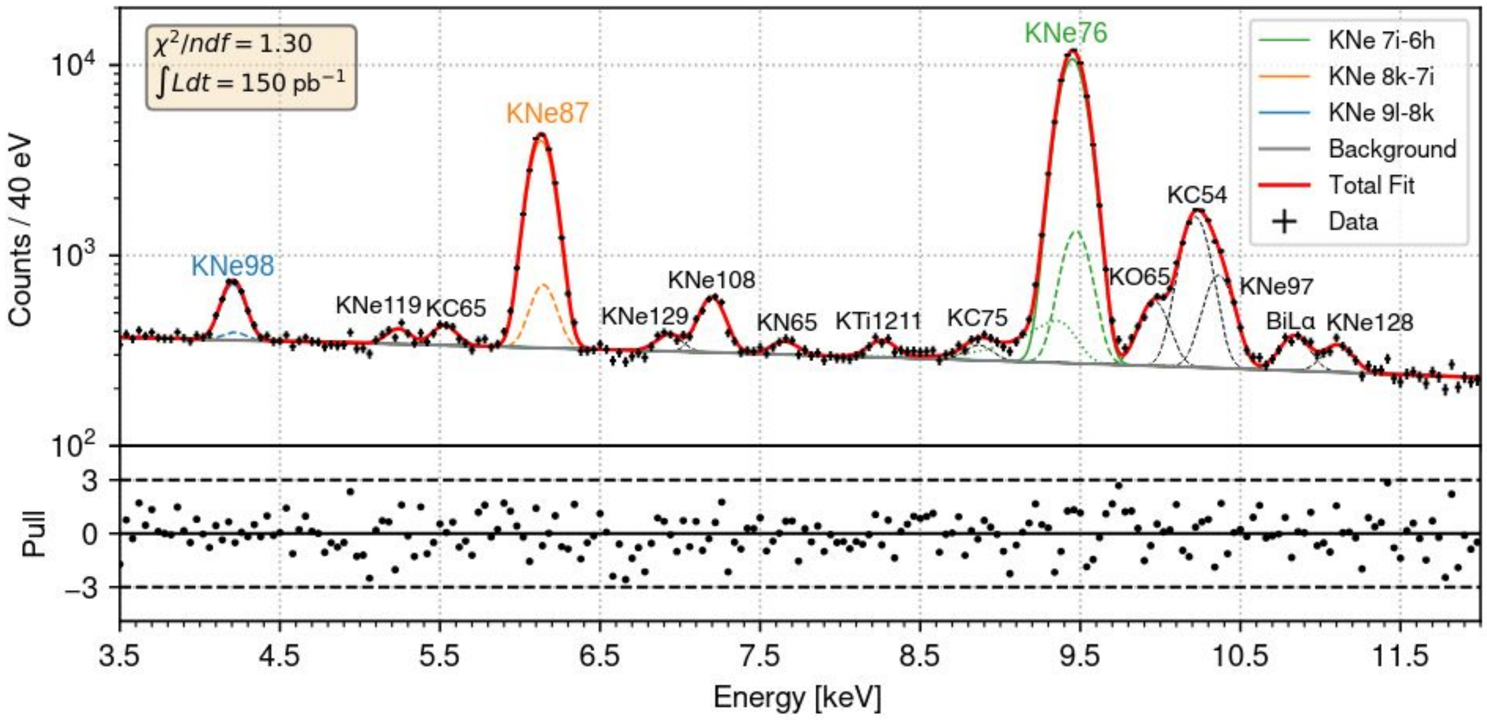
QED Self-Energy & Vacuum Polarization



Parameters for Atomic Cascade

High Precision X-ray Spectroscopy in KNe

Dirac-Fock Calculations are employed to determine the transition energies of the spectral peaks. **Nuclear Shifts and Widths** for n>4 are negligible ($\varepsilon_{4f}^{Ne} \approx 0 \quad \Gamma_{4f}^{Ne} \approx 0$)



High Precision X-ray Spectroscopy in KNe

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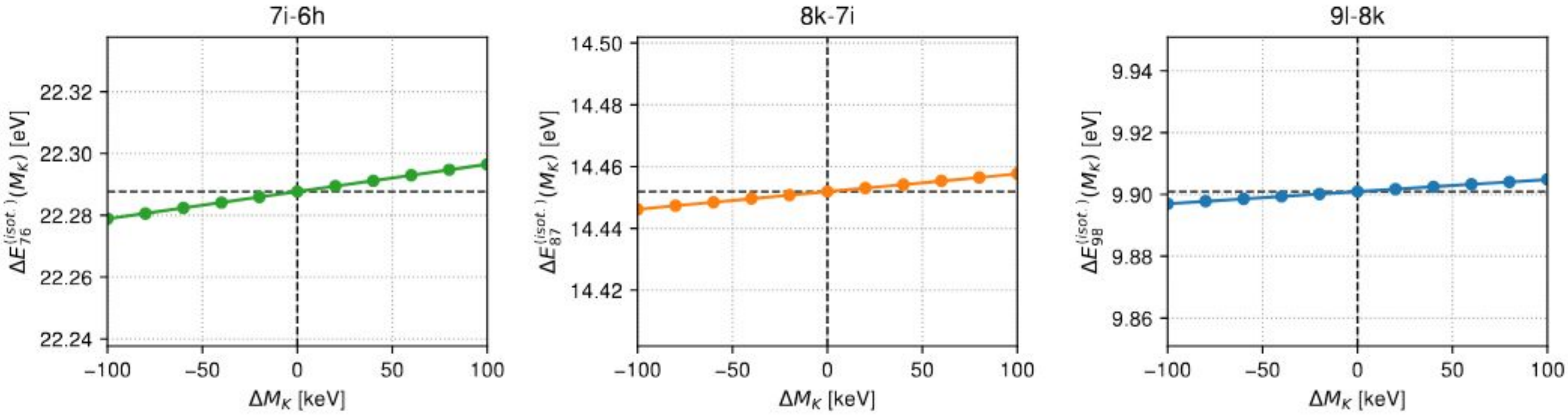
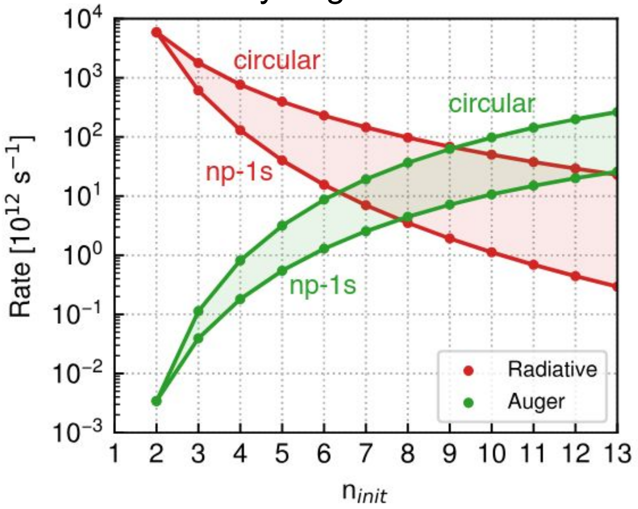


Calculations to Reveal the Cascade Mechanism in KNe

- **Qualitative trends** in **kaonic X-ray Yields** can be explained by the interplay between **Radiative** and **Auger decay** rates
  → X-ray emission dominates for n<10

- **Quantitative predictions** require accurate **Cascade Calculations**
  → Currently in development

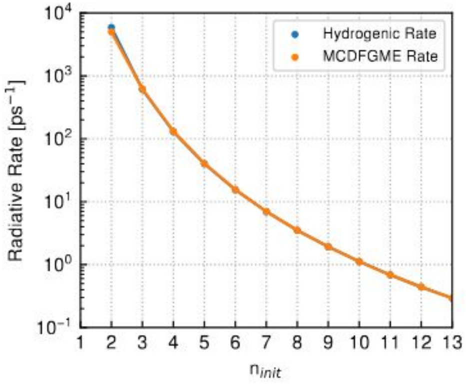
Scaled Hydrogenic Rates formulas

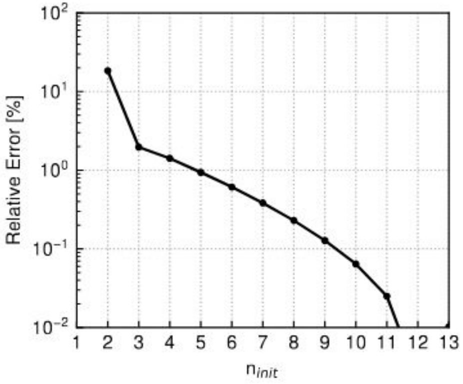


Burbidge, G.R., de Borde, A.H., 1953. Phys. Rev. 89, 189–193.

$$\Gamma^R_{n,\,l\to n',\,l\pm1} = \frac{4\mu Z^4}{3}\alpha^3 \left|R^{n,\,l}_{n',\,l\pm1}\right|^2 (\Delta E_{if})^3$$

$$\Gamma^A_{n,\,l\to n',\,l\pm1} = \frac{32}{3}\left(\frac{Z_e}{Z}\right)^2 \frac{\pi}{\mu^2}\frac{l}{2l+1}\left|R^{n,\,l}_{n',\,l\pm1}\right|^2 \frac{y^2}{1+y^2}\frac{\exp\left[y(4\tan^{-1}y-\pi)\right]}{\sinh(\pi y)}$$



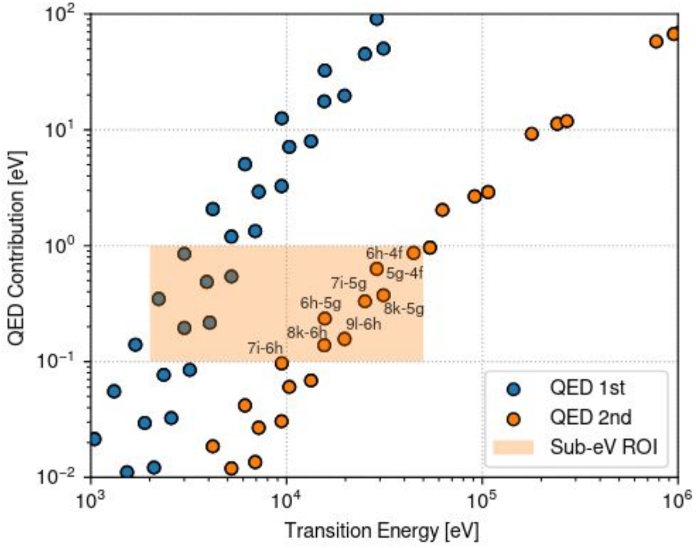


Kaonic Atoms for Bound State QED Studies

Transition	$E_{if}^{(\mathrm{exp.})}$	$\delta E_{if}^{(\mathrm{stat.})}$	$\delta E_{if}^{(\mathrm{sys.})}$	$E_{if}^{(\mathrm{calc.})}$	$E_{if}^{(\mathrm{QED})}$	$E_{if}^{(\mathrm{QED1})}$	$E_{if}^{(\mathrm{QED2})}$	$\Delta E_{if}^{(\mathrm{isot.})}$	$\Delta E_{if}^{(\mathrm{screen.})}$	$\Delta E_{if}^{(\mathrm{PDG})}$
9l-8k	4206.97	3.43	2.00	4201.45	2.09	2.07	0.02	9.90	-0.38	0.11
8k-7i	6130.57	0.65	1.50	6130.31	5.09	5.05	0.04	14.45	-0.27	0.16
7i-6h	9450.23	0.37	1.50	9450.28	12.66	12.56	0.10	22.28	-0.18	0.24
6h-5g[a]	15673.30	0.52	9.00	15685.39	32.75	32.51	0.24	37.01	-0.11	0.40

- **MCDF Calculations** include transition energies, QED effects, Electron Screening, and Isotopic Shifts ($K^{20}Ne$,$K^{22}Ne$)
- **Sub-eV Statistical Uncertainties** match the scale of **QED corrections** (for some even the 2nd QED contribution!)
- **$K^{20}Ne$ 7i → 6h transition:**
  - **Exp:** 9450.23 ± 0.37 (stat) ± 1.50 (syst) eV
  - **Theory:** 9450.28 ± 0.18 (screen) ± 0.21 (pdg) eV

  → Useful for validating **BSQED predictions**.



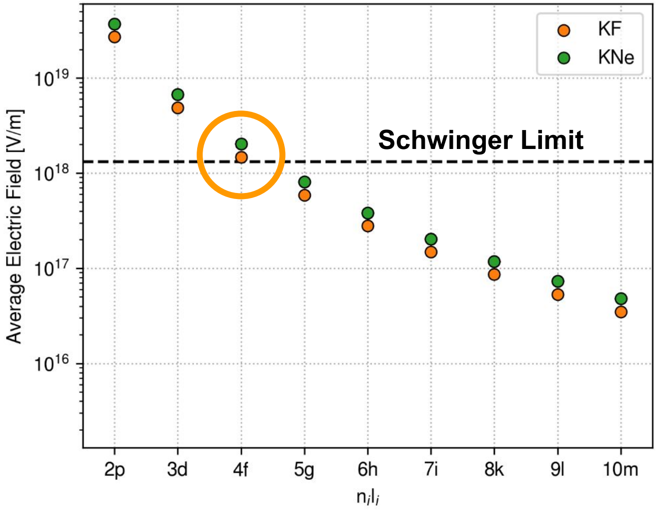
Kaonic Atoms as Probes of Strong Fields QED: Schwinger Limit for KF

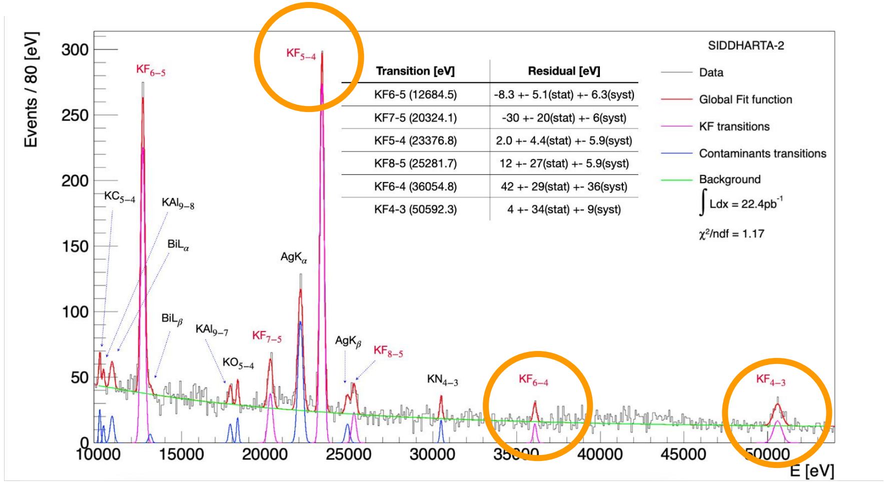
- **Exotic atoms** (like **KNe** and **KF**) enable experimental access to **Strong Electric Fields** [Paul *et al.*, PRL 126, 173001 (2021)]

- The **Schwinger Limit** for spontaneous **e⁺- e⁻ Pair Creation** is:

$$E_c = \frac{m_e^2 c^3}{q_e \hbar} \approx 1.32 \times 10^{18} V/m \qquad \langle E \rangle_{nl} = \int d^3 r \, |\psi_{nl}(\mathbf{r})|^2 E(\mathbf{r})$$

- The transition **KF 4f → 3d, E = 50.6 keV**, the **Average Electric Field** in kaonic orbitals approaches Ec





Transition [eV]	Residual [eV]
KF6-5 (12684.5)	-8.3 +- 5.1(stat) +- 6.3(syst)
KF7-5 (20324.1)	-30 +- 20(stat) +- 6(syst)
KF5-4 (23376.8)	2.0 +- 4.4(stat) +- 5.9(syst)
KF8-5 (25281.7)	12 +- 27(stat) +- 5.9(syst)
KF6-4 (36054.8)	42 +- 29(stat) +- 36(syst)
KF4-3 (50592.3)	4 +- 34(stat) +- 9(syst)

Kaonic Atoms for Bound State QED Studies: Effects of Kaon Mass

Transition	$E_{if}^{(exp.)}$	$\delta E_{if}^{(stat.)}$	$\delta E_{if}^{(sys.)}$	$E_{if}^{(calc.)}$	$E_{if}^{(QED)}$	$E_{if}^{(QED1)}$	$E_{if}^{(QED2)}$	$\Delta E_{if}^{(isot.)}$	$\Delta E_{if}^{(screen.)}$	$\Delta E_{if}^{(PDG)}$
9l-8k	4206.97	3.43	2.00	4201.45	2.09	2.07	0.02	9.90	-0.38	0.11
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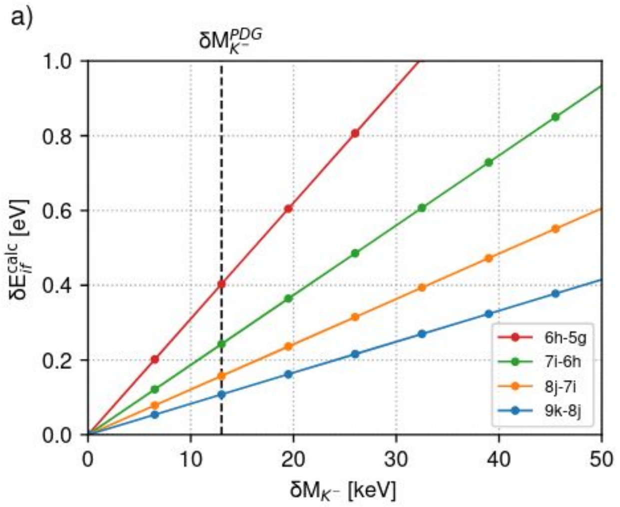
- **MCDF Calculations** include transition energies, QED effects, Electron Screening, and Isotopic Shifts ($K^{20}Ne$,$K^{22}Ne$)

- **Sub-eV Statistical Uncertainties** match the scale of **QED corrections** (for some even the 2nd QED contribution!)

- Investigate how result are robust against the uncertainty on the **Kaon Mass** (MK = 469.677 +- 0.013 MeV)



Precision Kaon Mass Determination: Application to KNe

Transition	$E_{if}^{(exp.)}$	$\delta E_{if}^{(stat.)}$	$\delta E_{if}^{(sys.)}$	$E_{if}^{(calc.)}$	$E_{if}^{(QED)}$	$E_{if}^{(QED1)}$	$E_{if}^{(QED2)}$	$\Delta E_{if}^{(isot.)}$	$\Delta E_{if}^{(screen.)}$	$\Delta E_{if}^{(PDG)}$
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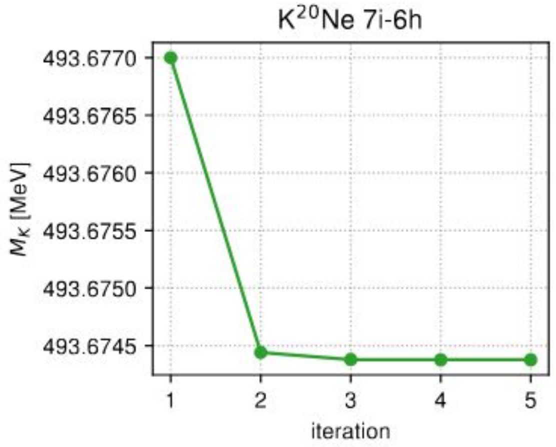
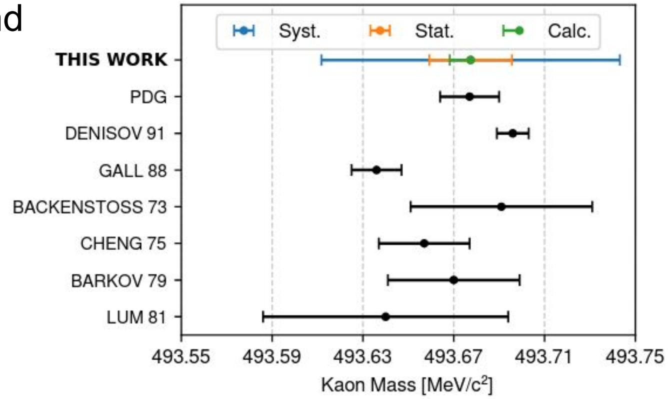


- **MCDF calculations** are used to iteratively extract the Kaon Mass from measured transitions as in **GAL88:**

$$M'_{K^-} = M_{K^-}\frac{E^{exp}}{E^{calc}} \qquad \delta M_{K^-} = M_{K^-}\frac{\delta E^{exp}}{E^{exp}}$$

- **Theoretical Uncertainties** stem from **Electron Screening** and depend on occupation of a **K-shell** electron in the 1s during the transition

Transition	M_{K^-} [MeV]	$\delta M_{K^-}^{stat.}$ [keV]	$\delta M_{K^-}^{syst.}$ [keV]	$\delta M_{K^-}^{calc.}$ [keV]
7i-6h	493.674	19	78	9
8k-7i	493.699	52	121	21
7i-6h + 8k-7i	493.677	18	66	11



Kaonic Atoms for Physics Beyond the Standard Model (BSM)

Transition	$E_{if}^{(\mathrm{exp.})}$	$\delta E_{if}^{(\mathrm{stat.})}$	$\delta E_{if}^{(\mathrm{sys.})}$	$E_{if}^{(\mathrm{calc.})}$	$E_{if}^{(\mathrm{QED})}$	$E_{if}^{(\mathrm{QED1})}$	$E_{if}^{(\mathrm{QED2})}$	$\Delta E_{if}^{(\mathrm{isot.})}$	$\Delta E_{if}^{(\mathrm{screen.})}$	$\Delta E_{if}^{(\mathrm{PDG})}$
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- Assume an effect on the Energy of an **Exotic Atom** level:

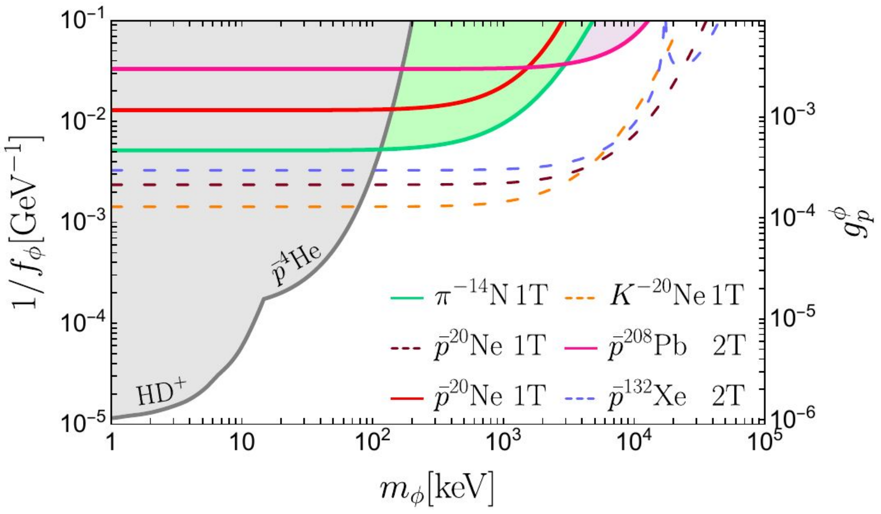
$$E_n^{\mathrm{th}} = \underbrace{E_n^{\mathrm{SM-NPol}} + E_n^{\mathrm{NPol}}}_{E_n^{\mathrm{SM}}} + E_n^X ,$$

- With **Yukawa Coupling:**

$$V_X(\boldsymbol{r}) = (-1)^s \frac{g_N^X g_H^X}{4\pi} \frac{e^{-m_X r}}{r} ,$$

$$E_n^X = (-1)^s \frac{g_H^X g_N^X}{4\pi} \frac{1}{r_n} \frac{1}{\left(1 + \frac{m_X r_n}{2n}\right)^{2n}} \equiv g_H^X g_N^X h_n^X ,$$

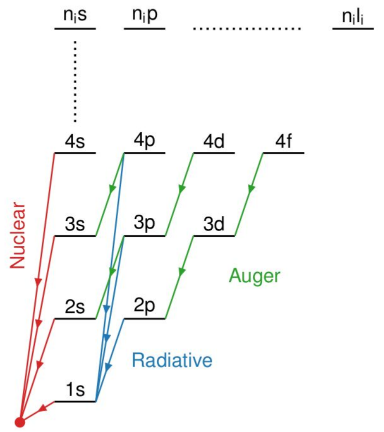
Constrain uds-scalar [arXiv:2502.03537]



Constraining Cascade Parameters with KNe (WIP)

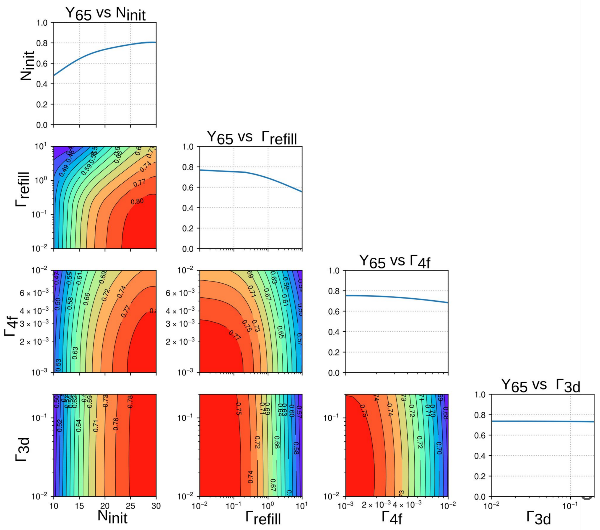
- Models are employed to predict **X-ray yields** and **Electron Populations** during the **Cascade**.

- **Cascade Calculations** depend on several **Parameters**:
  - **Initial conditions** of the cascade (n_{init})
  - Probabilities of **Electron Shell Refilling** (Γ_{refill})
  - **Nuclear Absorption Widths** (Γ_{3d} and Γ_{4f})



- **KNe** is the best system for constraining these parameters, thanks to its **numerous and accurate transition yields** by the **SIDDHARTA2 experiment**

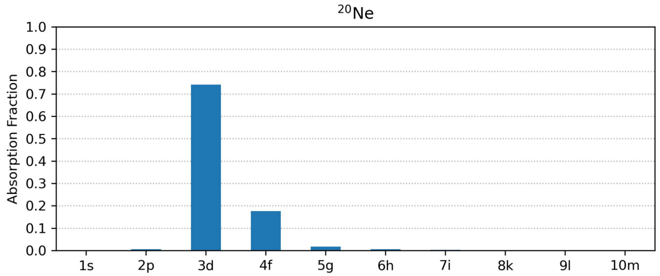
K-Ne Transition	Yield
9 - 8	(13.72 +/- 1.16)%
8 - 7	(22.79 +/- 0.38)%
10 - 8	(1.00 +/- 0.08)%
7 - 6	(27.74 +/- 0.24)%
10 - 7	(0.38 +/- 0.04)%
6 - 5	(30.84 +/- 0.26)%



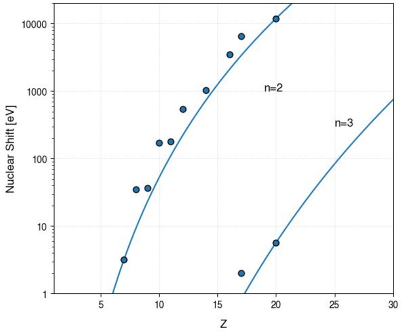
Connection between Nuclear Models and Experiment with Cascade Calculations (WIP)

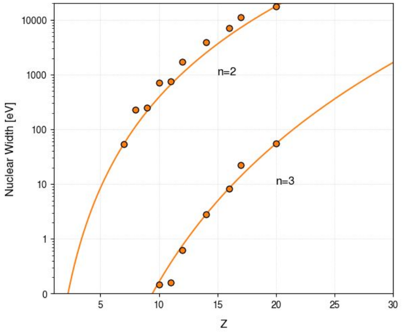
- Inputs from **Nuclear Models** for **Nuclear Shift and Widths** for Cascade Calculations

- **Nuclear Absorption Fractions** as function of the Widths



Z	Width 2p [eV]	Shift 2p [eV]	E32 [keV]	Width 3d [eV]	Shift 3d [eV]	E43 [keV]	Width 4f [eV]	Shift 4f [eV]
7Li	54	-3,2	15,33	0,0081	~0	5,355	~0	~0
8Be	225	-34.8	27,71	0,026	~0	9,677	~0	~0
9Be	247	-36,8	27,71	0,028	~0	9,677	~0	~0
10B	718	-170	43,568	0,146	~0	15,214	~0	~0
11B	757	-179	43,768	0,157	~0	15,284	~0	~0
12C	1723	-542	63,305	0,618	~0	22,105	~0	~0
14N	3906	-1028	86,731	2,8	~0	30,285	~0	~0
16O	7030	-3493	113,831	8,3	-0,39	39,755	0,0017	~0
17F	11249	-6534	144,759	22	-2	50,592	0,0065	~0
20Ne	17703	-11949	178,887	55	-5.7	62,568	0,0194	~0





Nuclear Shifts and Widths Calculations from:

Óbertová, J., Friedman, E., Mareš, J., 2022. First application of a microscopic K^-NN absorption model in calculations of kaonic atoms. Phys. Rev. C 106, 065201. https://doi.org/10.1103/PhysRevC.106.065201

Conclusion

- Comparison **Experimental Transition Energies** with **MCDF Calculations** for Kaonic Neon

- **Kaonic Atoms** can be used for **BSQED studies**

- **Kaonic Neon** can be used for **Extracting the Kaon Mass**

+

- **Kaonic Atoms** for Probing **Beyond Standard Model Physics**

- Work In Progress: **Cascade Calculations**