The X(3872) puzzle: insight from EFT

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Summary

What is an exotic hadron?

- How hadrons are made
- $X(3872)$

Non-relativistic EFT

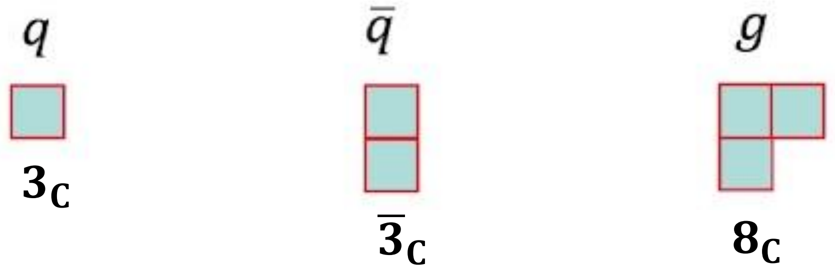
- Main features of a NREFT
- Low energy scattering in QM

X(3872): new-old ideas

- A toy model: nucleon scattering
- Future prospects

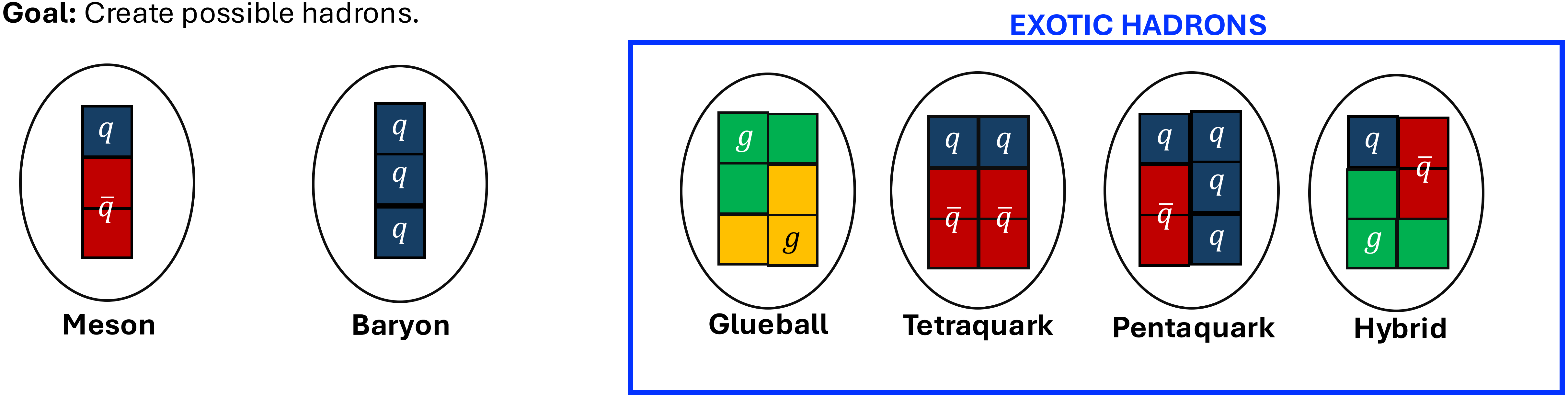
Let’s play a (group) game

- **Pieces:** **Quarks**, **Anti-quarks** and **gluons**



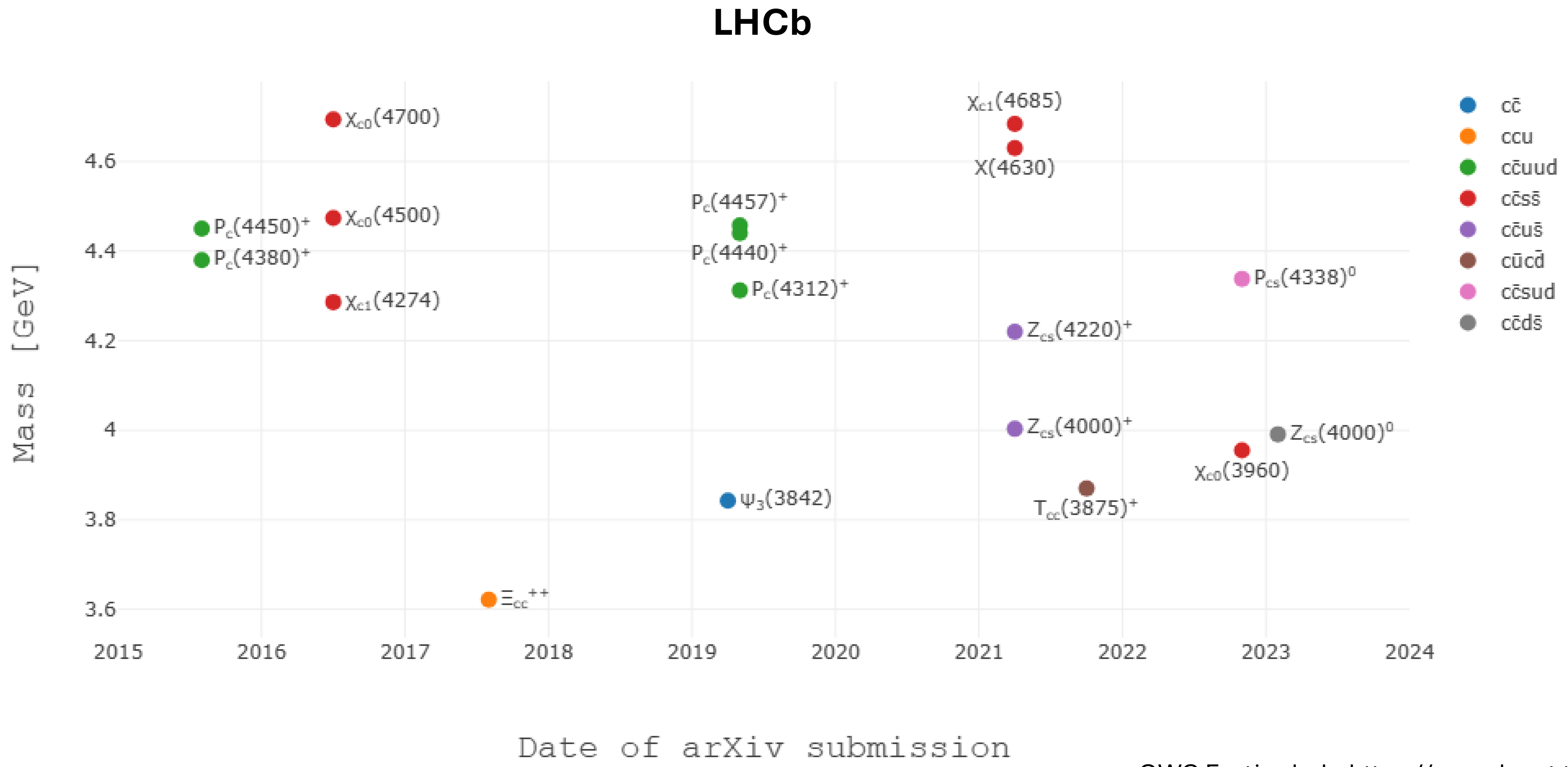
- **Rules:** Every combination is allowed **as long as** it respects **confinement** → **all boxes are in column of three elements**

- **Goal:** Create possible hadrons.



Exotic Hadrons

Exotic hadrons are subatomic particles composed of quarks and gluons, but which consist of more than three valence quarks or have an explicit valence gluon content.



QWG Exotics hub: https://qwg.ph.nat.tum.de/exoticshub/

Who is the *X(3872)*?

$$I(J^{PC}) = 0(1^{++})$$

Isosinglet

SEGNI PARTICOLARI:

Main decay channel:

$$X(3872) \longrightarrow D^0\bar{D}^{0*} \longrightarrow D^0\bar{D}^0\pi^0 \quad (\mathbf{55 \pm 28})\%$$

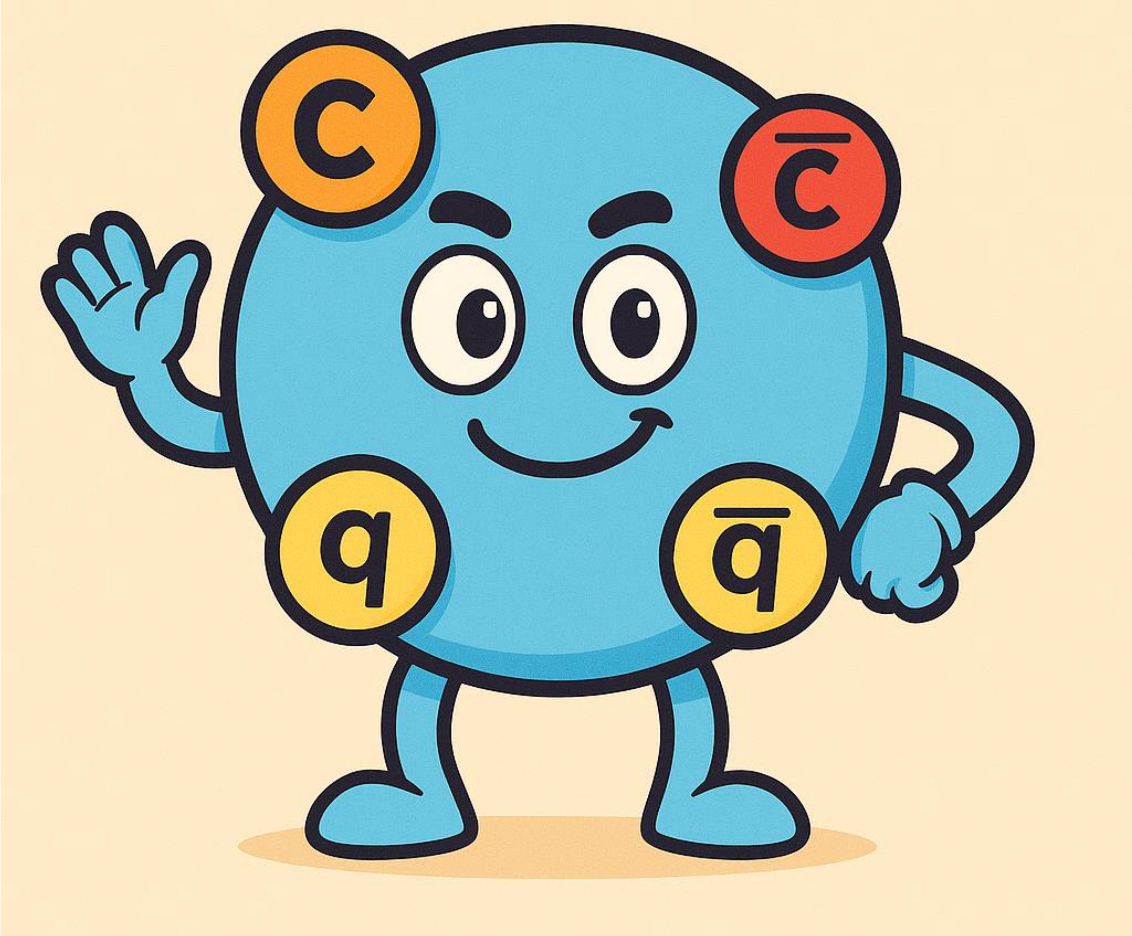
Isospin violating decay:

$$\frac{\mathcal{B}\,(X \to J/\psi\,\omega)}{\mathcal{B}\,(X \to J/\psi\,\rho)} \simeq 1$$

Threshold distance:

$$\boxed{(m_D + m_{D^*}) - m_X \sim O(\text{keV})}$$

Exotic Meson $[Q\bar{Q}q\bar{q}]$



Mass: 3871.65 ± 0.06 MeV

Compact objects vs Molecules

The X(3872)'s closeness to the $D^0 D^{*0}$ threshold makes it challenging to clearly determine whether the state is molecular or compact.

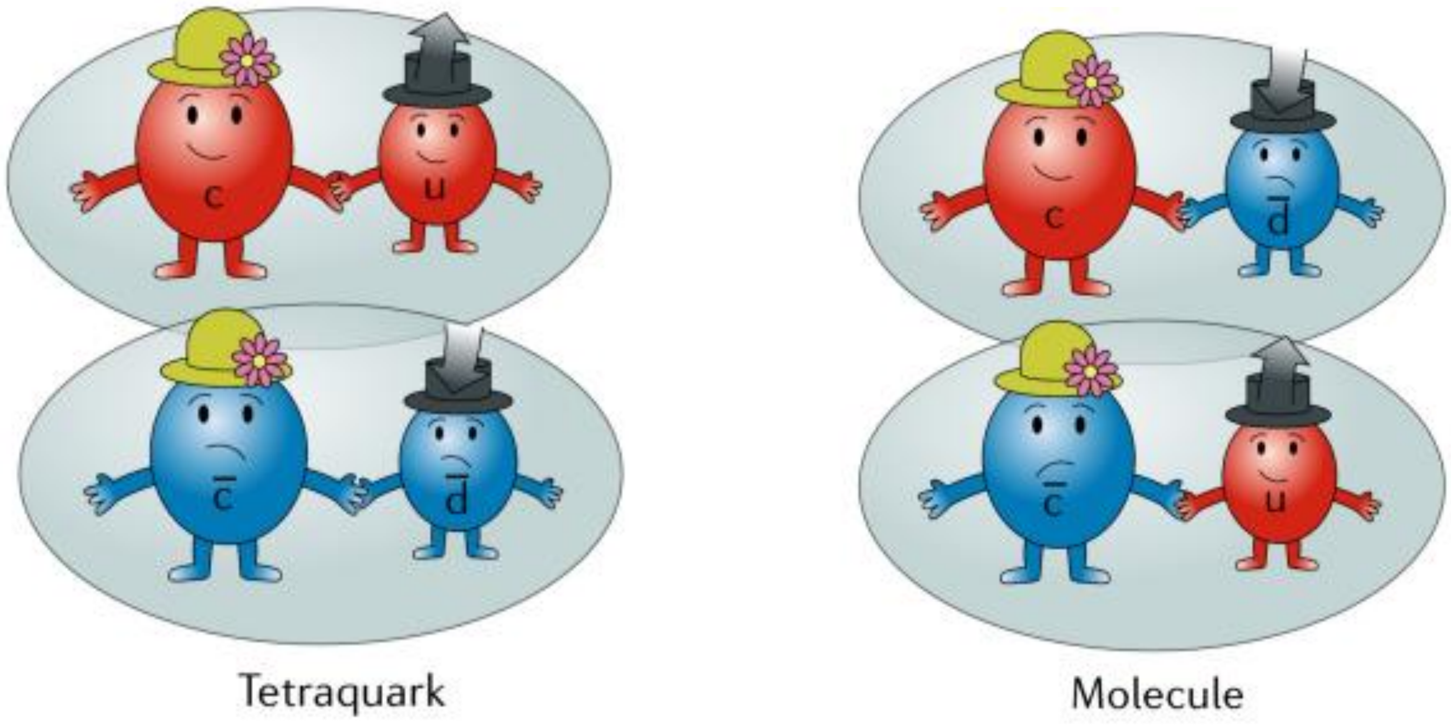


Figure from Nat Rev Phys 1, 480-494 (2019)

A Challenge but also an Opportunity

- **Cons:** The proximity to the meson threshold makes it difficult to determine whether the particle is a meson–antimeson molecule or a compact state.

- **Pro:** Systems near threshold can be effectively studied using the tools of non-relativistic field theory (aka quantum mechanics on steroids).

It’s not a new idea! The same strategy is used for low-energy proton–neutron scattering, in particular to study the properties of the deuteron.

Before focusing on the X(3872), let's briefly review non-relativistic EFTs and scattering theory in quantum mechanics.

Non-relativistic EFT

- EFT: the theory is valid up to a certain scale $\boldsymbol{\Lambda}$;
- **Non-relativistic:** The momenta $\boldsymbol{p_i}$ of the particles satisfy $p_i \ll m_i$ ⟹ $E_i = \sqrt{m_i^2 + p_i^2} \simeq m_i + \frac{p_i^2}{2m_i}$

Where quantum field theory meets quantum mechanics

EX: propagator $EFT \rightarrow NREFT$

$$\Pi_0(p_0, \vec{p}) = -\frac{i}{-p_0^2 + p^2 + m^2 - i\varepsilon} \underset{p_0 = E + m}{=} \frac{i}{(m+E)^2 - p^2 - m^2 + i\varepsilon} \underset{E/m \ll 1}{\simeq} \frac{i}{2mE - p^2 + i\varepsilon} = \frac{1}{2m} \frac{i}{\boxed{E - \frac{p^2}{2m} + i\varepsilon}}$$

Classical dispersion relation

$$\Longrightarrow \quad \boxed{\phi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \phi}$$

Kinetic term

Non-relativistic EFT: Fields

In a NREFT, the fields ϕ annihilate particles, while the fields $\phi^\dagger$ create them. Particles and antiparticles are created by different fields.

Intuitive idea behind this: Scalar field

$$\phi = \phi_p + \phi_{\bar{p}} = e^{-imt}\tilde{\phi}_p + e^{imt}\tilde{\phi}_{\bar{p}} \quad \Rightarrow \quad \phi^\dagger\phi = \boxed{\tilde{\phi}_p^\dagger\tilde{\phi}_p + \tilde{\phi}_{\bar{p}}^\dagger\tilde{\phi}_{\bar{p}}} + \cancel{e^{-2imt}}\,\tilde{\phi}_{\bar{p}}^\dagger\tilde{\phi}_p + \cancel{h.c.}$$

Relativistic field

The field of the particle and the one of the anti-particle decouple

In the non-relativistic limit they vanish

Starting from the relativistic Lagrangian

$$\mathcal{L}_{kin} = \tilde{\phi}_p^\dagger\left(i\partial_t + \frac{\nabla^2}{2m} - \boxed{\mu}\right)\tilde{\phi}_p + \tilde{\phi}_{\bar{p}}^\dagger\left(i\partial_t + \frac{\nabla^2}{2m} - \mu\right)\tilde{\phi}_{\bar{p}}$$

$$[\tilde{\phi}] = \frac{3}{2}$$

In a free theory this is the difference between the mass of the particle and the threshold

Non-relativistic EFT: Fields

In a classical theory, I can shift the energy offset without changing the physics.

$$\mathscr{L}_{kin} = \tilde{\phi}_p^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} - \mu - \lambda \right) \tilde{\phi}_p + \tilde{\phi}_{\bar{p}}^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} - \mu - \lambda \right) \tilde{\phi}_{\bar{p}}$$

$\Rightarrow$ $\tilde{\phi} \mapsto e^{-i\lambda t}\tilde{\phi}$ 'Gauge transformation' $\Rightarrow$ $i\partial_t \left(e^{-i\lambda t}\tilde{\phi} \right) = \lambda\tilde{\phi} + e^{-i\lambda t}\, i\partial_t\tilde{\phi}$

$$\Rightarrow \mathscr{L}_{kin} = \tilde{\phi}_p^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} - \mu \right) \tilde{\phi}_p + \tilde{\phi}_{\bar{p}}^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} - \mu \right) \tilde{\phi}_{\bar{p}}$$

From the Lagrangian, The equation of motion can be obtained: the Schrödinger equation.

$$\boxed{\frac{d}{dt}\frac{\partial \mathscr{L}}{\partial(\partial_t \tilde{\phi}^\dagger)} = \frac{\partial \mathscr{L}}{\partial \tilde{\phi}^\dagger} \Rightarrow i\partial_t\tilde{\phi} = -\frac{\nabla^2}{2m}\tilde{\phi}}$$

Then I can add interactions and compute Cross Sections!

Non-relativistic EFT: Scattering Amplitude

Scattering in a NREFT is related to scattering in QM, for which many results are well known.

Spherical and finite range Potential + Elastic scattering

$$d\sigma = \boxed{|f(\theta)|^2} d\Omega$$

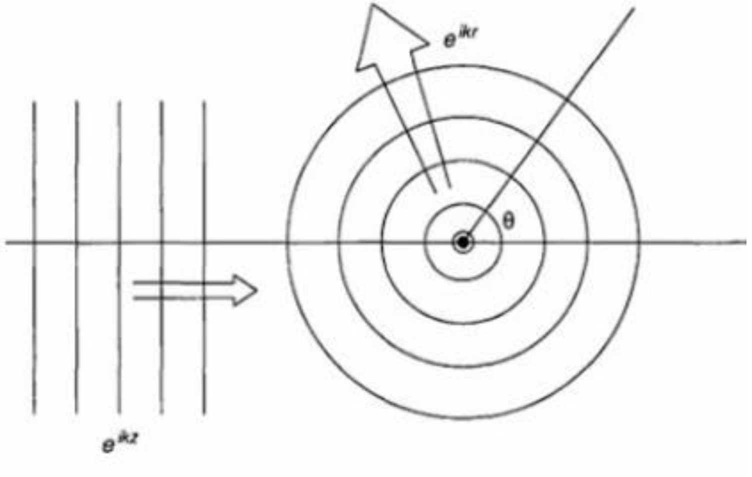
Differential cross section

Scattering amplitude

$$\psi(r) \xrightarrow{r\to+\infty} e^{ikz} + f(\theta)\frac{e^{ikr}}{r}$$

Expansion in spherical harmonics

$$f(\theta) = \sum_{l=0}^{\infty}(2l+1) f_l \, P_l(\cos\theta)$$



Low energy scattering:

$$f(\theta) \simeq f_0 = \frac{1}{k\cot\boxed{\delta_0} - ik}$$

Scattering phase shift

Outside the region where the potential is present, the wavefunction satisfies the free-particle equation. The scattering solution differs from the unperturbed (no-scattering) solution by a phase shift, since both solve the same equation.

Non-relativistic EFT: Scattering Length

At low energies, the scattering amplitude depends on two parameters.

Limit $k \to 0$:

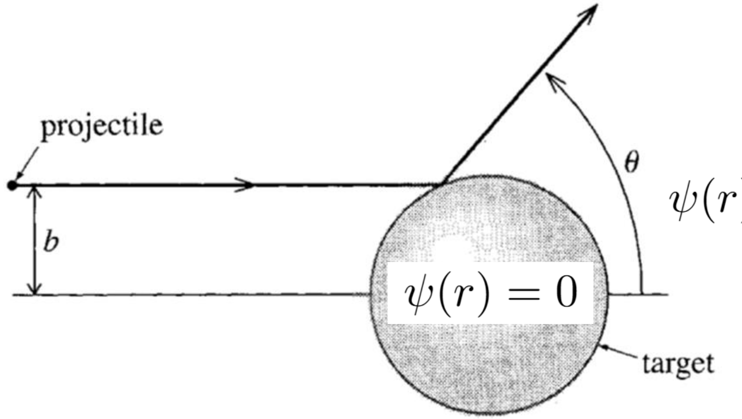
$$\boxed{f(\theta) \simeq f_0 = \frac{1}{k\cot\delta_0 - ik}}$$

At low momenta, the particle cannot resolve the potential. The cross section will depend on the size of the scattering center.

$$\boxed{\lim_{k\to 0^+} k\cot\delta_0 = -\frac{1}{a_s}} \Longrightarrow \lim_{k\to 0^+} \sigma = 4\pi a_s^2$$

Scattering Length: a_s is the radius of the sphere that defines the interaction region.

Esercizio (facile): $V(r) = \begin{cases} +\infty & r \leq a \\ 0 & r > a \end{cases}$



$$\psi(r) \propto \frac{\sin(kr - ka)}{r} \Longrightarrow \boxed{\delta_0 = -ka} \Longrightarrow \lim_{k\to 0^+} k\cot(-ka) = -\frac{1}{a}$$

Non-relativistic EFT: Scattering Length & Poles

Single particle states of the Hamiltonian can cause the formation of a pole in scattering amplitude. But not all the poles are due to single particle eigenstates.

There is a pole close to the threshold ($E \simeq 0$):

$$k \cot \delta_s - ik = 0 \xrightarrow[E_{pole} = -B]{} -\frac{1}{a_s} + \sqrt{-2mE_{pole} - i\varepsilon} = -\frac{1}{a_s} + \sqrt{2mB} = 0$$

$$\longrightarrow \boxed{\sqrt{2mB} = \frac{1}{a_s}}$$

- $a_s > 0$: The equation has a solution and the pole is related to a **real particle;**
- $a_s < 0$: The equation has no solution however there is a pole at $k = i/a_s$. This pole is in the lower half of the complex plane and it is not related to any particle in the spectrum. This is called a **virtual particle** (not in the sense of field theory!).

Non-relativistic EFT: Effective range & Nature of Poles

At low energies, the scattering amplitude depends on two parameters.

$$f(\theta) \simeq f_0 = \frac{1}{k \cot \delta_0 - ik}$$

Limit $k \to 0$:

$$\boxed{k \cot \delta_0 = -\frac{1}{a_s} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4)}$$

Effective Range: r_0 is the 'real' range of the scattering potential

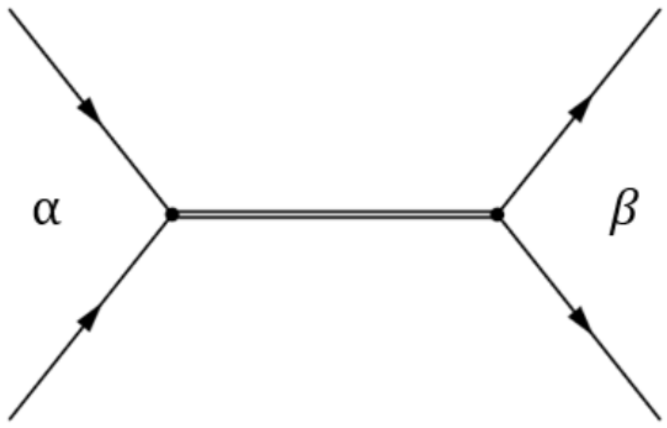
Due to quantum effects the 'real' range of the scattering potential differs from its scattering length.

The sign of r_0 is related to whether the particle is a molecule or a compact state (Weinberg 1965).

Weinberg Phys. Rev. 137, B672 (1965)

- $r_0 \geq 0$: The particle is a **bound state**;
- $r_0 < 0$: The particle is a **compact state**.

Idea



Low energy scattering

From the cross section I can infer $f(\theta)$

$a_s > 0$: real particle

$a_s < 0$: virtual particle

$r_0 \geq 0$: bound state

$r_0 < 0$: compact state

Proton – Neutron scattering

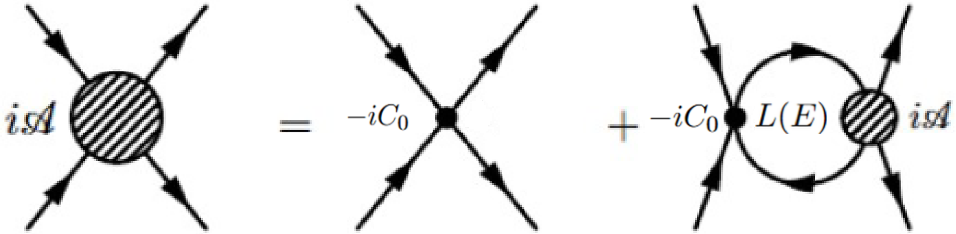
In QM, we compute scattering from a potential. In NREFT, we can calculate scattering between particles.

Simplified version: N stands for nucleon

$$\mathcal{L} = N^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) N - \frac{1}{2} C_0 (N^\dagger N)^2 - \frac{1}{2} C_2 (N^\dagger \nabla N)^2 + \dots$$

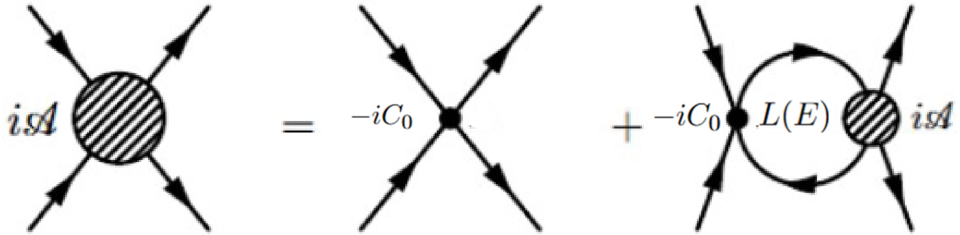
D. B. Kaplan Nucl. Phys. B 494(1997), 471–484, nucl-th/9610052.

I have a cut-off scale Λ and I expect that $C_{2n} \sim \Lambda^{-(2n+2)}$. Let's keep it simple and consider only C_0 dropping all the higer-derivative terms.



$$\boxed{i\mathcal{A} = -iC_0 + (-iC_0) L(E) i\mathcal{A}}$$

Proton – Neutron: scattering amplitude



$$\boxed{i\mathcal{A} = -iC_0 + (-iC_0)L(E)i\mathcal{A}}$$

$$\mathcal{A} = -C_0\left(1 + iC_0L(E)\right)^{-1} = -\frac{1}{\frac{1}{C_0} - \frac{M}{4\pi}\sqrt{-ME - i\varepsilon}}$$

$$\boxed{f = -\frac{M}{4\pi}\frac{1}{\frac{1}{C_0} + i\frac{M}{4\pi}k}}$$

$$\boxed{f(\theta) \simeq f_0 = \frac{1}{k\cot\delta_0 - ik}}$$

$$\boxed{k\cot\delta_0 = -\frac{1}{a_s} + \frac{1}{2}r_0k^2 + \mathcal{O}(k^4)}$$

Proton – Neutron: scattering length

$$\boxed{\frac{1}{a_s} = \frac{4\pi}{MC_0}}$$

MA…

$$\boxed{\sqrt{2mB} = \frac{1}{a_s}}$$

For the deuteron $B \simeq 2.2$ MeV so $1/a_s \ll \Lambda \simeq m_\pi$!!!

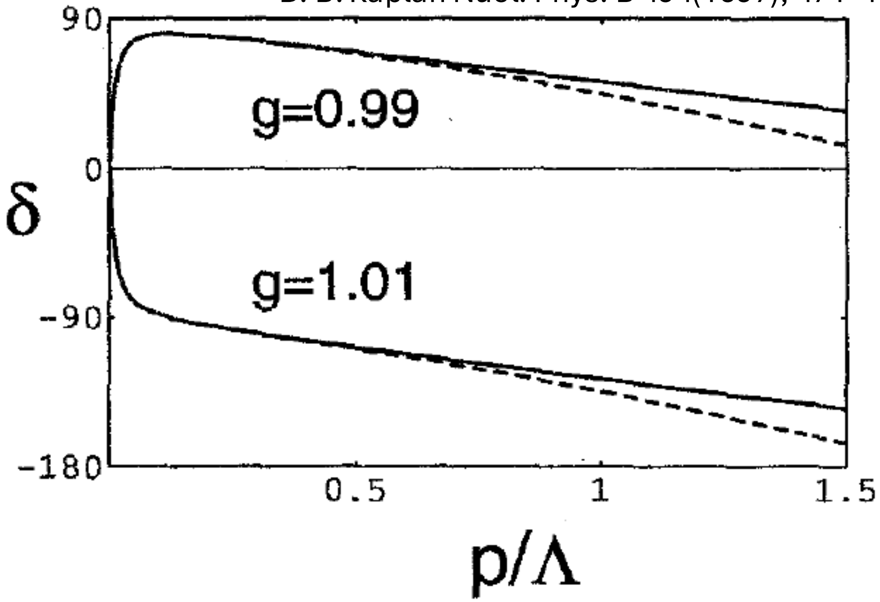
$a_s = 5.36$ fm while $1/m_\pi \sim 1.4$ fm

a_s (or C_0) è unnatural… That's a problem!

D. B. Kaplan Nucl. Phys. B 494(1997), 471–484, nucl-th/9610052.

Radius of convergence:

$$p \sim \sqrt{\frac{\Lambda}{a_s}}$$

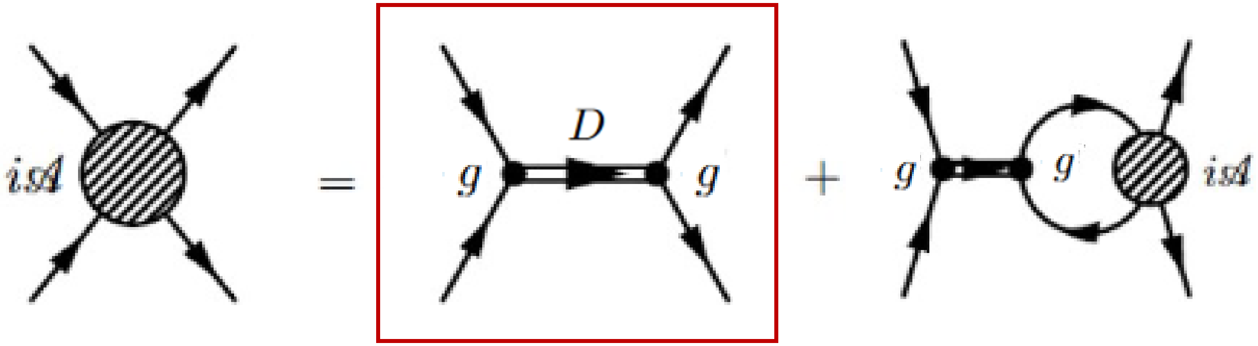


D. B. Kaplan and J. V. Steele Phys. Rev. C 60 (1999), 064002, nucl-th/9905027.

Proton – Neutron: the dibarion field (aka deuteron)

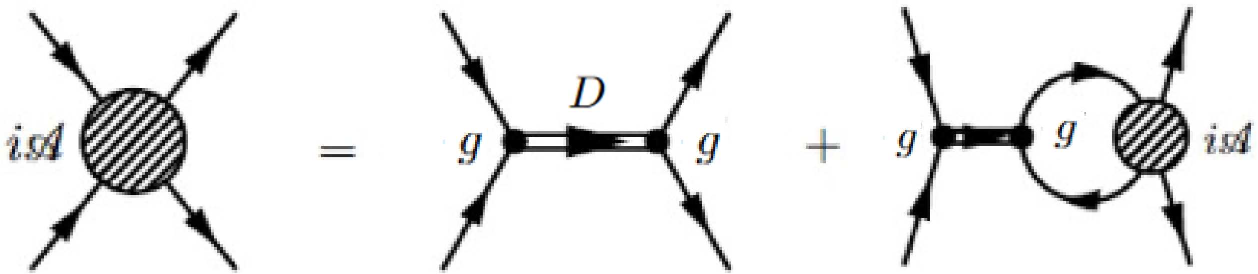
$$\mathcal{L} = N^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) N + \sigma\, D^\dagger \left(i\partial_t + \frac{\nabla^2}{4M} - \mu \right) \underbrace{D}_{\text{Dibarion field}} - \boxed{g\, D^\dagger (NN)} + h.c.$$

The dibarion's kinetic term is defined up to a sign $\sigma = \pm 1$



Trilinear coupling between the two nucleons and the dibarion

Proton – Neutron: the dibarion field (aka deuteron)



$$\boxed{f = -\frac{M}{4\pi}\frac{\sigma\, g^2}{E - \mu + i\,\sigma g^2 \frac{M}{4\pi} k}}$$

$$\boxed{\frac{1}{a_s} = -\sigma \frac{4\pi}{M g^2}\mu}$$

$$\boxed{r_0 = -\sigma \frac{8\pi}{g^2 M^2}}$$

- $\sigma = +1$: 'Classical' kinetic term and $r_0 < 0$. The field represents a **compact particle** if $\mu < 0$.

- $\sigma = -1$: 'Inverted' kinetic term and $r_0 > 0$. The field represents a **bound state** if $\mu > 0$.

$$\boxed{f(\theta) \simeq f_0 = \frac{1}{k\cot\delta_0 - ik}}$$

$$\boxed{k\cot\delta_0 = -\frac{1}{a_s} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4)}$$

What have we learned?

- If the EFT has an unnatural scattering length (i.e., a pole very close to threshold), the derivative expansion breaks down because the radius of convergence scales like $\sqrt{\Lambda/a_s}$;

- To reproduce the dynamics around the pole, it is possible to introduce an additional field in the Lagrangian. This new degree of freedom also allows us to resum all effects up to second order (in fact, we can also compute r_0);

- The nature of the pole is determined by the parameters of the field in the Lagrangian. The sign of the kinetic term tells us whether we are dealing with a possible compact particle or a bound state. Whether the resonance is real or not depends on the sign of the 'mass' term, which affects the sign of the scattering length.

$$\boxed{\frac{1}{a_s} = -\sigma \frac{4\pi}{Mg^2}\mu}$$

$$\boxed{r_0 = -\sigma \frac{8\pi}{g^2 M^2}}$$

The X(3872): compact model

A. Esposito et al. arXiv:2502.02505 [hep-ph]

$$\begin{aligned}\mathcal{L}_{kin} = {} & D^\dagger\left(i\partial_t + \frac{\nabla^2}{2m_D} - \begin{pmatrix}\Delta_1 & 0\\ 0 & 0\end{pmatrix}\right)D + \bar{D}^\dagger\left(i\partial_t + \frac{\nabla^2}{2m_D} - \begin{pmatrix}0 & 0\\ 0 & \Delta_1\end{pmatrix}\right)\bar{D} + \\ & + \boldsymbol{D}^\dagger\left(i\partial_t + \frac{\nabla^2}{2m_{D^*}} - \begin{pmatrix}\Delta_2 & 0\\ 0 & 0\end{pmatrix}\right)\boldsymbol{D} + \bar{\boldsymbol{D}}^\dagger\left(i\partial_t + \frac{\nabla^2}{2m_{D^*}} - \begin{pmatrix}0 & 0\\ 0 & \Delta_2\end{pmatrix}\right)\bar{\boldsymbol{D}} + \\ & + \boldsymbol{X}^\dagger\left(i\partial_t + \frac{\nabla^2}{2(m_D + m_{D^*})} - m_F\right)\boldsymbol{X}\,.\end{aligned}$$

A lot of fields…

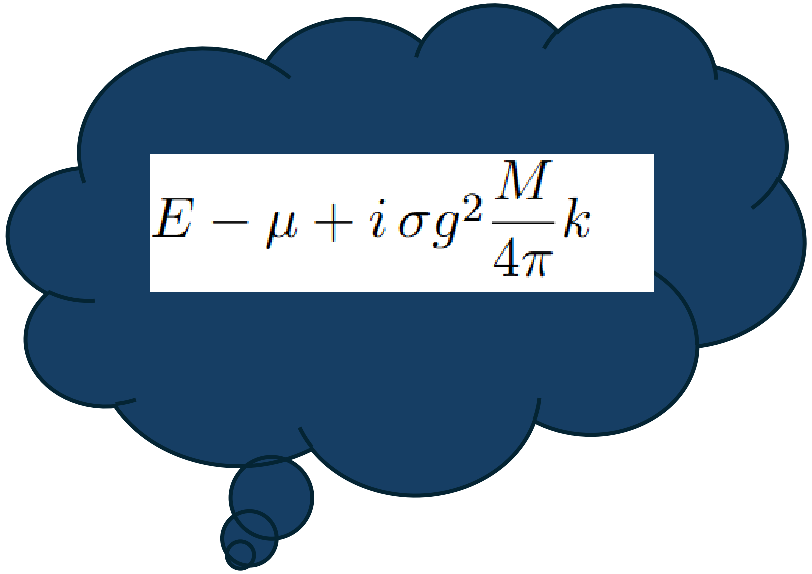
and a lot of interactions

$$\begin{aligned}\mathcal{L}_{\rm int} \supset {} & -\frac{\lambda_S}{2}(\bar{D}\boldsymbol{D})^\dagger_+\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}(\bar{D}\boldsymbol{D})_+ - \frac{\lambda_I}{2}(\bar{D}\boldsymbol{D})^\dagger_+\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}(\bar{D}\boldsymbol{D})_+ \\ & \boxed{-\frac{g}{\sqrt{2}}\boldsymbol{X}^\dagger(\bar{D}^0\boldsymbol{D}^0)_+ + \frac{g}{\sqrt{2}}\boldsymbol{X}^\dagger(D^-\boldsymbol{D}^+)_+}\,.\end{aligned}$$

The X(3872): what can we do?

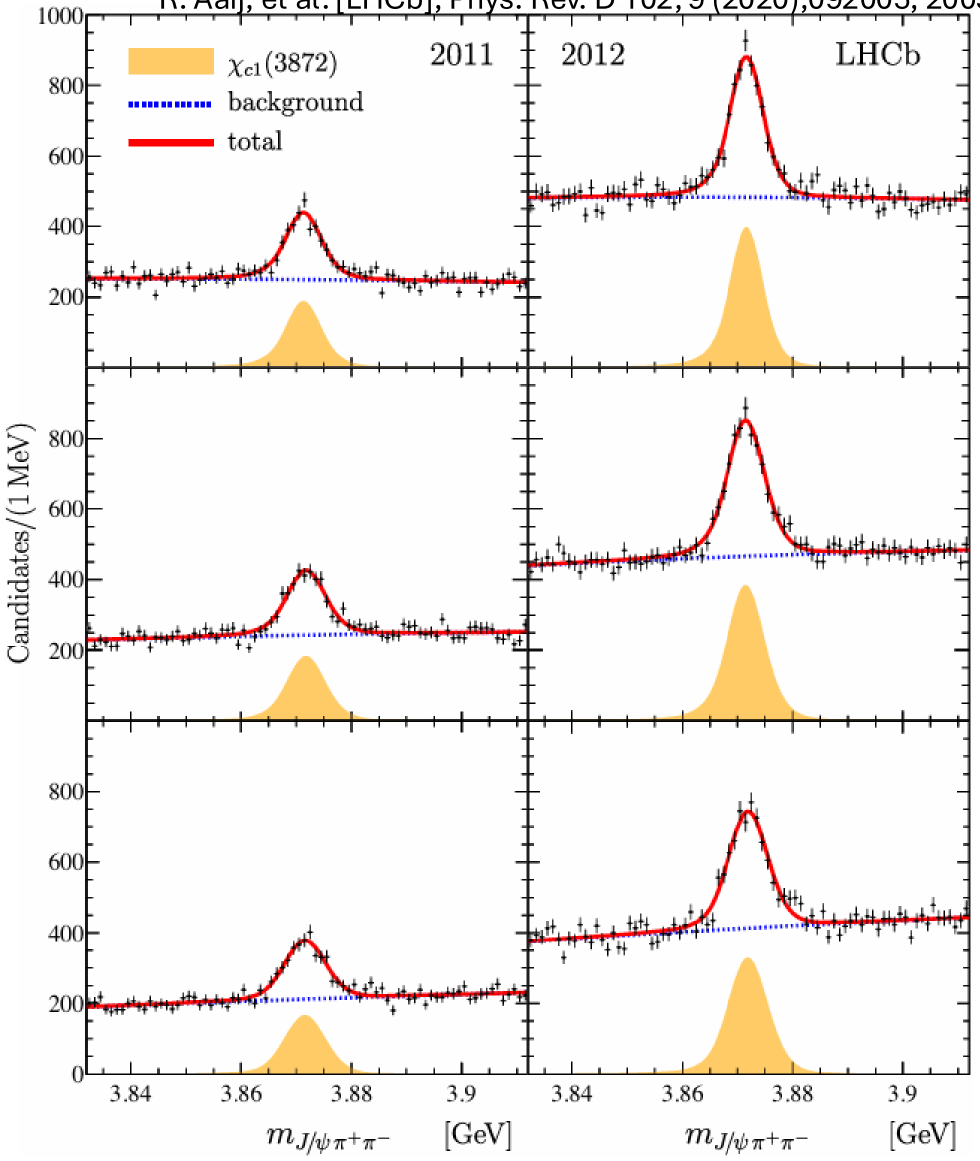
$$\frac{dR(J/\psi\pi^+\pi^-)}{dE} \propto \frac{\Gamma_\rho(E)}{|D(E)|^2}$$

$$D(E) = E - E_f + \frac{i}{2}\left[g(k_1 + k_2) + \Gamma_\rho(E) + \Gamma_\omega(E) + \Gamma_0\right].$$



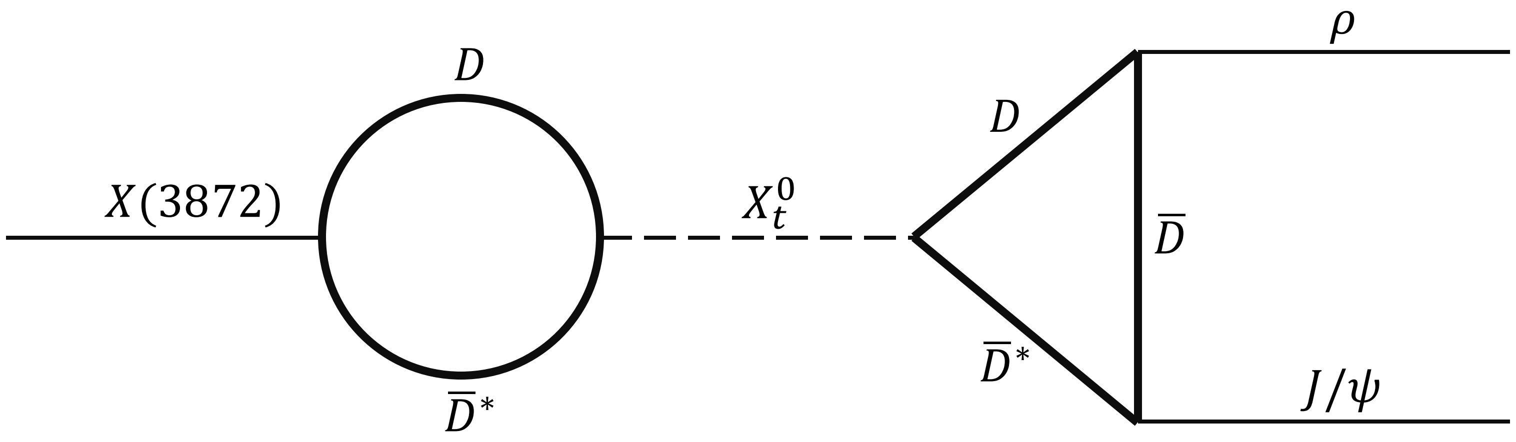
$$E - \mu + i\,\sigma g^2 \frac{M}{4\pi} k$$

R. Aaij, et al. [LHCb], Phys. Rev. D 102, 9 (2020),092005, 2005.13419.



The X(3872): what can we do (maybe)?

In the previous Lagrangian, it is also possible to add a field for the isospin triplet X_t, that we expect since SU(2) is a good symmetry of QCD. By doing so, we can generate loops that 'convert' the isosinglet $X(3872)$ into its neutral isotriplet partner X_t^0.



The enhancement of theisospin violating decay could be due to mixing between the isosinglet and isotriplet, induced by D-meson loops.

Conclusions

- NREFTs are a powerful tool for studying low-energy scattering. Thanks to their connection with quantum mechanics, many results from scattering theory can be directly applied.

- Low-energy scattering is governed by two parameters: the scattering length a_s and the effective range r_0. At low energies, the scattering amplitude therefore takes on a universal form, regardless of the specific potential involved.

- The sign of a_s gives us information about the nature of the pole: if $a_s > 0$, the particle is **real**; if $a_s < 0$, the state is **virtual**.

- The sign of r_0 gives us information about the nature of the particle: if $r_0 \geq 0$, the particle is a **bound state**; if $r_0 < 0$, it is **compact**.

- From the study of nucleon-nucleon scattering (i.e., proton-neutron), we learn that we can capture the dynamics around the pole by introducing an additional field for the resonance. The sign of the kinetic term is linked to r_0.

- Using NREFTs to study the $X(3872)$ can help us unravel its mysteries: from its nature to its ‘impossible’ decays.