Efficiently simulating **quarkonium evolution** beyond the dipole approximation

Jorge M. Mtz-Vera

Miguel Ángel Escobedo, Andrea Beraudo, Paolo Parotto

Quarkonia as a probe

1. Well-known probe. Experimentally, clean signal through dilepton decays.
2. Hard scale. Quarkonia mass $m_{Q\bar{Q}}, m_Q \gg \Lambda_{QCD}$. Suited to EFT description.
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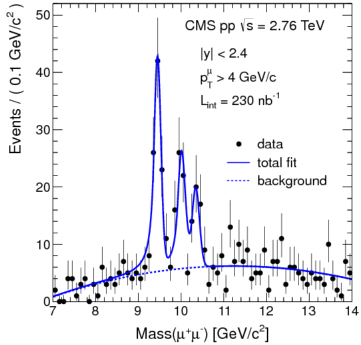
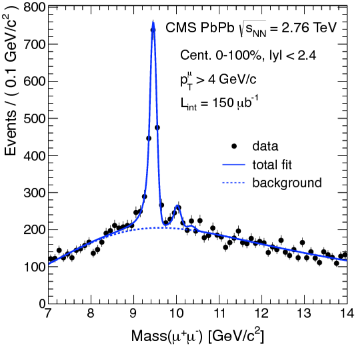
Target observable:





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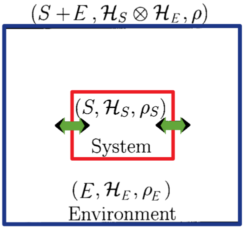
The full quantum dynamics of the subsystem is kept whereas the environment is traced out.



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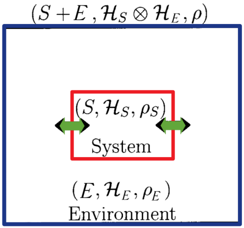

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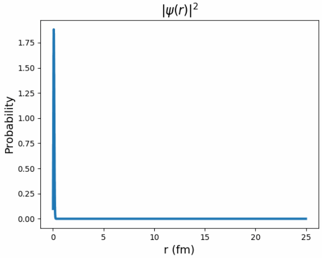

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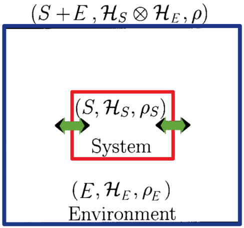

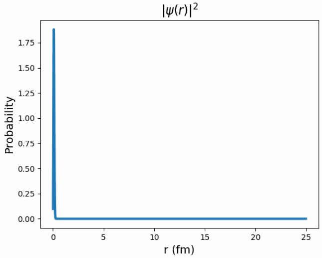

How do we simulate it *efficiently*?
By using a Monte Carlo Wavefunction approach: *quantum trajectories*.

OQS for Quarkonia

Density matrix, ρ_T and **observables** $\langle\mathcal{O}\rangle$:

$$\rho_T = \sum_i p_i \left|\psi_i\right\rangle \left\langle\psi_i\right| \longrightarrow \langle\mathcal{O}\rangle = Tr\{\rho_T \mathcal{O}\}. \quad (1)$$

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$$H_T = H_Q \otimes \mathbb{I}_E + \mathbb{I}_Q \otimes H_E + H_I, \text{ where } H_I = J_Q \otimes V_E. \quad (2)$$

$$\begin{aligned} H_Q &= T^Q + T^{\bar{Q}} + V^{vac}(|\mathbf{x}_{\bar{Q}} - \mathbf{x}_Q|) \\ H_E &= H_{q+A} \\ H_I &= \int_{\mathbf{x}} [\delta(\mathbf{x} - \mathbf{x}_Q) t^a_Q - \delta(\mathbf{x} - \mathbf{x}_{\bar{Q}}) t^{a*}_{\bar{Q}}] \otimes g A^a_0(\mathbf{x}) \end{aligned}$$

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$$Tr_E\Big[T[A^a_0(t_1,\mathbf{x}_1)A^b_0(t_2,\mathbf{x}_2)]\rho_E\Big] = -i\delta^{ab}\Delta(t_1-t_2,\mathbf{x}_1-\mathbf{x}_2) \quad (3)$$

$$V(\mathbf{r}) = -\Delta^R(\omega=0,\mathbf{r}), \qquad W(\mathbf{r}) = -\Delta^<(\omega=0,\mathbf{r}) \quad (4)$$

(Blaizot and Escobedo, 2017)

Timescales

Evolution	**Medium correlation**	**Relaxation** $Q\bar{Q}$
$\tau_Q \sim 1/\Delta E$	$\tau_E \sim 1/T$	$\tau_R \sim m_{Q\bar{Q}}/T^2$

$\Delta E \rightarrow$ Spacing between the energy levels.

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- Trace over environment degrees of freedom: $\text{tr}_E\,\{d\rho_T/dt\}$.
- Born, Markov and Born-Oppenheimer approximations $\longrightarrow$ Brownian motion regime.

Lindblad equation

$$\frac{d\rho_Q}{dt} = -i\left[H_Q^{\mathrm{scr}}, \rho_Q\right] + \sum_k \left(C_k \rho_Q C_k^\dagger - \frac{1}{2}\{C_k^\dagger C_k, \rho_Q\}\right). \qquad (5)$$

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High computational cost, *time* $\propto N^2$. $\implies$ Rearrange the terms. Define a pseudo-hamiltonian by putting the subsystem hamiltonian together with the anti-commutators. It gives rise to a **non-hermitian** operator.

(Akamatsu, 2022; Blaizot and Escobedo, 2018; Yao and Mehen, 2019)

$$H_{eff} = \boxed{H_Q^{scr} - \frac{i}{2}\sum_i \int_{\boldsymbol{q}} C^\dagger_{\boldsymbol{q},i} C_{\boldsymbol{q},i}} = H_Q^{scr} - \frac{i}{2}\Gamma.$$

$$\boxed{\frac{d\rho_Q}{dt} = -i(H_{\text{eff}}\rho_Q - \rho_Q H^\dagger_{\text{eff}})} + \sum_i \int_{\boldsymbol{q}} C_{\boldsymbol{q},i}\rho_Q C^\dagger_{\boldsymbol{q},i}, \text{ where } \rho_Q = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix} \quad (6)$$

Schrödinger-like evolution + Stochastic transitions.

Quantum trajectories and QTRAJ

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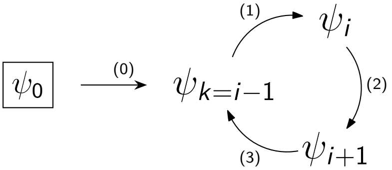


(0) Deterministic evolution of the initial state.
(1) A jump is randomly triggered.
(2) Projection and normalization.
(3) Deterministic evolution until next jump is triggered.

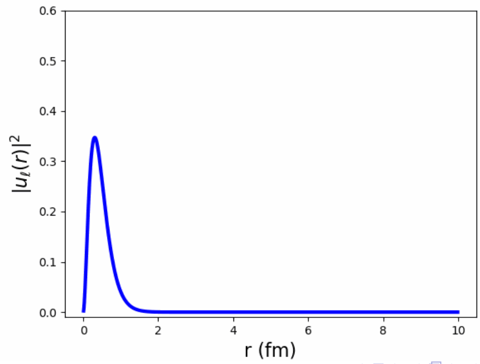
When is a jump triggered?

Non-hermitian hamiltonian $\Longrightarrow$ **norm decreases**.

A zeroth random number is drawn r_0. When:

$$r_0 > |\langle\psi(t_i)|\psi(t_i)\rangle|, \quad (8)$$

the jump is triggered and the selection rules come into play.



Upgrade from QTRAJ 1.0

QTRAJ 1.0 (dipolar)		QTRAJ 1.1 (one gluon exchange)
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Enact the stochastic evolution!

New selection rules, **beyond the dipole approximation**, need to be developed.

Selection rules.

QTRAJ 1.1 $\rightarrow$ 4 selection rules depending on the shape of the wavefunction.

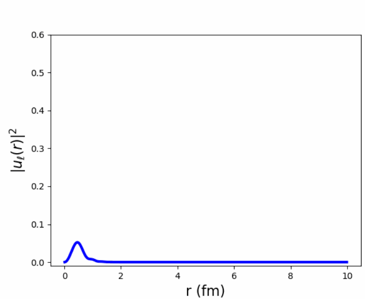
1. Color state: singlet/octet $\rightarrow c$.
2. Linear impulse exchanged by the propagator$\rightarrow |\boldsymbol{q}|$.
3. Maximum angular momentum exchanged, $\rightarrow t$.
4. Angular momentum of the final state$\rightarrow \ell_f$.

$$p(|\boldsymbol{q}|, c, t, \ell_f,) = p(c \cap |\boldsymbol{q}| \cap t \cap \ell_f) =$$

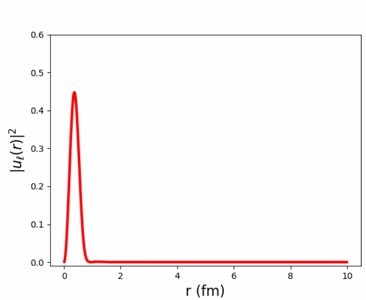
$$\underbrace{p(c|\Gamma)}_{r_1} \cdot \underbrace{p(|\boldsymbol{q}||\Gamma, c)}_{r_2} \cdot \underbrace{p(t|\Gamma, c, |\boldsymbol{q}|)}_{r_3} \cdot \underbrace{p(\ell_f|\Gamma, c, |\boldsymbol{q}|, t)}_{r_4} . \qquad (12)$$

Final state

$$|\psi_{new}\rangle = \frac{C^n_{\boldsymbol{q}} |\psi_{old}\rangle}{\sqrt{\langle\psi_{old}| \Gamma^n_{\boldsymbol{q}} |\psi_{old}\rangle}} \tag{13}$$



Before jump



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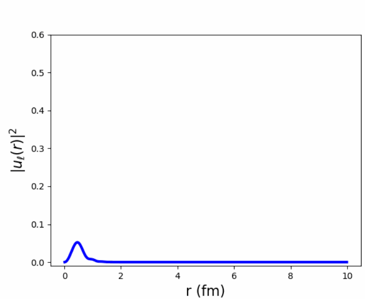

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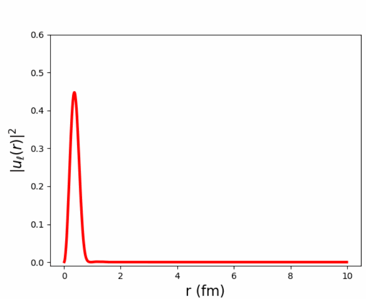
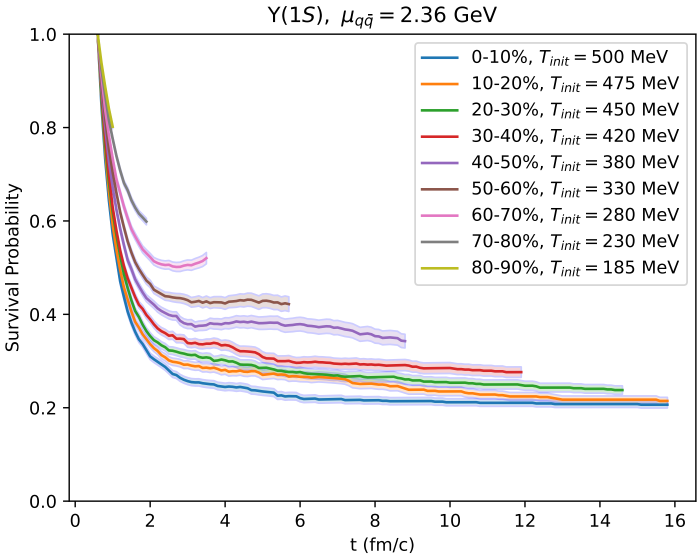


After jump

The deterministic evolution starts back again!

Overlaps 1S, 2S, 3S



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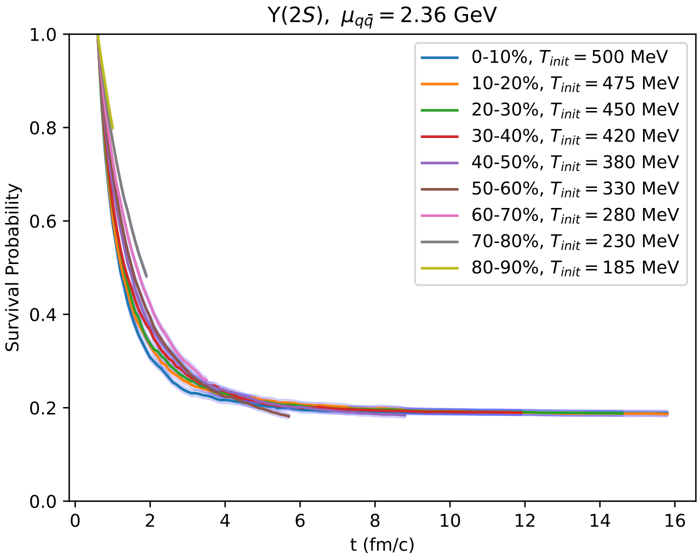

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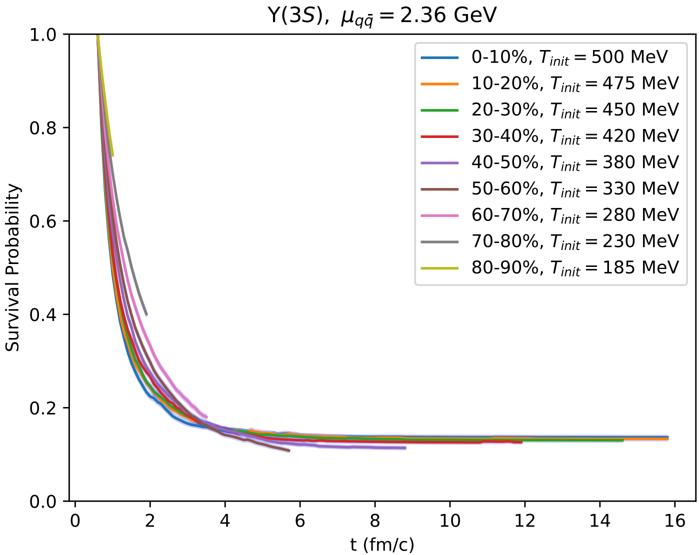

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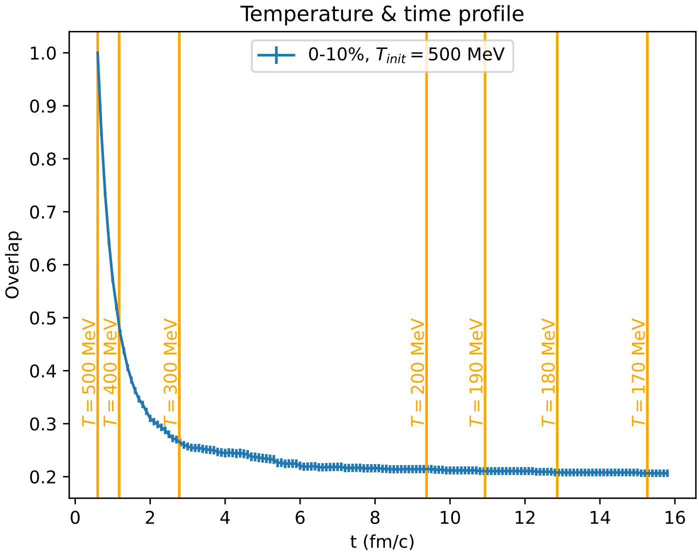

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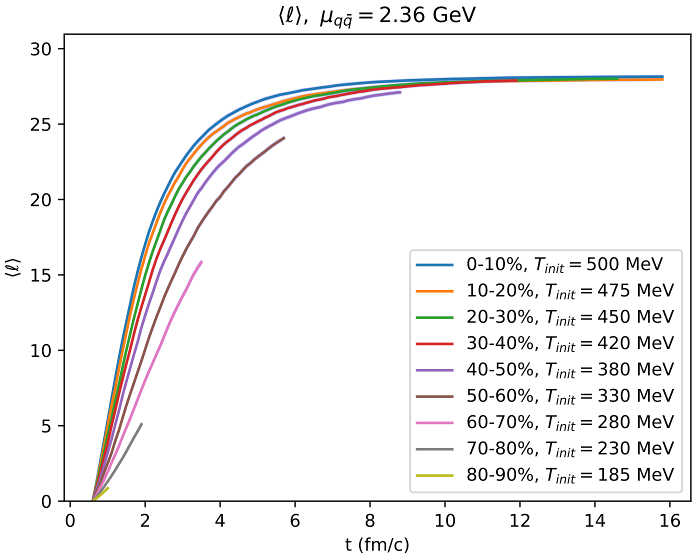

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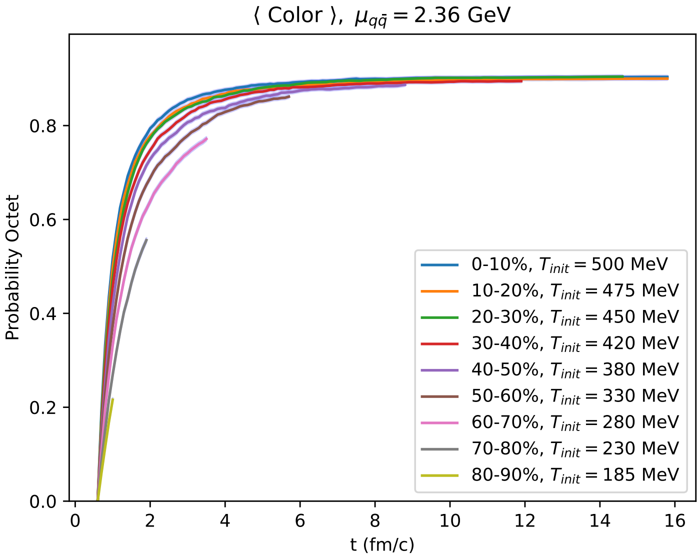

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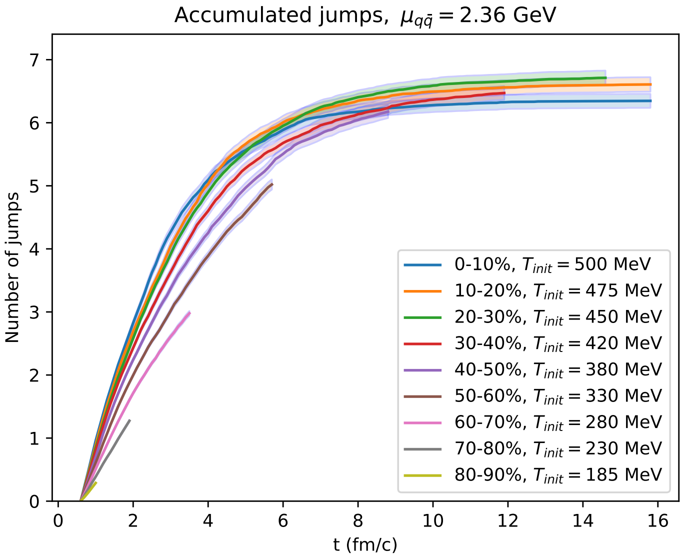
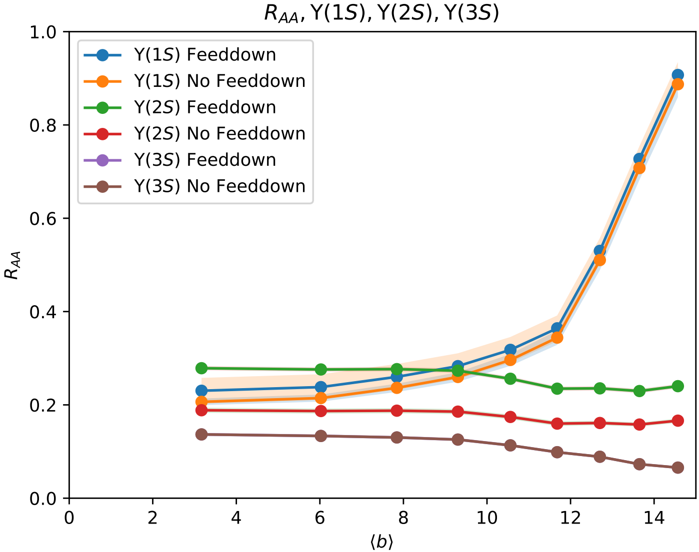


R_{AA} preliminary



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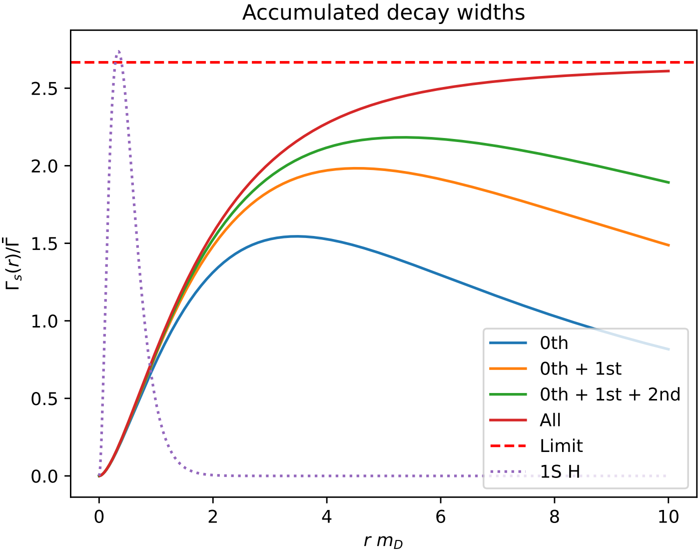
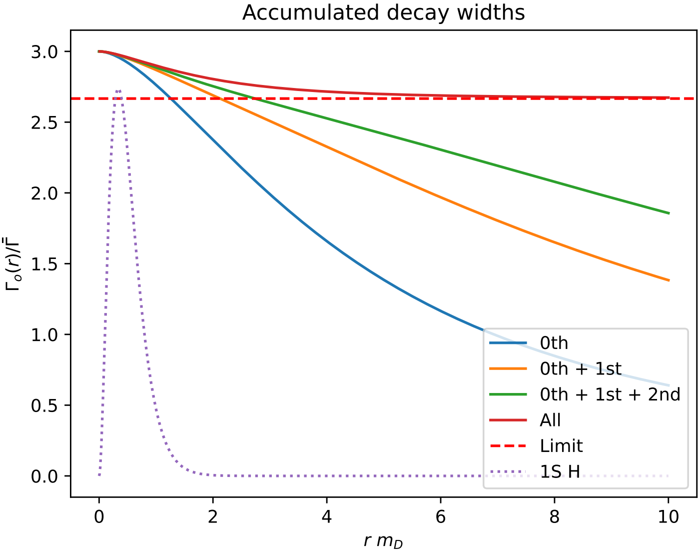
Thank you!

Further contact: `jorgemanuel.mtzvera@gmail.com`

Decay width in QTRAJ 1.1



Decay width in QTRAJ 1.1



The Dipolar Limit

Short distance behaviour of QTRAJ 1.1:

$$\Gamma_s(\hat{r}) \approx C_F \frac{\alpha_s m_D^2 T}{3} \left[\log\left(\frac{2}{m_D \hat{r}}\right) + \frac{1}{3}(4 - 3\gamma_E - 3\log 2) \right] \hat{r}^2. \qquad (14)$$

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Short distance behaviour of QTRAJ 1.0:

$$\Gamma_s(\hat{r}) = \hat{\kappa} T^3 \hat{r}^2 \qquad (15)$$

The Dipolar Limit

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Short distance behaviour of QTRAJ 1.0:

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How to recover the $\hat{r}^2$ behaviour?

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How to recover the $\hat{r}^2$ behaviour? $\rightarrow$ applying a UV cutoff $\Lambda = q_{\text{max}}/m_D$

$$\boxed{\Gamma^\Lambda = \frac{\alpha_s m_D^2 T}{6} \left[\log\left(\Lambda^2 + 1\right) - \frac{\Lambda^2}{\Lambda^2 + 1} \right] \hat{r}^2.} \quad (16)$$

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How to enforce the dipolar limit?

The Dipolar Limit

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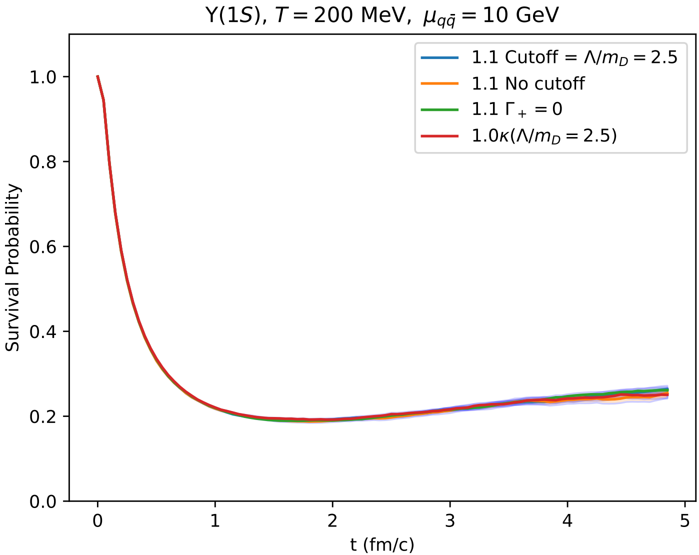
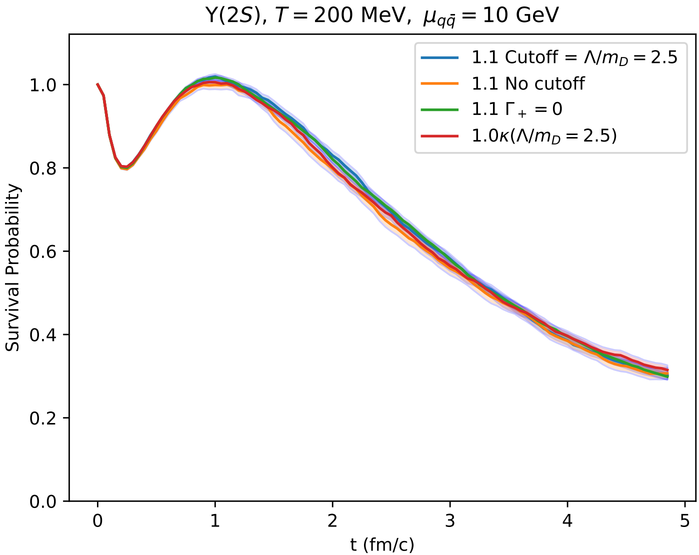
How to enforce the dipolar limit? $\rightarrow$ choose a very massive pair for which Bohr radius $a_0 \ll r_D$.

The Dipolar Limit (2)



The Dipolar Limit (2)



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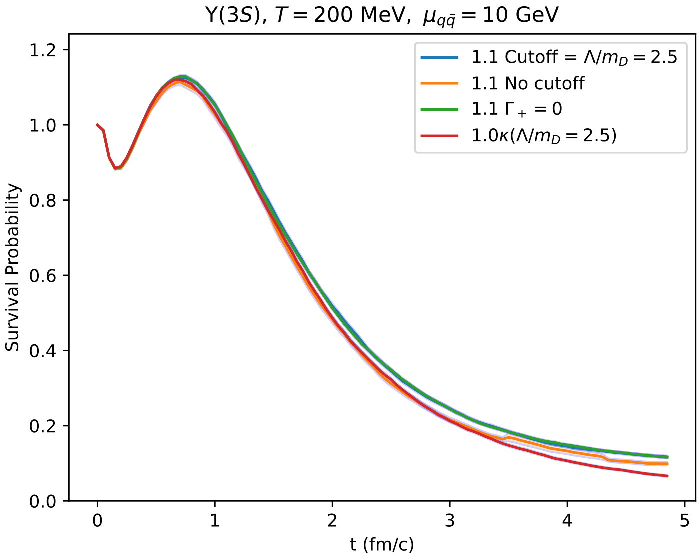

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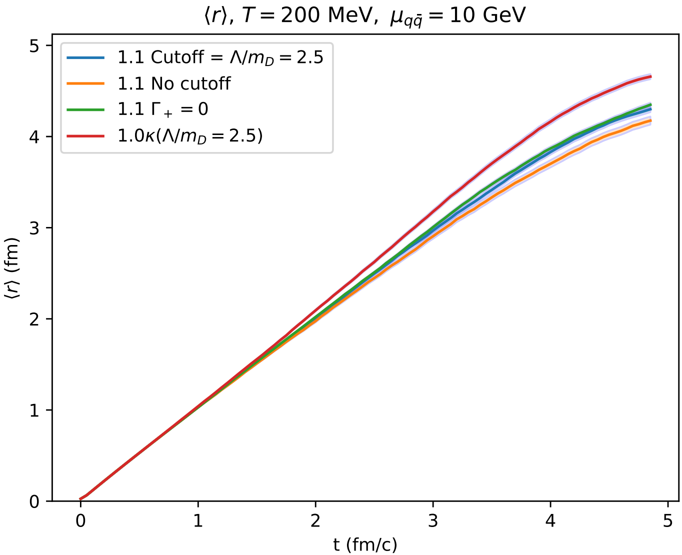

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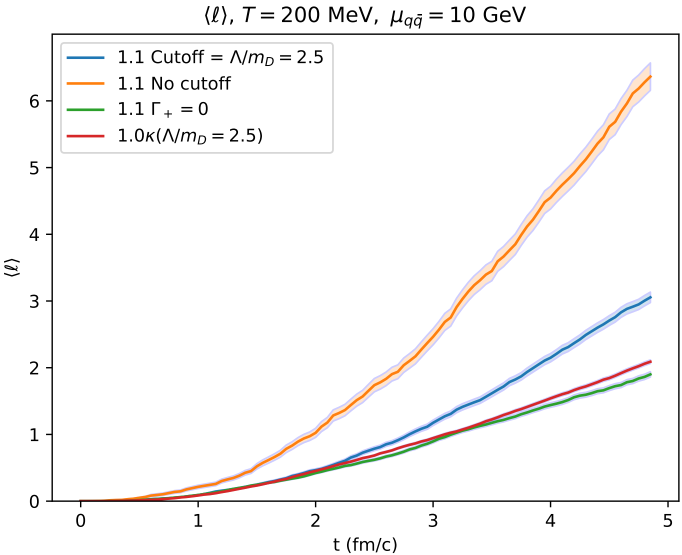
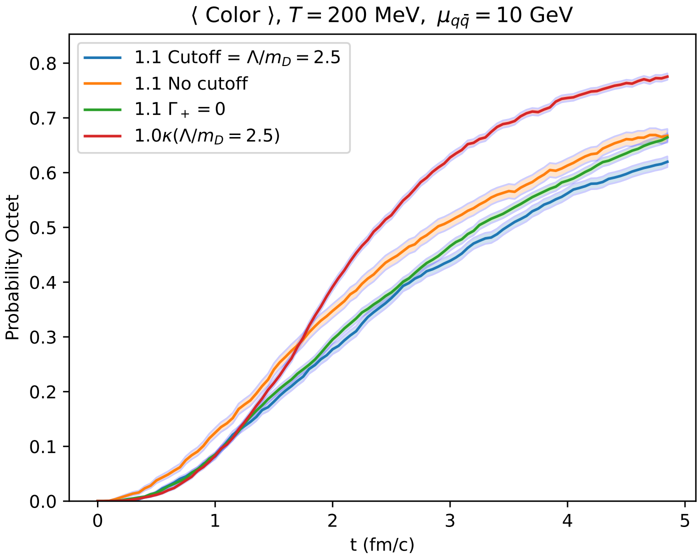


The Dipolar Limit (2)



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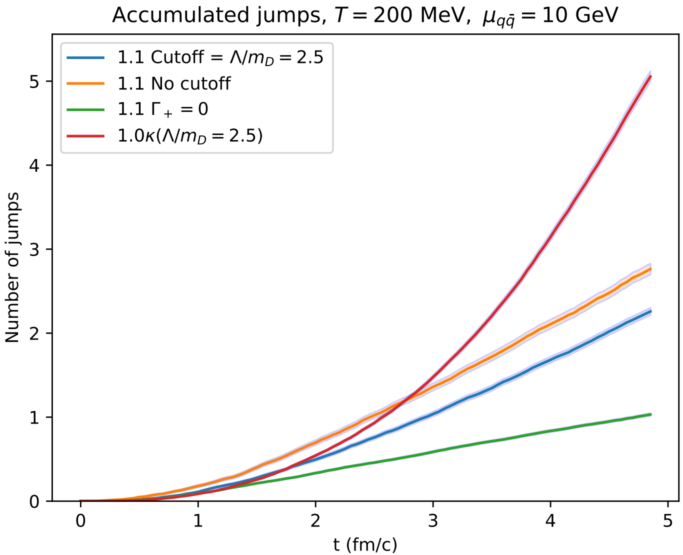
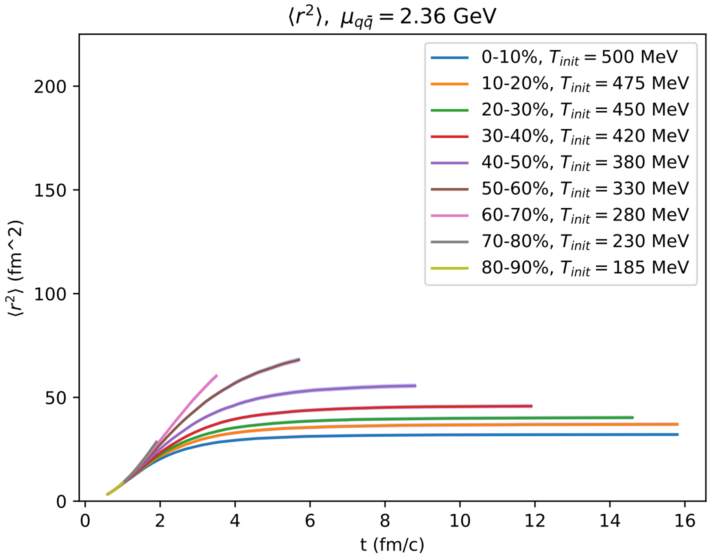
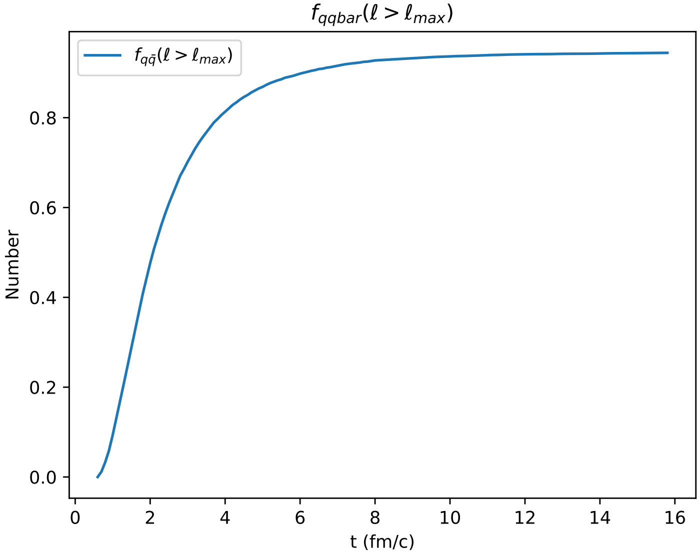


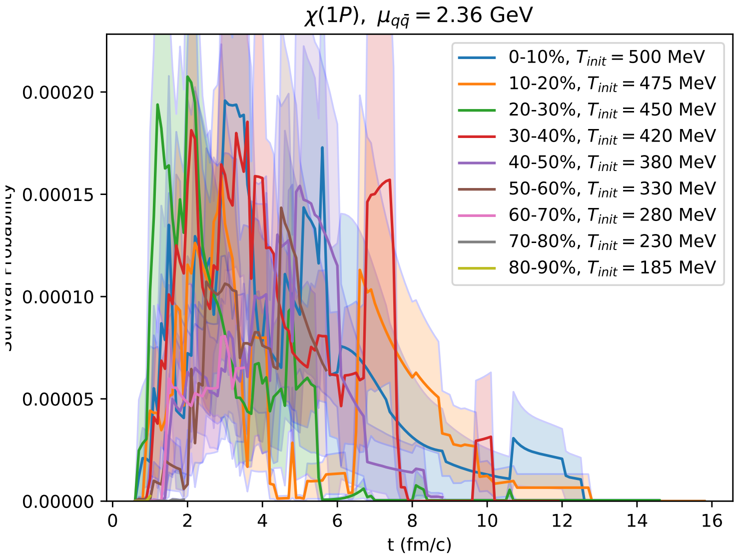
$\langle r^2 \rangle$



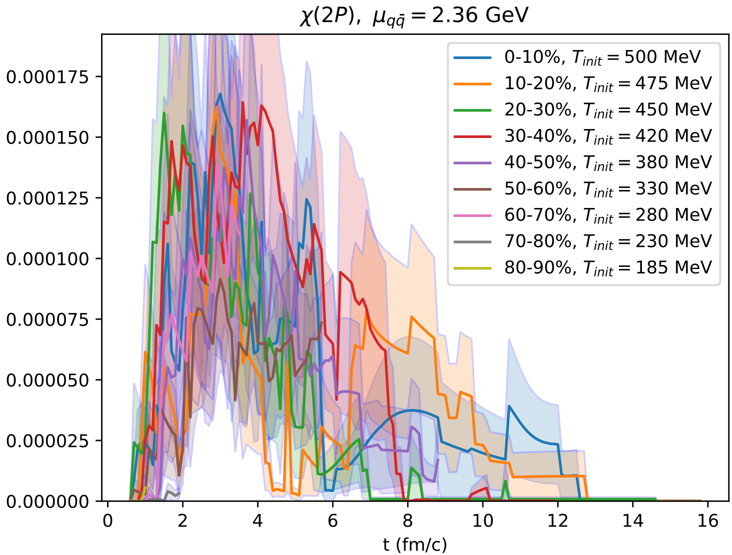
Accumulated over ℓ_{max}



1P, 2P, 1D

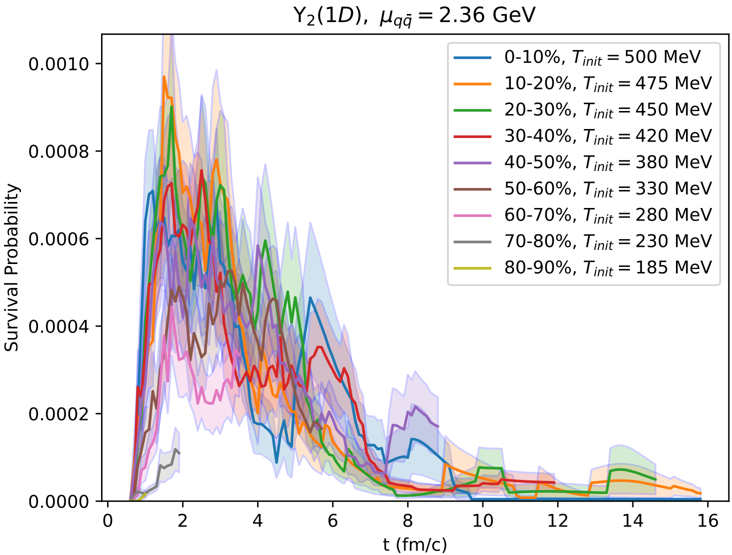

$\chi(1P)$, $\mu_{q\bar{q}} = 2.36$ GeV
0-10%, $T_{init} = 500$ MeV
10-20%, $T_{init} = 475$ MeV
20-30%, $T_{init} = 450$ MeV
30-40%, $T_{init} = 420$ MeV
40-50%, $T_{init} = 380$ MeV
50-60%, $T_{init} = 330$ MeV
60-70%, $T_{init} = 280$ MeV
70-80%, $T_{init} = 230$ MeV
80-90%, $T_{init} = 185$ MeV
0.00000
0.00005
0.00010
0.00015
0.00020
0
2
4
6
8
10
12
14
16
t (fm/c)

1P, 2P, 1D



$\chi(2P),\ \mu_{q\bar{q}} = 2.36$ GeV
0-10%, $T_{init} = 500$ MeV
10-20%, $T_{init} = 475$ MeV
20-30%, $T_{init} = 450$ MeV
30-40%, $T_{init} = 420$ MeV
40-50%, $T_{init} = 380$ MeV
50-60%, $T_{init} = 330$ MeV
60-70%, $T_{init} = 280$ MeV
70-80%, $T_{init} = 230$ MeV
80-90%, $T_{init} = 185$ MeV
0.000175
0.000150
0.000125
0.000100
0.000075
0.000050
0.000025
0.000000
0
2
4
6
8
10
12
14
16
t (fm/c)

1P, 2P, 1D


$\Upsilon_2(1D),\ \mu_{q\bar{q}} = 2.36$ GeV
Survival Probability
t (fm/c)
0.0000
0.0002
0.0004
0.0006
0.0008
0.0010
0
2
4
6
8
10
12
14
16
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