

Charm and beauty quarks in the initial stages of proton-Ion Collisions

Gabriele Parisi, Lucia Oliva, Vincenzo Greco, Marco Ruggieri



Meeting of the SIM Project, Turin
July 2, 2025

Outline

General Topic

Heavy Ion Collisions

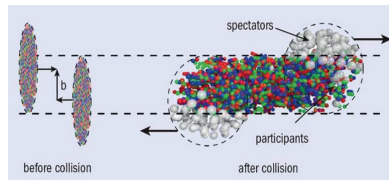
Specific Topic

Initial Stages of Heavy Ion Collisions

More Specific Topic

Heavy Quarks in Initial Stages of Heavy Ion Collisions

(my work)



Outline

General Topic

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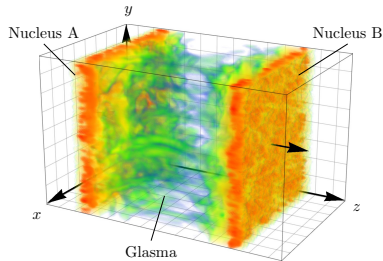
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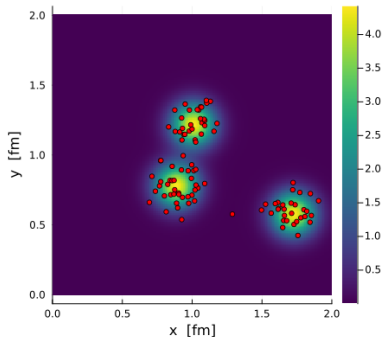
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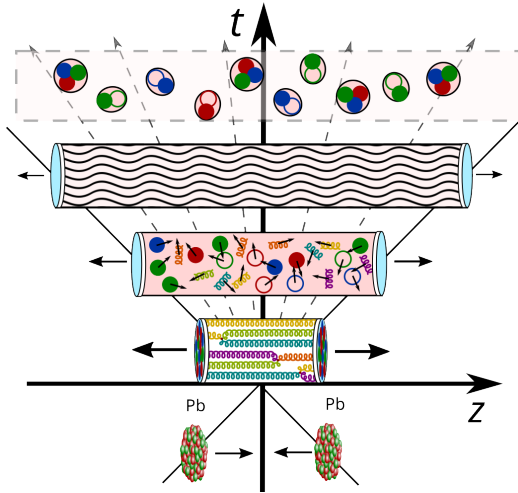
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Heavy-ion collisions



Final stages $\tau \geq 10 \text{ fm}/c$

↑

Local equilibrium $\tau \sim 10 \text{ fm}/c$

↑

QGP phase $\tau \sim 1 \text{ fm}/c$

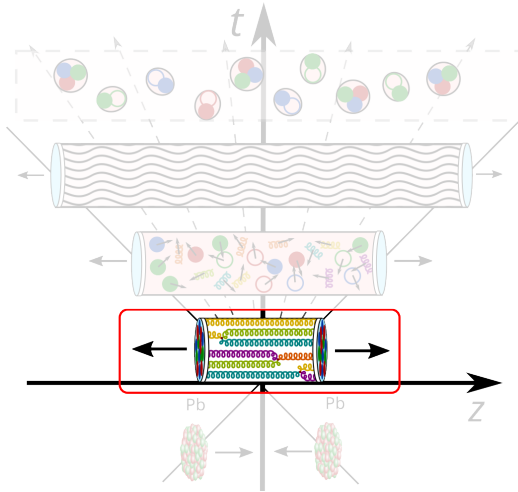
↑

Initial stage $\tau \sim 0.1 \text{ fm}/c$

↑

Before the collision $\tau \leq 0 \text{ fm}/c$

Heavy-ion collisions



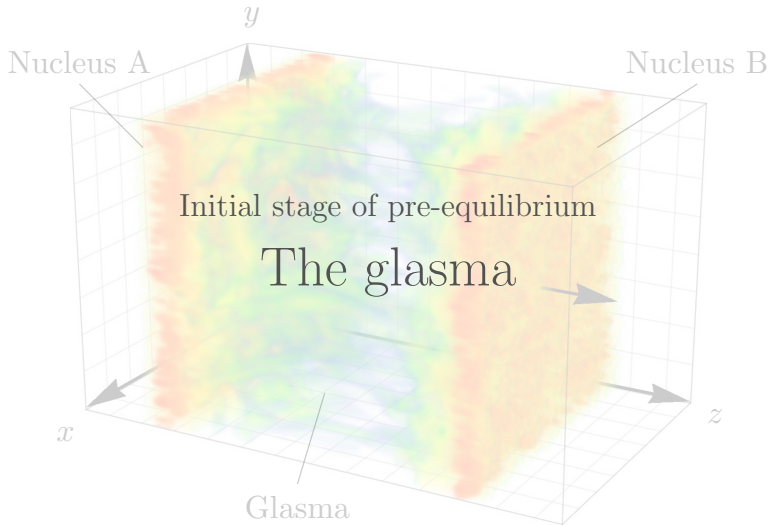
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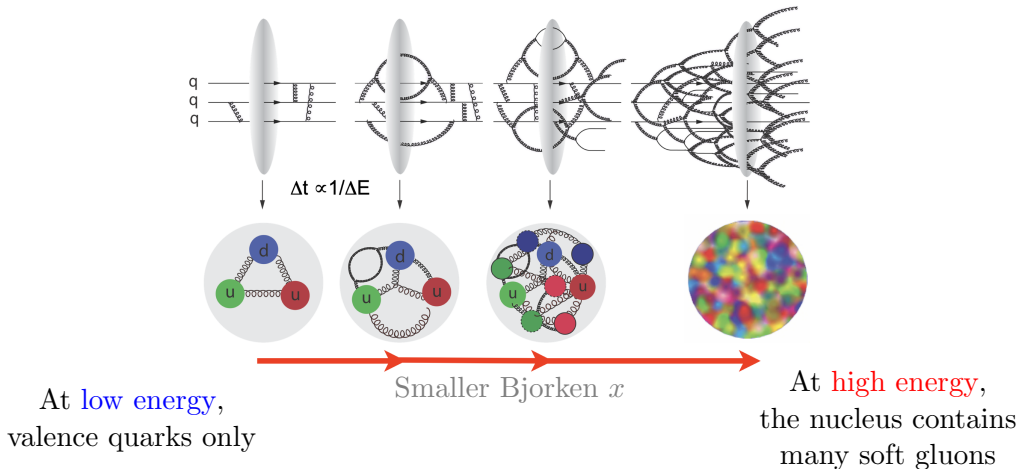
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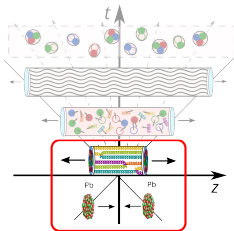
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BFKL evolution



QCD at high energy



At LHC energies, collision among **dense gluon distributions**

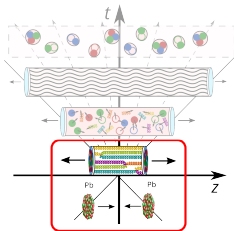


Emergence of a **saturation scale** $Q_s \sim 2 \text{ GeV}$: **Glasma**



described by the **Color Glass Condensate** formalism

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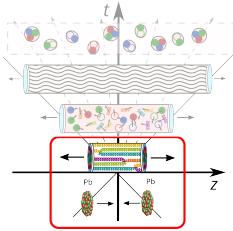


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Emergence of a **saturation scale** $Q_s \sim 2 \text{ GeV}$: **Glasma**



described by the **Color Glass Condensate** formalism

Color Glass Condensate

Color

QCD degrees of freedom

Condensate

High-gluon density,
classical dynamics

Glass

Quarks are static sources,
due to time dilation

Color Glass Condensate in 2+1D

McLerran-Venugopalan (MV) initial conditions

$$\langle \rho^a(\mathbf{x}_T) \rangle = 0,$$

$$\langle \rho^a(\mathbf{x}_T) \rho^b(\mathbf{y}_T) \rangle = (g\mu)^2 \delta^{ab} \delta^{(2)}(\mathbf{x}_T - \mathbf{y}_T),$$

J^μ by the **hard** partons $\implies A^\mu$ of the **soft** partons

Classical dynamics of glasma, Yang Mills equations

$$\mathcal{D}_\mu F^{\mu\nu} = 0$$

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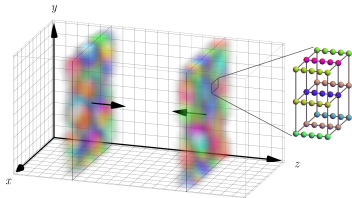
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Numerical implementation



- ▶ Classical Yang-Mills discretized on lattice:
color sheets
- ▶ Real-time lattice gauge theory techniques

Gauge links

$$V^\dagger(\mathbf{x}_T, x^-) = \mathcal{P} \exp \left[-ig \int_{-\infty}^{x^-} dz^- A^+(z^-, \mathbf{x}_T) \right]$$

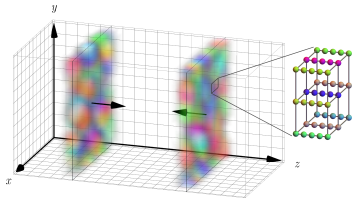
Plaquettes

$$U_{x,\mu\nu} = U_{x,\mu} U_{x+\mu,\nu} U_{x+\mu+\nu,-\mu} U_{x+\nu,-\nu}$$

Wilson lines

$$U_{\mathbf{x}_T,i} = V(\mathbf{x}_T) V^\dagger(\mathbf{x}_T + \Delta x_i)$$

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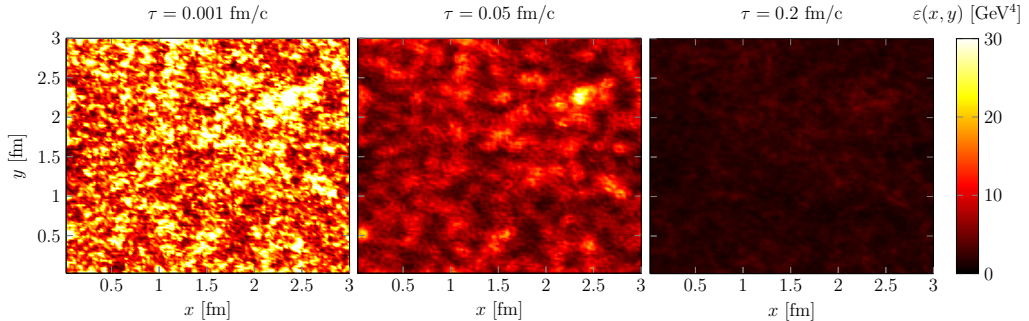
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Features of the glasma

Energy density in nucleus-nucleus collisions

$$\varepsilon = \text{Tr}[E_L^2 + B_L^2 + E_T^2 + B_T^2]$$

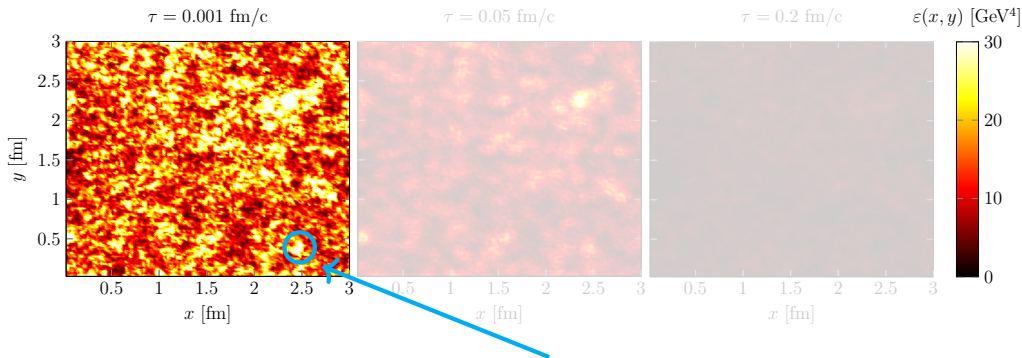


Fields dilute after $\delta\tau \sim Q_s^{-1} \sim \mathcal{O}(0.1 \text{ fm/c})$

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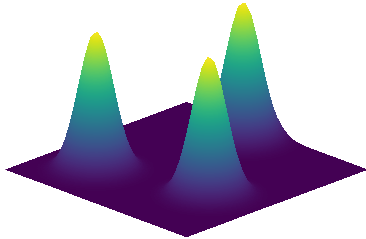


Hotspots of size $\delta x_T \sim Q_s^{-1} \sim \mathcal{O}(0.1 \text{ fm})$

Color charge generation in pA

$$\langle \rho^a(\mathbf{x}_T) \rho^b(\mathbf{y}_T) \rangle = (g\mu)^2 \delta^{ab} \delta^{(2)}(\mathbf{x}_T - \mathbf{y}_T)$$

We can have μ space-dependent:
proton structure



- ▶ Three hotspots $\bar{\mathbf{x}}_T^i$ in a width $\sqrt{B_{qc}}$.
- ▶ Then:

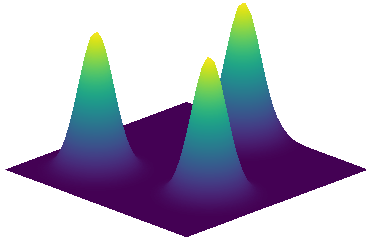
$$\mu(\mathbf{x}_T) \propto \frac{1}{3} \sum_{i=1}^3 \frac{1}{2\pi B_q} \exp \left[-\frac{(\mathbf{x}_T - \bar{\mathbf{x}}_T^i)^2}{2B_q} \right].$$

$$B_{qc} = (0.4 \text{ fm})^2, \quad B_q = (0.11 \text{ fm})^2$$

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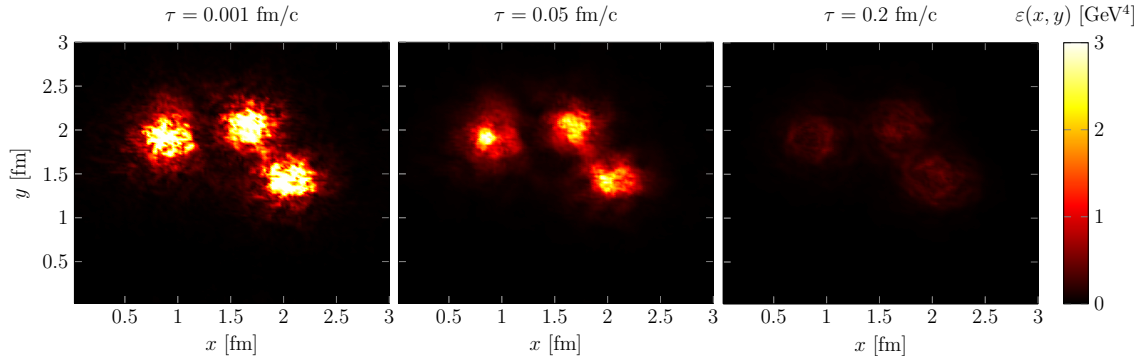
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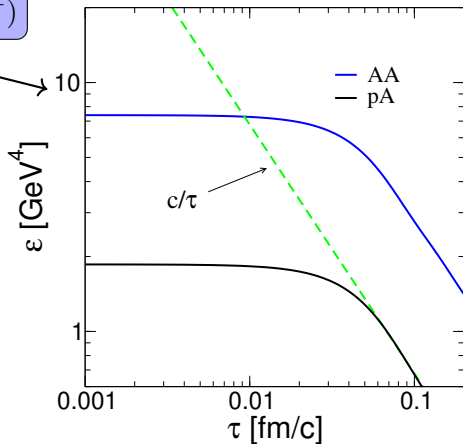
Energy density in pA vs (x, y)

$$\varepsilon = \text{Tr}[E_L^2 + B_L^2 + E_T^2 + B_T^2]$$



Energy density in pA vs τ

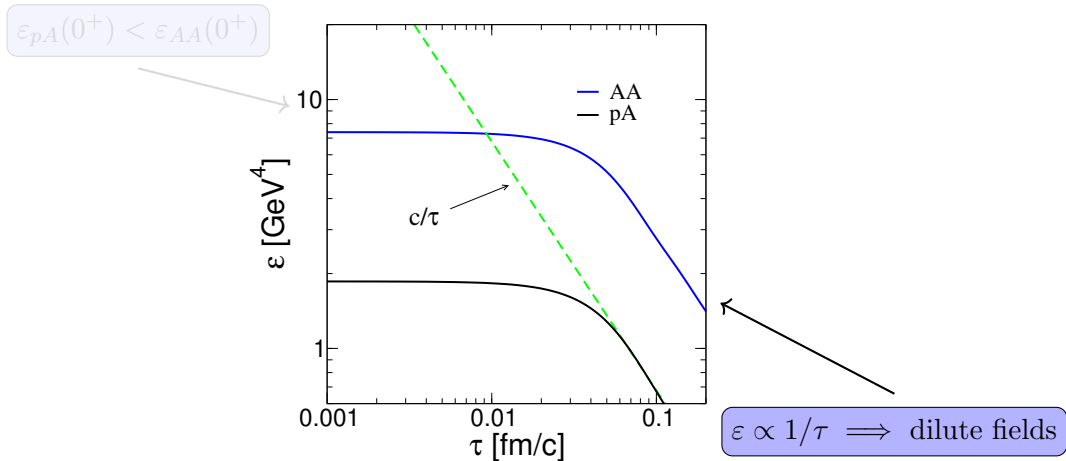
$$\varepsilon_{pA}(0^+) < \varepsilon_{AA}(0^+)$$



$\varepsilon \propto 1/\tau \Rightarrow$ dilute fields

32x32x32 lattice, $N_\tau = 400$

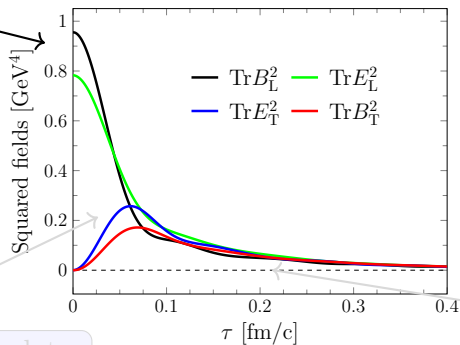
Energy density in pA vs τ



32x32x32 lattice, $N_\tau = 400$

Fields in pA vs τ

Longitudinal fields $\neq 0$



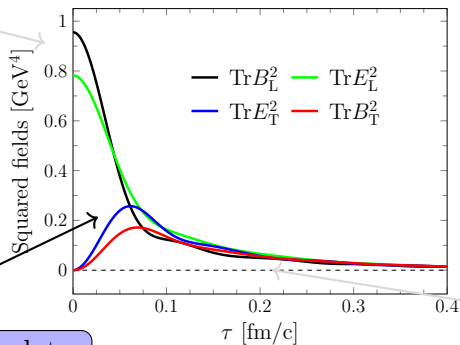
Transverse fields emerge later

Dilute fields after ~ 0.1 fm/c

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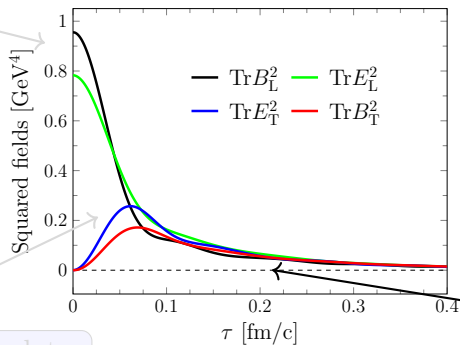
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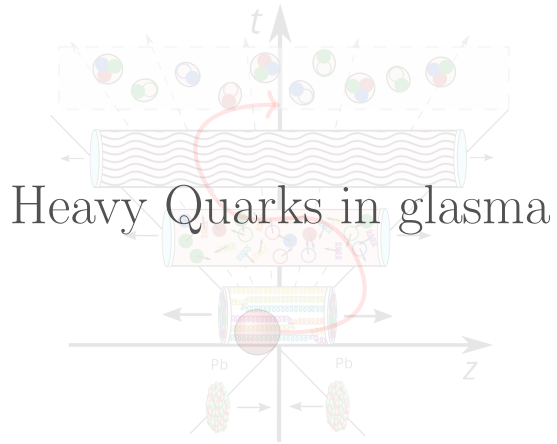
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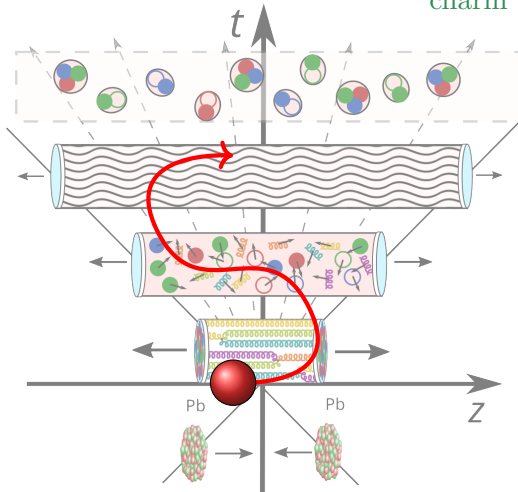
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Heavy quarks

charm and beauty



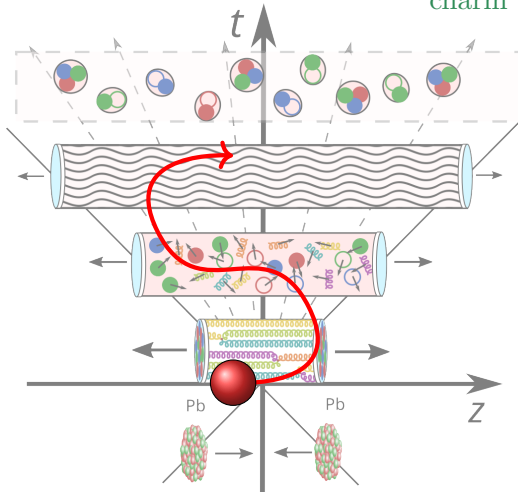
High mass: useful probes of initial stages

- ▶ $M \gg \Lambda_{\text{QCD}}$:
early pQCD production
- ▶ $M \gg T_{\text{QGP}}$:
no thermal production
- ▶ $\tau_{\text{creation}} \ll \tau_{\text{QGP}}$:
probes of the collision

$$M_{\text{charm}} = 1.3 \text{ GeV}, M_{\text{beauty}} = 4.2 \text{ GeV}$$
$$\tau_{\text{creation}} \sim 1/2M$$

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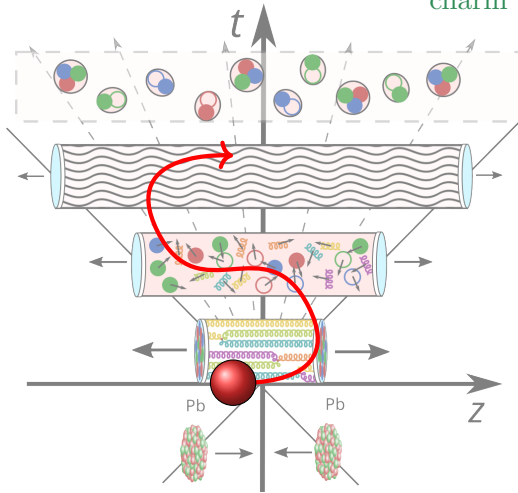
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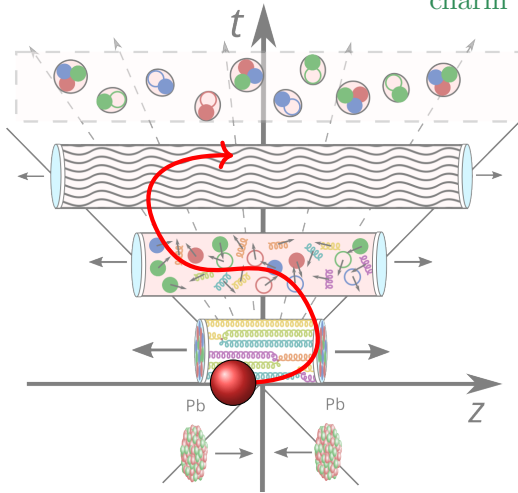
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Heavy Quarks in Yang-Mills fields

Wong equations

Classical transport in Yang-Mills background field A^μ

Coordinate evolution

$$\frac{d}{d\tau} x^\mu = \frac{p^\mu}{m}$$

Momentum evolution

$$\frac{d}{d\tau} p^\mu = \frac{1}{T_R} g \operatorname{Tr} \{ Q F^{\mu\nu} \} \frac{p_\nu}{m}$$

Color charge evolution

$$\frac{d}{d\tau} Q = -ig[A_\mu, Q] \frac{p^\mu}{m}$$

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Basically $\mathbf{p} = m\mathbf{v}$

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Basically $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

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Basically Color Rotation in SU(3)

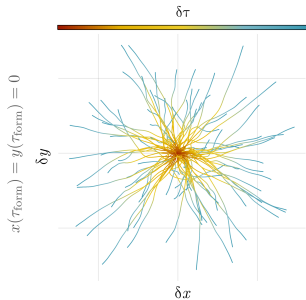
Preserves Casimir invariants $q_2 = Q^a Q^a$ and $q_3 = d_{abc} Q^a Q^b Q^c$

Heavy Quarks in Yang-Mills fields: trajectories

- Change in **coordinates** due to momentum kicks

- **Momentum** broadening due to color Lorentz force

- **Color charge** rotation in SU(3) with Wilson lines

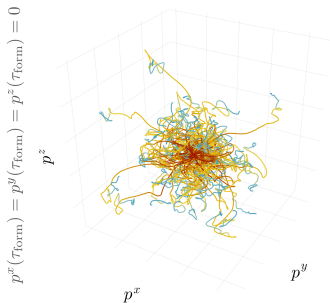


Heavy Quarks in Yang-Mills fields: trajectories

► Change in **coordinates** due to momentum kicks

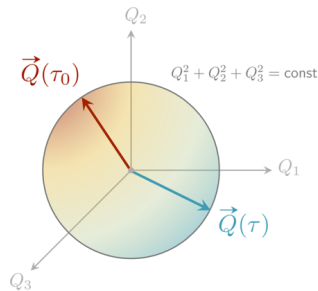
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Heavy Quarks in Yang-Mills fields: trajectories

- Change in **coordinates** due to momentum kicks
- **Momentum** broadening due to color Lorentz force
- **Color charge** rotation in SU(3) with Wilson lines



(Picture should actually be in 8 dims)

Heavy Quarks in Yang-Mills fields: initialization

- Center of Mass position:

$$P(\mathbf{x}_\perp) \propto \mu(\mathbf{x}_\perp)$$

- Relative position:

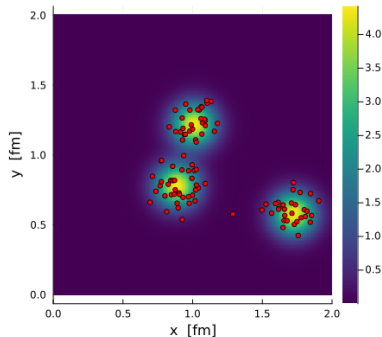
$$P(r_{\text{rel}}) \propto r_{\text{rel}} \exp(-r_{\text{rel}}^2/\sigma^2)$$

- Relative momentum:

$$P(p_{\text{rel}}) \propto p_{\text{rel}} \exp(-p_{\text{rel}}^2\sigma^2)$$

- Color charge:

$$\text{singlet, } Q_a = -\bar{Q}_a$$



Heavy Quarks in Yang-Mills fields: pair potential

$$\frac{dp^i}{d\tau} = \frac{1}{T_R} g \operatorname{Tr} \{ Q F^{i\nu} \} \frac{p_\nu}{m} - \frac{\partial V}{\partial x^i}$$

Classical projection of pQCD potential

$$\hat{V} = T^a \otimes \bar{T}^a \frac{\alpha_s}{r_{\text{rel}}} \implies V = \frac{Q_a \bar{Q}_a}{N_c} \frac{\alpha_s}{r_{\text{rel}}}$$

- ▶ For $\tau = \tau_{\text{form}}$, $Q_a \bar{Q}_a = -4$: attractive potential
- ▶ For $\tau > \tau_{\text{form}}$: dynamic potential



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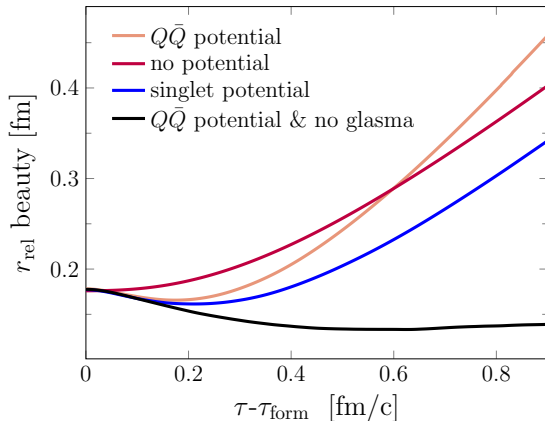
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Relative distance

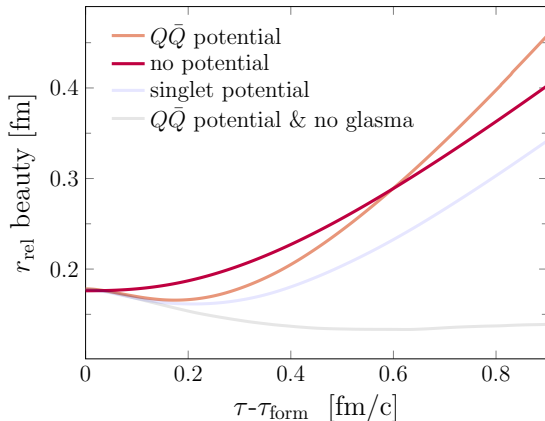
Effect of potential on beauty quark



- ▶ Glasma fields only: diffusion
- ▶ First attractive, then repulsive potential
- ▶ Without glasma, no color evolution, potential attraction only

Relative distance

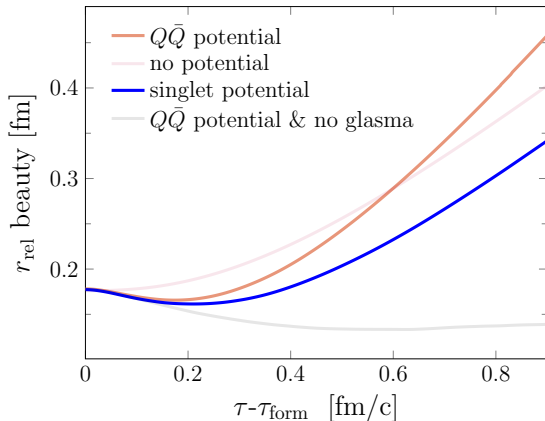
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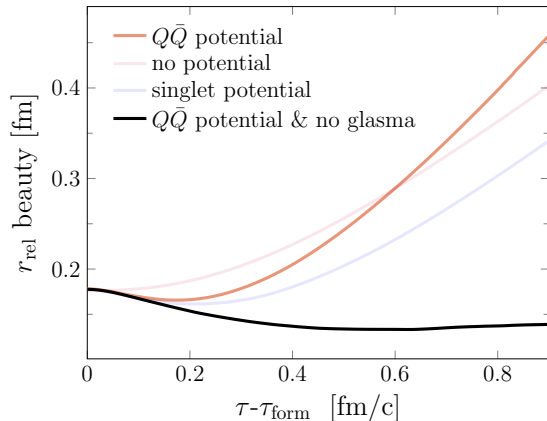
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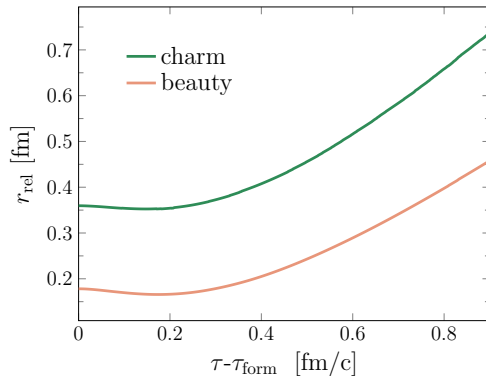
charm vs beauty

- ▶ Physical input: $r_{\text{rel}}^{\text{beauty}} < r_{\text{rel}}^{\text{charm}}$
- ▶ Higher mass, lower spread
- ▶ Initial balance between:

$Q\bar{Q}$ potential (attraction)



Glasma fields (diffusion)



Distance not the main driver of pair decorrelation
(within glasma timescales)

Relative distance

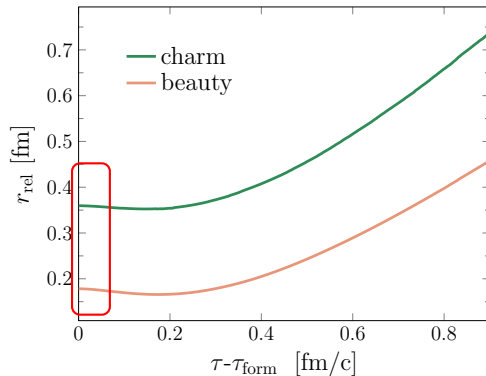
charm vs beauty

- ▶ Physical input: $r_{\text{rel}}^{\text{beauty}} < r_{\text{rel}}^{\text{charm}}$
- ▶ Higher mass, lower spread
- ▶ Initial balance between:

$Q\bar{Q}$ potential (attraction)



Glasma fields (diffusion)



Distance not the main driver of pair decorrelation
(within glasma timescales)

Relative distance

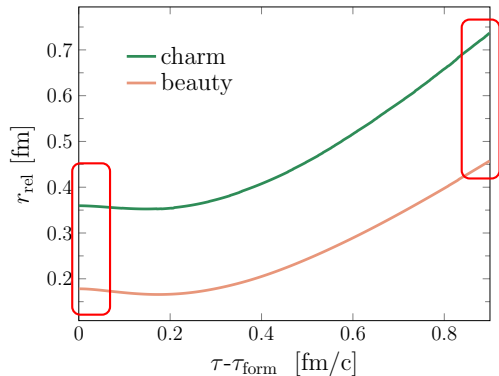
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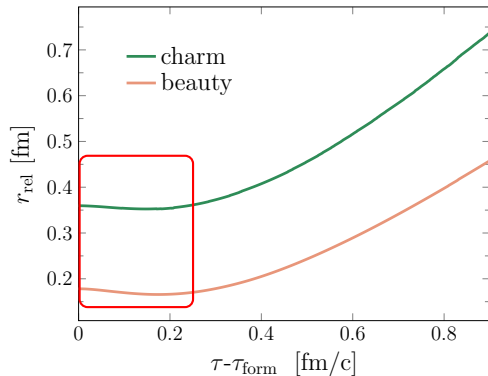
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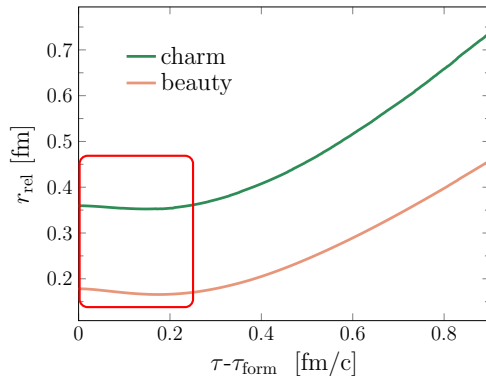
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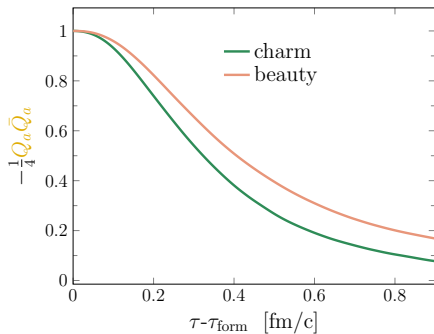
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Dissociation of pairs

Melting of quarkonia due to color decorrelation



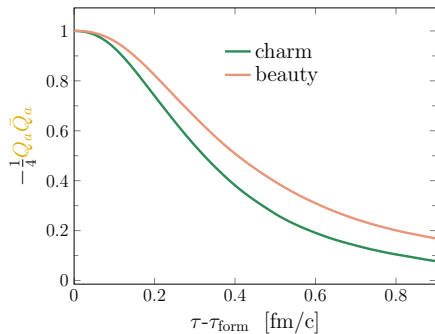
For each pair, at each time, we define:

$$\mathcal{P}_{\text{melting}} = 1 - \mathcal{P}_{\text{survival}},$$

$$\mathcal{P}_{\text{survival}} \equiv \exp \left[-\kappa \left(-\frac{1}{4} Q_a \bar{Q}_a - 1 \right)^2 \right].$$

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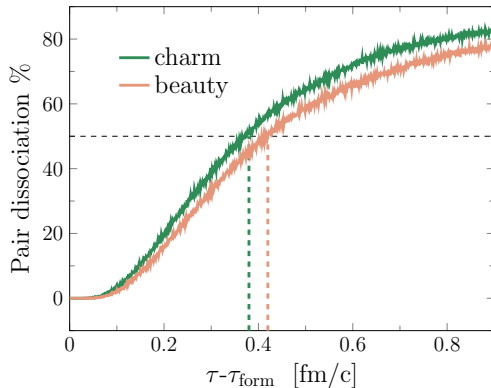
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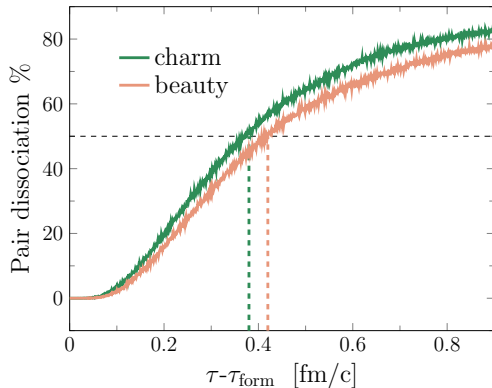
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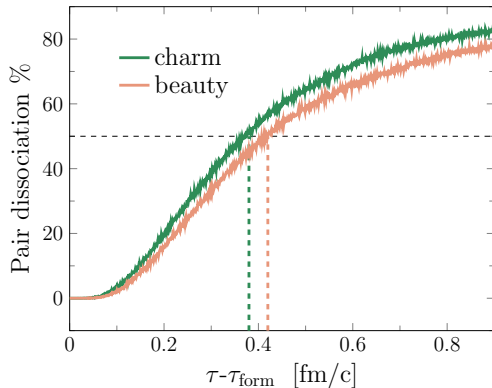
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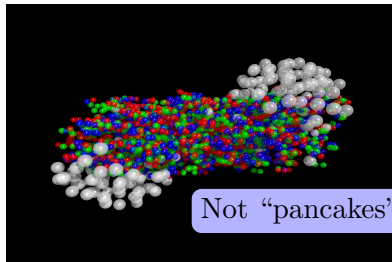


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3+1D simulations

Heavy Ion Collisions are **not** boost-invariant:

rapidity-dependence to be taken into
account.



Same initial conditions + η -dependent terms such that:

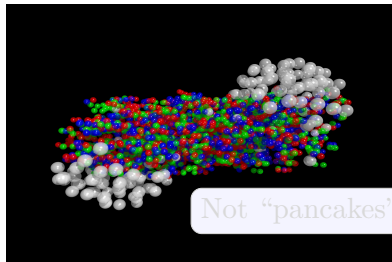
$$D_i E_i(\eta) + D_\eta E_\eta(\eta) = 0 \quad \text{Gauss' Law}$$

Explicit expressions for δE in backup

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Momentum broadening for $M_{\text{HQ}} \rightarrow +\infty$

Focus on **momentum broadening**:

$$\delta p_i^2(\tau) \equiv p_i^2(\tau) - p_i^2(\tau_{\text{form}}), \quad i = x, y, z.$$

One can prove that for $M_{\text{HQ}} \rightarrow +\infty$:

$$\begin{aligned} \langle \delta p_L^2(\tau) \rangle_\infty &= g^2 \int_{\tau_0}^{\tau} d\tau' \int_{\tau_0}^{\tau} d\tau'' \langle \text{Tr}[E_z(\tau') E_z(\tau'')] \rangle \\ \langle \delta p_T^2(\tau) \rangle_\infty &= g^2 \int_{\tau_0}^{\tau} d\tau' \int_{\tau_0}^{\tau} d\tau'' \frac{1}{\tau' \tau''} \langle \sum_{i=x,y} \text{Tr}[E_i(\tau') E_i(\tau'')] \rangle \end{aligned}$$

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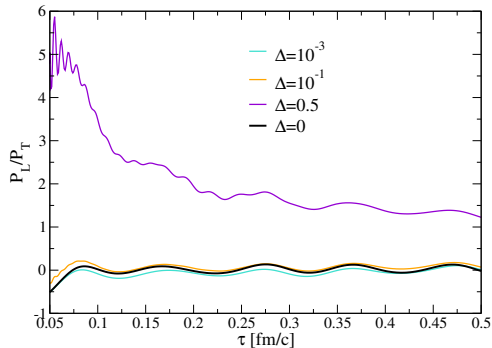
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Results: 3+1D simulations

Pressures Ratio

Fluctuations scaling as $\delta E_i \sim \Delta$



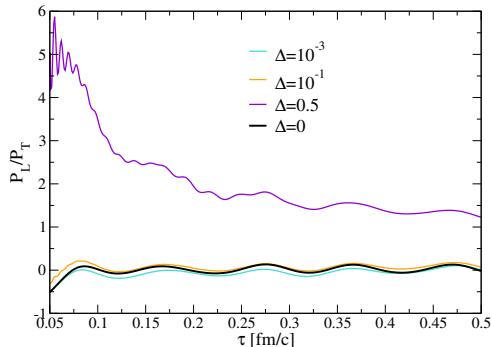
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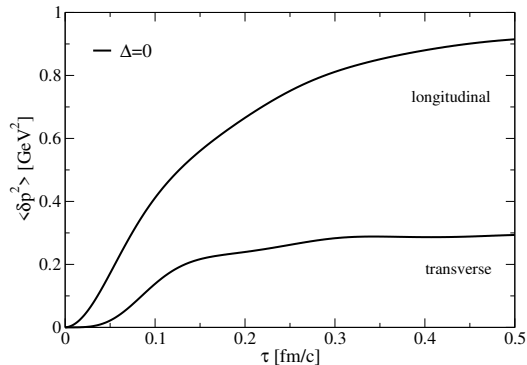
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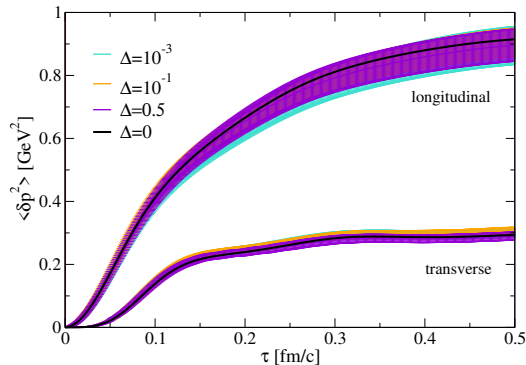
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Need different initial conditions to favor isotropization

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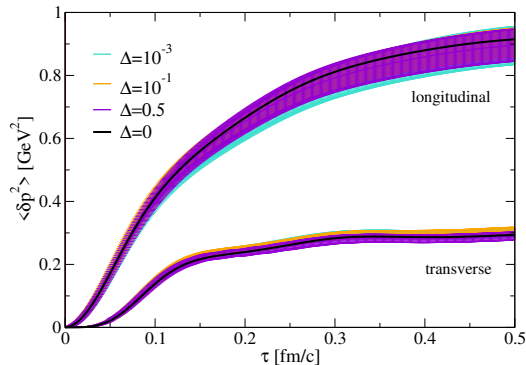
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Outro

Conclusions

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Outlook

- ▶ Anisotropic flows v_n of gluons and Heavy Quarks in Glasma
- ▶ Coupling to later stages of Heavy Ion Collisions

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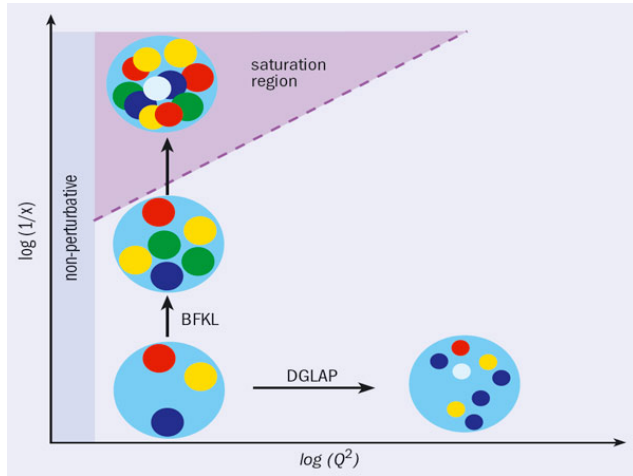
Thank you!

Any questions?



Back-up

BFKL and DGLAP evolutions



CYM on lattice

After gauge choice, YM equations reduce to:

$$\Delta_{\perp} \alpha(\mathbf{x}_{\perp}) = -\rho(\mathbf{x}_{\perp}) \implies \tilde{\alpha}_{n,k}^a = \frac{\tilde{\rho}_{n,k}^a}{\tilde{k}_{\perp}^2 + m^2}$$

where

$$\tilde{k}_{\perp}^2 = \sum_{i=x,y} \left(\frac{2}{a_{\perp}} \right)^2 \sin^2 \left(\frac{k_i a_{\perp}}{2} \right).$$

CYM on lattice

Solve previous eq. for A_a^+ in each nucleus (at $\tau = 0^-$). For $\tau = 0^+$?

$$A_{\text{tot}}^+ = A_1^+ + A_2^+ \stackrel{?}{\Longrightarrow} U_{\text{tot}} \propto \exp[iA_{\text{tot}}^+] = U_1 \cdot U_2 \quad \text{NO!!}$$

In SU(3) no exact formula for U_{tot} , rather we iteratively solve:

$$\text{Tr}[t_a(U_{x,i}^A + U_{x,i}^B)(\mathbb{I} + U_{x,i}) - \text{h.c.}] = 0.$$

Those U are needed for the magnetic fields:

$$B_L^2 = \frac{2}{g^2 a_\perp^4} \text{Tr}(\mathbb{I} - U_{xy}), \quad B_T^2 = \frac{2}{(ga_\eta a_\perp \tau)^2} \sum_{i=x,y} \text{Tr}(\mathbb{I} - U_{\eta i}).$$

CYM on lattice

► **Initial conditions:**

$E_x = E_y = 0$, $U_\eta = \mathbb{I}$, $U_{x/y} = U_{\text{tot},x/y}$ as in previous slide

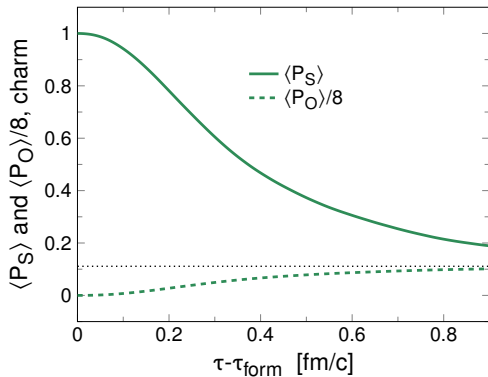
$$E^\eta = -\frac{i}{4ga_\perp^2} \sum_{i=x,y} \left[(U_i(\mathbf{x}_\perp) - \mathbb{I})(U_i^{B,\dagger}(\mathbf{x}_\perp) - U_i^{A,\dagger}(\mathbf{x}_\perp)) + \right. \\ \left. (U_i^\dagger(\mathbf{x}_\perp - \hat{i}) - \mathbb{I})(U_i^B(\mathbf{x}_\perp - \hat{i}) - U_i^A(\mathbf{x}_\perp - \hat{i})) - h.c. \right].$$

► **Eqs of motion:**

$$\partial_\tau U_i(x) = \frac{-iga_\perp}{\tau} E^i(x) U_i(x), \quad \partial_\tau U_\eta(x) = -iga_\eta \tau E^\eta(x) U_\eta(x).$$

Octet and Singlet projectors

$$P_S = -\frac{2}{3} \frac{Q_a \bar{Q}_a}{N_c} + \frac{1}{9}, \quad P_O = \frac{2}{3} \frac{Q_a \bar{Q}_a}{N_c} + \frac{8}{9},$$



At late times

$$\langle P_S \rangle \approx \langle P_O \rangle / 8 \approx 1/9.$$

Qualitative agreement with other approaches:

Brownian motion in QGP

Explicit expressions for δE

$$\begin{aligned}\delta E^i &= a_\eta^{-1} [F(\eta - a_\eta) - F(\eta)] \xi_i(\mathbf{x}_\perp), \\ \delta E^\eta &= -a_\perp^{-1} F(\eta) \sum_{i=x,y} [U_i^\dagger(\mathbf{x}_\perp - \hat{i}) \xi_i(\mathbf{x}_\perp - \hat{i}) U_i(\mathbf{x}_\perp - \hat{i}) - \xi^i(\mathbf{x}_\perp)],\end{aligned}$$

where ξ such that:

$$\langle \xi_i(\mathbf{x}_\perp) \xi_j(\mathbf{y}_\perp) \rangle = \delta_{ij} \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp).$$

Explicit expressions for δE

Actual shape of the fluctuation given by $F(\eta)$. We can choose:

- Random gaussian function in η .
- Oscillating function $F(\eta) = \frac{\Delta}{N_{\perp}} \cos\left(\frac{2\pi\eta}{L_{\eta}} \cdot \nu_0\right)$ for some ν_0 .
- Superposition of oscillating functions:

$$F(\eta) = \sum_{\nu} \frac{\Delta}{N_{\perp}} \cos\left(\frac{2\pi\eta}{L_{\eta}} \cdot \nu\right).$$

The ‘seed’ Δ regulates the amplitude.