Electric conductivity and flavor diffusion in a viscous, resistive quark-gluon plasma for weak and strong magnetic fields

Ferdinando Frascà

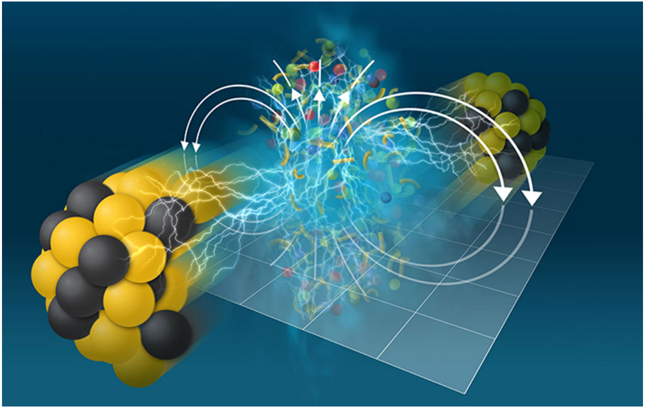
Istituto Nazionale di Fisica Nucleare - Sezione di Torino

July 2-3, 2025

Contents

Strongly magnetized systems in plasma physics

Evidence of strong magnetic fields in HIC's

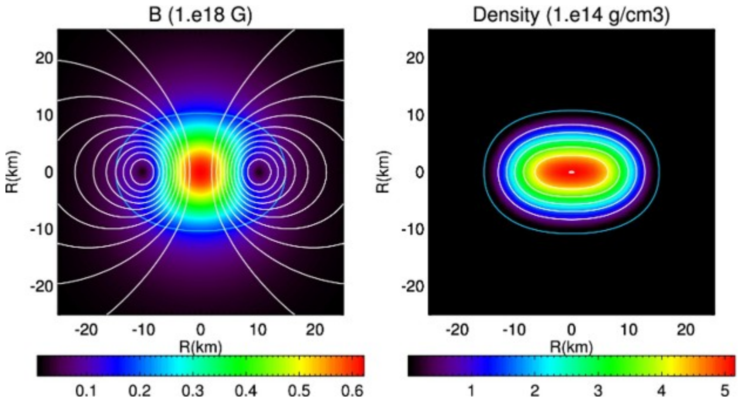


"Non-central collisions can produce **strong magnetic fields** on the order of 10^{18} G, which offers a probe into the **electrical conductivity** of the **QGP**"

M. I. Abdulhamid et al., Phys.Rev.X 14 (2024) 1, 011028

- Conductivity: crucial parameter to determine the **time-evolution of magnetic field**

Strongly magnetized astrophysical systems



Strong magnetic fields are expected to exist in several **astrophysical objects**, like in extremely magnetized neutron stars (magnetars) where $B \gtrsim 10^{16}$ G

A. G. Pili et al., MNRAS 439, 3541–3563 (2014)

How to quantify the strength of the magnetic field?

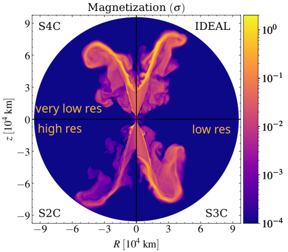
Magnetic effects vs **matter contribution**:

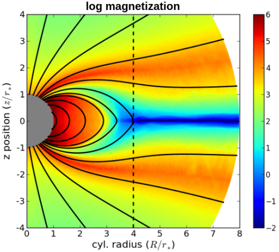
$$\underbrace{\beta_V \equiv \frac{P}{B^2/2}}_{\text{plasma beta-value}} \quad , \quad \underbrace{\sigma \equiv \frac{B^2}{\varepsilon + P}}_{\text{magnetization}} \longrightarrow \begin{cases} \text{weakly magnetized: } \beta_V \gg 1\,,\ \sigma \ll 1 \\ \text{strongly magnetized: } \beta_V \ll 1\,,\ \sigma \gg 1 \end{cases}$$

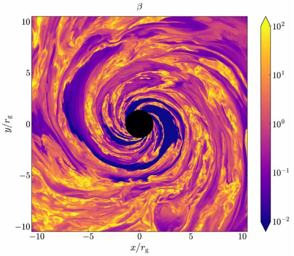
How to quantify the strength of the magnetic field?

Magnetic effects vs **matter contribution**:

$$\underbrace{\beta_V \equiv \frac{P}{B^2/2}}_{\text{plasma beta-value}} \quad , \quad \underbrace{\sigma \equiv \frac{B^2}{\varepsilon + P}}_{\text{magnetization}} \longrightarrow \begin{cases} \textbf{weakly magnetized: } \beta_V \gg 1\,,\ \sigma \ll 1 \\ \textbf{strongly magnetized: } \beta_V \ll 1\,,\ \sigma \gg 1 \end{cases}$$

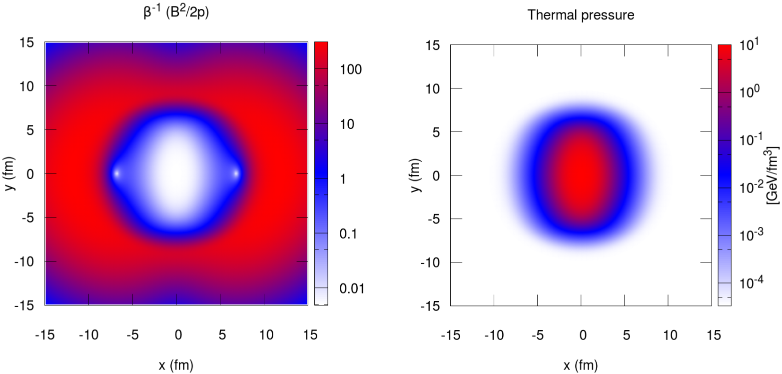






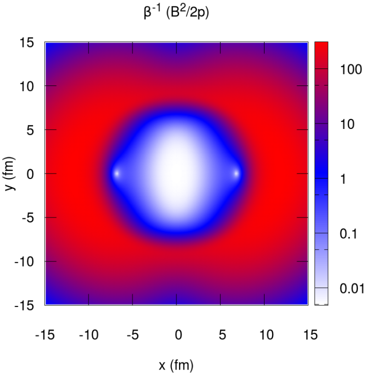
Strongly magnetized plasmas in astrophysics: post BNS-merger jets (G. Mattia et al., A&A, 691, A105 (2024)), pulsars magnetosphere (M. A. Belyaev, MNRAS 449, 2759–2767 (2015)), black hole accretion disks (B. Ripperda et al., Astrophys.J.Lett. 924 (2022) 2, L32) and solar atmosphere (Ph.-A. Bourdin, ApJL 850:L29, 201)

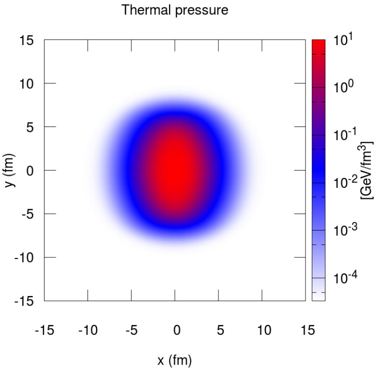
Is the QGP a strongly-magnetized system?



Peripheral (impact param. $b = 10$ fm) **Au-Au collisions** at $\sqrt{s_{NN}} = 200$ GeV: **strong magnetization** ($\beta_V^{-1} \gg 1$) at **early times** ($\tau_0 = 0.4$ fm/c) where the fireball is **very rarefied** (**G. Inghirami et al., Eur.Phys.J.C 76 (2016) 12, 659**)

Is the QGP a strongly-magnetized system?





Peripheral (impact param. $b = 10$ fm) **Au-Au collisions** at $\sqrt{s_{NN}} = 200$ GeV: **strong magnetization** ($\beta_V^{-1} \gg 1$) at **early times** ($\tau_0 = 0.4$ fm/c) where the fireball is **very rarefied** (**G. Inghirami et al., Eur.Phys.J.C 76 (2016) 12, 659**)

- **Magnetic-field evolution** depends on **transport coefficients** (viscosity, resistivity, etc.)
- **Can transport coefficients be also affected by strong magnetic fields?**

Magnetohydrodynamics of a weakly magnetized QGP

General setup and (G)RMHD in HIC's

- **conformal** plasma (EoS: $\boldsymbol{\varepsilon = 3P}$) composed of three massless quark species
- system **close to thermodynamic equilibrium**: small density gradients $\Delta_\mu \alpha_f$ and electric field $e^\mu \equiv F^{\mu\nu} u_\nu$ (Debye screening)
- **Landau-Lifshitz frame**: $u_\nu \, T_{\mathrm{m}}^{\mu\nu} \equiv -\varepsilon \, u^\mu$

General setup and (G)RMHD in HIC's

- **conformal** plasma (EoS: $\varepsilon = 3P$) composed of three massless quark species
- system **close to thermodynamic equilibrium**: small density gradients $\Delta_\mu \alpha_f$ and electric field $e^\mu \equiv F^{\mu\nu} u_\nu$ (Debye screening)
- **Landau-Lifshitz frame**: $u_\nu T^{\mu\nu}_{\rm m} \equiv -\varepsilon\, u^\mu$

Dissipative corrections to **net-flavor current** and **matter stress-energy tensor**:

$$J^\mu_f \equiv n_f\, u^\mu + \boldsymbol{j^\mu_f} \quad , \quad T^{\mu\nu}_{\rm m} \equiv \varepsilon\, u^\mu u^\nu + P\,\Delta^{\mu\nu} + \boldsymbol{\pi^{\mu\nu}} \quad \text{with} \quad u_\mu\, j^\mu_f = u_\mu\, \pi^{\mu\nu} = 0$$

Evolution according to the conservation laws (∇_μ : covariant derivative)

$$\boldsymbol{\nabla_\mu T^{\mu\nu}_{\rm m} = -(J_Q)_\mu\, F^{\mu\nu}} \quad , \quad F^{\mu\nu} = u^\mu e^\nu - u^\nu e^\mu + \epsilon^{\mu\nu\lambda\rho} b_\lambda\, u_\rho$$
$$\boldsymbol{\nabla_\mu J^\mu_f = 0} \quad , \quad f = u, d, s \quad \longleftrightarrow \quad \nabla_\mu J^\mu_q = 0 \quad , \quad q = \mathcal{B}, Q, S$$

and Maxwell's equations ($b^\mu \equiv {}^\star F^{\mu\nu}\, u_\nu$: magnetic field)

$$\boldsymbol{\nabla_\mu {}^\star F^{\mu\nu} = 0} \quad , \quad \boldsymbol{\nabla_\mu F^{\mu\nu} = -J^\nu_Q} \quad , \quad J^\mu_Q \,:\, \text{electric current}$$

Transformation laws from flavor to charge space:

$$J^\mu_q \equiv \sum_f \mathcal{M}_{qf}\, J^\mu_f \quad , \quad \mathcal{M} \equiv \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3}|e| & -\frac{1}{3}|e| & -\frac{1}{3}|e| \\ 0 & 0 & -1 \end{pmatrix}$$

General setup and (G)RMHD in HIC's

Fireball dynamics described by means of the **energy-momentum conservation** projections along the fluid velocity u^μ ($D \equiv u^\mu \nabla_\mu$, $\Theta \equiv \nabla_\mu u^\mu$)

$$D\varepsilon + (\varepsilon + P)\,\Theta + \pi^{\mu\nu}\sigma_{\mu\nu} = \underbrace{e^\mu\,(j_Q)_\mu}_{\text{Joule heating}} \quad , \quad \sigma_{\mu\nu} : \text{shear tensor}$$

and transversally, using $\Delta_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu$ ($a^\mu \equiv Du^\mu$, $\Delta_\mu \equiv \Delta_{\mu\nu}\,\partial^\nu$)

$$(\varepsilon + P)\,a_\mu + \Delta_\mu P + \Delta_{\mu\beta}\,\Delta_\alpha \pi^{\alpha\beta} + a^\nu\,\pi_{\nu\mu} = n_Q\,e_\mu + \underbrace{\epsilon_{\mu\nu\lambda\rho}\,j_Q^\nu\,b^\lambda\,u^\rho}_{\text{first order if } b^\lambda \sim \mathcal{O}(1)}$$

and the **continuity equation** for the flavor current

$$Dn_f + n_f\,\Theta + \nabla_\mu\,j_f^\mu = 0$$

General setup and (G)RMHD in HIC's

Fireball dynamics described by means of the **energy-momentum conservation** projections along the fluid velocity u^μ ($D \equiv u^\mu \nabla_\mu$, $\Theta \equiv \nabla_\mu u^\mu$)

$$D\varepsilon + (\varepsilon + P)\,\Theta + \pi^{\mu\nu}\sigma_{\mu\nu} = \underbrace{e^\mu\,(j_Q)_\mu}_{\text{Joule heating}} \quad , \quad \sigma_{\mu\nu} : \text{shear tensor}$$

and transversally, using $\Delta_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu$ ($a^\mu \equiv Du^\mu$, $\Delta_\mu \equiv \Delta_{\mu\nu}\,\partial^\nu$)

$$(\varepsilon + P)\,a_\mu + \Delta_\mu P + \Delta_{\mu\beta}\,\Delta_\alpha \pi^{\alpha\beta} + a^\nu\,\pi_{\nu\mu} = n_Q\,e_\mu + \underbrace{\boldsymbol{\epsilon_{\mu\nu\lambda\rho}\,j_Q^\nu\,b^\lambda\,u^\rho}}_{\textbf{first order if } b^\lambda \sim \mathcal{O}(1)}$$

and the **continuity equation** for the flavor current

$$Dn_f + n_f\,\Theta + \nabla_\mu\,j_f^\mu = 0$$

Constitutive relations for the dissipative corrections:

- **macroscopic approach**: $\nabla_\mu \mathcal{S}^\mu \geq 0$ (MIS-type transient theories)
- **microscopic approach**: Boltzmann-Vlasov equation (allows to get also the expression of transport coefficients)

Generalized Ohm's law for multiple conserved charges

Requirement of **non-negative entropy production rate**

$$\boldsymbol{\nabla_\mu \mathcal{S}^\mu \geq 0} \quad , \quad \mathcal{S}^\mu \equiv s u^\mu - \sum_f \alpha_f \, j_f^\mu$$

satisfied if $\boldsymbol{\alpha_f = \mu_f / T}$ (dimensionless chemical potential) and leads to the first-order constitutive relations ($\boldsymbol{\beta \equiv 1/T}$)

$$\pi^{\mu\nu} = -2\eta \, \sigma^{\mu\nu} \quad , \quad \boldsymbol{j_f^\mu} = \sum_{f'} \kappa_{f f'} \big(\underbrace{-\Delta^\mu \alpha_{f'}}_{\text{diffusion}} + \underbrace{\beta \, Q_{f'} \, |e| \, e^\mu}_{\text{conduction}} \big) \equiv \boldsymbol{\sum_{f'} \kappa_{f f'} \, \widetilde{e}^\mu_{f'}}$$

Generalized Ohm's law for multiple conserved charges

Requirement of **non-negative entropy production rate**

$$\boldsymbol{\nabla_\mu \mathcal{S}^\mu \geq 0} \quad , \quad \mathcal{S}^\mu \equiv s u^\mu - \sum_f \alpha_f \, j_f^\mu$$

satisfied if $\boldsymbol{\alpha_f = \mu_f / T}$ (dimensionless chemical potential) and leads to the first-order constitutive relations ($\boldsymbol{\beta \equiv 1/T}$)

$$\pi^{\mu\nu} = -2\eta \, \sigma^{\mu\nu} \quad , \quad \boldsymbol{j_f^\mu} = \sum_{f'} \kappa_{ff'} \Big(\underbrace{-\Delta^\mu \alpha_{f'}}_{\text{diffusion}} + \underbrace{\beta \, Q_{f'} \, |e| \, e^\mu}_{\text{conduction}} \Big) \equiv \boldsymbol{\sum_{f'} \kappa_{ff'} \, \widetilde{e}^\mu_{f'}}$$

such that

$$\boldsymbol{T \nabla_\mu \mathcal{S}^\mu = \frac{\pi^{\mu\nu}\pi_{\mu\nu}}{2\eta} + T \sum_{f,f'} j_f^\mu \left(\kappa^{-1}\right)_{ff'} (j_{f'})_\mu \geq 0}$$

- $\kappa_{ff'}$: flavor diffusion matrix (**possible off-diagonal response**)
- η: interpreted as the shear viscosity

$\Rightarrow$ here we focus on the **weakly-magnetized regime** (more details below)

The Wiedemann-Franz law for a multi-component plasma

In the **macroscopic-charge basis** the diffusion current reads

$$j_q^\mu = -\sum_{q'} \underbrace{\sum_{f,f'} \mathcal{M}_{qf}\,\kappa_{ff'}\,(\mathcal{M}^{\mathrm{T}})_{f'q'}}_{\equiv\,\kappa_{qq'}} \Delta^\mu \alpha_{q'} + \underbrace{\beta \sum_{f,f'} \mathcal{M}_{qf}\,\kappa_{ff'}\,Q_{f'}\,|e|}_{\equiv\,\sigma_q}\, e^\mu$$

Leading to the **generalized Wiedemann-Franz law** ($q = \mathcal{B}$, Q, S)

$$\boldsymbol{\sigma_q = \beta \sum_{f,f'} \mathcal{M}_{qf}\,\kappa_{ff'}\,Q_{f'}\,|e|} \quad : \text{charge conductivity}$$

Limit of ideal electric conductor:

- $\boldsymbol{\sigma_Q} = \beta \sum_{f,f'} |e|^2\, Q_f\,\kappa_{ff'}\,Q_{f'} \to \infty$ and $\boldsymbol{e^\mu = 0}$
- infinite quark diffusion and $\boldsymbol{\Delta_\mu \alpha_f = 0}$

The Wiedemann-Franz law for a multi-component plasma

In the **macroscopic-charge basis** the diffusion current reads

$$j_q^\mu = -\sum_{q'} \underbrace{\sum_{f,f'} \mathcal{M}_{qf}\, \kappa_{ff'}\, (\mathcal{M}^{\mathrm{T}})_{f'q'}}_{\equiv\, \kappa_{qq'}} \Delta^\mu \alpha_{q'} + \underbrace{\beta \sum_{f,f'} \mathcal{M}_{qf}\, \kappa_{ff'}\, Q_{f'}\, |e|}_{\equiv\, \sigma_q}\, e^\mu$$

Leading to the **generalized Wiedemann-Franz law** ($q = \mathcal{B}$, Q, S)

$$\boldsymbol{\sigma_q = \beta \sum_{f,f'} \mathcal{M}_{qf}\, \kappa_{ff'}\, Q_{f'}\, |e|} \quad : \text{charge conductivity}$$

Limit of ideal electric conductor:

- $\boldsymbol{\sigma_Q} = \beta \sum_{f,f'} |e|^2 Q_f\, \kappa_{ff'}\, Q_{f'} \to \boldsymbol{\infty}$ and $\boldsymbol{e^\mu = 0}$
- infinite quark diffusion and $\boldsymbol{\Delta_\mu \alpha_f = 0}$

Kinetic theory in RTA (weakly-magnetized) $\Longrightarrow$ calculate transport coefficients

$$\eta = \boldsymbol{\tau_R} \frac{\varepsilon + P}{5} \quad , \quad \sigma_Q \sim \boldsymbol{\tau_R}\, |e|^2\, T^2$$

A **viscous ideally-conducting plasma** "**cannot be realized in a fully consistent manner** in a system whose microscopic dynamics is described by the Boltzmann-Vlasov equation" (**G. S. Denicol et al., Phys.Rev.D 99 (2019) 5, 056017**)

(G)RMHD limit of the Boltzmann-Vlasov equation

RTA-kinetic equation for the (anti-)particle distribution $f_f^\pm$ in the presence of **electromagnetic fields** (Minkowskian spacetime, for simplicity)

$$\left[\boldsymbol{p^\mu \partial_\mu + Q_f^\pm \, |e| \, F^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu}}\right] \boldsymbol{f_f^\pm = \frac{p \cdot u}{\tau_R}(f_f^\pm - f_{0f}^\pm)} \quad , \quad f = u, d, s$$

whose solution can be formally recast into the form of a Neumann series

$$f_f^\pm = \sum_{n=0}^{\infty} \left[\frac{\tau_R}{p \cdot u} \left(p^\mu \partial_\mu + Q_f^\pm \, |e| \, F^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu} \right) \right]^n f_{0f}^\pm \ , \ \underbrace{f_{0f}^\pm \equiv \frac{1}{e^{-[\beta\,(p\cdot u) \pm \alpha_f]} + 1}}_{\text{equilibrium: Fermi-Dirac}}$$

(G)RMHD limit of the Boltzmann-Vlasov equation

RTA-kinetic equation for the (anti-)particle distribution $f_f^{\pm}$ in the presence of **electromagnetic fields** (Minkowskian spacetime, for simplicity)

$$\left[p^\mu \partial_\mu + Q_f^{\pm} \, |e| \, F^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu} \right] f_f^{\pm} = \frac{p \cdot u}{\tau_R} (f_f^{\pm} - f_{0f}^{\pm}) \quad , \quad f = u, d, s$$

whose solution can be formally recast into the form of a Neumann series

$$f_f^{\pm} = \sum_{n=0}^{\infty} \left[\frac{\tau_R}{p \cdot u} \left(p^\mu \partial_\mu + Q_f^{\pm} \, |e| \, F^{\mu\nu} \, p_\nu \frac{\partial}{\partial p^\mu} \right) \right]^n f_{0f}^{\pm} \ , \ \underbrace{f_{0f}^{\pm} \equiv \frac{1}{e^{-[\beta \, (p \cdot u) \pm \alpha_f]} + 1}}_{\text{equilibrium: Fermi-Dirac}}$$

Conditions to fulfill in order for the truncated expansion ($\sim$ **Chapman-Enskog**) to be meaningful and well defined:

- $\mathbf{Kn} \equiv \lambda_{\text{mfp}} / L \sim \tau_R \, \partial \ll 1$: **small space-time gradients**
- $\xi \equiv \tau_R \, \beta \, |e| \, E \ll 1$: **negligible energy gain** between two collisions
- $\chi \equiv \tau_R \, \beta \, |e| \, B \sim \lambda_{\text{mfp}} / r_{\text{Larm}} \ll 1$: **negligible bending** between two subsequent collisions

$\Rightarrow E$, B norms of the electric and magnetic fields

Magnetic expansion parameter and plasma beta-value

Back-of-the envelope estimates for a conformal plasma of classical particles:

$$\tau_R = 5\,(\eta/s)\frac{1}{T}\,, \qquad P = \frac{g_{\rm dof}}{\pi^2}T^4\,, \qquad \beta_V \equiv \frac{P}{B^2/2}$$

Hence, for $g_{\rm dof} \approx 50$ and $\eta/s = 0.2$ one obtains this relation

$$\boldsymbol{\chi^2 = \frac{50\,g_{\rm dof}\,(\eta/s)^2\,4\pi\,\alpha_{\rm em}}{\pi^2}\,\beta_V^{-1} \approx \beta_V^{-1}} \quad \text{with} \quad \alpha_{\rm em} \equiv \frac{|e|^2}{4\pi} \approx \frac{1}{137}$$

Magnetic expansion parameter and plasma beta-value

Back-of-the envelope estimates for a conformal plasma of classical particles:

$$\tau_R = 5\,(\eta/s)\frac{1}{T}\,, \qquad P = \frac{g_{\rm dof}}{\pi^2}T^4\,, \qquad \beta_V \equiv \frac{P}{B^2/2}$$

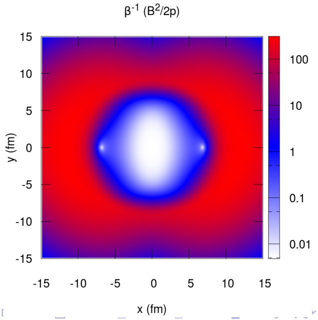
Hence, for $g_{\rm dof} \approx 50$ and $\eta/s = 0.2$ one obtains this relation

$$\boldsymbol{\chi^2 = \frac{50\, g_{\rm dof}\,(\eta/s)^2\, 4\pi\,\alpha_{\rm em}}{\pi^2}\,\beta_V^{-1} \approx \beta_V^{-1}} \quad \text{with} \quad \alpha_{\rm em} \equiv \frac{|e|^2}{4\pi} \approx \frac{1}{137}$$

- **Bulk** of the fireball: **weakly magnetized** ($\chi \ll 1$: perturbative methods)
- **Corona**: **strongly magnetized** ($\chi \gtrsim 1$)

$\Downarrow$

self-consistent **resummation of magnetic effects**



Dissipative current in a weakly magnetized QGP

Quark-flavor currents expressed in terms of off-equilibrium distributions

$$\boldsymbol{j_f^\mu \equiv \sum_{f'} \kappa_{ff'}\, \widetilde{e}^{\mu}_{f'} = g_f\, \Delta^\mu{}_\nu \int d\Pi\, p^\nu \left[\delta f_f^+ - \delta f_f^-\right]}, \quad \text{with} \quad d\Pi \equiv \frac{d^3p}{(2\pi)^3}\frac{1}{\epsilon_p}$$

where the **first-order correction** to the distribution function reads

$$\boldsymbol{\delta f_f^\pm} = \frac{\tau_R}{p\cdot u}\left(p^\mu \partial_\mu + Q_f\,|e|\,F^{\mu\nu}\,p_\nu \frac{\partial}{\partial p^\mu}\right) f_{0f}^\pm = \ldots = \boldsymbol{p^\lambda \sum_{f'} A^\pm_{ff'}\, g_{\lambda\rho}\, \widetilde{e}^{\rho}_{f'}}$$

having introduced the following matrix in flavor space

$$\boldsymbol{A^\pm_{ff'} \equiv \frac{\tau_R}{(-p\cdot u)}\left[f^\pm_{0f}\widetilde{f}^\pm_{0f}\right]\left(\pm\delta_{ff'} - \frac{n_{f'}\,(-p\cdot u)}{\varepsilon + P}\right)} \quad , \text{with} \quad \widetilde{f}^\pm_{0f} \equiv 1 - f^\pm_{0f}$$

Dissipative current in a weakly magnetized QGP

Quark-flavor currents expressed in terms of off-equilibrium distributions

$$j^{\mu}_{f} \equiv \sum_{f'} \kappa_{ff'}\,\widetilde{e}^{\mu}_{f'} = g_f\,\Delta^{\mu}{}_{\nu}\int d\Pi\, p^{\nu}\,\left[\delta f^{+}_{f} - \delta f^{-}_{f}\right], \quad \text{with} \quad d\Pi \equiv \frac{d^3p}{(2\pi)^3}\frac{1}{\epsilon_p}$$

where the **first-order correction** to the distribution function reads

$$\delta f^{\pm}_{f} = \frac{\tau_R}{p\cdot u}\left(p^{\mu}\partial_{\mu} + Q_f\,|e|\,F^{\mu\nu}\,p_{\nu}\frac{\partial}{\partial p^{\mu}}\right) f^{\pm}_{0f} = \ldots = p^{\lambda}\sum_{f'} A^{\pm}_{ff'}\,g_{\lambda\rho}\,\widetilde{e}^{\rho}_{f'}$$

having introduced the following matrix in flavor space

$$A^{\pm}_{ff'} \equiv \frac{\tau_R}{(-p\cdot u)}\left[f^{\pm}_{0f}\widetilde{f}^{\pm}_{0f}\right]\left(\pm\delta_{ff'} - \frac{n_{f'}\,(-p\cdot u)}{\varepsilon + P}\right) \quad , \text{with} \quad \widetilde{f}^{\pm}_{0f} \equiv 1 - f^{\pm}_{0f}$$

Consequently, in the **LRF**, one obtains:

$$\kappa_{ff'} = \frac{g_f}{3}\int d\Pi\,\vec{p}^{\,2}\,(A^{+}_{ff'} - A^{-}_{ff'}) = \tau_R\left[\frac{g_f T^3}{18}\left(1 + \frac{3}{\pi^2}\alpha_f^2\right)\delta_{ff'} - \frac{n_f\,n_{f'}\,T}{\varepsilon + P}\right]$$

$\Rightarrow$ **non-diagonal**, **symmetric** flavor-diffusion matrix ($\propto \tau_R$)

Electric conductivity: numerical estimates for $\alpha_f = 0$

For **zero quark density** the flavor-diffusion matrix is **diagonal** and from the **generalized Wiedemann-Franz law** one gets ($g_f = 6$)

$$\sigma_Q = \sum_{f,f'} |e|^2 \frac{Q_f \, \kappa_{ff'} \, Q_{f'}}{T} = \frac{\tau_R}{T} \frac{T^3}{3} |e|^2 \sum_f Q_f^2 \equiv \tau_R \frac{T^2}{3} C_{\rm em}$$

Electric conductivity: numerical estimates for $\alpha_f = 0$

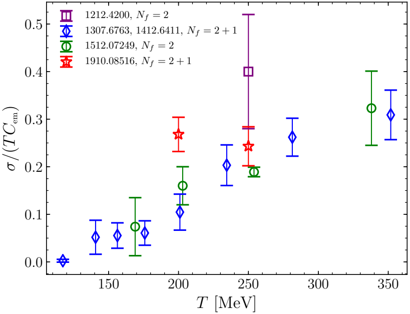
For **zero quark density** the flavor-diffusion matrix is **diagonal** and from the **generalized Wiedemann-Franz law** one gets ($g_f = 6$)

$$\sigma_Q = \sum_{f,f'} |e|^2 \frac{Q_f\, \kappa_{ff'}\, Q_{f'}}{T} = \frac{\tau_R}{T}\frac{T^3}{3}\,|e|^2 \sum_f Q_f^2 \equiv \tau_R \frac{T^2}{3}\, C_{\mathrm{em}}$$

Exploting the conformal relation $\tau_R = 5\,(\eta/s)\frac{1}{T}$ one finds

$$\frac{\sigma_Q}{T\, C_{\mathrm{em}}} = \frac{5}{3}\,(\eta/s)$$

comparable, for $0.1 \lesssim \eta/s \lesssim 0.2$, to **lattice-QCD calculations** (**G. Aarts et al., Eur.Phys.J.A 57 (2021) 4, 118**)



Kinetics of QGP in strong magnetic fields

Boltzmann-Vlasov equation: strongly-magnetized case

To **first order in Kn and ξ**, but for $\chi \gtrsim 1$: implicitly resum all the terms in the BV through a proper ansatz for the off-equilibrium fluctuation ($b^{\mu\nu} \equiv \epsilon^{\mu\nu\lambda\rho}\, b_\lambda u_\rho$)

$$\frac{\tau_R}{p\cdot u}\left[p^\mu \partial_\mu + Q_f^\pm\,|e|\,(e^\nu p_\nu)\, u^\mu\, \frac{\partial}{\partial p^\mu}\right] f_{0f}^\pm + \underbrace{\boldsymbol{\frac{\tau_R}{p\cdot u}\, Q_f^\pm\, |e|\, b^{\mu\nu}\, p_\nu\, \frac{\partial}{\partial p^\mu} \delta f_f^\pm}}_{\textbf{(I): first order if } B\,\sim\,\mathcal{O}(1)} = \delta f_f^\pm$$

Boltzmann-Vlasov equation: strongly-magnetized case

To **first order in Kn and $\boldsymbol{\xi}$**, but for $\chi \gtrsim 1$: implicitly resum all the terms in the BV through a proper ansatz for the off-equilibrium fluctuation ($b^{\mu\nu} \equiv \epsilon^{\mu\nu\lambda\rho}\, b_\lambda u_\rho$)

$$\frac{\tau_R}{p\cdot u}\left[p^\mu \partial_\mu + Q_f^\pm\,|e|\,(e^\nu p_\nu)\,u^\mu\,\frac{\partial}{\partial p^\mu}\right] f_{0f}^\pm + \underbrace{\frac{\tau_R}{p\cdot u}\,Q_f^\pm\,|e|\,b^{\mu\nu}\,p_\nu\,\frac{\partial}{\partial p^\mu}\delta f_f^\pm}_{\text{(I): first order if } B\sim\mathcal{O}(1)} = \delta f_f^\pm$$

To all orders in χ the first term receives a contribution from the magnetic part in the Euler equation for the fluid acceleration (**LRF**: $u^i = e^0 = b^0 = 0$)

$$\frac{\tau_R}{p\cdot u}\left[p^\mu \partial_\mu + Q_f^\pm\,|e|\,(e^\nu p_\nu)\,u^\mu\,\frac{\partial}{\partial p^\mu}\right] f_{0f}^\pm = \underbrace{p^i \sum_{f'} A_{ff'}^\pm\,\delta^{ij}\,\tilde{e}_{f'}^j}_{\text{weak-field result}} +$$

$$\underbrace{+\frac{\tau_R}{p\cdot u}\left[-f_{0f}^\pm \tilde{f}_{0f}^\pm\right]\frac{\beta\,(p\cdot u)}{\varepsilon + P}\,p^i \sum_{f''} Q_{f''}\,|e|\,B\,\epsilon^{ilk} j_{f''}^l\,\hat{b}^k}_{\text{(II): first order if } B\sim\mathcal{O}(1)}$$

where we decomposed the **magnetic field** into its norm and direction $b^i \equiv B\,\hat{b}^i$

The ansatz for the dissipative corrections to distributions

We assume the **off-equilibrium fluctuations** to be of the form (**A. Harutyunyan et al., Phys.Rev.C 94 (2016) 2, 025805**)

$$\delta f_f^{\pm}\Big|_{\mathrm{LRF}} \equiv p^i \sum_{f'} \Big[\underbrace{E^{\pm}_{ff'}\,\Xi^{ij}}_{\text{perpendicular}} + \underbrace{L^{\pm}_{ff'}\,\hat{b}^i\hat{b}^j}_{\text{parallel}} + \underbrace{H^{\pm}_{ff'}\,\epsilon^{ijk}\hat{b}^k}_{\text{Hall}} \Big] \tilde{e}^{j}_{f'}$$

and the flavor-diffusion matrix gains a <u>tensor structure</u> ($\Xi^{ij} \equiv \delta^{ij} - \hat{b}^i\hat{b}^j$)

$$\kappa^{ij}_{ff'} = \kappa^{\perp}_{ff'}\,\Xi^{ij} + \kappa^{||}_{ff'}\,\hat{b}^i\hat{b}^j + \kappa^{\times}_{ff'}\,\epsilon^{ijk}\hat{b}^k \quad \text{such that} \quad j^i_f \equiv \sum_{f'} \kappa^{ij}_{ff'}\,\tilde{e}^j_{f'}$$

The ansatz for the dissipative corrections to distributions

We assume the **off-equilibrium fluctuations** to be of the form (**A. Harutyunyan et al., Phys.Rev.C 94 (2016) 2, 025805**)

$$\delta f_f^{\pm}\Big|_{\rm LRF} \equiv p^i \sum_{f'} \Big[\underbrace{E^{\pm}_{ff'}\, \Xi^{ij}}_{\text{perpendicular}} + \underbrace{L^{\pm}_{ff'}\, \hat{b}^i \hat{b}^j}_{\text{parallel}} + \underbrace{H^{\pm}_{ff'}\, \epsilon^{ijk} \hat{b}^k}_{\text{Hall}} \Big]\, \tilde{e}^j_{f'}$$

and the flavor-diffusion matrix gains a tensor structure ($\Xi^{ij} \equiv \delta^{ij} - \hat{b}^i \hat{b}^j$)

$$\kappa^{ij}_{ff'} = \kappa^{\perp}_{ff'}\, \Xi^{ij} + \kappa^{||}_{ff'}\, \hat{b}^i \hat{b}^j + \kappa^{\times}_{ff'}\, \epsilon^{ijk} \hat{b}^k \quad \text{such that} \quad j^i_f \equiv \sum_{f'} \kappa^{ij}_{ff'}\, \tilde{e}^j_{f'}$$

It is possible to show that $L^{\pm}_{ff'} = A^{\pm}_{ff'}$ and

$$\kappa^{||}_{ff'} = \frac{g_f}{3} \int d\Pi\, \vec{p}^{\,2} \left(L^{+}_{ff'} - L^{-}_{ff'} \right) = \kappa_{ff'} \quad \text{(weak field)}$$

$$\kappa^{\perp}_{ff'} = \frac{g_f}{3} \int d\Pi\, \vec{p}^{\,2} \left(E^{+}_{ff'} - E^{-}_{ff'} \right)$$

$$\kappa^{\times}_{ff'} = \frac{g_f}{3} \int d\Pi\, \vec{p}^{\,2} \left(H^{+}_{ff'} - H^{-}_{ff'} \right)$$

Flavor-diffusion tensor in a strongly magnetized plasma

One eventually obtains a **linear system of coupled equations** to be solved in order to determine the expression of each component of the diffusion tensor

$$\begin{cases} \kappa^{\perp}_{ff'} = \widetilde{\kappa}^{\perp}_{ff'} - \sum_{f''} \mathcal{T}_{ff''}\, \kappa^{\times}_{f''f'} - \sum_{f''} X_{ff''}\, \kappa^{\perp}_{f''f'} \\ \kappa^{\times}_{ff'} = \widetilde{\kappa}^{\times}_{ff'} - \sum_{f''} X_{ff''}\, \kappa^{\times}_{f''f'} + \sum_{f''} \mathcal{T}_{ff''}\, \kappa^{\perp}_{f''f'} \end{cases}$$

Flavor-diffusion tensor in a strongly magnetized plasma

One eventually obtains a **linear system of coupled equations** to be solved in order to determine the expression of each component of the diffusion tensor

$$\begin{cases} \kappa^{\perp}_{ff'} = \widetilde{\kappa}^{\perp}_{ff'} - \sum_{f''} \mathcal{T}_{ff''}\,\kappa^{\times}_{f''f'} - \sum_{f''} X_{ff''}\,\kappa^{\perp}_{f''f'} \\ \kappa^{\times}_{ff'} = \widetilde{\kappa}^{\times}_{ff'} - \sum_{f''} X_{ff''}\,\kappa^{\times}_{f''f'} + \sum_{f''} \mathcal{T}_{ff''}\,\kappa^{\perp}_{f''f'} \end{cases}$$

mixing the contributions from orthogonal and Hall components, via the matrices coming from the term (II) in the BV equation

$$\mathcal{T}_{ff''} \equiv \frac{g_f}{3}\int d\Pi\,\vec{p}^{\,2}\,\frac{\overline{A}^{+}_{ff''} - \overline{A}^{-}_{ff''}}{1+\left(Q_f\,\frac{\tau_R}{\tau_L}\right)^2}\,,\quad X_{ff''} \equiv \frac{g_f}{3}\int d\Pi\,\vec{p}^{\,2}\,\frac{\left(\frac{\tau_R}{\tau_L}\right)\left(Q^{+}_f\,\overline{A}^{+}_{ff''} - Q^{-}_f\,\overline{A}^{-}_{ff''}\right)}{1+\left(Q_f\,\frac{\tau_R}{\tau_L}\right)^2}$$

where $\overline{A}^{\pm}_{ff''} \equiv \frac{\beta\,\tau_R}{\varepsilon+P}\left[-f^{\pm}_{0f}\widetilde{f}_{0f}\pm\right] Q_{f''}\,|e|\,B$ and $\boldsymbol{\tau_L} \equiv \boldsymbol{\epsilon^*_p}/\left(|\boldsymbol{e}|\,\boldsymbol{B}\right)$.

Flavor-diffusion tensor in a strongly magnetized plasma

One eventually obtains a **linear system of coupled equations** to be solved in order to determine the expression of each component of the diffusion tensor

$$\begin{cases} \kappa^{\perp}_{ff'} = \widetilde{\kappa}^{\perp}_{ff'} - \sum_{f''} \mathcal{T}_{ff''}\,\kappa^{\times}_{f''f'} - \sum_{f''} X_{ff''}\,\kappa^{\perp}_{f''f'} \\ \kappa^{\times}_{ff'} = \widetilde{\kappa}^{\times}_{ff'} - \sum_{f''} X_{ff''}\,\kappa^{\times}_{f''f'} + \sum_{f''} \mathcal{T}_{ff''}\,\kappa^{\perp}_{f''f'} \end{cases}$$

mixing the contributions from orthogonal and Hall components, via the matrices coming from the term (II) in the BV equation

$$\mathcal{T}_{ff''} \equiv \frac{g_f}{3}\int d\Pi\,\vec{p}^{\,2}\,\frac{\overline{A}^{+}_{ff''} - \overline{A}^{-}_{ff''}}{1+\left(Q_f\frac{\tau_R}{\tau_L}\right)^2}\,,\quad X_{ff''} \equiv \frac{g_f}{3}\int d\Pi\,\vec{p}^{\,2}\,\frac{\left(\frac{\tau_R}{\tau_L}\right)\left(Q^{+}_f\,\overline{A}^{+}_{ff''} - Q^{-}_f\,\overline{A}^{-}_{ff''}\right)}{1+\left(Q_f\frac{\tau_R}{\tau_L}\right)^2}$$

where $\overline{A}^{\pm}_{ff''} \equiv \frac{\beta\,\tau_R}{\varepsilon+P}\left[-f^{\pm}_{0f}\widetilde{f}_{0f}\pm\right] Q_{f''}\,|e|\,B$ and $\tau_L \equiv \epsilon^{*}_{p}/(|e|\,B)$. Moreover

$$\widetilde{\kappa}^{\perp}_{ff'} = \frac{g_f}{3}\int d\Pi\,\vec{p}^{\,2}\,\frac{A^{+}_{ff'} - A^{-}_{ff'}}{1+\left(Q_f\frac{\tau_R}{\tau_L}\right)^2}\,,\quad \widetilde{\kappa}^{\times}_{ff'} = \frac{g_f}{3}\int d\Pi\,\vec{p}^{\,2}\,\frac{\left(\frac{\tau_R}{\tau_L}\right)\left(Q^{+}_f\,A^{+}_{ff'} - Q^{-}_f\,A^{-}_{ff'}\right)}{1+\left(Q_f\frac{\tau_R}{\tau_L}\right)^2}$$

arise from the term (I) in the BV equation

Flavor-diffusion tensor in a strongly magnetized plasma

One eventually obtains a **linear system of coupled equations** to be solved in order to determine the expression of each component of the diffusion tensor

$$\begin{cases} \kappa^{\perp}_{ff'} = \widetilde{\kappa}^{\perp}_{ff'} - \sum_{f''} \mathcal{T}_{ff''}\, \kappa^{\times}_{f''f'} - \sum_{f''} X_{ff''}\, \kappa^{\perp}_{f''f'} \\ \kappa^{\times}_{ff'} = \widetilde{\kappa}^{\times}_{ff'} - \sum_{f''} X_{ff''}\, \kappa^{\times}_{f''f'} + \sum_{f''} \mathcal{T}_{ff''}\, \kappa^{\perp}_{f''f'} \end{cases}$$

mixing the contributions from orthogonal and Hall components, via the matrices coming from the term (II) in the BV equation

$$\mathcal{T}_{ff''} \equiv \frac{g_f}{3}\int d\Pi\, \vec{p}^{\,2}\, \frac{\overline{A}^{+}_{ff''} - \overline{A}^{-}_{ff''}}{1+\left(Q_f \frac{\tau_R}{\tau_L}\right)^2}\,,\quad X_{ff''} \equiv \frac{g_f}{3}\int d\Pi\, \vec{p}^{\,2}\, \frac{\left(\frac{\tau_R}{\tau_L}\right)\left(Q_f^{+}\, \overline{A}^{+}_{ff''} - Q_f^{-}\, \overline{A}^{-}_{ff''}\right)}{1+\left(Q_f \frac{\tau_R}{\tau_L}\right)^2}$$

where $\overline{A}^{\pm}_{ff''} \equiv \frac{\beta\,\tau_R}{\varepsilon+P}\left[-f^{\pm}_{0f}\widetilde{f}_{0f}\pm\right] Q_{f''}\,|e|\,B$ and $\tau_L \equiv \epsilon^{*}_{p}/(|e|\,B)$. Moreover

$$\widetilde{\kappa}^{\perp}_{ff'} = \frac{g_f}{3}\int d\Pi\, \vec{p}^{\,2}\, \frac{A^{+}_{ff'} - A^{-}_{ff'}}{1+\left(Q_f \frac{\tau_R}{\tau_L}\right)^2}\,,\quad \widetilde{\kappa}^{\times}_{ff'} = \frac{g_f}{3}\int d\Pi\, \vec{p}^{\,2}\, \frac{\left(\frac{\tau_R}{\tau_L}\right)\left(Q_f^{+}\, A^{+}_{ff'} - Q_f^{-}\, A^{-}_{ff'}\right)}{1+\left(Q_f \frac{\tau_R}{\tau_L}\right)^2}$$

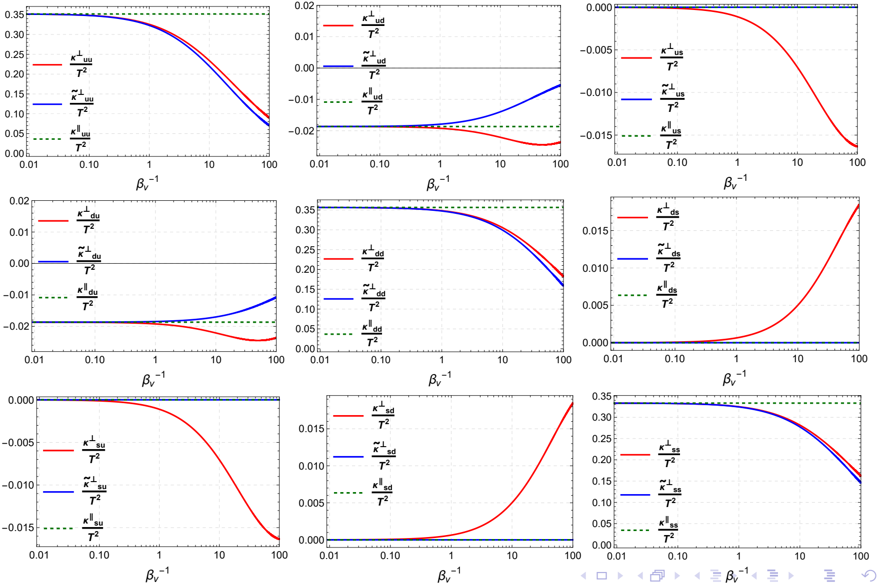
arise from the term (I) in the BV equation

- Conformality: dimensionless components κ/T^2, σ/T **functions of the scaling variable** β_V^{-1} only (no need to specify neither T nor B)

Numerical results

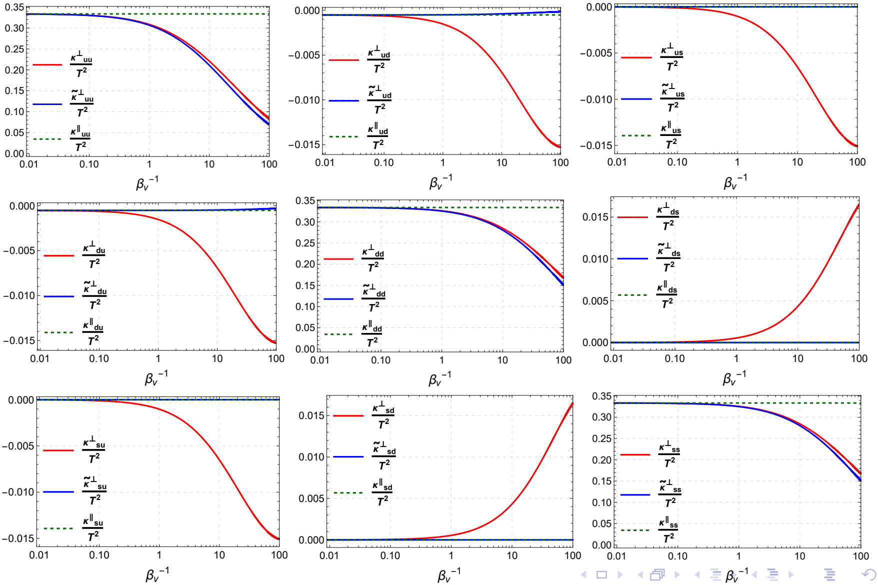
Transverse diffusion in strongly-magnetized QGP (Pt. 1)

- results: $\boldsymbol{s/n_B = 50}$ (<u>intermediate</u> RHIC), $n_Q/n_B = 0.4\,|e|$ and $n_S = 0$



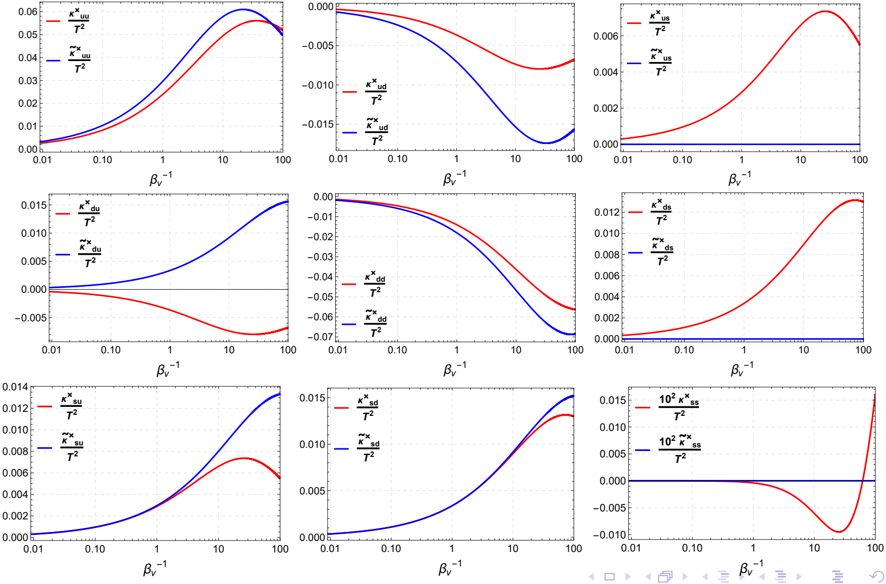
Transverse diffusion in strongly-magnetized QGP (Pt. 2)

- results: $\boldsymbol{s/n_B = 300}$ (top RHIC), $n_Q/n_B = 0.4\,|e|$ and $n_S = 0$



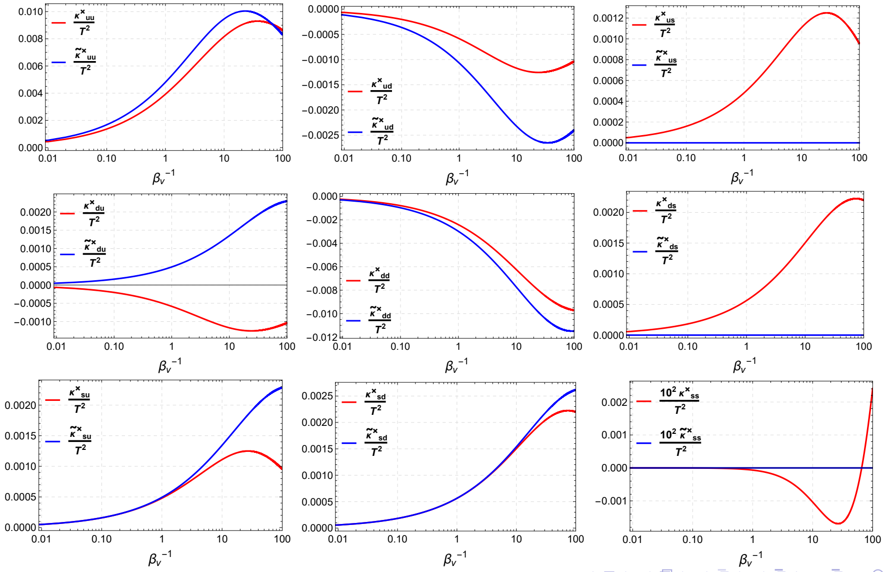
Hall diffusion in strongly-magnetized QGP (Pt. 1)

- results: $\boldsymbol{s/n_B = 50}$ (<u>intermediate</u> RHIC), $n_Q/n_B = 0.4\,|e|$ and $n_S = 0$



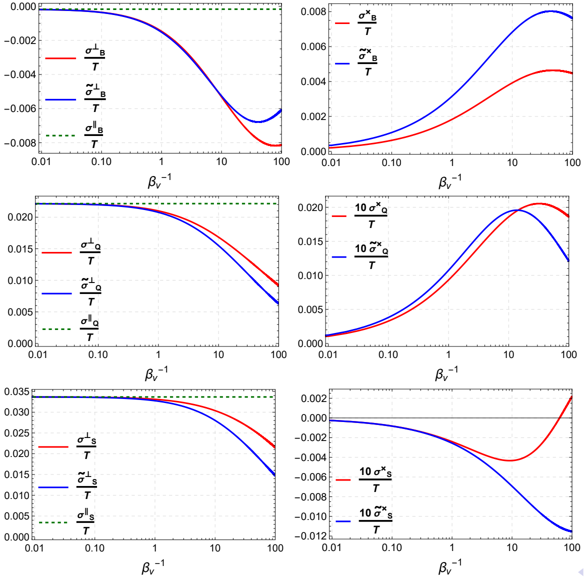
Hall diffusion in strongly-magnetized QGP (Pt. 2)

- results: $s/n_B = 300$ (top RHIC), $n_Q/n_B = 0.4\,|e|$ and $n_S = 0$



Charge conductivity in strongly-magnetized QGP (Pt. 1)

- results: $\boldsymbol{s/n_B = 50}$ (<u>intermediate</u> RHIC), $n_Q/n_B = 0.4\,|e|$ and $n_S = 0$

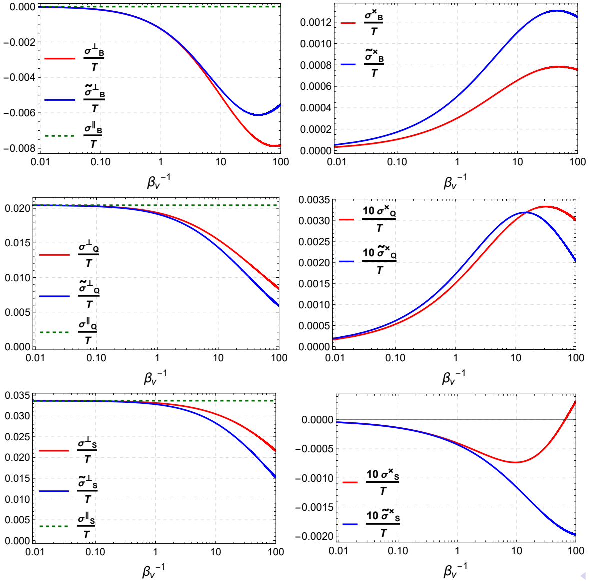


$$\sigma_q^a = \sum_{f,f'} \mathcal{M}_{qf}\, \frac{\kappa^a_{ff'}}{T}\, Q_{f'}|e|$$

$$(a = \parallel, \perp, \times)$$

Charge conductivity in strongly-magnetized QGP (Pt. 2)

- results: $\boldsymbol{s/n_B} = \mathbf{300}$ (top RHIC), $n_Q/n_B = 0.4\,|e|$ and $n_S = 0$



$$\sigma_q^a = \sum_{f,f'} \mathcal{M}_{qf} \frac{\kappa^a_{ff'}}{T} Q_{f'}|e|$$

$$(a = \parallel, \perp, \times)$$

Repercussions on entropy production ($s/n_B = 300$)

Contribution of diffusion processes to the **entropy production rate** (LRF)

$$\partial_\mu \mathcal{S}^\mu = \ldots + \sum_{f,f'} j_f^i \left(\kappa^{-1}\right)^{il}_{ff'} j^l_{f'} \quad \text{with} \quad \left(\kappa^{-1}\right)^{il}_{ff'} \equiv \mathcal{C}^{\parallel}_{ff'} \hat{b}^i \hat{b}^l + \mathcal{C}^{\perp}_{ff'} \Xi^{il} + \mathcal{C}^{\times}_{ff'} \hat{b}^{il}$$

components of the inverse diffusion tensor (implicit sum over repeated flavor indices)

$$\mathcal{C}^{\parallel}_{ff'} = \left(\kappa^{\parallel}\right)^{-1}_{ff'}, \; \mathcal{C}^{\perp}_{ff'} = \left(\kappa^{\perp}\right)^{-1}_{f\bar{f}} \left[1 + \left(\kappa^{\times}\left(\kappa^{\perp}\right)^{-1}\right)^2\right]^{-1}_{\bar{f}f'}, \; \mathcal{C}^{\times}_{ff'} = -\mathcal{C}^{\perp}_{ff''} \kappa^{\times}_{f''\bar{f}} \left(\kappa^{\perp}\right)^{-1}_{\bar{f}f'}$$

Repercussions on entropy production ($s/n_B = 300$)

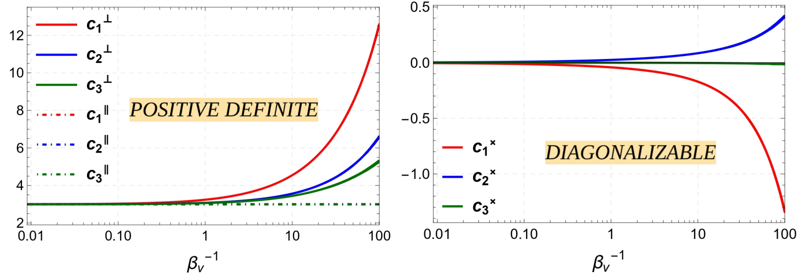
Contribution of diffusion processes to the **entropy production rate** (LRF)

$$\partial_\mu \mathcal{S}^\mu = \ldots + \sum_{f,f'} j_f^i \left(\kappa^{-1}\right)^{il}_{ff'} j_{f'}^l \quad \text{with} \quad \left(\kappa^{-1}\right)^{il}_{ff'} \equiv \mathcal{C}^{\parallel}_{ff'}\,\hat{b}^i\hat{b}^l + \mathcal{C}^{\perp}_{ff'}\,\Xi^{il} + \mathcal{C}^{\times}_{ff'}\,\hat{b}^{il}$$

components of the inverse diffusion tensor (implicit sum over repeated flavor indices)

$$\mathcal{C}^{\parallel}_{ff'} = \left(\kappa^{\parallel}\right)^{-1}_{ff'}, \quad \mathcal{C}^{\perp}_{ff'} = \left(\kappa^{\perp}\right)^{-1}_{f\bar{f}}\left[1 + \left(\kappa^{\times}\left(\kappa^{\perp}\right)^{-1}\right)^2\right]^{-1}_{\bar{f}f'}, \quad \mathcal{C}^{\times}_{ff'} = -\mathcal{C}^{\perp}_{ff''}\,\kappa^{\times}_{f''\bar{f}}\left(\kappa^{\perp}\right)^{-1}_{\bar{f}f'}$$

whose eigenvalues are (**Hall** part provides a **zero-entropy contribution**)



So one finds that our results automatically ensure **positive entropy production**

$$\partial_\mu \mathcal{S}^\mu = \ldots + \Xi^{il}\, j_f^i\, \mathcal{C}^{\perp}_{ff'}\, j_{f'}^l + \hat{b}^i\hat{b}^l\, j_f^i\, \mathcal{C}^{\parallel}_{ff'}\, j_{f'}^l \geq 0 \quad \text{since} \quad \epsilon^{ilk}\,\hat{b}_k\, j_f^i\, \mathcal{C}^{\times}_{ff'}\, j_{f'}^l = 0$$

Conclusions and outlook

Summary and motivations

- In **HIC's** for most of the fireball 4-volume $\kappa_{ff'}$ just a scalar and σ_Q from RTA-BV in agreement with lattice-QCD, but in peripheral regions **B-induced breaking of spatial isotropy** has to be considered at **early times**;
- In **astrophysics** strongly-magnetized plasmas are much more common and hopefully the **RTA-BV approach can be a good guidance**;
- Our prescription leads to a **flavor-diffusion tensor compatible with the second law of thermodynamics**, so in principle it could be employed in Maxwell-Cattaneo approaches to **causally** (and more accurately) describe the dynamics of dissipative currents in **strong magnetic fields**

$$\tau_R \, D j_f^i + j_f^i \underset{\mathrm{LRF}}{=} \sum_{f'} \kappa_{ff'}^{il} \, \widetilde{e}_{f'}^{l} \; ;$$

- Next non-trivial step in order to get a causal set of RMHD equations: going to **second order** (in **Kn** and $\boldsymbol{\xi}$) within the **generalized Chapman-Enskog expansion**, including a **non-perturbative resummation** of magnetic effects.

This presentation is based on the pre-print article:

F. Frascà, A. Beraudo and L. Del Zanna, *Electric conductivity and flavor diffusion in a viscous, resistive quark-gluon plasma for weak and strong magnetic fields*, [`hep-ph/2506.10783`**]**

Thanks for your attention!