… A career devoted to the richness of nuclear many-body physics

Torino 3-4 July 2025

Celebrating

Wanda Alberico

at '75

CONSTRAINTS on the NEUTRON STAR MATTER EQUATION-OF-STATE

Wolfram Weise
Technische Universität München
PHYSIK DEPARTMENT

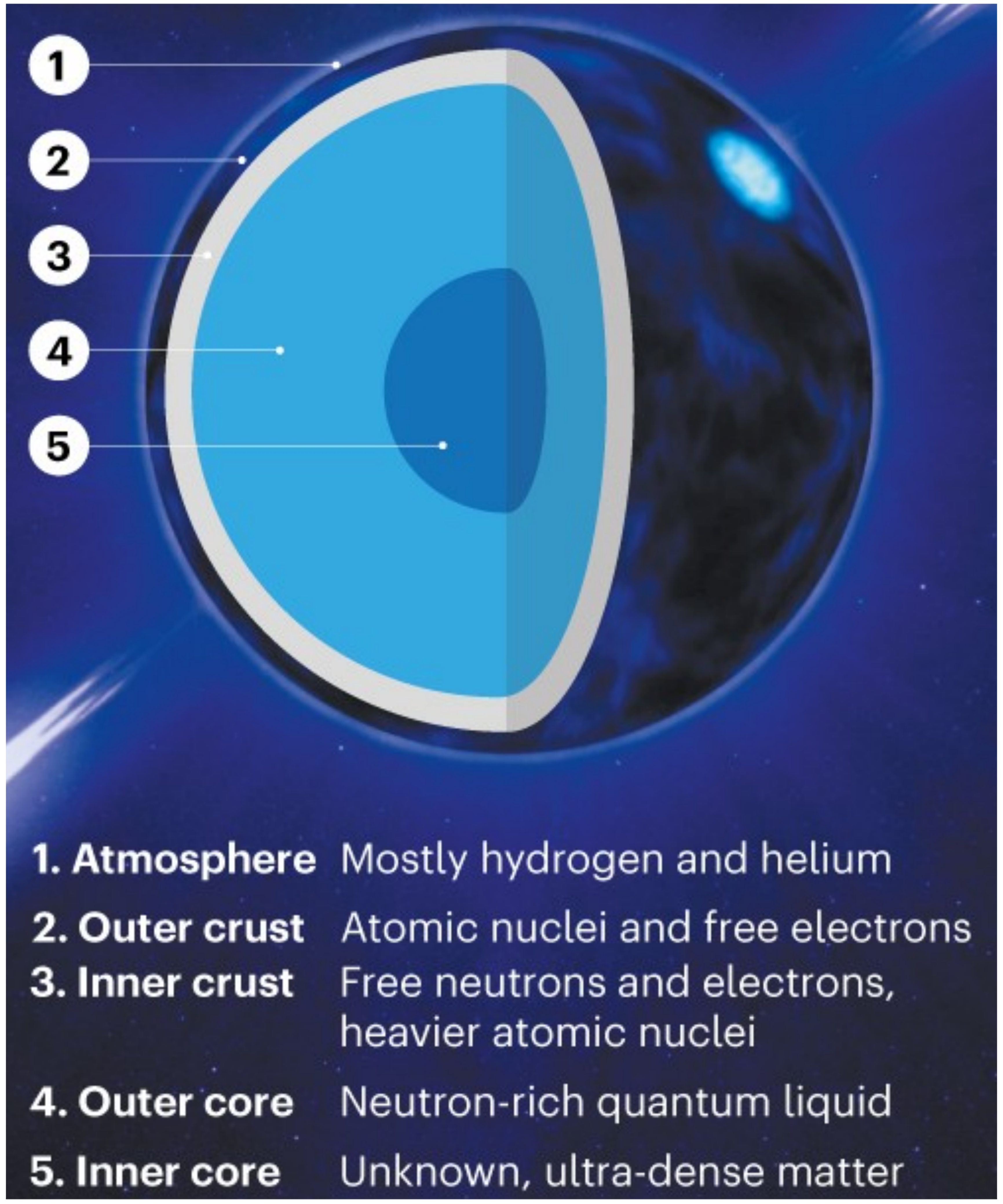
- **Dense Matter in Neutron Stars: Speed of Sound and Equation of State**
  - Empirical constraints from heavy neutron stars and binary mergers
  - Bayesian inference results and constraints on phase transitions
- **Phenomenology and Models for Dense Baryonic Matter**
  - Neutron star core matter as a (relativistic) Fermi liquid
  - Low-energy nucleon structure and hadron-quark continuity

Part One

✷ Constraints on the **Equation-of-State** of **COLD** and **DENSE BARYONIC MATTER**

- **Neutron star mass** measurements (Shapiro delay, Radioastronomy)
- **Neutron star radius** measurements (Neutron star Interior Composition ExploreR : **NICER telescope** @ ISS)
- **Gravitational wave** signals of **neutron star mergers** (LIGO and Virgo collaborations)

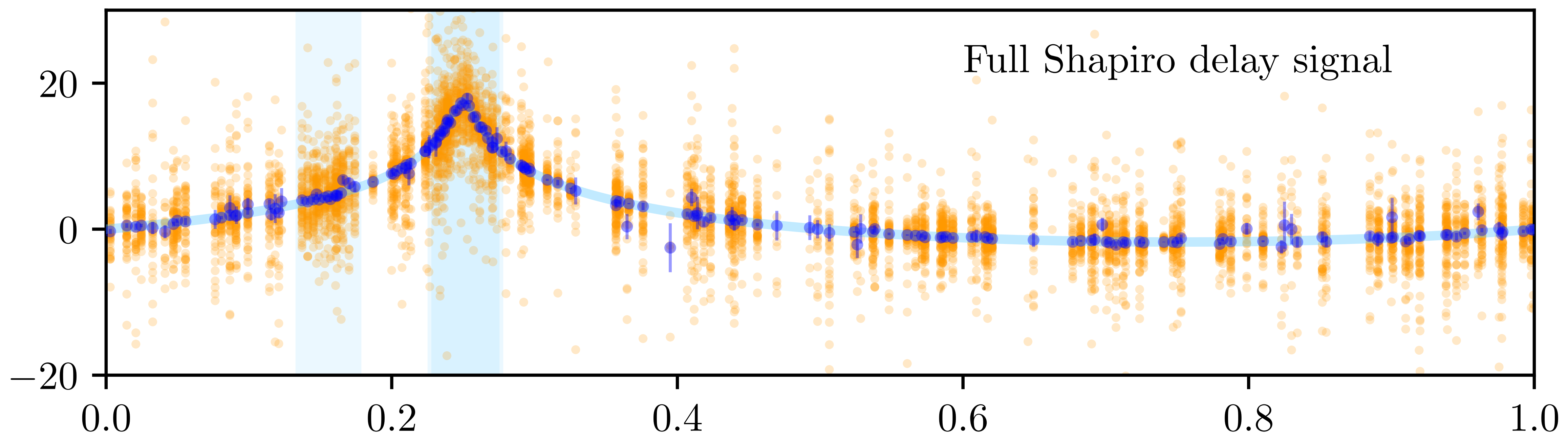
Neutron Star Matter



Layers of a **Neutron Star**

NEUTRON STARS : DATA

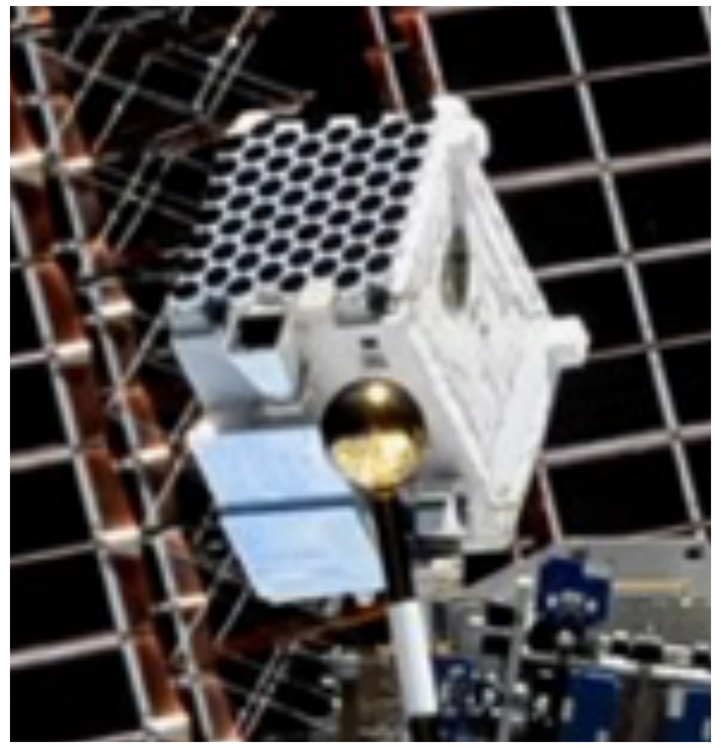
- Database for **inference of Equation-of-State** and other properties of neutron stars



- **Neutron star masses**

  Shapiro delay measurements
  (Green Bank Telescope)

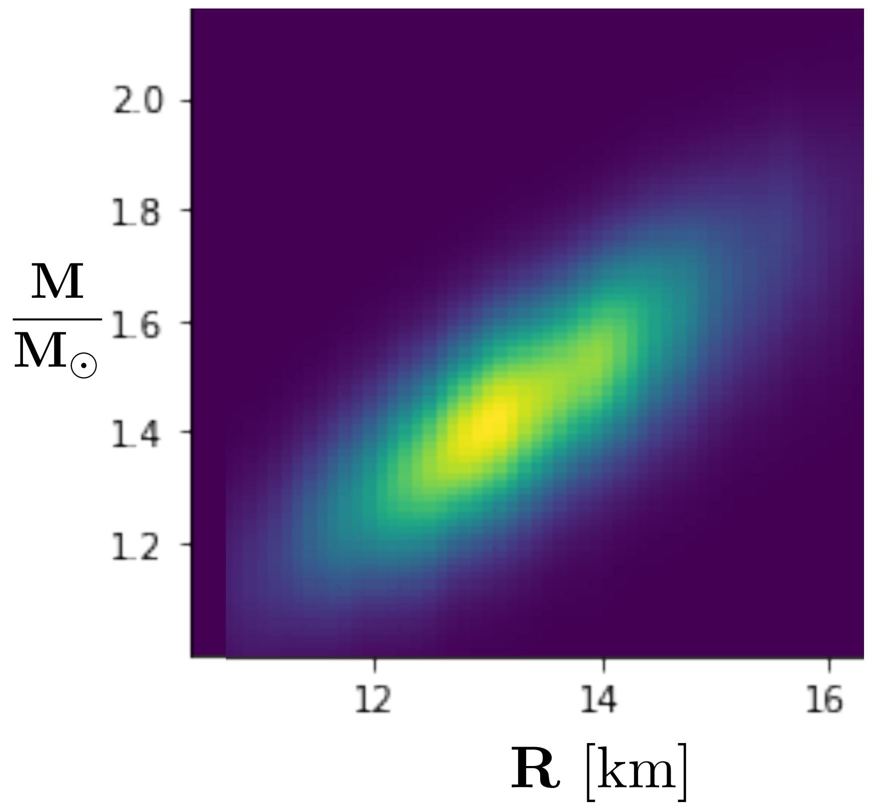
  Radio astronomy

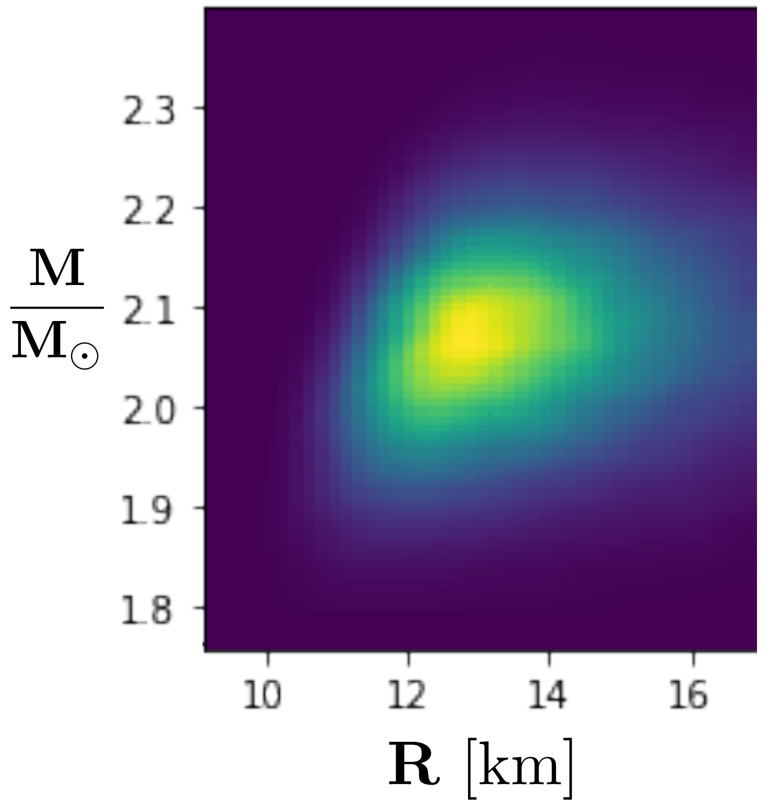


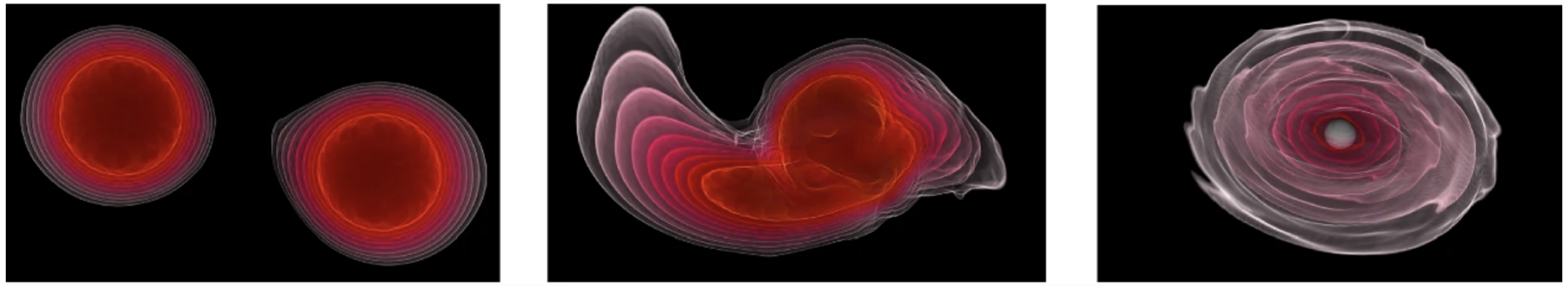
- **Masses and radii**

  X rays from hot spots on the surface of rotating neutron stars

  (NICER Telescope @ ISS)







- **Tidal deformabilities**

  Gravitational wave signals of neutron star mergers

  (LIGO and Virgo Collab.)

NEUTRON STARS : DATA BASE

- **Masses of $2\,M_\odot$ stars**
  (Shapiro delay & radio observations)

PSR J0348+0432

$$M = 2.01 \pm 0.04\ M_\odot$$

J. Antoniadis et al.: Science 340 (2013) 1233232

PSR J1614-2230

$$M = 1.908 \pm 0.016\ M_\odot$$

Z. Arzoumanian et al., Astrophys. J. Suppl. 235 (2018) 37

PSR J0740+6620

$$M = 2.08 \pm 0.07\ M_\odot$$

E. Fonseca et al., Astrophys. J. Lett. 915 (2021) L12

- **Masses and Radii** (NICER, XMM Newton)

PSR J0030+0451

$$M = 1.34 \pm 0.16\,M_\odot \qquad R = 12.71^{+1.14}_{-1.19}\ \mathrm{km}$$

T.E. Riley et al. (NICER), Astroph. J. Lett. 887 (2019) L21

PSR J0030+0451 update

Model	Mass [$M_\odot$]	Radius (km)
ST+PDT	$1.40^{+0.13}_{-0.12}$	$11.71^{+0.88}_{-0.83}$
PDT-U	$1.70^{+0.18}_{-0.19}$	$14.44^{+0.88}_{-1.05}$

… questions remaining

S.Vinciguerra et al.
Astroph. J. 961 (2024) 62

PSR J0740+6620

$$M = 2.073 \pm 0.069\,M_\odot \qquad R = 12.76^{+1.49}_{-1.02}\ \mathrm{km}$$

T.E. Riley et al. (NICER + XMM Newton), Astroph. J. Lett. 918 (2021) L27
T. Salmi et al. (NICER + XMM Newton), Astroph. J. 974 (2024) 294
A.J. Dittmann et al. (NICER + XMM Newton), Astroph. J. 974 (2024) 295

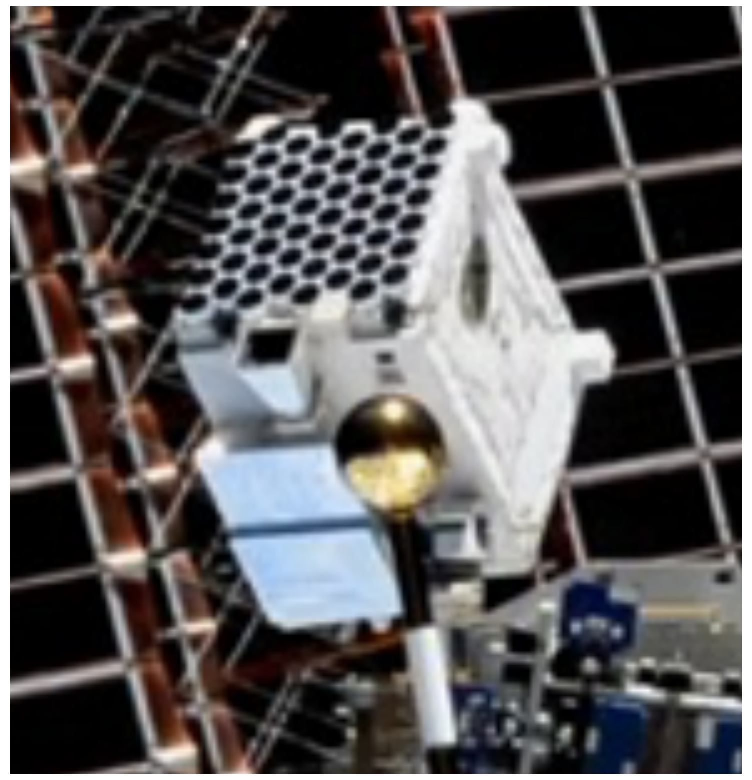
NEUTRON STARS : DATA (contd.)

- **New data from NICER** (analysis ongoing)

PSR J0437-4715

$$\mathrm{M} = 1.418 \pm 0.037\,\mathrm{M}_\odot \qquad \mathrm{R} = 11.36^{+0.95}_{-0.63}\ \mathrm{km}$$

D. Choudhury et al. : Astroph. J. Lett. 971 (2024) L20



- **Very massive** and **fast rotating galactic neutron star**

PSR J0952-0607

$$\mathrm{M} = 2.35 \pm 0.17\ \mathrm{M}_\odot$$

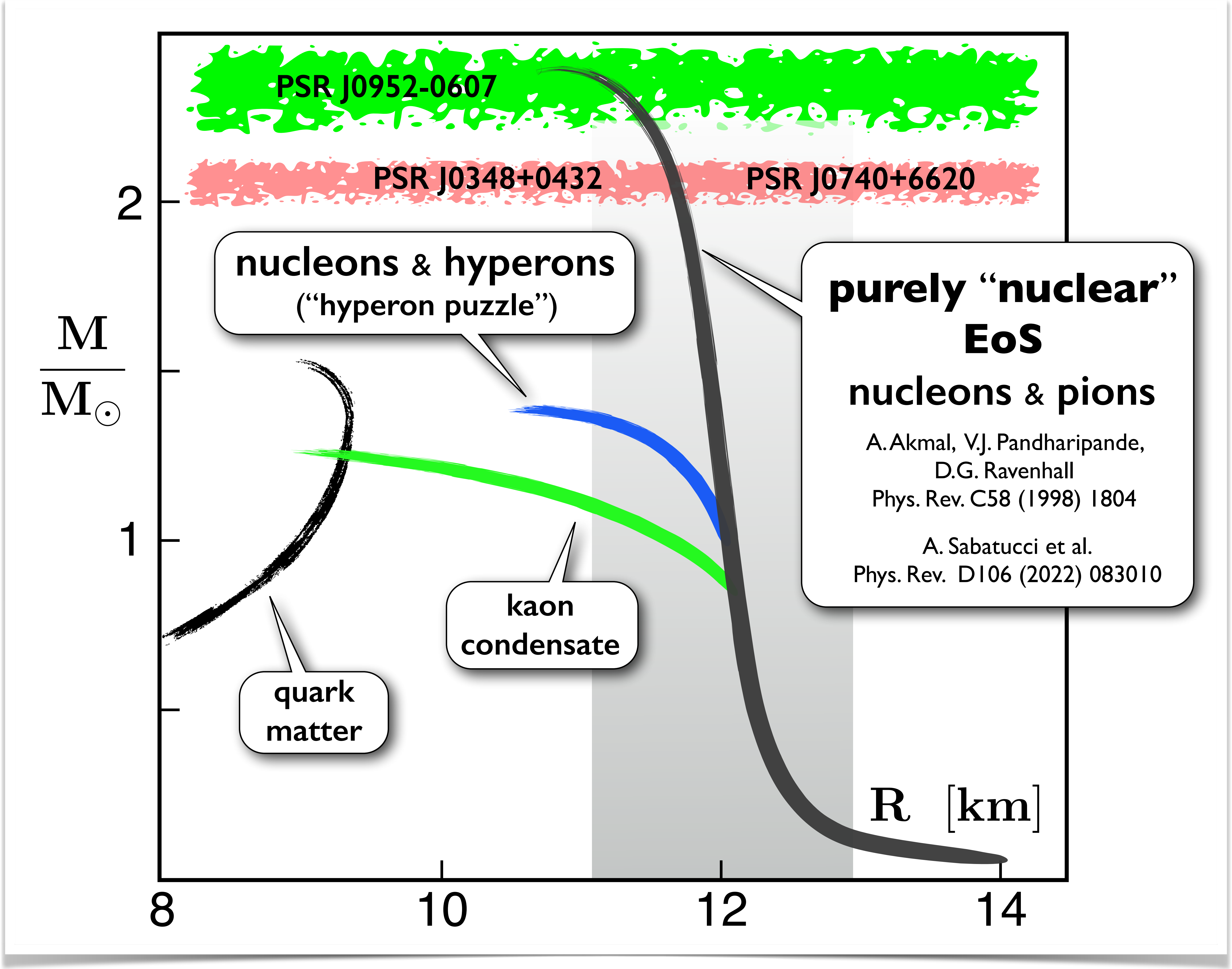
R.W. Romani et al. : Astroph. J. Lett. 934 (2022) L17

→ **equivalent non-rotating mass after rotational correction :** $\mathrm{M} = 2.3 \pm 0.2\ \mathrm{M}_\odot$

(Keck Observatory)

CONSTRAINTS on EQUATION of STATE $P(\varepsilon)$

- from observations of massive neutron stars



Tolman - **O**ppenheimer - **V**olkov Equations

$$\frac{dP(r)}{dr} = \frac{G\,[\varepsilon(r)+P(r)]\,\big[m(r)+4\pi r^3 P(r)\big]}{r\,[r-2\,G\,m(r)]}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

$$M = m(R) = 4\pi \int_0^R dr\, r^2 \varepsilon(r)$$

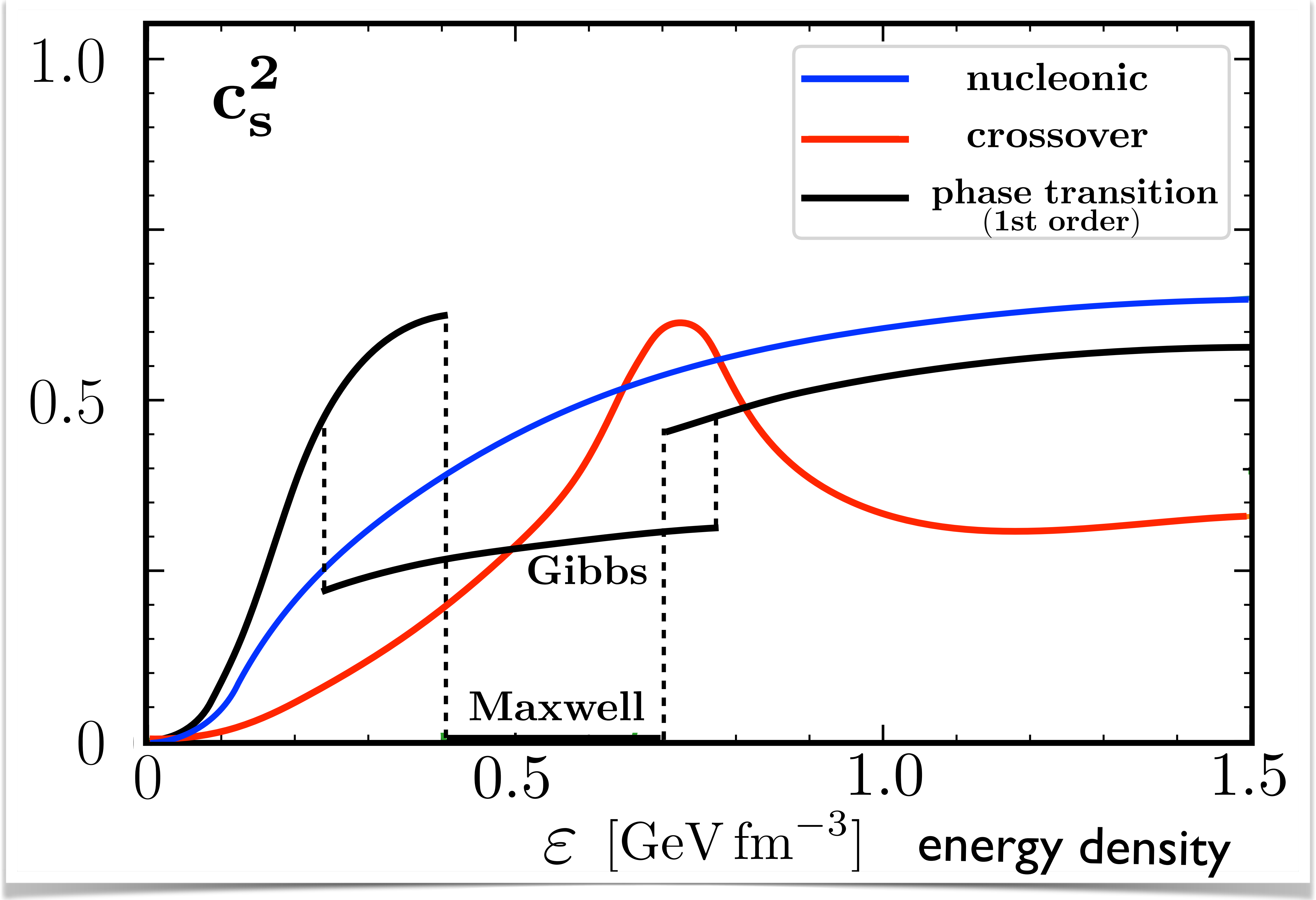
- **Stiff equation of state $P(\varepsilon)$ required**
- **Simplest forms of exotic matter** (kaon condensate, quark matter, ...) **ruled out**

SOUND VELOCITY and EQUATION of STATE

- Key quantity : **Speed of Sound**

$$c_s^2(\varepsilon) = \frac{\partial P(\varepsilon)}{\partial \varepsilon}$$

displays characteristic signature of
phase transition or **crossover**



- Equation of State :

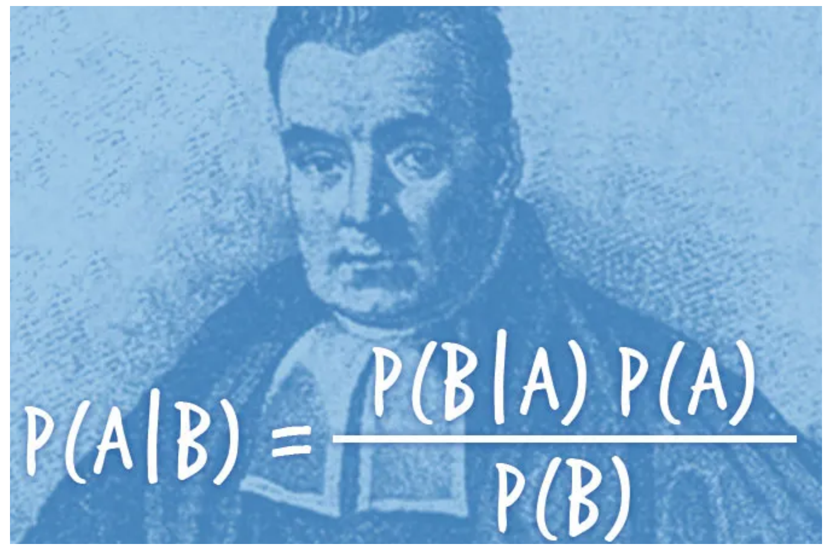
$$P(\varepsilon) = \int_0^{\varepsilon} d\varepsilon' \, c_s^2(\varepsilon')$$

- Gibbs - Duhem equation (T=0)

$$P + \varepsilon = \mu_B \, n_B = \sum_i \mu_i \, n_i$$

- Baryon density $\quad n_B = \partial P / \partial \mu_B$
- Baryon chemical potential

$$\mu_B = \partial \varepsilon / \partial n_B$$



INFERENCE of SOUND SPEED and RELATED PROPERTIES of NEUTRON STARS

- Introduce **general parametrization of sound velocity** : segment-wise representation

$$c_s^2(\varepsilon,\theta) = \frac{(\varepsilon_{i+1}-\varepsilon)c_{s,i}^2 + (\varepsilon-\varepsilon_i)c_{s,i+1}^2}{\varepsilon_{i+1}-\varepsilon_i} \quad , \quad \text{parameter set} \quad \theta = (c_{s,i}^2, \varepsilon_i) \quad (i = 1,\dots,N)$$

- Constrain parameters θ by **Bayesian inference** using **nuclear** and **astrophysical data** $\mathscr{D}$

$$\Pr(\theta|\mathscr{D}) \propto \Pr(\mathscr{D}|\theta)\,\Pr(\theta)$$

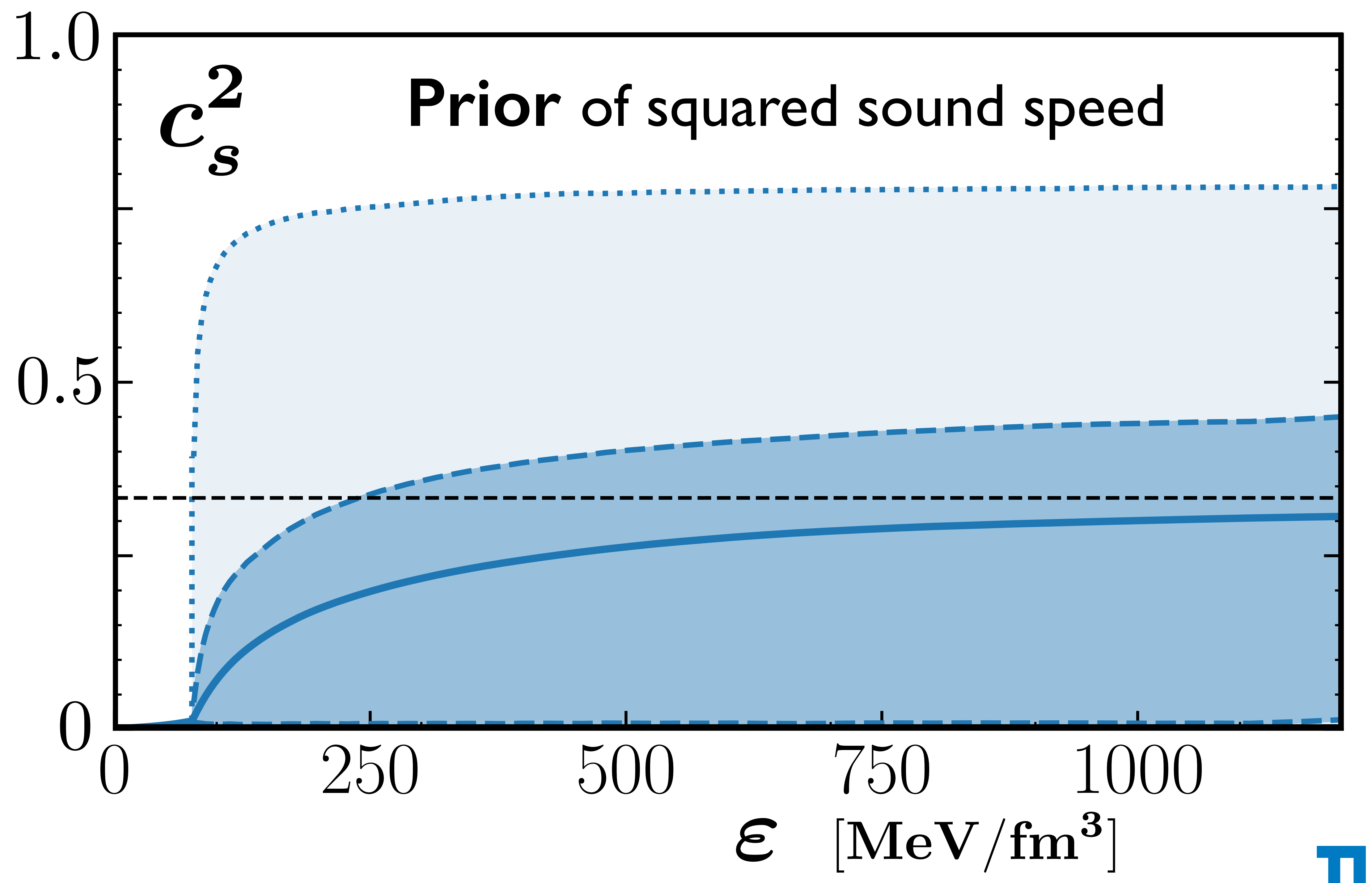
→ Choose **Prior** $\Pr(\theta)$

→ Compute **Posterior** $\Pr(\theta|\mathscr{D})$

from **Likelihood** $\Pr(\mathscr{D}|\theta)$

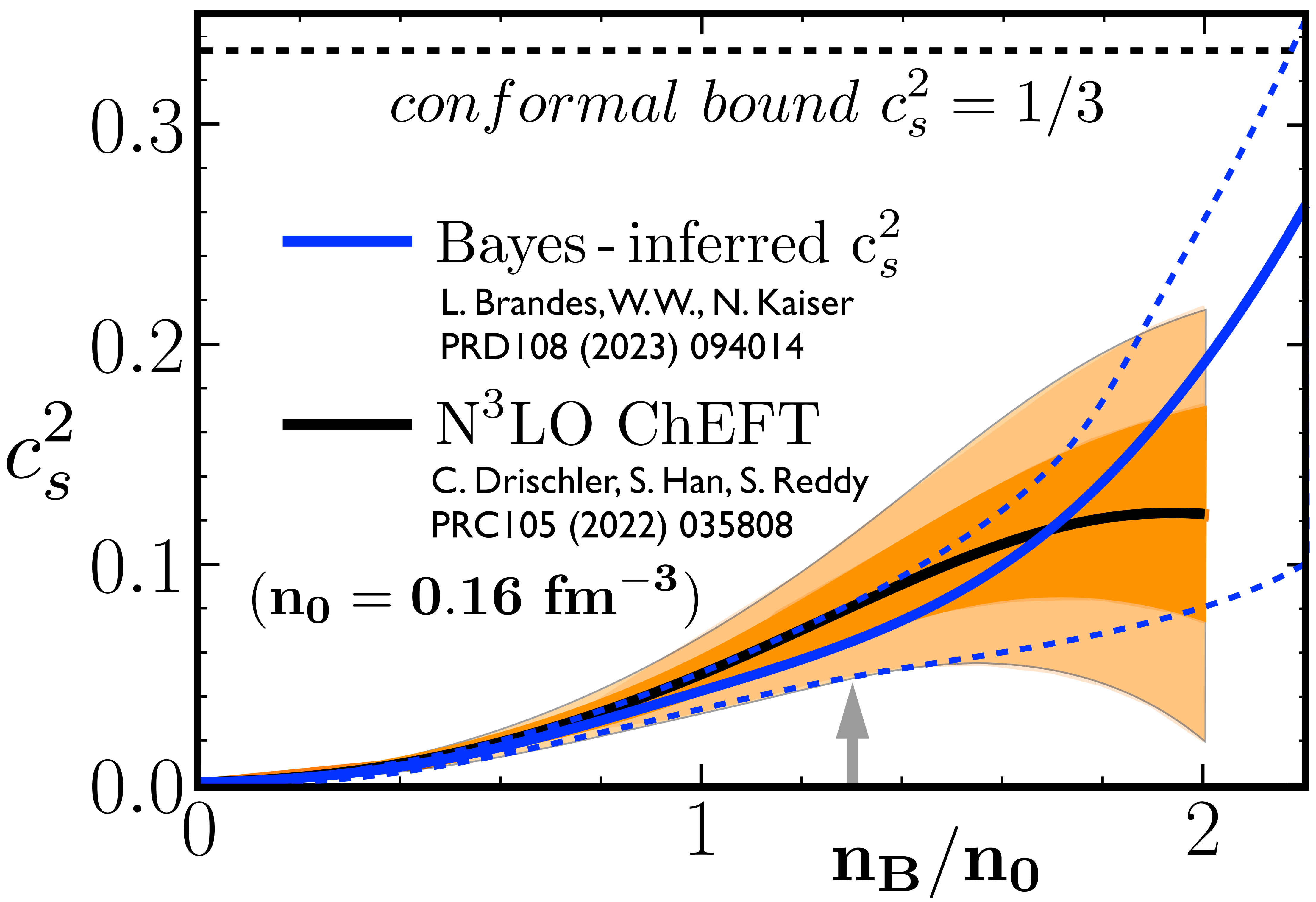
→ Quantify **Evidences** for hypotheses H_0 vs. H_1

in terms of **Bayes factors** $\mathscr{B}^{H_1}_{H_0} = \dfrac{\Pr(\mathscr{D}|H_1)}{\Pr(\mathscr{D}|H_0)}$

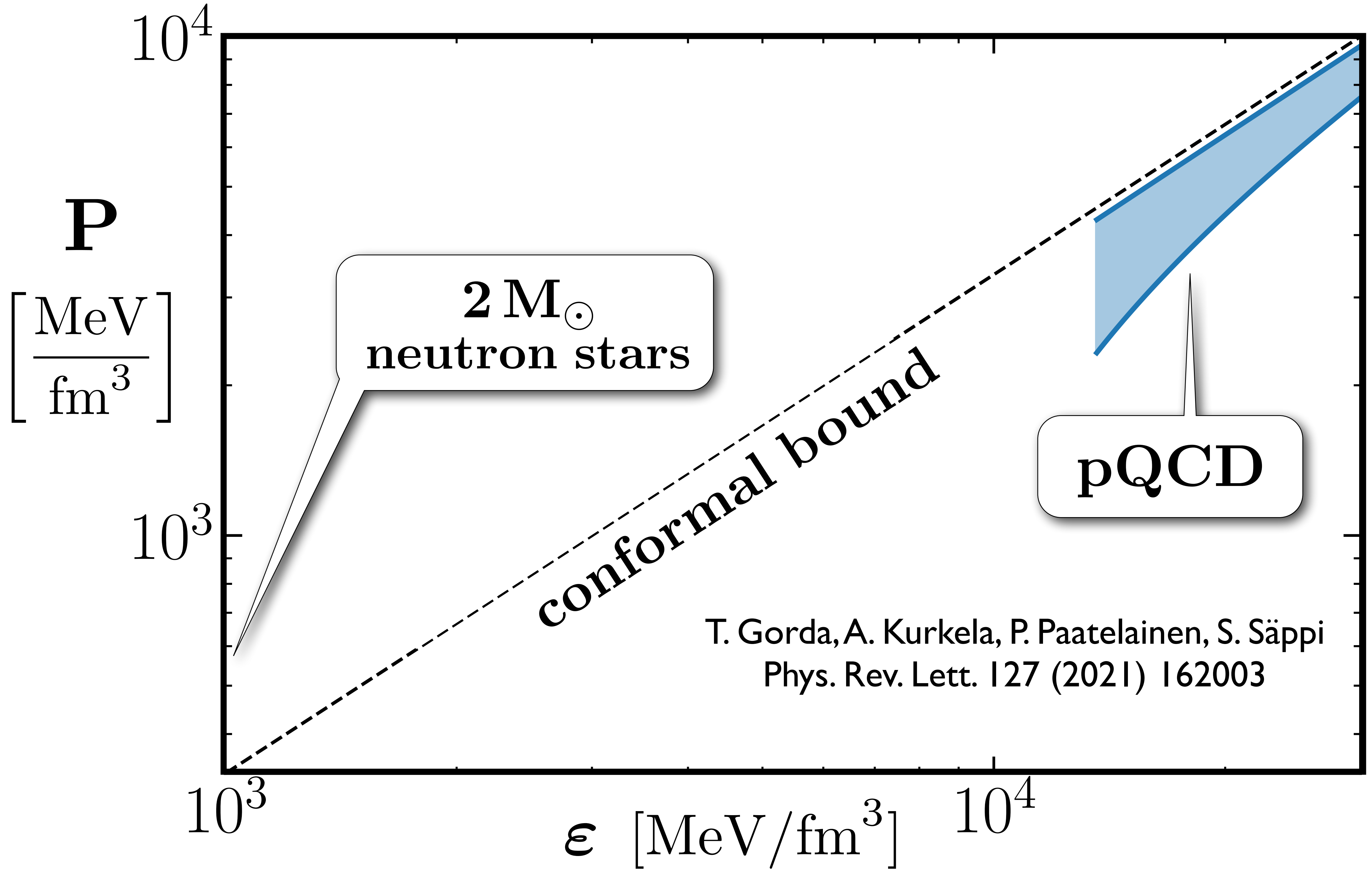


EQUATION of STATE and SOUND VELOCITY
- boundary conditions -

- **Low densities : Chiral EFT @ $n_B \lesssim 2\, n_0$**



- **Extremely high densities : $n_B >> n_{core}(2M_\odot)$**

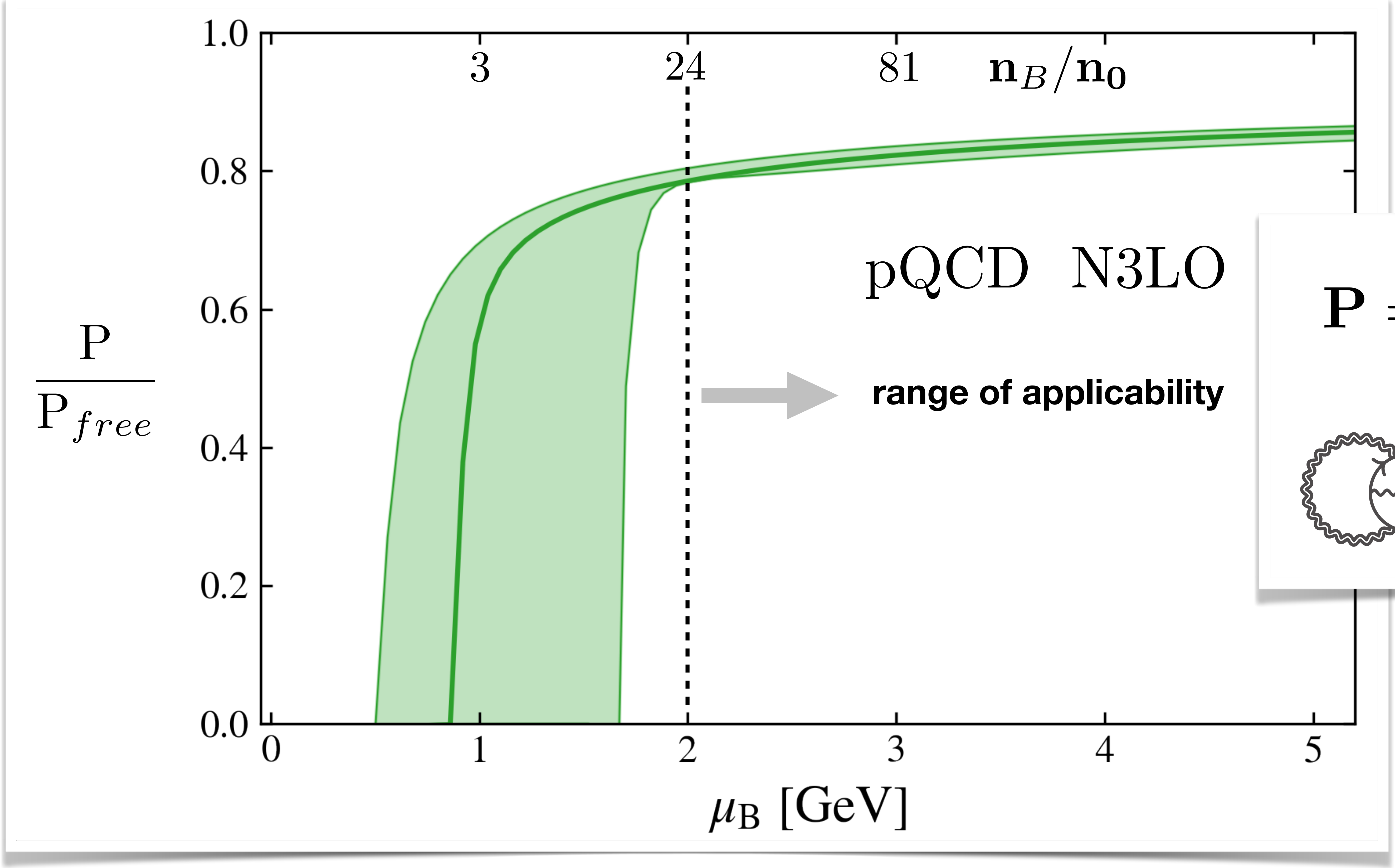


- Employ ChEFT constraint at $n_B = 1.3\, n_0$ in Bayes inference as **Likelihood, NOT Prior**

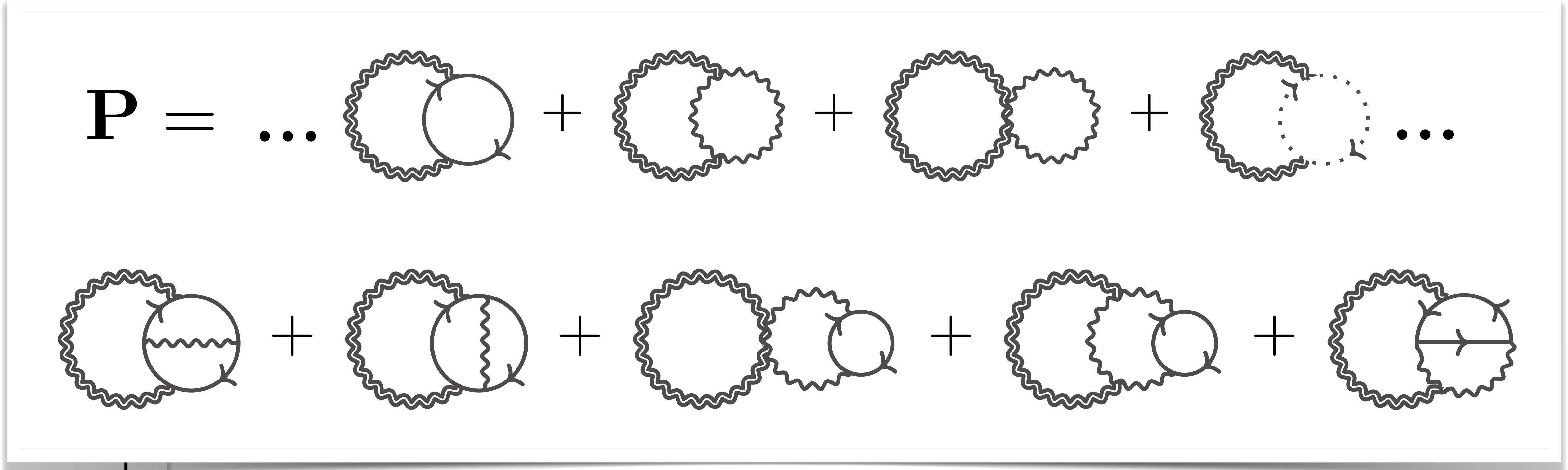
- **Conformal bound** $c_s^2 = \frac{1}{3}$ reached asymptotically

COLD QUARK MATTER in pQCD

- State-of-the-art : next-to-next-to-next-to leading order in α_s ($N_f = 3$ massless quarks)
- Pressure $$\frac{P}{P_{free}} = 1 - \frac{\alpha_s}{\pi} - 3\left(\frac{\alpha_s}{\pi}\right)^2 f\left[\ln\left(\frac{\alpha_s}{\pi}\right), \mu\right] + 9\left(\frac{\alpha_s}{\pi}\right)^3 g\left[\ln\left(\frac{\alpha_s}{\pi}\right), \mu\right]$$



$$P_{free} = \frac{3\,\mu^4}{4\pi^2} \qquad \mu = \frac{\mu_B}{3}$$



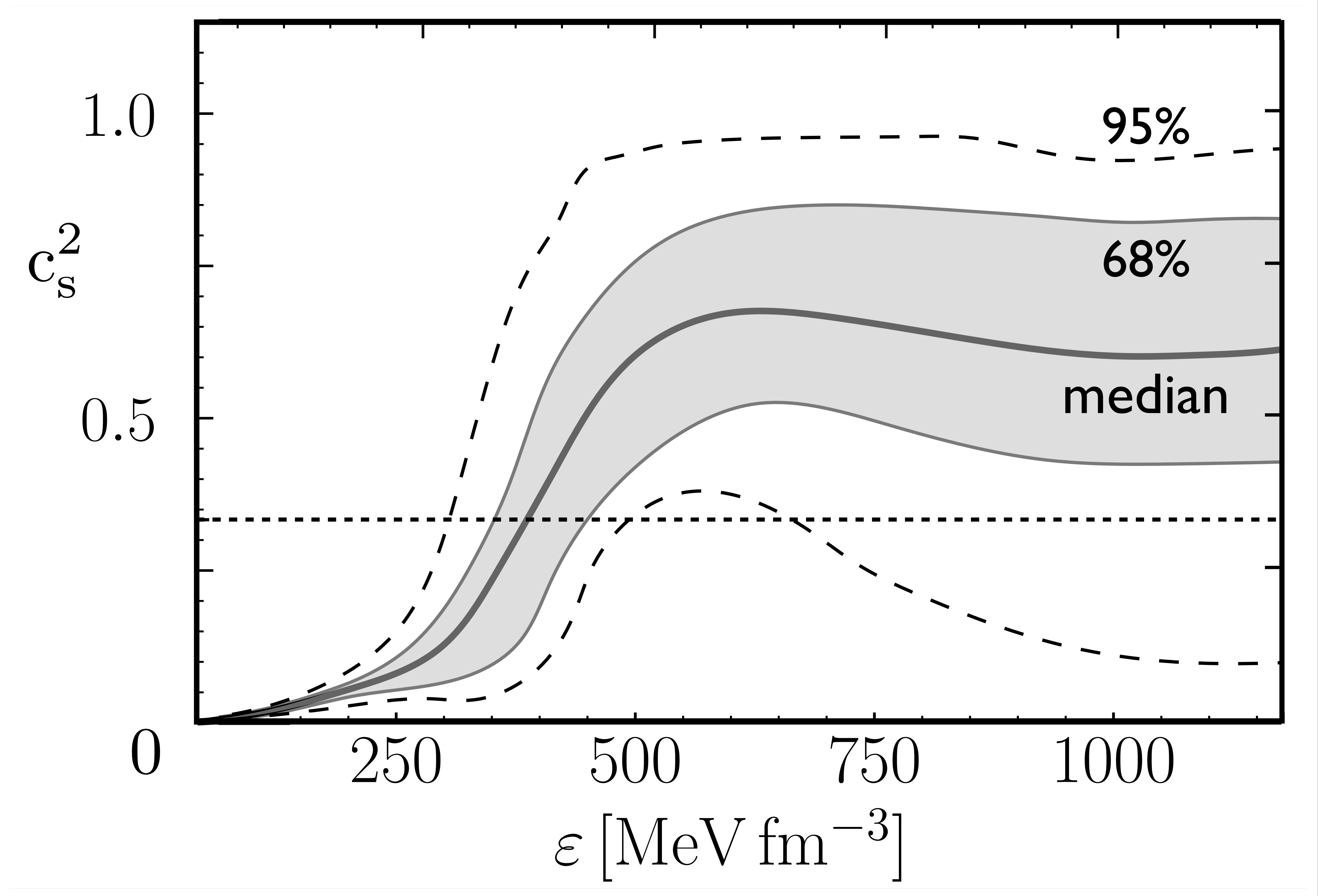
T. Gorda, R. Paatelainen, S. Säppi, K. Seppänen
Phys. Rev. Lett. 131 (2023) 181902

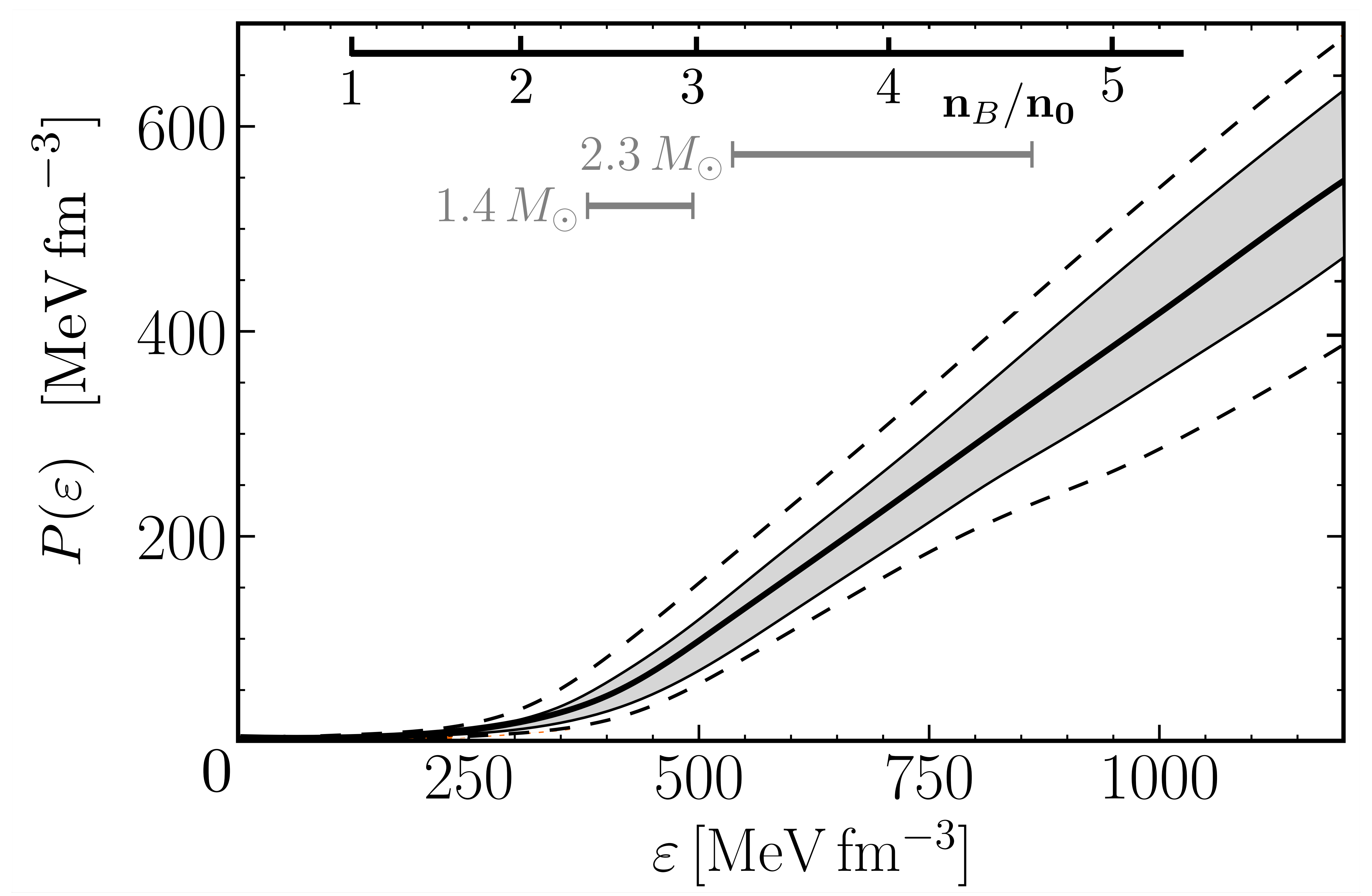
NEUTRON STAR MATTER : EQUATION of STATE

- Bayesian inference of **sound speed** and **EoS**

PSR masses, NICER & GW data, low-density constraints (ChEFT), asymptotic constraints (pQCD)

L. Brandes, W.W., N. Kaiser : Phys. Rev. D 107 (2023) 014011 ; Phys. Rev. D 108 (2023) 094014
L. Brandes, W.W.: Symmetry 16 (2024) 111 ; Phys. Rev. D 111 (2025) 034005



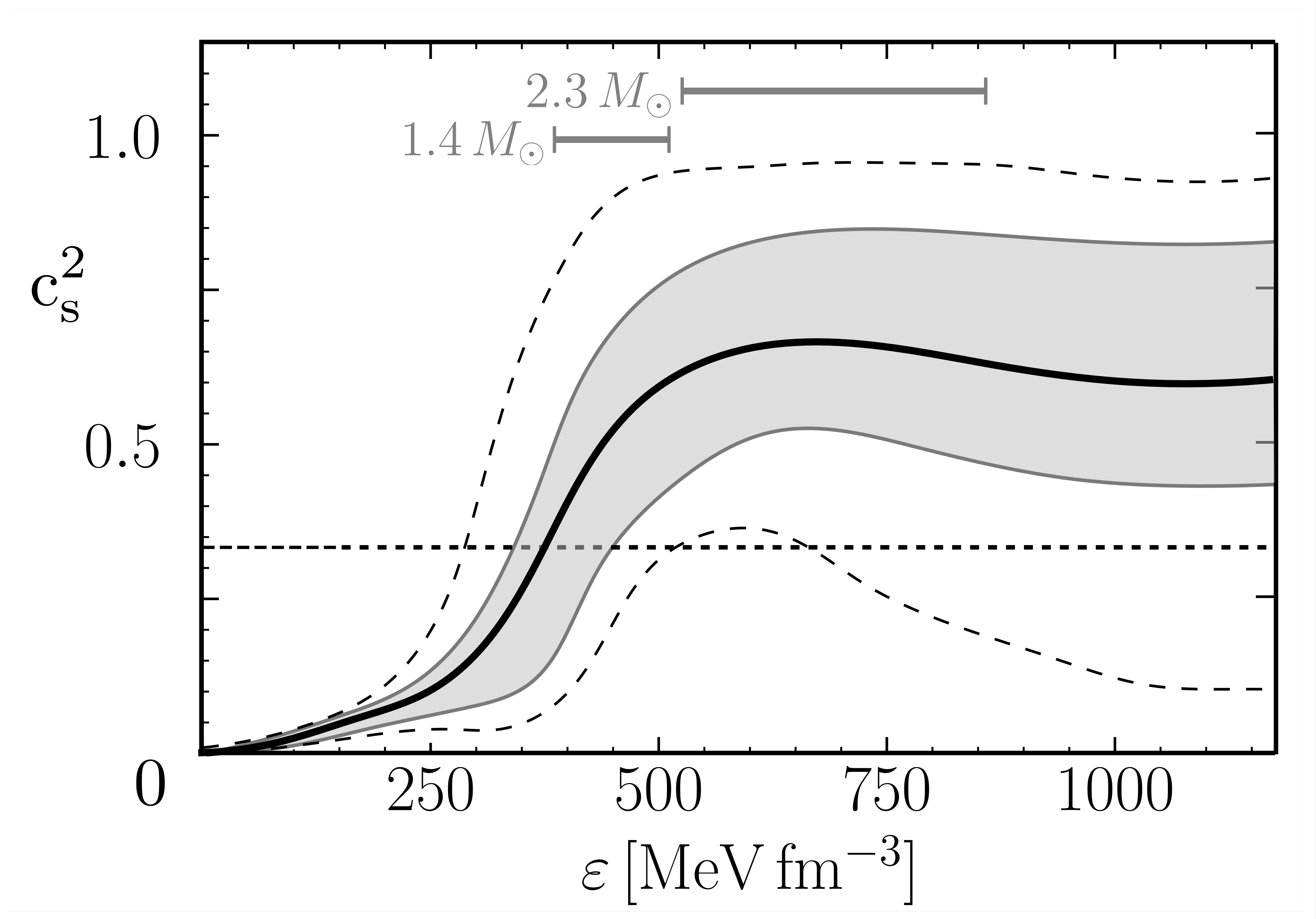


- **Speed of sound** exceeds conformal bound $c_s = 1/\sqrt{3}$ at baryon densities $n_B > 2 - 3\, n_0$
- **Strongly repulsive correlations** in dense baryonic matter

Comment : **SPEED of SOUND** exceeding **CONFORMAL BOUND**

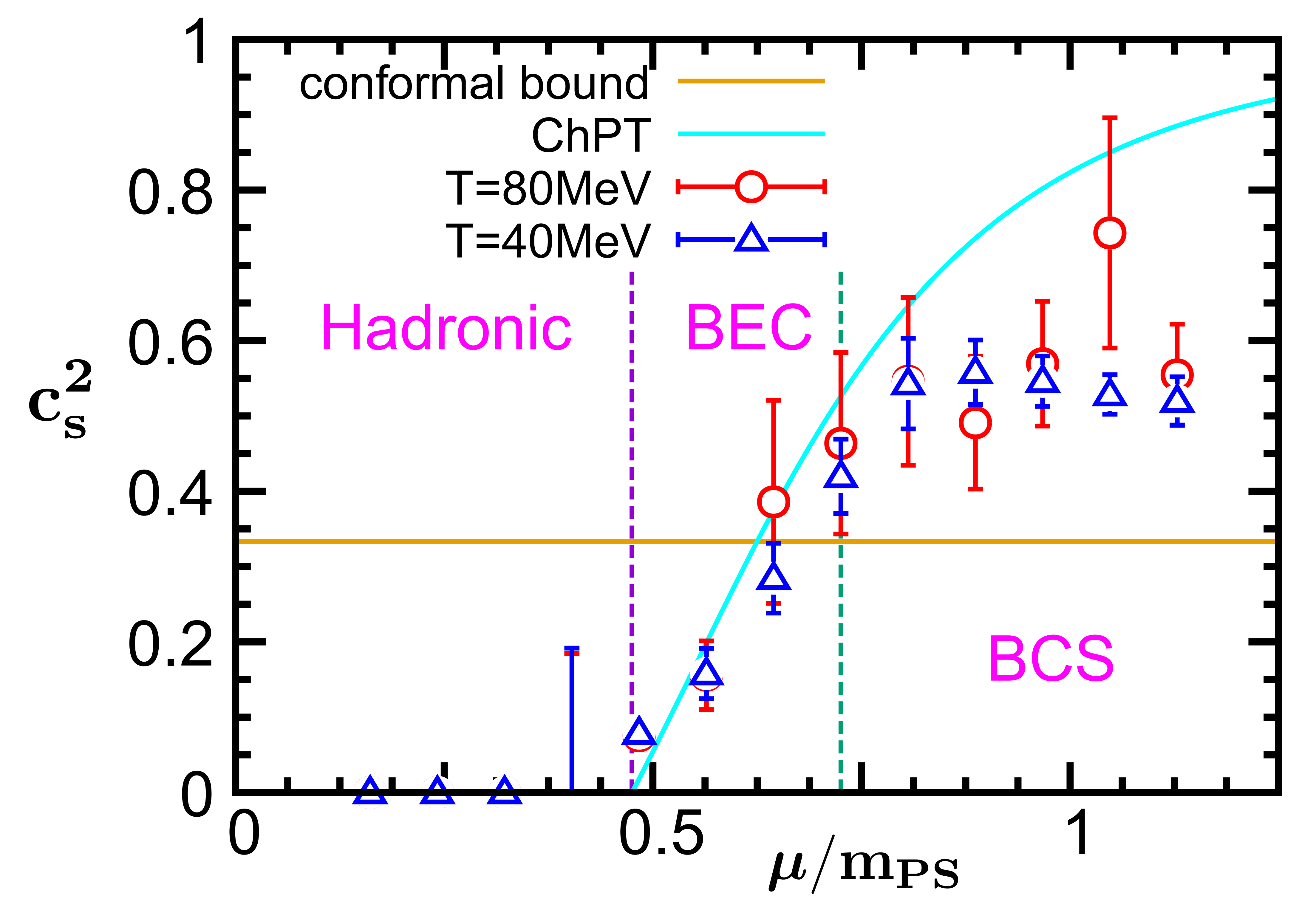
- Bayesian inference of **sound speed** in neutron star matter

L. Brandes, W.W., N. Kaiser : Phys. Rev. D 108 (2023) 094014



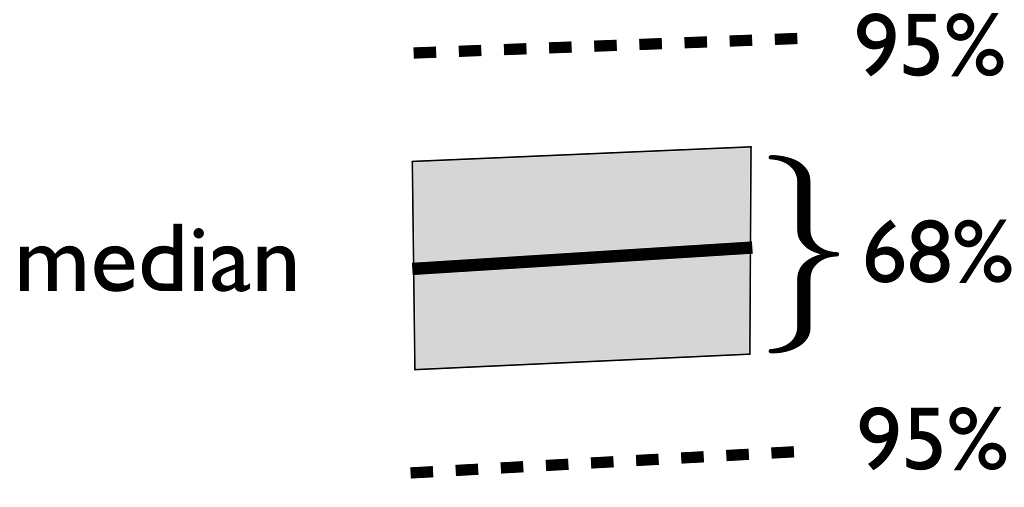
- Sound speed as function of baryon chemical potential in $N_c = 2$ LQCD

K. Iida, E. Itou, K. Murakami, D. Suenaga : JHEP 10 (2024) 022



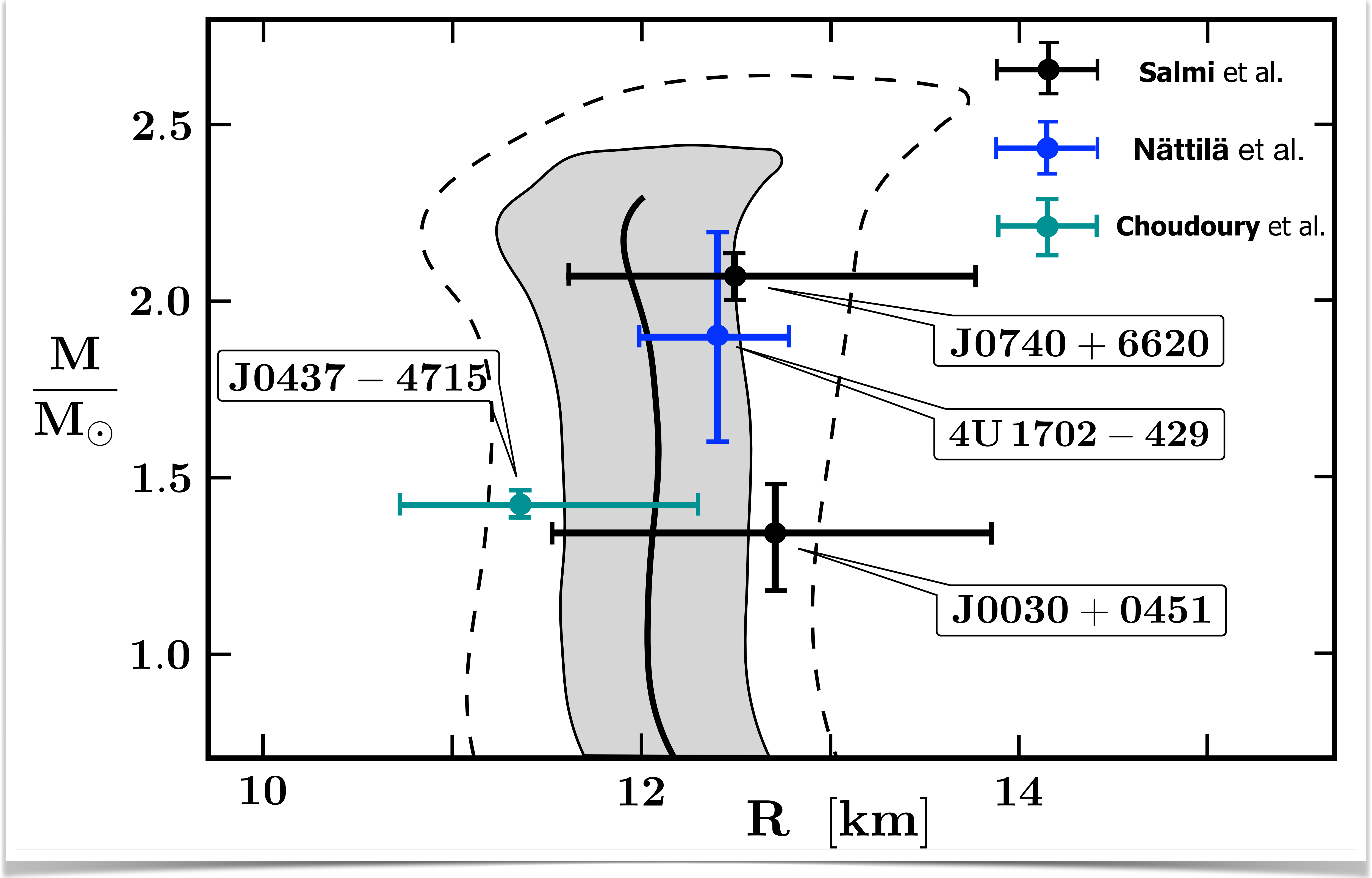
- **Speed of sound** exceeds conformal bound $c_s = 1/\sqrt{3}$ at baryon densities $n_B > 2 - 3\,n_0$

NEUTRON STAR PROPERTIES

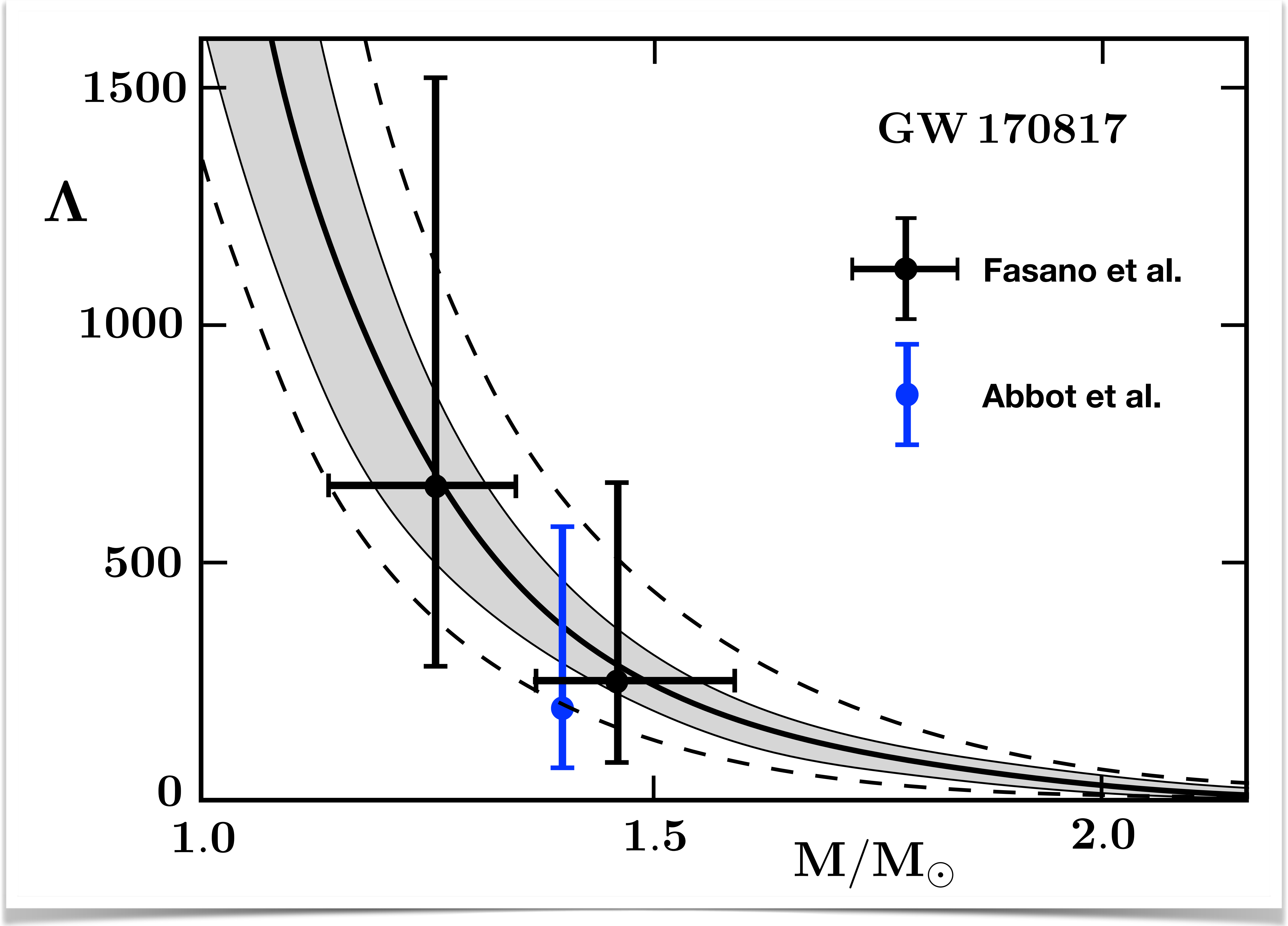


- Bayesian inference posterior bands (68% and 95% c.l)

- Mass - Radius relation (TOV)



- Tidal deformability



L. Brandes, W. W., N. Kaiser : Phys. Rev. D 107 (2023) 014011 ; Phys. Rev. D 108 (2023) 094014

L. Brandes, W. W. Phys. Rev. D 111 (2025) 034005

NEUTRON STAR PROPERTIES (contd.)

- **Baryon chemical potential**

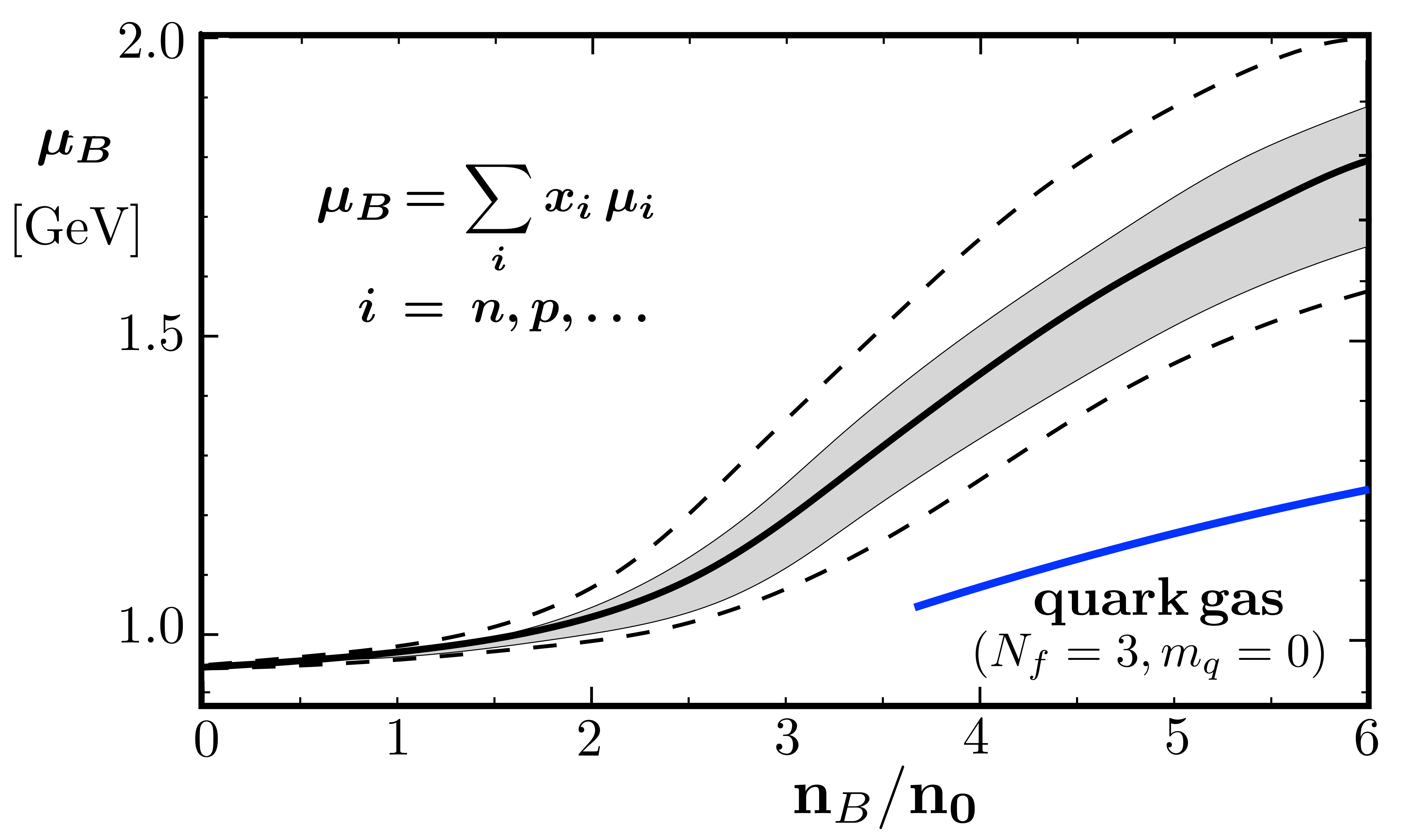
$$\mu_B = \frac{\partial \varepsilon}{\partial n_B} = \frac{P + \varepsilon}{n_B}$$

- **Stiff equation of state**

  ↓

  strongly repulsive correlations at work between baryons / quarks

- **Quark gas ruled out at densities** $n_B \lesssim 6\, n_0$



L. Brandes, W. W., N. Kaiser : Phys. Rev. D 108 (2023) 094014

L. Brandes, W. W. : Phys. Rev. D 111 (2025) 034005

QCD TRACE ANOMALY and CONFORMALITY in NEUTRON STARS

Y. Fujimoto, K. Fukushima, L.D. McLerran, M. Praszalowicz : Phys. Rev. Lett. 129 (2022) 252702

- **Trace of energy-momentum tensor :** $T^\mu_\mu = \Theta = \frac{\beta}{2g} G^a_{\mu\nu} G^{\mu\nu}_a + (1+\gamma_m) \sum_f m_f \, \bar{q}_f q_f$
- **Finite T and μ_B :**

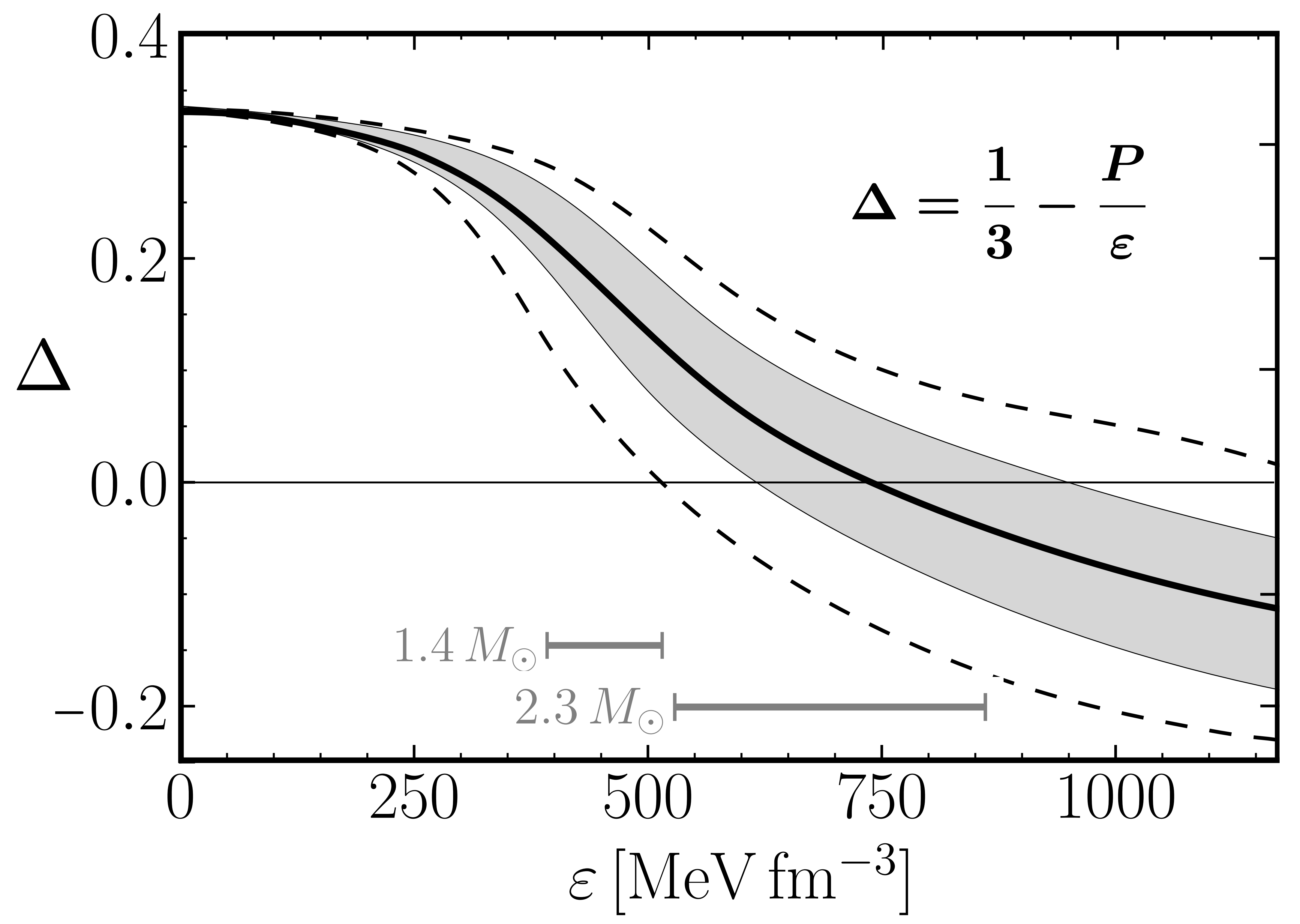
$$\langle \Theta \rangle_{T,\mu_B} = \varepsilon - 3P$$

- **Trace anomaly measure**

$$\Delta \equiv \frac{\langle \Theta \rangle_{T,\mu_B}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$$

- **Conformal limit :** $\Delta \to 0$
- **Bayes factor analysis:**
  Strong evidence for
  $\Delta < 0 \quad (P > \varepsilon/3)$

at densities $n_B \gtrsim 4\, n_0$



L. Brandes, W.W., N. Kaiser : Phys. Rev. D 108 (2023) 094014
L. Brandes, W.W. : Phys. Rev. D 111 (2025) 034005

NEUTRON STAR PROPERTIES (contd.)

- **Density profiles of neutron stars**
  using inferred median of $P(\varepsilon)$

- **Central core densities**

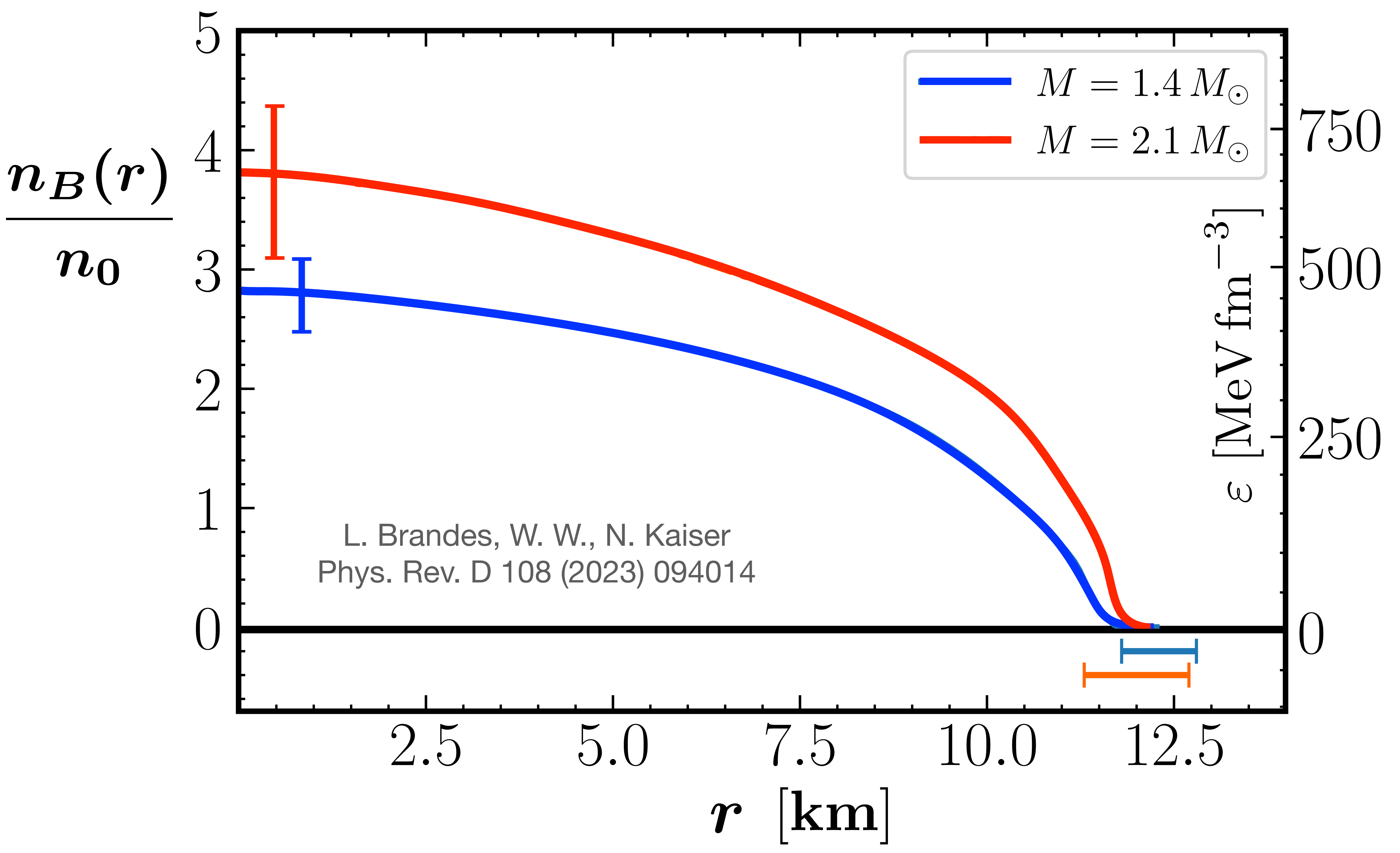
$$n_c = n_B(r=0)$$

in neutron stars are **NOT** extreme

- **Average distance between baryons :**

$$d \gtrsim 1\,\mathrm{fm}$$

even for the heaviest neutron stars



$$n_c(1.4\,M_\odot) = 2.8 \pm 0.3\,n_0 \qquad n_c(2.1\,M_\odot) = 3.8^{+0.6}_{-0.7}\,n_0 \qquad n_c(2.3\,M_\odot) = 4.0^{+0.6}_{-0.9}\,n_0$$

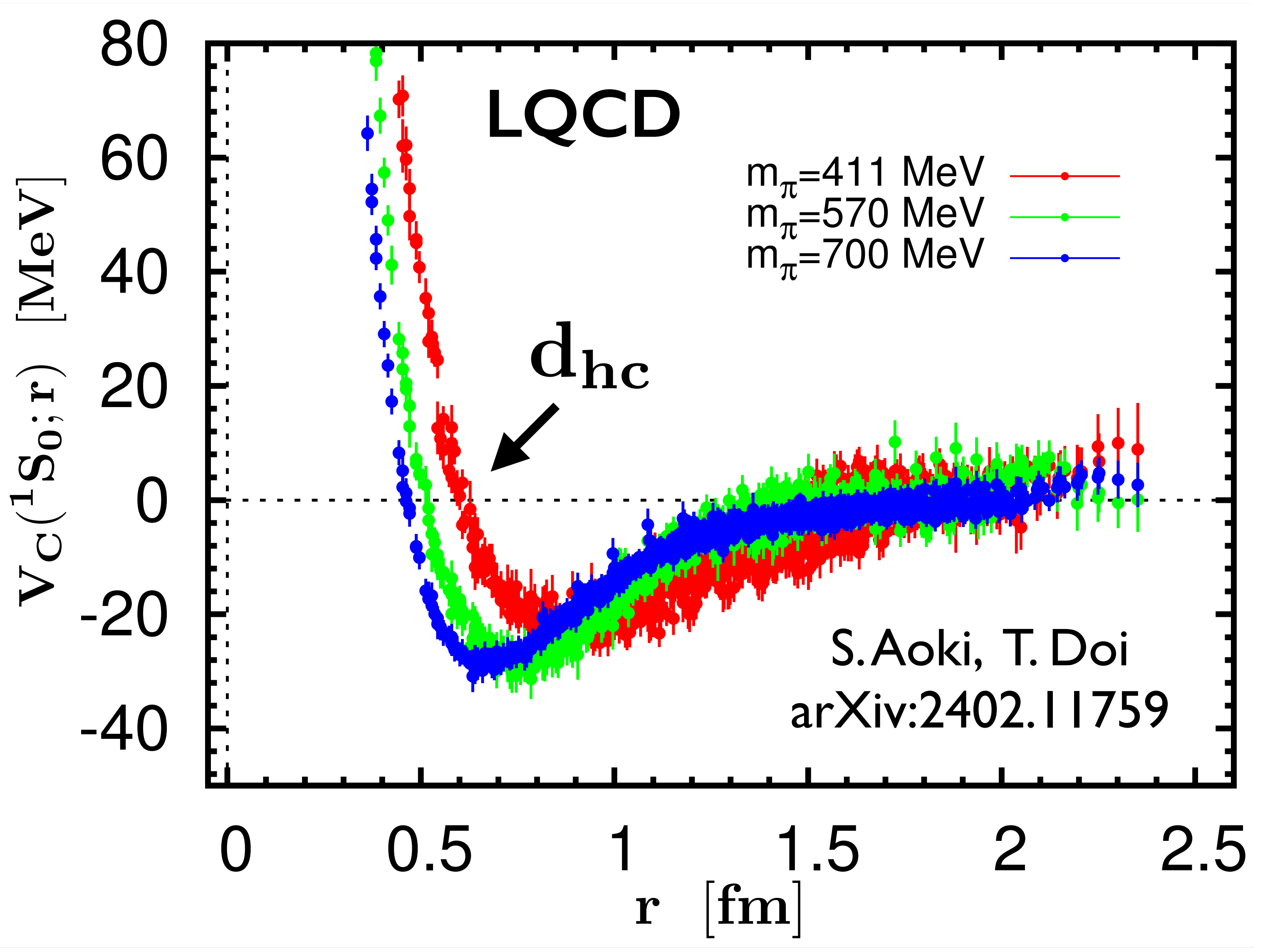
(68% c.l. — including new NICER data and "black widow" PSR J0952-0607)

L. Brandes, W. W. : Phys. Rev. D 111 (2025) 034005

BARYONIC MATTER

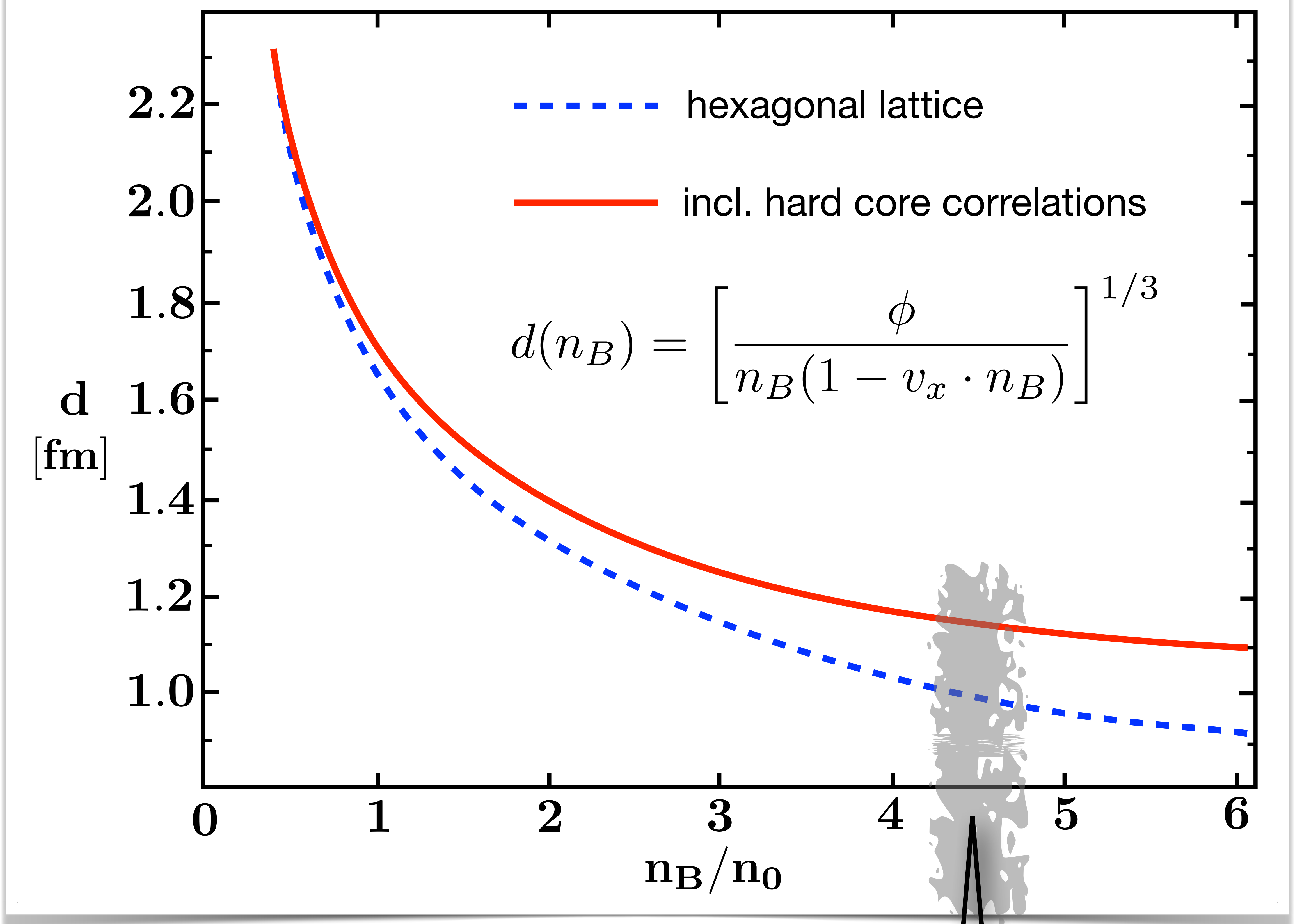
- Example:
  hexagonal lattice arrangement
  packing fraction $\phi = \dfrac{\pi}{3\sqrt{2}} = 0.74$

- Repulsive short-range correlations



→ excluded volume $v_x = \dfrac{2\pi}{3} d_{hc}^3$

- Average distance between baryons



max. range of central densities in neutron star cores

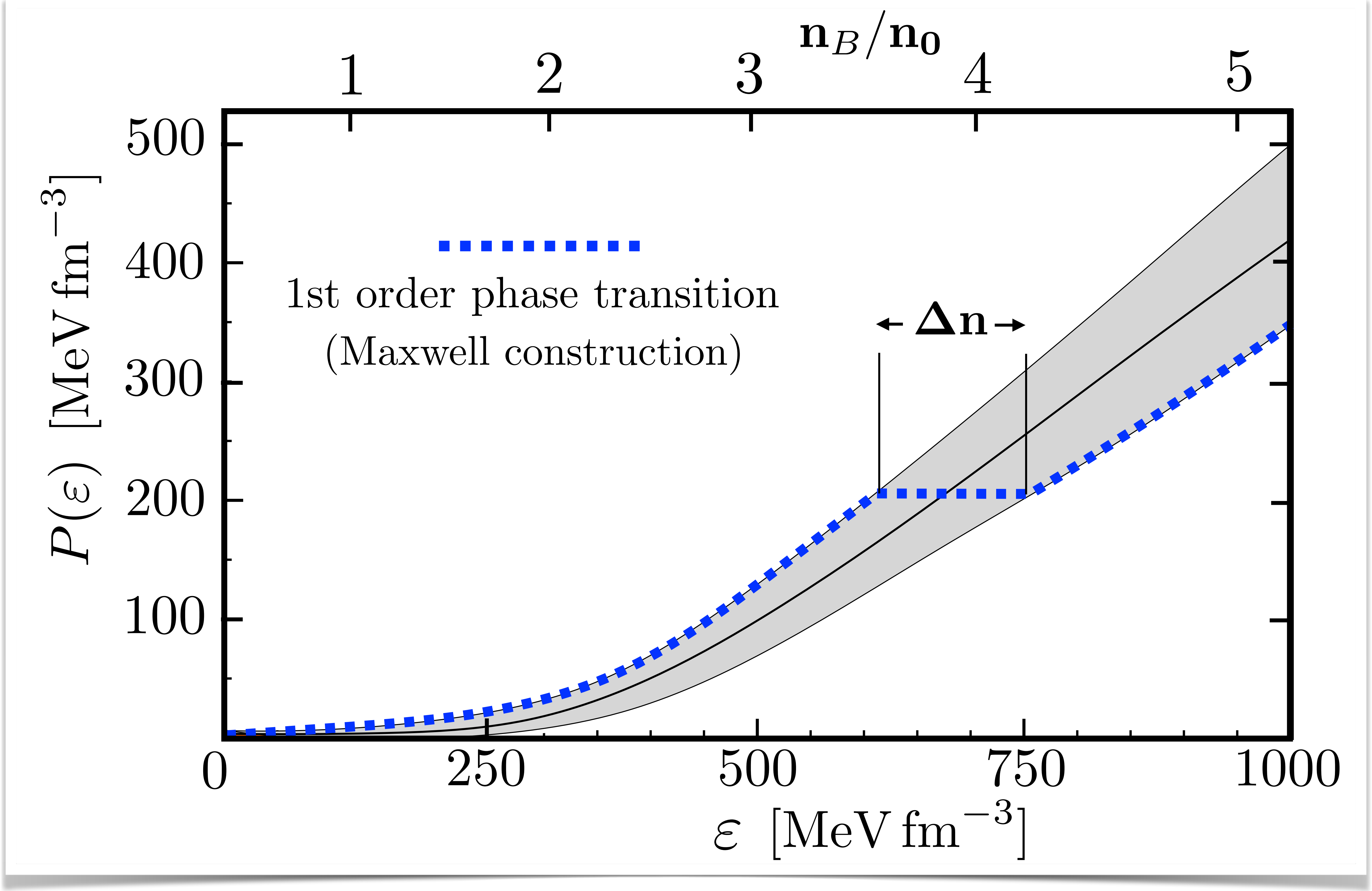
Constraints on
FIRST-ORDER PHASE TRANSITIONS in NEUTRON STAR MATTER

- **Bayes factor analysis :**

→ **Extreme evidence for sound velocities $c_s^2 > 1/3$ in cores of all neutron stars with $1.4 \leq M/M_\odot \leq 2.3$**

- **Evidence against strong 1st order phase transition :**

→ **Maximum possible extension of phase coexistence domain**
$$\Delta n/n_B < 0.2 \text{ (68\% c.l.)}$$



L. Brandes, W.W., N. Kaiser : Phys. Rev. D 108 (2023) 094014 - L. Brandes, W.W. : Symmetry 16 (2024) 111

→ **For comparison :**
Maxwell construction for nuclear liquid-gas first-order phase transition $(\Delta n/n_B > 1)$

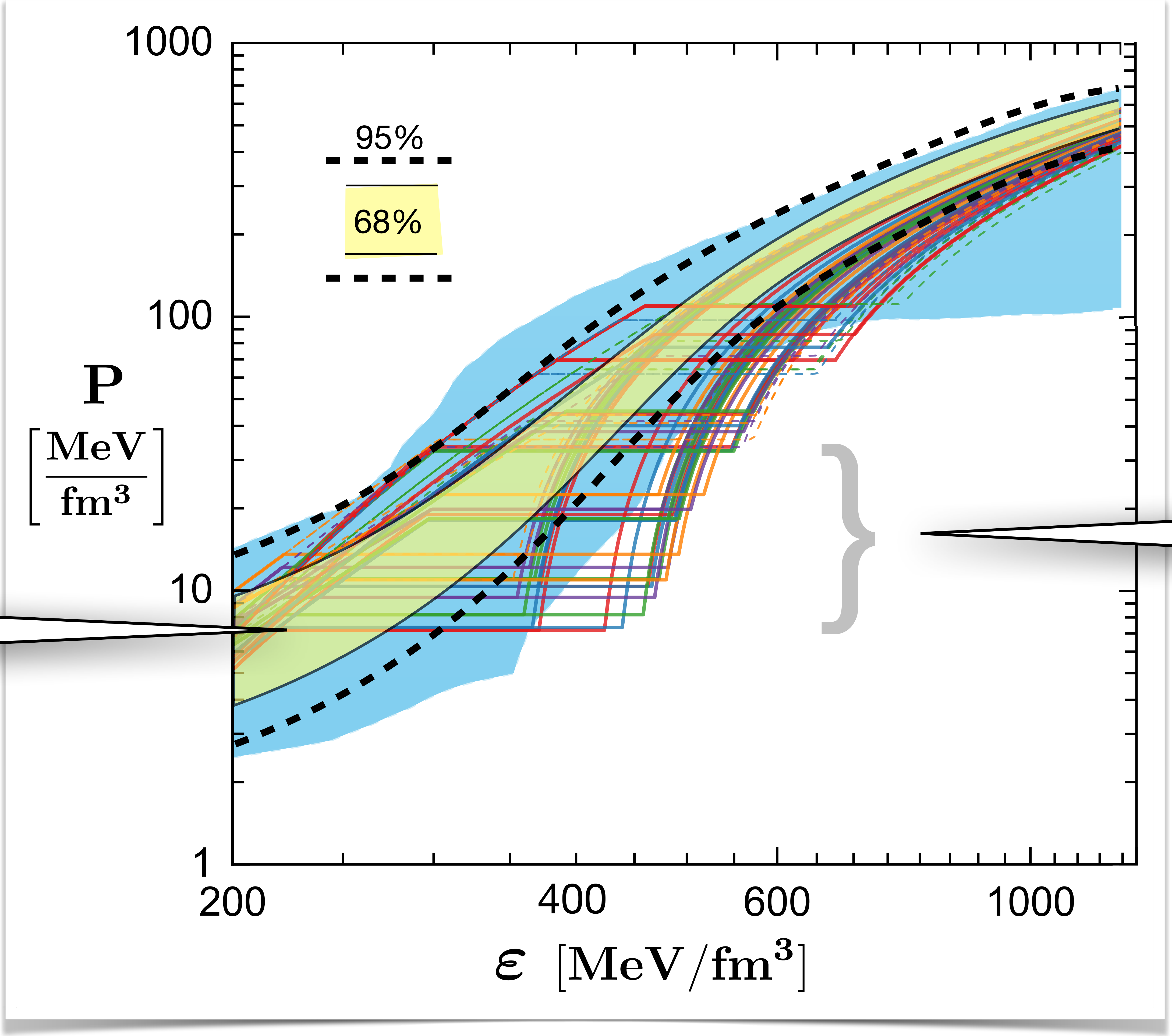
Constraints on **FIRST-ORDER PHASE TRANSITIONS** in **NEUTRON STAR MATTER** (contd.)

- **Restrictions at work:**

  Narrowing down the likelihoods for exotics (twins, hybrids)

empirically constrained band (Bayes inference)

L. Brandes, W. W.
Phys. Rev. D 111 (2025) 034005



various scenarios involving first-order phase transitions at low densities

$n_B \sim 1.5 - 2.5\ n_0$

J.J. Li, A. Sedrakian, M. Alford
Astroph. J. 967 (2024) 116

Part Two

Phenomenology, Models

and

Possible Dense Matter Scenarios

CHIRAL PHASE TRANSITION in DENSE BARYONIC MATTER ?

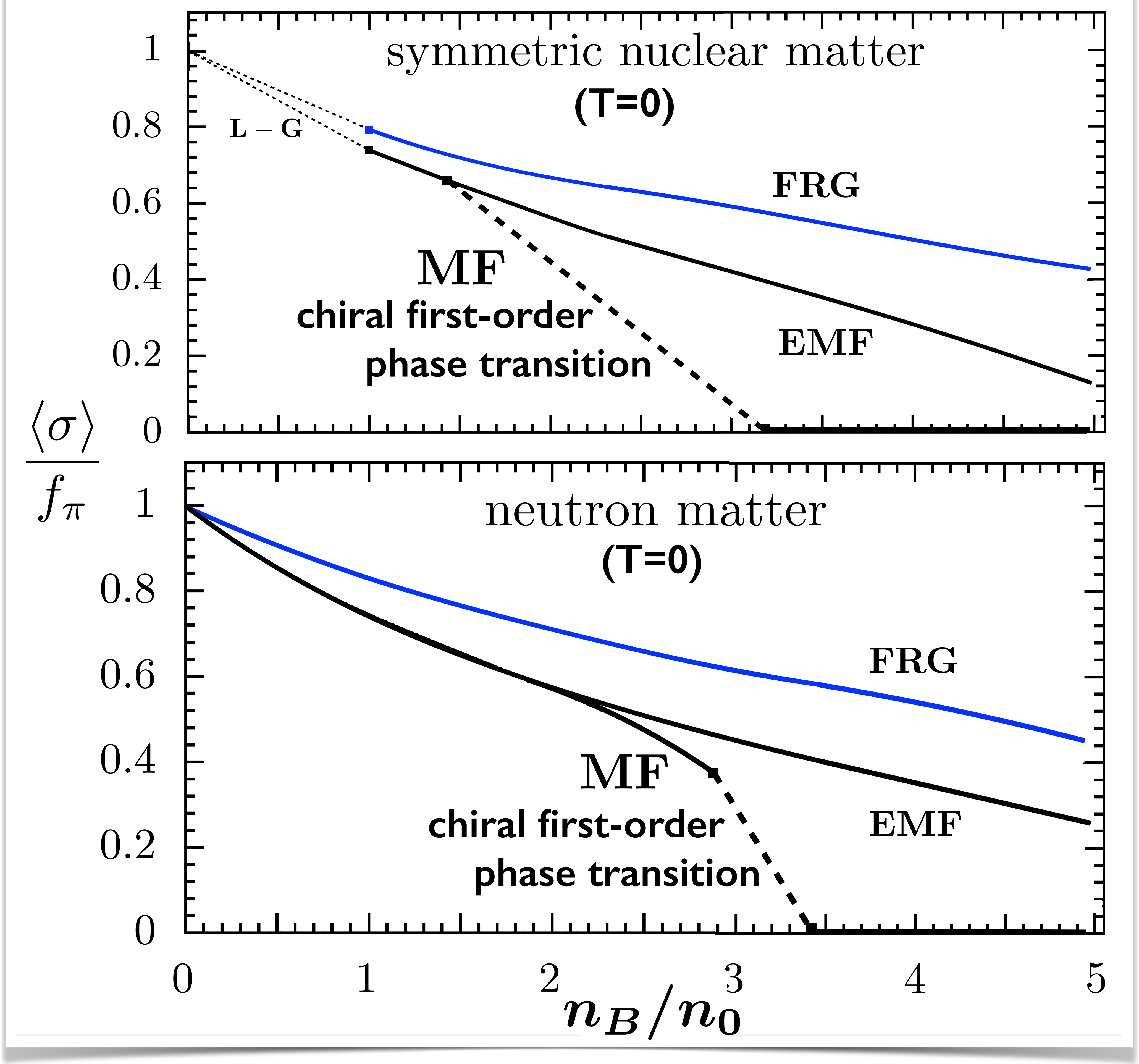
✱ Studies in chiral nucleon-meson field theory

M. Drews, W.W.: Prog. Part. Nucl. Phys. 93 (2017) 69 — L. Brandes, N. Kaiser, W.W.: Eur. Phys. J. A57 (2021) 243

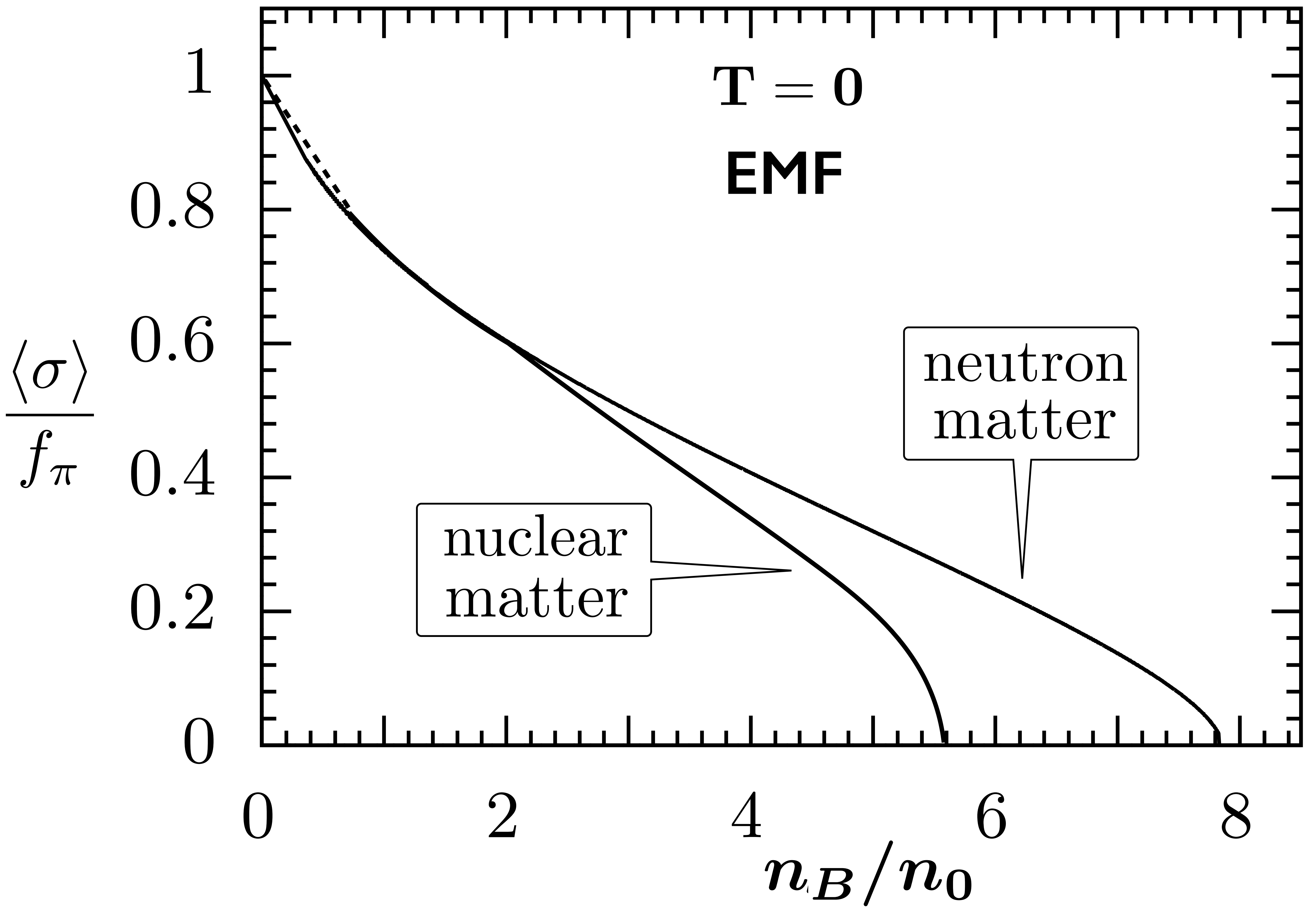
- **Mean-field** approximation **(MF)** :
  chiral first-order phase transition
  at baryon densities $n_B \sim 2 - 3\, n_0$
- **Vacuum fluctuations** (EMF) :
  shift chiral transition to **high density**
  ➜ **smooth crossover**
- **Functional Renormalisation Group** (FRG) :
  non-perturbative loop corrections
  involving **pions** & **nucleon-hole** excitations
  ➜ further reinforcement of **stabilising effects**

Chiral crossover transition at $n_B > 6\, n_0$
beyond core densities in neutron stars

Chiral order parameter



CHIRAL LIMIT $(m_\pi \to 0)$
2nd order chiral phase transition in nuclear and neutron matter



L. Brandes, N.. Kaiser, W.W.: Eur. Phys. J. A57 (2021) 243

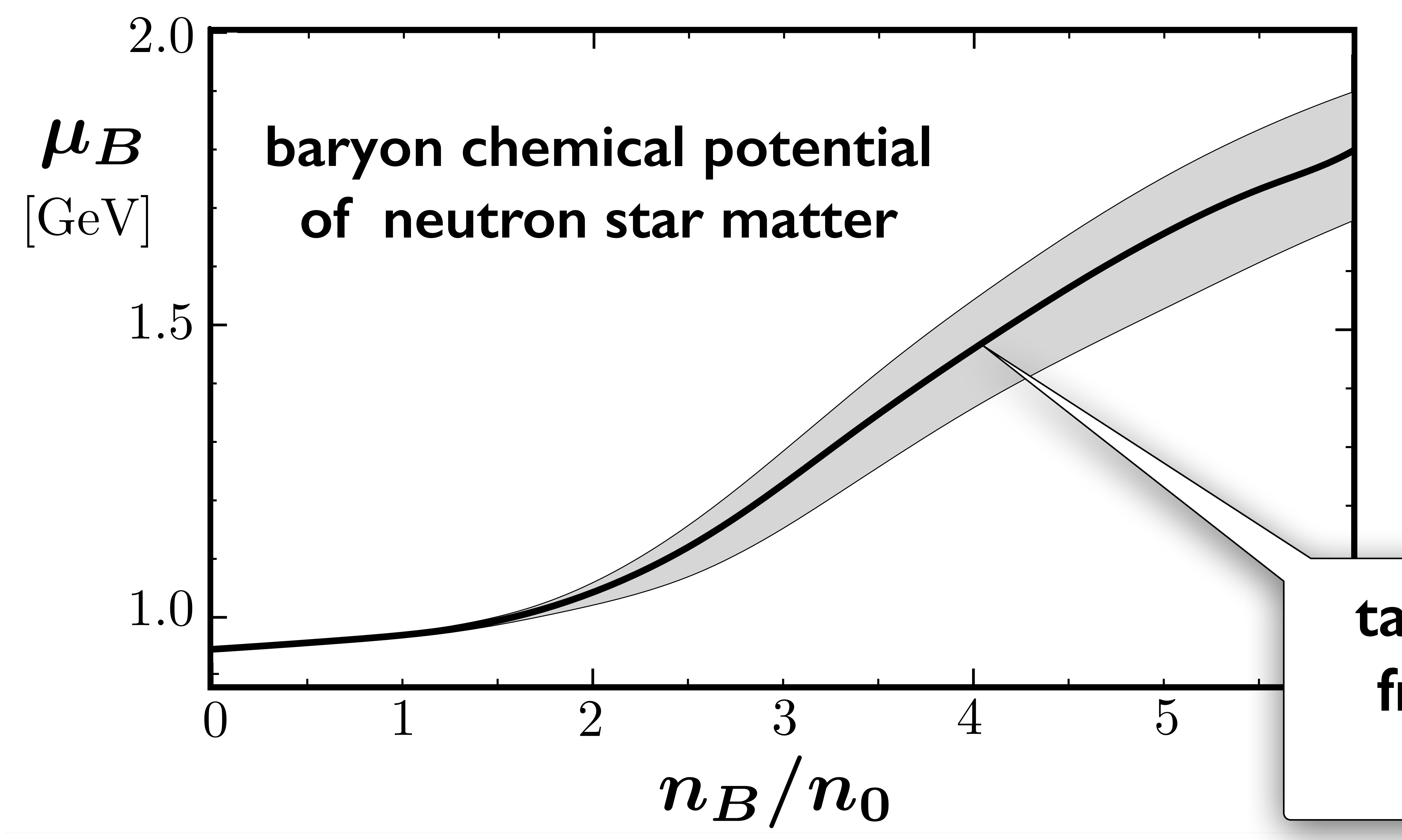
- **Chiral Nucleon-Meson Field Theory**
- **EMF calculations :** Extended Mean-Field including logarithmic vacuum fluctuations
- **Critical densities** (chiral limit) $n_B^{cr} > 5\, n_0$
- Alternative approach: **Parity-doublet model** $\{N(1/2^+) - N^*(1/2^-)\}$ Critical densities $n_B^{cr} > 8\, n_0$

J. Eser, J.-P. Blaizot : Phys. Rev. C109 (2024) 045201, Phys. Rev. C110 (2024) 065205

DENSE BARYONIC MATTER in NEUTRON STARS as a RELAVISTIC FERMI LIQUID

B. Friman, W.W. : Rhys. Rev. C100 (2019) 065807 L. Brandes, W.W. : Symmetry 16 (2024) 111

- **Neutron Star Matter : Fermi liquid** / dominantly neutrons + ca. 5 % protons
- **Baryonic Quasiparticles :**
  baryons "dressed" by their **strong interactions** and imbedded in **mesonic (multi-pion) field**



- **Landau effective mass**

$$m_L^*(n_B) = \sqrt{p_F^2 + M_N^2(n_B)}$$

- **Baryon chemical potential**

$$\mu_B = m_L^*(n_B) + \mathcal{U}(n_B)$$

take median of $\mu_B(n_B)$ from Bayesian-inferred neutron star EoS

quasiparticle potential

Basics of (Relativistic) Fermi-Liquid Theory

G. Baym, S.A. Chin : Nucl. Phys. A262 (1976) 527 T. Matsui : Nucl. Phys. A370 (1981) 365

- **Variation of the energy** (T = 0)

$$\delta E = V\delta\mathcal{E} = \sum_p \varepsilon_p\,\delta n_p + \frac{1}{2V}\sum_{pp'} \mathcal{F}_{pp'}\delta n_p \delta n_{p'} + \ldots \qquad n_p = \Theta(\mu - \varepsilon_p)$$

quasiparticle energy $\varepsilon_p = \frac{\delta E}{\delta n_p}$

quasiparticle interaction

$$\mathcal{F}_{pp'} = V\,\frac{\delta^2 E}{\delta n_p \delta n_{p'}} = f_{pp'} + g_{pp'}\,\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'$$

- **Landau effective mass**

$$m_L^* = \sqrt{p_F^2 + M^2(n)}$$

- **Density of states** at the **Fermi surface**

$$N_0 = \frac{m_L^*\,p_F}{\pi^2}$$

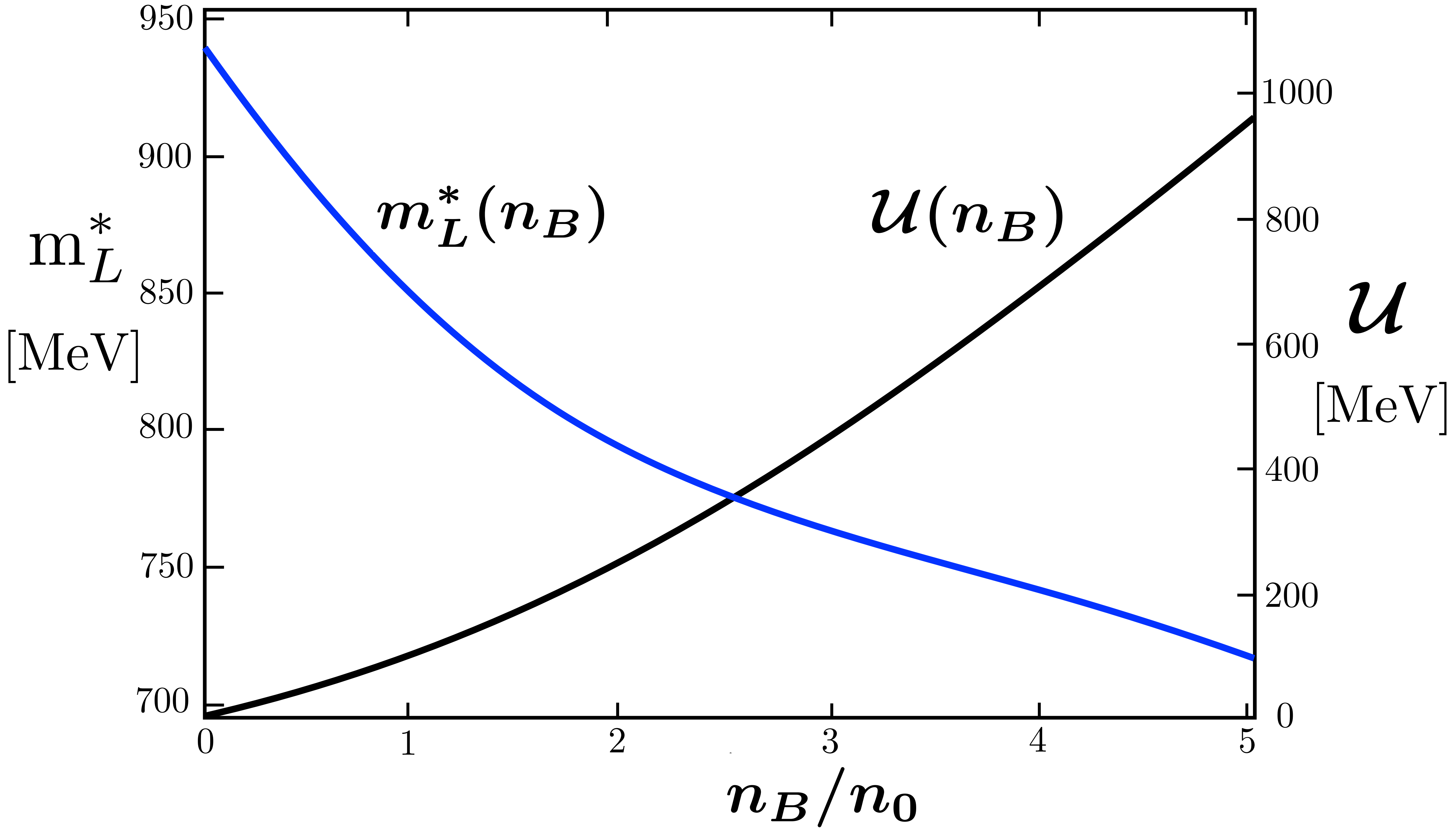
- **Landau parameters** **Quasiparticle interaction** expanded in **Legendre series**

$$f_{pp'} = \sum_{\ell=0}^{\infty} f_\ell\,P_\ell(\cos\theta_{pp'}) \qquad \boldsymbol{F_\ell = N_0\,f_\ell}$$

QUASIPARTICLE POTENTIAL and FERMI-LIQUID PARAMETERS

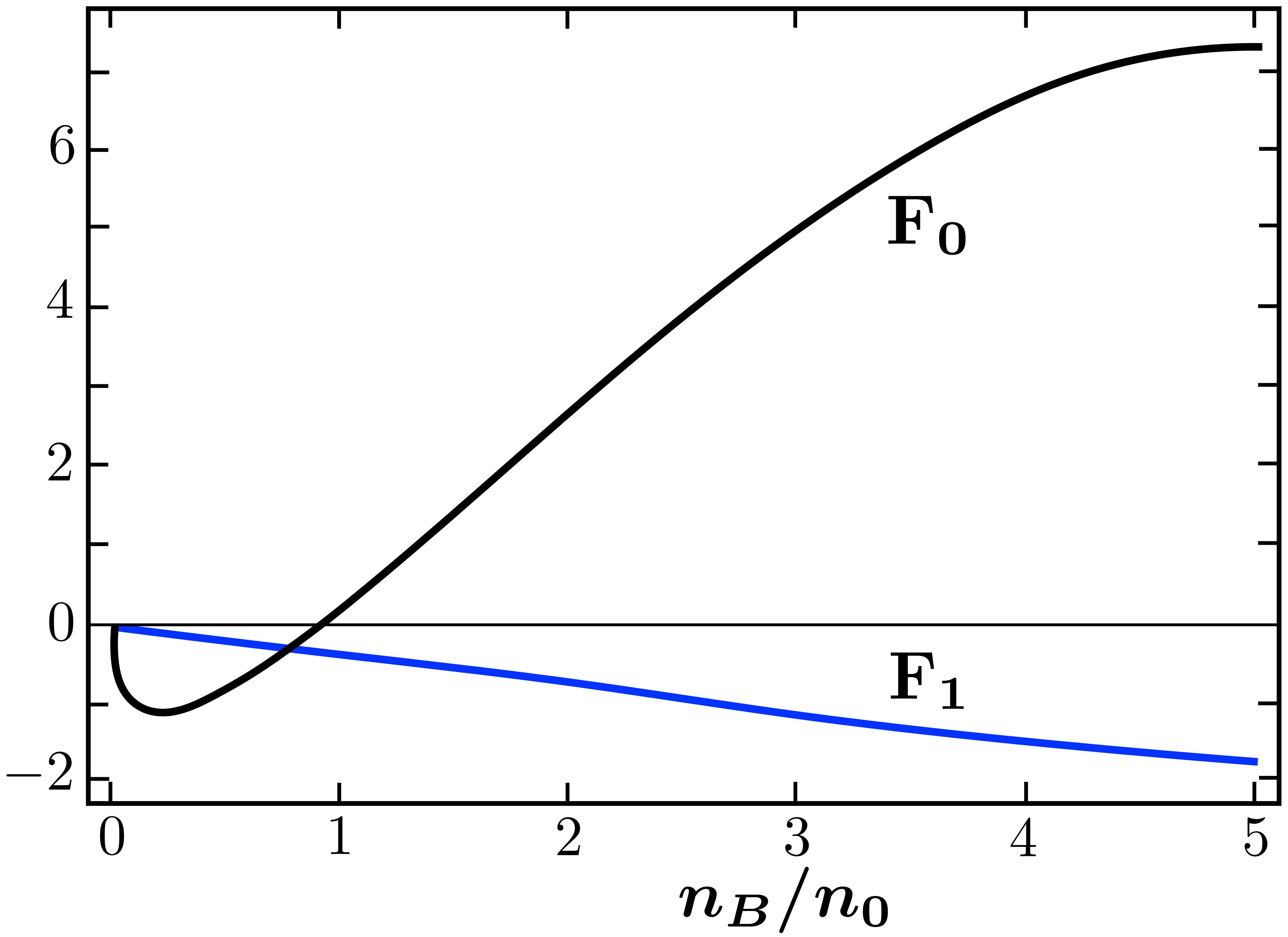
- $m_L^*(n_B)$ from **chiral nucleon-meson field theory** & Functional Renormalisation Group
- Quasiparticle effective potential

$$\mathcal{U}(n_B) = \sum_\nu u_\nu \left(\frac{n_B}{n_0}\right)^\nu$$



- Landau Fermi-Liquid parameters

$$F_0 = \frac{m_L^*\, p_F}{\pi^2}\frac{\partial \mu_B}{\partial n_B} - 1 \qquad F_1 = -\frac{3\mathcal{U}}{\mu_B}$$



→ **Strongly repulsive correlations** including **many-body forces** with $\nu \geq 2$

QUASIPARTICLE POTENTIAL and FERMI-LIQUID PARAMETERS

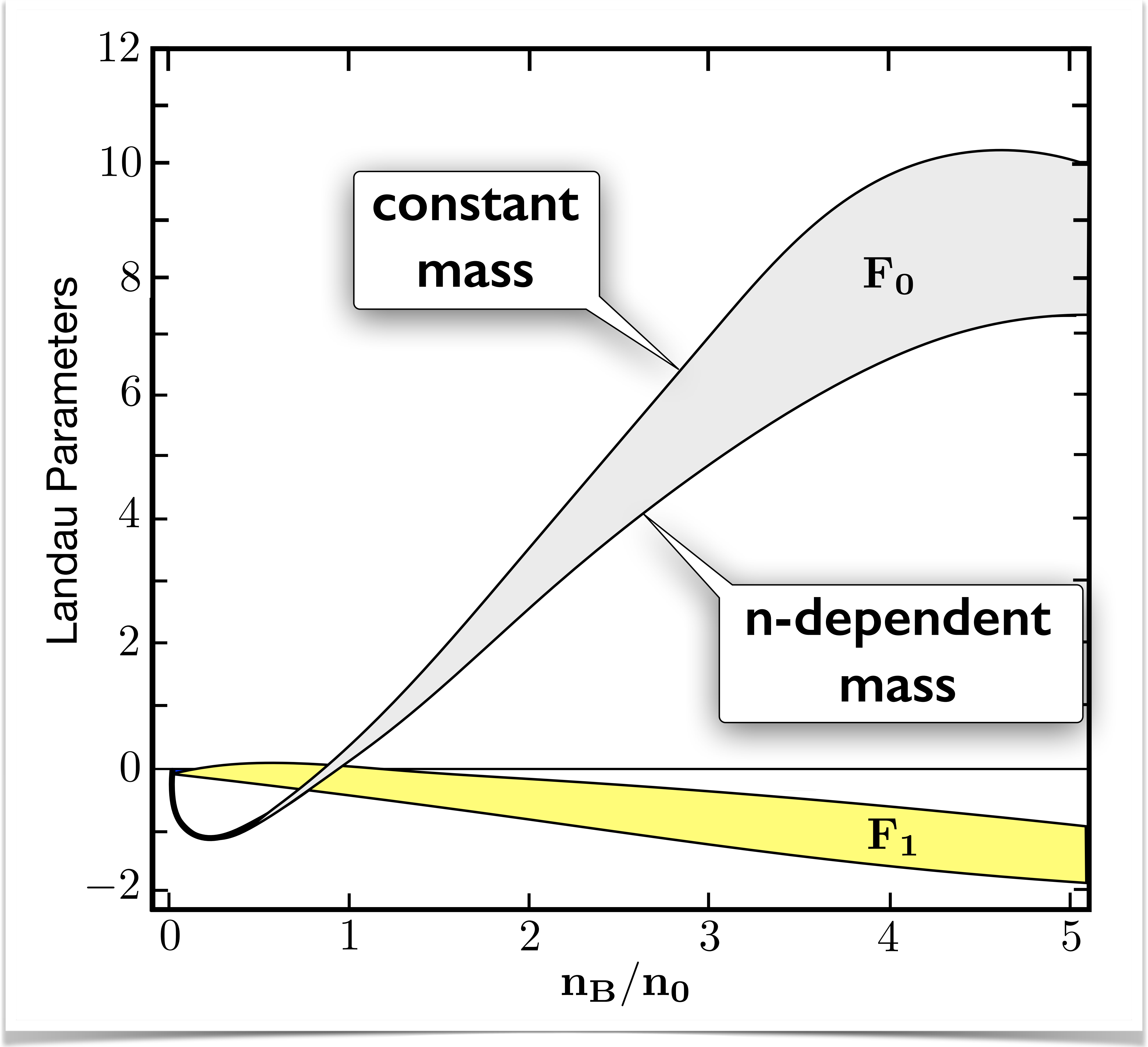
$$F_0 = \frac{m_L^* \, p_F}{\pi^2} \frac{\partial \mu_B}{\partial n_B} - 1$$

$$F_1 = -\frac{3\mathcal{U}}{\mu_B}$$

- Upper bound for Landau parameters : limiting case of constant Fermion mass

$$M(n_B) = M_0 = const.$$

$$\mu_B = m_L^* + \mathcal{U} = \sqrt{p_F^2 + M_0^2} + \mathcal{U}$$



LANDAU FERMI LIQUID PARAMETERS (contd.)

- **Comparison with atomic liquid helium-3 in its normal phase at low temperature** (3 K)

  G. Baym, Ch. Pethick : Landau Fermi-Liquid Theory (1991)

- **Interaction** between **He-3 atoms:**
  attractive van der Waals potential plus strongly **repulsive short-range core**

- **Landau Fermi Liquid parameters** of **liquid helium-3** at pressures P = (0 - 30) bar:

$$\mathbf{F_0(^3He) \sim 10 - 70} \qquad\qquad \mathbf{F_1(^3He) \sim 5 - 13}$$

D. S. Greywall, Phys. Rev. B33 (1986) 7520

… **generally much larger** in magnitude than Landau parameters of **neutron star matter** !

- **Neutron star matter** at central densities is a **strongly correlated Fermi system**
  … but not as extreme as one might have thought !

CONCLUSIONS & OUTLOOKS

- **Constraints on EoS and phases in neutron star matter**
  - → **stiff equation of state** implied by Bayesian inference results
  - → **strong first-order transition** unlikely in neutron star cores
  - → **central baryon densities** in neutron stars : $n_c \lesssim 5\, n_0$ (68% c.l.)

- **Scenarios for cold dense matter in the core of neutron stars**
  - → **neutron - dominated baryonic** matter :

    e.g. relativistic **Fermi liquid** featuring strongly repulsive **many-body correlations** between **baryonic quasiparticles**
  - → **hadron-quark** continuity with "core (qqq) + cloud $(q\bar{q})$" baryons :

    two-scales scenario: soft-surface delocalisation (percolation) followed by hard-core deconfinement at densities $n_B \gtrsim n_c$

Happy Birthday, Wanda

with

all best wishes

for

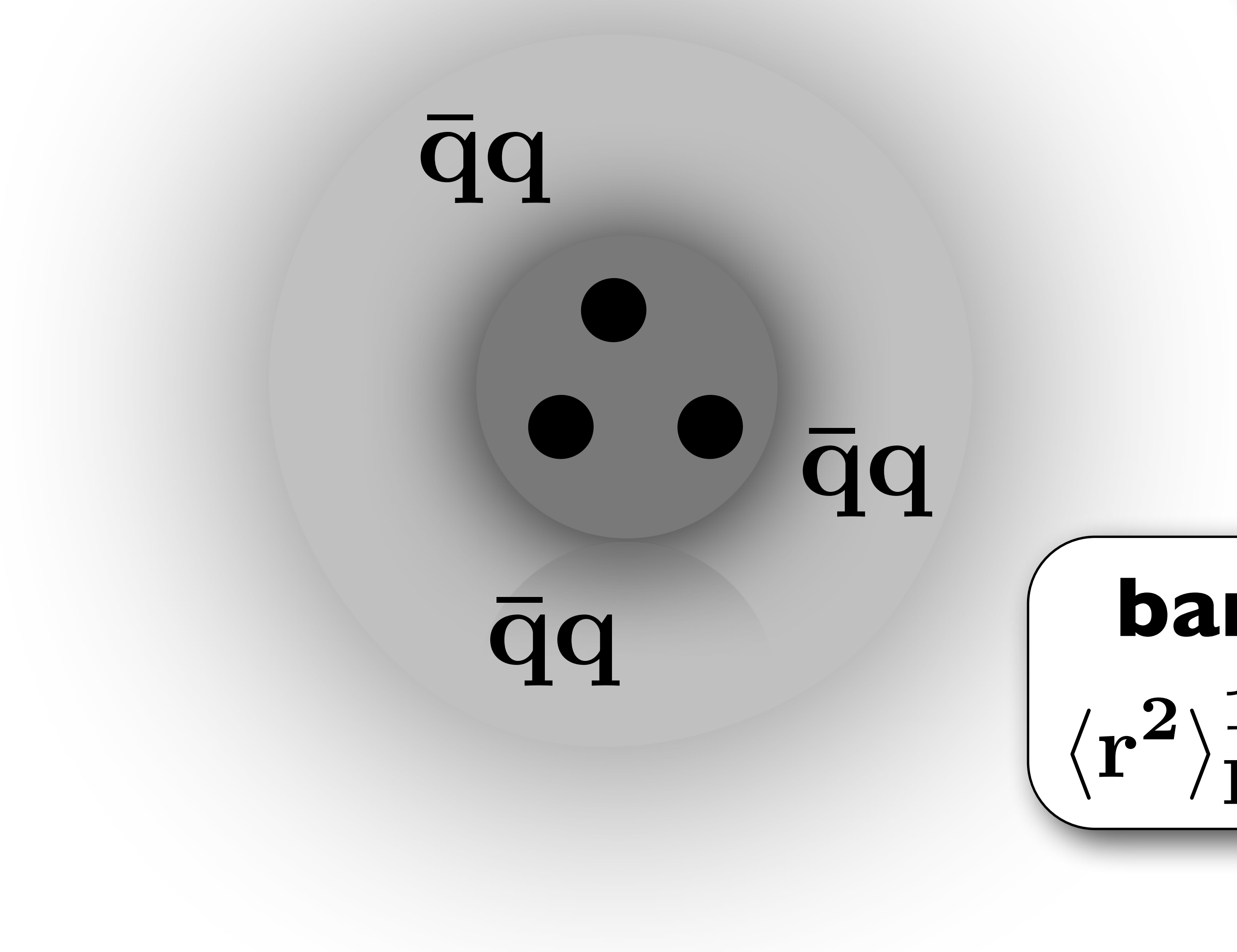
many decades to come

Supplementary Materials

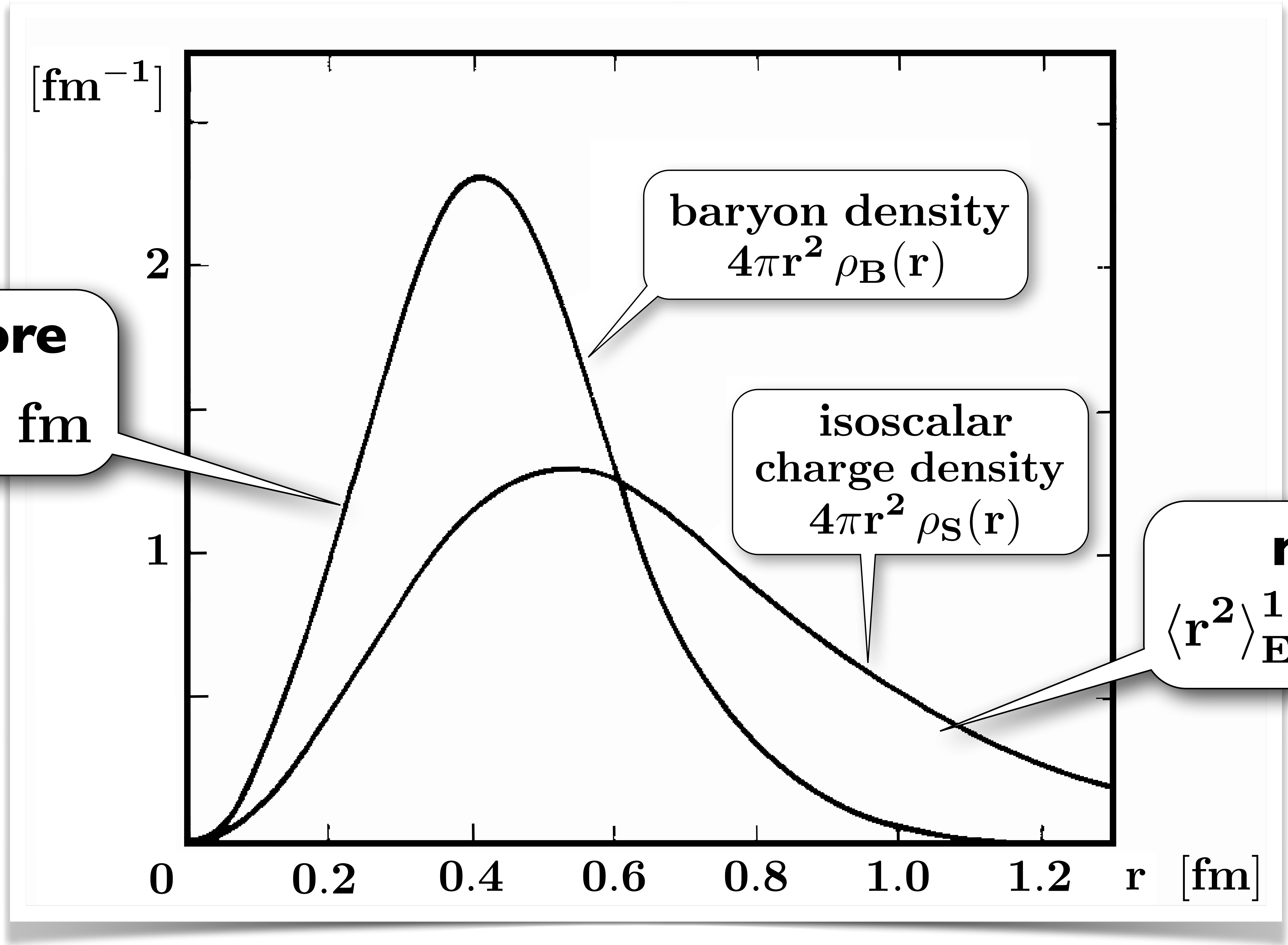
SIZES of the NUCLEON

Low-energy QCD: **spontaneously broken chiral symmetry + localisation (confinement)**

- **NUCLEON** : **compact valence quark core + mesonic** (multi $\bar{q}q$) **cloud**
  - Historic example: Chiral Soliton Model of the Nucleon



baryonic core
$\langle r^2 \rangle_B^{1/2} \simeq 0.5$ fm



mesonic cloud
$\langle r^2 \rangle_{E,isoscalar}^{1/2} \simeq 0.8$ fm

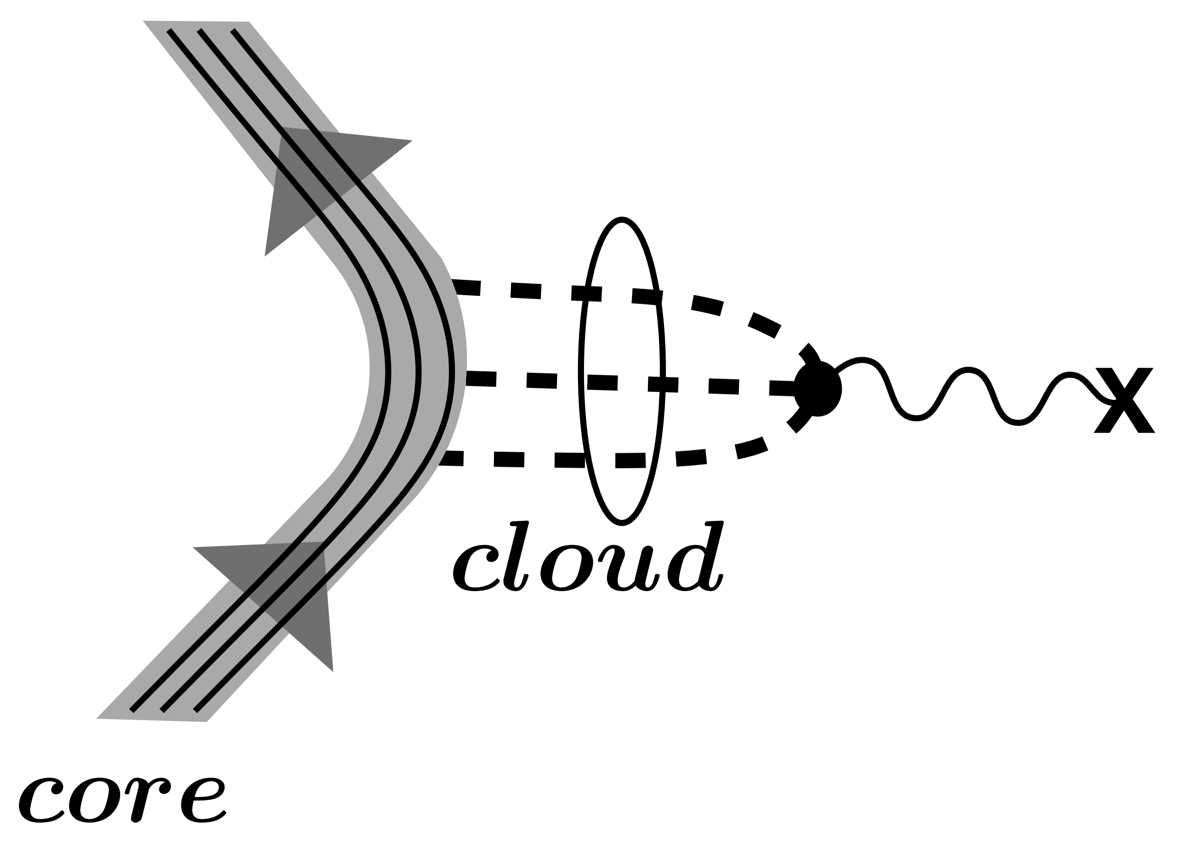
N. Kaiser,
U.-G. Meißner,
W.W.
Nucl. Phys. A466 (1987) 685

- **Separation of scales**

$$\left(\frac{R_{cloud}}{R_{core}}\right)^3 \gg 1$$

FORM FACTORS of the NUCLEON

$$G_i(q^2) = G_i(0) + \frac{q^2}{\pi}\int_{t_0}^{\infty} dt \frac{Im\, G_i(t)}{t(t-q^2-i\epsilon)} \qquad \langle r_i^2 \rangle = \frac{6}{G_i(0)} \frac{dG_i(q^2)}{dq^2}\Big|_{q^2=0} = \frac{6}{\pi}\int_{t_0}^{\infty} \frac{dt}{t^2} S_i(t)$$



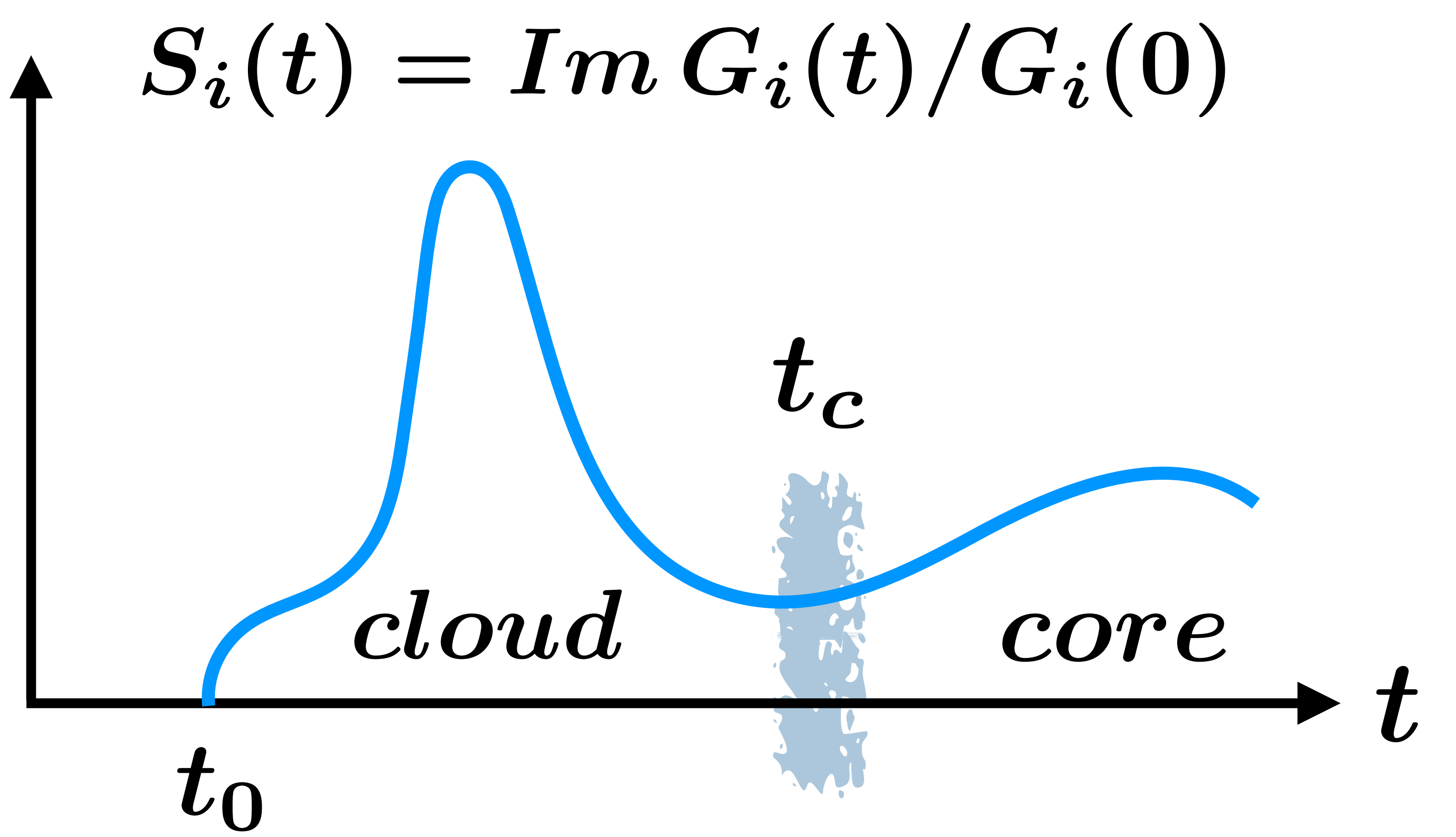
- Delineation of

  valence quark (qqq) **CORE**

  and

  mesonic (multi $\bar{q}q$) **CLOUD**

  $$t_c \simeq 1\,\mathrm{GeV}^2$$



$$\langle r_i^2 \rangle = \langle r_i^2 \rangle_{cloud} + \langle r_i^2 \rangle_{core} = \frac{6}{\pi}\left[\int_{t_0}^{t_c} \frac{dt}{t^2} S_i(t) + \int_{t_c}^{\infty} \frac{dt}{t^2} S_i(t)\right]$$

- **Detailed spectral analysis of accurately determined empirical form factors**

N. Kaiser, W.W. : Phys. Rev. C110 (2024) 015202

FORM FACTORS of the NUCLEON (contd.)

form factor		J^π(cloud)	empirical rms radii
isoscalar electric	$G_E^S(q^2)$	1^-	$\langle r_S^2\rangle^{1/2} = 0.78 \pm 0.01\,\mathrm{fm}$ Y.H. Lin, H.-W. Hammer, U.-G. Meißner PRL 128 (2022) 052002
isovector electric	$G_E^V(q^2)$	1^-	$\langle r_V^2\rangle^{1/2} = 0.90 \pm 0.01\,\mathrm{fm}$
isovector axial	$G_A(q^2)$	1^+	$\langle r_A^2\rangle^{1/2} = 0.67 \pm 0.01\,\mathrm{fm}$ $(\langle r_A^2\rangle^{1/2} = 0.68 \pm 0.11\,\mathrm{fm})$ R.J. Hill et al. : Rep. Prog. Phys. 81 (2018) 096301
mass	$G_m(q^2) = \langle p'\vert T^\mu_\mu\vert p\rangle$	0^+	$\langle r_m^2\rangle^{1/2} = 0.55 \pm 0.03\,\mathrm{fm}$ D. Kharzeev : Phys. Rev. D104 (2021) 054015 $\langle r_m^2\rangle^{1/2} = 0.53 \pm 0.04\,\mathrm{fm}$ S. Adhikari et al. : Phys. Rev. C108 (2023) 025201

extracted core radii

N. Kaiser, W.W. : Phys. Rev. C110 (2024) 015202

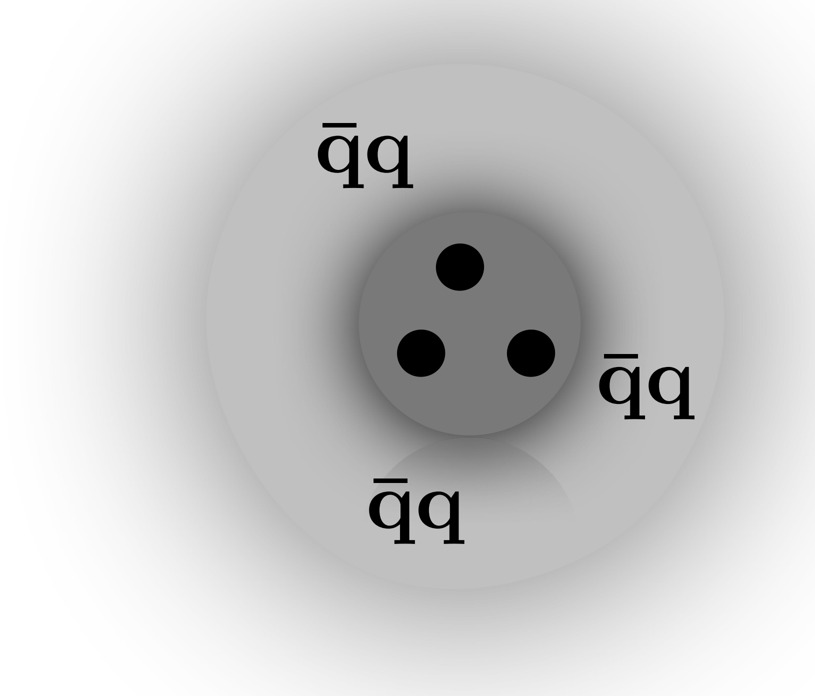
$$\langle r_S^2\rangle^{1/2}_{core} = 0.50 \pm 0.01\,\mathrm{fm}$$

$$\langle r_V^2\rangle_{core} \simeq 0\,(\pm 0.02)\,\mathrm{fm}^2\ !!$$

$$\langle r_A^2\rangle^{1/2}_{core} = 0.53 \pm 0.02\,\mathrm{fm}$$

$$(0.5 \pm 0.2)$$

$$\langle r_m^2\rangle^{1/2}_{core} = 0.48 \pm 0.05\,\mathrm{fm}$$



TWO-SCALES Picture of the NUCLEON : Implications for DENSE BARYONIC MATTER

$$\langle r_S^2\rangle^{1/2}_{core} \simeq \langle r_A^2\rangle^{1/2}_{core} \simeq \langle r_m^2\rangle^{1/2}_{core} \equiv R_{core} \simeq \frac{1}{2}\,\mathrm{fm}$$

$R_{core} \sim \frac{1}{2}\,\mathrm{fm}$ $\quad R_{cloud} \sim 1\,\mathrm{fm}$

- **Separation of scales** $\left(\frac{R_{cloud}}{R_{core}}\right)^3 \gg 1$

- **Soft mesonic (multi-pion) cloud**
  expected to **expand** with increasing baryon density along with decreasing **in-medium pion decay constant** $f_\pi^*(n_B)$
- **Hard baryonic core** governed by **gluon dynamics**
  expected to remain approximately **stable** with increasing baryon density up until hard compact cores begin to touch and overlap

(example: extended NJL model calculation $R_{core}(n_B = 5n_0)/R_{core}(n_B = 0) \simeq 1.1$)

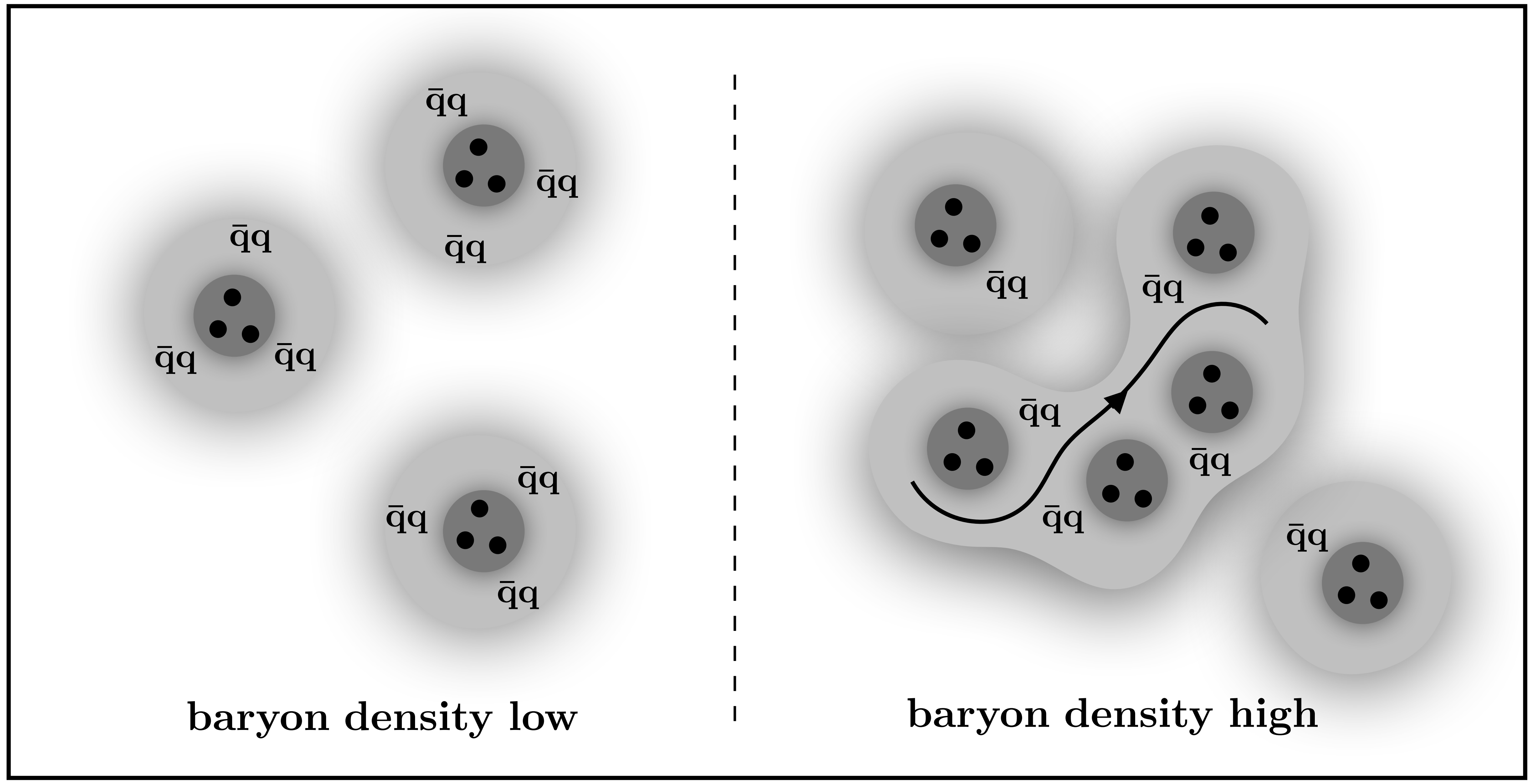
W. Bentz, I.C. Cloet : arXiv:2503.20564

TWO-SCALES Scenario for DENSE BARYONIC MATTER

- Baryon densities

  $$n_B \sim n_0 = 0.16\,\mathrm{fm}^{-3}$$

  Tails of mesonic clouds overlap :
  two-body boson exchange forces
  between nucleons



- $n_B \gtrsim 2 - 3\,n_0$

  Soft $\bar{q}q$ clouds delocalize :
  percolation → many-body forces
  baryonic cores still separated, but subject to increasingly strong repulsive Pauli effects

K. Fukushima, T. Kojo, W.W.
Phys. Rev. D 102 (2020) 096017

- $n_B > 5\,n_0$ (beyond central densities of neutron stars)

  Compact nucleon cores begin to touch and overlap at distances $d \lesssim 1\,\mathrm{fm}$
  (but still have to overcome the repulsive NN hard core)

Example 1: **ISOSCALAR ELECTRIC FORM FACTOR** of the **NUCLEON**

- **Isoscalar electric form factor** $G_E^S(q^2) = \frac{1}{2}\left[G_E^p(q^2) + G_E^n(q^2)\right]$ $\quad \langle r_S^2 \rangle = \langle r_p^2 \rangle + \langle r_n^2 \rangle$

Empirical :

$$\langle r_p^2 \rangle^{1/2} = 0.840 \pm 0.004\,\mathrm{fm}$$

$$\langle r_n^2 \rangle = -0.105 \pm 0.006\,\mathrm{fm}^2$$

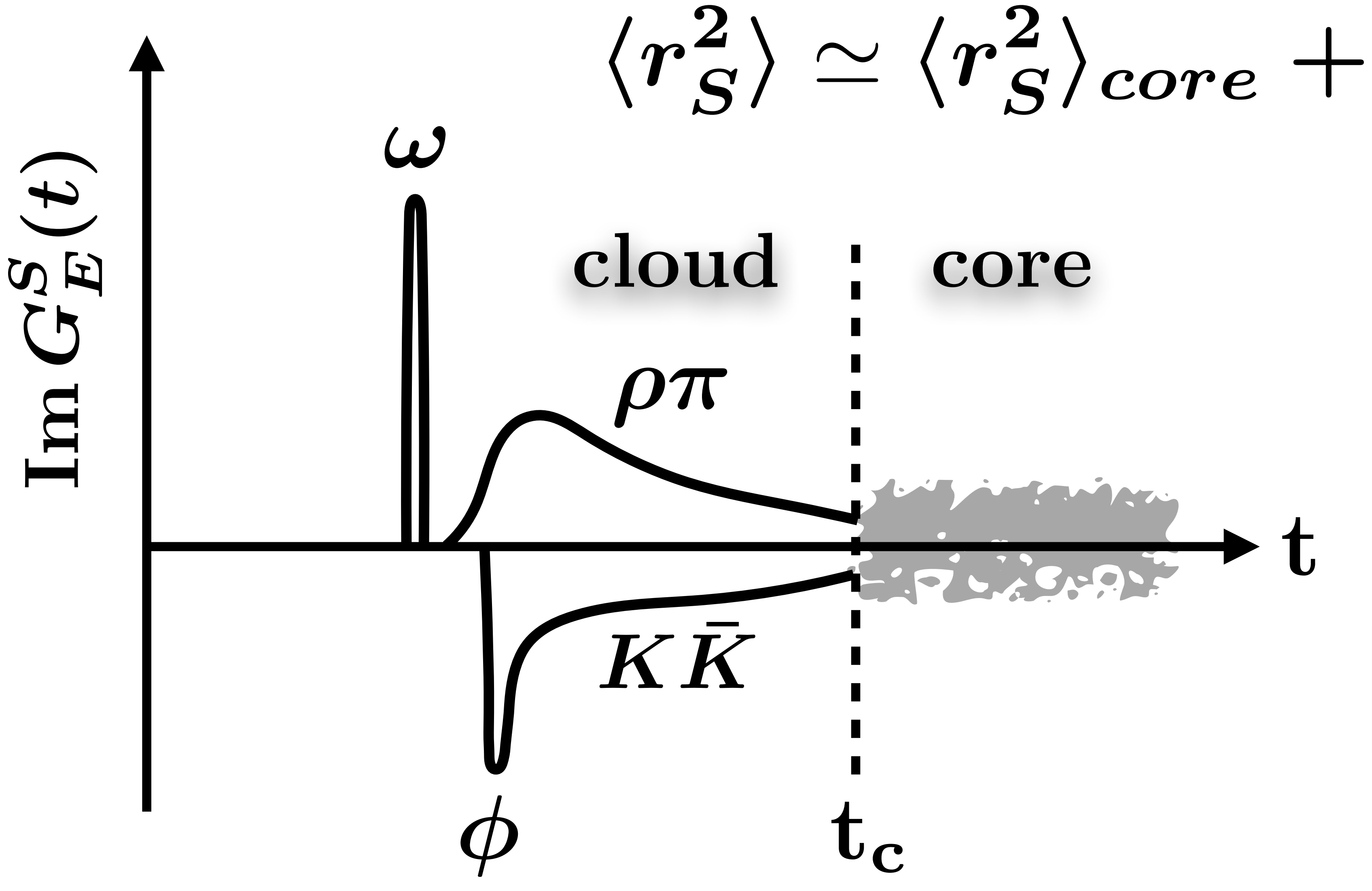
$$\langle r_S^2 \rangle^{1/2} = 0.775 \pm 0.011\,\mathrm{fm}$$

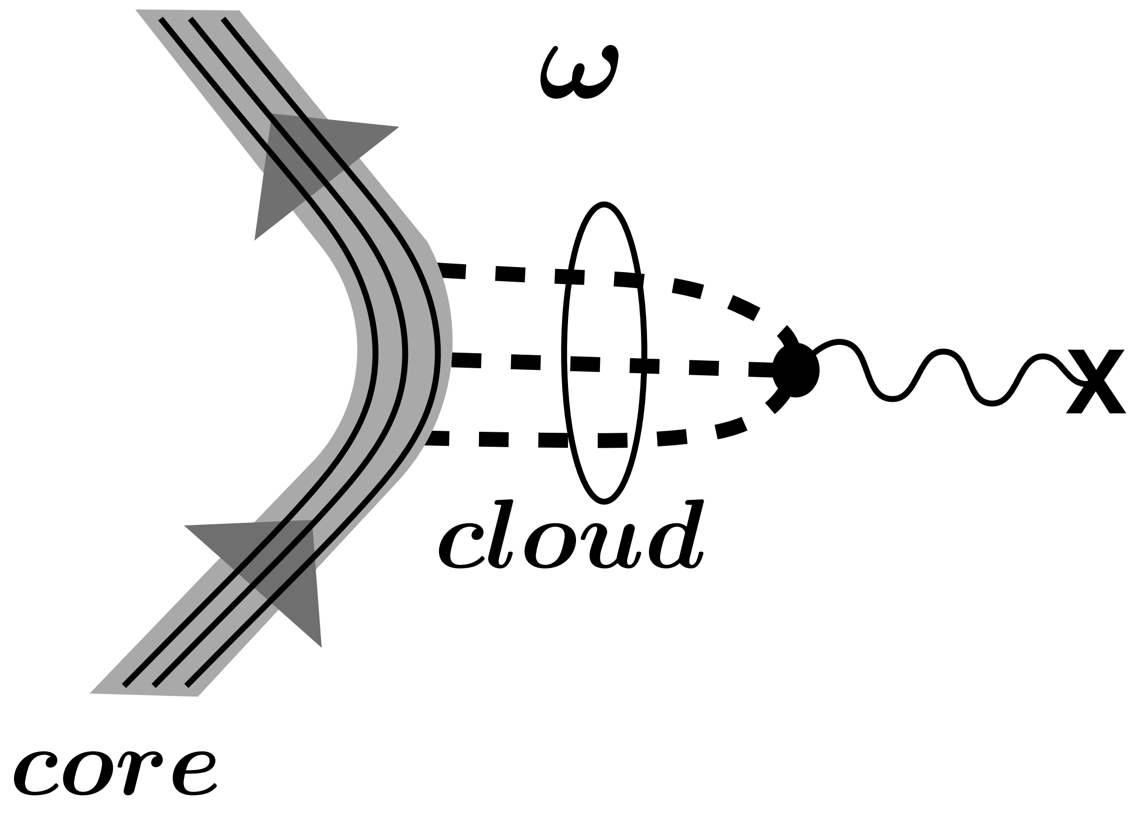
Y.H. Lin,
H.-W. Hammer,
U.-G. Meißner
PRL 128 (2022) 052002

… based on **precision fits** to form factors at both **spacelike and timelike** q^2

- **Simplest Vector Dominance Model:** "cloud" dominated by ω meson

$$\langle r_S^2 \rangle \simeq \langle r_S^2 \rangle_{core} + \frac{6}{m_\omega^2} \;\rightarrow\; \langle r_S^2 \rangle_{core}^{1/2} \simeq 0.47\,\mathrm{fm}$$





- **Detailed analysis using best-fit spectral functions :**

$$\langle r_S^2 \rangle_{core}^{1/2} = \langle r_B^2 \rangle^{1/2} = 0.50 \pm 0.01\,\mathrm{fm}$$

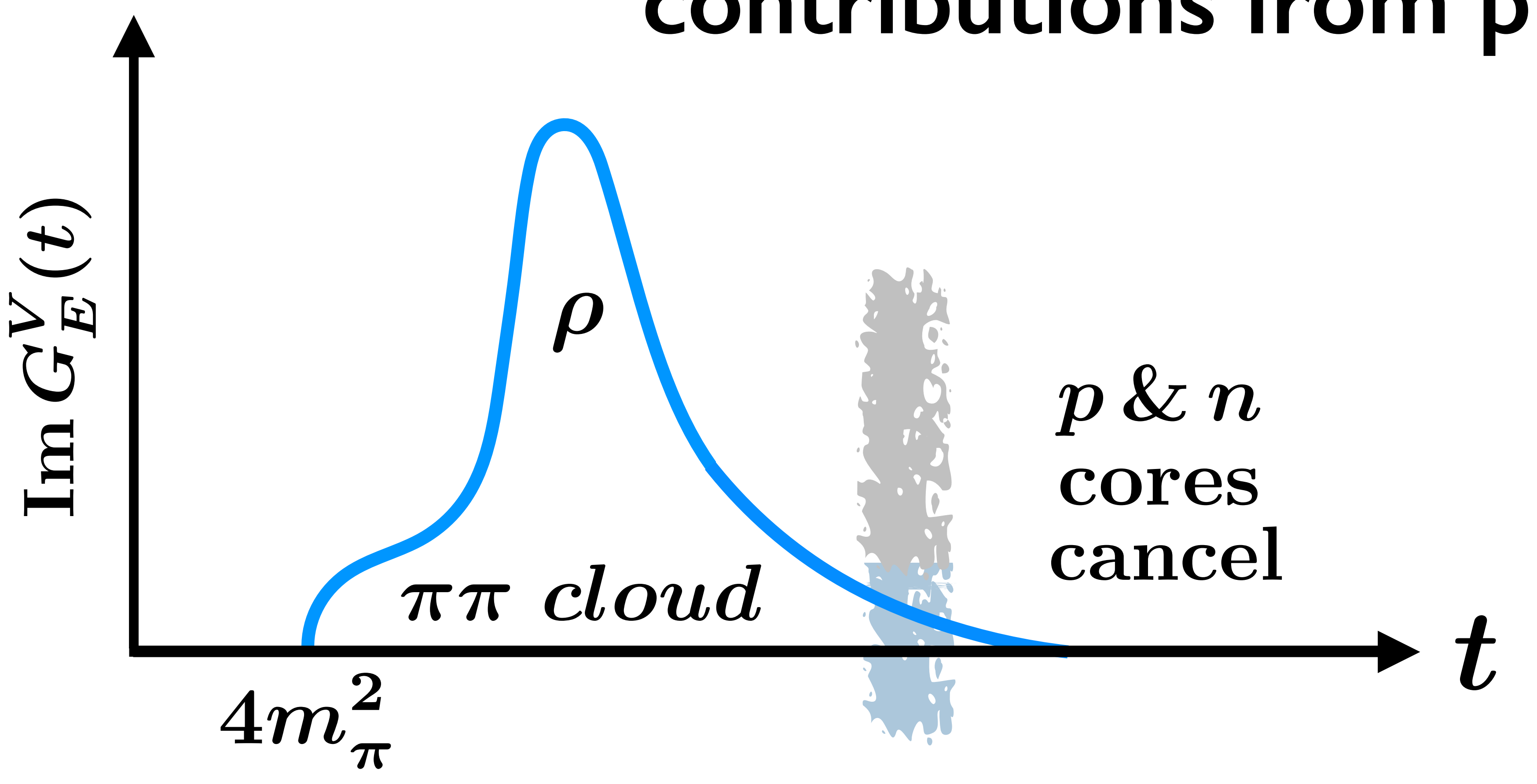
N. Kaiser, W.W.
Phys. Rev.
C110 (2024) 015202

Example II: **ISOVECTOR ELECTRIC FORM FACTOR** of the **NUCLEON**

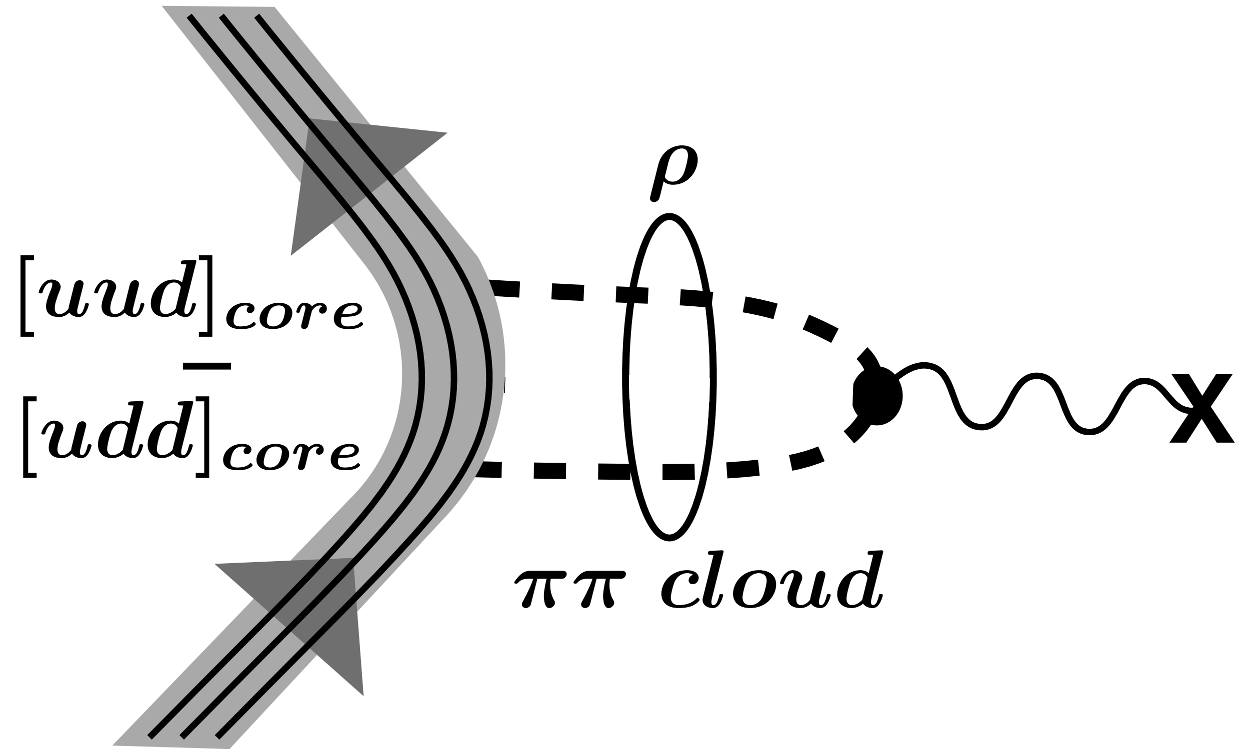
- Isovector electric form factor $G_E^V(q^2) = \frac{1}{2}\left[G_E^p(q^2) - G_E^n(q^2)\right]$ $\quad \langle r_V^2\rangle = \langle r_p^2\rangle - \langle r_n^2\rangle$

Empirical : $\langle r_V^2\rangle^{1/2} = 0.901 \pm 0.009$ fm — Y.H. Lin, H.-W. Hammer, U.-G. Meißner PRL 128 (2022) 052002

… clue and test case : in the limit of exact isospin symmetry, contributions from proton and neutron valence quark cores **CANCEL**



- Detailed analysis using best-fit spectral functions :



$$\langle r_V^2\rangle_{core} = \langle r_p^2\rangle_{core} - \langle r_n^2\rangle_{core} = -0.025\ \mathrm{fm}^2$$ … almost vanishing

N. Kaiser, W.W. Phys. Rev. C110 (2024) 015202

- **Isovector charge radius** almost entirely determined by **two-pion cloud**

Example III: **ISOVECTOR AXIAL FORM FACTOR** of the **NUCLEON**

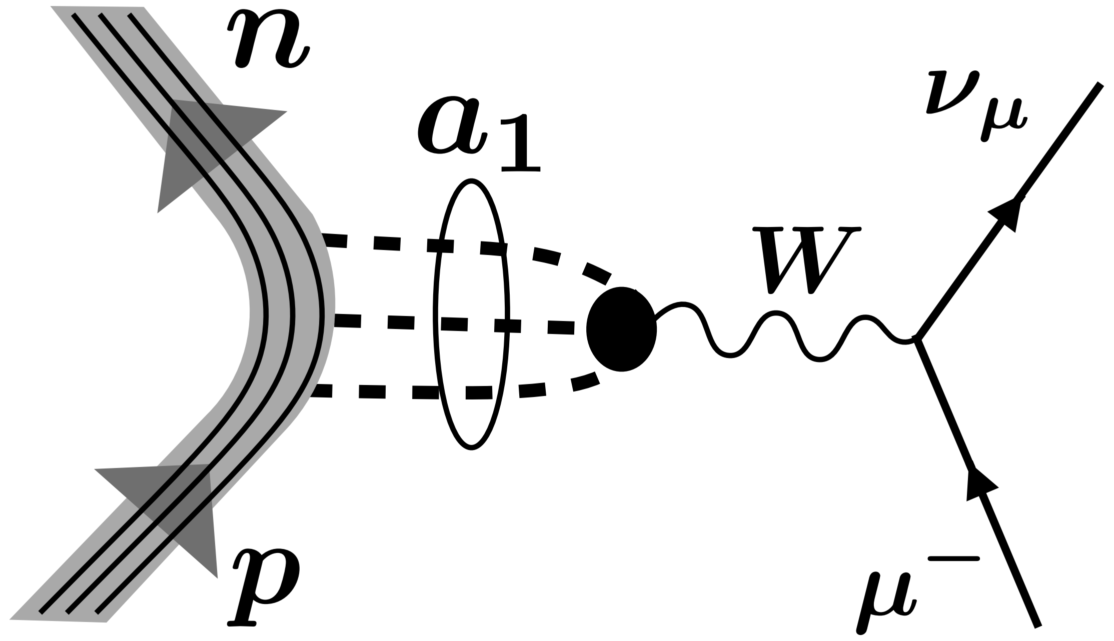
- **Axial form factor** $G_A(q^2) = g_A \left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \dots\right]$

R.J. Hill, P. Kammel, W.C. Marciano, A. Sirlin
Rep. Prog. Phys. 81 (2018) 096301

Empirical :

a) $\langle r_A^2\rangle = 0.454 \pm 0.013\ \mathrm{fm}^2$

(from νd scattering and $e\,p \to e\,n\pi^+$ dipole fits)



b) $\langle r_A^2\rangle = 0.46 \pm 0.16\ \mathrm{fm}^2$

(from μp capture and νd scattering analysis)

Axial radius significantly smaller than proton charge radius $\left(\langle r_p^2\rangle = 0.71 \pm 0.01\,\mathrm{fm}^2\right)$

- **Detailed analysis using three-pion spectrum dominated by broad a_1 meson :**

$$\langle r_A^2\rangle = \langle r_A^2\rangle_{core} + \frac{6}{m_a^2}(1+\delta_a) \qquad \delta_a = -\frac{m_a^3}{\pi}\int_{9m_\pi^2}^{t_{max}} dt \frac{\Gamma_a(t)}{t^2(t-m_a^2)}$$

➡ $\langle r_A^2\rangle_{core}^{1/2} = 0.53 \pm 0.02\ \mathrm{fm}$

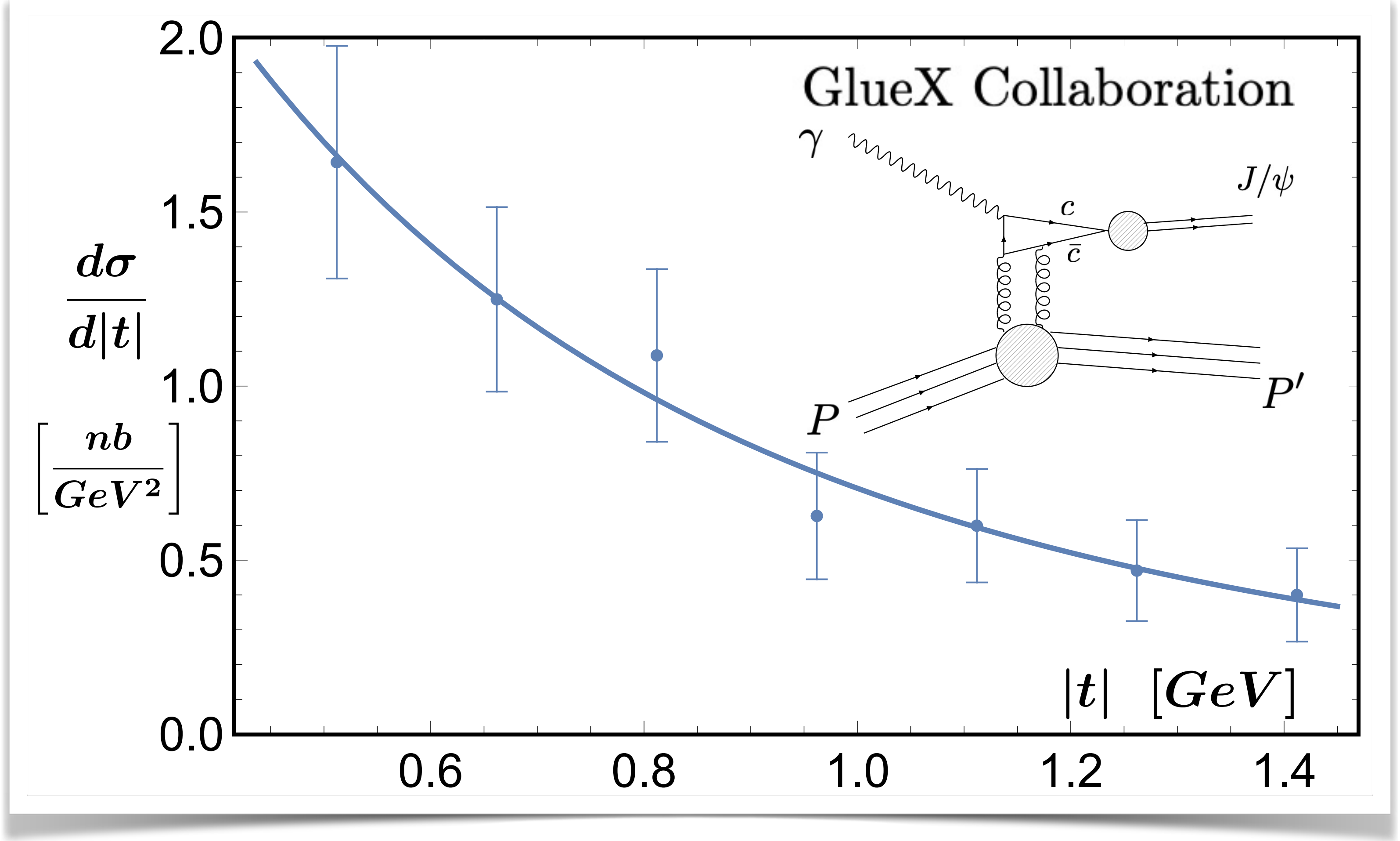
N. Kaiser, W.W.
Phys. Rev. C110 (2024) 015202

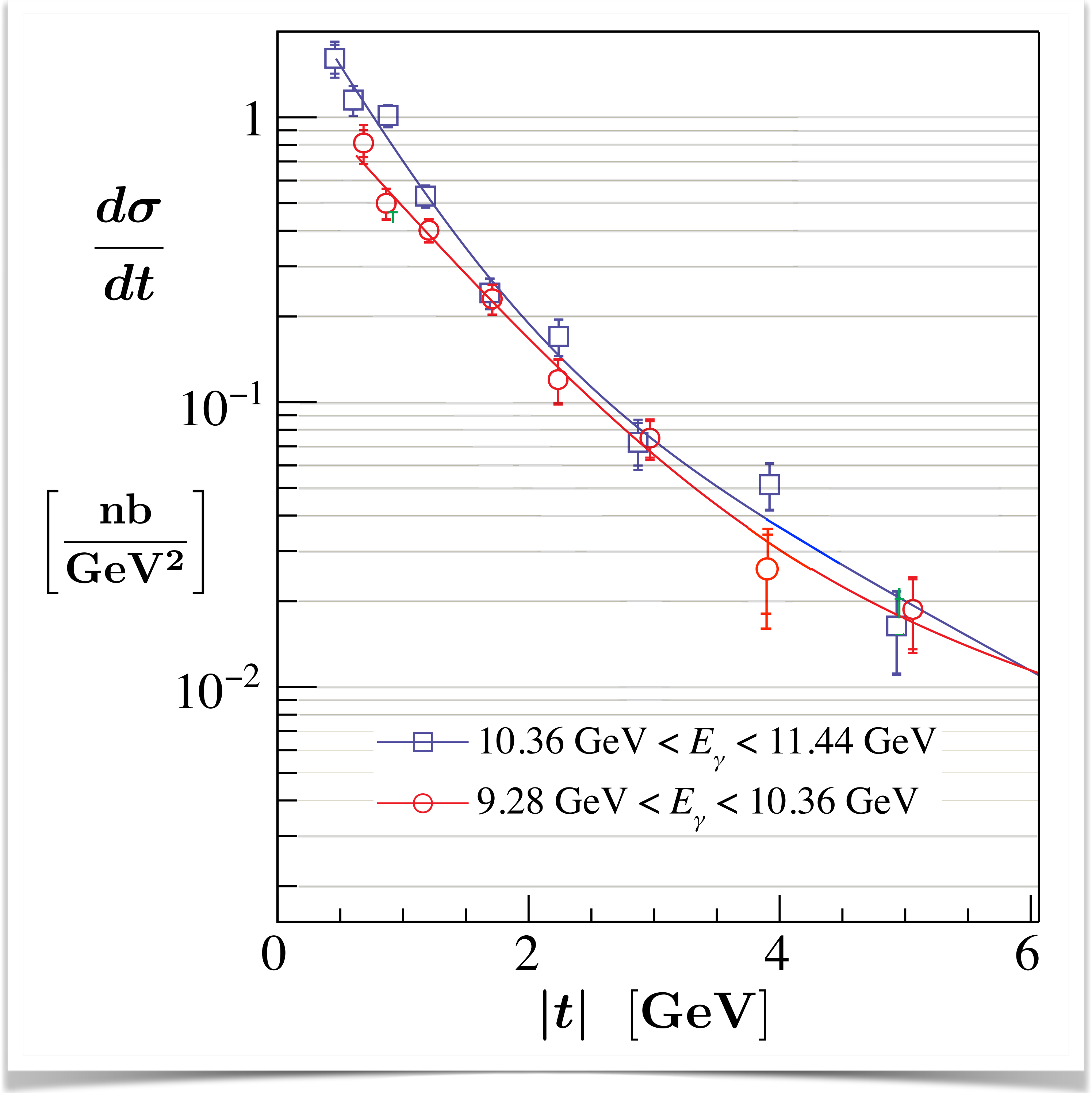
[based on **a)** ; correspondingly larger uncertainty when using **b)**]

Example IV: MASS RADIUS of the NUCLEON

- Mass ("gravitational") form factor
- Trace of QCD energy-momentum tensor

$$G_m(q^2) = \langle P'|T^\mu_\mu|P\rangle = \langle P'|\frac{\beta(g)}{2g}G^{\mu\nu}_a G^a_{\mu\nu} + m_q(\bar{u}u + \bar{d}d) + m_s\bar{s}s|P\rangle$$





$$G_m(0) = M_N \simeq 0.94\,\text{GeV}$$

$$M_N = M_0 + \sigma_N + \sigma_s$$

$$(M_0 \gtrsim 0.9\,M_N)$$

$$\langle r_m^2\rangle = \frac{6}{M_N}\frac{dG_m(q^2)}{dq^2}\Big|_{q^2=0}$$

- Empirical mass radius

$$\langle r_m^2\rangle^{1/2} = (0.55 \pm 0.03)\,\text{fm}$$

$$\langle r_m^2\rangle^{1/2} = (0.53 \pm 0.04)\,\text{fm}$$

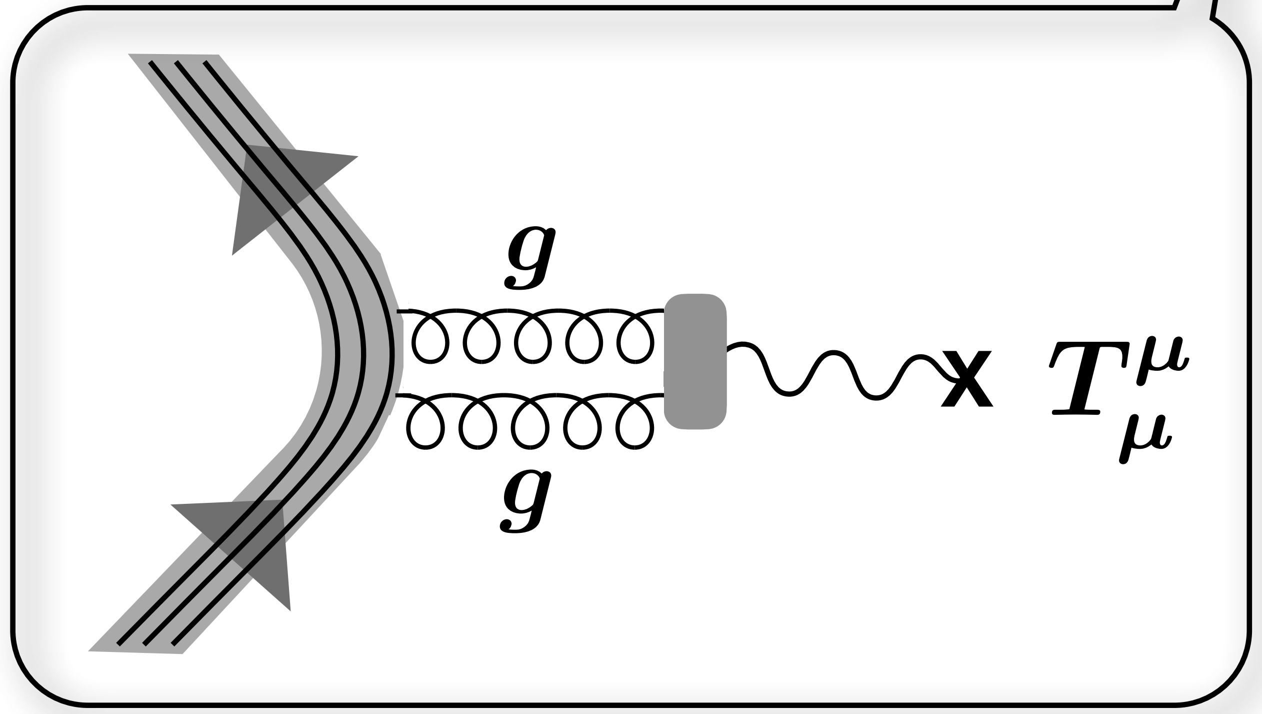
D. Kharzeev : Phys. Rev. D104 (2021) 054015

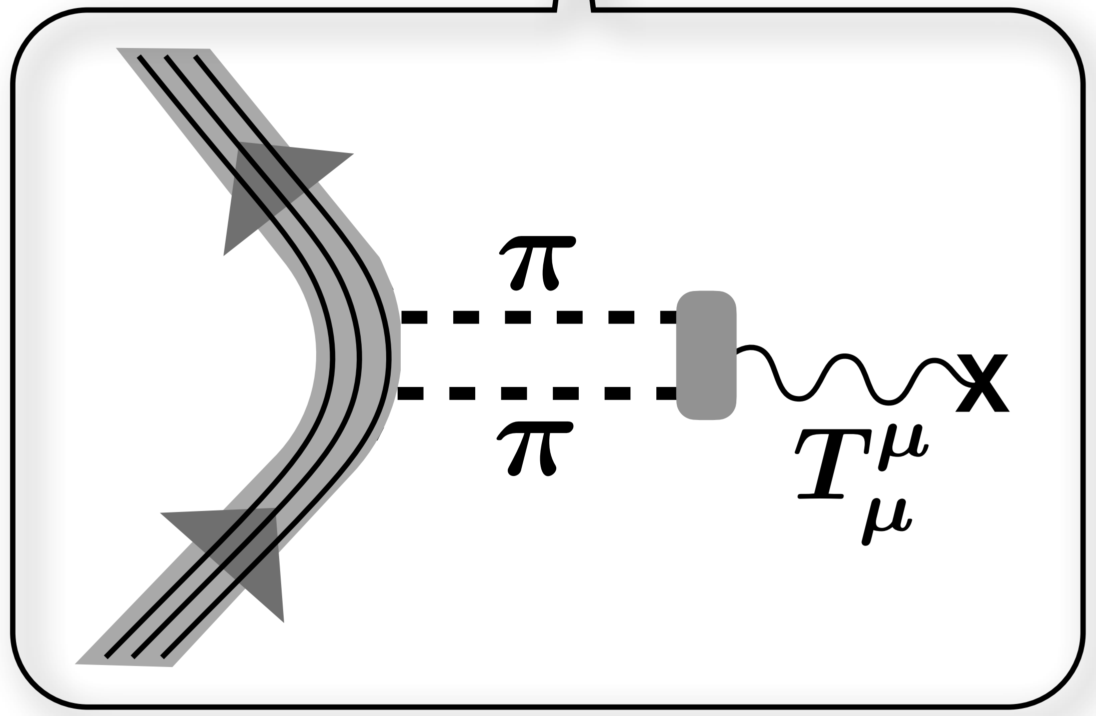
Recent GlueX update: S. Adhikari et al. ; arXiv:2304.03845

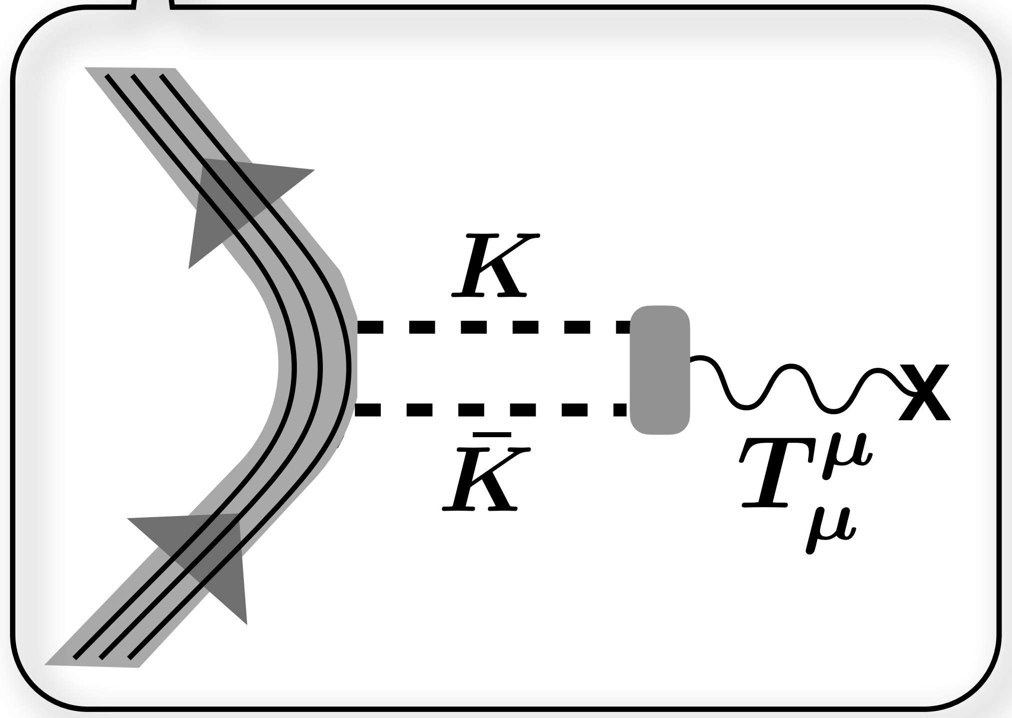
Example IV: MASS RADIUS of the NUCLEON (contd.)

- Core (gluon) dominance plus small corrections from sigma terms

$$\langle r_m^2\rangle = \left[\frac{M_0}{M_N}\langle r_m^2\rangle_{core} + \frac{\sigma_N}{M_N}\langle r_{\pi\pi}^2\rangle + \frac{\sigma_s}{M_N}\langle r_{K\bar{K}}^2\rangle\right]$$







- Estimates of sigma terms and associated radii from Lattice QCD and ChPT

$$\sigma_N \simeq 40 - 60\ \mathrm{MeV}\ ,\ \sigma_s \simeq 30\ \mathrm{MeV} \qquad \langle r_{\pi\pi}^2\rangle^{1/2} \simeq 1.3\ \mathrm{fm}\ ,\ \langle r_{K\bar{K}}^2\rangle \sim (m_\pi/m_K)^2\langle r_{\pi\pi}^2\rangle$$

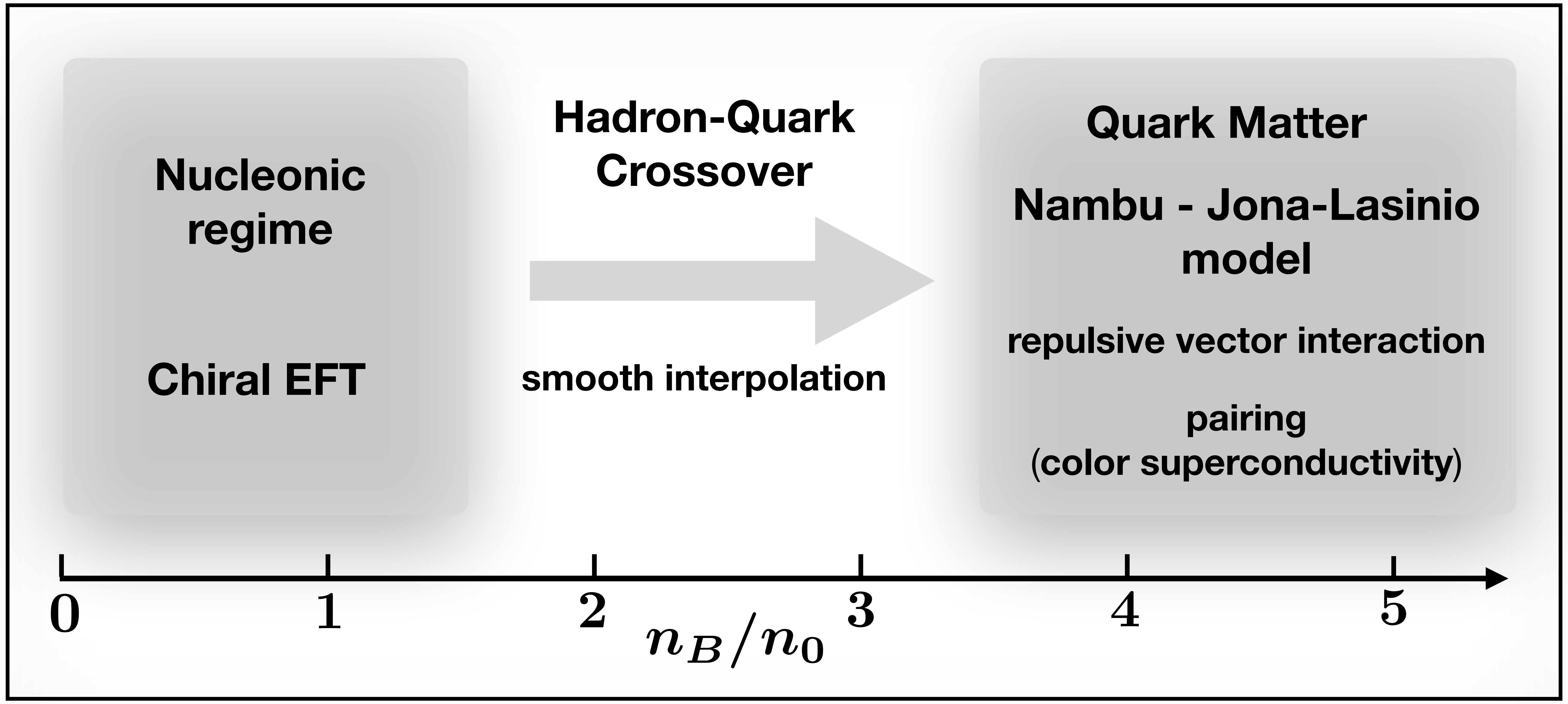
→ $$\langle r_m^2\rangle_{core}^{1/2} = 0.48 \pm 0.05\ \mathrm{fm}$$

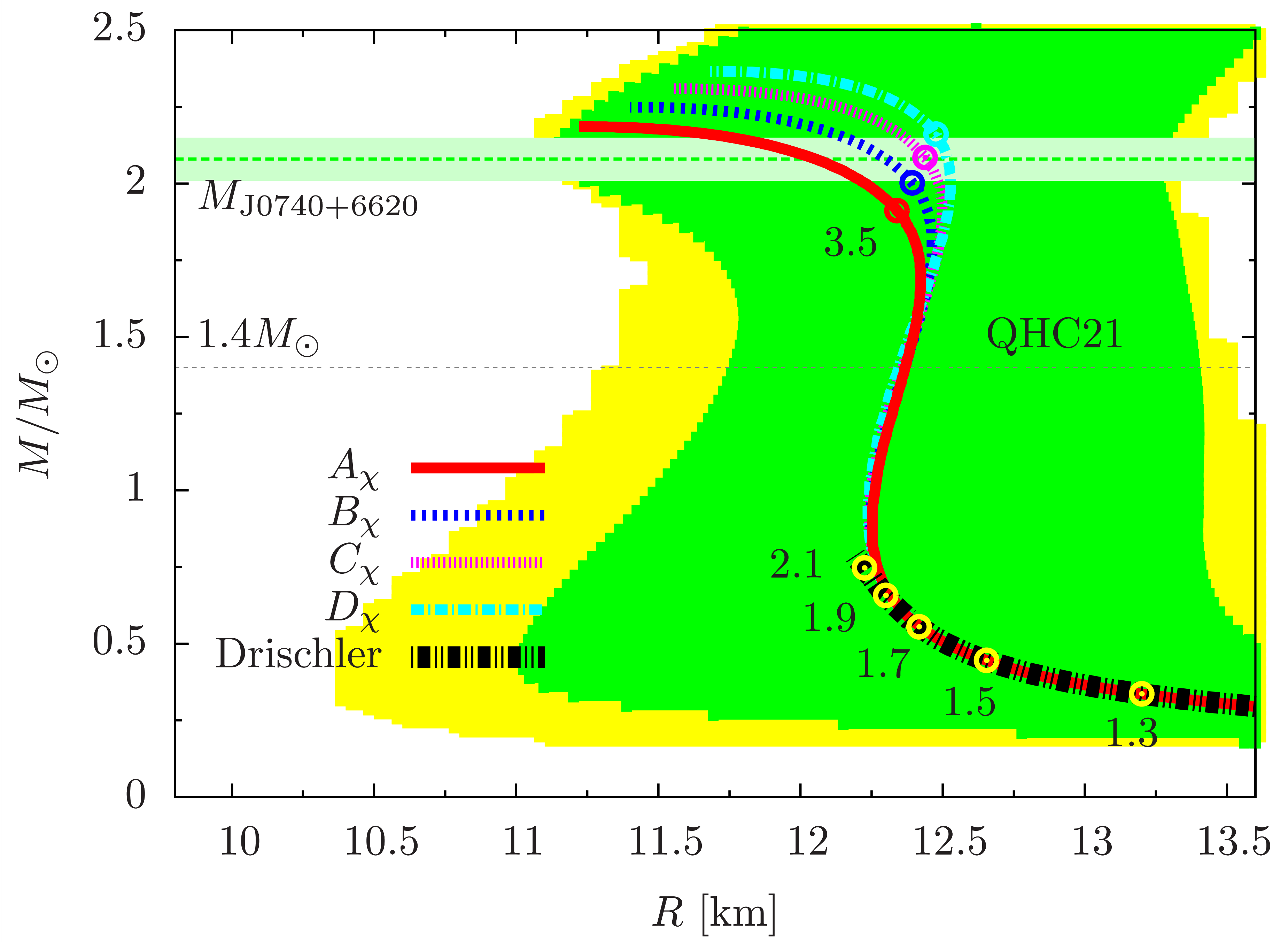
N. Kaiser, W.W.
Phys. Rev. C110 (2024) 015202

COLD MATTER at EXTREME DENSITIES
Hadron - Quark Continuity

- **QHC21 Equation-of-State**

T. Kojo, G. Baym, T. Hatsuda : Astroph. J. 934 (2022) 46



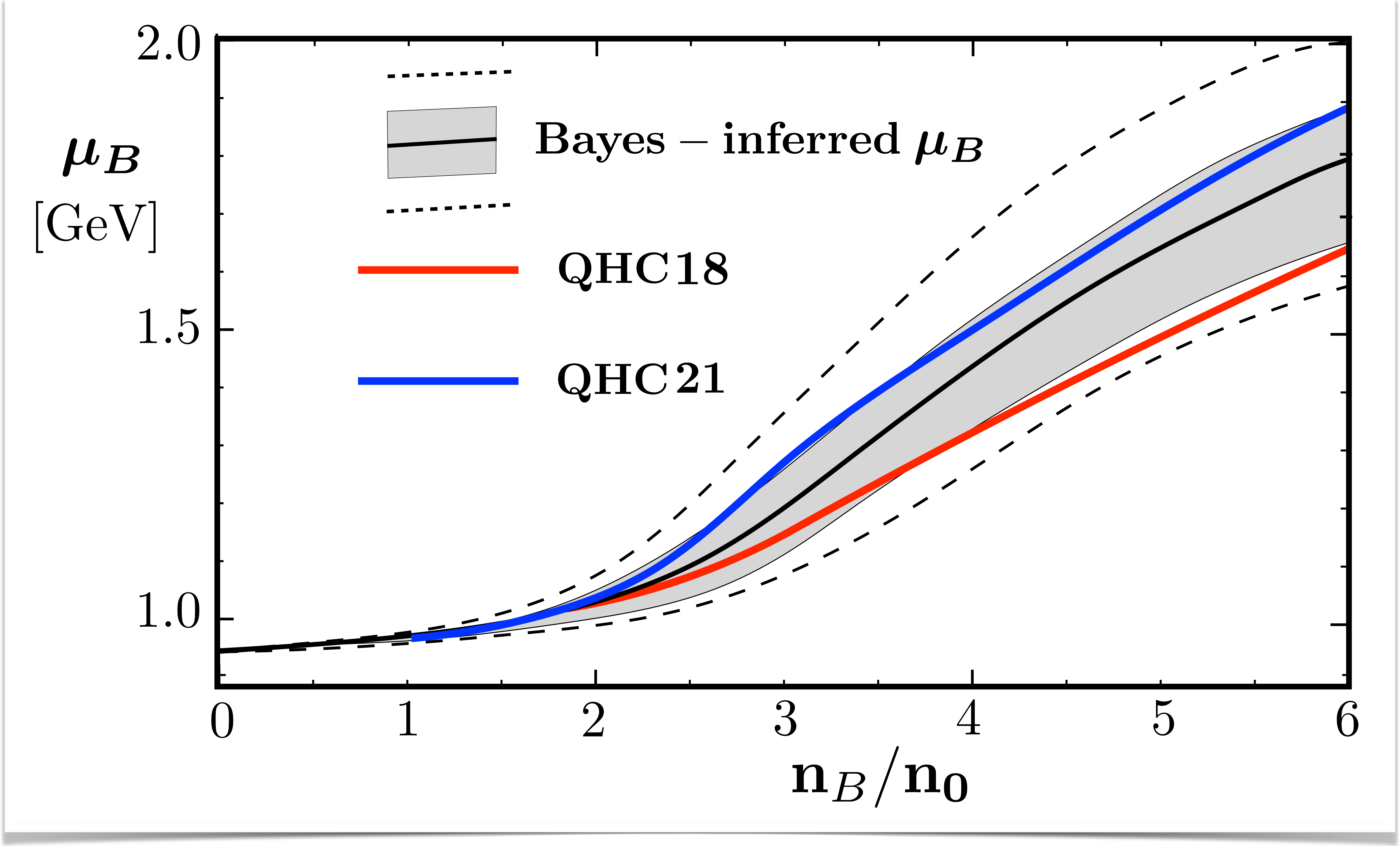


- **NJL model features** : Chiral symmetry restoration   Scalar-pseudoscalar coupling G

  Vector coupling $g_V/G \simeq 1.0 - 1.3$ Pairing interaction $H/G \simeq 1.5 - 1.6$

Outlook : **How Bayes-inferred baryon chemical potential can help improving EoS models**

- Example: QHC equation of state from **QHC18** to **QHC21**



QHC21

T. Kojo, G. Baym, T. Hatsuda
Astroph. J. 934 (2022) 46

increasing ↑ repulsion

QHC18

G. Baym, T. Hatsuda, T Kojo,
P.D. Powell, Y. Song, T. Takatsuka
Rept. Prog. Phys. 81 (2018) 056902