Towards a fluid-dynamic description of an entire heavy-ion collision: from the colliding nuclei to the quark-gluon plasma phase

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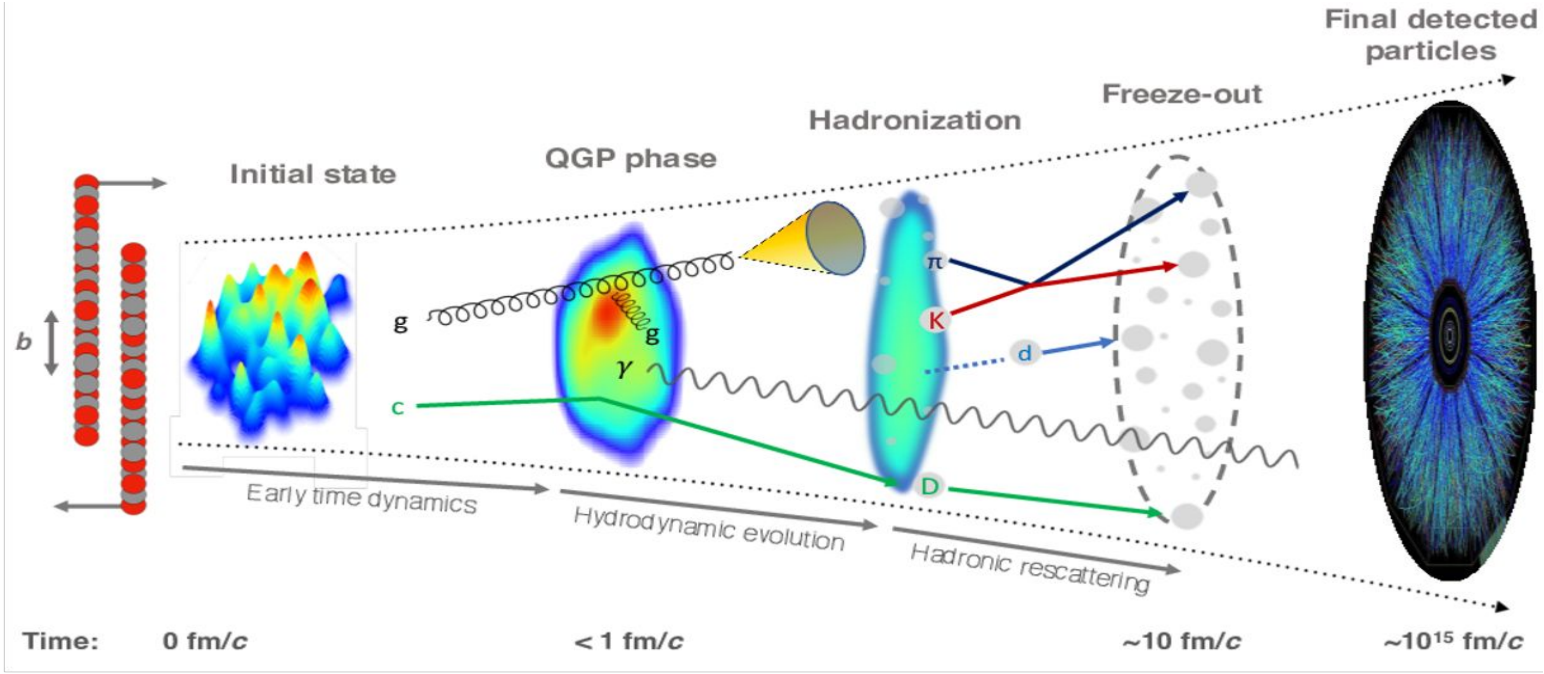
Torino, Celebrating Wanda's birthday, 4.07.2025

Based on 2410.08169 [nucl-th], accepted by PRC
In collaboration with A. Kirchner, E. Grossi and S. Floerchinger

Standard model of heavy-ion collisions



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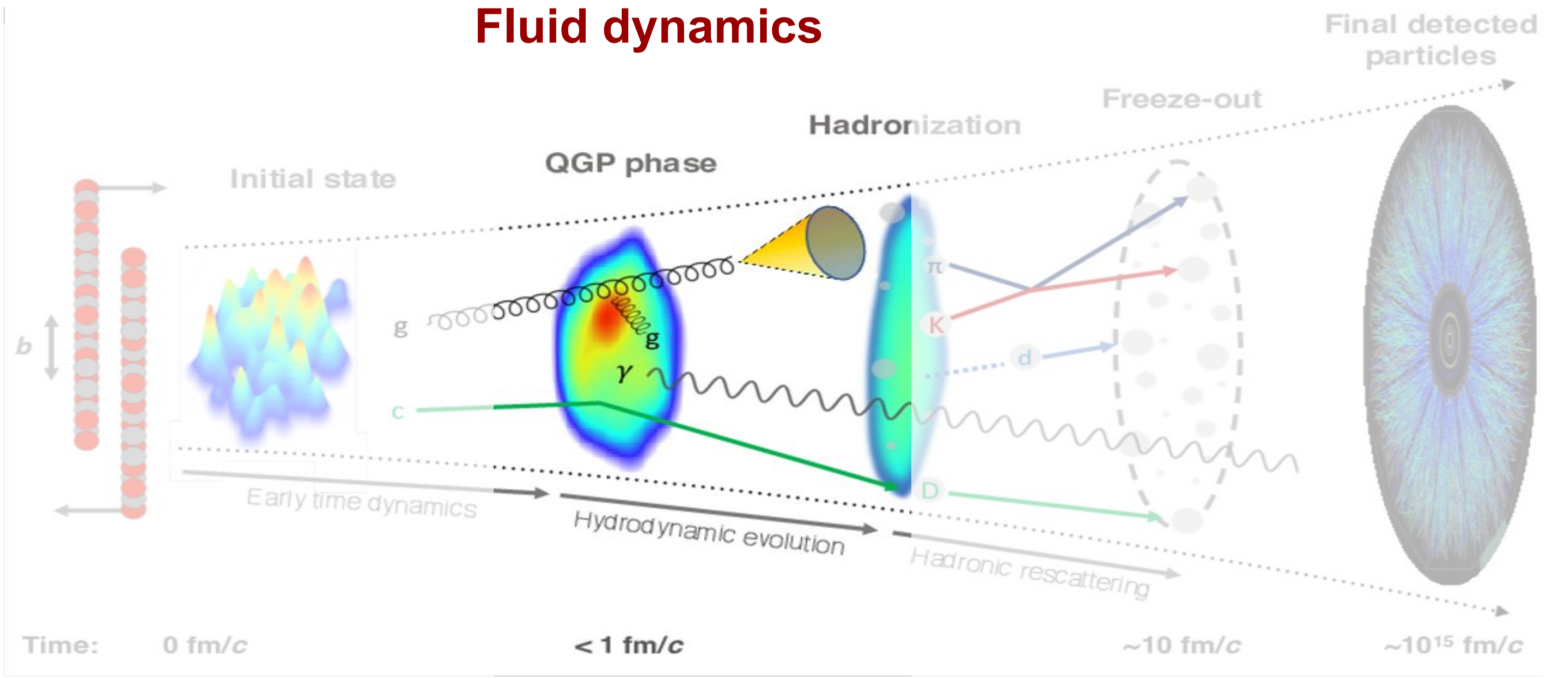

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Fluid dynamics for heavy ions: what do we need?

- Conservation laws:

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu N^\mu = 0$$

- Equation of State (EoS):

$$P = P(T, \mu)$$

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$$\partial_t \pi_b = \ldots \qquad \partial_t \pi^{\mu\nu} = \ldots$$

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- A working numerical code :)

Observables

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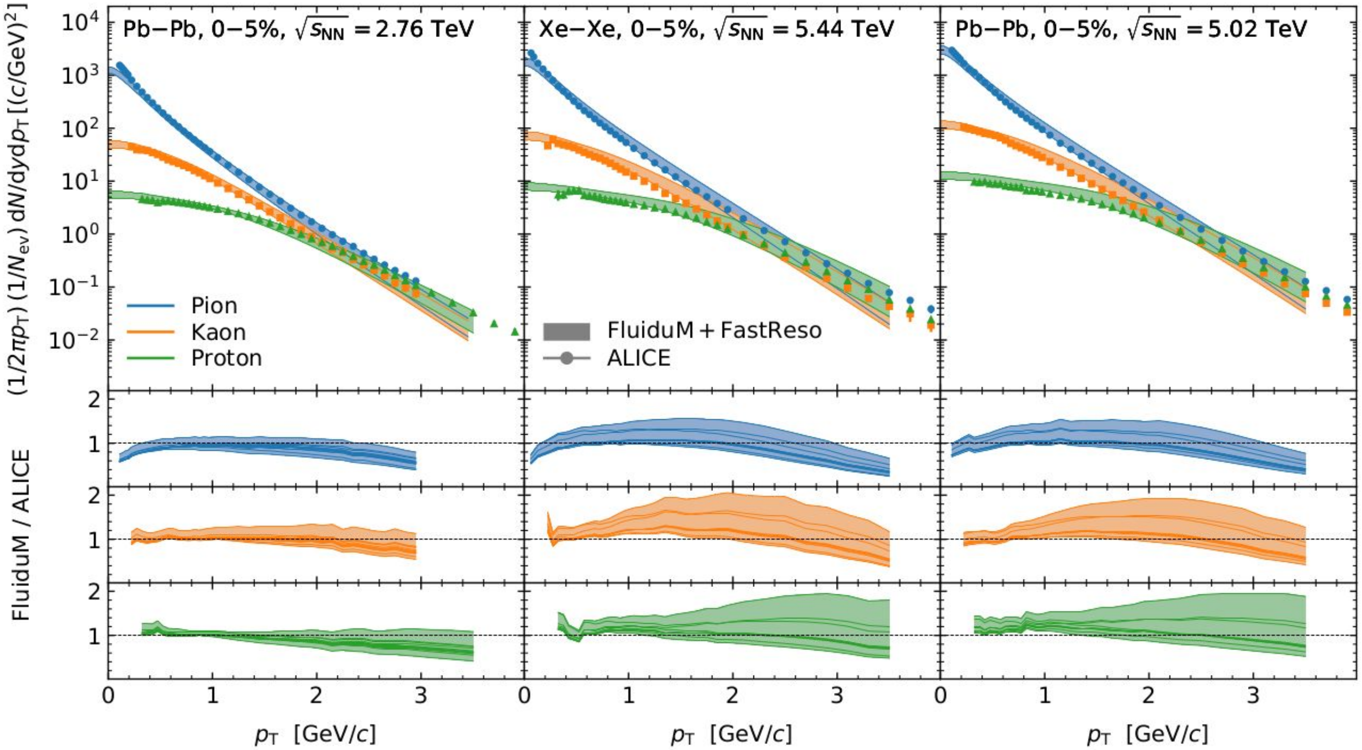
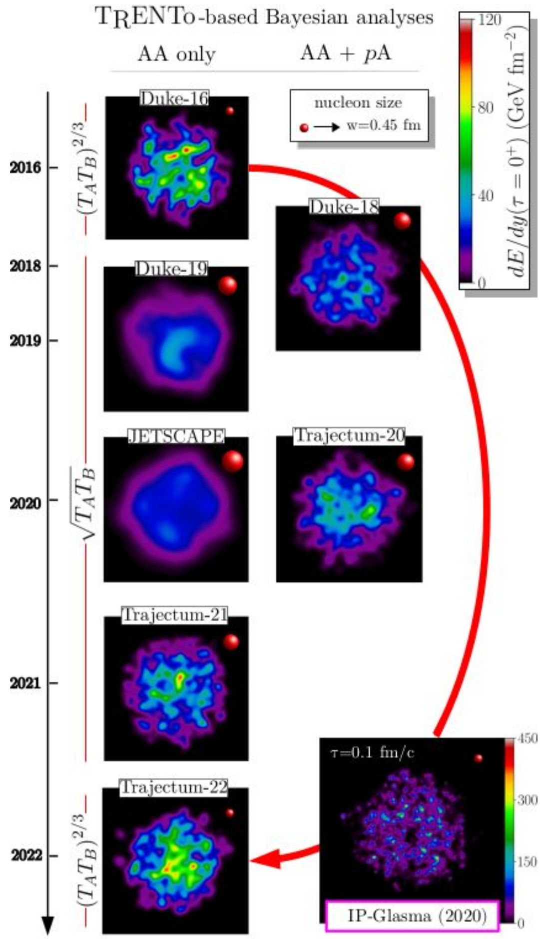


- Agreement between data and fluid dynamics across different energies and systems
- Largest uncertainty from fluid fields in initial state

Initial state models



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- Many models for initial conditions (TrenTo, IP-Glasma, CGC, free-streaming,...)
- Require additional parameters (e.g. Normalization, τhydro ...)
- Fixed by fit/Bayesian analysis

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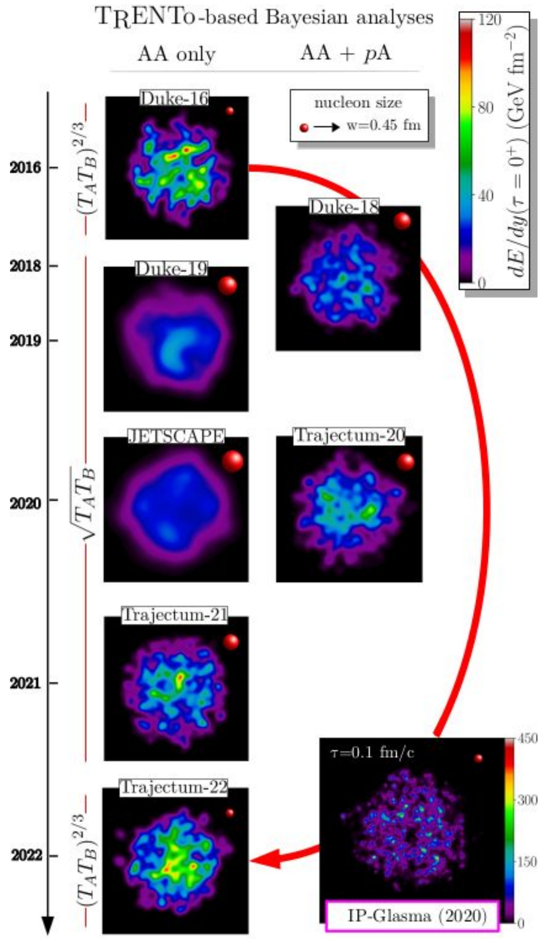

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Can we extend the fluid dynamic description to describe the soft aspects of QCD in the initial state?

See e.g. Weizaecker 1938 (liquid drop model), Landau 1953, Stoecker et al., Katscher et a., Huovinen et al., Karpenko et al. (multifluid)

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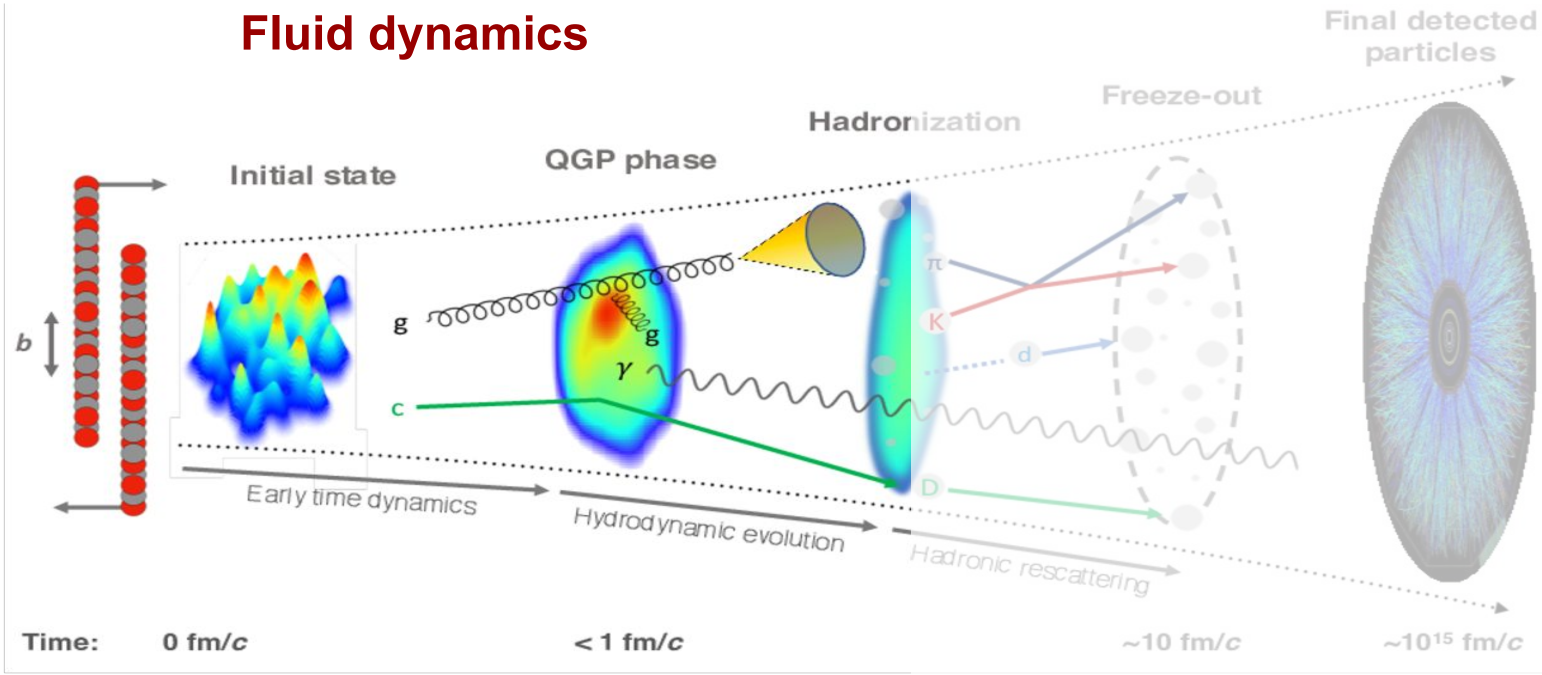

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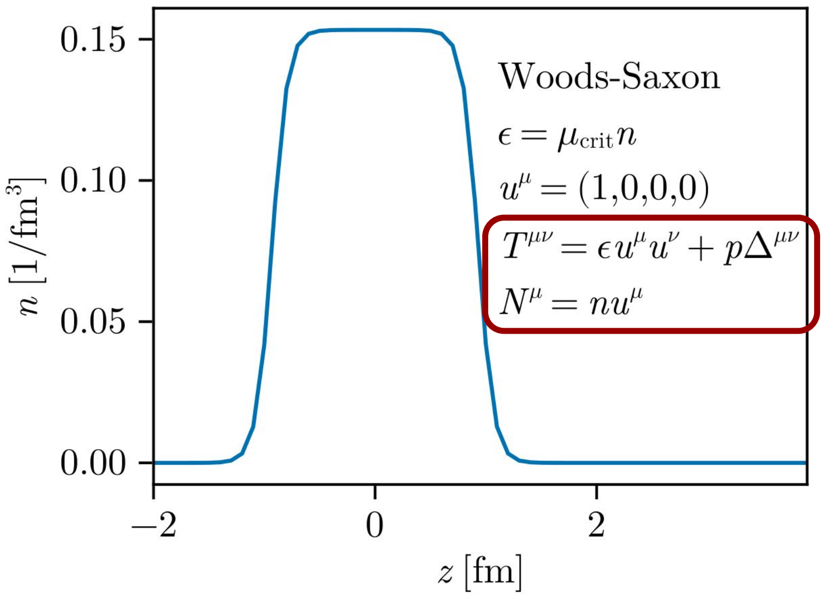
Setup of one nucleus



Fluid description

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu N^\mu = 0$$

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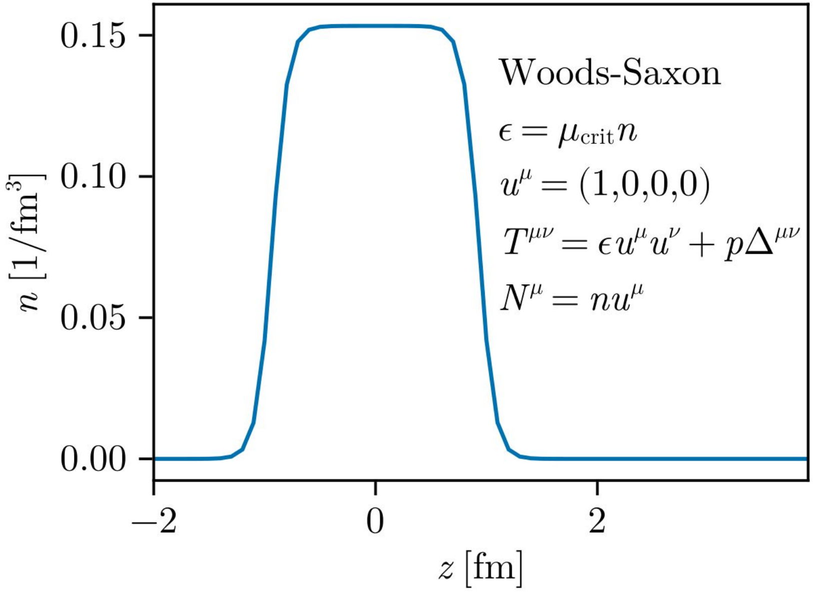

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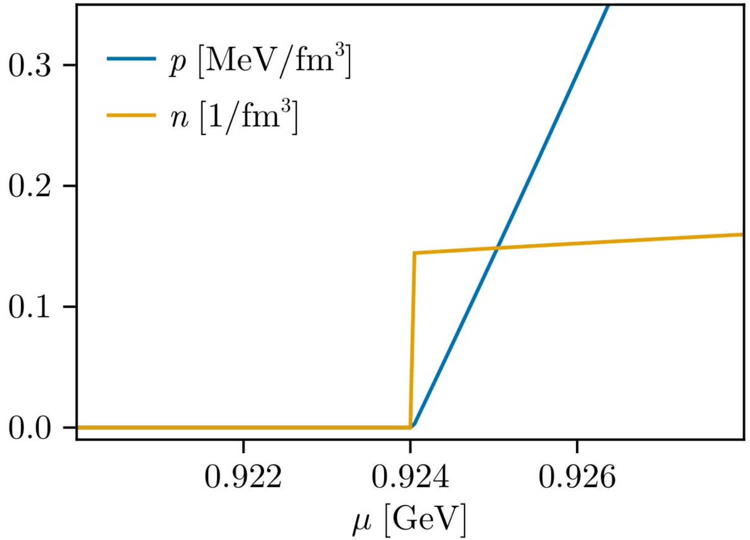
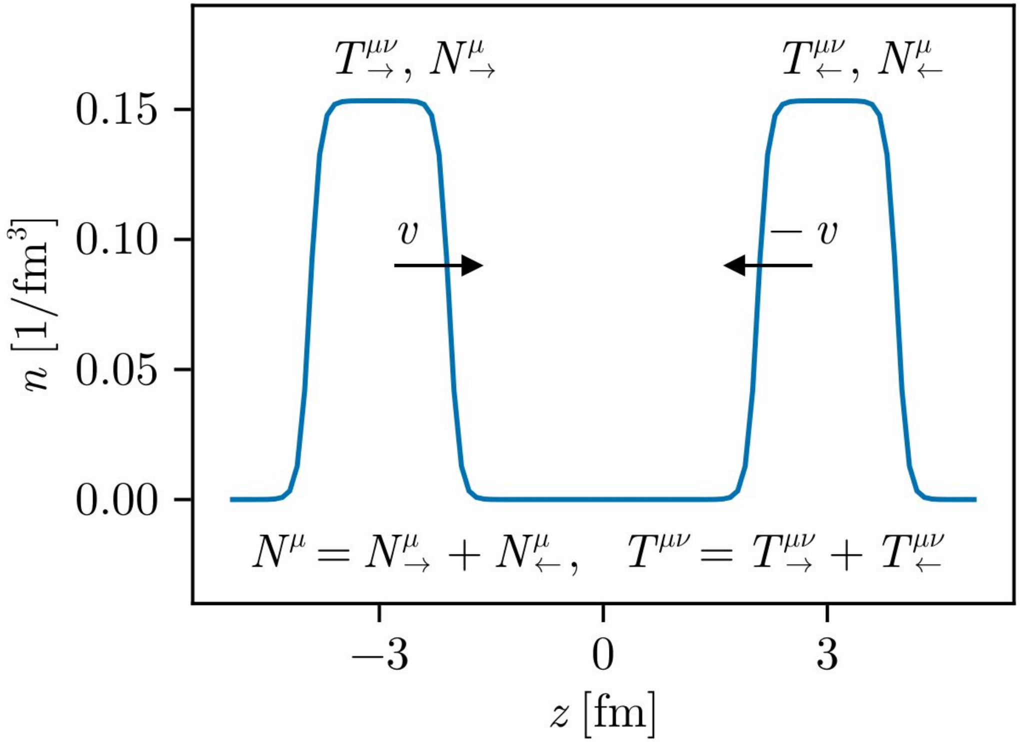


Nuclei at 1st order phase transition line

$$T = 0, \quad \mu = \mu_{\text{crit}}$$

$\Rightarrow$ stability (zero pressure)

Setup of colliding nuclei



System of colliding nuclei

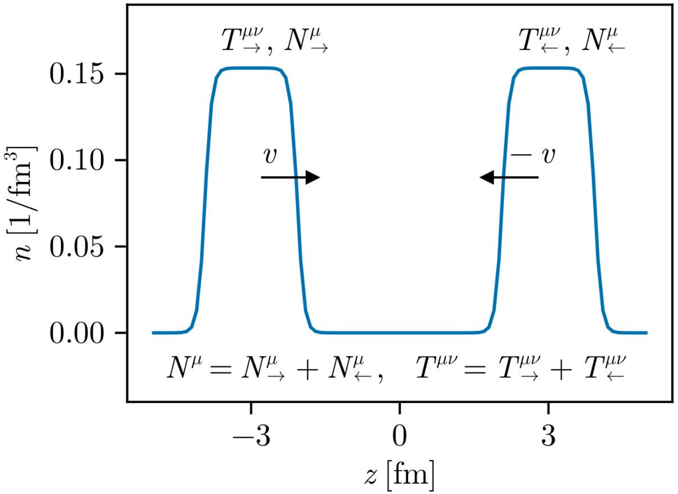
$$T^{\mu\nu} = T^{\mu\nu}_{\rightarrow} + T^{\mu\nu}_{\leftarrow}$$

$$N^{\mu} = N^{\mu}_{\rightarrow} + N^{\mu}_{\leftarrow}$$

Fluid description

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu N^{\mu} = 0$$

Consistency of the fluid-dynamic description



- Nucleus

  $T = 0,\ \mu = \mu_{\mathrm{crit}},\ \epsilon = \mu_{\mathrm{crit}} n, n > 0$

- Vacuum

  $T = 0,\ \mu = \mu_{\mathrm{crit}},\ \epsilon = 0, n = 0$

$$\boxed{T^{\mu\nu} = \epsilon u^\mu u^\nu + (p + \pi_b)\Delta^{\mu\nu} + \pi^{\mu\nu}}$$

Is it valid in the vacuum region?

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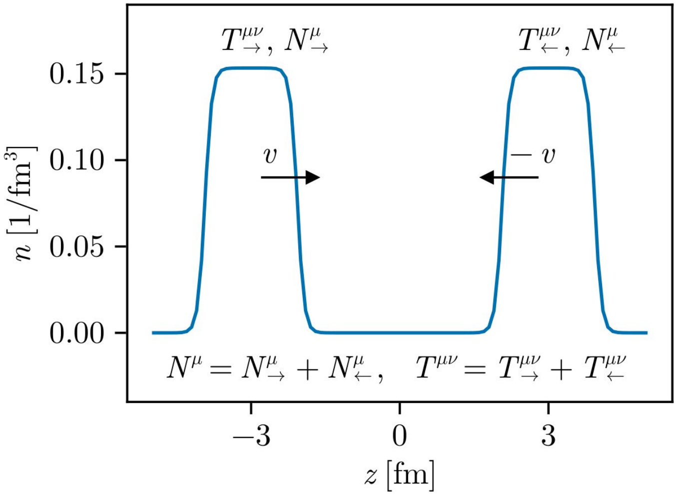

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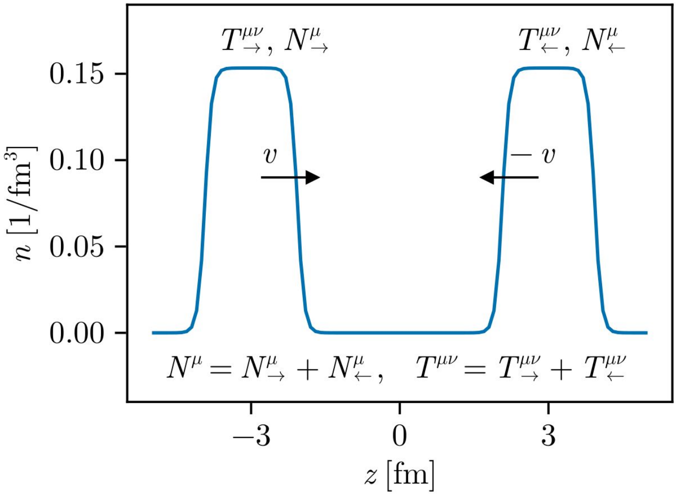

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$$\pi_b = -\zeta\theta$$

$$\pi^{\mu\nu} = -2\eta\sigma^{\mu\nu}$$

= gradients of fluid velocity

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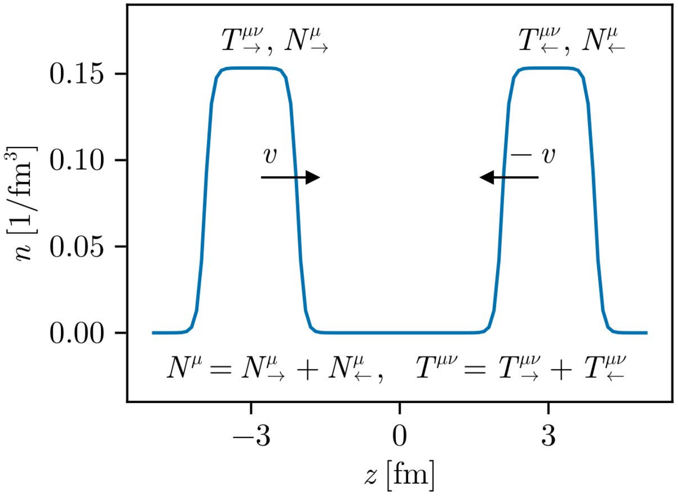

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$\rightarrow$ non-vanishing gradients of fluid velocity in vacuum

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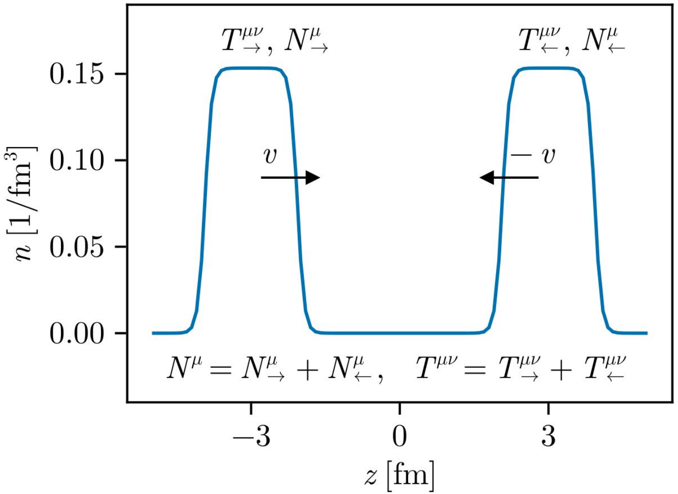

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✓

$$\pi_b = -\zeta\theta$$

$$\pi^{\mu\nu} = -2\eta\sigma^{\mu\nu}$$

→ non-vanishing gradients of fluid velocity in vacuum → $\eta = 0\,, \zeta = 0$

Fluid dynamics

Equation of motion for dissipative fields from entropy principle

- entropy production

$$\nabla_\mu S^\mu = \frac{\pi_b^2}{\zeta T} + \frac{\pi^{\mu\nu}\pi_{\mu\nu}}{2\eta T} + \frac{(\epsilon + p)^2 \nu^\mu \nu_\mu}{\kappa (nT)^2} \geq 0$$

- Transport coefficients η, ζ, κ and relaxation times $\tau_{\mathrm{shear}}, \tau_{\mathrm{bulk}}, \tau_{\mathrm{heat}}$ ensure validity of equations outside of local equilibrium

$$\tau_{\mathrm{bulk}} u^\mu \partial_\mu \pi_b + \pi_b = \ldots$$

Transparency

- Limiting factor of previous ideal single-fluid models
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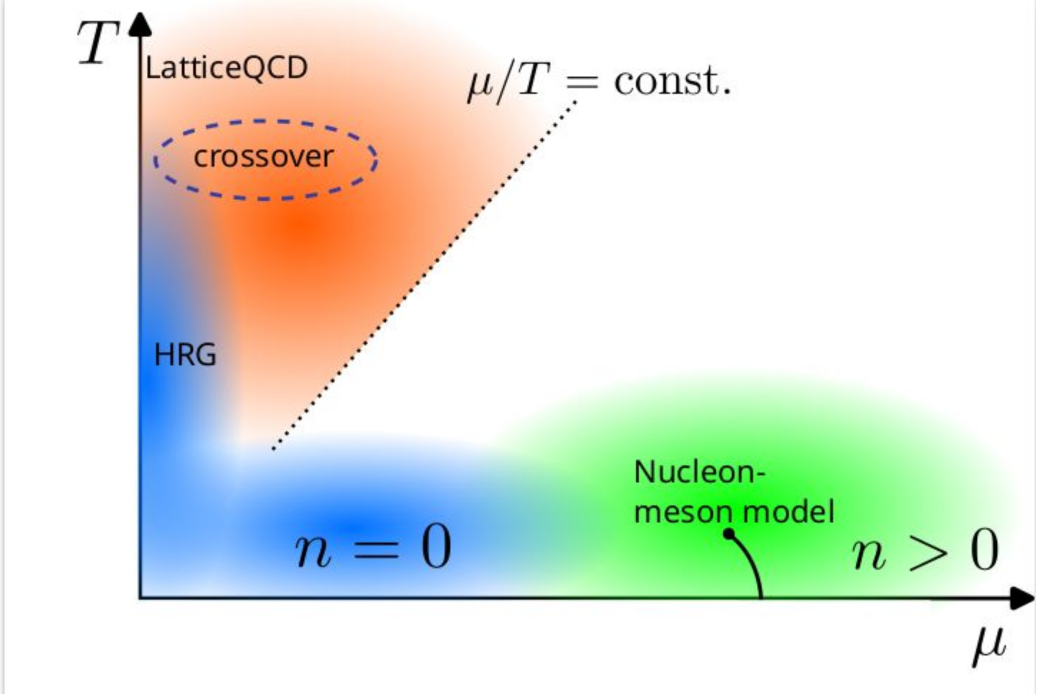
$\Rightarrow$ **Finite relaxation time allows for non-complete transparency in a <u>viscous</u> single-fluid model!**

Equation of State



Composition of

- LQCD
- Hadron Resonance Gas (HRG)
- Walecka model

LQCD and HRG

Lattice EoS

$$p_{\mathrm{LQCD}}(T,\mu) = p_{\mathrm{eq}}(T) + \sum_{n=2,4,6} \frac{\chi_n(T)}{n!}\mu^n$$

Hadron Resonance Gas EoS

$$p_{\mathrm{HRG}}(T,\mu) = \sum_{\mathrm{baryons}} d_i p_{\mathrm{FG}}(T, B_i\mu; m_i) + \sum_{\mathrm{mesons}} d_i p_{\mathrm{BG}}(T, 0; m_i)$$

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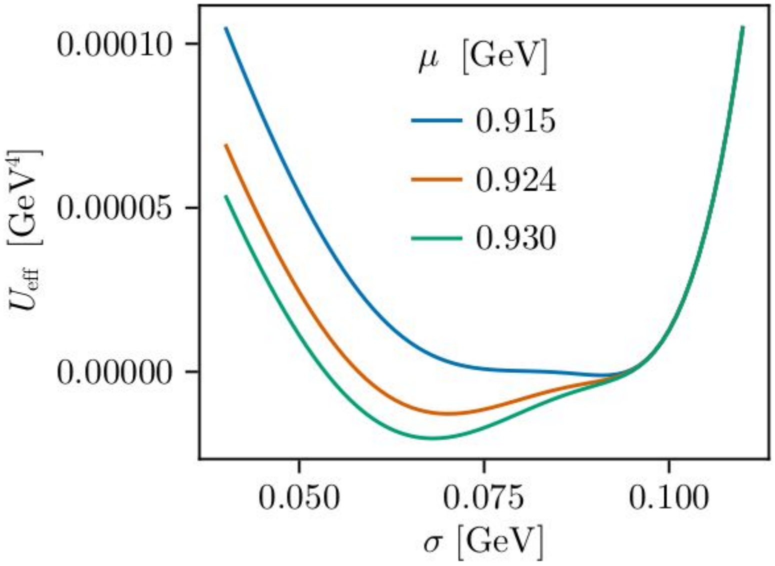
Composition

$$p = \frac{1}{2}(1-f)P_{\mathrm{HRG}} + \frac{1}{2}(1+f)p_{\mathrm{LQCD}}$$

$$f(T,\mu) = \tanh\left(\frac{T - T_{\mathrm{trans}}(\mu)}{\Delta T_{\mathrm{trans}}}\right)$$

Walecka model



- Effective model for **cold, dense nuclear matter**
- Interactions via scalar, pseudo-scalar and vector meson exchange
- Our case: isospin symmetric

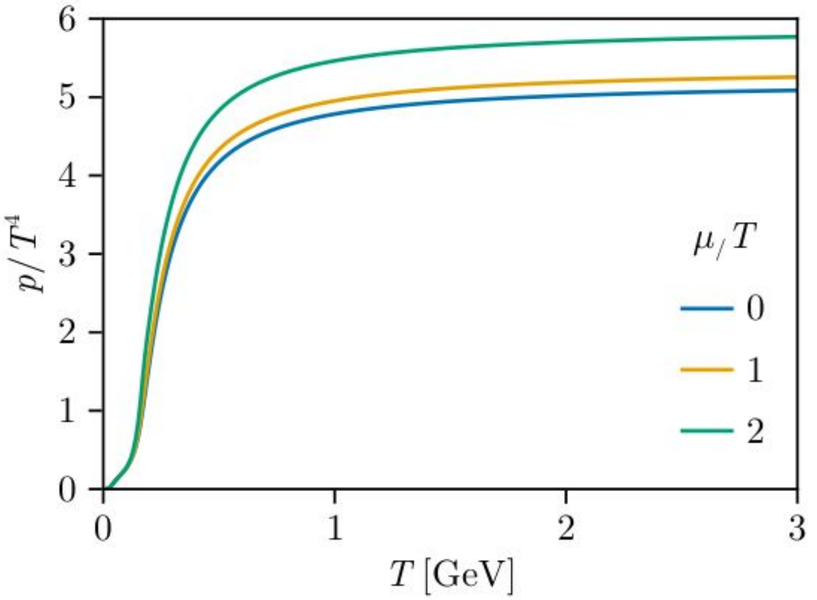
Mean-field approximation

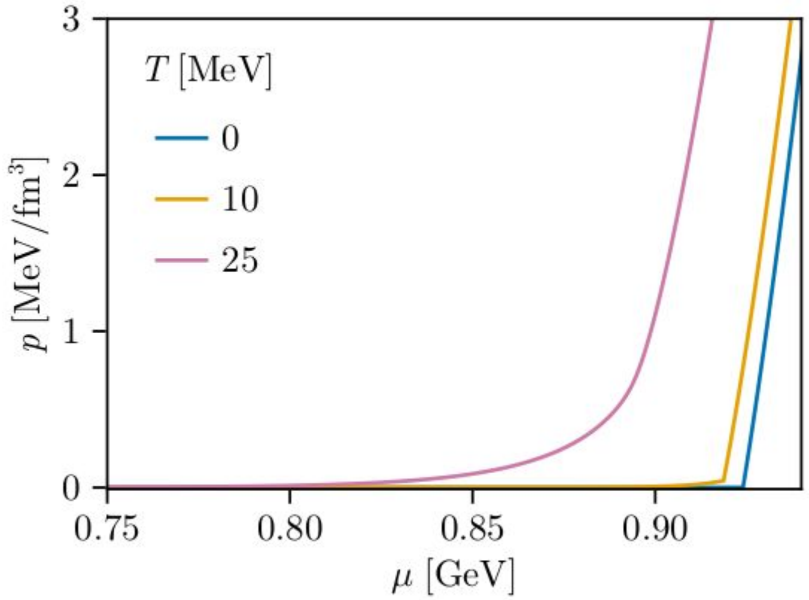
→ Fields self-consistently determined by gap equations

⇒ First-order phase transition captured

EoS composition

- Smooth transition between LQCD & HRG
- Includes first order phase transition





Phase transition dynamics

volume ratio between 2 phases

$$\boxed{r = V''/V'}$$

- Massieu potential $\omega(T, \mu) = p/T$
- Phase coexistence $\omega = (1-r)\omega' + r\omega''$

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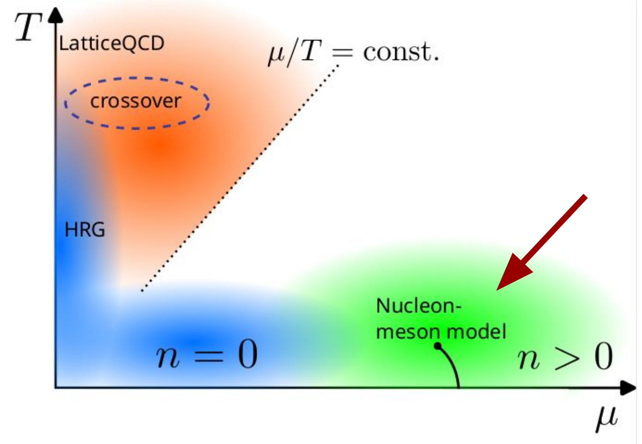
- Massieu potential $\omega(T, \mu) = p/T$
- Phase coexistence $\omega = (1 - r)\omega' + r\omega''$
- (β, α) constrained on phase transition line:
  $\Rightarrow$ Equation for volume ratio parameter

$$u^\mu \partial_\mu r = \quad \ldots$$



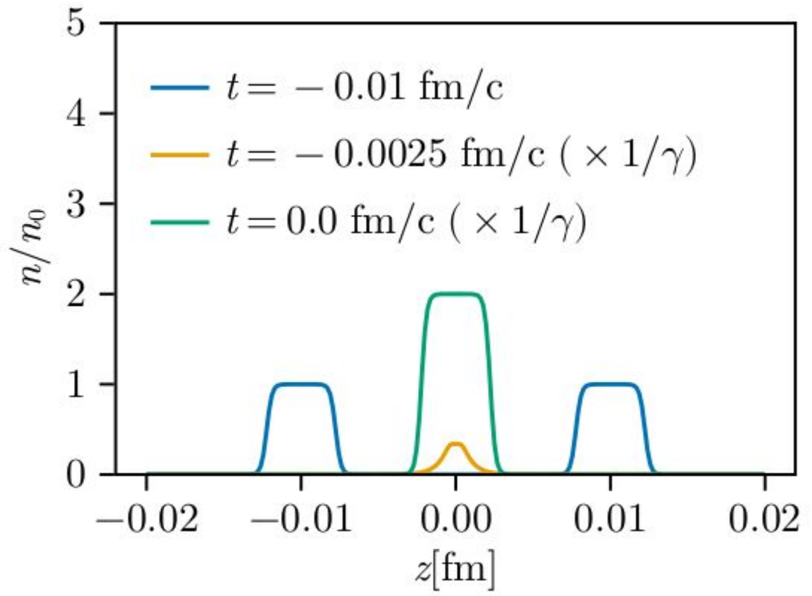
- Modified equation of motion

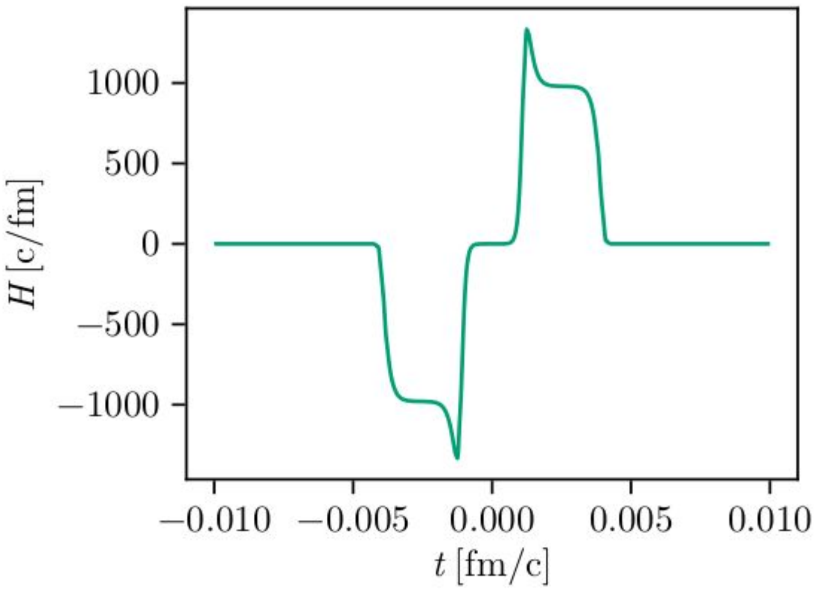
$$\partial_\mu T = \ldots \qquad \partial_\mu \mu = \ldots$$

Application: Hubble universe

- High compression & expansion of nuclear matter during initial moments of collision
- Homogeneous universe filled with nuclear matter
$$\mathrm{ds}^2 = -\mathrm{dt}^2 + \mathrm{a(t)}^2\left(\mathrm{dx}^2 + \mathrm{dy}^2 + \mathrm{dz}^2\right)$$
- Reduction to ODE instead of PDE: $\Phi(t, x, y, z) \rightarrow \Phi(t)$
- Fluid fields reduce to: $\Phi(t) = (T, \mu, \pi\mathrm{bulk})$

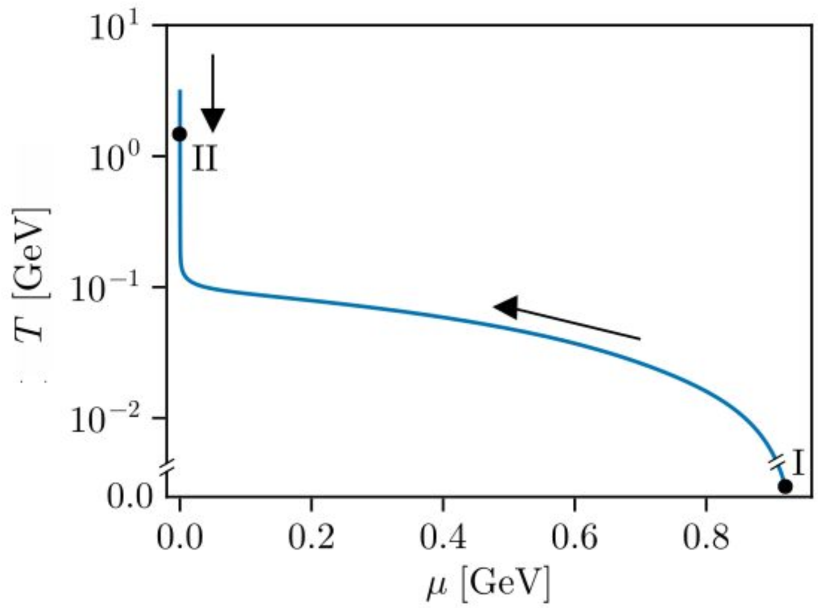
Hubble rate

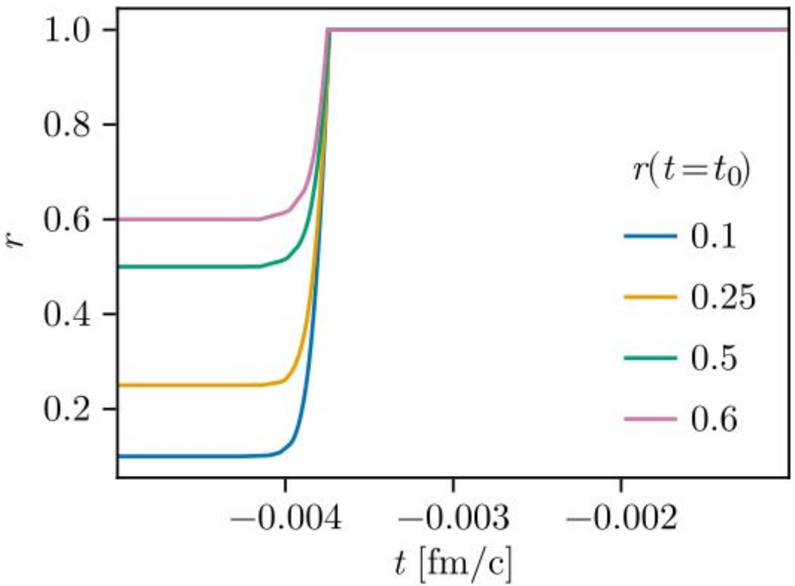




Hubble rate from: $H(t) = \frac{\dot{a}(t)}{a(t)} = -\frac{\partial_t n}{3n}$

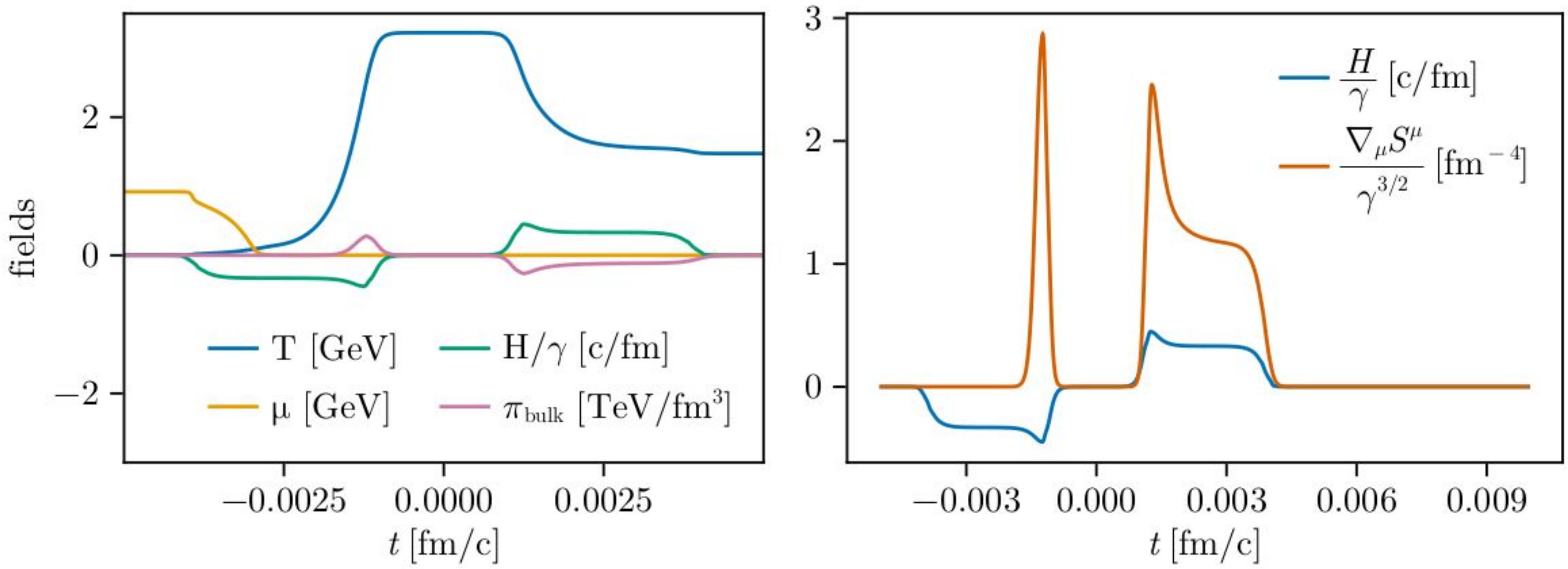
Phase transition





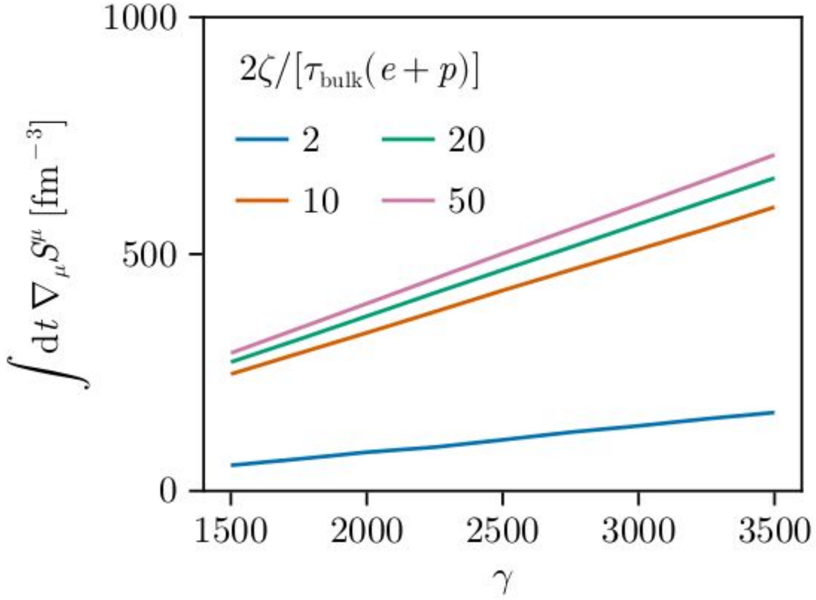
- Initial & final point of trajectory in phase diagram are not same due to viscosity
- Dynamics independent of *r*

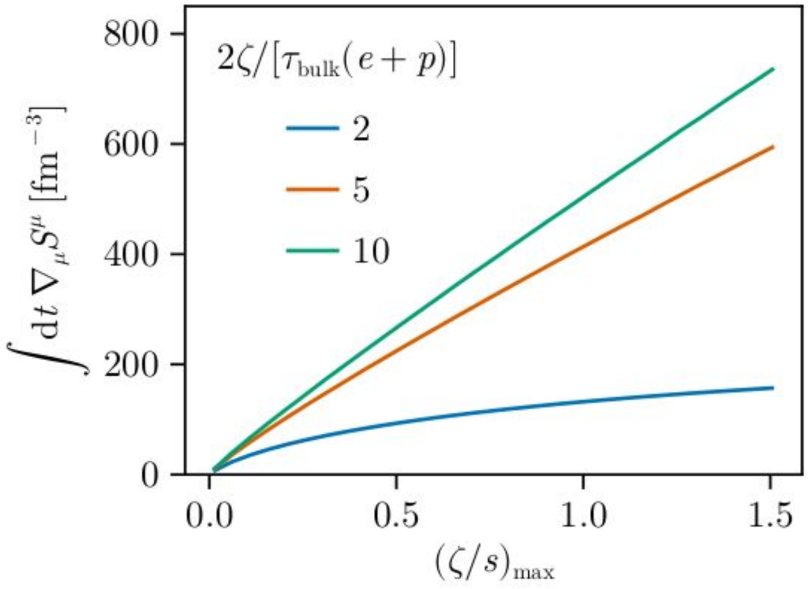
Fields evolution and entropy production



- Viscosity creates heat and viscous bulk pressure
- Entropy production follows Hubble rate after delay

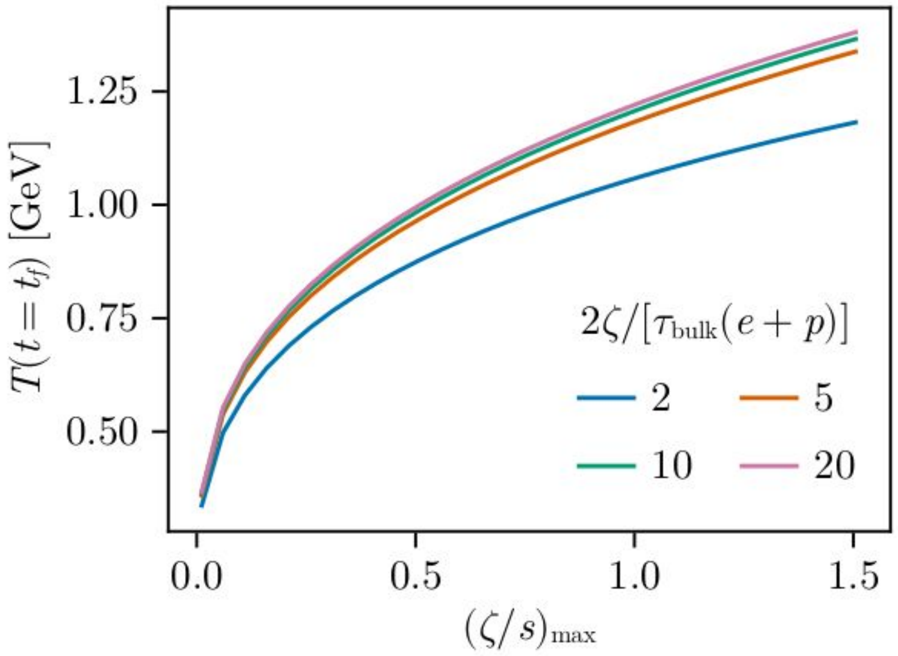
Entropy production





- More entropy produced for higher γ and viscosity
- Dynamics almost independent of $\tau_{\rm bulk}$ for $\tau_{\rm bulk}$ small enough

Final temperature



- Final temperature scales with viscosity & inverse relaxation time
- $T(t = tf\,)$ independent of relaxation time for $\tau_{\mathbf{bulk}}$ small enough

Conclusions

First steps toward fluid-dynamic description:

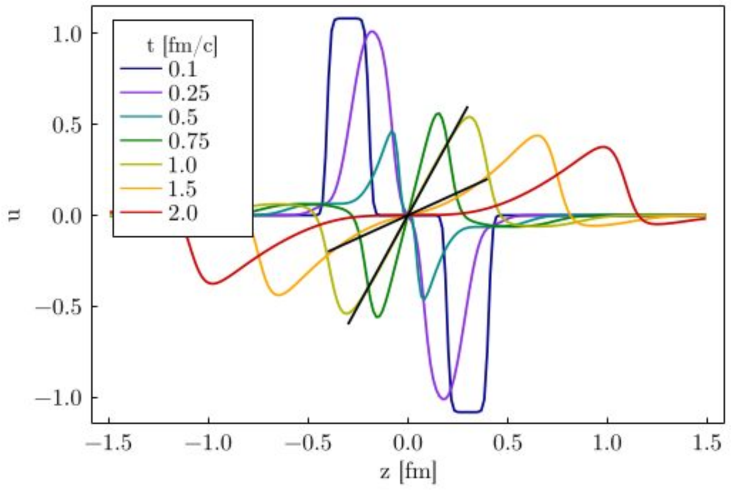
- Established set of equations & equation of state to describe soft part of HIC
- Study entropy production during compression & expansion
- Impact of bulk viscosity & relaxation on $T(t = tf)$

Outlook

- More realistic 1 + 1D setup

→ Includes shear viscosity & baryon diffusion

- Include fluctuations & correlations



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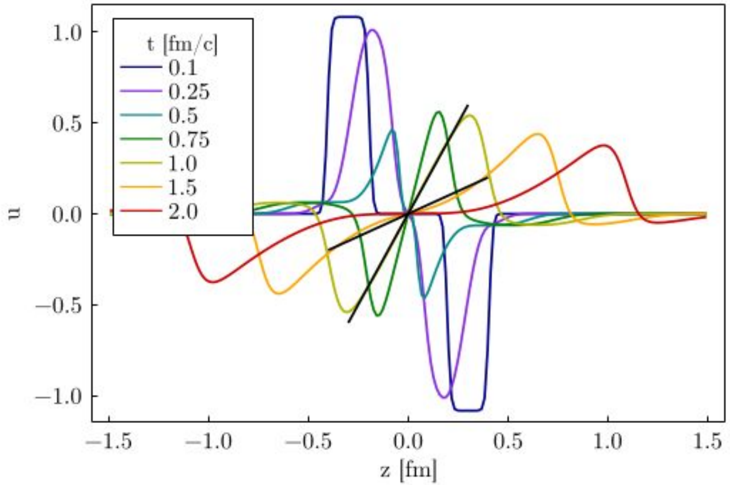


Thank you for your attention...

and happy birthday Wanda!

Back up

Interactionless collision

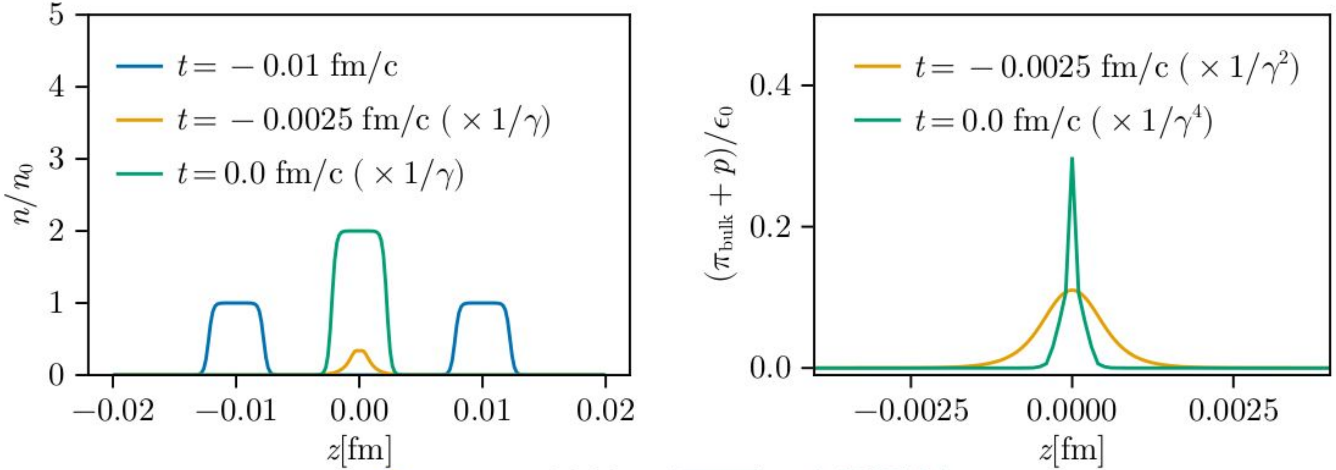
Assume no interaction between colliding nuclei

- Fields obtained from matching procedure at each time step

  - $T^{\mu}{}_{\nu} u^{\nu} = -\epsilon u^{\mu}$
  - $p + \pi_{\text{bulk}} = \frac{1}{3} \Delta^{\mu\nu} T_{\mu\nu}$
  - $\pi^{\mu\nu} = T^{\mu\nu} - \epsilon u^{\mu} u^{\nu} - [p + \pi_{\text{bulk}}] \Delta^{\mu\nu}$
  - Solve $n u^{\mu} + \nu^{\mu} = N^{\mu}_{\rightarrow} + N^{\mu}_{\leftarrow}$ for n, ν^{i}

Interactionless collision: fields

From Kirchner: GGI workshop on hydrodynamics 2025



$\gamma_{\rightarrow/\leftarrow} \approx 2960, \ \sqrt{s_{NN}} = 2.76\text{TeV}$

Large densities and pressure through overlap

$$n = \gamma\gamma_{\rightarrow/\leftarrow}\left[(1 - v_{\rightarrow}v)n_{\rightarrow} + (1 + v_{\leftarrow}v)n_{\leftarrow}\right]$$
$$= 2\gamma_{\rightarrow/\leftarrow}n_{\rightarrow/\leftarrow} \quad \text{(full overlap)}$$

Interactionless collision: fields

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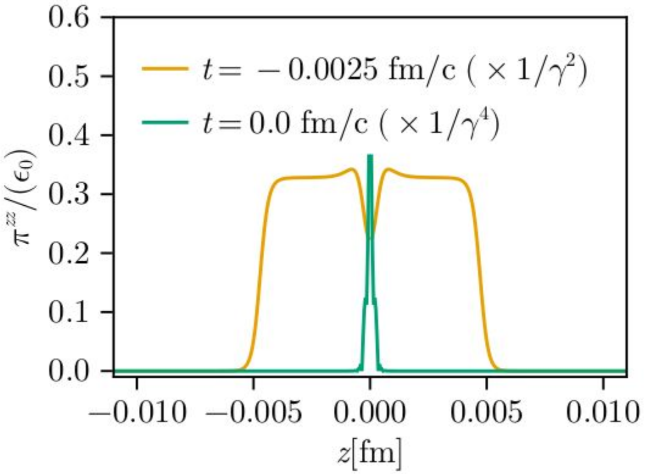

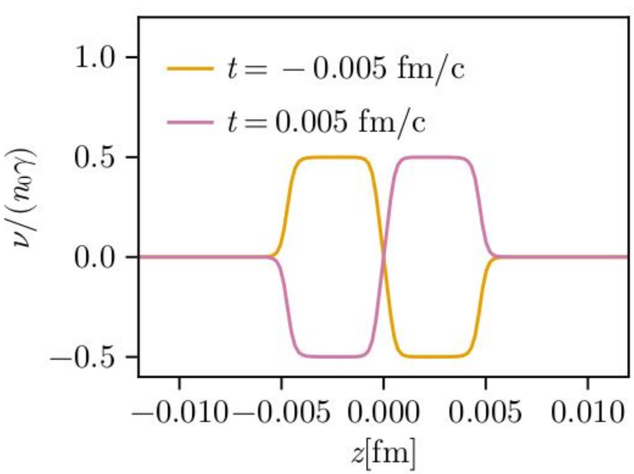

$$\gamma \approx 2960,\ \sqrt{s_{NN}} = 2.76\text{TeV}$$

Large viscous corrections $\Rightarrow$ careful treatment of evolution equations (causality, stability, formation of shocks, ...)

Interactionless collision: fields

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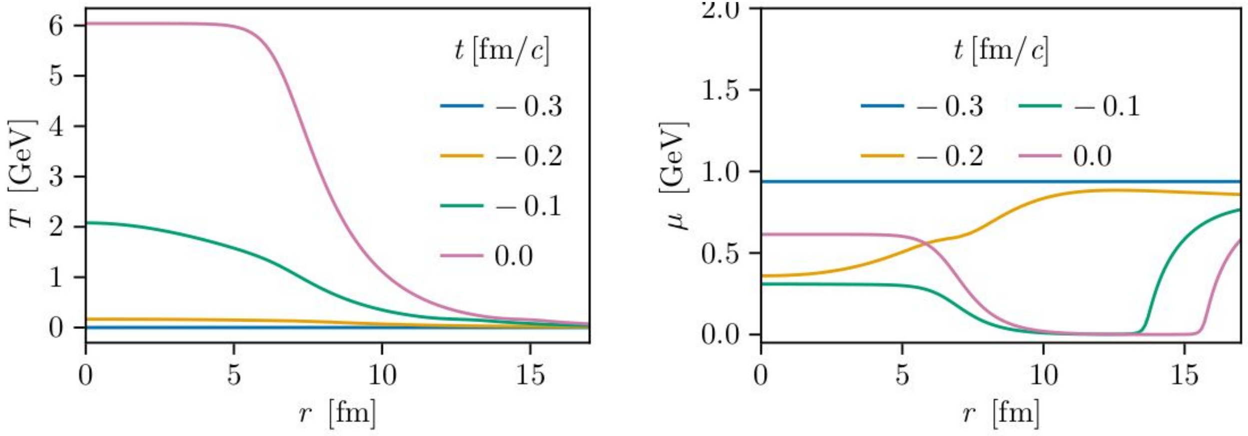

Apply equation of state to obtain T & μ:

- $T \to 0$ & $\mu \to 0.93\,\text{GeV}$ for low densities
- Indications of expected trajectory, but T much too high
- Description including interactions needed