From relativistic ion collisions to nuclear structure, and back

Vittorio Somà
CEA Paris-Saclay, France

Celebrating Wanda's birthday

3-4 July 2025, Torino

It all started with high-energy collisions

○ **Phenomenological study of non-linear and collective effects in proton rapidity spectra**

→ Departure from ideal QGP described in terms of non-extensive statistics (Tsallis)

→ Spectra computed via a relativistic diffusion equation (Fokker-Planck) incorporating Tsallis statistics

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Signals of non-extensive statistical mechanics in high energy nuclear collisions

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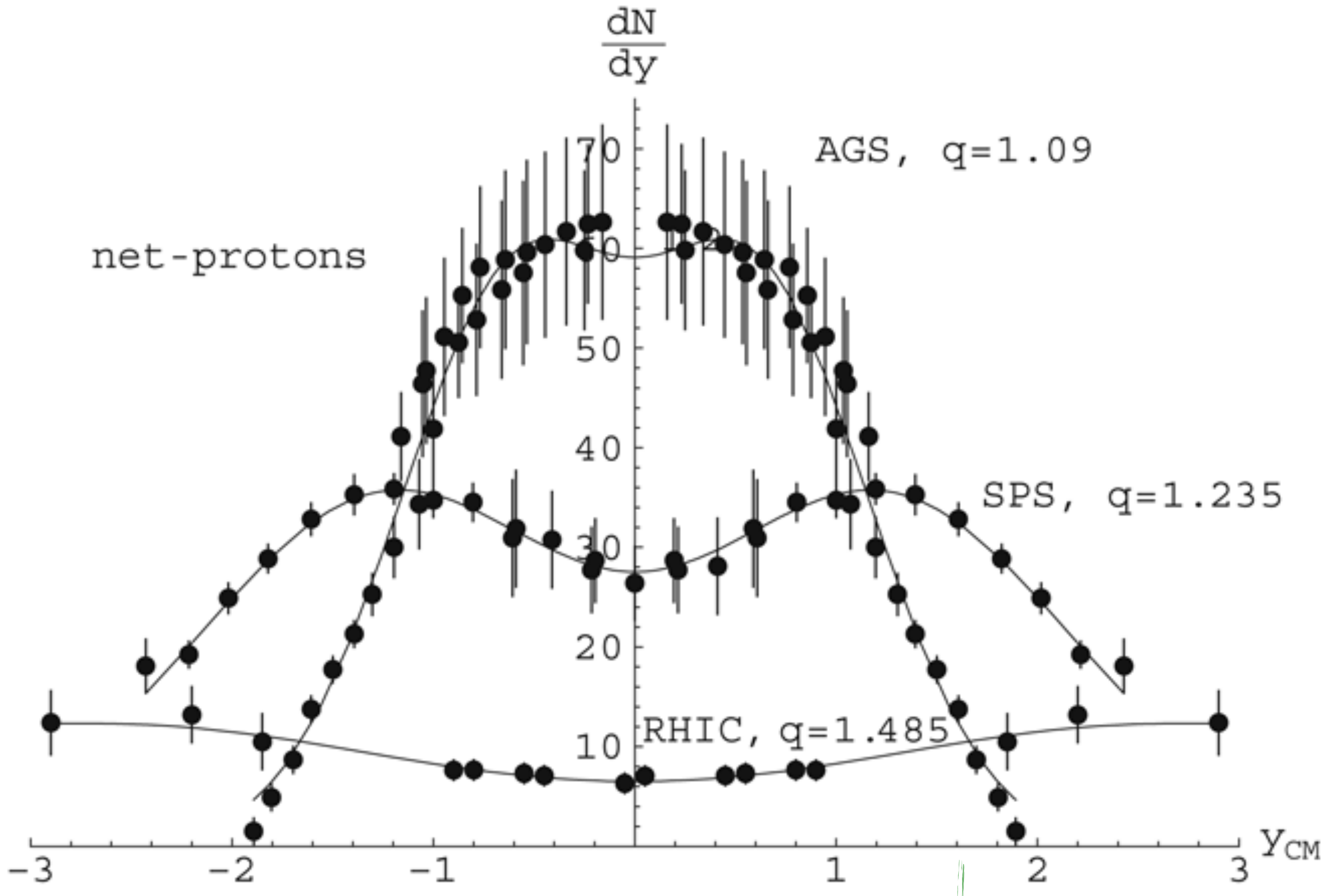
Abstract

We investigate, from a phenomenological point of view, the relevance of non-conventional statistical mechanics effects on the rapidity spectra of net proton yield at AGS, SPS and RHIC. We show that the broad rapidity shape measured at RHIC can be very well reproduced in the framework of a non-linear relativistic Fokker–Planck equation which incorporates non-extensive statistics and anomalous diffusion.



○ Good reproduction of rapidity distributions
○ Non-linear effects grow with beam energy
→ Microscopic justification?

Outline

- **Part I: nuclear structure**
  - Progress in ab initio nuclear structure
  - Some examples of recent applications

    [Somà et al, EPJA **57** 135 (2021); Frosini et al. EPJA **58** 63 (2022); Porro at al. EPJA **60** 134 (2024), …]

- **Part II: high-energy collisions**
  - O-O & Ne-Ne collisions

    [Giacalone et al., Phys. Rev. Lett. **135** 012302 (2025)]
  - Fixed-target Pb-O & Pb-Ne collisions

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  - Triaxial shape of ^{129}Xe

    [Bally et al., Phys. Rev. Lett. **128** 082301 (2022)]

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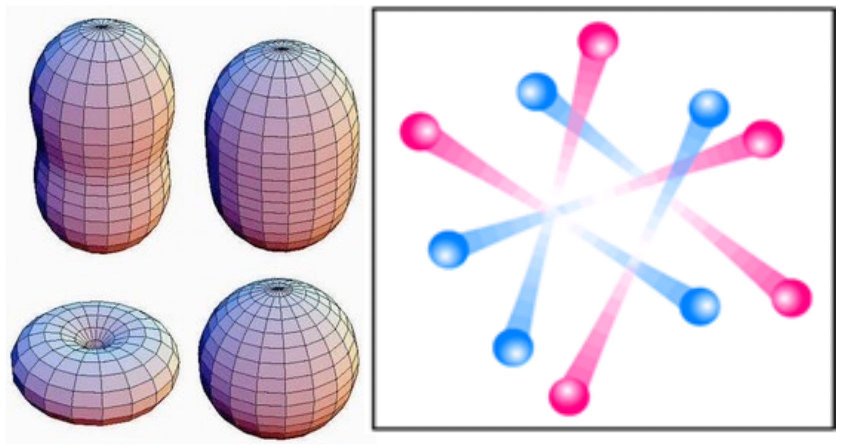
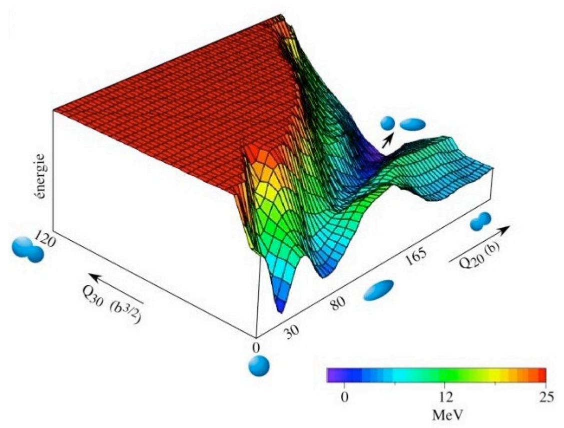
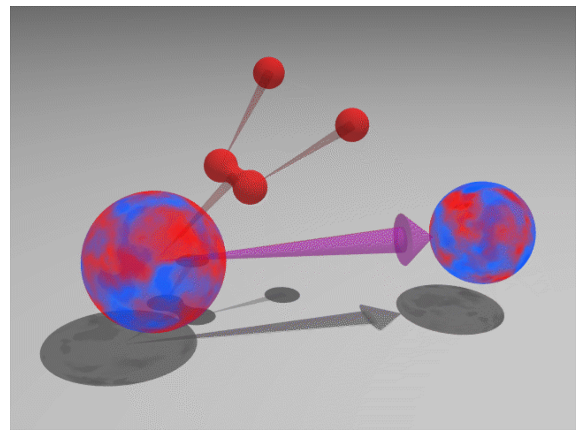
Diversity of nuclear phenomena

Ground state
Mass, size, superfluidity, …

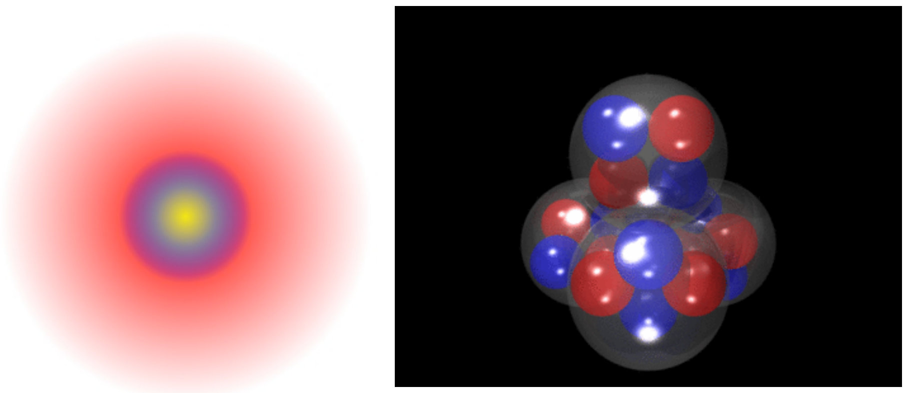


Radioactive decays
β, 2β, α, p, 2p, fission, …



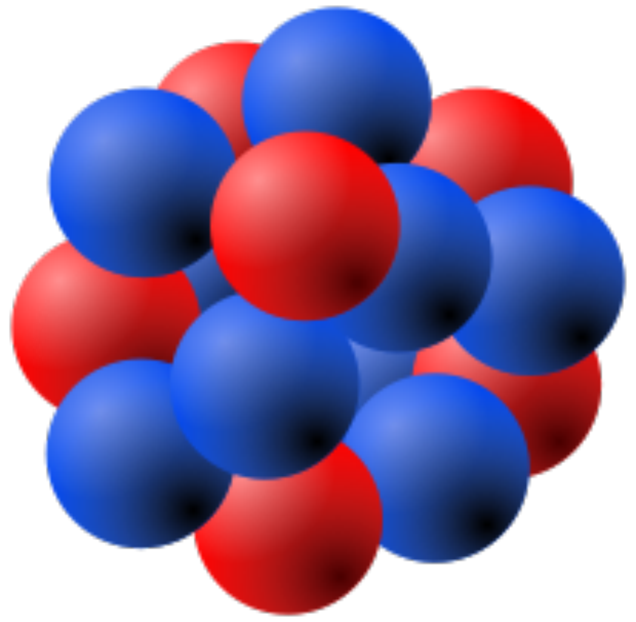


Exotic structures
Clusters, halos, …



Strongly-correlated systems

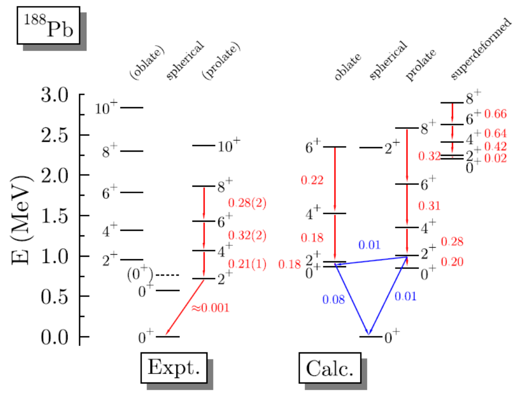
Angular corr. → Deformation
Pairing corr. → Superfluidity
Quartet corr. → Clustering

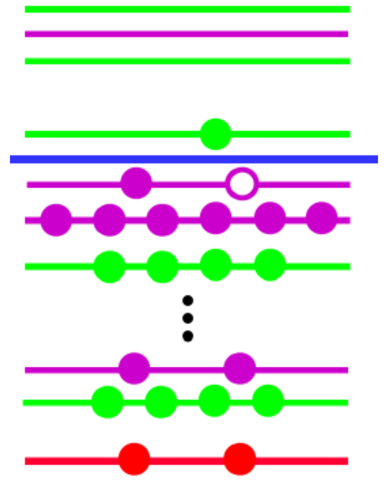


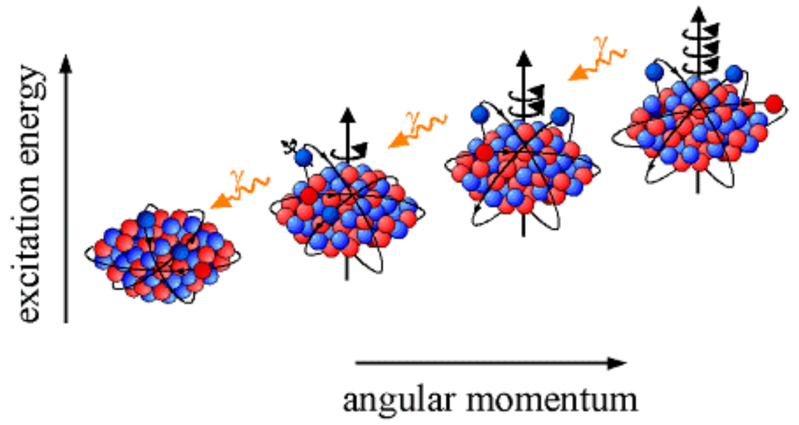
Several scales at play

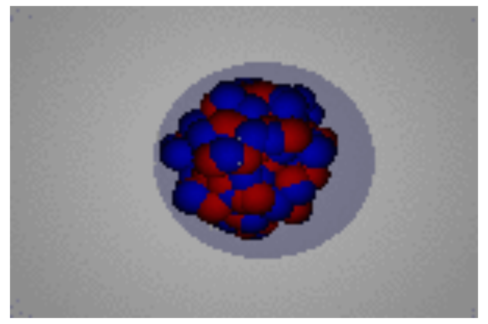
Nucleon momenta ~ **100** MeV
Separation energies ~ **10** MeV
Vibration modes ~ **1** MeV
Rotation modes ~ **0.01-few** MeV

Spectroscopy
Excitation modes

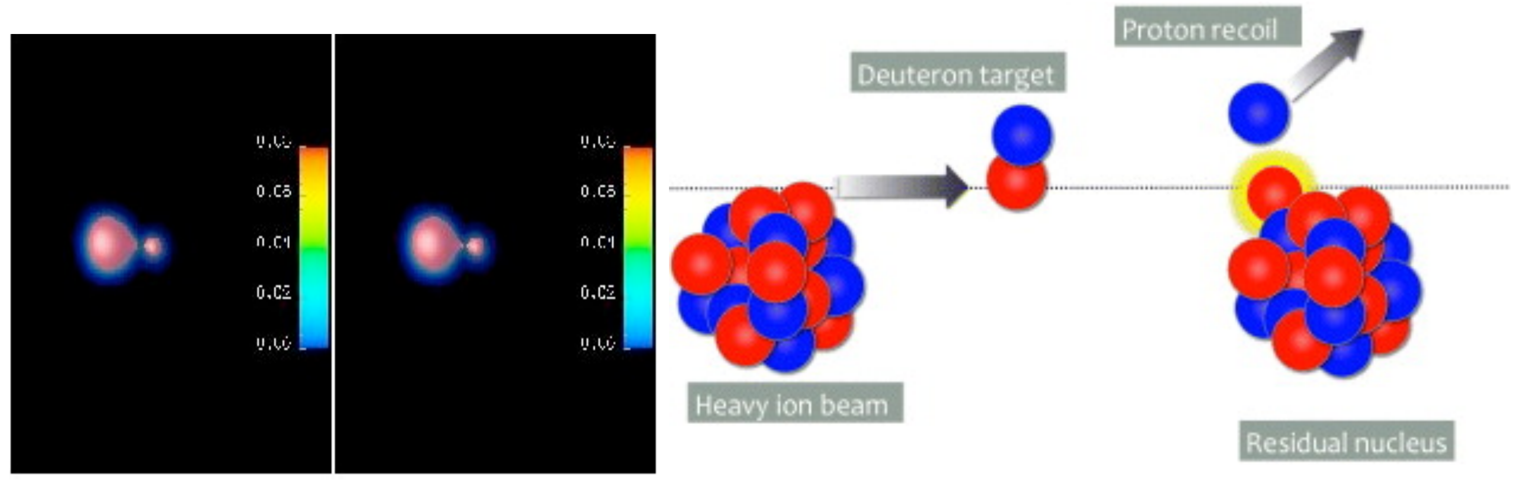






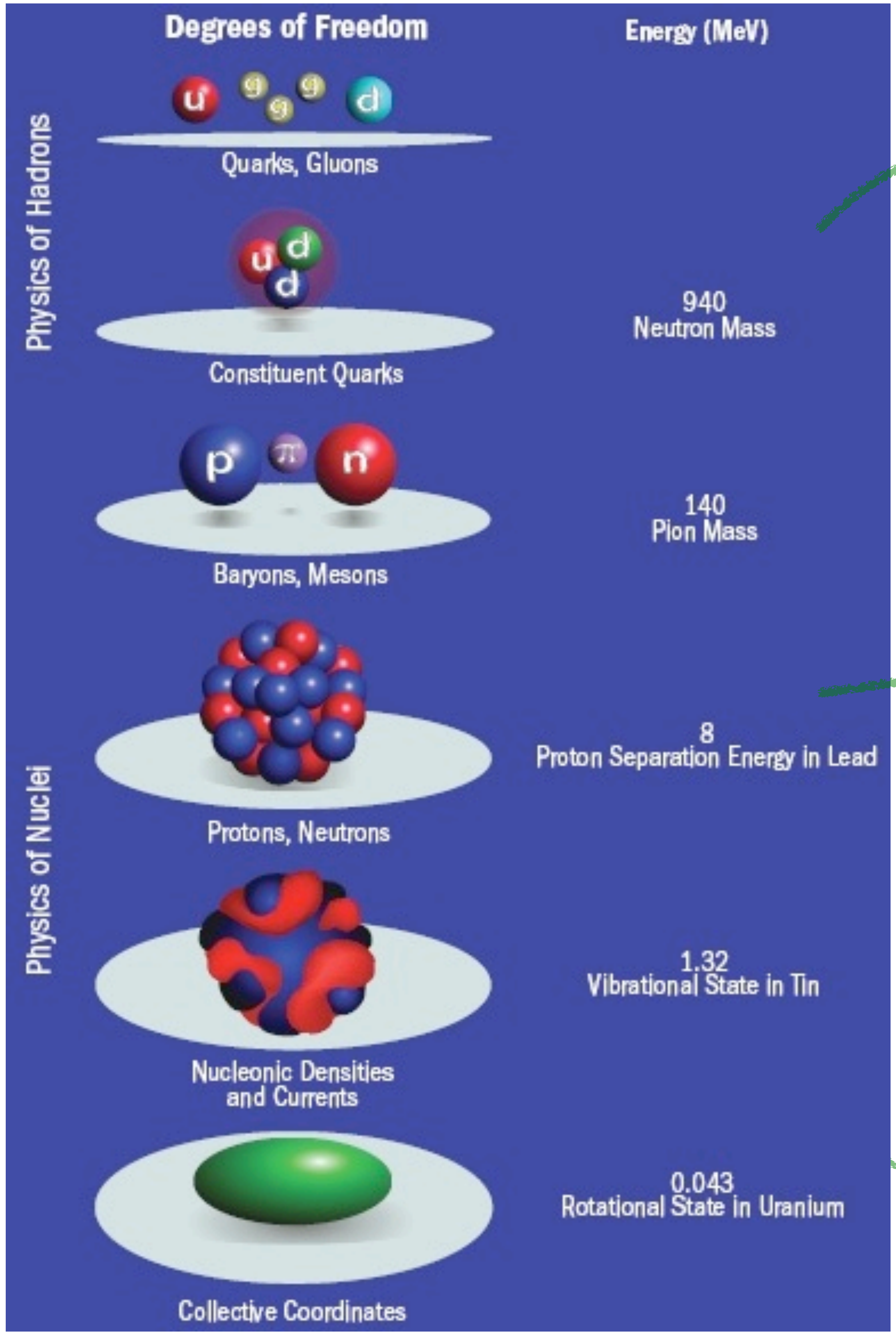


Reaction processes
Fusion, transfer, knockout, …



What is the most appropriate theoretical description?

More reductionist/elementary/"fundamental" description



Emergent phenomena amenable to effective descriptions

QCD

Microscopic nuclear model (nucleons d.o.f.)

Collective models

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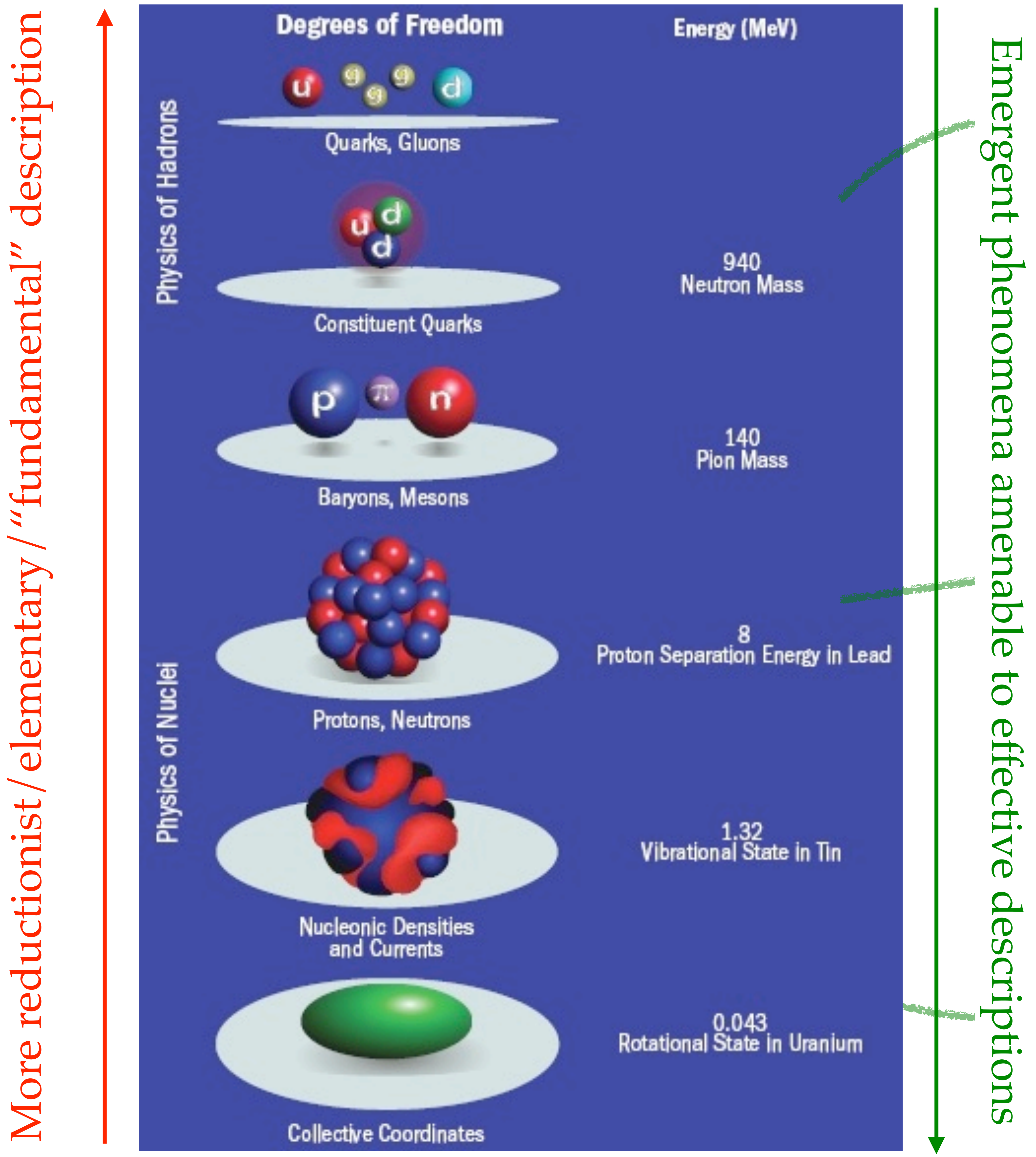

QCD

Modern view: "tower" of EFT

Microscopic nuclear model (nucleons d.o.f.)

Collective models

Ab initio approach to nuclear structure

○ **A systematic approach to describe nuclei**

$$H|\Psi_k^A\rangle = E_k^A|\Psi_k^A\rangle$$

Ab initio approach to nuclear structure

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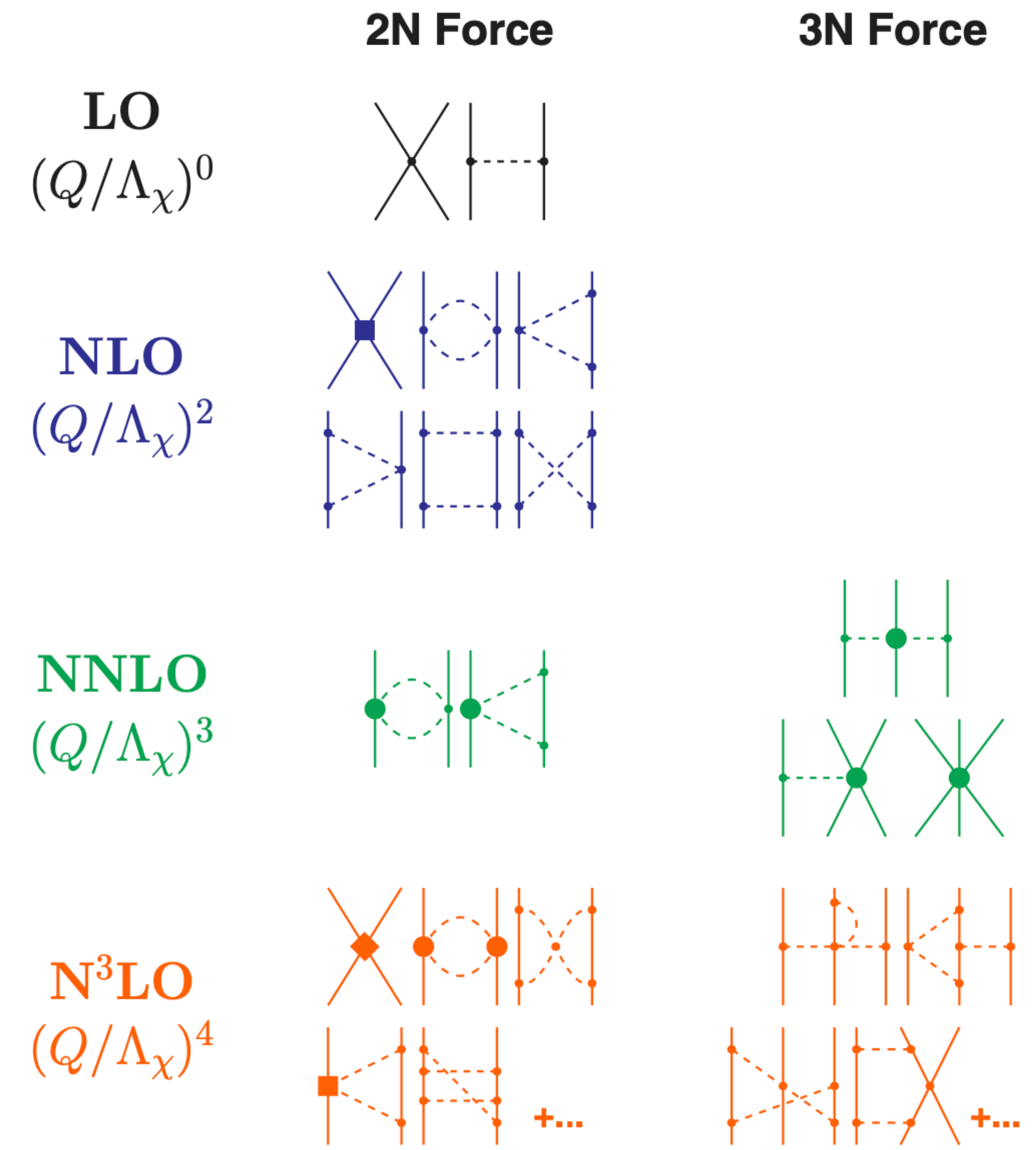
$$H|\Psi_k^A\rangle = E_k^A|\Psi_k^A\rangle$$

1. Model Hamiltonian

Inter-nucleon forces from chiral EFT

- Low-energy limit of QCD
- Nucleons and pions as d.o.f.
- Power counting → expansion of H



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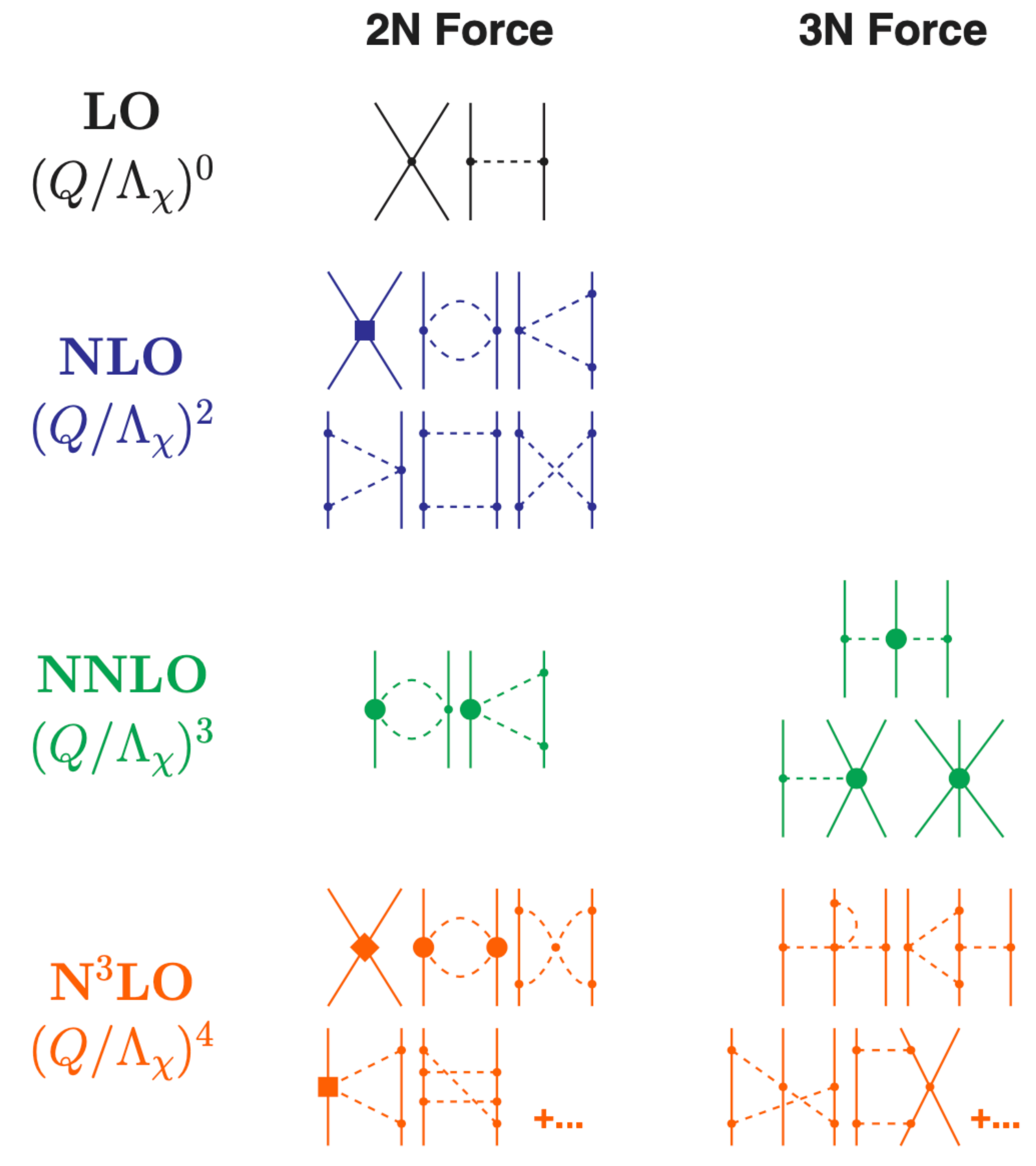

Option 1: Exact solutions have factorial or exponential scaling $\mathbf{e}^n$ → limited to light nuclei (A ≤ 20)

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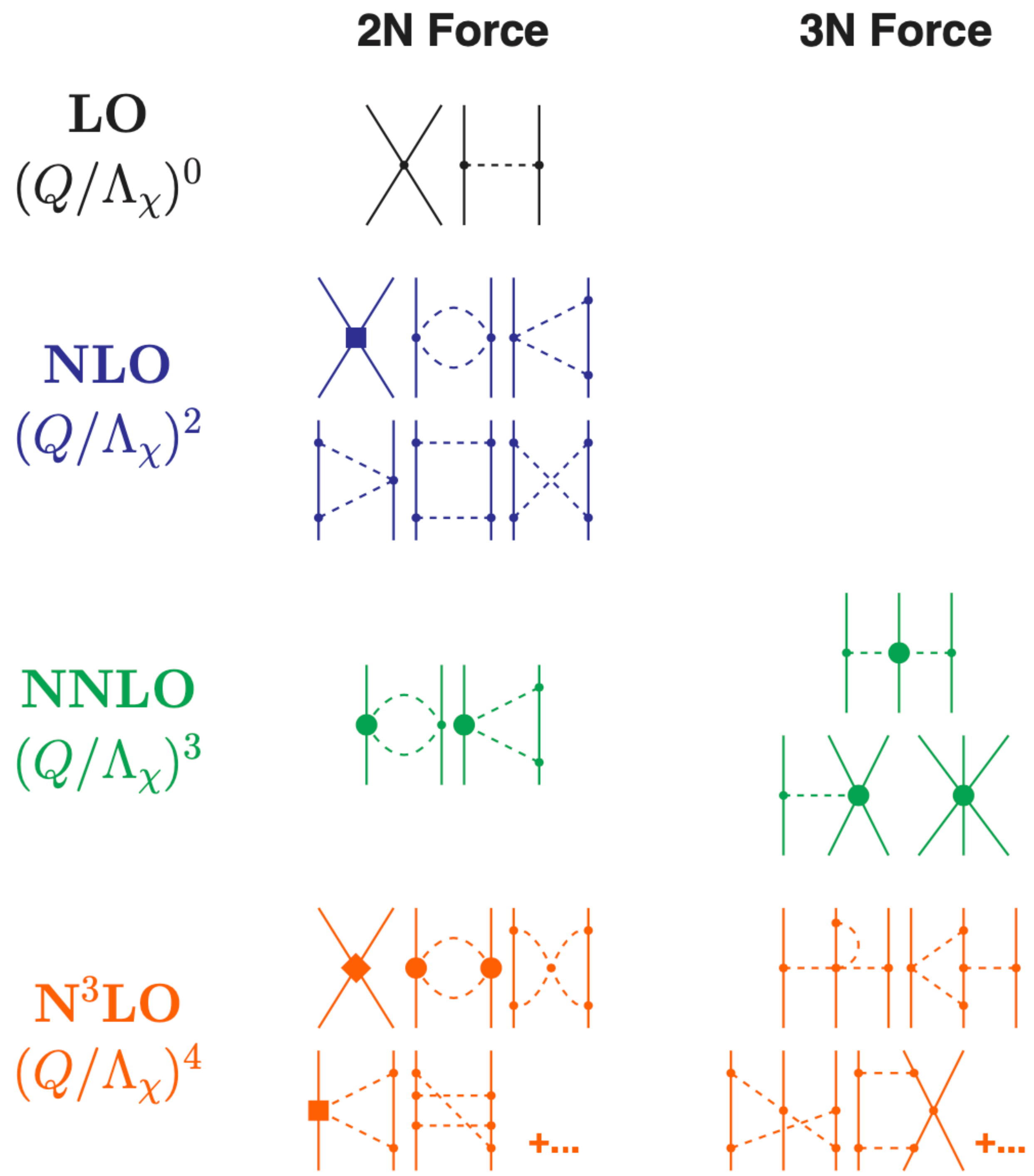

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Option 2: Correlation-expansion methods to achieve **polynomial** scaling

- Hamiltonian partitioning $H = H_0 + H_1$
- Reference state $H_0|\Phi_k^{(0)}\rangle = E_k^{(0)}|\Phi_k^{(0)}\rangle$
- Wave-operator expansion $|\Psi_k^A\rangle = \Omega_k|\Phi_k^{(0)}\rangle = |\Phi_k^{(1)}\rangle + |\Phi_k^{(1)}\rangle + |\Phi_k^{(2)}\rangle + \ldots$

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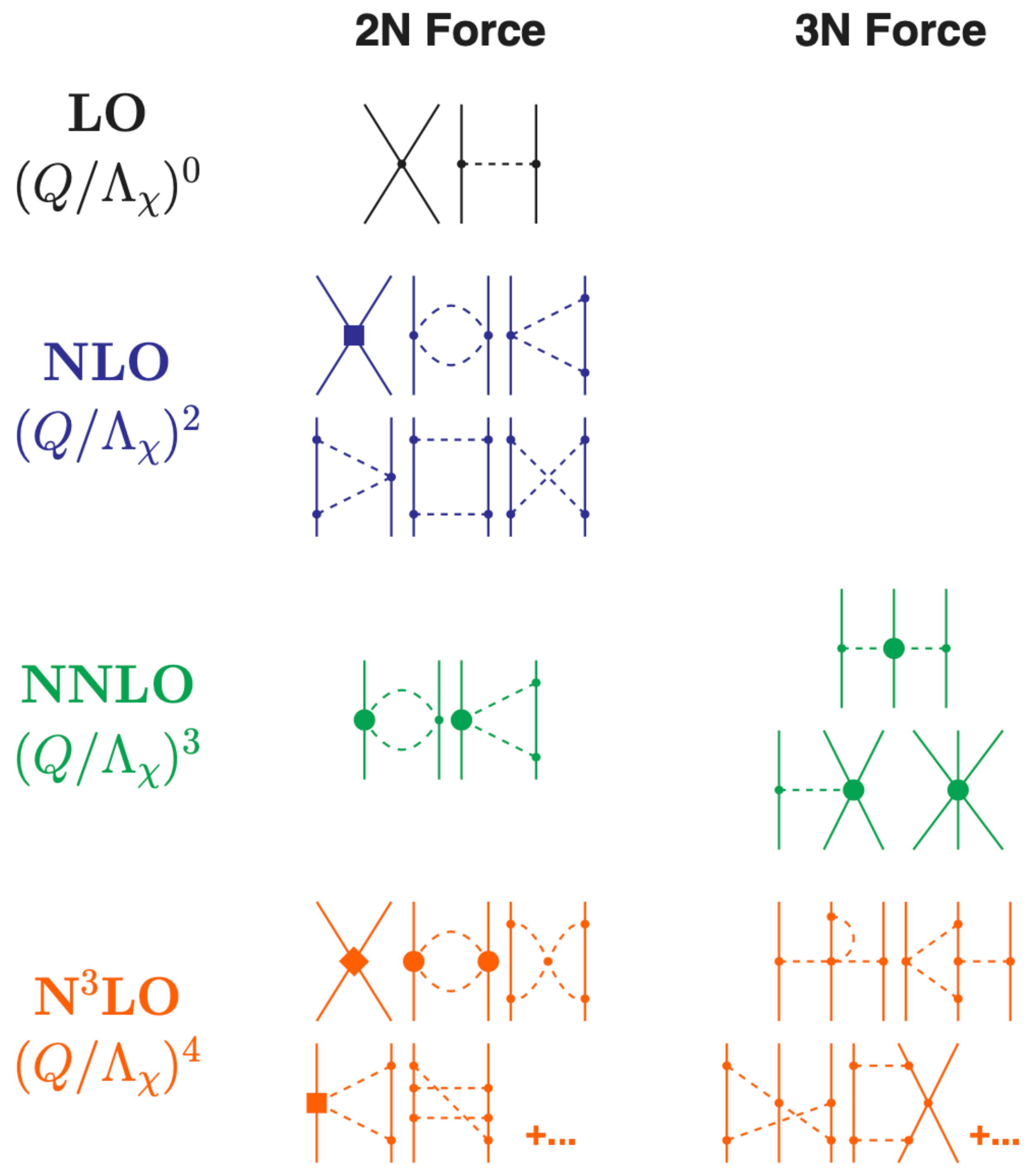

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○ Wave-operator expansion $|\Psi_k^A\rangle = \Omega_k|\Phi_k^{(0)}\rangle = |\Phi_k^{(0)}\rangle + |\Phi_k^{(1)}\rangle + |\Phi_k^{(2)}\rangle + \dots$ → scaling n^α with $\alpha > 4$

CPU-scalable to **heavy masses**?

The Segrè chart

Nuclear structure approaches

- ○ **Energy density functionals**
- ○ Empirical shell model
- ○ **Ab initio**
- ○ …

[Figure: B. Bally]

Data partly taken from:
M. Wang et *al.*, Chin. Phys. C 45, 030003 (2021)
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Diversity of many-body techniques

○ Correlation expansion performed in terms of **particle-hole excitations** → **Breaks down in open-shell systems**

95%

Solution: start from a **symmetry-breaking reference state**

→ At some point, necessary to **restore symmetries**

$U(1)_N \times U(1)_Z$	→	Superfluidity
$SU(2)$	→	Deformation

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○ Keep **polynomial cost**

○ **Many different strategies exist**

→ Break which symmetries?

→ Restore then expand or expand then restore?

Most efficient option will depend on

○ Nucleus

○ Observables

○ Required precision

○ …

Necessity to develop many different, complementary approaches

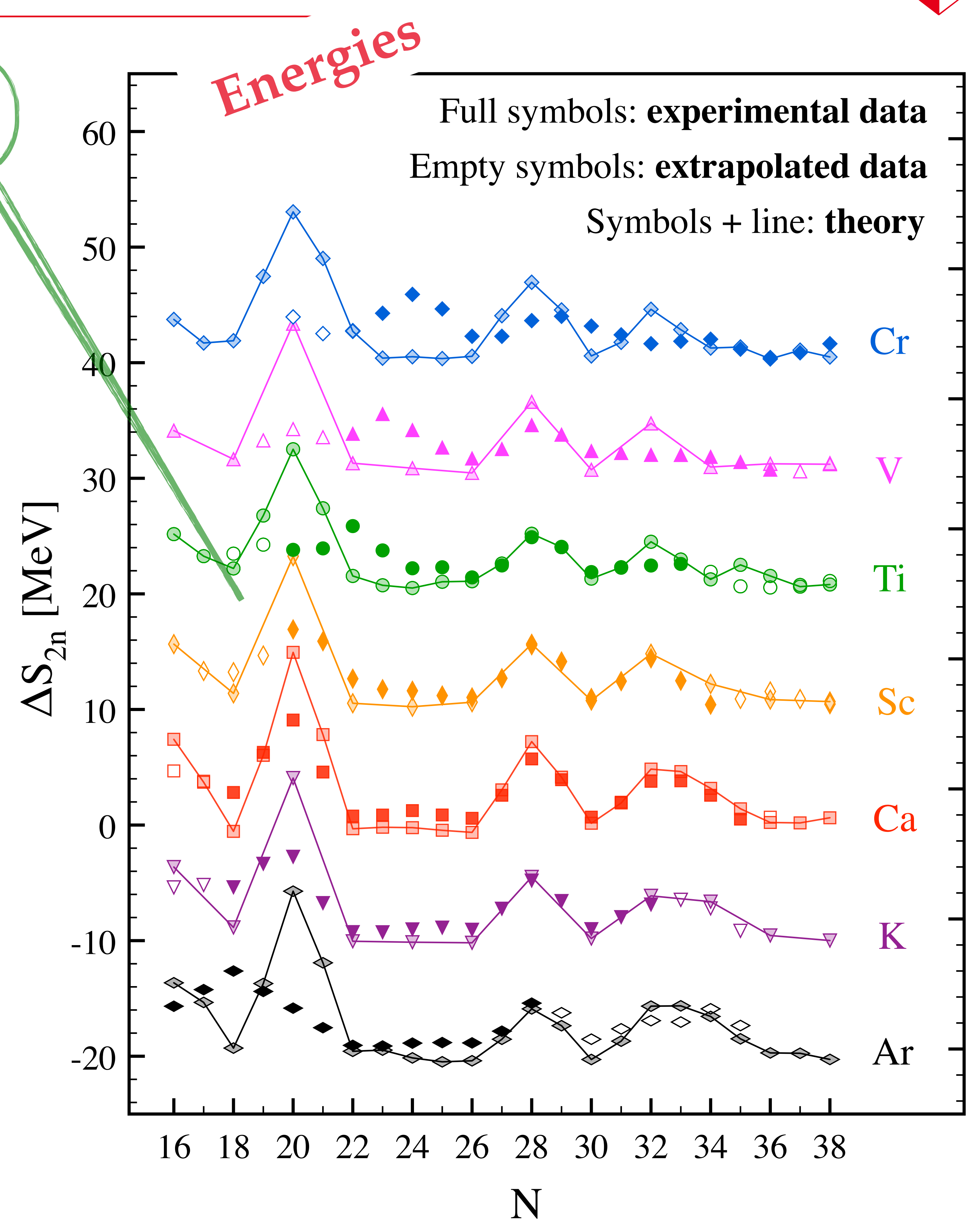
First example: spherical superfluid calculations

○ Self-consistent Green's functions

○ Symmetry breaking: particle number

→ **G.s. properties of singly open-shell**

Magic numbers emerge "ab initio"



[Somà *et al.*, 2021]

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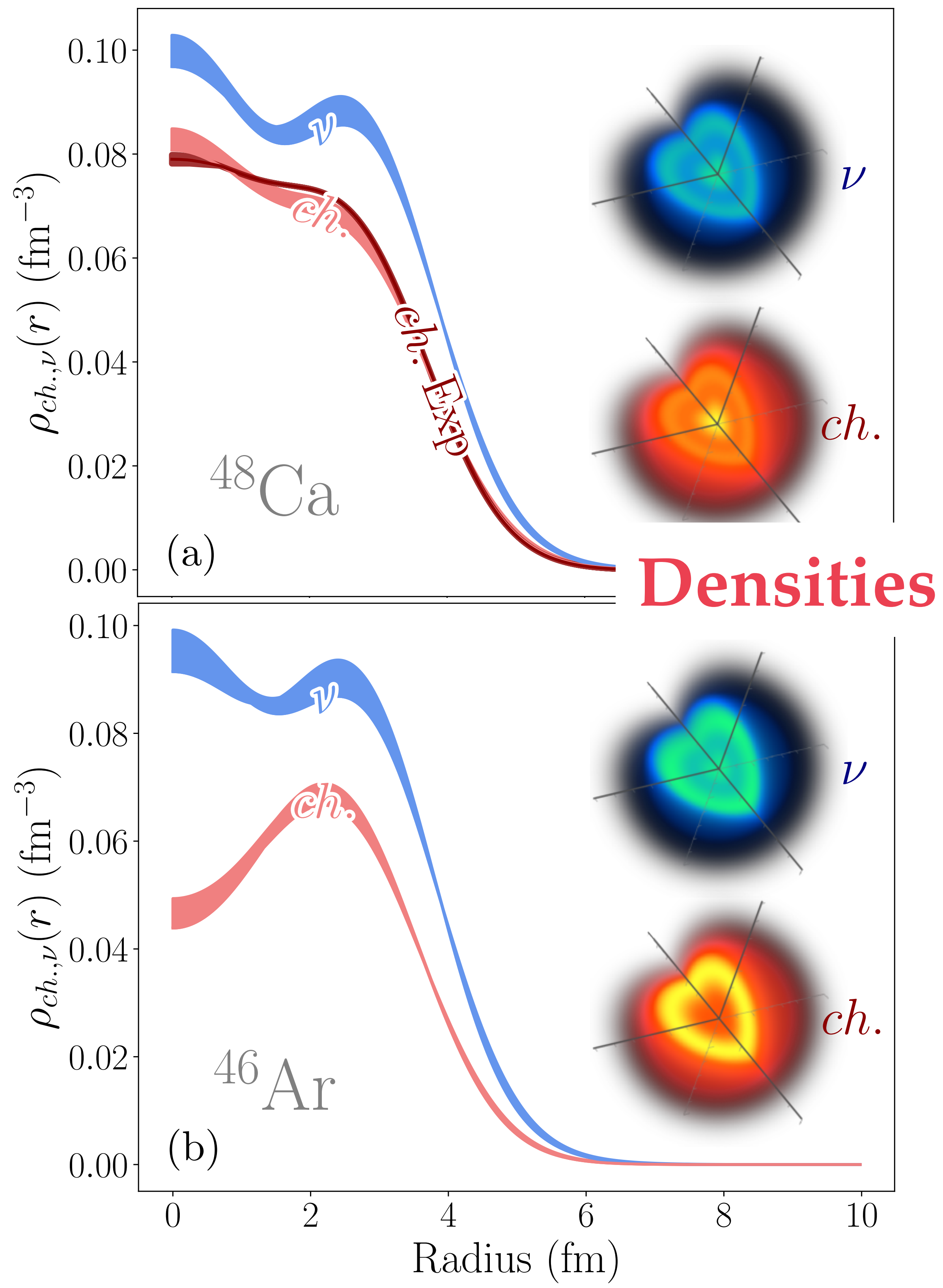
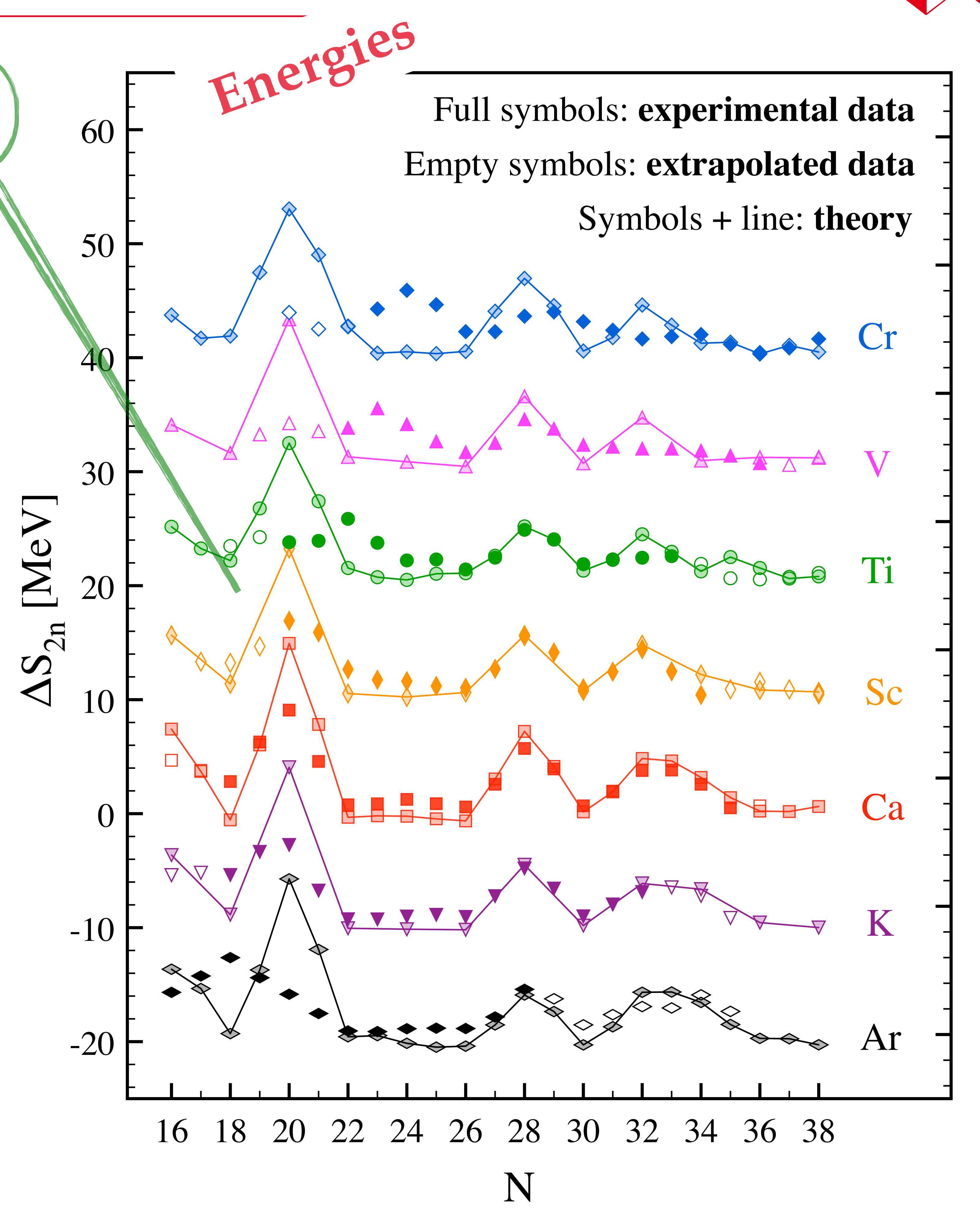
[Brugnara *et al.*, submitted]





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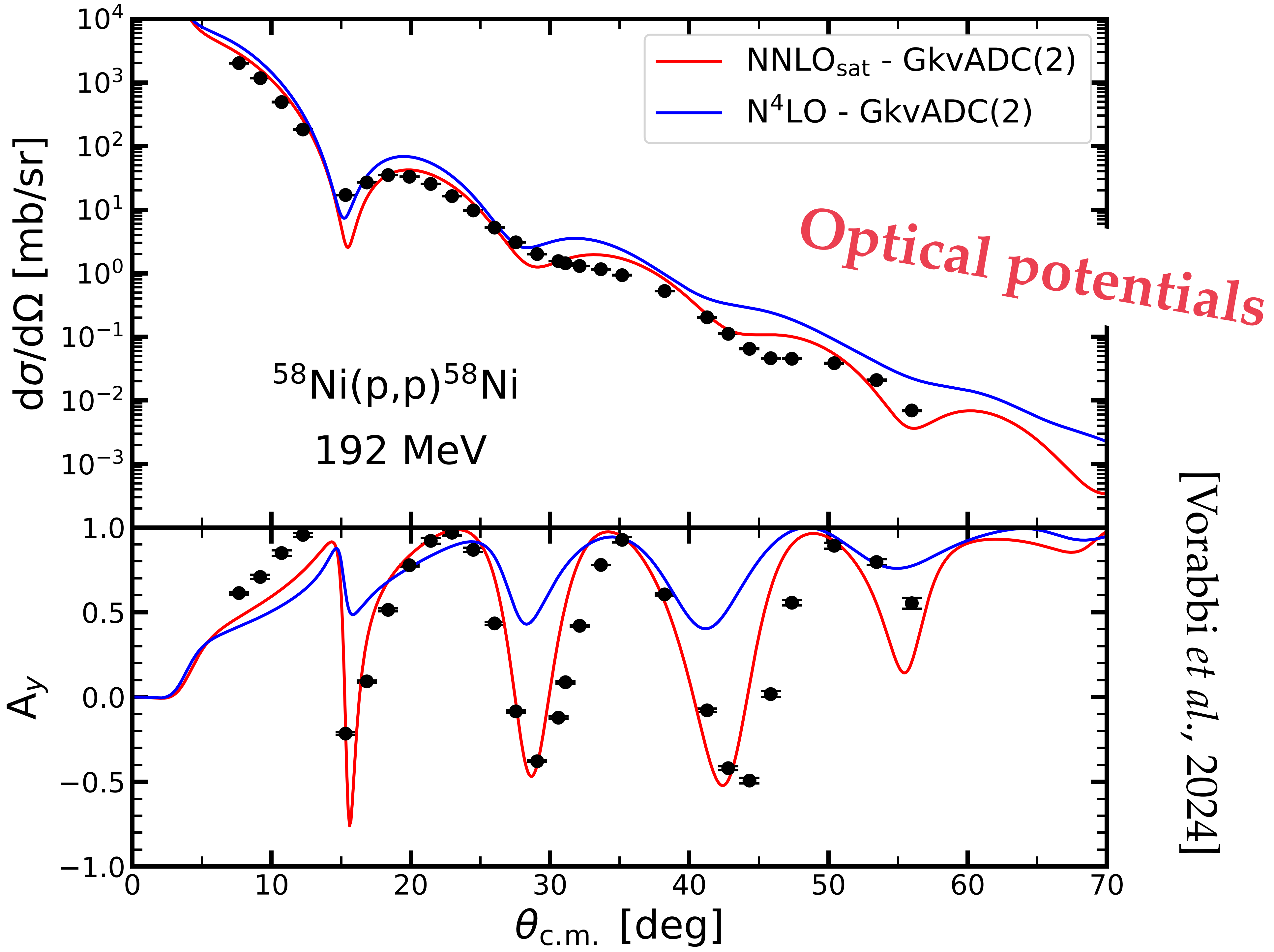

[Vorabi *et al.*, 2024]

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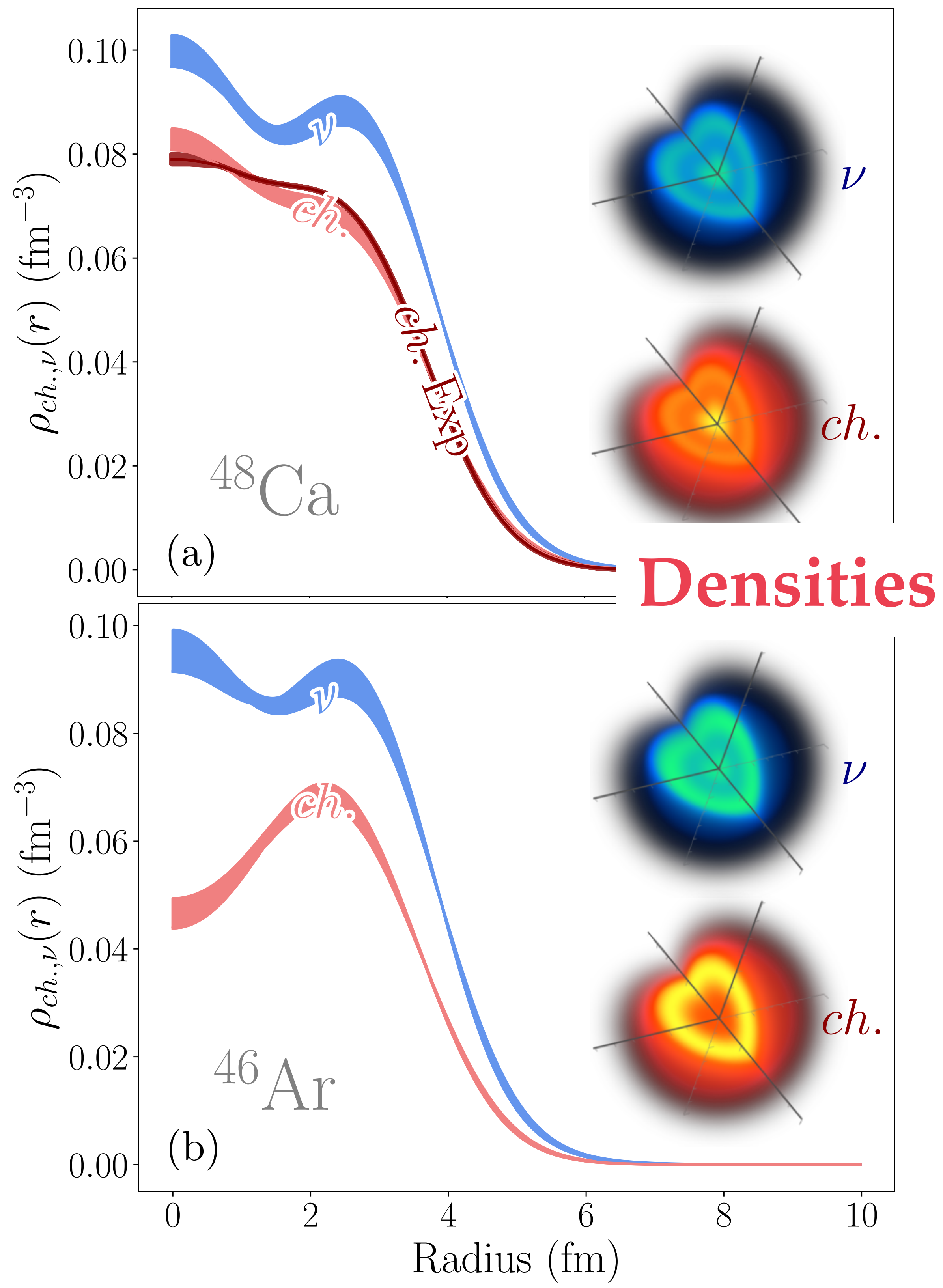

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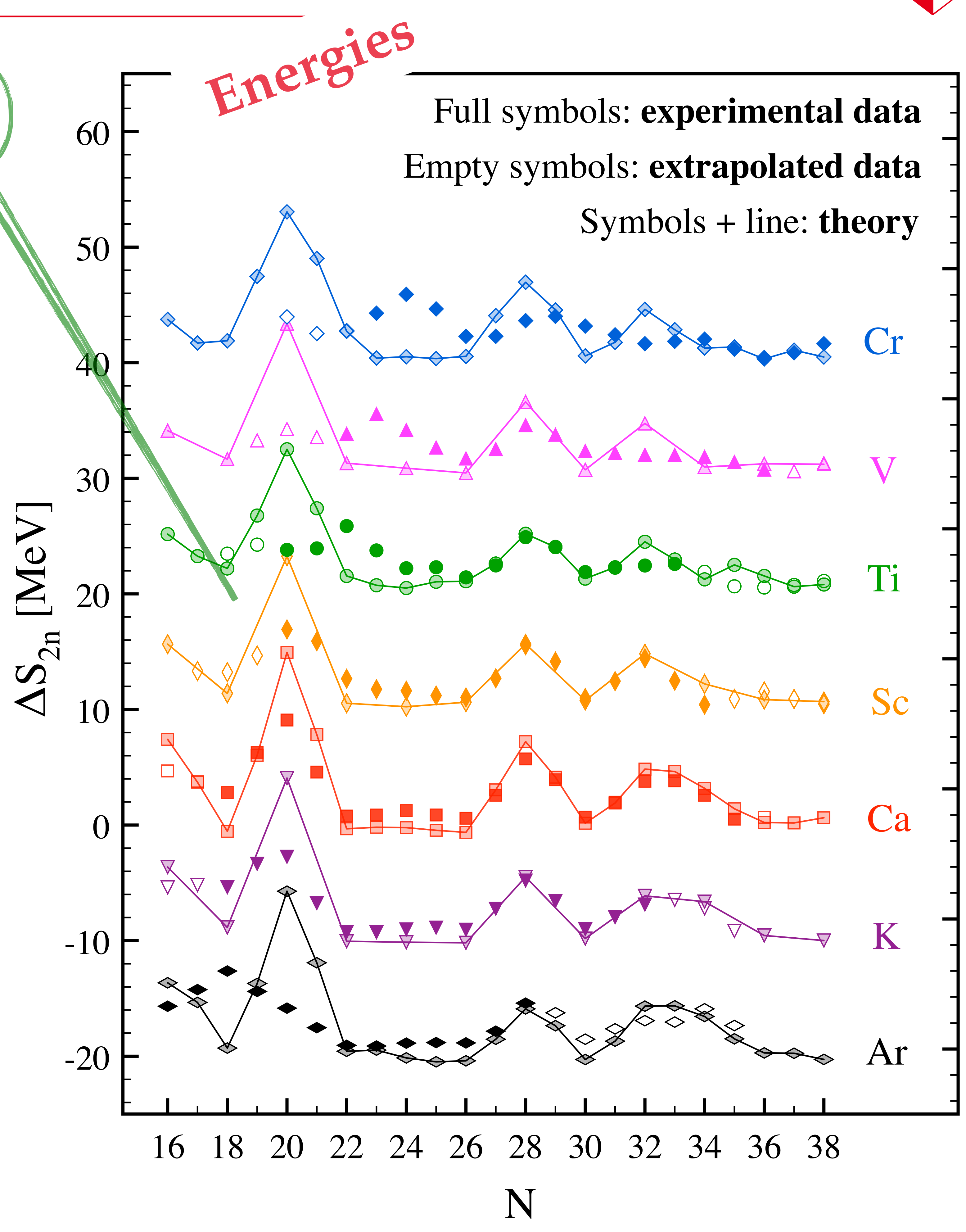

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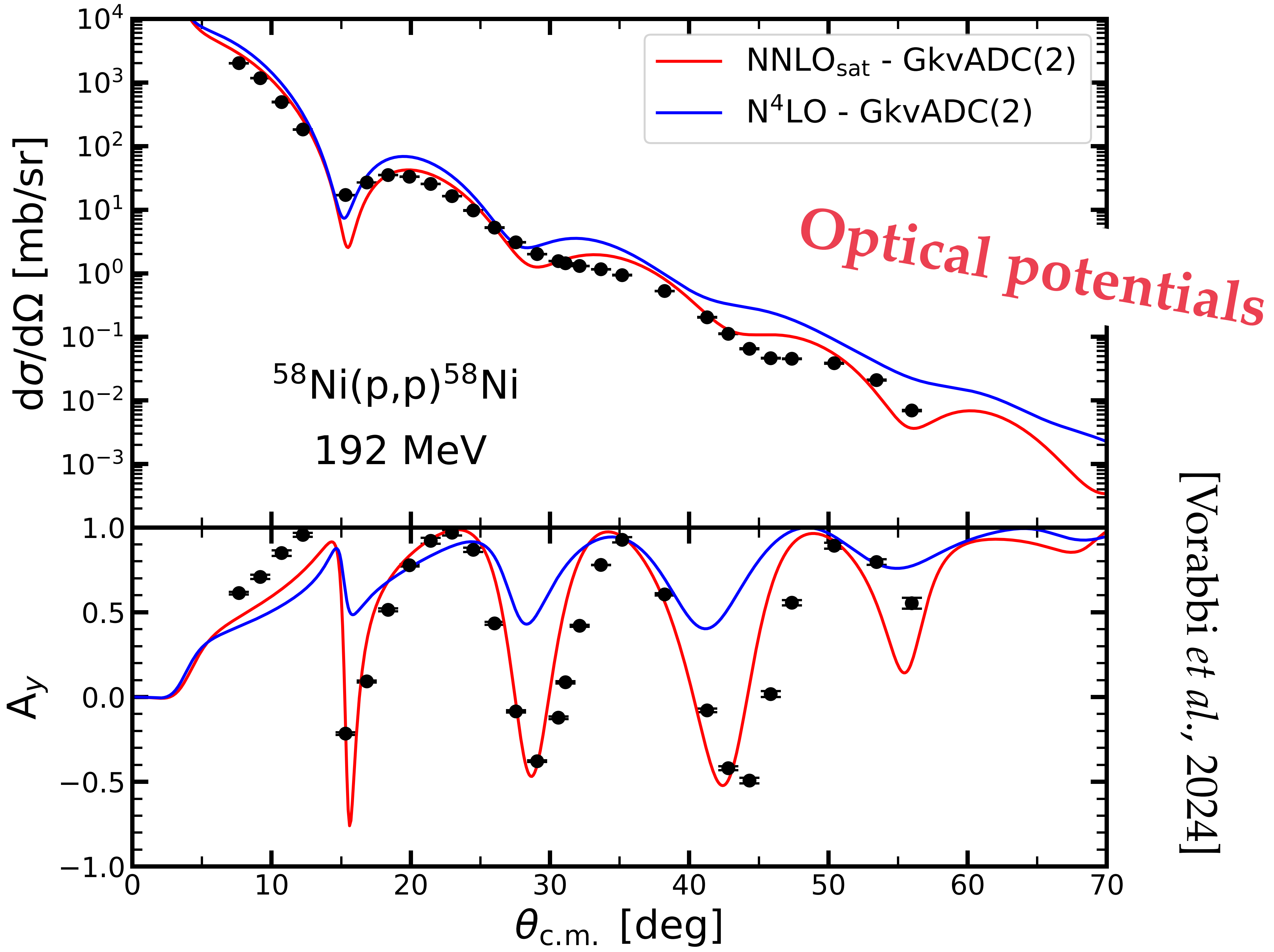

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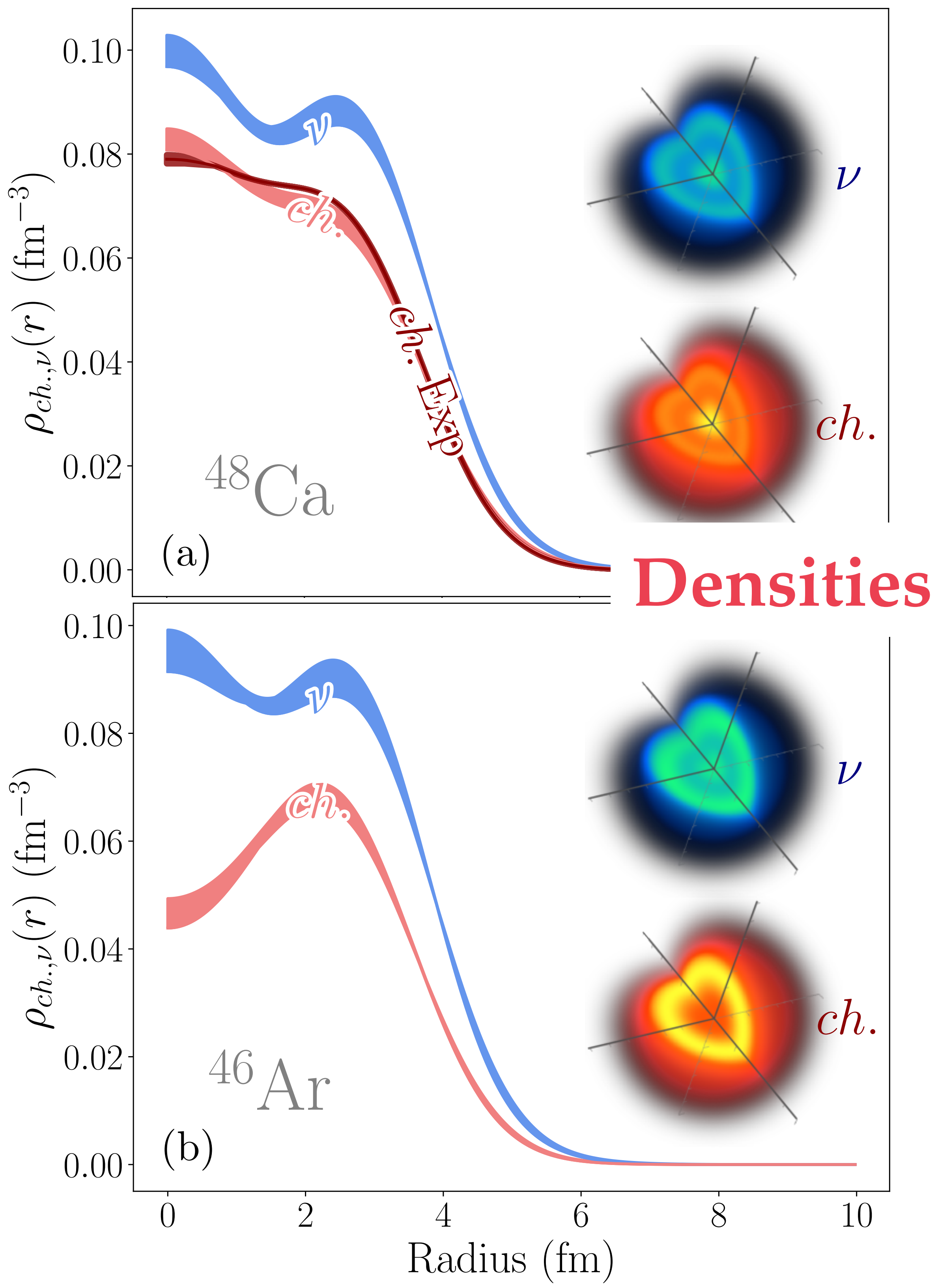
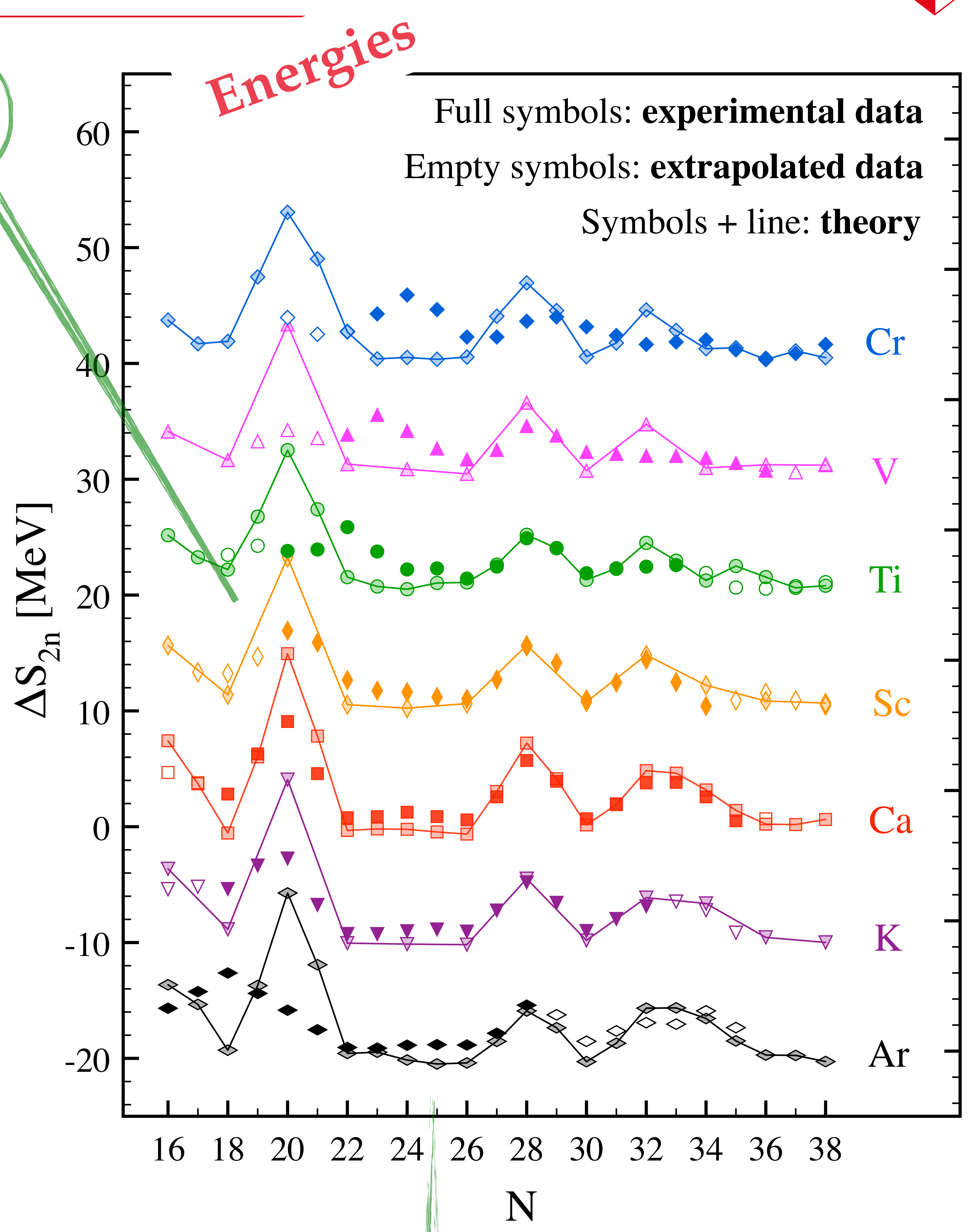




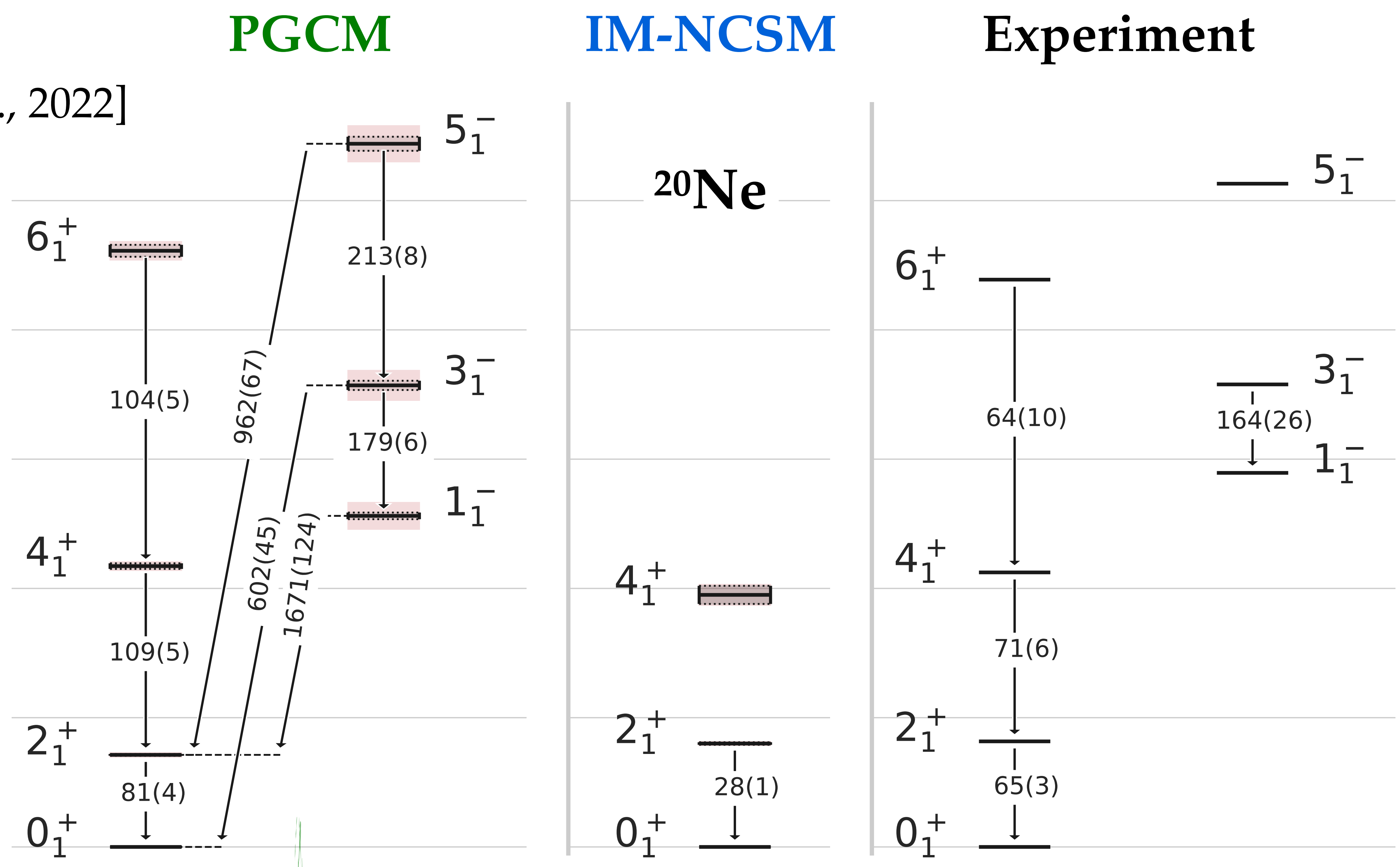
[Somà *et al.*, 2021]

- ○ Accuracy degrades away from semi-magic Ca
- ○ Correlation with nuclear deformation
- → Calls for **explicit inclusion of deformation**

Second example: deformed calculations

- ○ Projected generator coordinate method
- ○ Symmetry breaking & restoration
  - → particle number
  - → rotational invariance (axial)
  - → parity
- → **Excitation spectra & collective properties**

[Frosini *et al.*, 2022]



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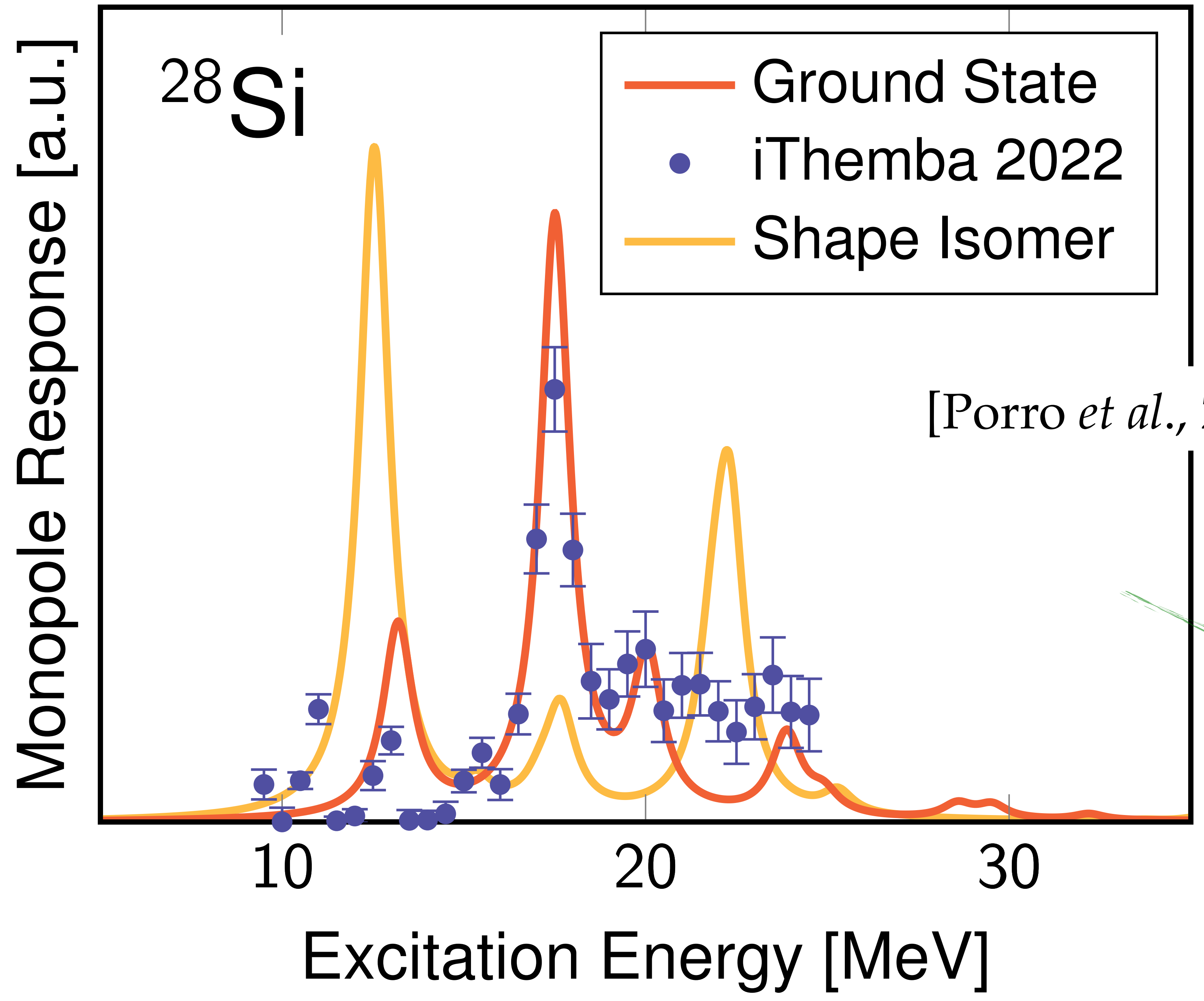
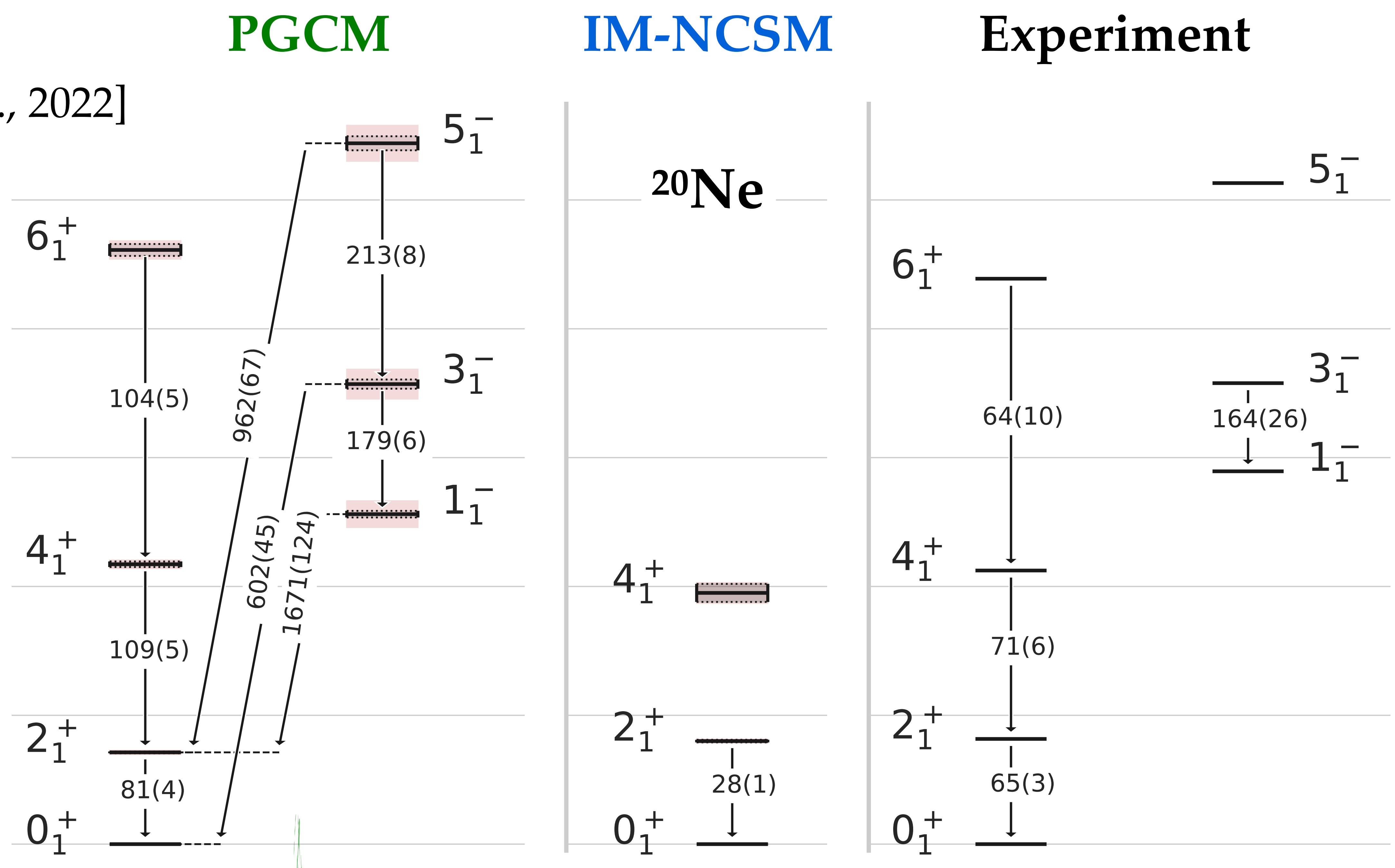


[Porro *et al.*, 2024]

[Frosini *et al.*, 2022]



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- Oblate ground state & low-lying prolate isomer
  - → Shape coexistence (but weak mixing)

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High-energy collisions of small systems - motivations

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→ Observation of hydrodynamic behaviour would be a clear signature

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$$v_2\{2\}_{d^{197}\mathrm{Au}} > v_2\{2\}_{p^{197}\mathrm{Au}}$$ [RHIC]

Geometry relies on poorly-known low-x proton structure

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Different origin of elliptic flow (geometry vs. fluctuations)

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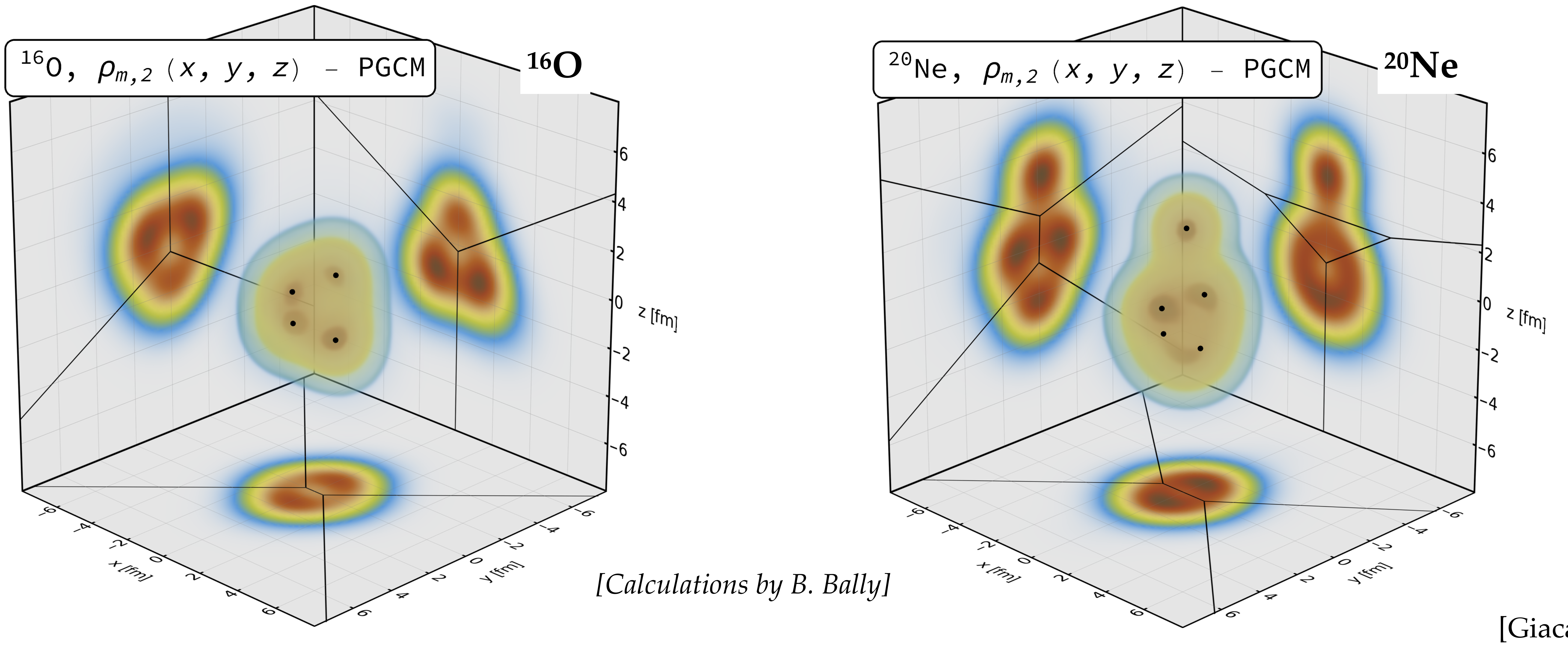
Different origin of elliptic flow (geometry vs. fluctuations)

○ Inconclusive evidence

→ **Suggestion: complement ^{16}O-^{16}O collisions with a ^{20}Ne-^{20}Ne run** [Giacalone *et al.*, 2025]

High-energy collisions of small systems - structure input

○ **Nuclear densities (PGCM & NLEFT) → Hydro simulation (Trajectum) → Hadronization (SMASH)**



[Calculations by B. Bally]

[Giacalone *et al.*, 2025]

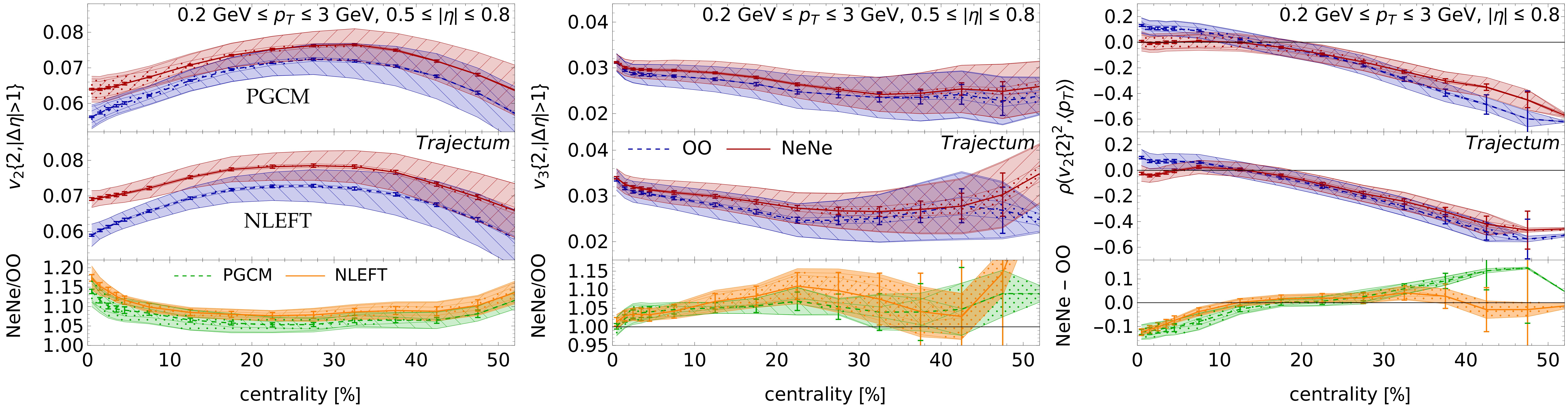
○ Nucleon configurations directly computed (NLEFT) or sampled from nucleon density (PGCM)

○ Configurations randomly oriented + random impact parameter assigned → Trajectum + SMASH

○ Careful assessment of statistical and systematic uncertainties

High-energy collisions of small systems - hydro results

[Giacalone *et al.*, 2025]



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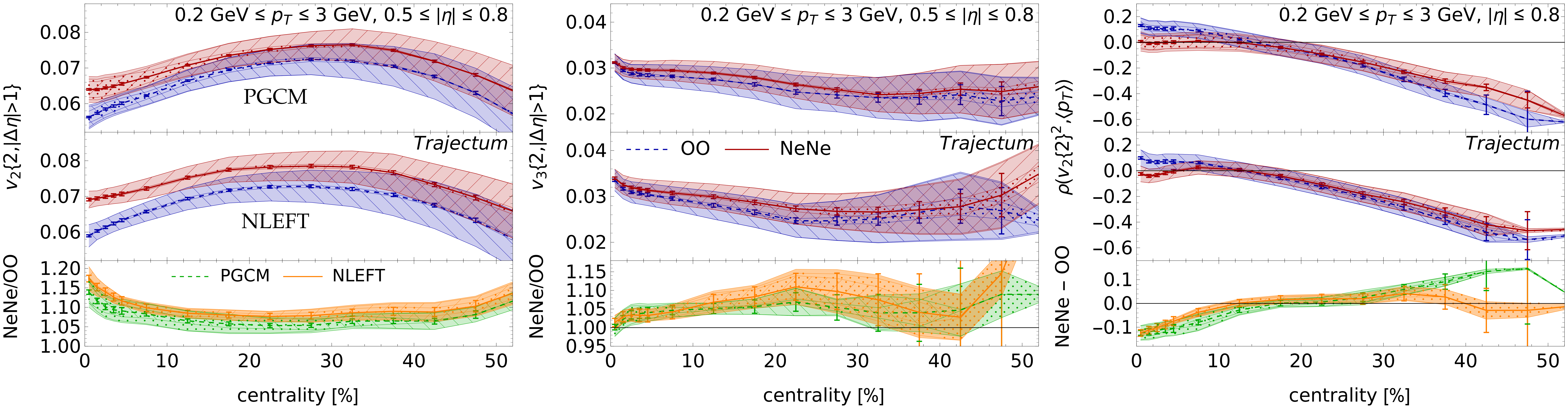

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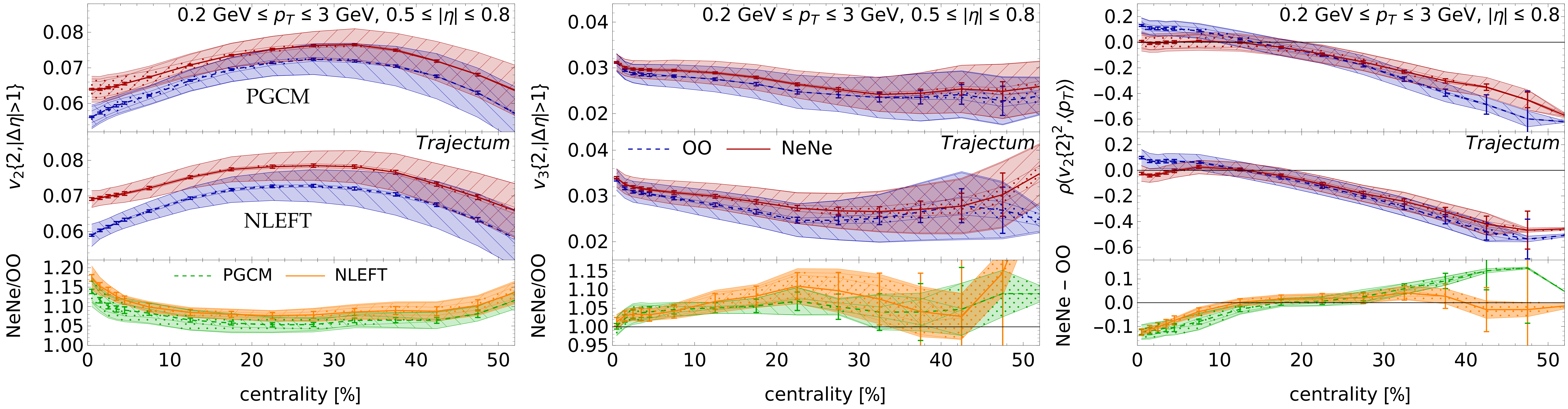

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○ **Enhanced elliptic flow** in Ne collisions vs. O baseline

$$\frac{v_2\{2\}_{\rm NeNe}}{v_2\{2\}_{\rm OO}} = \begin{cases} 1.174(8)_{\rm stat.}(31)^{Traj.}_{\rm syst.}(4)^{\rm str.}_{\rm syst.} & ({\rm NLEFT}) \\ 1.139(6)_{\rm stat.}(27)^{Traj.}_{\rm syst.}(28)^{\rm str.}_{\rm syst.} & ({\rm PGCM}) \end{cases}$$

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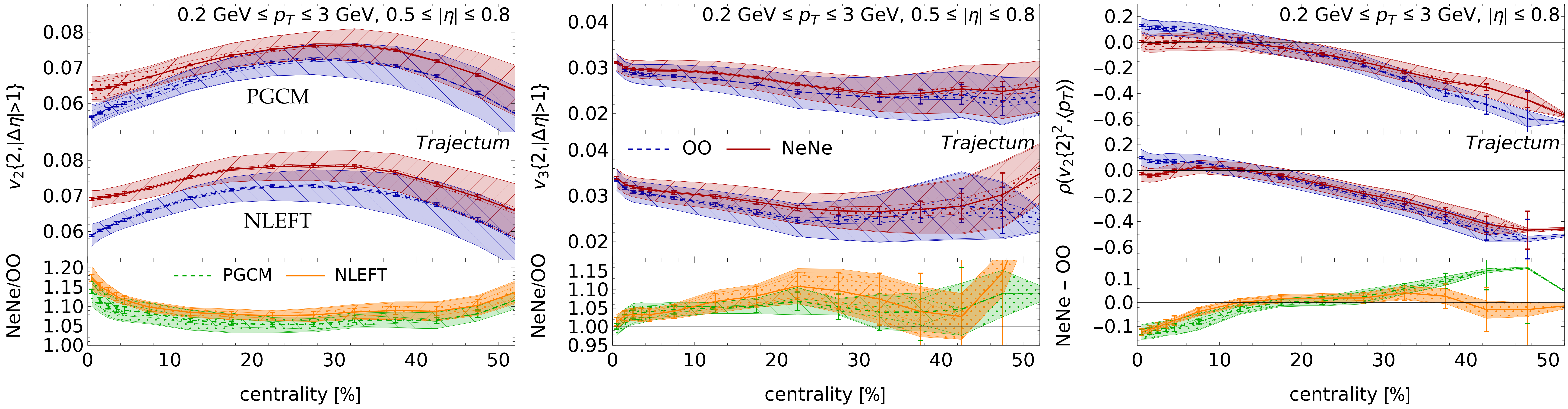

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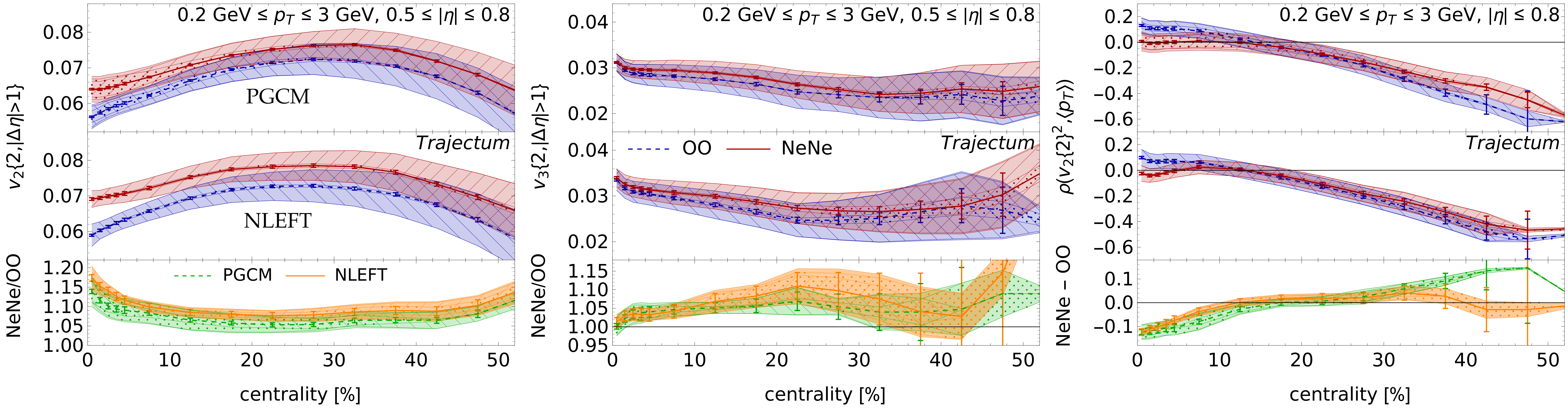
- Bowling-pin shape of ^{20}Ne also leads to negative ratio of Pearson coefficients for central events

High-energy collisions of small systems - hydro results

[Giacalone *et al.*, 2025]



○ **Systematic uncertainties** dominate absolute error bars, but to a large extent **cancel in ratios**

○ **Enhanced elliptic flow** in Ne collisions vs. O baseline

$$\frac{v_2\{2\}_{\mathrm{NeNe}}}{v_2\{2\}_{\mathrm{OO}}} = \begin{cases} 1.174(8)_{\mathrm{stat.}}(31)^{Traj.}_{\mathrm{syst.}}(4)^{\mathrm{str.}}_{\mathrm{syst.}} & \text{(NLEFT)} \\ 1.139(6)_{\mathrm{stat.}}(27)^{Traj.}_{\mathrm{syst.}}(28)^{\mathrm{str.}}_{\mathrm{syst.}} & \text{(PGCM)} \end{cases}$$

○ Bowling-pin shape of ^{20}Ne also leads to negative ratio of Pearson coefficients for central events

○ Triggered change in LHC schedule → **^{20}Ne-^{20}Ne will be run in July 2025!**

Fixed-target collisions

○ **SMOG2 @LHCb offers further insight** via fixed-target collisions

→ Heavy (e.g. ^{208}Pb) - light ion collisions optimal to vary initial-state geometry

→ Large flexibility in the choice of the light species (← nuclear structure input)

→ ^{208}Pb-^{16}O and ^{208}Pb-^{20}Ne collisions can provide useful complementary information at lower energy

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○ Same nuclear structure calculations applied to another set of simulations (PGCM/NLEFT + MUSIC + UrQMD)

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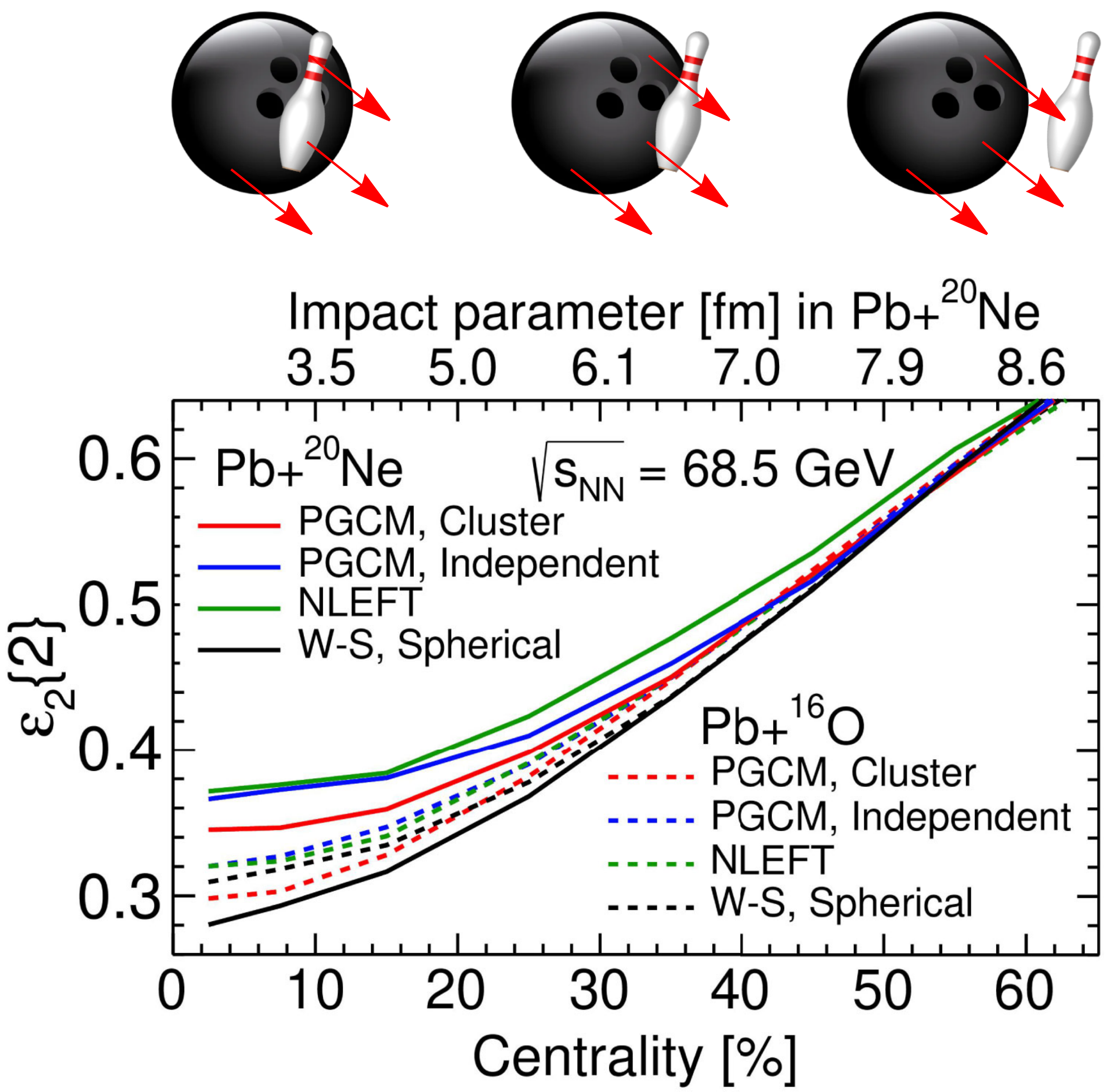

Initial-state ellipticity

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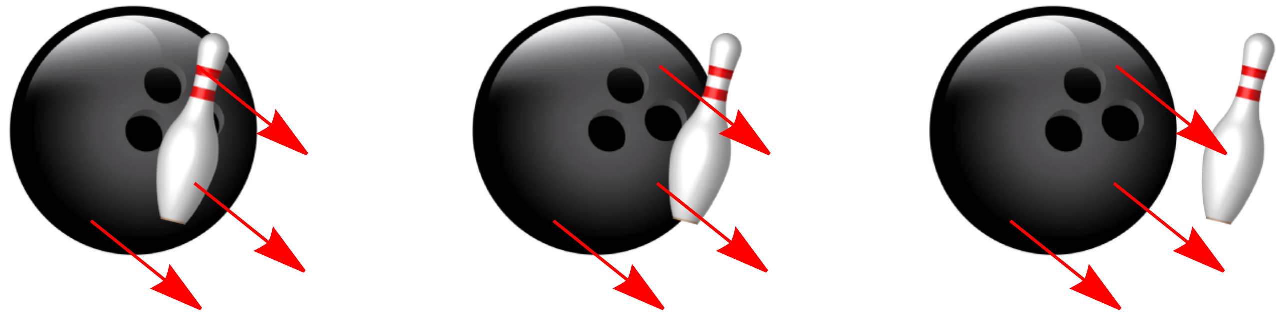
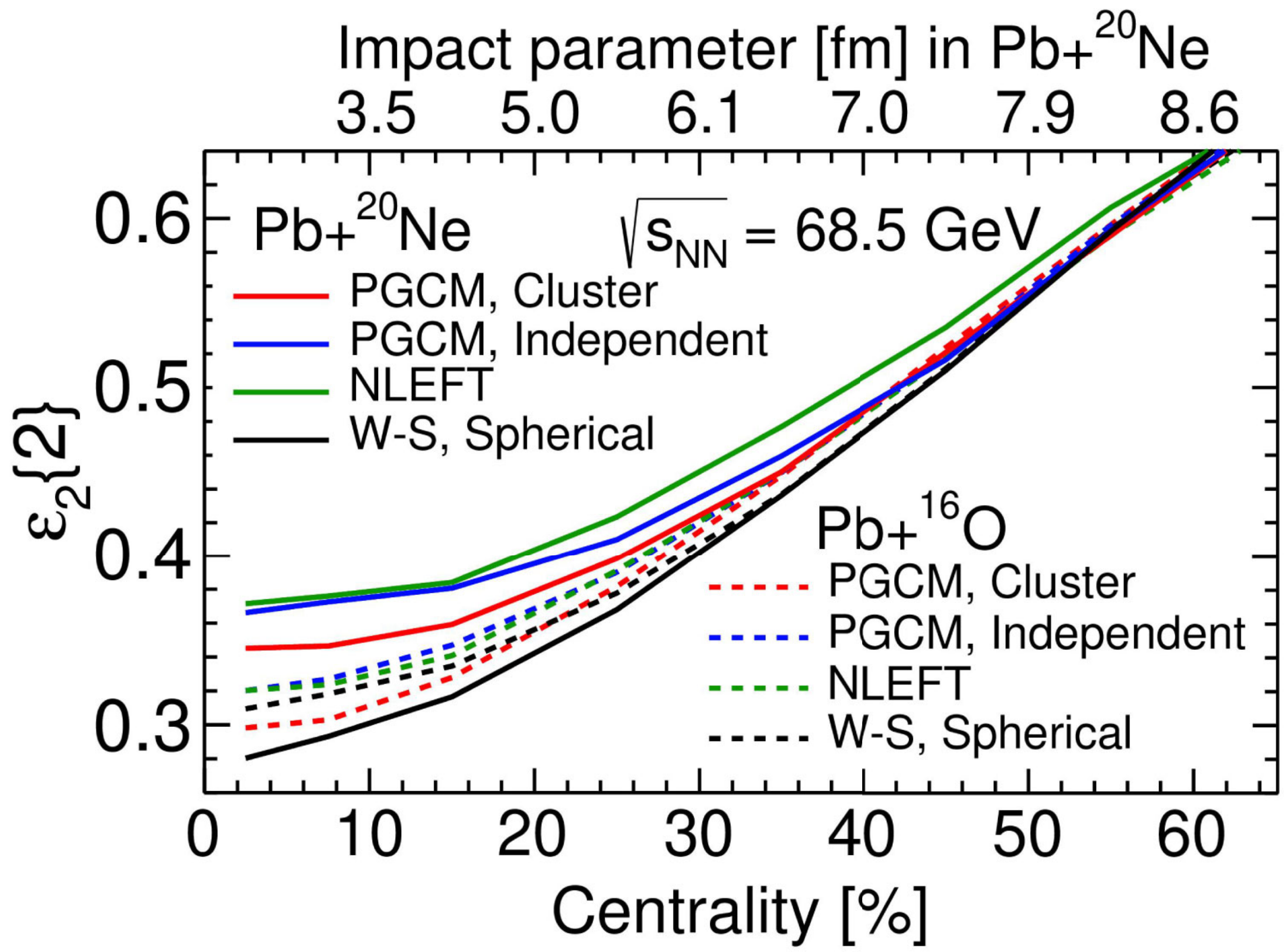
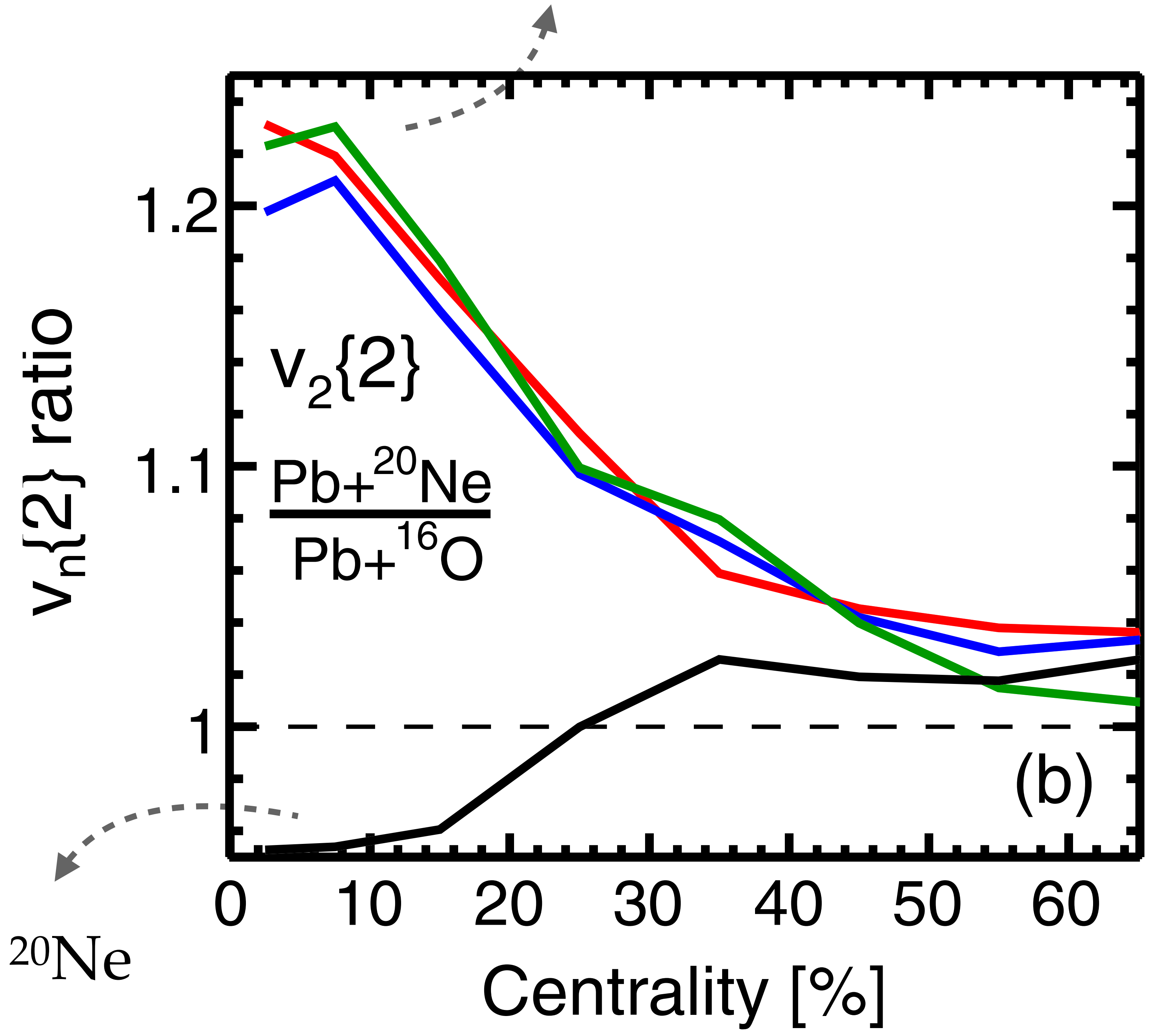




Initial-state ellipticity

Large increase of v_2 ratio

deformed ^{20}Ne

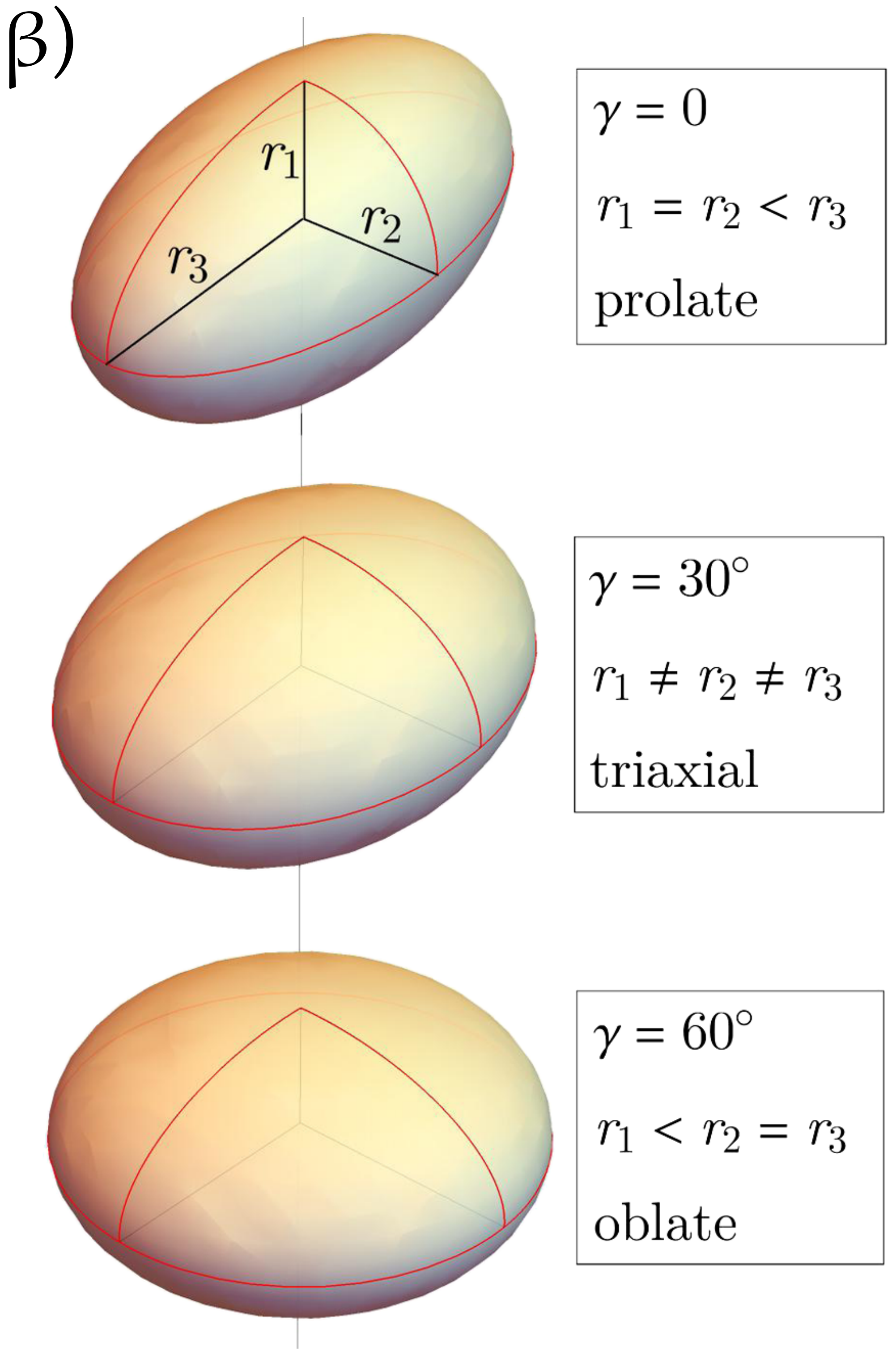


spherical ^{20}Ne

Triaxial shape of ^{129}Xe

- **How precisely can nuclear deformation be determined in relativistic collisions?**
- Compare prolate, oblate and triaxial systems → Parametrise surface as $R(\theta,\varphi) = R_0\{1+\beta[\cos\gamma Y_{20}(\theta,\varphi)+\sin\gamma Y_{22}(\theta,\varphi)]\}$

(Fix β)



[Bally *et al.*, 2022]

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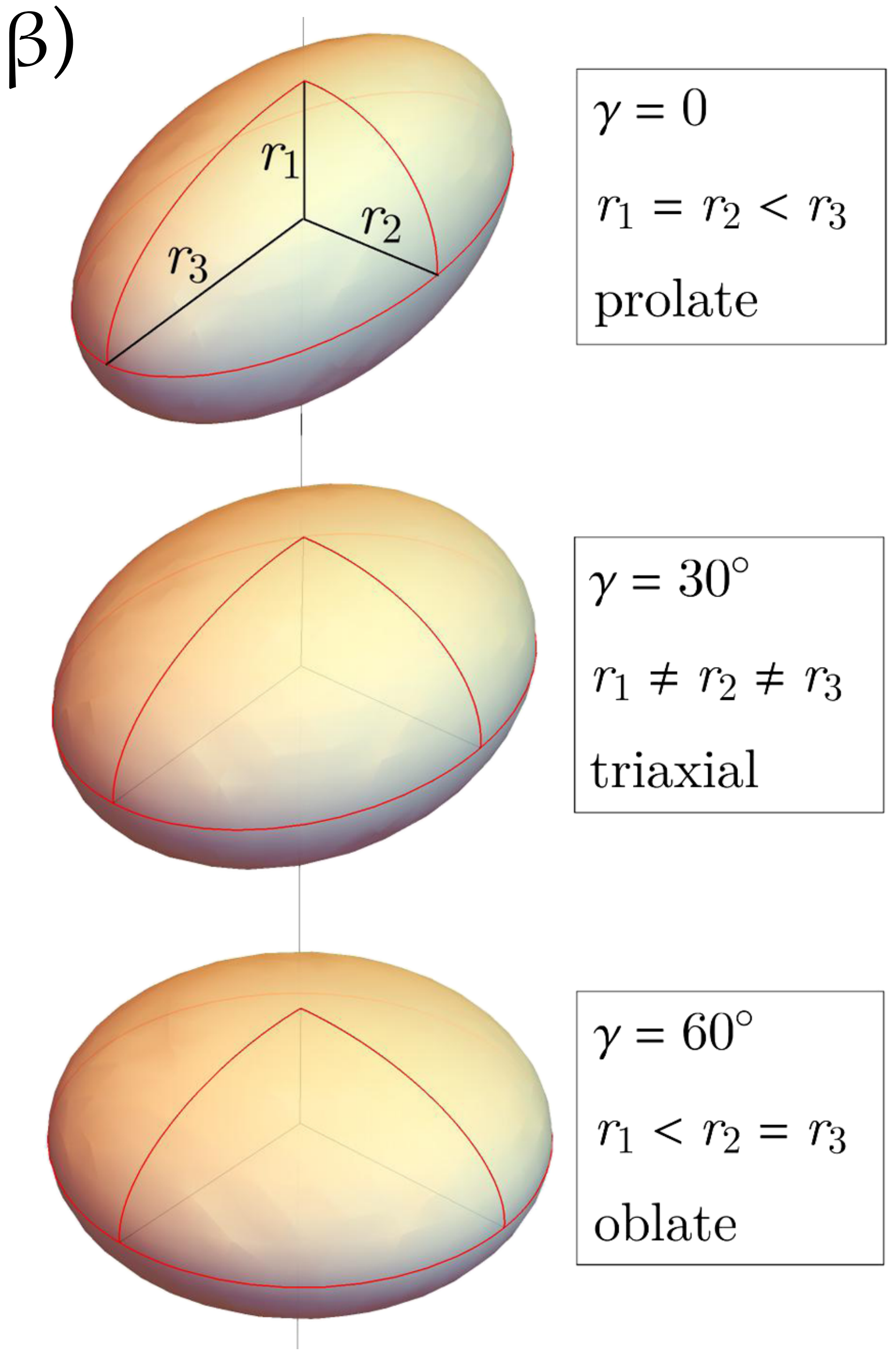
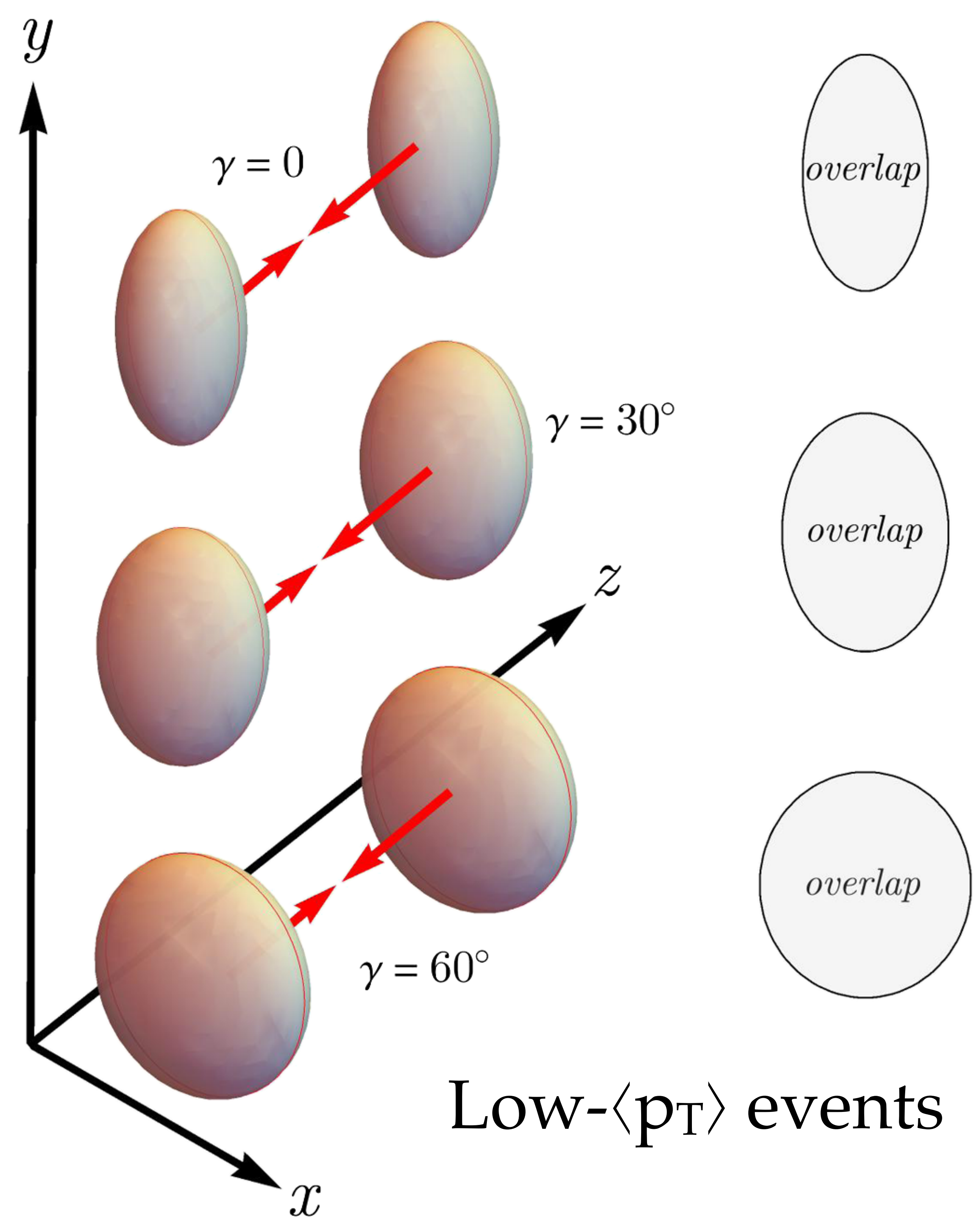


Collisions that minimise $\langle p_T \rangle$
↔ maximise transverse area

(Low-$\langle p_T \rangle$ events)



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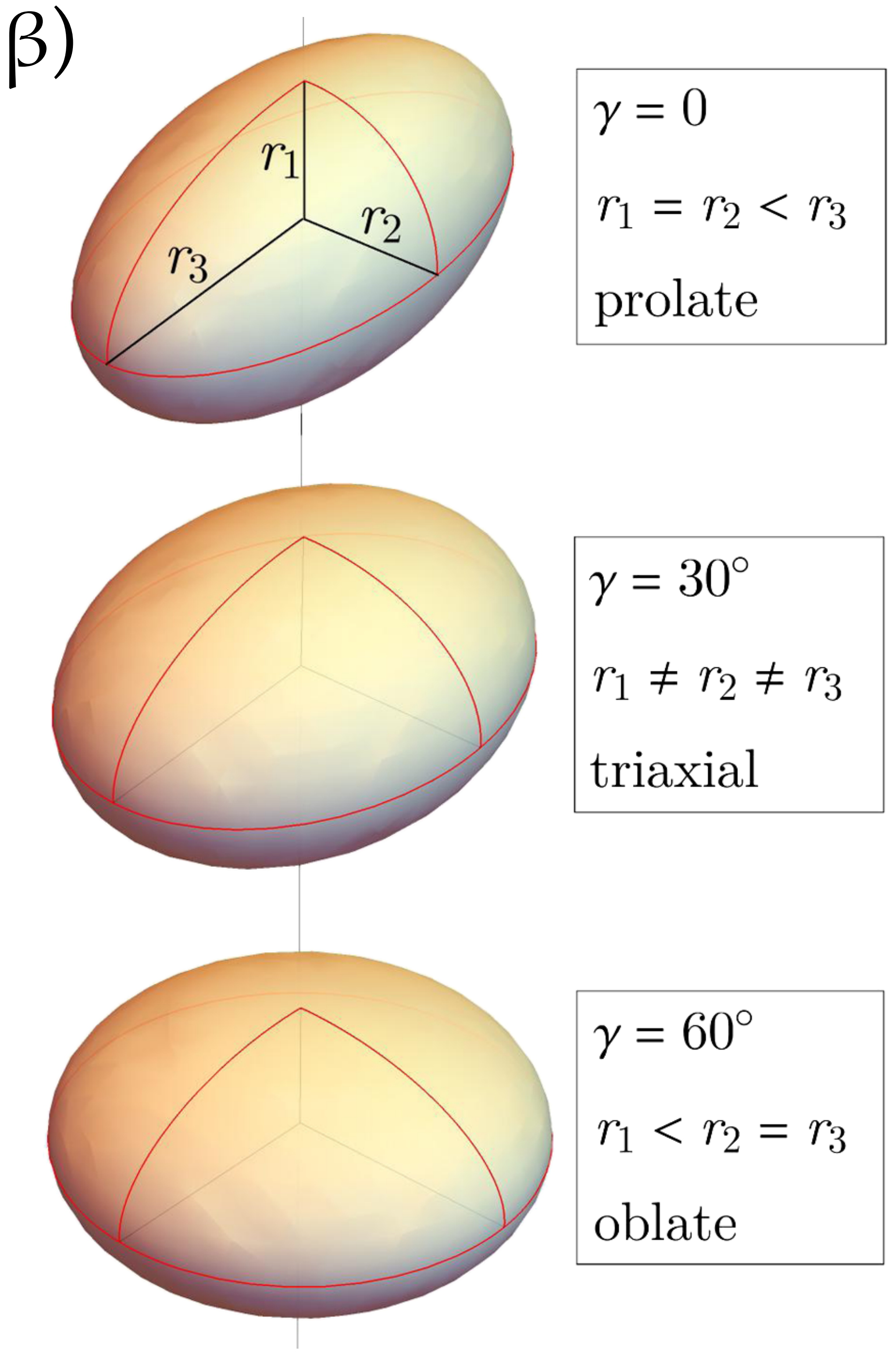

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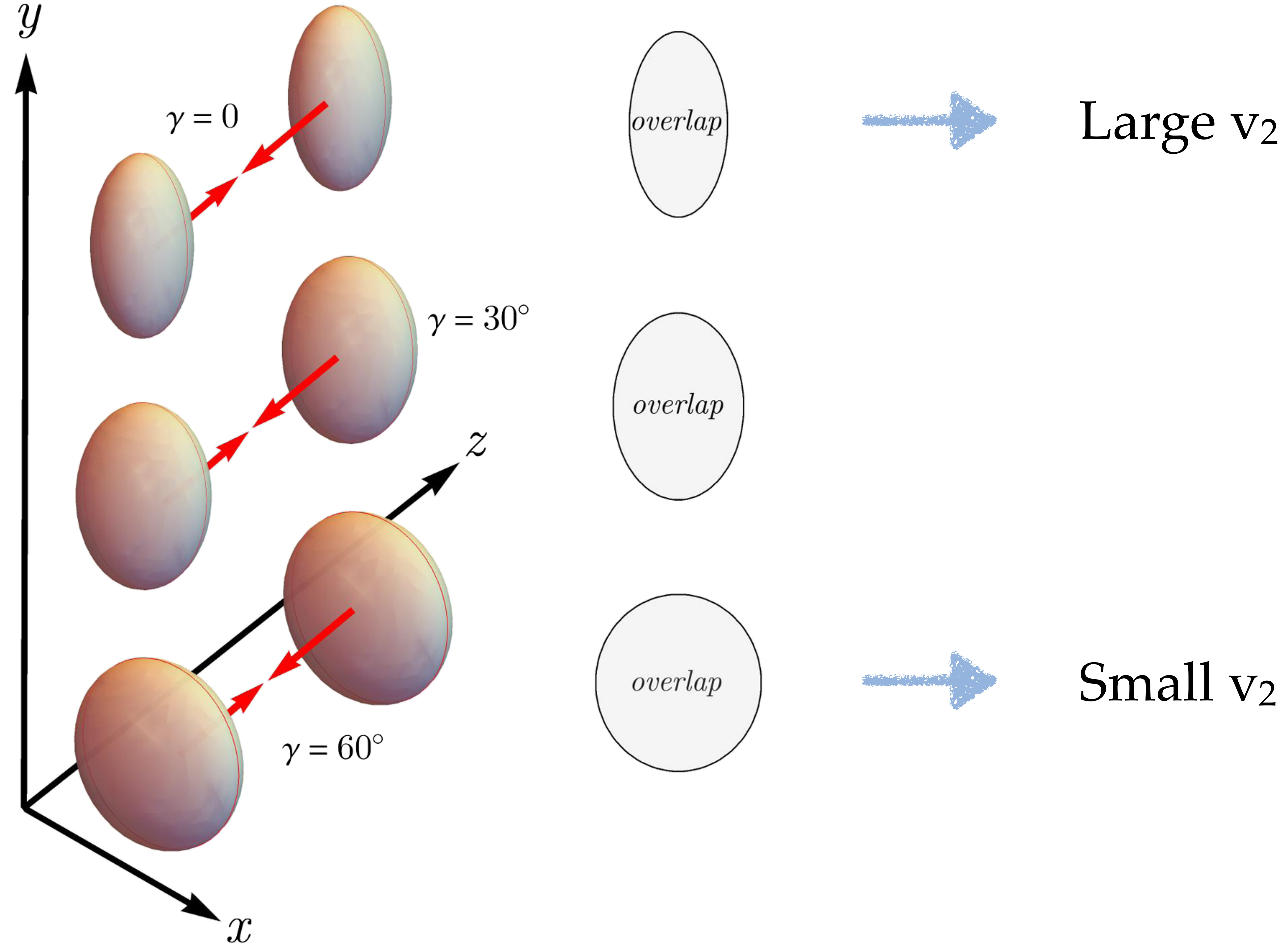

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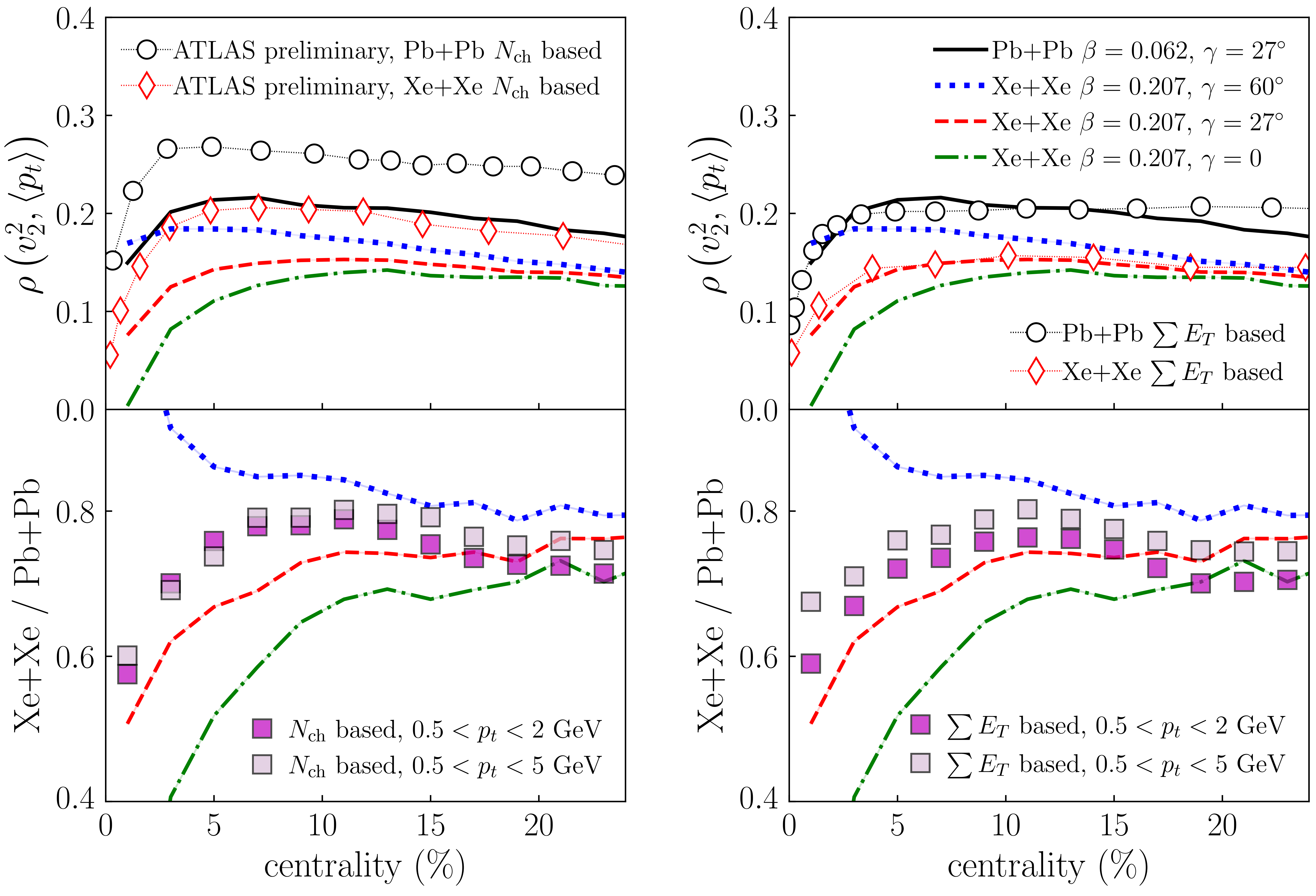
- **Pearson coefficient** captures such correlation $$\rho(v_2^2, \langle p_t \rangle) = \frac{\langle \delta v_2^2 \delta \langle p_t \rangle \rangle}{\sqrt{\langle (\delta v_2^2)^2 \rangle \langle (\delta \langle p_t \rangle)^2 \rangle}}$$

Triaxial shape of ^{129}Xe

○ **Analysis of ^{129}Xe-^{129}Xe data from ATLAS vs nuclear structure calculations (PGCM with phenomenological interactions)**

[Bally *et al.*, 2022]



○ Overall sensitivity to quadrupole def. β

○ Central collisions **sensitive to triaxiality γ**

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○ **Analysis of ^{129}Xe-^{129}Xe data from ATLAS vs nuclear structure calculations (PGCM with phenomenological interactions)**

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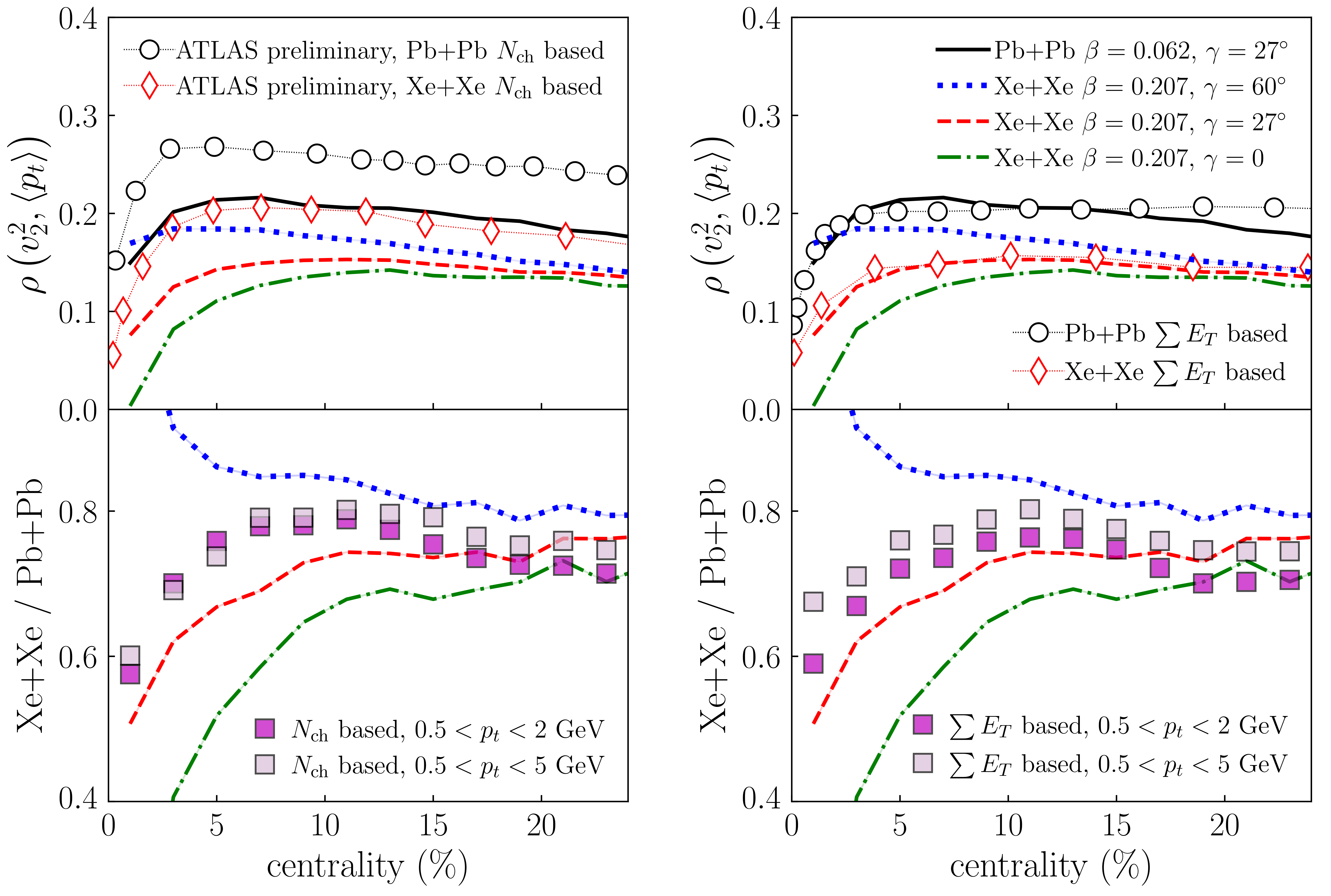

○ Overall sensitivity to quadrupole def. β

○ Central collisions **sensitive to triaxiality γ**

○ Best fit from χ^2 analysis → β=0.20 γ=30°

○ PGCM calculation → β=0.21 γ=27°

Conclusions

- **Nuclear structure**
  - Progress in ab initio nuclear structure
  - Some examples of recent applications
    → Spherical vs deformed; "precision era" of ab initio calculations

- **High-energy collisions**
  - O-O & Ne-Ne collisions
    → Different shapes allow to pinpoint role of hydrodynamic evolution
  - Fixed-target Pb-Ne & PB-Ne collisions
    → Complementary studies & more flexibility in the target species
  - Triaxial shape of ^{129}Xe
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BUON COMPLEANNO, WANDA!