Quasi-static modelling of relativistic beam-plasma instabilities

P. San Miguel Claveria[1]

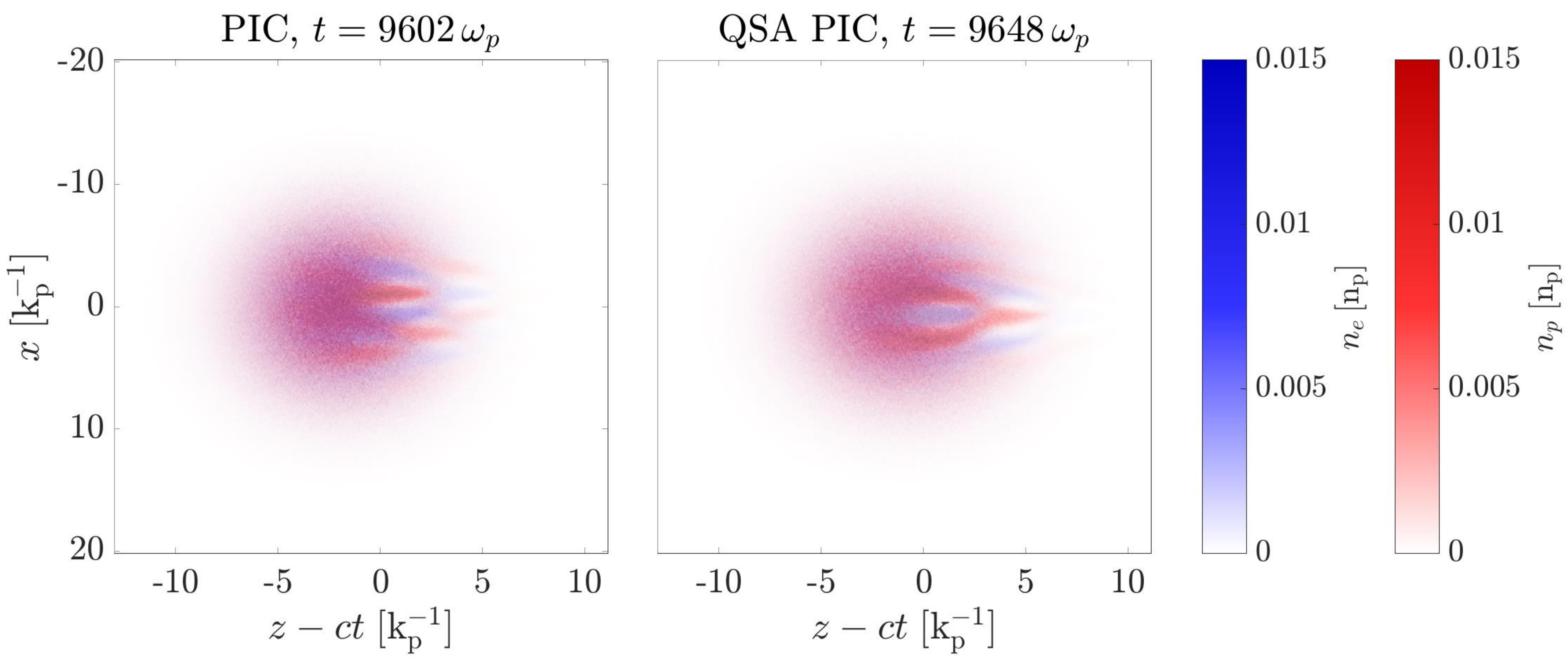
Q. Labro[2], X. Davoine[2], S. Corde[3]

L. Gremillet[2], F. Fiuza[1]

[1] GPA / Instituto de Plasmas e Fusão Nuclear
Instituto Superior Técnico, Lisbon, Portugal

[2] CEA, DAM, DIF, 91297 Arpajon, France

[3] LOA / CNRS, Ecole Polytechnique



Funded by the European Union

ERA-MSCA-PF 101180632

Beam-driven plasma wakefield acceleration (PWFA)

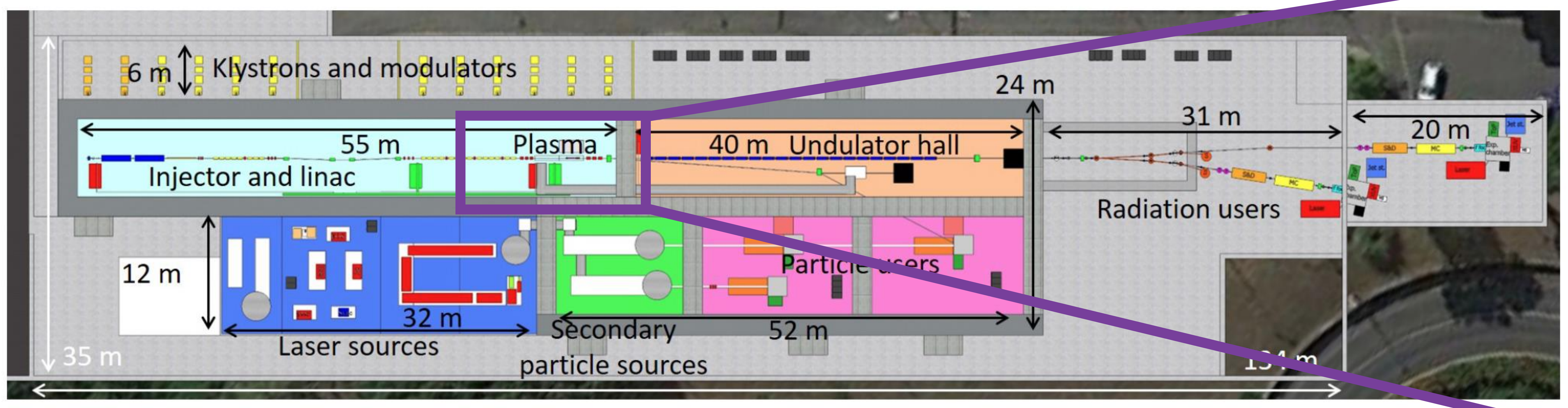
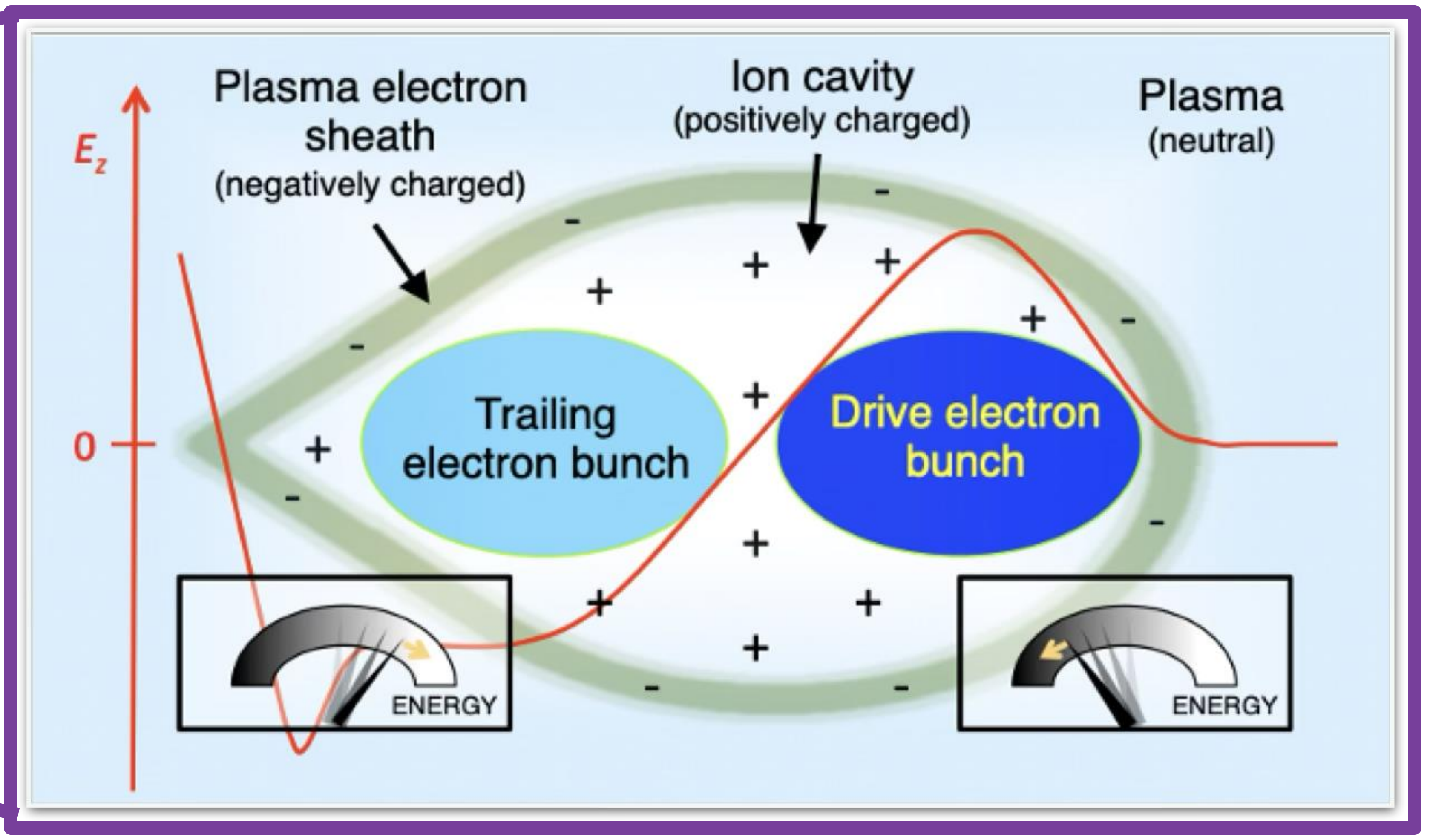


Figure 3.10: Layout of the EuPRAXIA@SPARC_LAB infrastructure.

Taken from EuPRAXIA CDR



Courtesy of S. Corde

For an efficient PWFA

1) Beam Lorentz factor $\gamma_b \gg 1$
2) Beam-to-plasma density ratio $\alpha = {}^{n_b}/_{n_p} > 1$
3) Beam sizes $\sigma_r, \sigma_z < k_p^{-1}$

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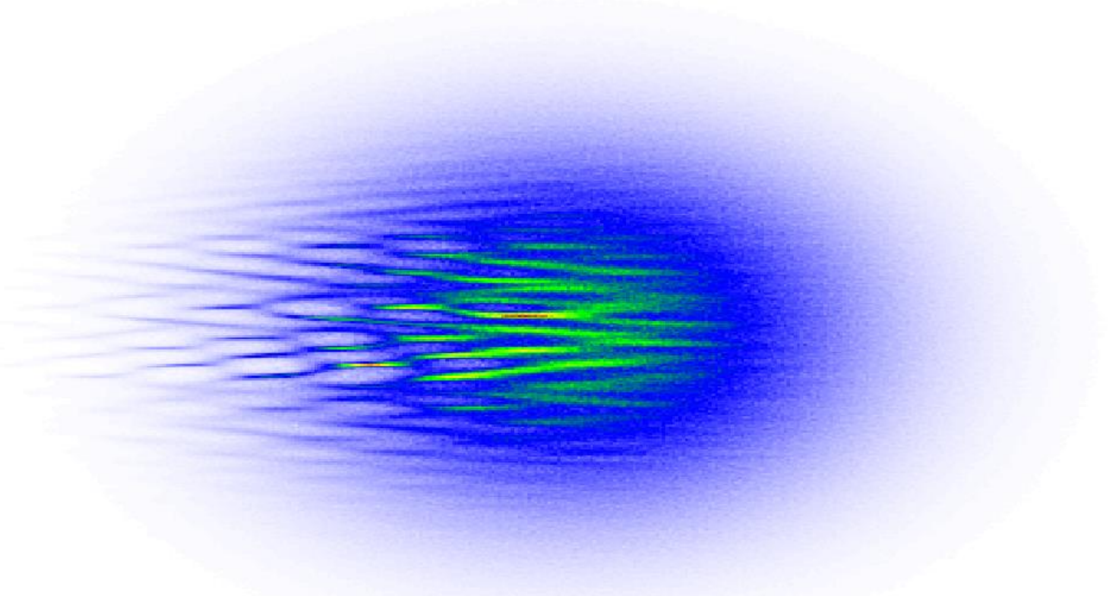

Filamented beam density due to exponential growth of EM fields

For an efficient PWFA

1) Beam Lorentz factor $\gamma_b \gg 1$
2) Beam-to-plasma density ratio $\alpha = {}^{n_b}/_{n_p} > 1$
3) Beam sizes $\sigma_r, \sigma_z < k_p^{-1}$

For beam-plasma instabilities

1) Beam Lorentz factor $\gamma_b \gg 1$
2) Beam-to-plasma density ratio $\alpha = {}^{n_b}/_{n_p} < 1$ *
3) Beam sizes $\sigma_r, \sigma_z > k_p^{-1}$

* Unless neutral beam, e.g. $e^{\pm}$ fireball beam [Arrowsmith, C.D., *Nat Commun* **15**, 5029 (2024)]

Why are beam-plasma instabilities important?

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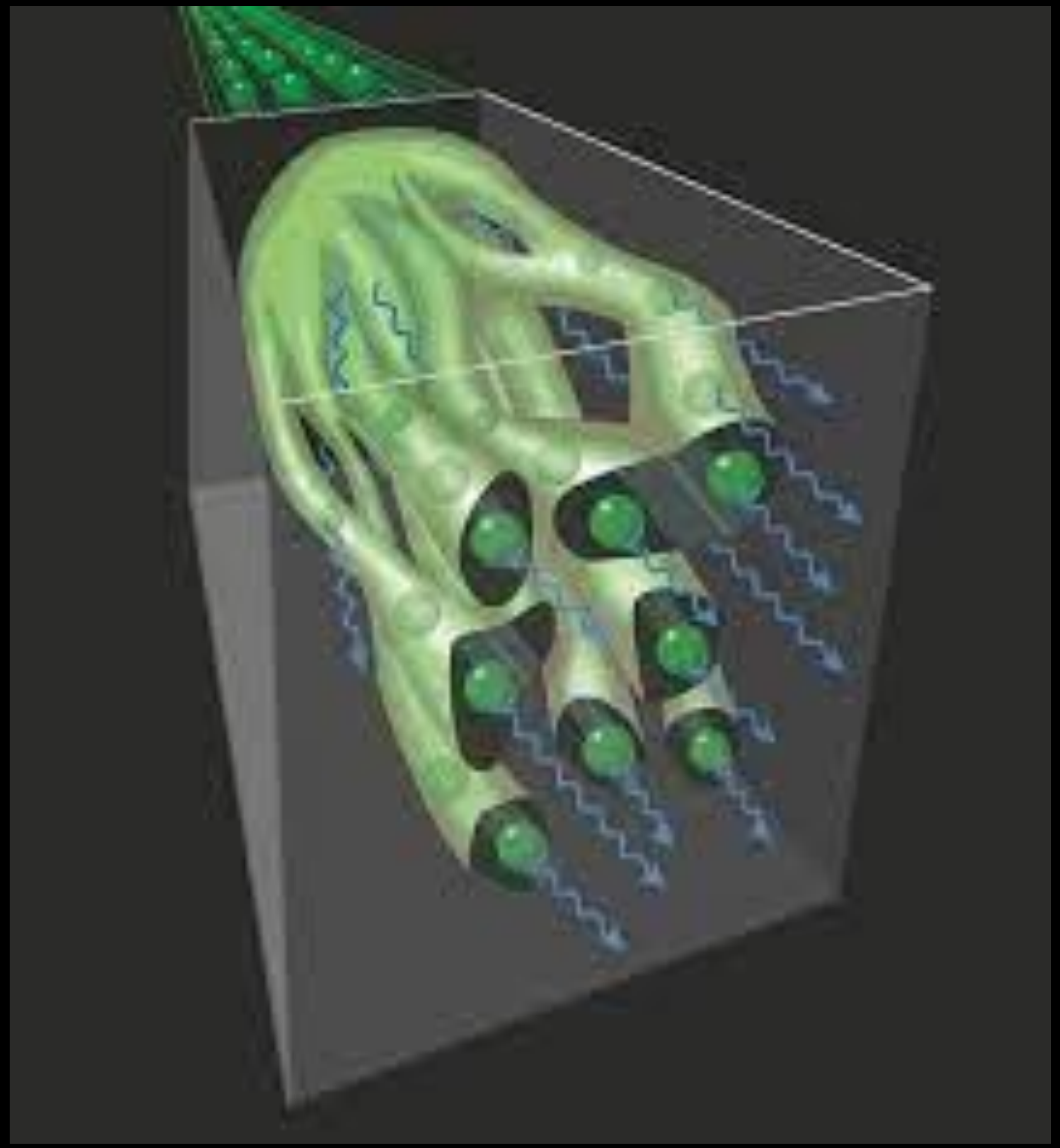
Laser-plasma interactions

Particle and radiation sources

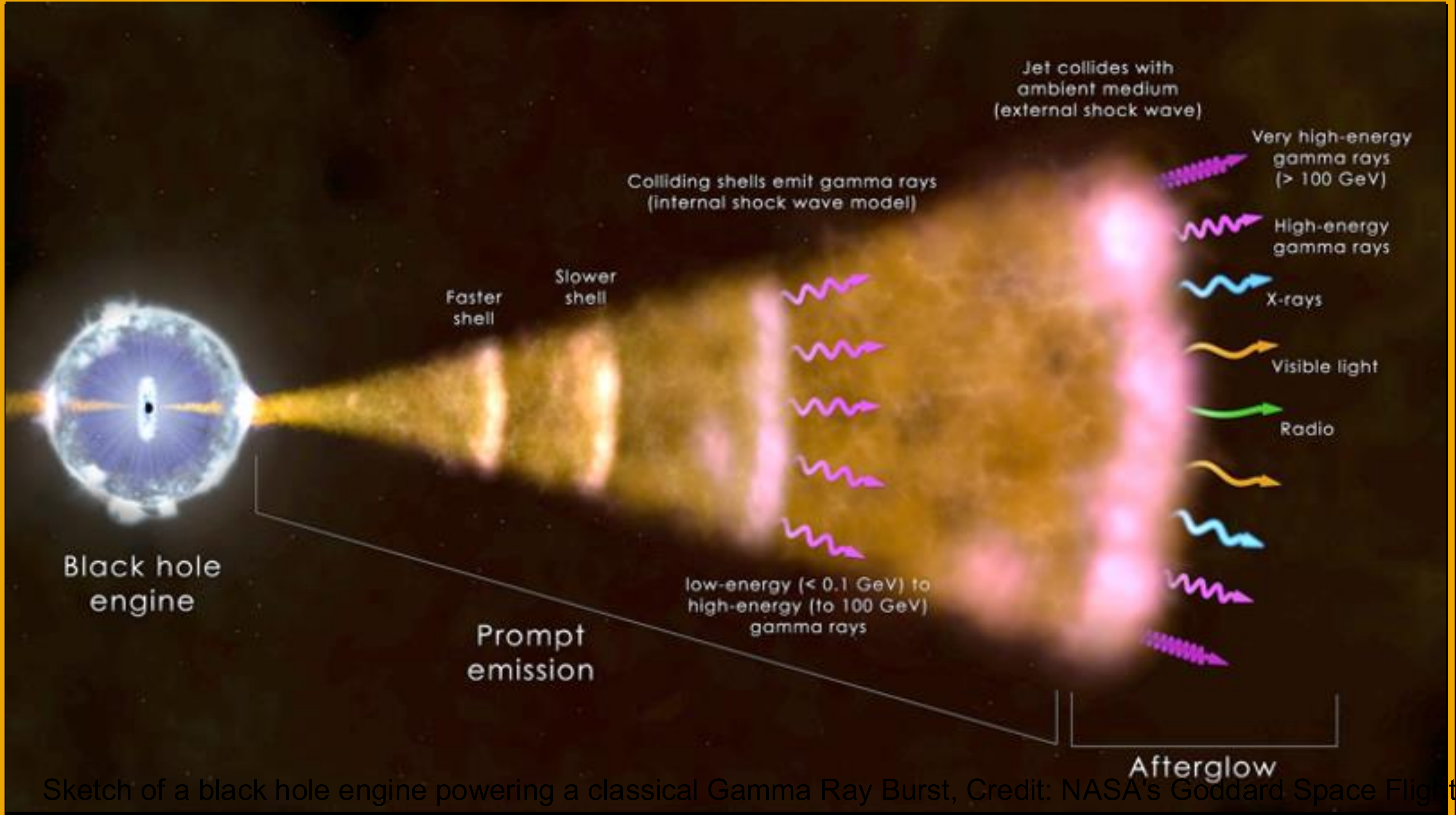


Giant collimated gamma-ray flashes
A. Benedetti, et. al. Nat. Phot. (2018),

NIF hohlraum. Credits to LLNL

Inertial Confinement Fusion

Astrophysics



Sketch of a black hole engine powering a classical Gamma Ray Burst, Credit: NASA's Goddard Space Flight Ce

Mediators of radiation-particle energy exchange

Contents

Contents

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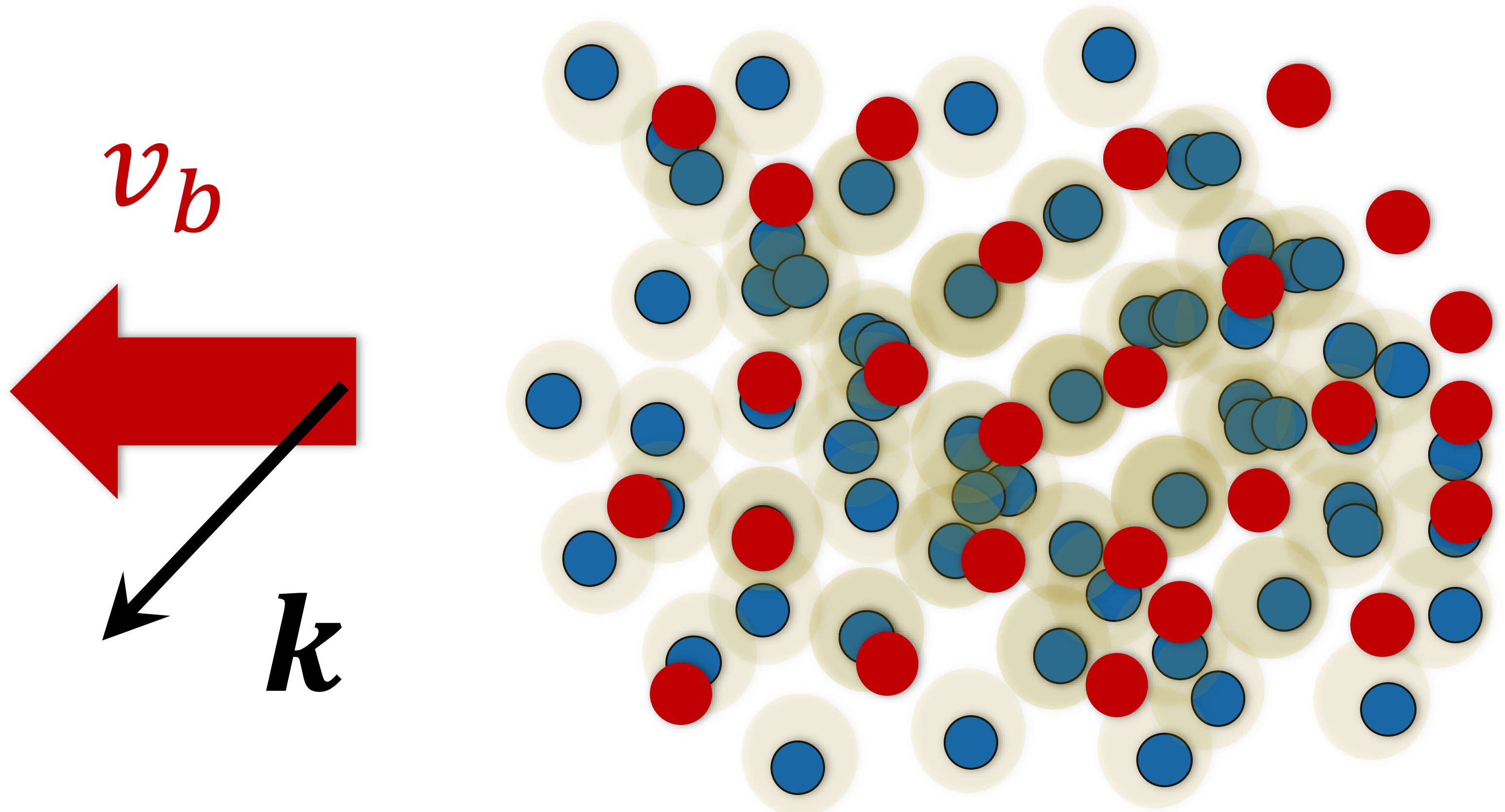
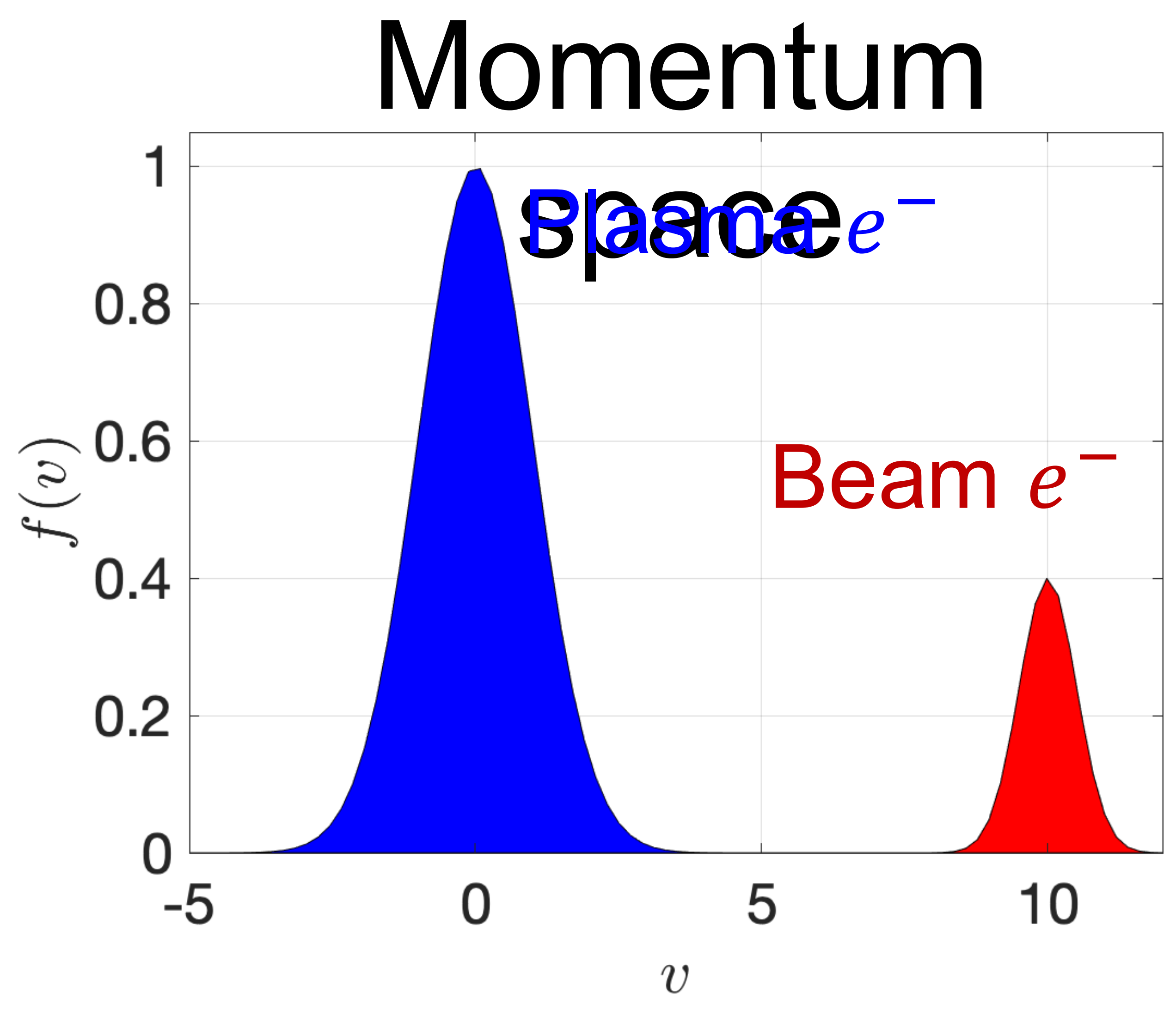
Beam-plasma instabilities

Unbounded systems

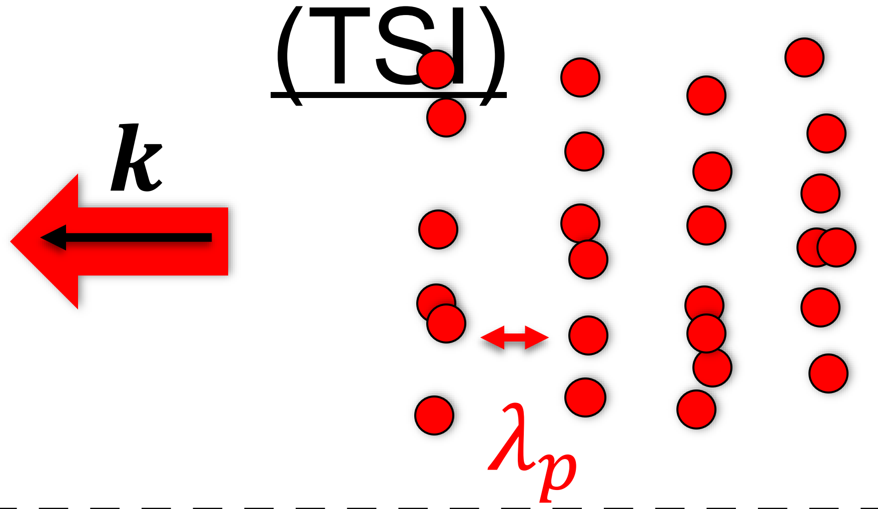
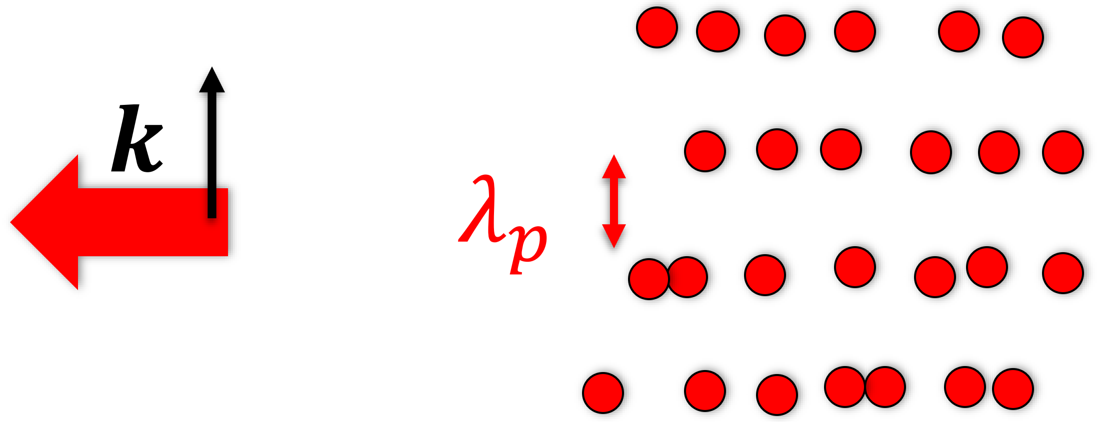
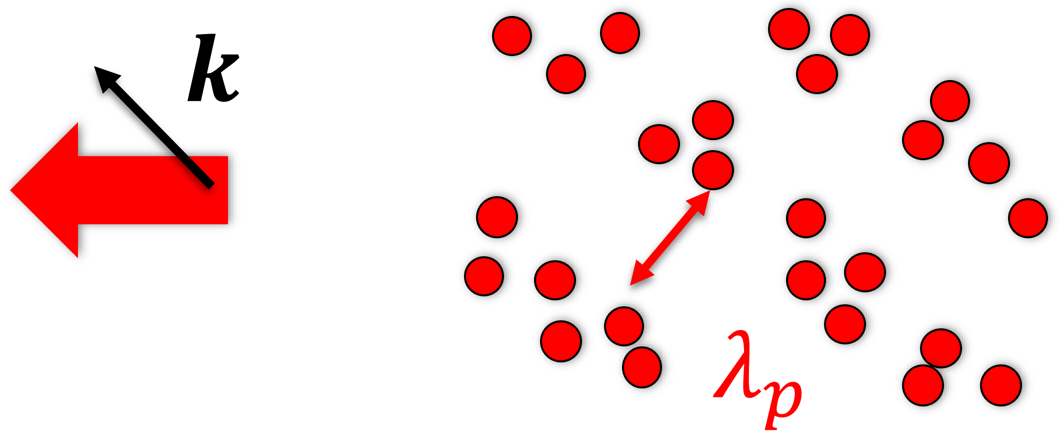
EM fields + three species:

- Plasma electrons
- Ions (stationary background)
- Beam particles (e^-)





"Three instabilities"

Two-Stream Instability (TSI)	Current Filament. Instability (CFI)	Oblique Instability (OTSI)
		
Electrostatic longitudinal mode	Magnetic transverse mode	(Quasi-)electrostatic oblique mode
$\Gamma_{\mathrm{TSI}} = \frac{\sqrt{3}}{2^{4/3}} \frac{\alpha^{1/3}}{\gamma_b} \omega_p$	$\Gamma_{\mathrm{CFI}} = \beta \sqrt{\frac{\alpha}{\gamma_b}} \omega_p$	$\Gamma_{\mathrm{Obl}} = \frac{\sqrt{3}}{2^{4/3}} \left(\frac{\alpha}{\gamma_b}\right)^{1/3} \omega_p$

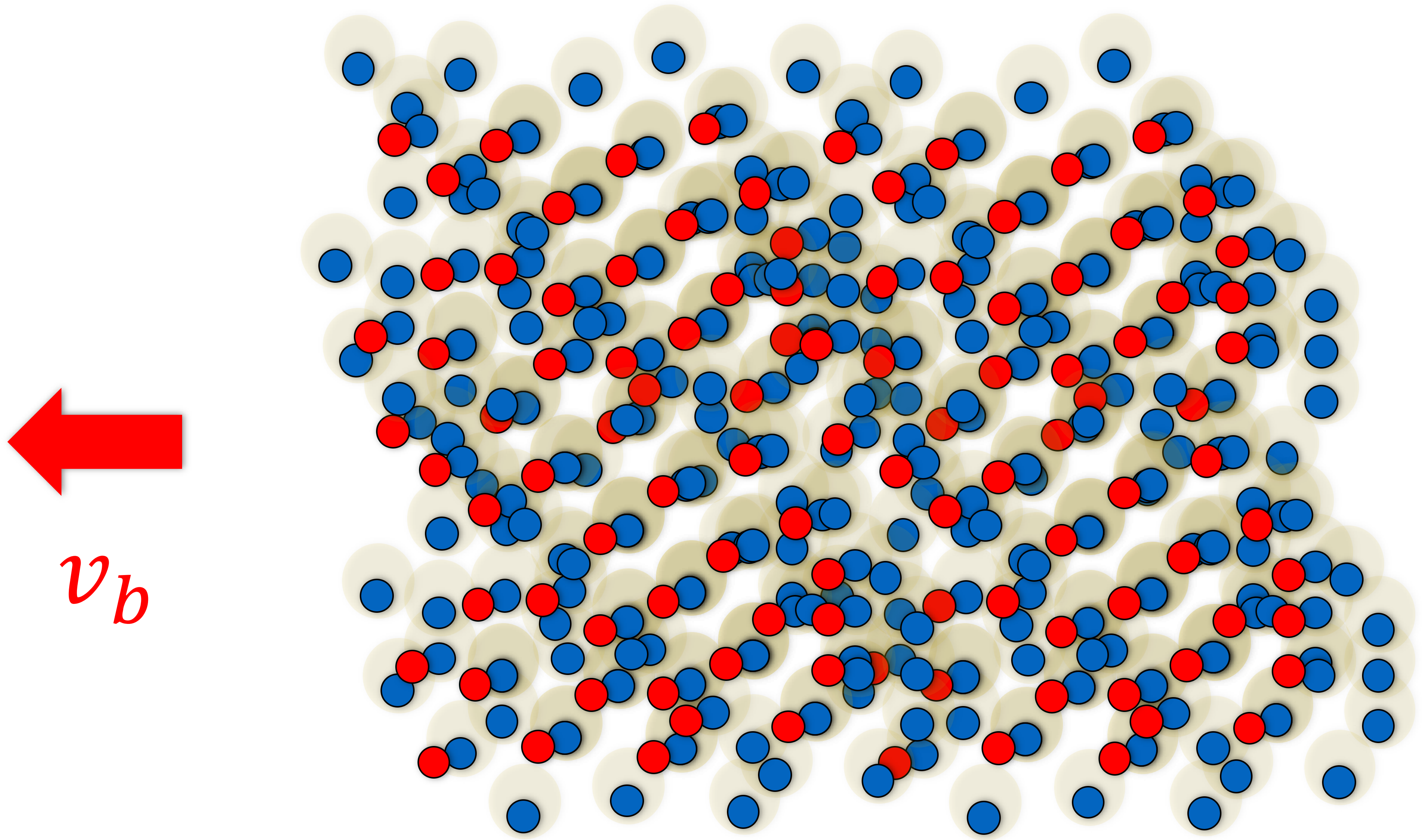
Growthrate (Kinetic theory for cold unbounded systems)

* A. Bret et al. *Phys. Plasmas* 17, 120501 (2010)

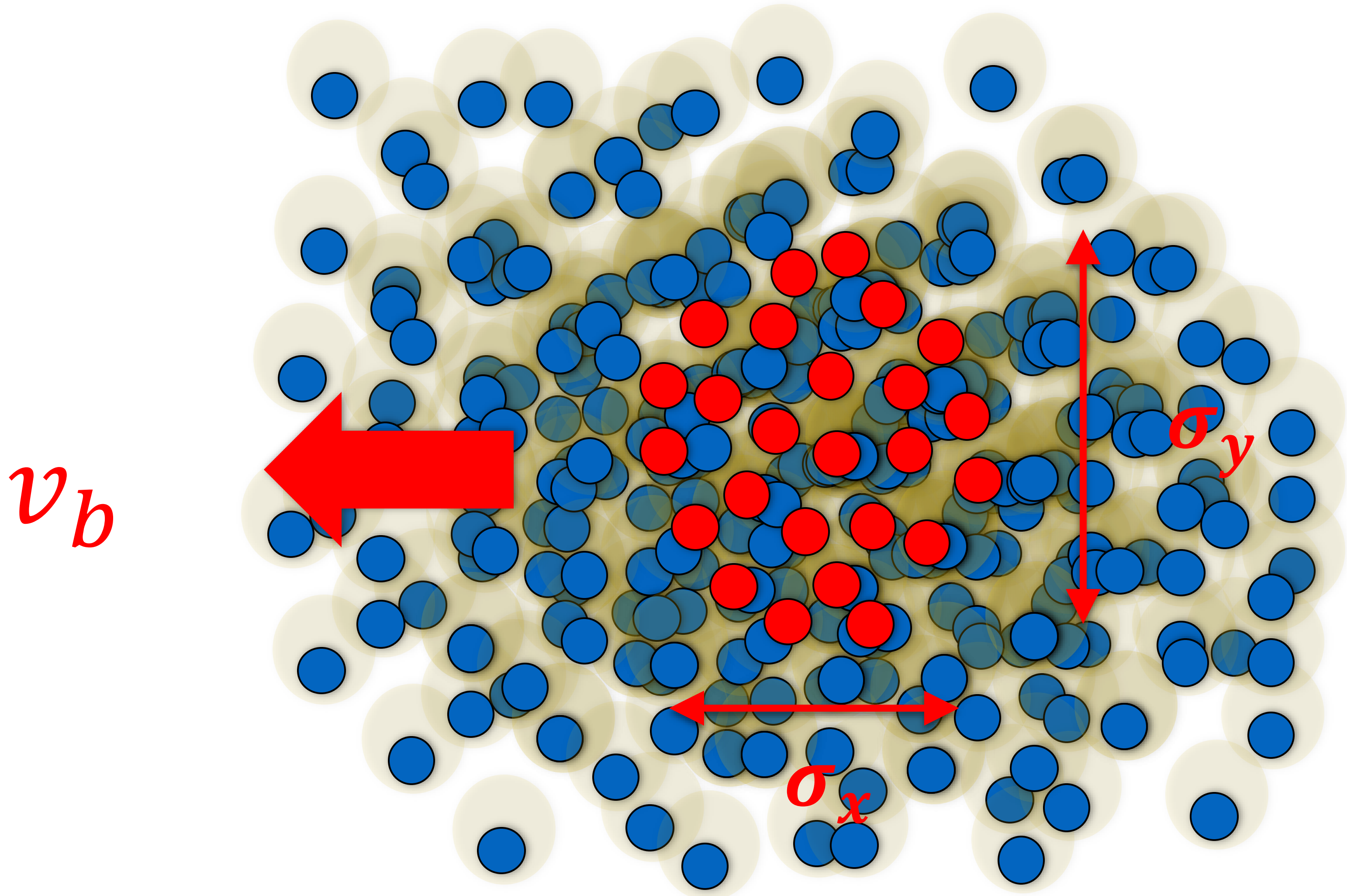
Beam-plasma instabilities

Bounded systems

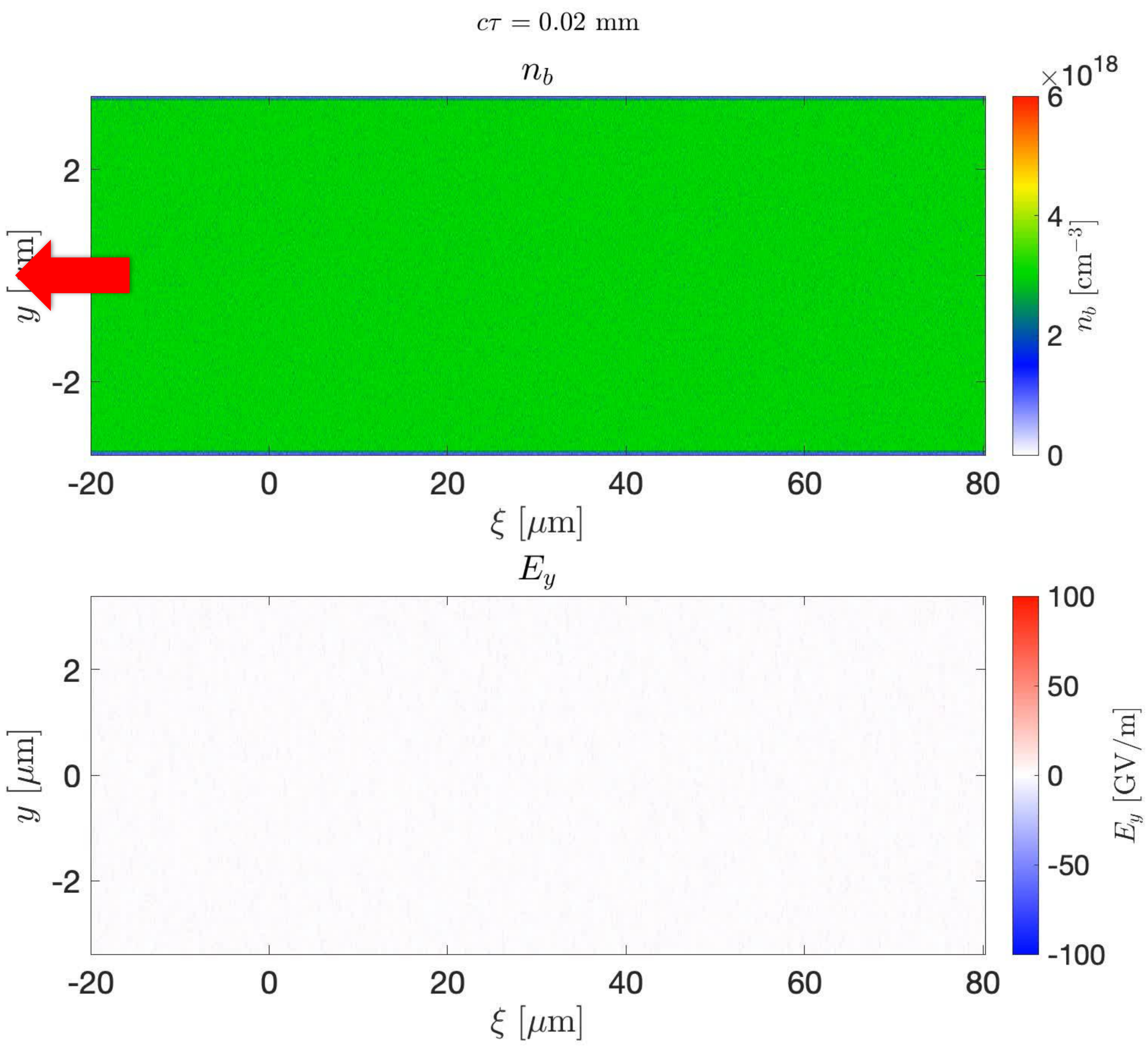
Unbounded system



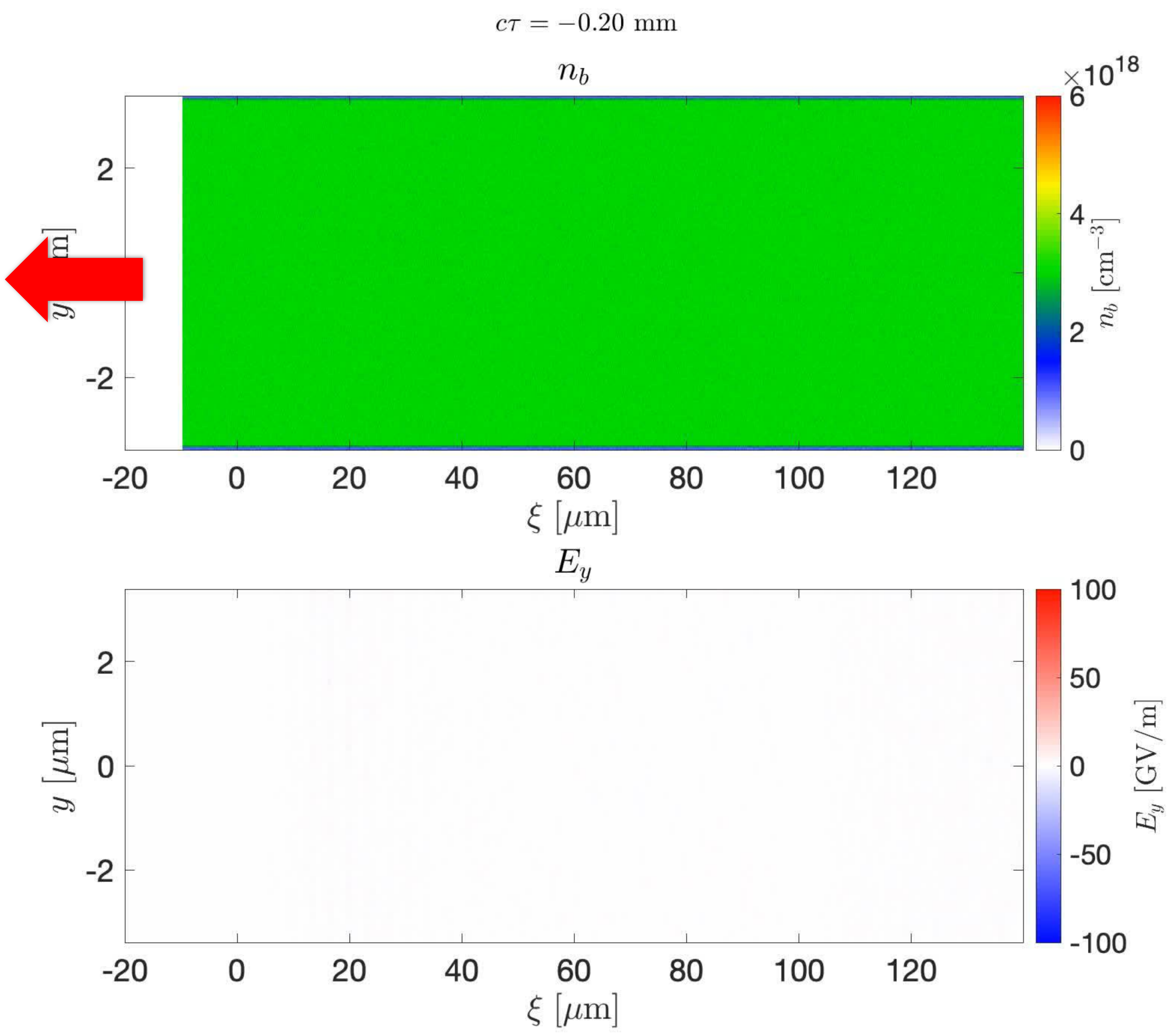
Finite beam



Unbounded system



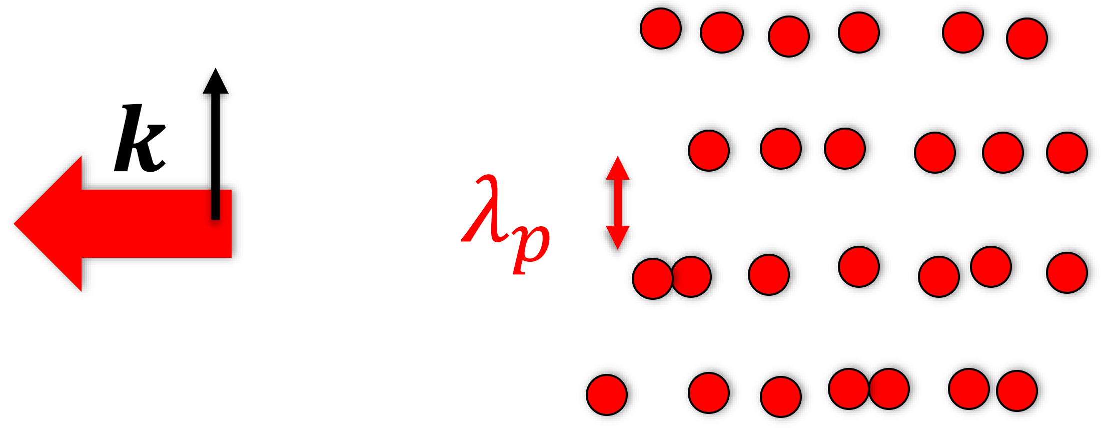
Finite beam



Spatiotemporal growth with finite-size beams

Co-moving variables
$\xi = z - v_b t$
$\tau = t$

Current Filament. Instability (CFI)



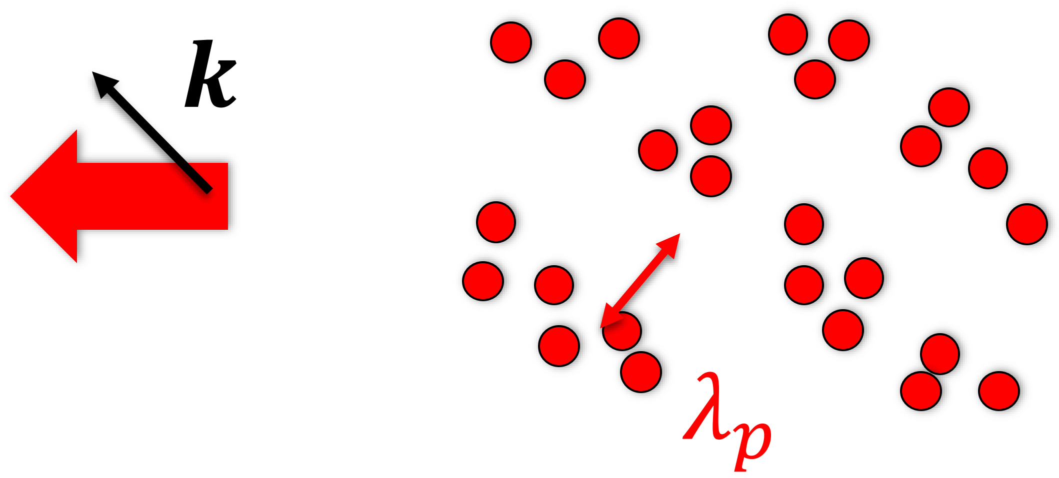
Purely **electromagnetic** model*

$$\left(\partial_\tau^2 + Q \cdot \partial_\xi \partial_\tau - \Gamma_{\mathrm{CFI}}^2\right) \delta n_b^{(1)} = 0 \qquad \left(Q \propto \frac{1}{\gamma_b^3}\right)$$

Asymptotic solutions:

$$n_b^{(1)}(\xi, y, \tau) \propto \cosh(\Gamma_{\mathrm{CFI}} \tau) + e^{\Gamma_{\mathrm{CFI}} \tau^{\frac{1}{2}} \left[\frac{\xi}{Q}\right]^{\frac{1}{2}}}$$

Oblique Instability (OTSI)



Purely **electrostatic** model**

$$\left(\partial_\tau^3 + \partial_\xi \partial_\tau^2 - i\Gamma_{\mathrm{Obl}}^3\right) \delta n_b^{(1)} = 0$$

Asymptotic solutions:

$$n_b^{(1)}(\xi, y, \tau) \propto \begin{cases} e^{\Gamma_{\mathrm{(O)TSI}} \tau} & \text{for } v_b \tau < \xi \\ e^{\Gamma_{\mathrm{(O)TSI}} \tau^{2/3} \xi^{1/3}} & \text{for } v_b \tau > \xi \end{cases}$$

*V B Pathak et al 2015 New J. Phys. 17 043049

**P. San Miguel Claveria, et al., Phys. Rev. Research 4, 023085, 2022

Contents

Contents

TÉCNICO LISBOA

Contents

Quasi-static PIC codes:

Q. Labro et al. Simulating ultrarelativistic beam-plasma instabilities with a quasistatic particle-in-cell code, https://arxiv.org/abs/2506.18567

Contents

Quasi-static model of beam-plasma instabilities

Plasma parameters

Uniform density

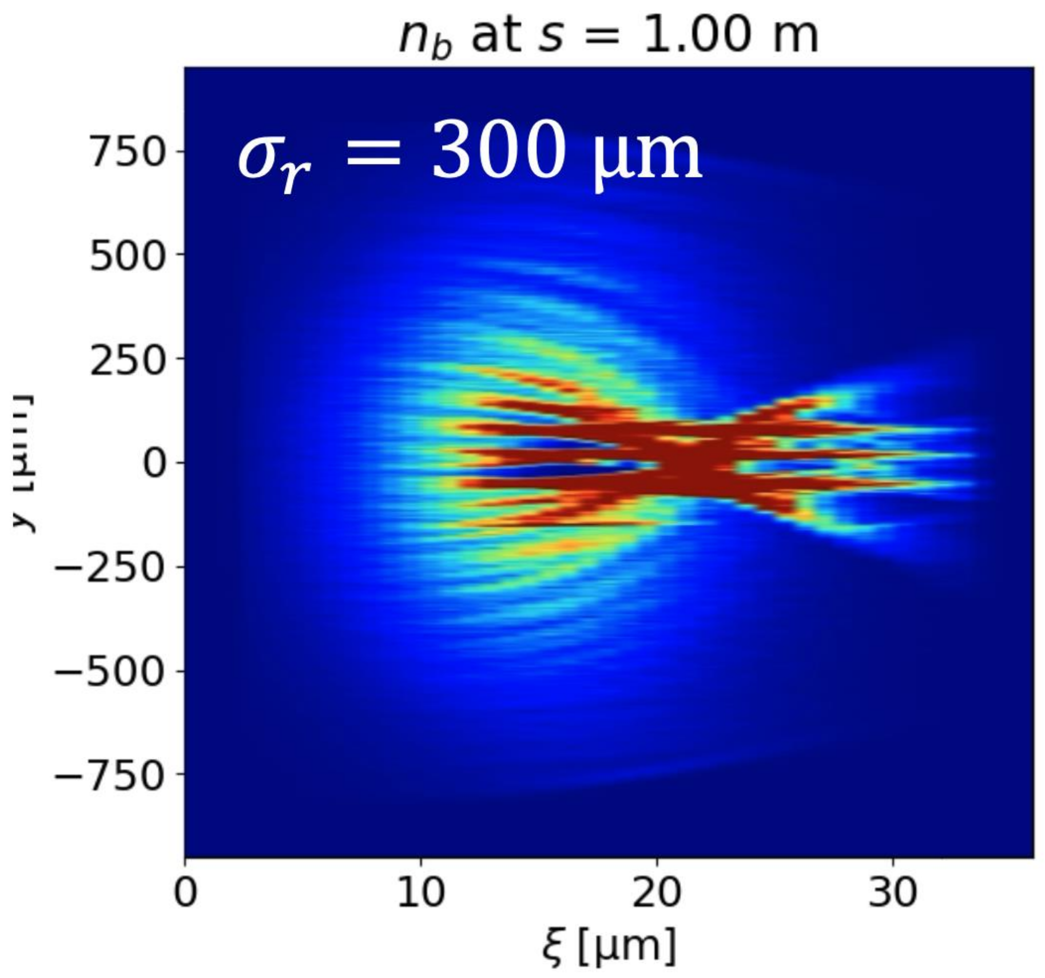
- $n_p = 10^{17}\ \mathrm{cm}^{-3}$
- $L = 1\ \mathrm{m} \approx 50\ c\Gamma^{-1}$

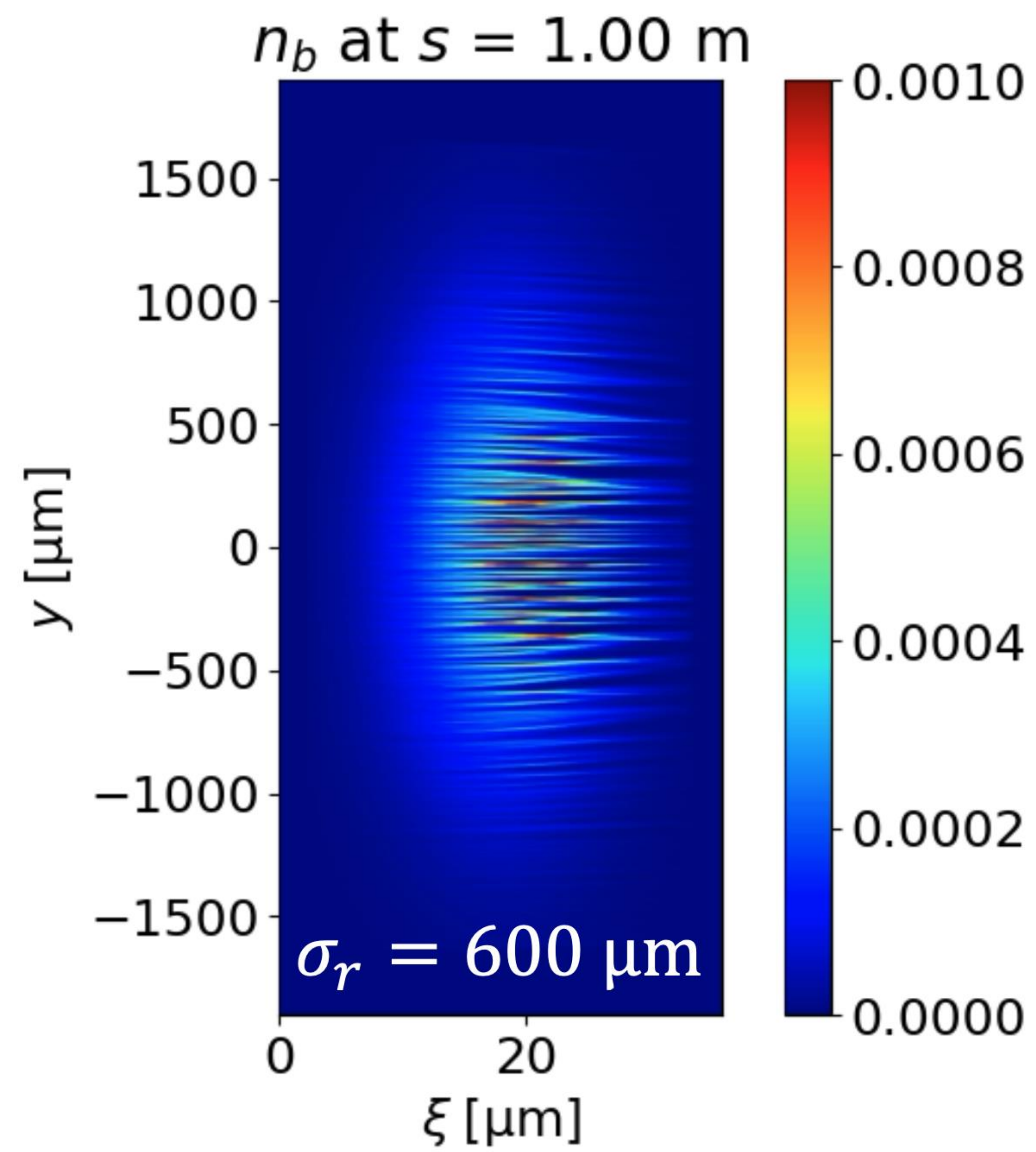
Beam parameters

Gaussian e^- beam

- $E = 500\ \mathrm{MeV}$ — $\gamma_b \approx 10^3$
- $Q = 200\ pC$ — $\alpha \approx 0.001$
- $\sigma_z = 10\ \mu\mathrm{m}$ — $k_p\sigma_z \approx 1$
- $\sigma_r = 300\ \mu\mathrm{m}, 600\ \mu\mathrm{m}$ — $k_p\sigma_r \approx 30, 60$
- $\varepsilon_n = 5\ \mathrm{mm} \cdot \mathrm{mrad}$

QuaSSis simulations





Reaching saturation of CFI modes at the core of the beam

Unique opportunity to study non-linear evolution!