Rome seminar, 15th April 2025

Higgs metastability and the hierarchy problem

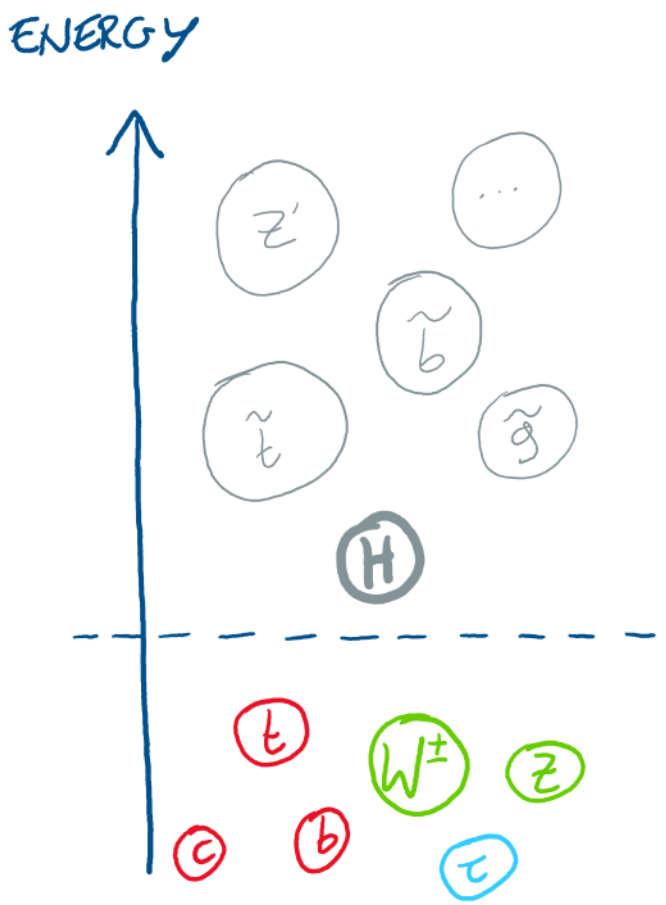
Tevong You

Contents

Contents

The hierarchy problem

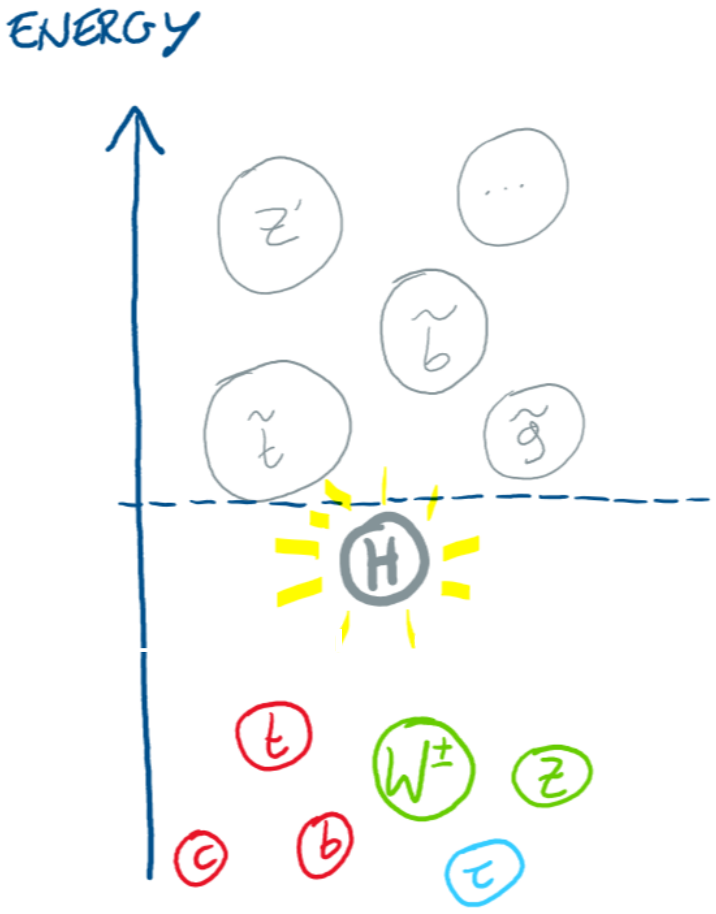
- Until now, there had been a **clear roadmap**



Pre-LHC: **high anticipation** of accompanying BSM particles expected to appear together with the Higgs.

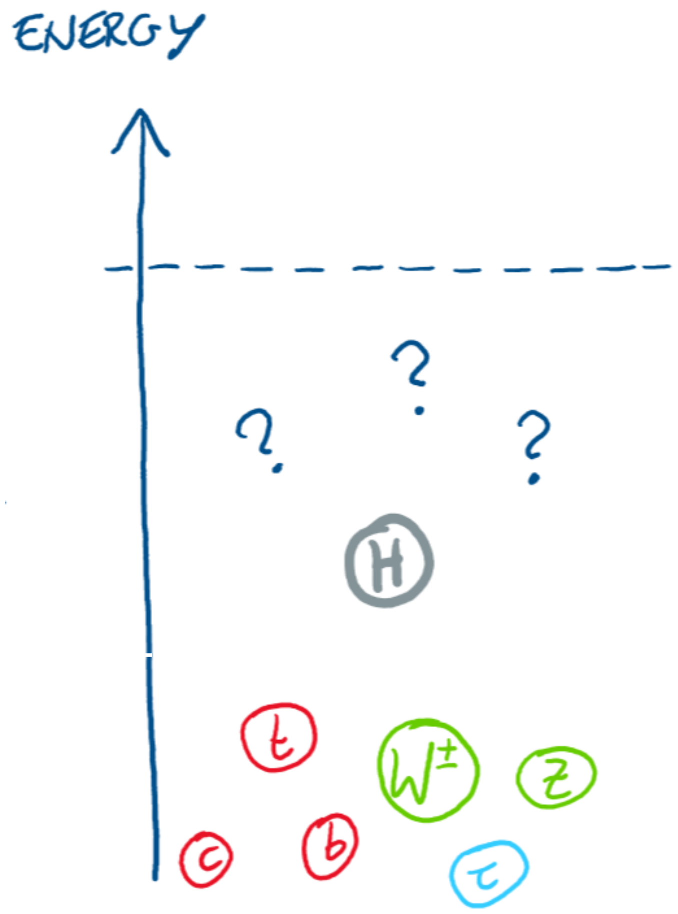
The hierarchy problem

- Until now, there had been a **clear roadmap**



The hierarchy problem

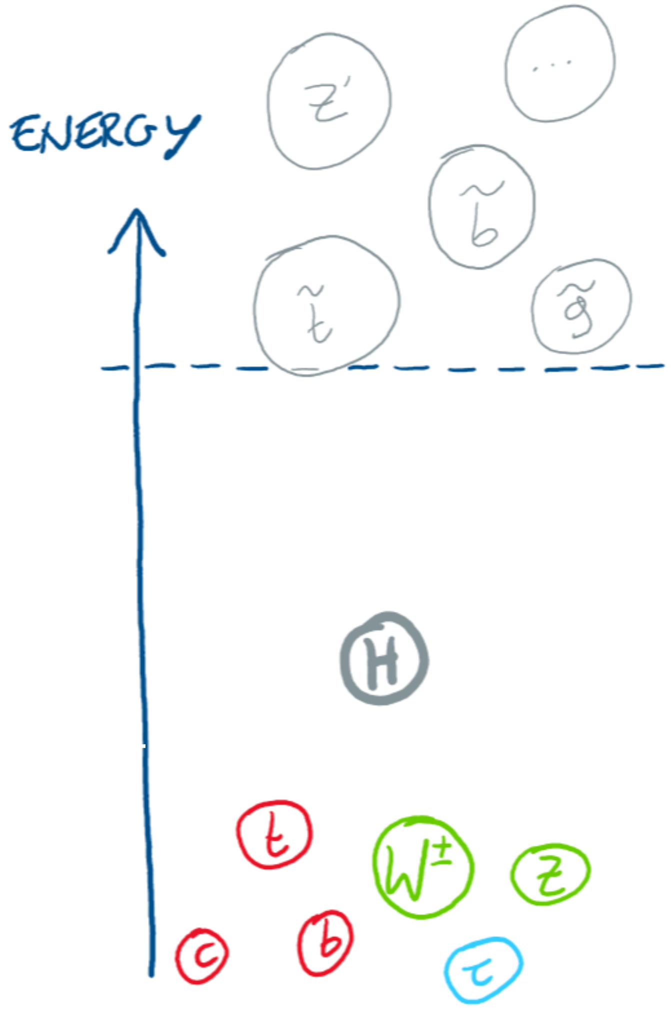
- Until now, there had been a **clear roadmap**



Tevong You

The hierarchy problem

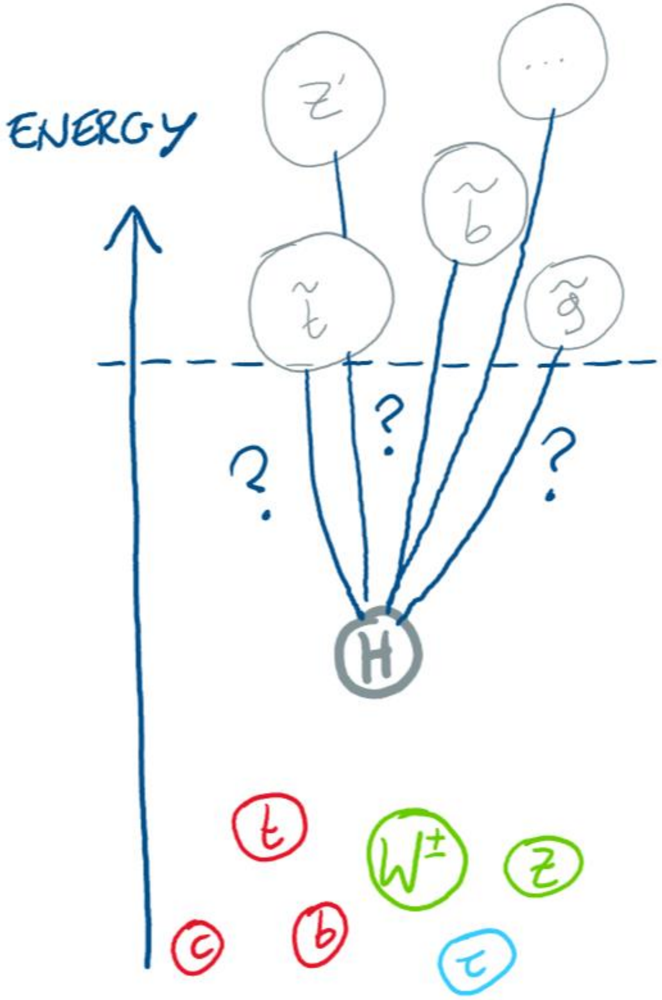
- Until now, there had been a **clear roadmap**



Maybe **just around the corner…**

The hierarchy problem

- Until now, there had been a **clear roadmap**



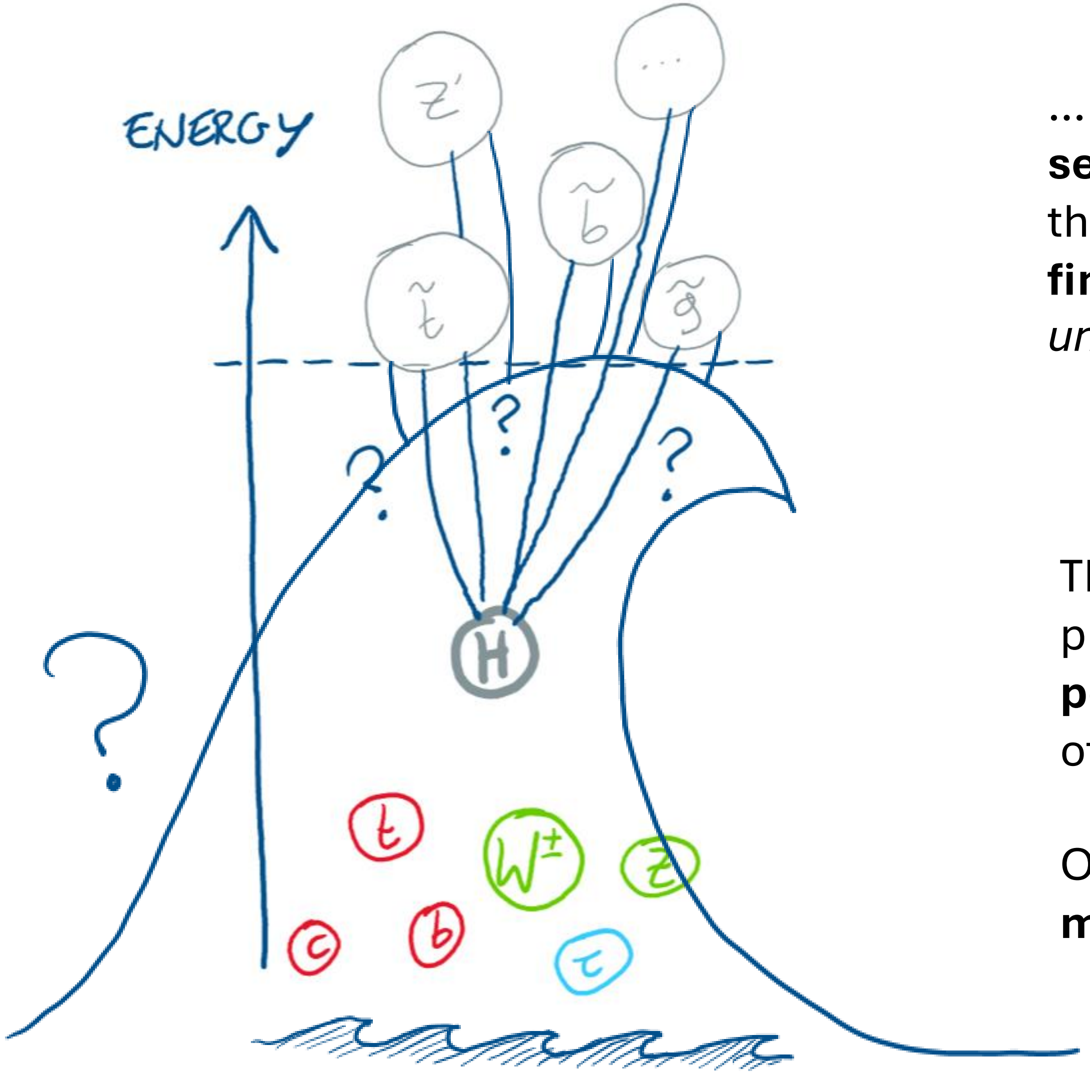
...but the larger the **separation of scales**, the more unnaturally **fine-tuned** the *underlying* theory is!

The Higgs' naturalness problem is **even more perplexing** in the absence of new physics at the LHC.

Our **Michelson-Morley moment?**

The hierarchy problem

- Until now, there had been a **clear roadmap**



...but the larger the **separation of scales**, the more unnaturally **fine-tuned** the *underlying* theory is!

The Higgs' naturalness problem is **even more perplexing** in the absence of new physics at the LHC.

Our **Michelson-Morley moment?**

Cosmological constant problem!

The hierarchy problem

Take fine-tuning problems seriously.

e.g. 2205.05708 N. Craig - Snowmass review,
1307.7879 G. Giudice - Naturalness after LHC

Example 1

$$(m_e c^2)_{obs} = (m_e c^2)_{bare} + \Delta E_{\text{Coulomb}}. \qquad \Delta E_{\text{Coulomb}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_e}.$$

Avoiding cancellation between “bare” mass and divergent self-energy in classical electrodynamics requires new physics around

$$e^2/(4\pi\varepsilon_0 m_e c^2) = 2.8 \times 10^{-13}\ \text{cm}$$

Indeed, the positron and quantum-mechanics appears just before!

$$\Delta E = \Delta E_{\text{Coulomb}} + \Delta E_{\text{pair}} = \frac{3\alpha}{4\pi} m_e c^2 \log \frac{\hbar}{m_e c r_e}$$

The hierarchy problem

Take fine-tuning problems seriously.

e.g. 2205.05708 N. Craig - Snowmass review,
1307.7879 G. Giudice - Naturalness after LHC

Example 2

Divergence in pion mass: $m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{3\alpha}{4\pi}\Lambda^2$

Experimental value is $m_{\pi^\pm}^2 - m_{\pi_0}^2 \sim (35.5\,\mathrm{MeV})^2$

Expect new physics at $\Lambda \sim 850$ MeV to avoid fine-tuned cancellation.

ρ meson appears at 775 MeV!

The hierarchy problem

Take fine-tuning problems seriously.

e.g. 2205.05708 N. Craig - Snowmass review,
1307.7879 G. Giudice - Naturalness after LHC

Example 3

Divergence in Kaons mass difference in a theory with only up, down, strange:

$$m_{K_L^0} - m_{K_S^0} =\simeq \frac{1}{16\pi^2} m_K f_K^2 G_F^2 \sin^2\theta_C \cos^2\theta_C \times \Lambda^2$$

Avoiding fine-tuned cancellation requires $\Lambda < 3$ GeV.

Gaillard & Lee in 1974 predicted the charm quark mass!

The hierarchy problem

Take fine-tuning problems seriously.

e.g. 2205.05708 N. Craig - Snowmass review,
1307.7879 G. Giudice - Naturalness after LHC

<u>Higgs?</u>

Higgs also has a quadratically divergent contribution to its mass

$$\Delta m_H^2 = \frac{\Lambda^2}{16\pi^2}\left(-6y_t^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda\right)$$

Avoiding fine-tuned cancellation requires $\Lambda < O(100)$ GeV??

As Λ is pushed to the TeV scale by null results, tuning is around 10% - 1%.

Note: in the SM the Higgs mass is a parameter to be measured, not calculated. What the quadratic divergence represents (independently of the choice of renormalisation scheme) is the fine-tuning in an underlying theory in which we expect the Higgs mass to be calculable.

The hierarchy problem

Naturalness from an EFT perspective:

$$\boxed{\mathcal{L} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)}} + \xcancel{\frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)}} + \ldots$$

1960s point of view: renormalisability of a *finite* number of parameters is essential

Modern point of view: our QFTs are really EFTs - include *all* operators allowed by symmetries

Symmetries dictate EFT structure and natural expectations for sizes of coefficients

$$\mathcal{L}_{SM} = \mathcal{L}_m + \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_y \quad ,$$

$$\begin{aligned}
\mathcal{L}_m &= \bar{Q}_L i\gamma^\mu D^L_\mu Q_L + \bar{q}_R i\gamma^\mu D^R_\mu q_R + \bar{L}_L i\gamma^\mu D^L_\mu L_L + \bar{l}_R i\gamma^\mu D^R_\mu l_R \\
\mathcal{L}_G &= -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} \\
\mathcal{L}_H &= (D^L_\mu\phi)^\dagger(D^{L\mu}\phi) - V(\phi) \\
\mathcal{L}_Y &= y_d\bar{Q}_L\phi q^d_R + y_u\bar{Q}_L\phi^c q^u_R + y_L\bar{L}_L\phi l_R + \text{h.c.} \quad ,
\end{aligned}$$

The hierarchy problem

The **“Standard Model”**

$$\boxed{\mathcal{L} = \Lambda^4 + \Lambda^2\mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)}} + \cancel{\frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)}} + \ldots$$

1960s point of view: renormalisability of a *finite* number of parameters is essential

Modern point of view: our QFTs are really EFTs - include *all* operators allowed by symmetries

Symmetries dictate EFT structure and natural expectations for sizes of coefficients

The hierarchy problem

Naturalness from an EFT perspective:

$$\mathcal{L} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

1960s point of view: renormalisability of a *finite* number of parameters is essential

Modern point of view: our QFTs are really EFTs - include *all* operators allowed by symmetries

(“quantum totalitarian principle”)

Symmetries dictate EFT structure and natural expectations for sizes of coefficients

$$\mathcal{L}_{SM} = \mathcal{L}_m + \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_y + \frac{c_5}{\Lambda}\mathcal{O}^{(5)} + \frac{c_6}{\Lambda^2}\mathcal{O}^{(6)} + \frac{c_7}{\Lambda^3}\mathcal{O}^{(7)} + \frac{c_8}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

?

The hierarchy problem

$$\mathcal{L}_m = \bar{Q}_L i\gamma^\mu D^L_\mu Q_L + \bar{q}_R i\gamma^\mu D^R_\mu q_R + \bar{L}_L i\gamma^\mu D^L_\mu L_L + \bar{l}_R i\gamma^\mu D^R_\mu l_R$$

$$\mathcal{L}_G = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} \quad -\theta\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}$$

? **Strong-CP problem**

$$\mathcal{L}_H = (D^L_\mu\phi)^\dagger(D^{L\mu}\phi) - V(\phi)$$

$$\mathcal{L}_Y = y_d\bar{Q}_L\phi q^d_R + y_u\bar{Q}_L\phi^c q^u_R + y_L\bar{L}_L\phi l_R + \text{h.c.} \quad ,$$

$$\mathcal{L} = \Lambda^4 + \Lambda^2\mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

1960s point of view: renormalisability of a *finite* number of parameters is essential

Modern point of view: our QFTs are really EFTs - include *all* operators allowed by symmetries

("quantum totalitarian principle")

Symmetries dictate EFT structure and natural expectations for sizes of coefficients

The hierarchy problem

Naturalness from an EFT perspective:

$$\mathcal{L} = \Lambda^4 + \Lambda^2\mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \underbrace{\frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \ldots}_{\text{Suppressed!}}$$

1960s point of view: renormalisability of a *finite* number of parameters is essential

Modern point of view: our QFTs are really EFTs - include *all* operators allowed by symmetries

("quantum totalitarian principle")

Symmetries dictate EFT structure and natural expectations for sizes of coefficients

The hierarchy problem

$$\mathcal{L} = \overbrace{\Lambda^4 + \Lambda^2 \mathcal{O}^{(2)}}^{\text{Naturalness violation?}} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

1960s point of view: renormalisability of a *finite* number of parameters is essential

Modern point of view: our QFTs are really EFTs - include *all* operators allowed by symmetries

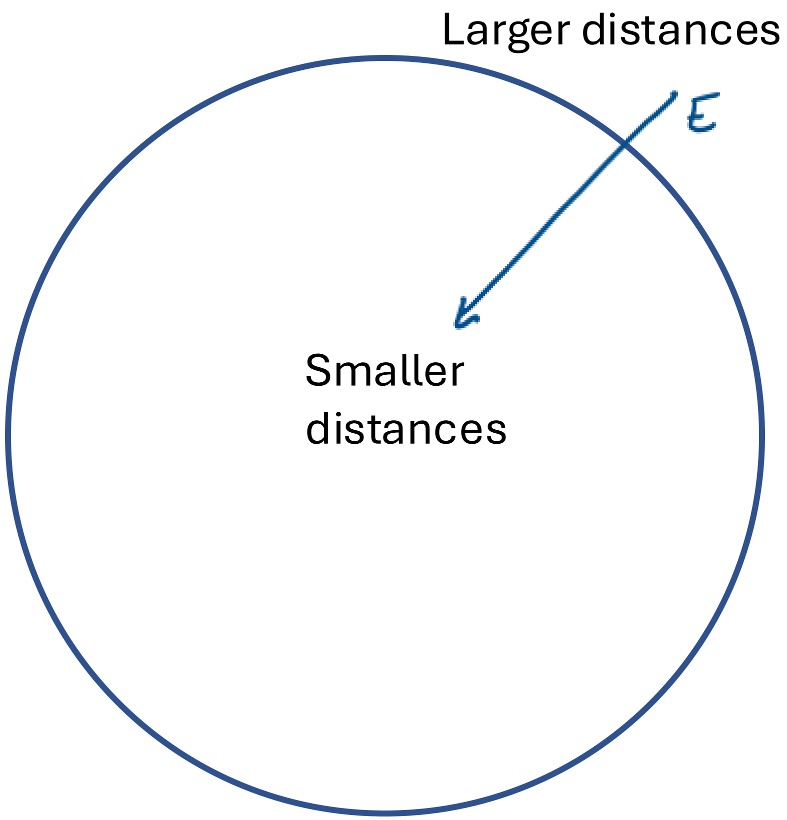
("quantum totalitarian principle")

Symmetries dictate EFT structure and natural expectations for sizes of coefficients

Naturalness is still a fundamental problem

- *Why is unnatural fine-tuning such a big deal?* An intuitive picture:

Physical theories govern a huge range of phenomena across vast scales

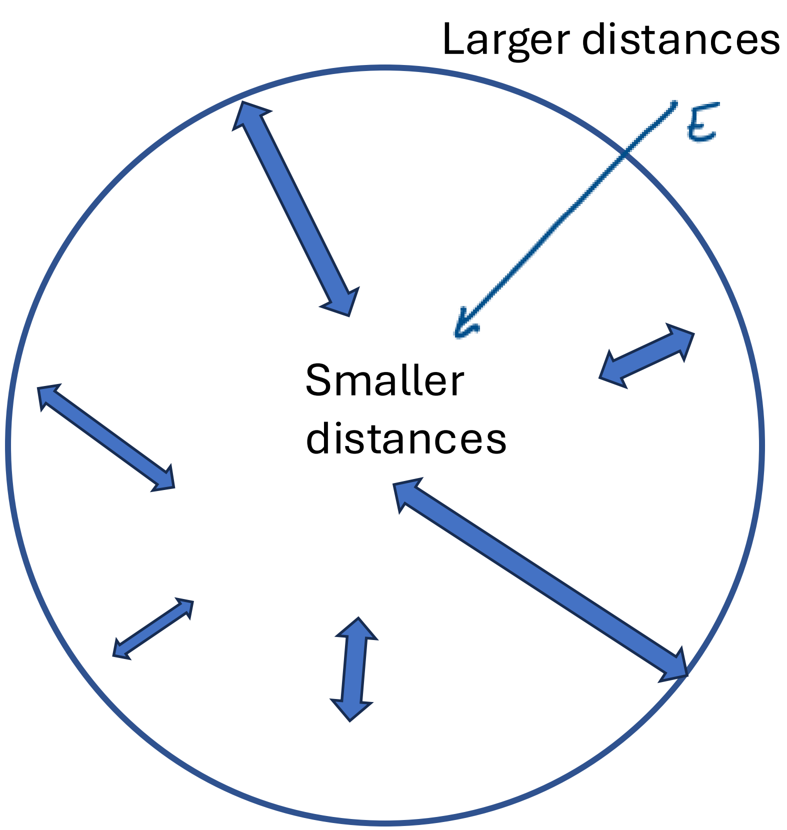


Naturalness is still a fundamental problem

- *Why is unnatural fine-tuning such a big deal?* An intuitive picture:

Everything does **not** depend on everything else equally.

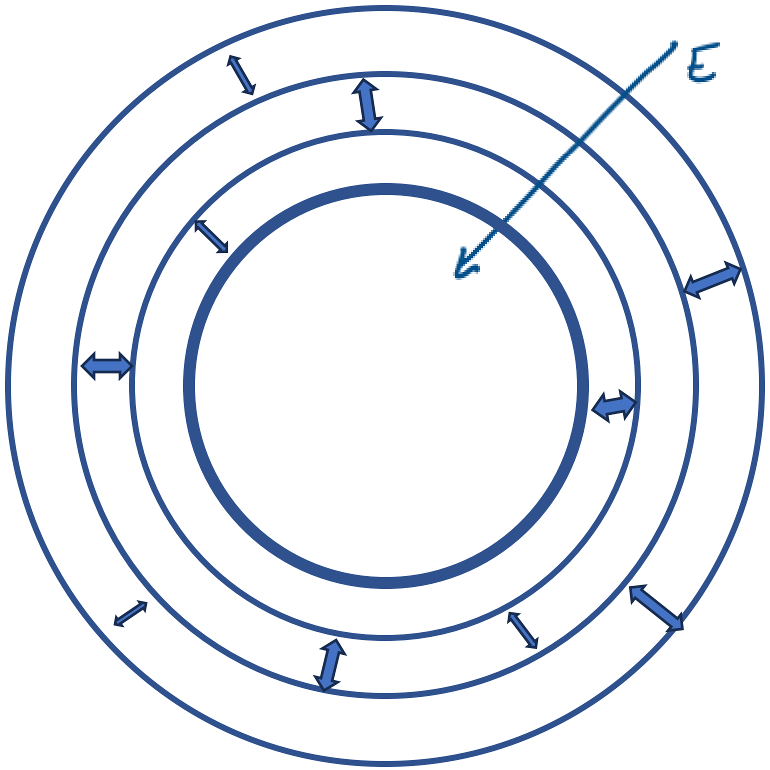
(Otherwise, we would need a Theory of Everything to calculate anything)



Naturalness is still a fundamental problem

- *Why is unnatural fine-tuning such a big deal?* An intuitive picture:

Effective theory at each energy scale E is **predictive** as a **self-contained** theory at that scale

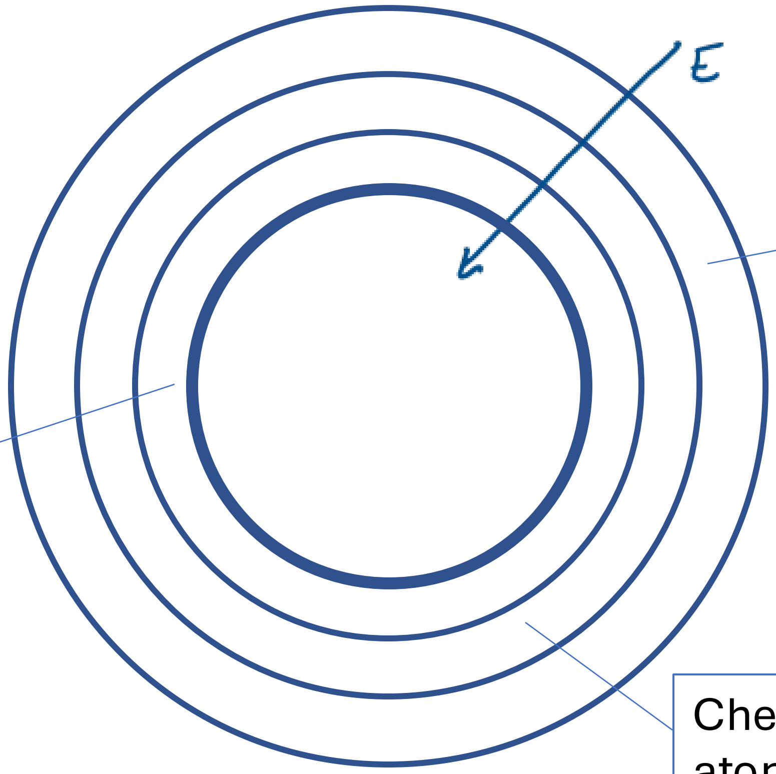


Tevong You

Naturalness is still a fundamental problem

- *Why is unnatural fine-tuning such a big deal?* An intuitive picture:

Effective theory at each energy scale E is **predictive** as a **self-contained** theory at that scale



Planetary dynamics, thermodynamics, fluid dynamics, …

Strong / weak interactions, …

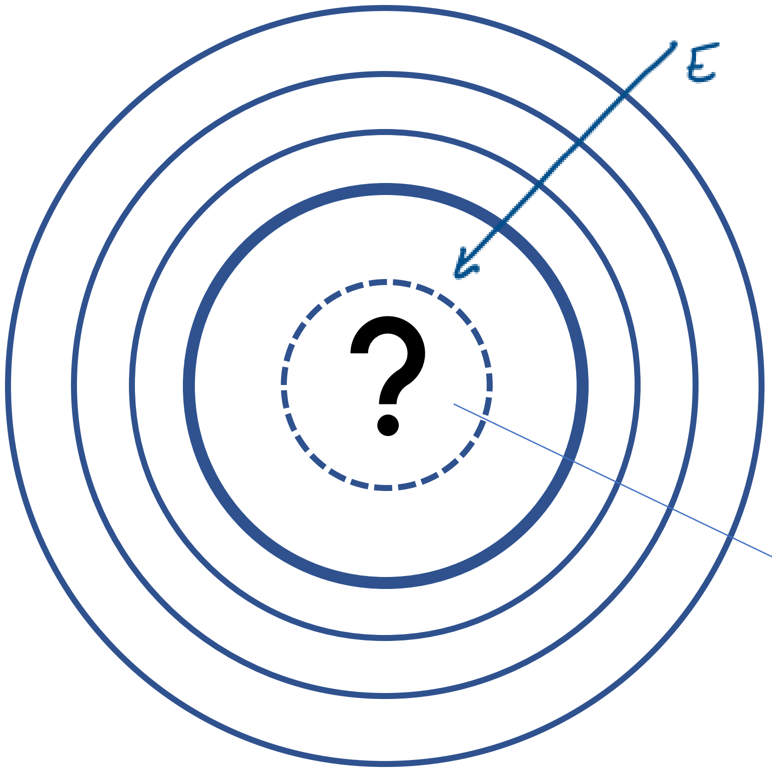
Chemistry, atomic physics, nuclear physics, …

In **all theories so far**, no contributions from **smaller scales** compete with **similar magnitude** to effects **on larger scales**

Naturalness is still a fundamental problem

- *Why is unnatural fine-tuning such a big deal?* An intuitive picture:

Effective theory at each energy scale E is **predictive** as a **self-contained** theory at that scale

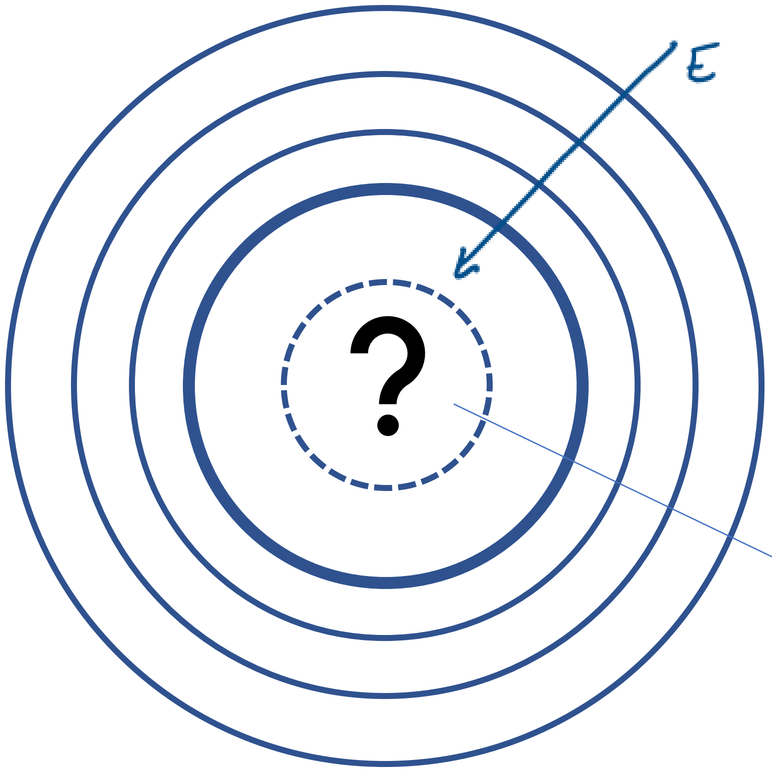


Unnatural Higgs means the next layer **is no longer predictive** without including contributions **from much smaller scales**

Naturalness is still a fundamental problem

- *Why is unnatural fine-tuning such a big deal?* An intuitive picture:
- Indicates **an unprecedented breakdown** of the **effective theory** structure of nature

Effective theory at each energy scale E is **predictive** as a **self-contained** theory at that scale

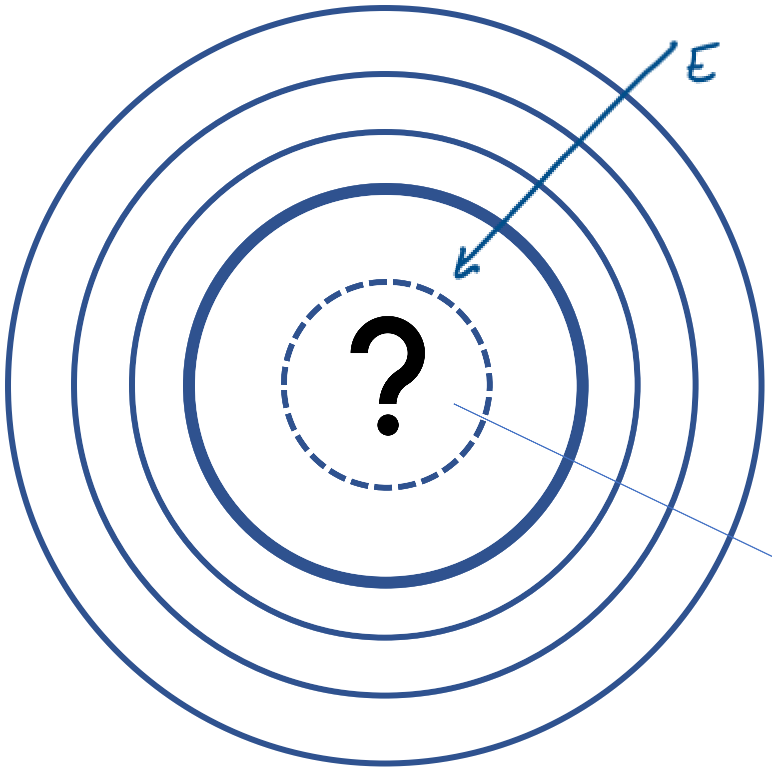


Unnatural Higgs means the next layer **is no longer predictive** without including contributions **from much smaller scales**

Naturalness is still a fundamental problem

- *Why is unnatural fine-tuning such a big deal?* An intuitive picture:
- Indicates **an unprecedented breakdown** of the **effective theory** structure of nature

Effective theory at each energy scale E is **predictive** as a **self-contained** theory at that scale



Unnatural Higgs means the next layer **is no longer predictive** without including contributions **from much smaller scales**

- **Beyond symmetry**: could **cosmology** play a fundamental role in **solving naturalness problems**?

Cosmological solutions to naturalness problems

- The **good**: QCD axion solution of strong CP problem
- The **bad**: Abbott relaxation of cosmological constant
- The **ugly**: Cosmological relaxation of weak scale
- The **exotic**: Self-Organised Localisation

Cosmological solutions to naturalness problems

- The **good**: QCD axion solution of strong CP problem
  - Most likely candidate for existing in nature
- The **bad**: Abbott relaxation of cosmological constant
  - Doesn’t work
- The **ugly**: Cosmological relaxation of weak scale
  - Works, but wouldn’t bet on it yet
- The **exotic**: Self-Organised Localisation
  - Measure problem in eternal inflation landscape

QCD axion

$$\mathcal{L} \supset \Lambda_p^4 \cos\left(\frac{\phi}{f_p}\right)$$

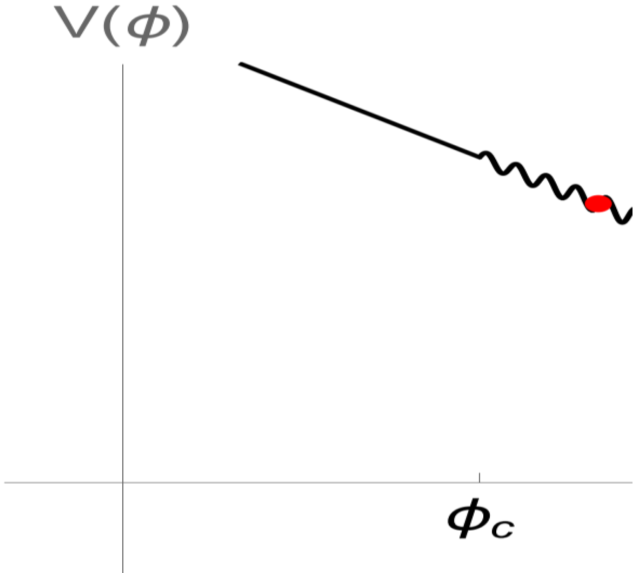
- Needs no introduction – **widely accepted** cosmological solution

- First incarnation (Weinberg-Wilczek axion) ruled out ⇒ DFSZ / KSVZ **invisible axion**

- Has a *'halo of truth'* to it, but also lack of attractive alternatives

- Still a PQ **quality problem**: requires additional UV model-building

Abbott model



L. F. Abbott, Phys. Lett. B 150 (1985) 427

$$\mathcal{L} \supset \epsilon M^3 \phi + \ldots + \Lambda_p^4 \cos\left(\frac{\phi}{f_p}\right)$$

- Vacuum energy relaxed by ϕ
- Periodic potential barriers **suppressed** by Hawking temperature
- **Unsuppressed** for small enough vacuum energy density ⇒ **trapped at small CC**

- However, ends in **cold empty universe**
- Reheating requires *e.g. null energy condition violation*

Alberte et al 1608.05715
Graham, Kaplan, Rajendran 1902.06793

Cosmological relaxation

P. W. Graham, D. E. Kaplan and S. Rajendran, [arXiv:1504.07551]

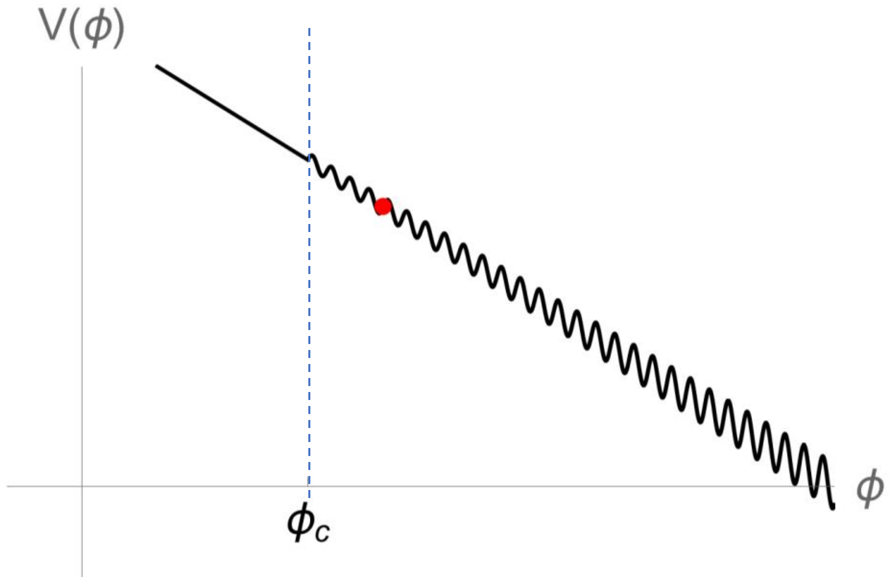
- Assume Higgs mass is naturally large at cut-off M

$$\mathcal{L} \supset (M^2 + \epsilon M\phi)|h|^2 + \epsilon M^3\phi + \ldots + \Lambda_p^{4-n} v^n \cos\left(\frac{\phi}{f_p}\right)$$

- Higgs quadratic term scanned by axion-like field ϕ during inflation

- ϕ protected by shift symmetry, explicitly broken by small parameter ε

- Backreaction when $< h > \sim v$ stops ϕ evolution at small electroweak scale v



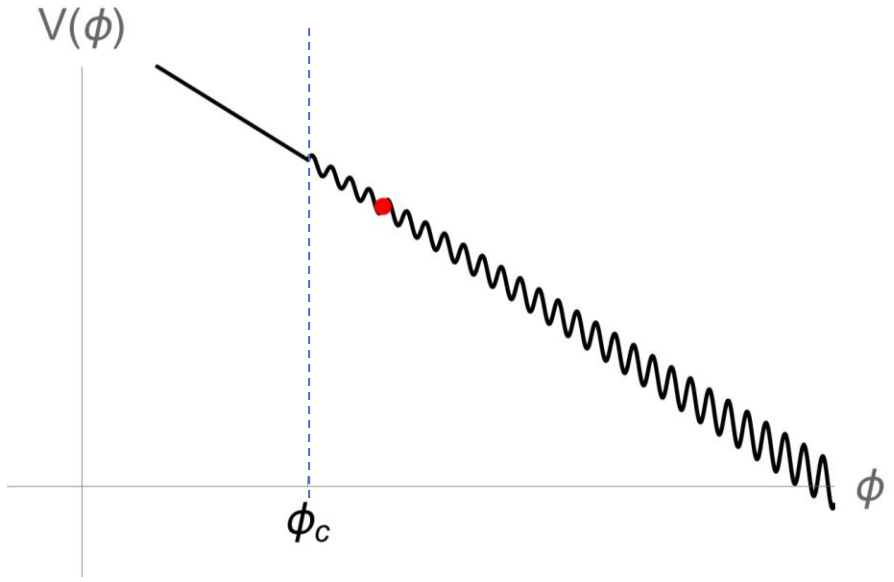
$$\epsilon M^3 \simeq \frac{\Lambda_p^{4-n} v^n}{f_p}$$

Cosmological relaxation

P. W. Graham, D. E. Kaplan and S. Rajendran, [arXiv:1504.07551]

- Assume Higgs mass is naturally large at cut-off $\boxed{M}$

$$\mathcal{L} \supset (\boxed{M^2} + \epsilon M \phi)|h|^2 + \epsilon M^3 \phi + \ldots + \Lambda_p^{4-n} v^n \cos\left(\frac{\phi}{f_p}\right)$$



- Higgs quadratic term scanned by axion-like field ϕ during inflation

- ϕ protected by shift symmetry, explicitly broken by small parameter ε

- Backreaction when $< h > \sim v$ stops ϕ evolution at small electroweak scale v

$$\epsilon M^3 \simeq \frac{\Lambda_p^{4-n} v^n}{f_p}$$

Cosmological relaxation

P. W. Graham, D. E. Kaplan and S. Rajendran, [arXiv:1504.07551]

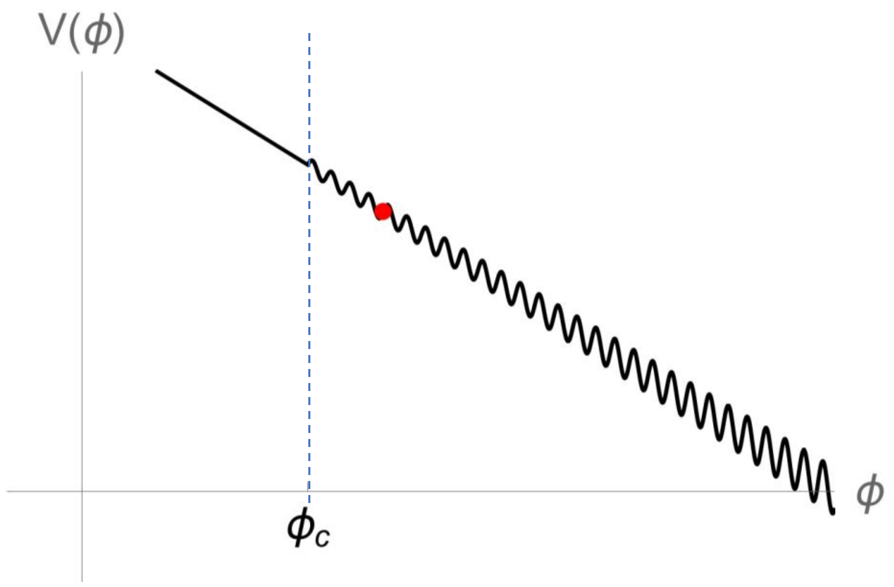
- Assume Higgs mass is naturally large at cut-off M

$$\mathcal{L} \supset \left(\boxed{M^2 + \epsilon M \phi}\right)|h|^2 + \epsilon M^3 \phi + \ldots + \Lambda_p^{4-n} v^n \cos\left(\frac{\phi}{f_p}\right)$$

- Higgs quadratic term scanned by axion-like field ϕ during inflation

- ϕ protected by shift symmetry, explicitly broken by small parameter ε

- Backreaction when $< h > \sim v$ stops ϕ evolution at small electroweak scale v



$$\epsilon M^3 \simeq \frac{\Lambda_p^{4-n} v^n}{f_p}$$

Cosmological relaxation

P. W. Graham, D. E. Kaplan and S. Rajendran,
[arXiv:1504.07551]

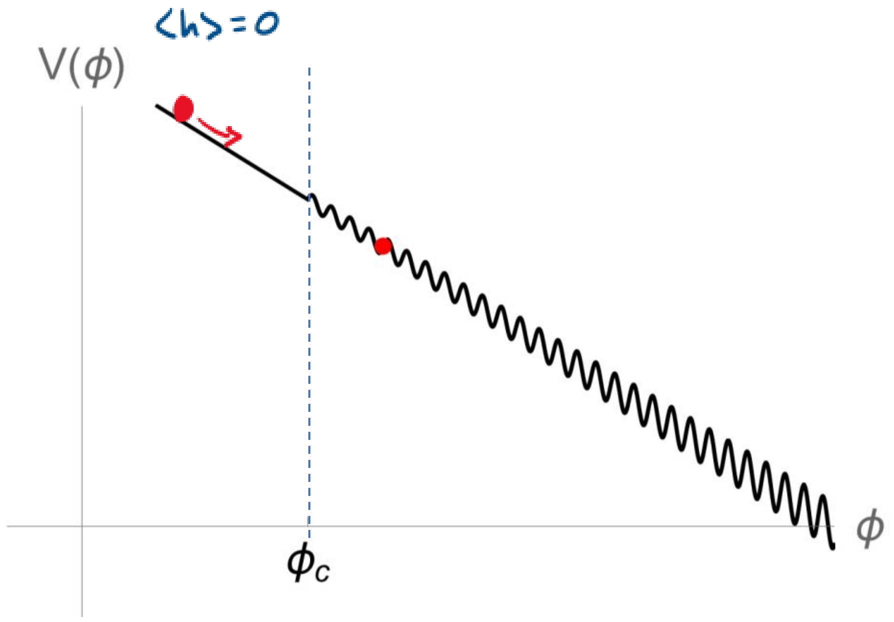
- Assume Higgs mass is naturally large at cut-off M

$$\mathcal{L} \supset (M^2 + \epsilon M \phi)|h|^2 + \epsilon M^3 \phi + \ldots + \Lambda_p^{4-n} v^n \cos\left(\frac{\phi}{f_p}\right)$$

- Higgs quadratic term scanned by axion-like field ϕ during inflation

- ϕ protected by shift symmetry, explicitly broken by small parameter ε

- Backreaction when $< h > \sim v$ stops ϕ evolution at small electroweak scale v



$$\epsilon M^3 \simeq \frac{\Lambda_p^{4-n} v^n}{f_p}$$

Cosmological relaxation

P. W. Graham, D. E. Kaplan and S. Rajendran, [arXiv:1504.07551]

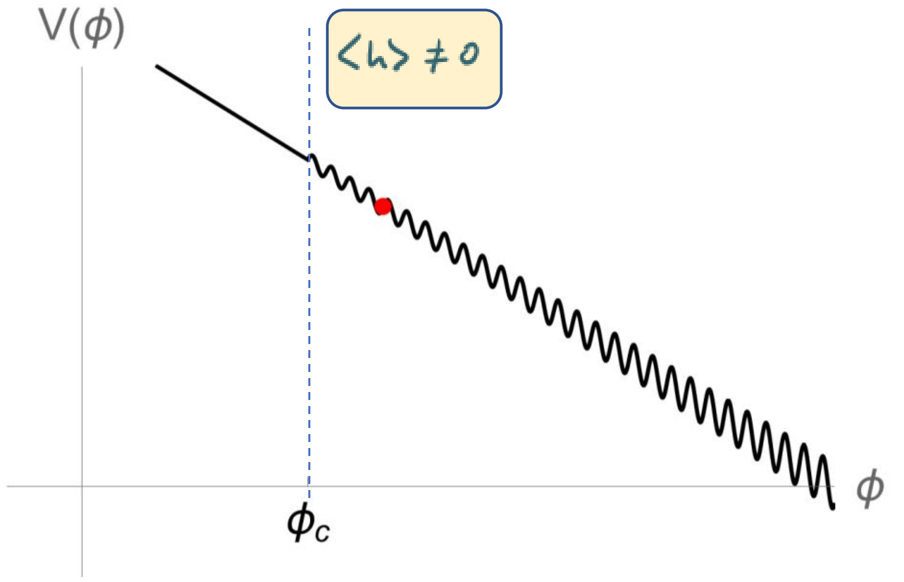
- Assume Higgs mass is naturally large at cut-off M

$$\mathcal{L} \supset (M^2 + \epsilon M\phi)|h|^2 + \epsilon M^3\phi + \dots + \Lambda_p^{4-n} v^n \cos\left(\frac{\phi}{f_p}\right)$$

- Higgs quadratic term scanned by axion-like field ϕ during inflation

- ϕ protected by shift symmetry, explicitly broken by small parameter ε

- Backreaction when $\boxed{< h > \sim v}$ stops ϕ evolution at small electroweak scale v



$$\epsilon M^3 \simeq \frac{\Lambda_p^{4-n} v^n}{f_p}$$

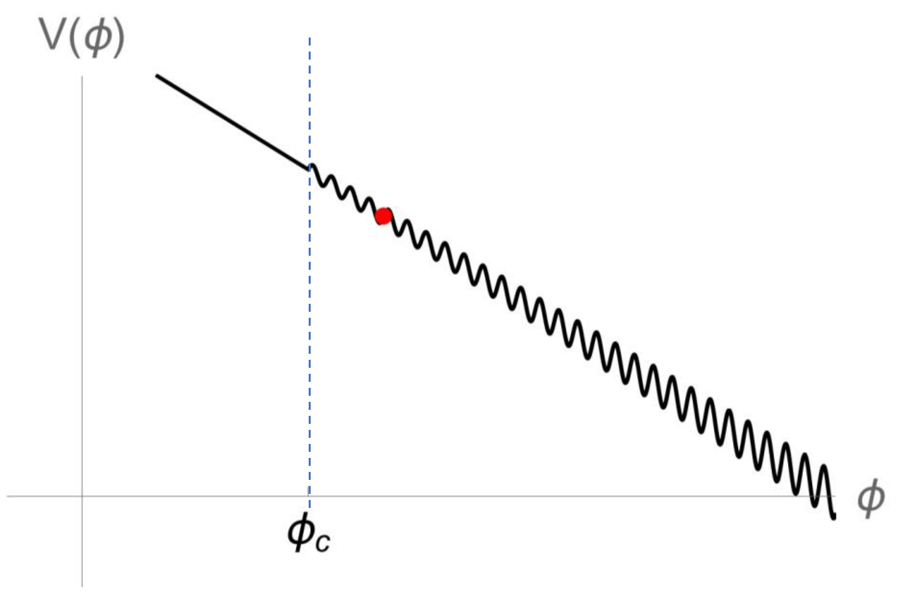
Cosmological relaxatio

Constraints: H < v, classical rolling vs quantum, inflaton energy density dominates relaxion, etc.

Very small ε and natural scanning range lead to super-planckian field excursions, exponential e-foldings…

- Assume Higgs mass is naturally l

$$\mathcal{L} \supset (M^2 + \epsilon M\phi)|h|^2 + \epsilon M^3\phi + \ldots + \Lambda_p^{4-n} v^n \cos\left(\frac{\phi}{f_p}\right)$$



- Higgs quadratic term scanned by axion-like field ϕ during inflation

- ϕ protected by shift symmetry, explicitly broken by small parameter ε

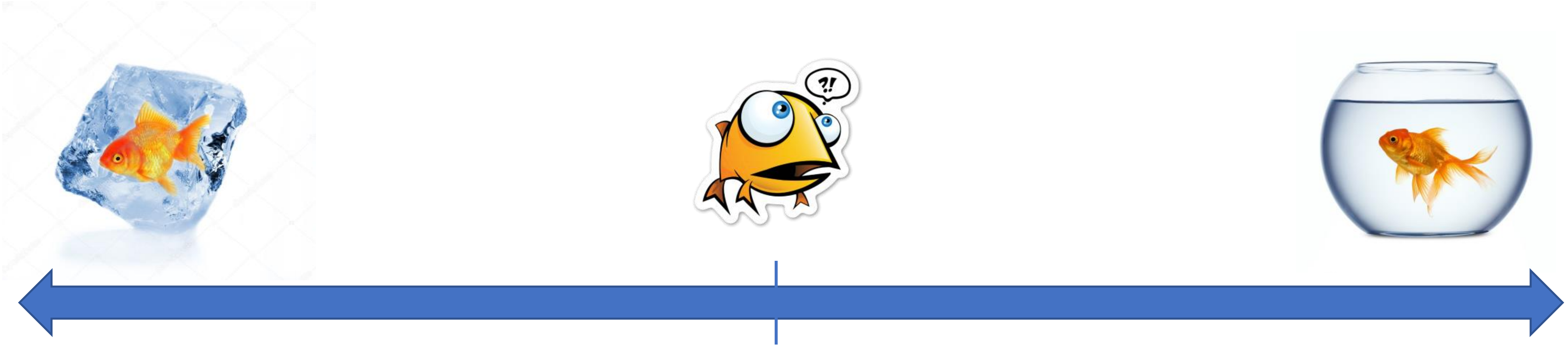
- Backreaction when $< h > \sim v$ stops ϕ evolution at small electroweak scale v

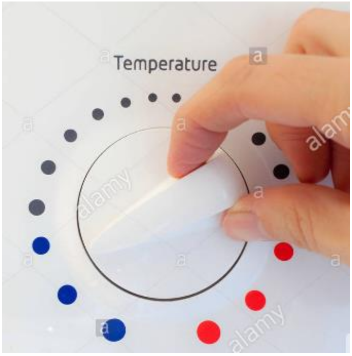
$$\epsilon M^3 \simeq \frac{\Lambda_p^{4-n} v^n}{f_p}$$

Contents

Critical points

- To be at the **critical point** of a classical phase transition **requires tuning**

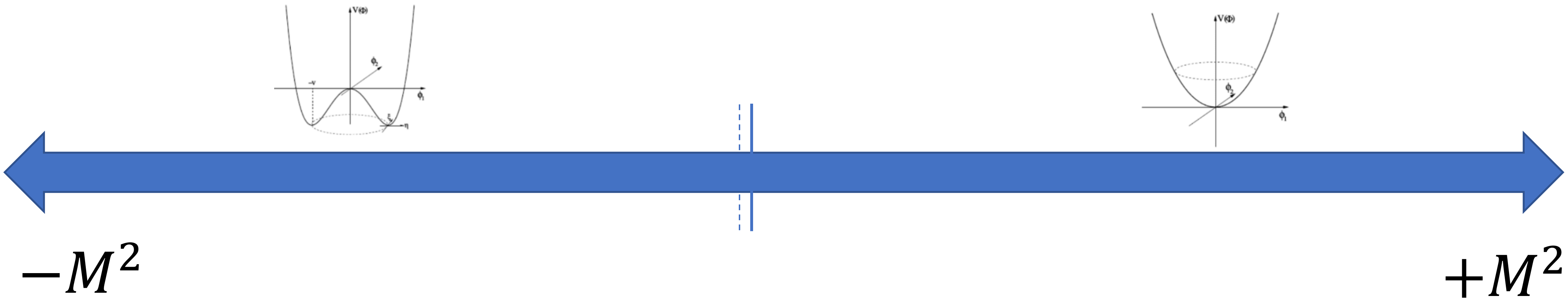




- Living **near criticality** is highly **non-generic**!

Higgs potential near criticality?

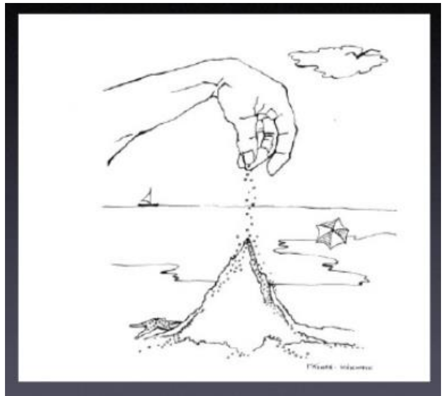
- 2) **Higgs mass**



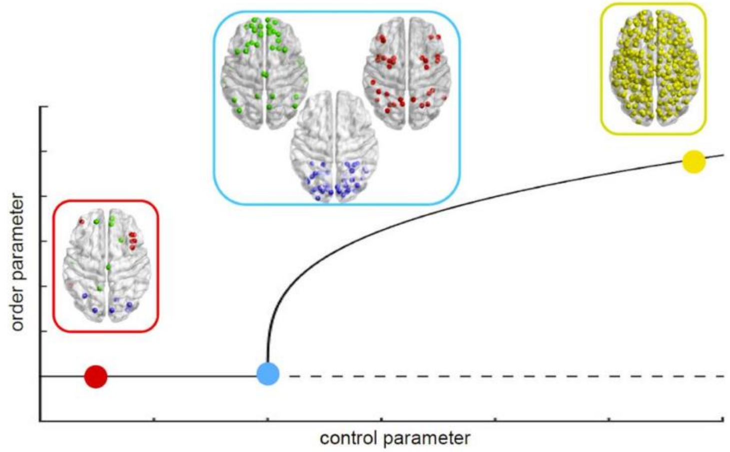
- Tuned close to boundary between **ordered** and **disordered** phase

Self-Organised Criticality

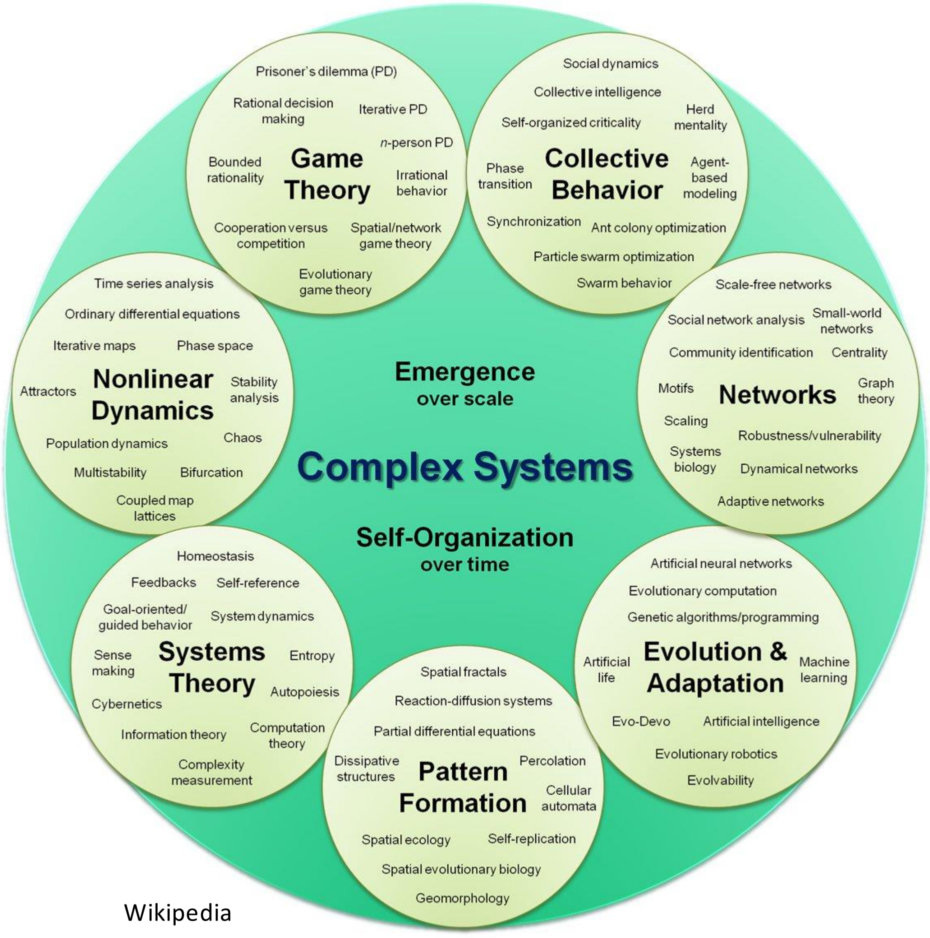
- Many systems in nature **self-tuned** to live near criticality



Bak, Tang and Weisenfeld (1989)



https://www.quantamagazine.org/toward-a-theory-of-self-organized-criticality-in-the-brain-20140403/



Wikipedia

Self-Organised Criticality

- **Fundamental** self-organised criticality in our universe?

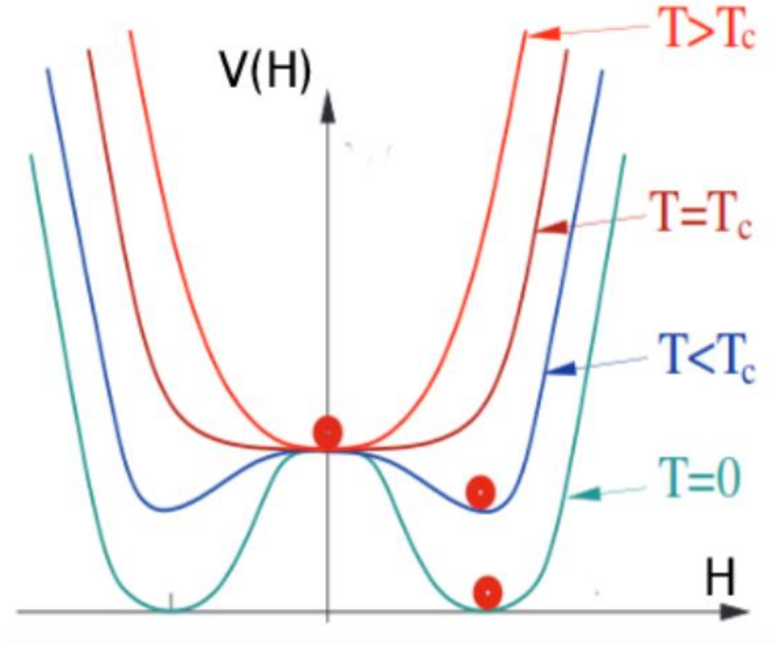
- Need a **mechanism** for self-organisation of **fundamental parameters**

  e.g. Self-Organized Criticality in eternal inflation landscape: J. Khoury et al 1907.07693, 1912.06706, 2003.12594

- **Self-Organised Localisation (SOL):**
  - cosmological **quantum phase transitions** localise fluctuating scalar fields during inflation at critical points

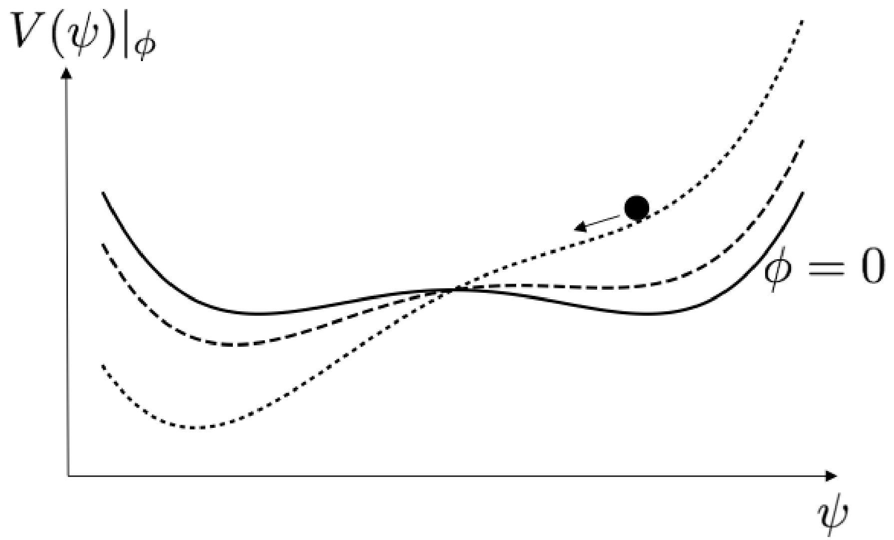
Phase Transitions (PT)

- **Classical** PT: varying background temperature



- **Quantum** PT: varying background field

$$V = \frac{\lambda}{4}\left(\psi^2 - \rho^2\right)^2 + \kappa\phi\psi$$



Fokker-Planck Volume (**FPV**) equation

- **Langevin equation**: classical slow-roll + Hubble quantum fluctuations

$$\phi(t+\Delta t) = \phi(t) - \frac{V'}{3H}\Delta t + \eta_{\Delta t}(t)$$

- Volume-averaged Langevin trajectories: **FPV for volume distribution** $P(\phi, t)$

$$\frac{\partial}{\partial \phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^3 P)}{\partial \phi} + \frac{V' P}{3H}\right] + 3HP = \frac{\partial P}{\partial t}$$

$$H(\phi) = \sqrt{\frac{V(\phi)}{3M_p^2}}$$

Quantum diffusion term

Classical drift term

Volume term

Fokker-Planck Volume (**FPV**) equation

- **Langevin equation**: classical slow-roll + Hubble quantum fluctuations

$$\phi(t+\Delta t) = \phi(t) - \frac{V'}{3H}\Delta t + \eta_{\Delta t}(t)$$

- Volume-averaged Langevin trajectories: **FPV for volume distribution** $P(\phi, t)$

$$\frac{\partial}{\partial\phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^{2+\xi}P)}{\partial\phi} + \frac{V'P}{3H^{2-\xi}}\right] + 3H^{\xi}P = H_0^{\xi-1}\frac{\partial P}{\partial t_\xi}$$

- **Ambiguity** in choosing time "gauge" $dt_\xi/dt = (H/H_0)^{1-\xi}$

FPV dynamics

- ϕ is *not* the inflaton: **apeiron** field scanning parameters
- Restrict to **EFT** field range f $\quad \varphi \equiv \frac{\phi}{f} \qquad V = 3H_0^2 M_P^2 + g_\epsilon^2 f^4 \omega(\varphi)\ , \qquad \omega(\varphi) = \sum_{n=1}^{\infty} \frac{c_n}{n!}\varphi^n$
- Assume sub-dominant energy density
- Expand around constant inflationary background H_0 $\qquad H(\varphi) \simeq H_0 \left(1 + \frac{\epsilon^2 f^4 \omega(\varphi)}{6M_p^2 H_0^2}\right)$

- FPV becomes

$$\frac{\alpha}{2}\frac{\partial^2 P}{\partial \varphi^2} + \frac{\partial(\omega' P)}{\partial \varphi} + \beta \omega P = \frac{\partial P}{\partial T}$$

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2 \epsilon^2 f^4}\ , \qquad \beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2}\ , \qquad T \equiv \frac{t}{t_R}\ , \qquad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha\beta S_{ds}}{3\xi H_0} \qquad S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

Quantum diffusion

Volume

Classical drift

FPV dynamics

- ϕ is *not* the inflaton: **apeiron** field scanning parameters
- Restrict to **EFT** field range f $\quad \varphi \equiv \frac{\phi}{f} \qquad V = 3H_0^2 M_P^2 + g_\epsilon^2 f^4 \omega(\varphi) \ , \qquad \omega(\varphi) = \sum_{n=1}^{\infty} \frac{c_n}{n!}\varphi^n$
- Assume sub-dominant energy density
- Expand around constant inflationary background H_0 $\quad H(\varphi) \simeq H_0 \left(1 + \frac{\epsilon^2 f^4 \omega(\varphi)}{6M_p^2 H_0^2}\right)$

- FPV becomes

$$\frac{\alpha}{2}\frac{\partial^2 P}{\partial \varphi^2} + \frac{\partial(\omega' P)}{\partial \varphi} + \beta\omega P = \frac{\partial P}{\partial T}$$

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2 \epsilon^2 f^4} \quad , \quad \beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2} \quad , \quad T \equiv \frac{t}{t_R} \quad , \quad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha\beta S_{ds}}{3\xi H_0} \qquad S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

- **Maximum** number of e-folds for **non-eternal** inflation: $N_{\text{e-folds}} < S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$

FPV dynamics

- **Stationary** FPV distributions $P(\varphi, T) = \sum_\lambda e^{\lambda T} p(\varphi, \lambda)$

$$\frac{\alpha}{2} p'' + \omega' p' + (\omega'' + \beta\omega - \lambda) p = 0$$

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2 \epsilon^2 f^4} \quad , \quad \beta \equiv \frac{3}{2} \frac{\xi f^2}{M_p^2} \quad , \quad T \equiv \frac{t}{t_R} \quad , \quad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha\beta S_{ds}}{3\xi H_0} \qquad S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

- Largest eigenvalue $\lambda = \lambda_{\max}$ inflates most
- **Eigenvalue** determines **peak location**

- Note: **boundary conditions** necessary input for solution

FPV dynamics

- **Stationary** FPV distributions $P(\varphi, T) = \sum_\lambda e^{\lambda T} p(\varphi, \lambda)$

$$\frac{\alpha}{2} p'' + \omega' p' + (\omega'' + \beta\omega - \lambda)\, p = 0 \quad \Rightarrow$$

Discriminant D>0 for **positive** solution:

$$D = \omega'^2 + 2\alpha(\lambda - \beta\omega - \omega'')$$

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2 \epsilon^2 f^4} \quad , \quad \beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2} \quad , \quad T \equiv \frac{t}{t_R} \quad , \quad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha\beta S_{ds}}{3\xi H_0} \qquad S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

- Largest eigenvalue $\lambda = \lambda_{\max}$ inflates most
- **Eigenvalue** determines **peak location**

- Note: **boundary conditions** necessary input for solution

FPV dynamics

- **Stationary** FPV distributions $P(\varphi, T) = \sum_{\lambda} e^{\lambda T} p(\varphi, \lambda)$

$$\frac{\alpha}{2} p'' + \omega' p' + (\omega'' + \beta\omega - \lambda) p = 0$$

Discriminant D>0 for **positive** solution:

$$D = \omega'^2 + 2\alpha(\lambda - \beta\omega - \omega'')$$

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2 \epsilon^2 f^4} \quad , \quad \beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2} \quad , \quad T \equiv \frac{t}{t_R} \quad , \quad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha\beta S_{ds}}{3\xi H_0} \qquad S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

e.g. D=0 at $\varphi = 1$ $\Rightarrow$ $\lambda_{\max} = \beta - \frac{\omega_1'^2}{2\alpha}$

- Largest eigenvalue $\lambda = \lambda_{\max}$ inflates most
- **Eigenvalue** determines **peak location**

- Note: **boundary conditions** necessary input for solution

FPV dynamics

- **Stationary** FPV distributions $P(\varphi, T) = \sum_{\lambda} e^{\lambda T} p(\varphi, \lambda)$

$$\frac{\alpha}{2} p'' + \omega' p' + (\omega'' + \beta\omega - \lambda)\, p = 0$$

Discriminant D>0 for **positive** solution:

$$D = \omega'^2 + 2\alpha(\lambda - \beta\omega - \omega'')$$

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2 \epsilon^2 f^4} \quad , \quad \beta \equiv \frac{3}{2} \frac{\xi f^2}{M_p^2} \quad , \quad T \equiv \frac{t}{t_R} \quad , \quad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha\beta S_{ds}}{3\xi H_0}$$

$$S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

e.g. D=0 at $\varphi = 1$ $\Rightarrow$ $\lambda_{\max} = \beta - \frac{\omega_1'^2}{2\alpha}$

- Largest eigenvalue $\lambda = \lambda_{\max}$ inflates most
- **Eigenvalue** determines **peak location**

$$\frac{\alpha}{2} p'' + \cancel{\omega' p'} + (\omega'' + \beta\omega - \lambda)\, p = 0 \quad \Rightarrow \quad \lambda = \beta\omega(\bar{\varphi}) + \omega''(\bar{\varphi}) - \frac{\alpha}{2\sigma^2}$$

- Note: **boundary conditions** necessary input for solution

FPV dynamics

- **Stationary** FPV distributions $P(\varphi, T) = \sum_{\lambda} e^{\lambda T} p(\varphi, \lambda)$

$$\frac{\alpha}{2} p'' + \omega' p' + (\omega'' + \beta\omega - \lambda)\, p = 0$$

Discriminant D>0 for **positive** solution:

$$D = \omega'^2 + 2\alpha(\lambda - \beta\omega - \omega'')$$

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2 \epsilon^2 f^4} \quad , \quad \beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2} \quad , \quad T \equiv \frac{t}{t_R} \quad , \quad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha\beta S_{ds}}{3\xi H_0}$$

$$S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

e.g. D=0 at $\varphi = 1$ $\Rightarrow$ $\lambda_{\max} = \beta - \frac{\omega_1'^2}{2\alpha}$

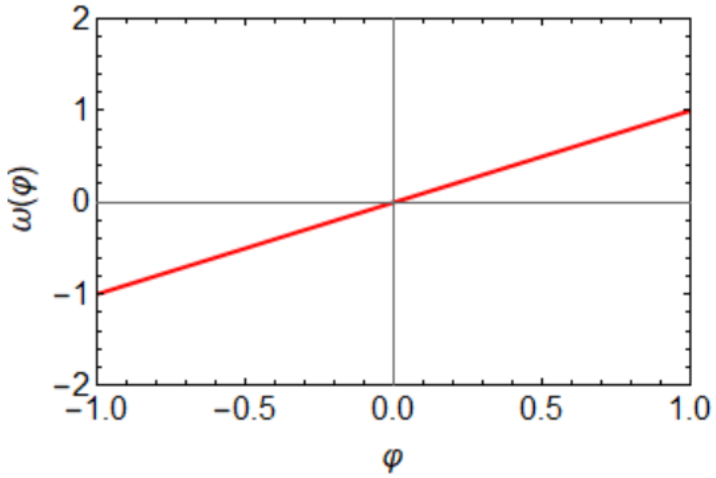
- Largest eigenvalue $\lambda = \lambda_{\max}$ inflates most
- **Eigenvalue** determines **peak location**

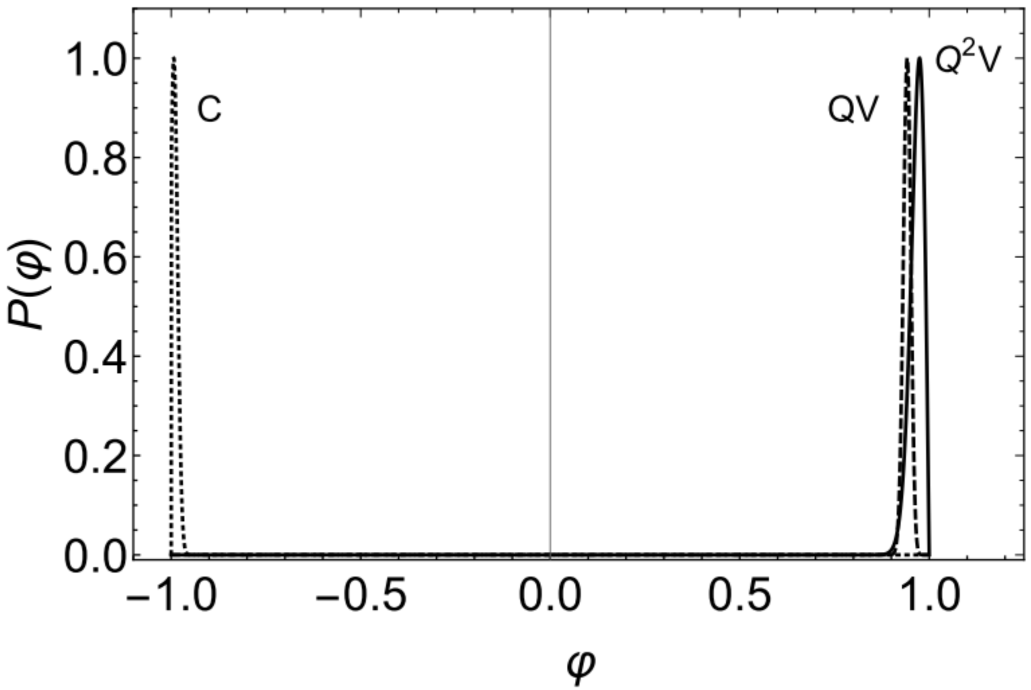
$$\frac{\alpha}{2} p'' + \cancel{\omega' p'} + (\omega'' + \beta\omega - \lambda)\, p = 0 \quad \Rightarrow \quad \lambda = \beta\omega(\bar{\varphi}) + \omega''(\bar{\varphi}) - \frac{\alpha}{2\sigma^2} \quad \Rightarrow \quad \omega(\bar{\varphi}) = 1 - \frac{\omega_1'^2}{2\alpha\beta}$$

- Note: **boundary conditions** necessary input for solution

FPV dynamics

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2\epsilon^2 f^4} \quad , \quad \beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2}$$



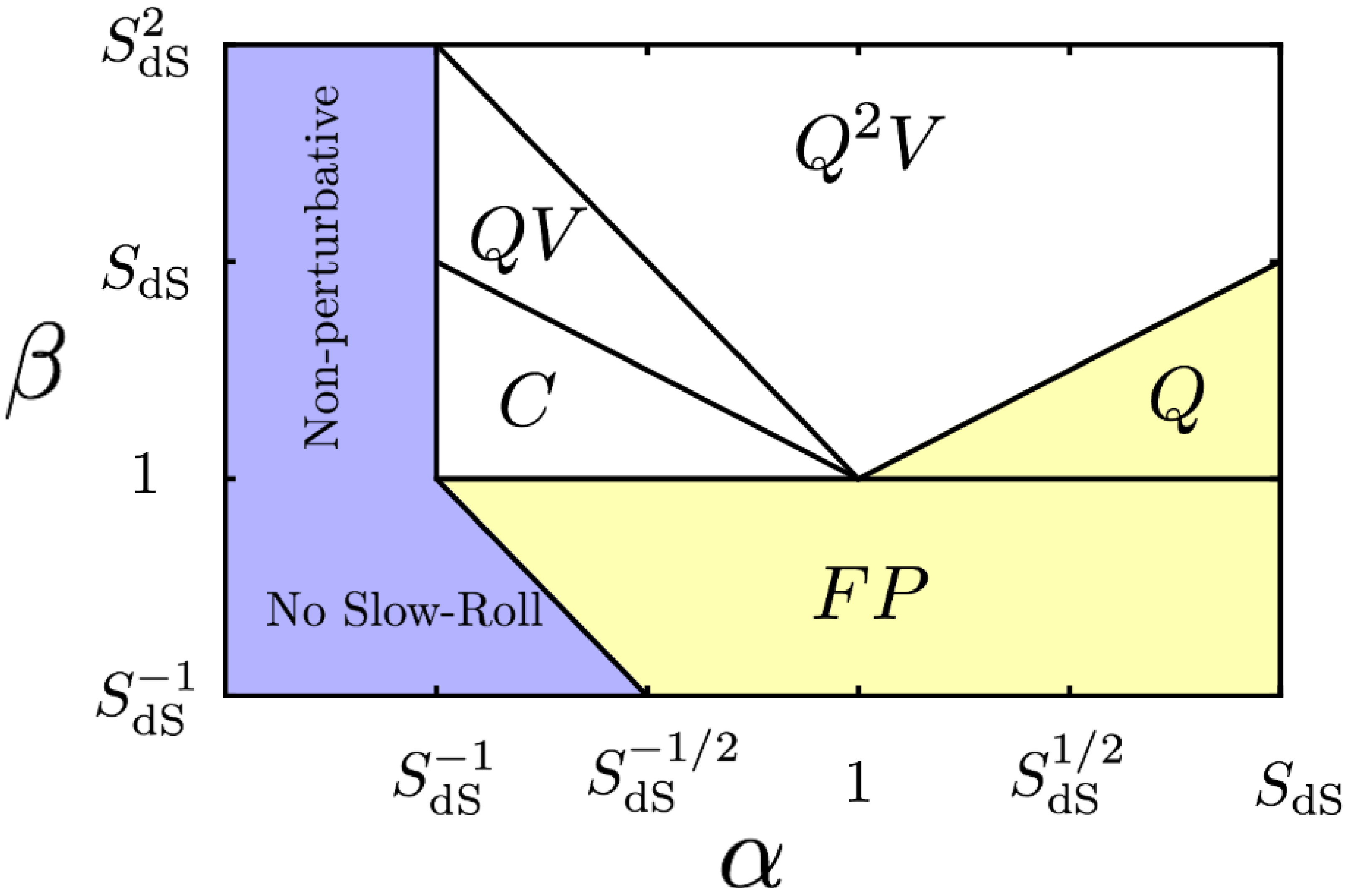


- C regime: $\alpha\beta \ll 1$. Peak is located as far down the potential as allowed by boundary condition.
- QV regime: $\alpha\beta \gg 1$, $\alpha^2\beta \ll 1$. Peak is a distance $1/(\alpha\beta)$ from the top with width $\sigma \simeq 1/\sqrt{\beta}$.
- Q^2V regime: $\alpha^2\beta \gg 1$. Peak as close to the top as possible, with a distance comparable to the width $\sigma \simeq (\alpha/\beta)^{1/3}$.

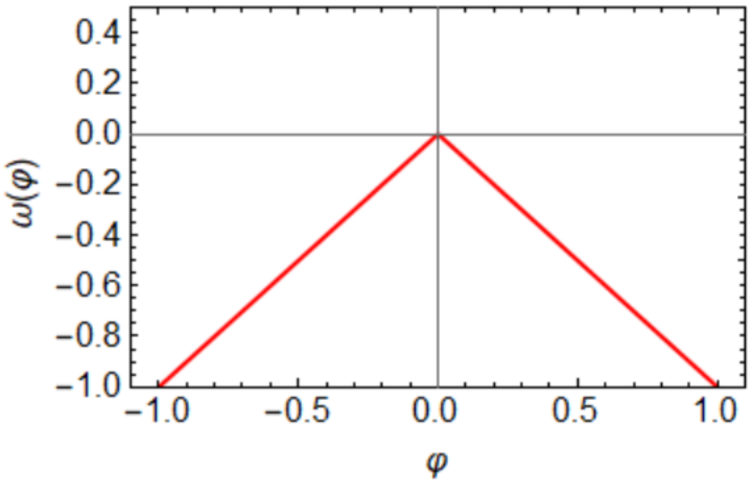
FPV dynamics

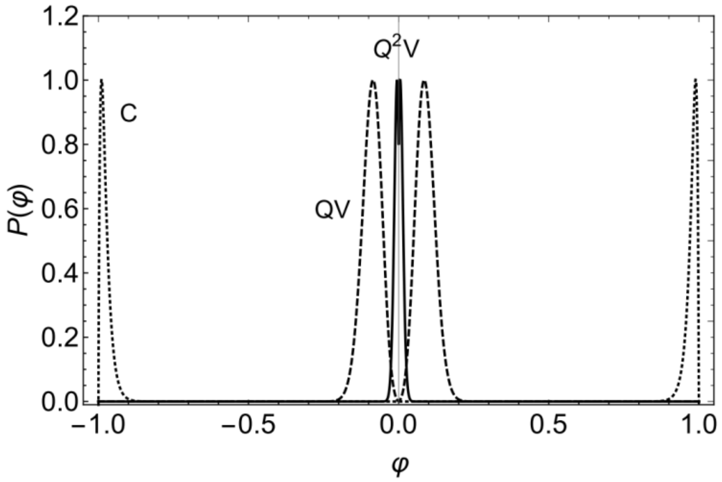
$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2\epsilon^2 f^4} \quad , \quad \beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2}$$

$$S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$



Junction conditions at phase transitions

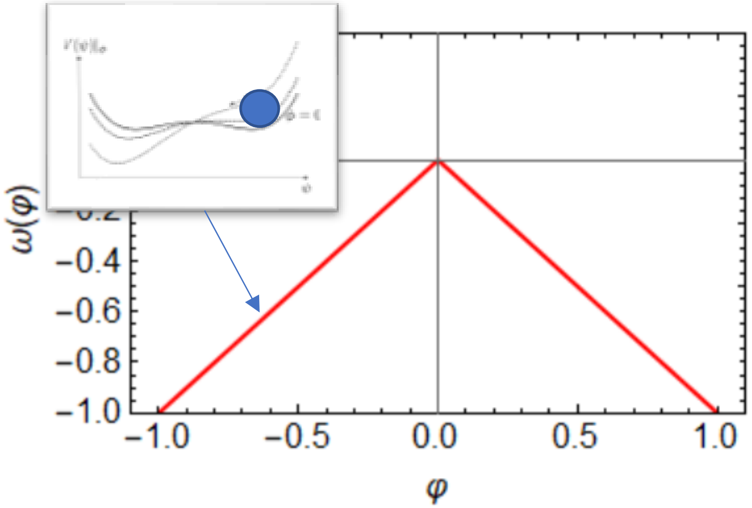


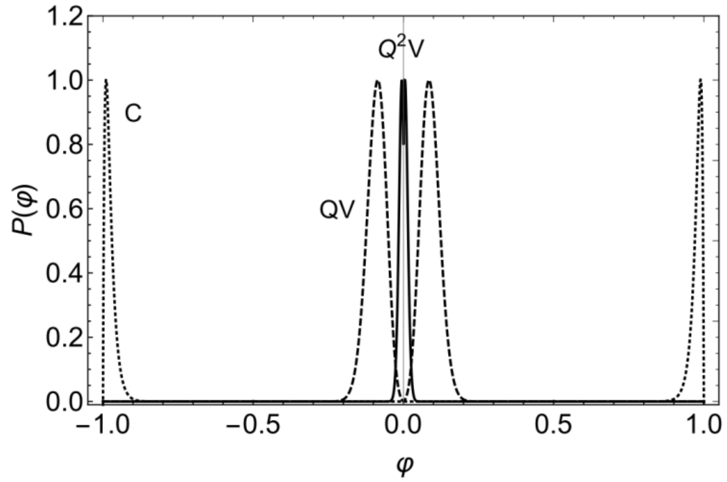


- ϕ triggers 1st order **quantum phase transition** at ϕ_c
- **Discontinuity** in V' leads to discontinuous P'
- Requiring **continuity of FPV** across the critical point gives a **junction condition** to satisfy

$$\lim_{\epsilon\to 0}\int_{\phi_c-\epsilon}^{\phi_c+\epsilon} d\phi \frac{\partial}{\partial\phi}\left[\frac{V'P}{3H}+\frac{\hbar}{8\pi^2}\frac{\partial}{\partial\phi}\left(H^3P\right)\right]=0 \quad \Rightarrow \quad \frac{\Delta P'}{P(\varphi_c)}=-\frac{2\Delta\omega'}{\alpha}$$

Junction conditions at phase transitions

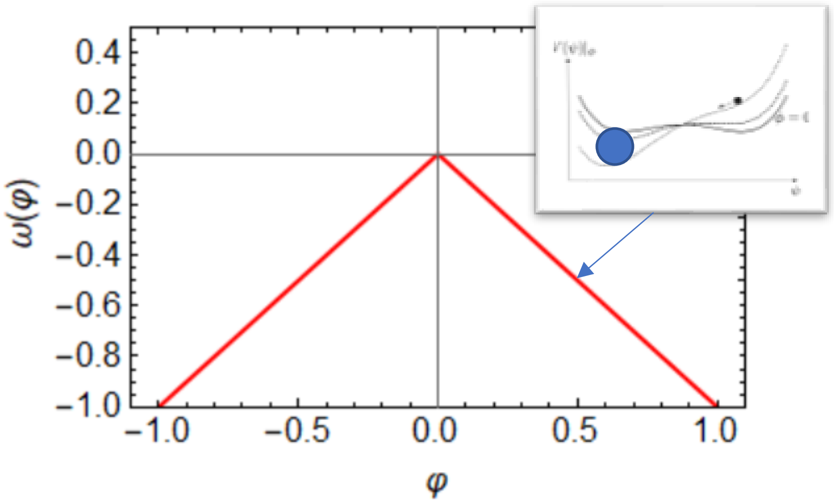


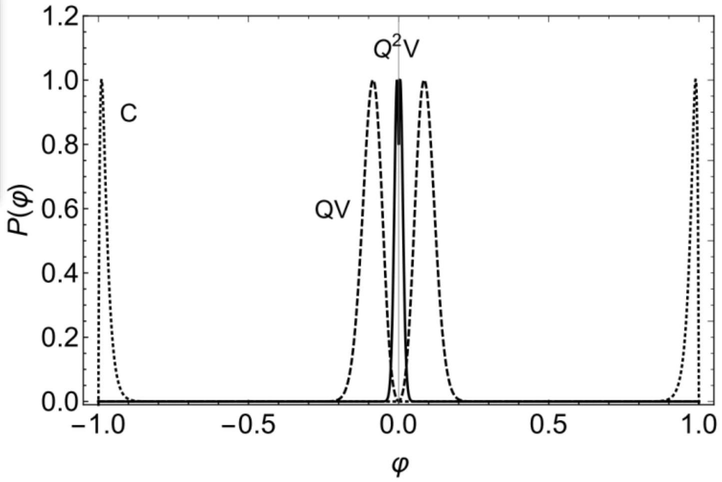


- ϕ triggers 1st order **quantum phase transition** at ϕ_c
- **Discontinuity** in V' leads to discontinuous P'
- Requiring **continuity of FPV** across the critical point gives a **junction condition** to satisfy

$$\lim_{\epsilon\to 0}\int_{\phi_c-\epsilon}^{\phi_c+\epsilon} d\phi \frac{\partial}{\partial\phi}\left[\frac{V'P}{3H}+\frac{\hbar}{8\pi^2}\frac{\partial}{\partial\phi}\left(H^3P\right)\right]=0 \quad\Rightarrow\quad \frac{\Delta P'}{P(\varphi_c)}=-\frac{2\Delta\omega'}{\alpha}$$

Junction conditions at phase transitions

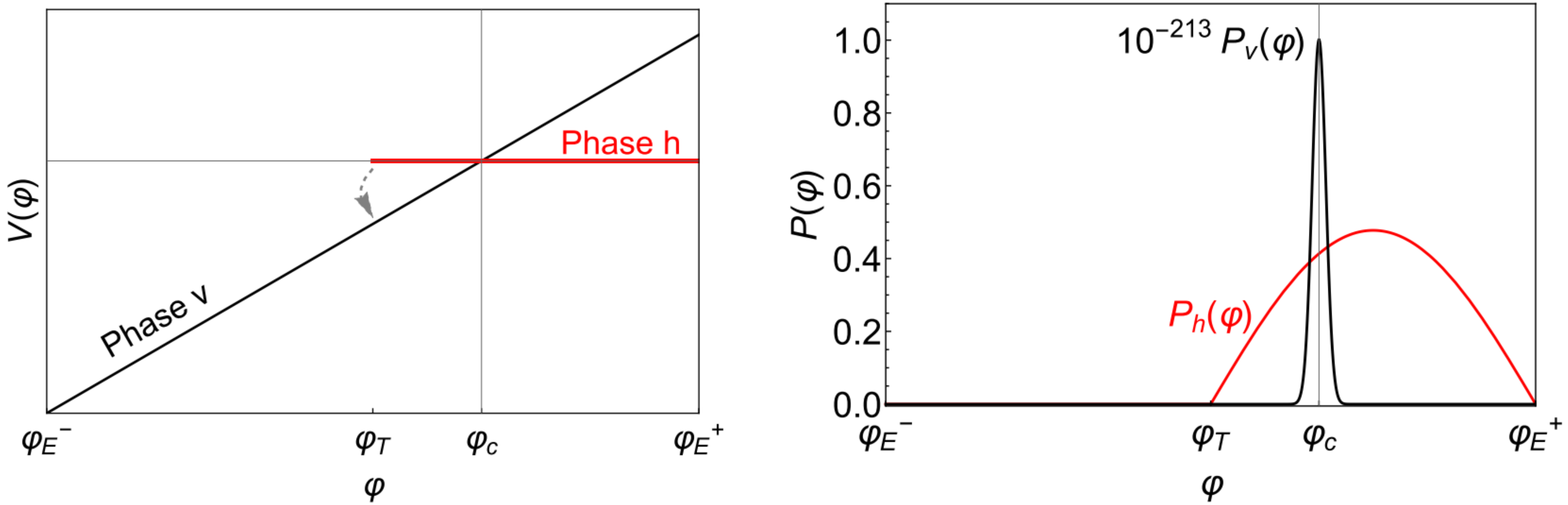




- ϕ triggers 1st order **quantum phase transition** at ϕ_c
- **Discontinuity** in V' leads to discontinuous P'
- Requiring **continuity of FPV** across the critical point gives a **junction condition** to satisfy

$$\lim_{\epsilon \to 0} \int_{\phi_c - \epsilon}^{\phi_c + \epsilon} d\phi \frac{\partial}{\partial \phi} \left[\frac{V' P}{3H} + \frac{\hbar}{8\pi^2} \frac{\partial}{\partial \phi} \left(H^3 P \right) \right] = 0 \quad \Rightarrow \quad \frac{\Delta P'}{P(\varphi_c)} = -\frac{2\Delta\omega'}{\alpha}$$

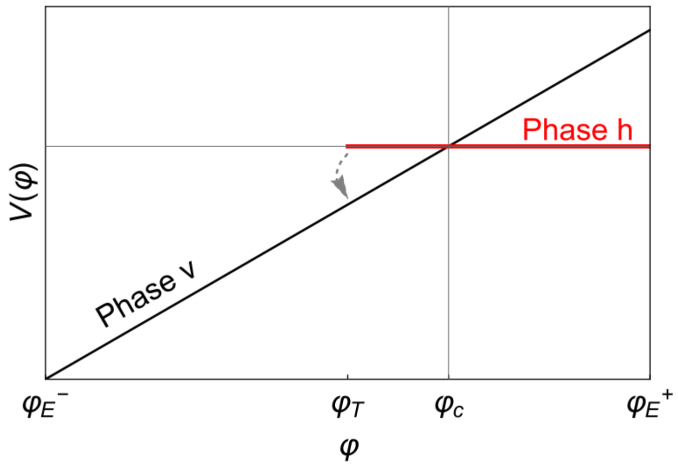
Junction conditions at phase transitions

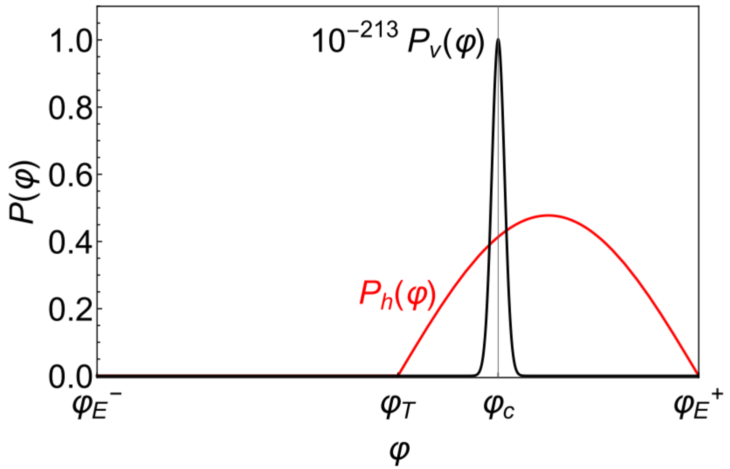


- **Coexistence** of branches of different phases, require continuity of P_V and $P_V + P_h$ in FPV at ϕ_T: **flux conservation** junction conditions

$$P_h(\phi_T) = 0 \qquad \Delta P'_v = -P'_h(\phi_T) \qquad \Delta P_v = 0$$

Junction conditions at phase transitions





- **Phase v** *must be in C regime*

- **Boundary conditions** pick out diffusionless solution over Gibbs solution

-Require **flux at boundary**

Solve FPV: $\frac{\alpha}{2}p'' + \omega' p' + (\omega'' + \beta\omega - \lambda)p = 0\,, \qquad \omega_h(\varphi) = 0 \quad , \quad \omega_v(\varphi) = \varphi\,.$

Phase h:

$$p_h(\varphi_E^+) = 0 \quad , \quad p_h(\varphi_T) = 0\,. \qquad p_h'(\varphi_T) = \kappa_h\,.$$

$$p_h(\varphi) = \frac{(\varphi_E^+ - \varphi_T)}{\pi}\kappa_h \sin\left(\frac{\pi(\varphi - \varphi_T)}{\varphi_E^+ - \varphi_T}\right) \quad , \quad (\varphi > \varphi_T)\,.$$

$$\lambda = -\frac{\alpha}{2}\frac{\pi^2}{(\varphi_E^+ - \varphi_T)^2}\,.$$

Phase v:

$$1)\ P_v^-(-1) = 0 \quad , \quad 2)\ P_v^{+\prime}(1) = -k_v\,,$$

$$3)\ P_v^+(\varphi_T) = P_v^-(\varphi_T) \quad , \quad 4)\ P_v^{+\prime}(\varphi_T) = P_v^{-\prime}(\varphi_T) - k_h\,,$$

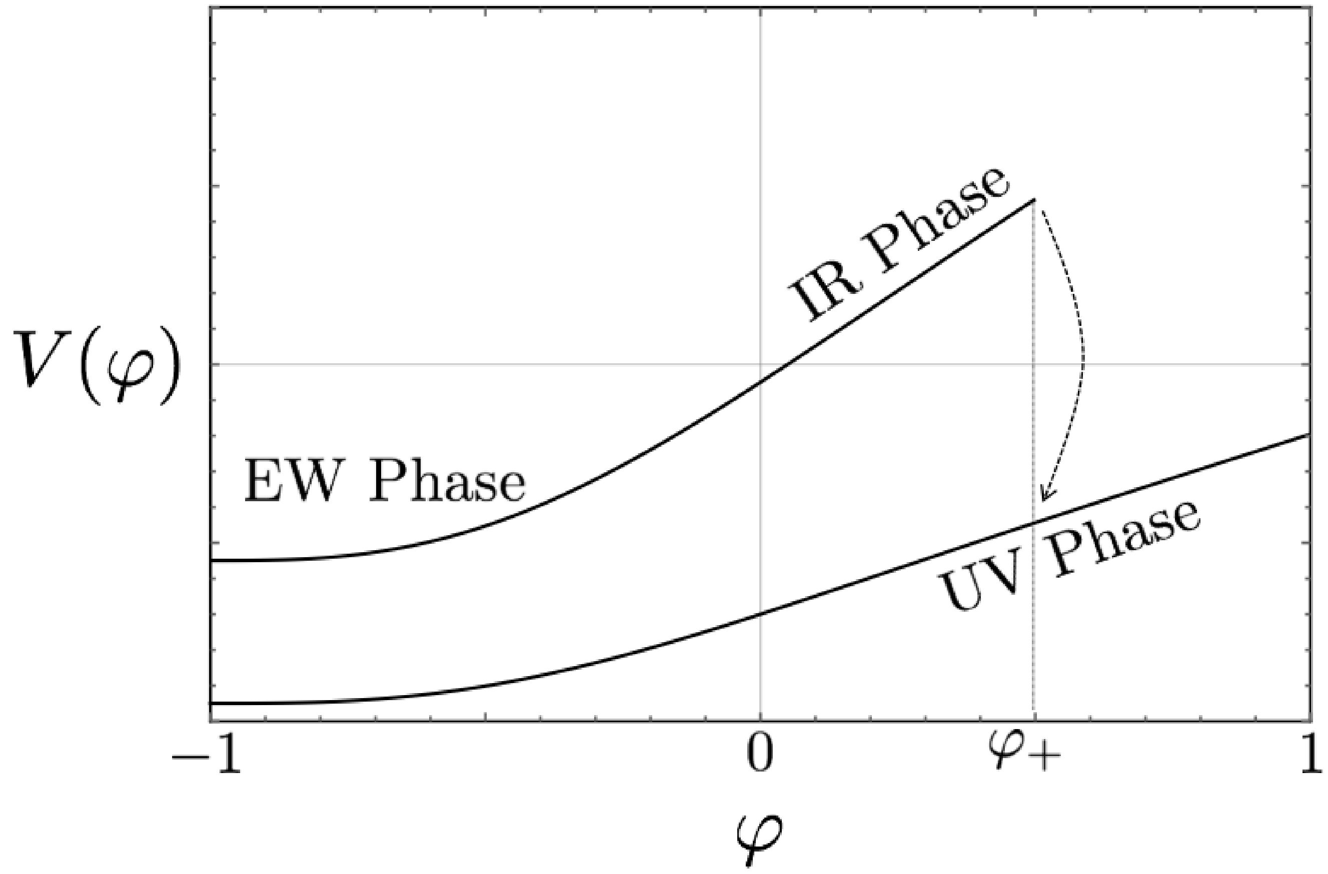
$$P_v^\pm(\varphi, \lambda) = e^{-\frac{\varphi}{\alpha}}\left[g_a^\pm(\lambda)Ai(x) + g_b^\pm(\lambda)Bi(x)\right]\,,$$
$$x = \frac{1 + 2\alpha\lambda - 2\alpha\beta\varphi}{(2\alpha^2\beta)^{2/3}}\,.$$

Higgs mass naturalness

$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) - \frac{\varphi M^2 h^2}{2} + \frac{\lambda(h)\,h^4}{4}$$

$$\frac{V(\varphi,\langle h\rangle)}{M^4} = \begin{cases} \kappa_{\rm EW}\varphi + \kappa_2\varphi^2 + \ldots & \text{for } \varphi<0 & (\text{unbroken EW: } \langle h\rangle = 0) \\ \kappa_{\rm EW}\varphi + \kappa_{\rm IR}\varphi^2 + \ldots & \text{for } 0<\varphi<\varphi_+ & (\text{IR phase: } \langle h\rangle = v) \\ -\kappa_0 + \kappa_{\rm UV}\varphi + \kappa_2\varphi^2 + \ldots & \text{for any } \varphi & (\text{UV phase: } \langle h\rangle = c_{\rm UV}M) \end{cases}$$

$$\kappa_{\rm EW} = \frac{\omega'(0)}{g_*^2}\,, \quad \kappa_2 = \frac{\omega''(0)}{2g_*^2}\,, \quad \kappa_{\rm IR} = \kappa_2 - \Delta\kappa\,, \quad \kappa_0 = \frac{-\lambda_{\rm UV}c_{\rm UV}^4}{4}\,, \quad \kappa_{\rm UV} = \kappa_{\rm EW} - \frac{c_{\rm UV}^2}{2}$$



- Unbroken to broken transition **not sufficient**

- Use **broken IR to broken UV** phase transition

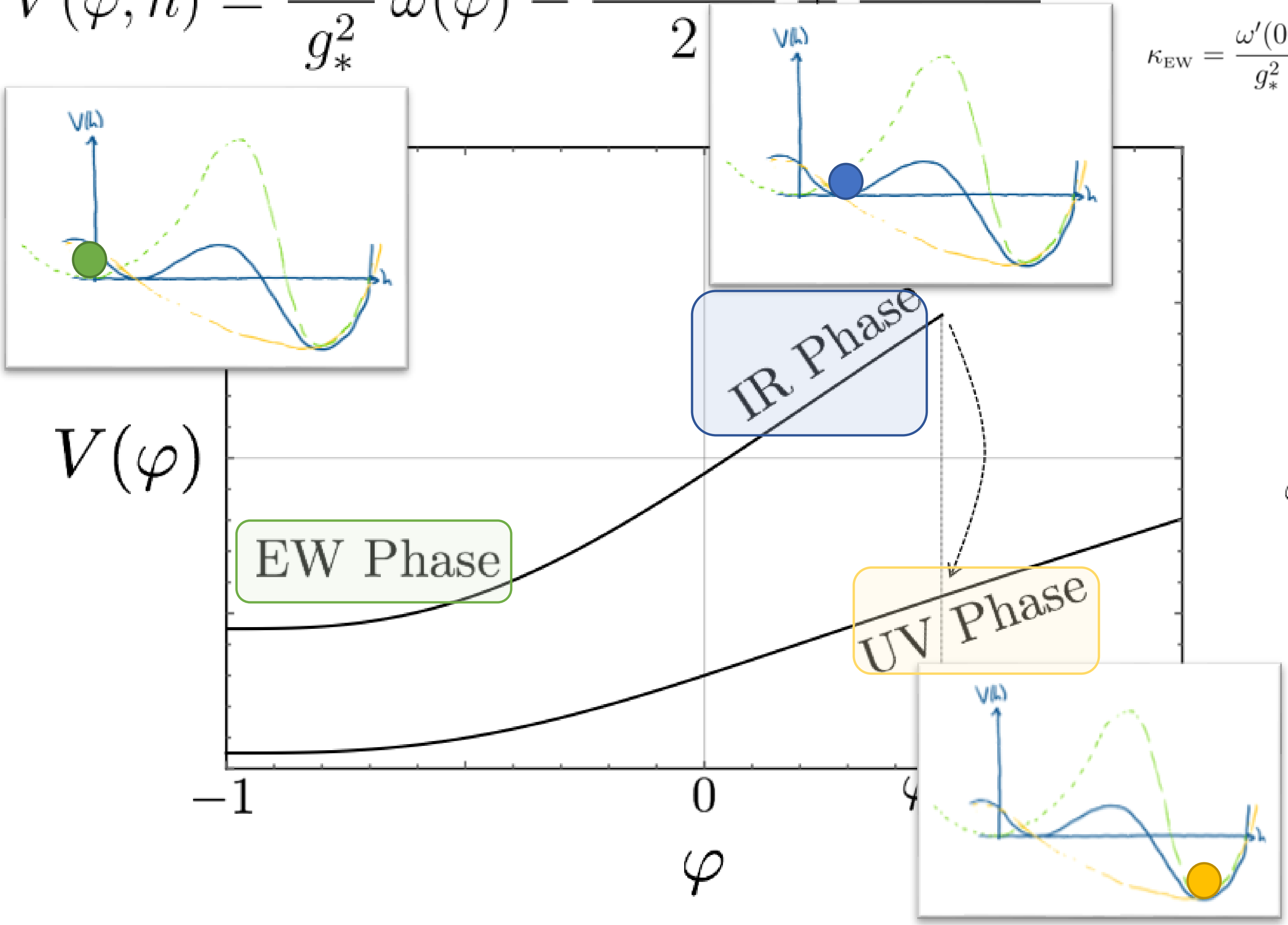
$$\varphi_+ = \frac{-\beta_I\, e^{-\frac{3}{2}}\Lambda_I^2}{M^2} \quad \Longrightarrow \quad v = e^{-\frac{3}{4}}\,\Lambda_I$$

- Need **lower instability scale Λ_I**: ~TeV through VL fermions

- (Naturalness motivation: scalars and vectors heavy, **only VL fermions at TeV scale**)

Higgs mass naturalness

$$V(\varphi,h)=\frac{M^4}{g_*^2}\,\omega(\varphi)-\frac{\varphi M^2 h^2}{2}+\frac{\lambda(h)\,h^4}{\cdots}$$



$$\frac{V(\varphi,\langle h\rangle)}{M^4}=\begin{cases}\kappa_{\rm EW}\varphi+\kappa_2\varphi^2+\dots & \text{for } \varphi<0 \quad \text{(unbroken EW: } \langle h\rangle=0)\\ \kappa_{\rm EW}\varphi+\kappa_{\rm IR}\varphi^2+\dots & \text{for } 0<\varphi<\varphi_+ \quad \text{(IR phase: } \langle h\rangle=v)\\ -\kappa_0+\kappa_{\rm UV}\varphi+\kappa_2\varphi^2+\dots & \text{for any } \varphi \quad \text{(UV phase: } \langle h\rangle=c_{\rm UV}M)\end{cases}$$

$$\kappa_{\rm EW}=\frac{\omega'(0)}{g_*^2}\,,\quad \kappa_2=\frac{\omega''(0)}{2g_*^2}\,,\quad \kappa_{\rm IR}=\kappa_2-\Delta\kappa\,,\quad \kappa_0=\frac{-\lambda_{\rm UV}c_{\rm UV}^4}{4}\,,\quad \kappa_{\rm UV}=\kappa_{\rm EW}-\frac{c_{\rm UV}^2}{2}$$

- Unbroken to broken transition **not sufficient**

- Use **broken IR to broken UV** phase transition

$$\varphi_+=\frac{-\beta_I\,e^{-\frac{3}{2}}\Lambda_I^2}{M^2} \;\Rightarrow\; v=e^{-\frac{3}{4}}\,\Lambda_I$$

- Need **lower instability scale Λ_I**: ~TeV through VL fermions

- (Naturalness motivation: scalars and vectors heavy, **only VL fermions at TeV scale**)

SOL take-home message

- Scalar fields undergoing quantum fluctuations during inflation can be **localised at the critical points** of quantum phase transitions: **SOL**

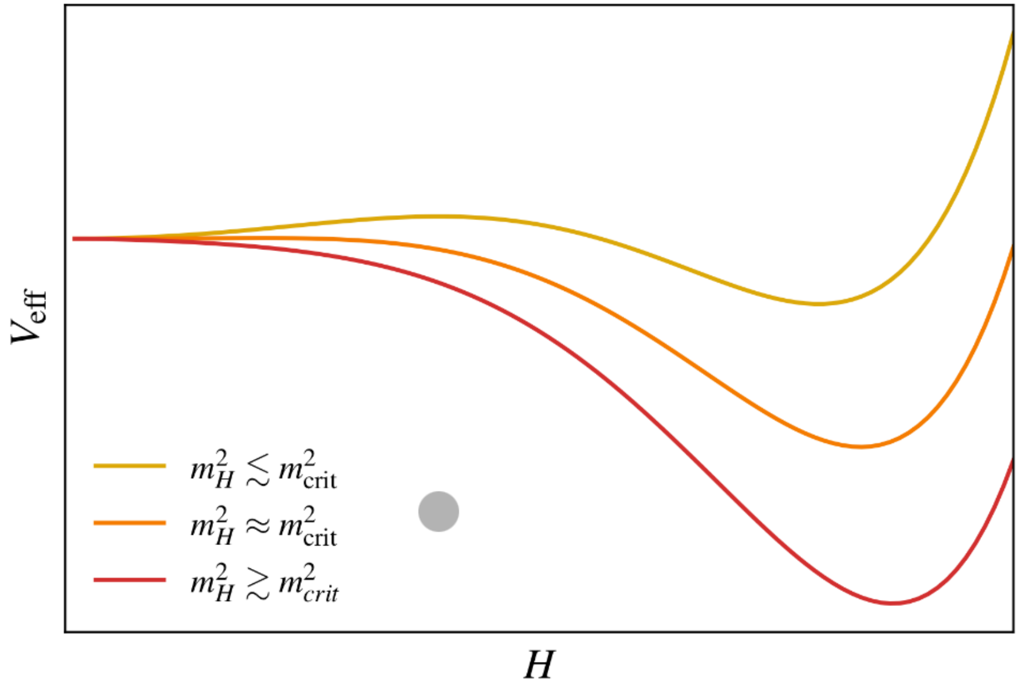
- SOL suggests **our Universe lives at the critical boundary** of coexistence of phases

- **Measure problem**: ambiguous choice of time parametrisation (recall $\beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2}$)
- Related to regularisation of **infinite reheating surface**
- We have **not specified** the inflaton sector: decoupled from our scalar
- SOL prediction is quantitative but dependent on chosen solution of measure problem: **exponential localisation can remain a feature**

Vacuum metastability bound

1307.3536 Buttazzo et al;
2105.08617 Giudice, McCullough, TY;
2108.09315 Khoury, Steingasser.

- Upper bound on Higgs mass landscape from vacuum metastability
- Agnostic about underlying mechanism; predicts light new physics



$$m_{\text{crit}}^2 = -\frac{1}{2}\,\beta_\lambda^{(1)}\Big|_{\mu_I}\, \mathrm{e}^{-3/2}\, \mu_I^2\,.$$

Contents

Axion-Higgs criticality model

2412.03542 Detering, **TY**

- Vector-like fermions previously used to lower vacuum instability scale

- Axions are also motivated, naturally light candidates for new physics

- Axion coupled to the Higgs can lower the vacuum metastability bound

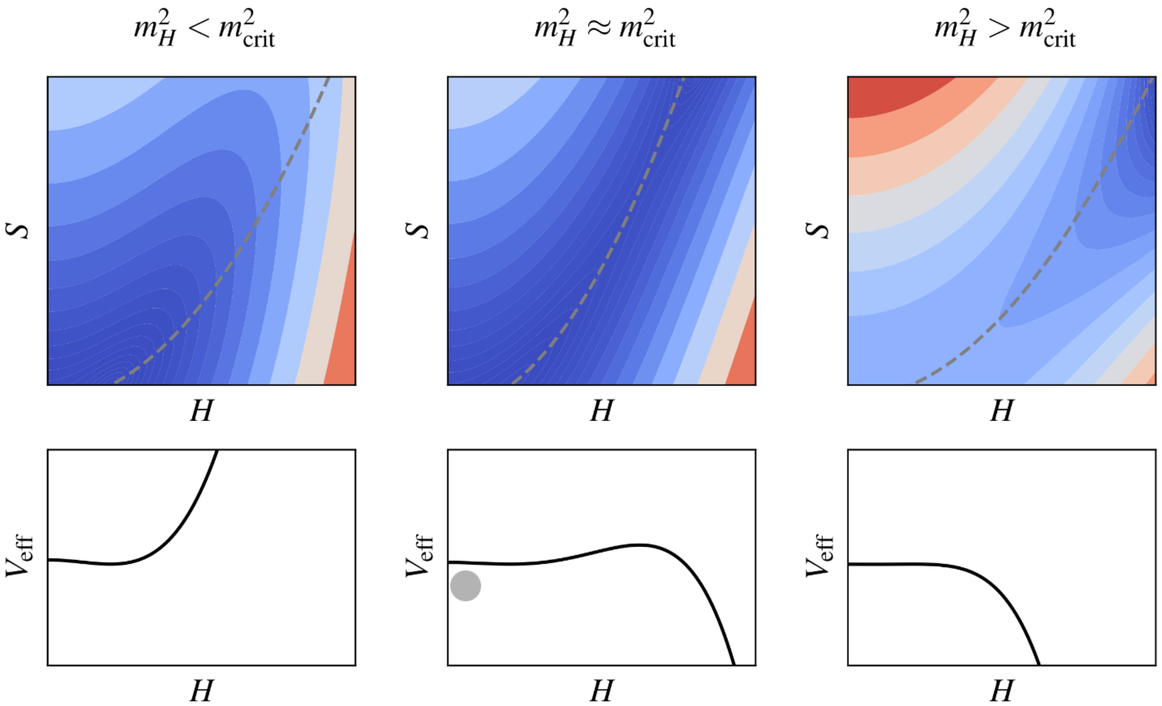
$$V(H,S) = -\frac{1}{2}m_H^2 H^2 + \frac{1}{4}\lambda H^4 + m_S^2 f^2 \left(1 - \cos\left(\frac{S}{f}\right)\right) - \frac{1}{2}Af(H^2 - v^2)\cos\left(\frac{S}{f} - \delta\right),$$

Axion-Higgs criticality model

2412.03542 Detering, **TY**

- For large decay constant f, axion-Higgs potential simplifies:

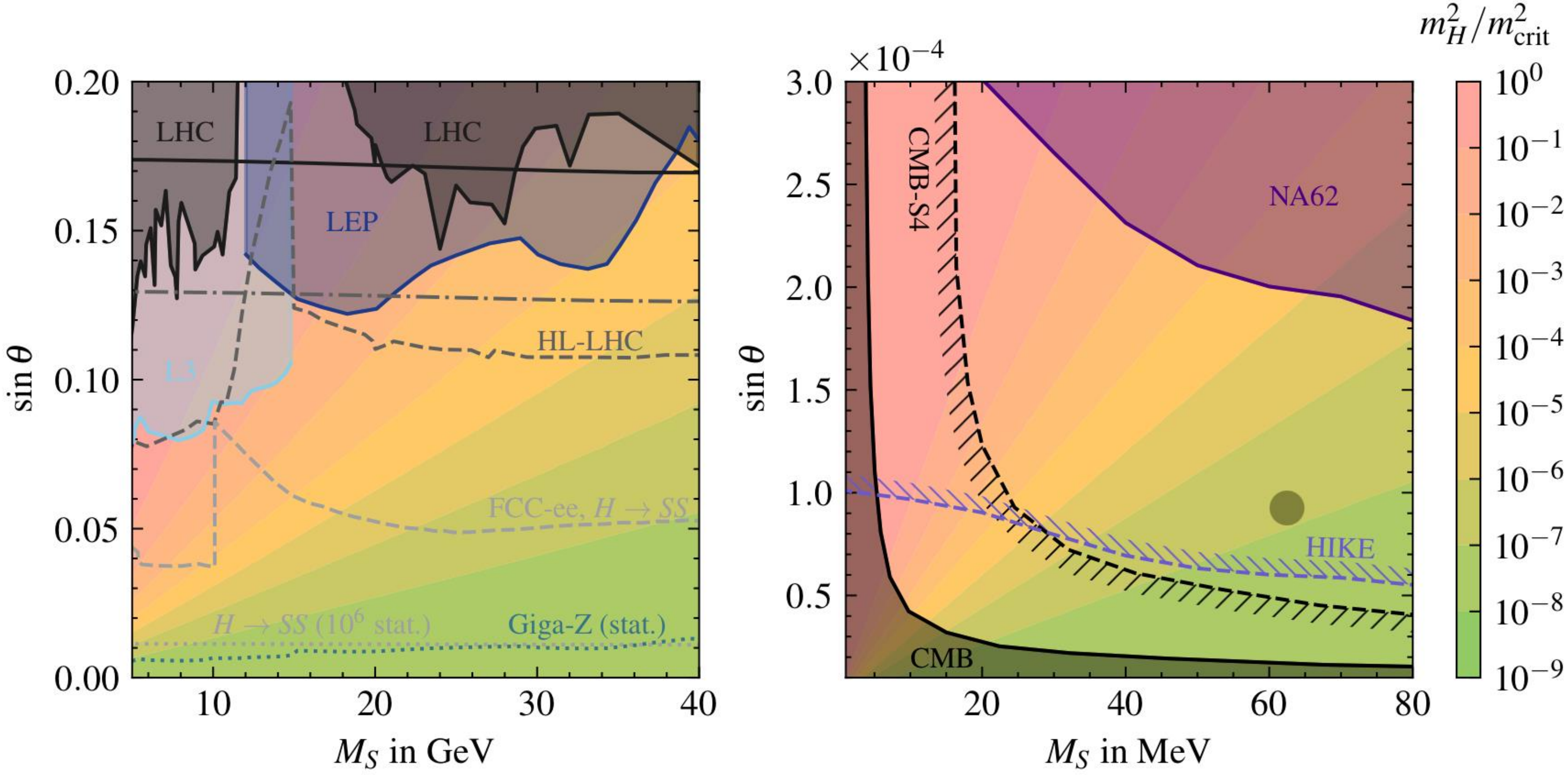
$$V(H,S) = -\frac{1}{2}m_H^2 H^2 + \frac{1}{4}\lambda H^4 + \frac{1}{2}m_S^2 S^2 - \frac{1}{2}A' S H^2 + \frac{1}{2}A' v^2 S,$$



Axion-Higgs criticality model

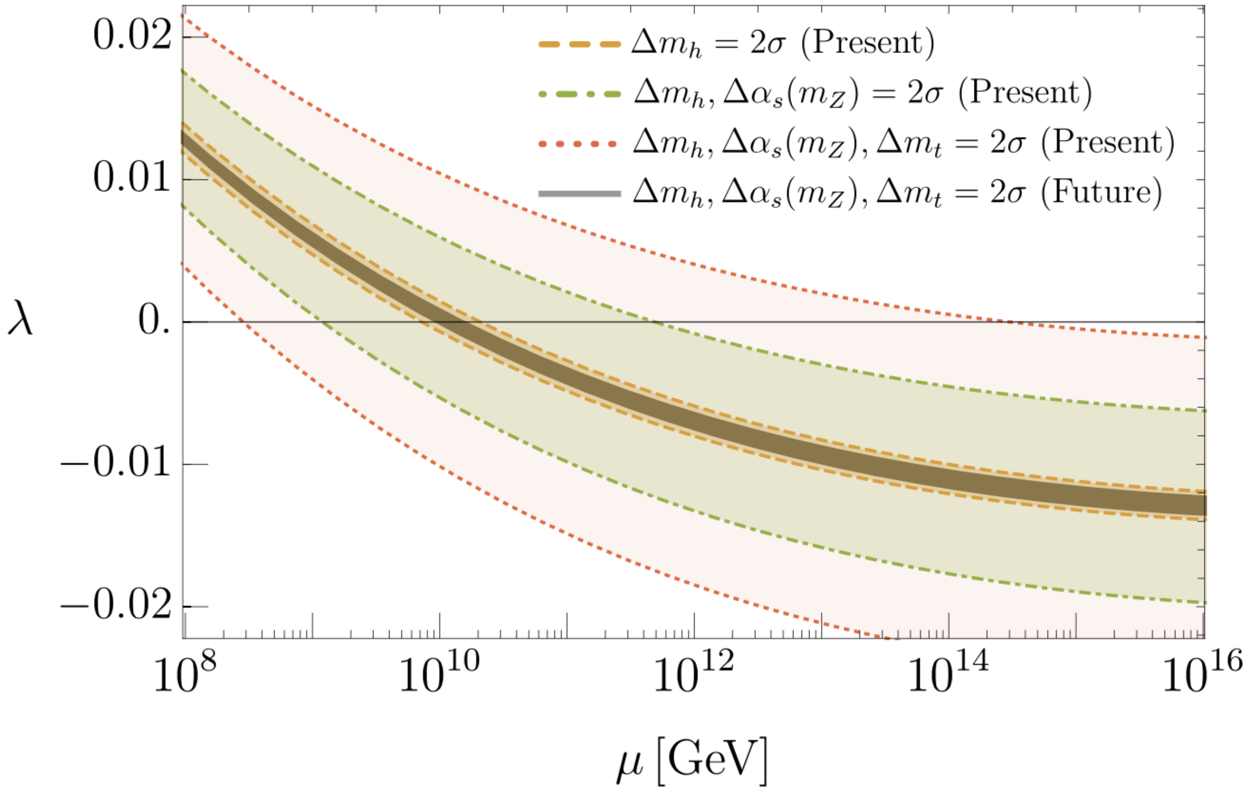
2412.03542 Detering, **TY**

- Natural parameter space, 10 MeV – 10 GeV, can be entirely covered!



Future colliders can answer definitive questions

e.g. What is the **vacuum instability scale** in the SM?



Snowmass 2021
Dunsky, Harigaya, Hall

See also e.g. 2203.17197
Franceschini, Strumia, Wulzer

Uncertainty can be reduced from $O(10^6)$ down to *a factor of **~2!*** Potential implications for BSM.

Conclusion

- The hierarchy problem is now an even bigger fundamental problem
- Self-organised criticality predicts Higgs mass set by metastability
- Lowered vacuum instability scale predicts accessible new physics
- See recent review of Higgs metastability bound:
  - 2503.22787 Detering, Enguita, Gavela, Steingasser, TY

Backup