Axion Physics Overview

Luca Di Luzio

LNGS 21/03/2025

luca.diluzio@pd.infn.it

Axion Physics Overview

1. Strong CP problem & QCD axions
2. Axion properties
3. Axion DM from missalignment
4. Wave DM detection concepts

1. Strong CP problem & QCD axions

$$\delta \mathcal{L}_{QCD} = \theta \frac{g_s^2}{32\pi^2} G\tilde{G}$$

- P and T violating ($\cancel{CP}$, via CPT theorem)
- allowed by gauge invariance
- total derivative $\rightarrow$ physical significance due to the non-perturbative vacuum structure of gauge theories

$$|\theta\rangle = \sum_{n=-\infty}^{+\infty} e^{i\theta n} |n\rangle$$

eigenstate of the Hamiltonian

vacuum states of $\mathcal{L}_{YM} = -\frac{1}{4} GG$ mix under time evolution due to the existence of instanton configurations

$$\langle \theta | \mathcal{O}_{\substack{\text{gauge} \\ \text{inv.}}} | \theta' \rangle \propto \delta(\theta - \theta')$$

$\theta \rightarrow$ labels the ground state of the theory

superselection rule.

In QCD things are slightly more involved due to quark masses:

$$\mathcal{L}_{QCD} \supset - \sum_q m_q \, e^{i\theta_q} \, \bar{q}_L q_R + h.c.$$

physical observable:

$$\bar{\theta} = \theta + \sum_q \theta_q$$ → invariant under $q \to e^{i\alpha\gamma_5} q$

$$= \theta + \arg\det M_q$$

$$= \theta + \arg\det Y_u Y_D$$

→ contribution to nEDM

non-relativ. limit

$$\mathcal{L} = - d_n \frac{i}{2} \bar{n} \sigma^{\mu\nu} \gamma_5 n F_{\mu\nu} \longrightarrow \mathcal{H} = - d_n \vec{E} \cdot \vec{S}$$

needs a phase because EDM imaginary

$$\mathcal{L} \supset \left(1 - c \frac{m_q e^{i\bar{\theta}}}{2 m_n} \right) \frac{e}{m_n} \bar{n} \sigma^{\mu\nu} \gamma_5 n F_{\mu\nu}$$

$$\Rightarrow d_n = \underbrace{c}_{\mathcal{O}(1)} \underbrace{\frac{m_q}{m_n}}_{10^{-2}} \underbrace{\frac{e}{m_n}}_{e\,\mathrm{GeV}^{-1}} \bar{\theta} \sim 10^{-2} \bar{\theta} \, e \, \underbrace{\mathrm{GeV}^{-1}}_{1.97 \times 10^{-14}\,\mathrm{cm}}$$

$$\sim 10^{-16} \bar{\theta} \, e \cdot \mathrm{cm}$$

$$|d_n^{exp}| \lesssim 10^{-26} \, e \cdot \mathrm{cm} \Rightarrow |\bar{\theta}| \lesssim 10^{-10}$$ strong CP problem

PQ mechanism $\rightarrow$ new spin-0 boson with pseudo-shift symmetry ; $a \rightarrow a + \alpha f_a$ broken only by a specific operator similar to θ term ;

$$\mathcal{L} \supset \frac{a}{f_a} \frac{g_s^2}{32\pi^2} G\tilde{G}$$

choose $\alpha = -\theta$ to remove θ.

Need to make sure that $\langle a \rangle = 0$ $\rightarrow$ ensured by Vafa-Witten theorem.

2. Axion properties

Some axion properties can be derived at the EFT level, without any reference to UV completion

$$\mathcal{L}_a^{EFT} = \frac{1}{2}(\partial_\mu a)^2 + \mathcal{L}(\partial_\mu a, \psi_{SM}) + \frac{g_s^2}{32\pi^2} \frac{a}{f_a} G\tilde{G}$$

$\frac{a}{f_a} G\tilde{G}$ $\rightarrow$ breaks axion shift symmetry $\rightarrow$ generates a potential

$$V = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2 \frac{a}{2 f_a}}$$

$a/f_a \ll 1$

$$\simeq -m_\pi^2 f_\pi^2 + \frac{1}{2} \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2} a^2 + O(a^4)$$

$$m_a \simeq 5.7 \text{ meV} \left(\frac{10^9 \text{ GeV}}{f_a} \right)$$

axion couplings to SM $\quad g_{ax} \sim \frac{1}{f_a}$

$\longrightarrow \quad f_a \simeq 10^{8 \div 9}$ GeV from astrophysics

Compton wave-lenght :

$$\lambda_a = \frac{2\pi\hbar}{m_a c} \sim 1 \text{ meter} \left(\frac{10^{-6} \text{ eV}}{m_a} \right)$$

$$\lambda \nu = c$$

$$\left[mc^2 = h\nu = h\frac{c}{\lambda} \longrightarrow \lambda = \frac{h}{mc} = \frac{2\pi\hbar}{mc} \right]$$

General properties of axions :

- weakly - coupled
- light (sub - eV)
- macroscopic wavelenght

From an experimental point of view, a crucial coupling for detection is the one to photons

$$\mathcal{L}_a \supset -\frac{1}{4} g_{a\gamma}\, a\, F\tilde{F}$$

$$\Downarrow$$

model - dependent contribution

model indep. contribution due to $aG\tilde{G}$

$$g_{a\gamma} = \frac{\alpha}{2\pi f_a} \underbrace{\left[\frac{E}{N} - 1.92(4) \right]}_{C_{a\gamma}}$$

3. Axion DM

Is the axion long-lived enough to be DM ?

$a \to \gamma\gamma$ kinematically open

$$\Gamma_{a\to\gamma\gamma} = \frac{g_{a\gamma}^2 m_a^3}{64\pi} \sim \frac{1}{10^{24}\,s}\left(\frac{m_a}{1\,eV}\right)^5$$

$$1/\Gamma_{a\to\gamma\gamma} \gg \tau_U \sim 10^{17}\,s$$ → stable on cosmological scales.

sub-eV DM axions cannot be produced thermally via scatterings off SM particles (like for WIMPs) because they would be relativistic at the time of the CMB

→ thermal axions rather contribute to dark radiation

Axion DM production proceeds via non-thermal mechanism known as misalignment

To be more general let us consider a scalar field ϕ

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 + \mathcal{L}_I$$ → interactions with SM or self-interactions

assume the universe underwent a period of inflation

$$H \equiv \frac{\dot{a}}{a} \gg m_\phi$$

after inflation the field is approximately spatially uniform and the initial state is characterized by the field value ϕ_i.

After inflation a period of reheating occurs, following a period of radiation domination

$a \propto t^{1/2}$ $\qquad H = \frac{\dot{a}}{a} = \frac{1}{2t}$

EOM : $\qquad \ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = 0$

in general $m_\phi = m_\phi(t)$ due to $\mathcal{L}_T$

solution can be separated in two regimes :

- $3H \gg m_\phi$: ϕ is an overdamped oscillator $\phi = \phi_i$
- t_1 : $3H(t_1) = m_\phi(t_1) \equiv m_1$, the damping becomes undercritical and the field starts to oscillate

WKB approximation (fast oscillation, slow amplitude)

$$\phi \approx \underbrace{\phi_1 \left(\frac{m_1 a_1^3}{m_\phi a^3} \right)^{1/2}}_{A} \cos \Big(\underbrace{\int_{t_1}^{t} m_\phi \, dt}_{\alpha(t)} \Big)$$

with $\phi_1 \simeq \phi_i$.

the energy density of the scalar field is

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m_\phi^2\phi^2$$

$$= \frac{1}{2}m_\phi^2 A^2 + O(\dot{A}^2) \qquad \dot{A} \ll m_\phi A$$

pressure :

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m_\phi^2\phi^2$$

$$= \frac{1}{2}\dot{A}^2\cos^2\alpha - \frac{1}{2}\dot{A}A\, m_\phi \sin 2\alpha - \frac{1}{2}m_\phi^2 A^2 \cos 2\alpha$$

at $t \gg t_1$ the oscillations in the pressure occur at timescales $1/m_\phi$ much faster than the cosmological evolution H^{-1} (since by definition $m_\phi \gg H$). Therfore, we can average the oscillations

$$\langle p_\phi \rangle = \ldots = \frac{1}{4}\dot{A}^2$$

$$\Rightarrow \quad w = \langle p \rangle / \langle \rho \rangle = \frac{1}{4}\frac{\dot{A}^2}{\frac{1}{2}m_\phi^2 A^2} \ll 1 \simeq 0$$

$\rightarrow$ non-relativistic matter.

energy density in a comoving volume

$$\rho a^3 \simeq \frac{1}{2}m_\phi^2 A^2 a^3$$

$$\simeq \frac{1}{2}\cancel{m_\phi^2} A_1^2 \frac{m_1 a_1^3}{\cancel{m_\phi}\cancel{a^3}}\cancel{a^3}$$

ρa^3 not conserved if m_ϕ changes in time

Hence, we define : $N = \frac{\rho a^3}{m_\phi} \simeq \frac{1}{2} \phi_1^2 m_1 a_1^3$

$\longrightarrow$ N (comoving number of quanta) is constant in time

To compute energy density today, we just need to use the conservation of N.

$$\rho_{\phi,0} = m_0 \frac{N}{a_0^3} \simeq \frac{1}{2} m_0 m_1 \phi_1^2 \left(\frac{a_1}{a_0}\right)^3$$

- switch from $t \leftrightarrow T$: $H = 1.66 \sqrt{g_*(T)} \frac{T^2}{M_{pl}}$ (rad. dom.)
- entropy conservation : $S = s a^3 = \frac{2\pi}{45} g_s(T) T^3 a^3$

$$\Rightarrow \left(\frac{a_1}{a}\right)^3 = \frac{g_s(T)}{g_s(T_1)} \frac{T^3}{T_1^3}$$

$$\rho_{\phi,0} \simeq 0.17 \frac{keV}{cm^3} \sqrt{\frac{m_0}{eV}} \sqrt{\frac{m_0}{m_1}} \left(\frac{\phi_1}{10^{10}\,GeV}\right)^2 \underbrace{F(T_1)}$$

$[1, 0.3]$ for $T \in [T_0, 200\,GeV]$

damping of oscillations

$\rho_{DM} = 1.17(6)$ keV/cm^3

in the QCD axion case : $m_a \propto 1/\phi_1$, $\frac{m_1}{m_a} = \left(\frac{T_c}{T}\right)^\beta$, $\beta \approx 4$

$$\Omega_a h^2 \simeq 0.12 \left(\frac{28\,\mu eV}{m_a}\right)^{7/6} \theta_i^2 \simeq 0.12 \left(\frac{f_a}{2 \cdot 10^{11}\,GeV}\right)^2 \theta_i^2$$

ρ_a/ρ_c $\qquad$ $\theta_i = \phi_1/f_a$

at the time of their production, particles from misalignment are semi-relativistic

$$p \sim H_1 \ll T_1 \qquad\qquad H = 1.66\, g_*^{1/2}\, T \cdot \left(\frac{T}{M_{pl}}\right)$$

$$m_1 v_1 \sim H_1 \quad \Rightarrow \quad v_1 \sim \frac{H_1}{m_1} \sim O(1)$$

accordingly we have a velocity distribution with a very narrow width today

$$\delta v(t) \sim v_1 \frac{a_1}{a_0} \sim \frac{H_1}{m_1} \frac{a_1}{a_0} \ll 1$$

combined with a high number density of particles

$$n_{\phi,0} = \frac{N}{a_0^3} = \frac{\rho_{DM}}{m_0},$$ this narrow distribution

leads to high occupation numbers for each quantum state

$$N_{occ.} \sim \underbrace{\frac{dN}{d^3x\, d^3p}}_{n} (2\pi\hbar)^3$$ quantum phase space volume ($\hbar = 1$)

$$\sim \frac{(2\pi)^3}{4\pi/3} \frac{n_{\phi,0}}{m_0^3\, \delta v^3} \sim 10^{42} \left(\frac{m_1}{m_0}\right)^{3/2} \left(\frac{eV}{m_0}\right)^{5/2}$$

→ huge occupation number, behaves like a classical field.

4. Wave DM detection concepts

DM density in the solar neighborhood $\rho_{DM} \sim 0.4 \ \mathrm{GeV/cm^3}$

de Broglie wavelenght : size of dwarf galaxies

$$\lambda_{dB} \equiv \frac{2\pi \hbar}{m_\phi v} \simeq 0.5 \ \mathrm{kpc} \left(\frac{10^{-22}\,\mathrm{eV}}{m_\phi}\right)\left(\frac{250\ \mathrm{km/s}}{v}\right)$$

$$\simeq 1.5 \ \mathrm{km} \left(\frac{10^{-6}\,\mathrm{eV}}{m_\phi}\right)\left(\frac{250\ \mathrm{km/s}}{v}\right)$$

$v \simeq 10^{-3} c \rightarrow$ velocity dispersion of DM in the galactic halo

Volume occupied by each DM state

$$V_{dB} = \lambda_{dB}^3 = \left(\frac{2\pi}{m_\phi v}\right)^3$$

# of states in a dB volume :

$$N_{dB} = n \cdot V_{dB} = \frac{(2\pi)^3 \rho_{DM}}{m_\phi^4 v^3} \simeq \left(\frac{34\ \mathrm{eV}}{m_\phi}\right)^4 \left(\frac{250\ \mathrm{km/s}}{v}\right)^3$$

$n = \rho_{DM}/m_\phi$

wave - particle transition of DM near the Earth $m_\phi \sim 30$ eV

$\rightarrow$ needs quantum detectors ?

For $m_\phi \ll 30$ eV, DM states begin to overlap, so that the collective description of DM as a classical wave is more appropriate.

Main features of wave DM :

i) such DM is necessarily bosonic

→ Pauli exclusion principle precludes multiple occupancies for fermions ($m_\psi^{DM} \gtrsim 0.1$ keV)

ii) we cannot rely on a detection technique based on energy deposition

iii) we leverage the large number density of the field and look for the associated coherent effects

→ analogy with detecting wind in the human context : better to have a windmill rather than looking at the scattering of the single wind's molecules .

How the "classical" DM field behave in the Galaxy ?

DM field seeds the growth of structures in the universe leading to Galaxy formation .

practically impossible to calculate the exact classical field today even if we are given some initial conditions, due to complexity of structure formation .

→ simply regard the classical field being random .

Still , we know some general properties :

Cold DM $\rightarrow$ non-relativistic

ϕ : oscillating scalar field, with oscillation occurring at $E \simeq m_\phi$ (non-relativistic)

$\phi(t) \sim \phi_0 \cos(m_\phi t)$

and amplitude fixed by average DM density

$$\rho_{DM} \sim \frac{1}{2} m_\phi^2 \phi^2 \quad \Rightarrow \quad \phi_0 \sim \sqrt{\frac{2\rho_{DM}}{m_\phi^2}}$$

so far, we ignored the spatial profile of the field

$\Rightarrow$ do not expect homogeneity in the Galaxy, the field will have random non-homogeneities.

although random, we can still define a correlation lenght:

given the value $\phi(\vec{x})$ at the point $\vec{x}$, how far we need to go before the field value is $O(1)$ different?

$\rightarrow$ think about the problem in Fourier space:

changing the value of the field in position corresponds to the field possessing momentum.

Thus, the distance we need to travel before the field value is $O(1)$ different is given by a dB wavelenght

$\lambda_{dB} \sim 1/mv$ with $v \sim 10^{-3}$ (virial velocity of DM in the Galaxy)

no matter what the scale of the DM is in the Galaxy we are guaranteed this minimal correlation lenght

$\to$ a shorter corr. lenght would correspond to a larger velocity and those particle would eventually not be gravitationally bound to the Galaxy.

$\longrightarrow$ Experiments can be devised to measure the coherent effects of the classical field ϕ.

For an experiment, we also care about the coherence time of the field!

how long can an experiment sit at a point and measure the value of the field, before this value changes by $O(1)$?

Relative velocity of experiments on the Earth and the stationary DM halo is also of $O(v)$

Mainly due to Sun velocity in the Galactic frame

$$\vec{v}_\odot \approx \{ \underbrace{(0, 220, 0)}_{\text{direction of disk rotation}} + (\underbrace{10}_{\text{Galactic center}}, 13, \underbrace{7}_{\text{Galactic North pole}}) \} \ \text{km/s}$$

+ annual modulation due to Earth rotation!

max (min) DM wind on June 2nd (Dec 2nd)

today (21st March) max velocity component towards the center of the Galaxy

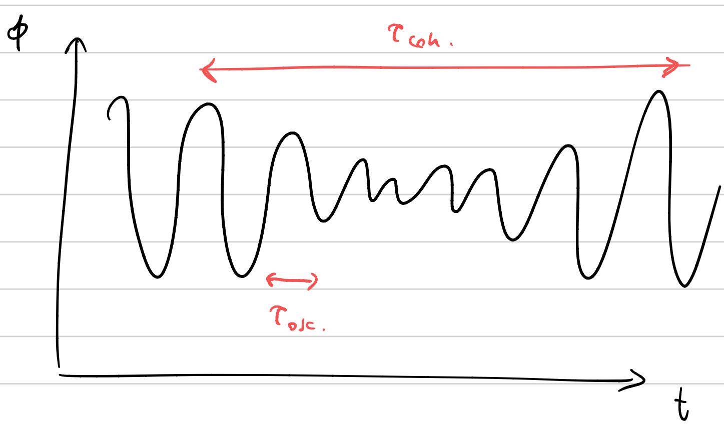
$$\Rightarrow \quad \tau_{coh.} \sim \lambda_{dB}/v \sim \frac{1}{m v^2} \cdot \frac{c^2}{c^2}$$

$v/c \simeq 10^{-3}$

$$\simeq 10^6 \, \tau_{comp} \longrightarrow$$ oscillation frequency of DM field

$$\simeq \underbrace{10^6 \cdot \underbrace{10^{-9}\,\text{sec}}_{\text{GHz}}}_{\text{kHz}} \left(\frac{10^{-6}\,\text{eV}}{m_\phi} \right)$$

time variation of ϕ at a fixed location $\vec{x}$



Take for instance a particle with $m_\phi \sim$ GHz (10^{-6} eV)

$\to$ oscillating field at GHz frequency, which remains coherent for $1/v^2 \sim 10^6$ periods $\to$ 1 msec.

Experimentally interesting: know how to build devices that respond to GHz frequencies and we are able to acquire signals for ~ 1 msec.

instead of trying to detect the (infinitesimal) energy deposited by a single particle, try to detect these oscillating fields, which oscillate at the unknown frequency $1/m_\phi$ with a coherence time $\sim \frac{1}{m_\phi v^2} \sim \frac{10^6}{m_\phi}$.

the fact that the oscillations of the field are coherent for 10^6 periods implies that one can conceive resonant schemes that will boost the DM signal

Possible interactions with feebly-interacting particles:

$$\underbrace{\frac{\phi}{f} F\tilde{F}}_{\text{photon production, currents in circuits}}, \quad \underbrace{\frac{\phi}{f} G\tilde{G}}_{\text{modulation of nuclear decays}}, \quad \frac{\partial_\mu \phi}{f} \bar{\psi}_{SM} \gamma^\mu \gamma_5 \psi_{SM}$$

photon production
currents in circuits

precession of electron and nucleon spins

modulation of nuclear decays

replace $\phi \to \phi_0 \cos(m_\phi t - \overbrace{m_\phi v}^{p} x)$

EOM of electrodynamics → possibility of resonance since DM field oscillates at a frequency m_ϕ with a width $\sim 10^{-6}\, m_\phi$

Main advantage of this kind of DM searches: the signal is narrow and persistent

→ for a persistent signal one can tune away to another frequency and see if the signal is persistent.

This fact distinguishes the search for oscillating ultra-light DM from conventional WIMP searches, since the latter need to combat backgrounds over a wide range of frequencies as their signal is truly DC.