

# Resonant emittance mixing of flat beams in plasma accelerators

S. Diederichs, C. Benedetti, A. Ferran Pousa, A. Sinn, J. Osterhoff, C. B. Schroeder, and M. Thévenet

**Maxence Thévenet** — DESY  
*Theory & simulation for plasma acceleration*  
MPL

PHYSICAL REVIEW LETTERS **133**, 265003 (2024)

## Resonant Emittance Mixing of Flat Beams in Plasma Accelerators

S. Diederichs<sup>1,2</sup>, C. Benedetti<sup>3</sup>, A. Ferran Pousa<sup>1</sup>, A. Sinn<sup>1</sup>, J. Osterhoff<sup>1,3</sup>, C. B. Schroeder<sup>3,4</sup> and M. Thévenet<sup>1,\*</sup>

<sup>1</sup>*Deutsches Elektronen-Synchrotron DESY, Notkestrasse 85, 22607 Hamburg, Germany*

<sup>2</sup>*CERN, Esplanade des Particules 1, 1211 Geneva, Switzerland*

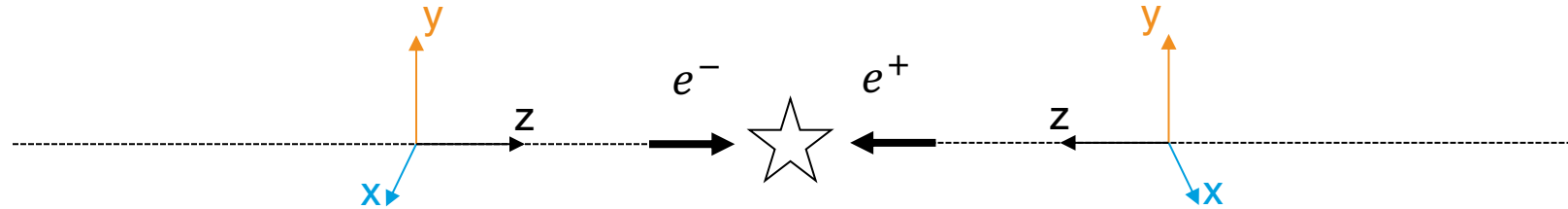
<sup>3</sup>*Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, California 94720, USA*

<sup>4</sup>*Department of Nuclear Engineering, University of California, Berkeley, California 94720, USA*



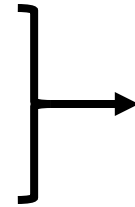
(Received 8 March 2024; revised 30 August 2024; accepted 22 November 2024; published 31 December 2024)

# Flat beams are preferred at the interaction point



Luminosity scales with<sup>2</sup>  $\sim 1/(\sigma_x \sigma_y)$  or  $\sim 1/\sqrt{\epsilon_x \epsilon_y}$

Beamstrahlung scales with<sup>1</sup>  $\sim 1/(\sigma_x + \sigma_y)$



Flat beams with  $\sigma_x \gg \sigma_y$  ( $\epsilon_x \gg \epsilon_y$ ) **minimize beamstrahlung and maximize luminosity**

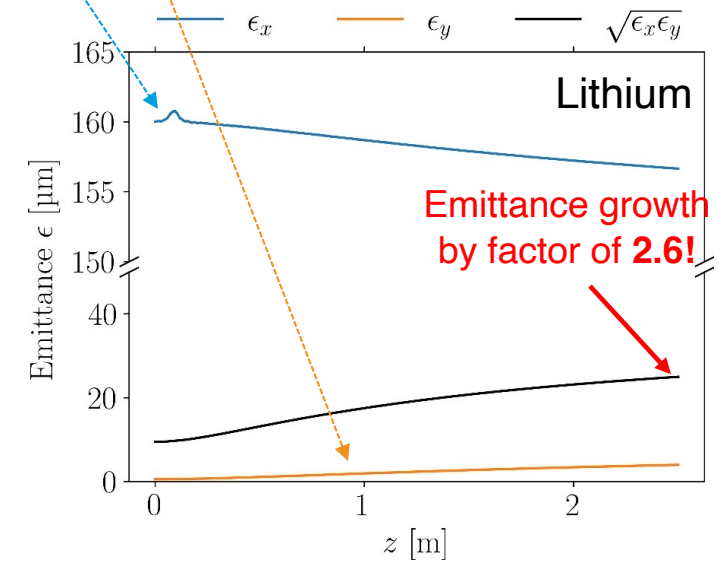
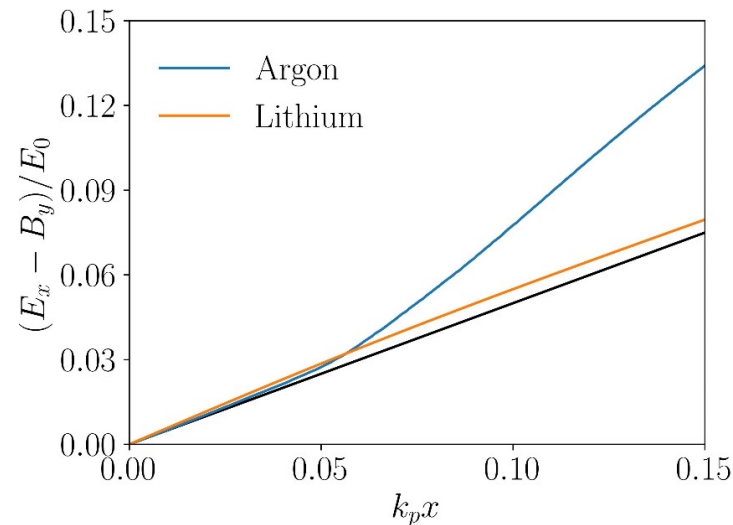
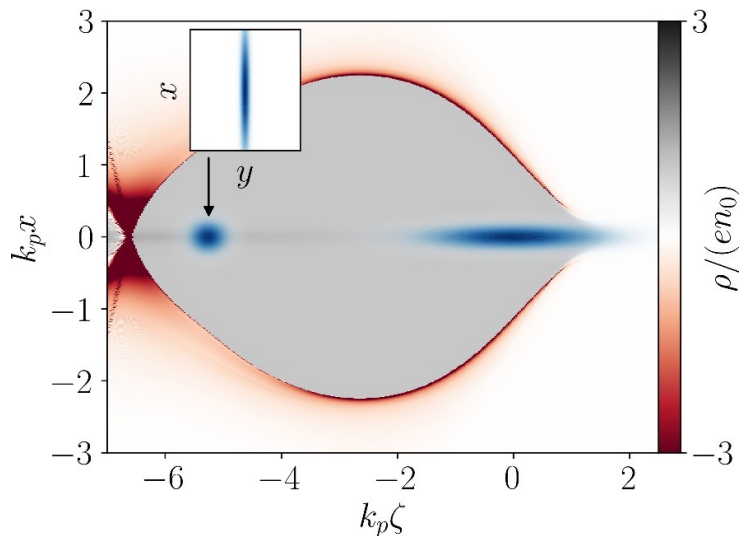
Acceleration of flat **beams not mentioned as a key R&D challenge** in ESPP

- [1] Schroeder et al, JINST 2022
- [2] Schulte, RAST 2016
- [3] Raubenheimer, SLAC PUB 1993



# Realistic plasma stage sees considerable emittance exchange

- **Drive bunch:** Charge: 4.28 nC, Length (rms):  $42 \mu\text{m}$ ,  $\epsilon_{[x,y]} = [60, 60] \mu\text{m}$ .
- **Witness bunch:** Charge: 1.6 nC, Length (rms):  $18 \mu\text{m}$ ,  $\epsilon_{[x,y]} = [160, 0.54] \mu\text{m}$ .
- **Plasma:**  $n_0 = 7 \times 10^{15} \text{ cm}^{-3}$ , Length: 2.5 m, Lithium.



- Mild ion motion or ionization causes considerable emittance exchange



# Mechanism: coupled wakefields give coupled x & y orbits

*Consider the dynamics of a single beam electron in wakefield  $\mathbf{W} = \mathbf{E} + c\mathbf{e}_z \times \mathbf{B}$*

**Ideal blowout regime**  
**Axisymmetric and linear**

$$W_r = E_0 \frac{k_p r}{2}$$

$$W_x = E_0 \frac{k_p x}{2}$$

$$W_y = E_0 \frac{k_p y}{2}$$

**→ x and y orbits fully decoupled**



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**Example**  
**Axisymmetric non-linear wakefields**

$$W_r = E_0 \frac{k_p r}{2} + \alpha r^2$$

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- Ion motion/ionization/... caused by an axisymmetric drive beam
  - Guiding channel for LPA
- *x and y orbits are coupled*

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**General**  
**non-axisymmetric, non-linear fields**

$$W_x = f(x, y)$$

$$W_y = g(x, y)$$

- Ion motion/ionization/... caused by a flat witness beam
  - Laser misalignment in guiding channel for LPA
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$$W_{[x,y]} = \frac{k_p[x,y]E_0}{2} \left[ 1 + \alpha_{[x,y]} H\left(\frac{r^2}{2L_{[x,y]}^2}\right) \right]$$

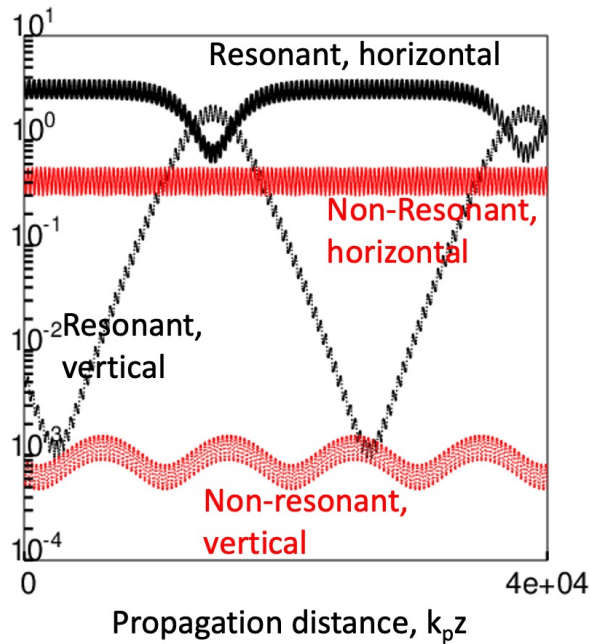
$$H(q) = [1 - \exp(-q)]$$

C. Benedetti *et al.*, PRAB 20, 111301 (2017)



# A fraction of beam particles are trapped in a resonance

Single particle betatron amplitudes



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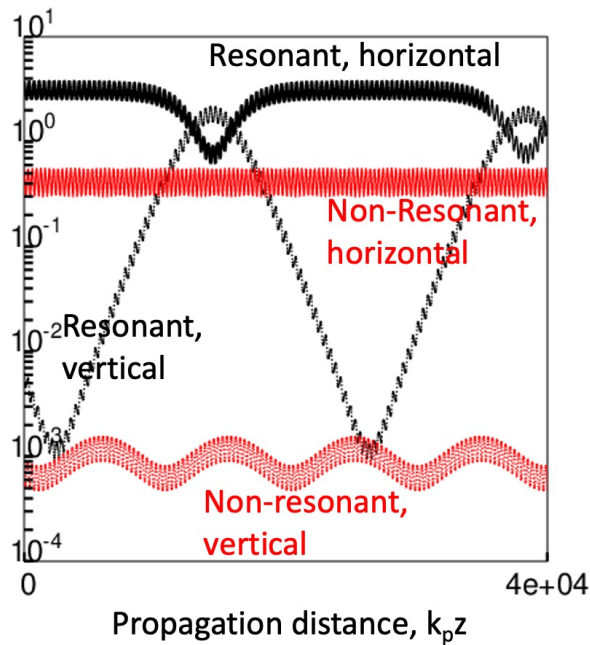
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C. Benedetti *et al.*, PRAB 20, 111301 (2017)

- Electron has different  $k_{\beta x}$  and  $k_{\beta y}$  depending on initial conditions.
- The resonance is located on the diagonal  $k_{\beta x} = k_{\beta y}$ .
- Electrons in a specific area are trapped in the resonance.
- Resonant electrons see periodic exchange in x and y orbits.
- They are responsible for the emittance exchange.

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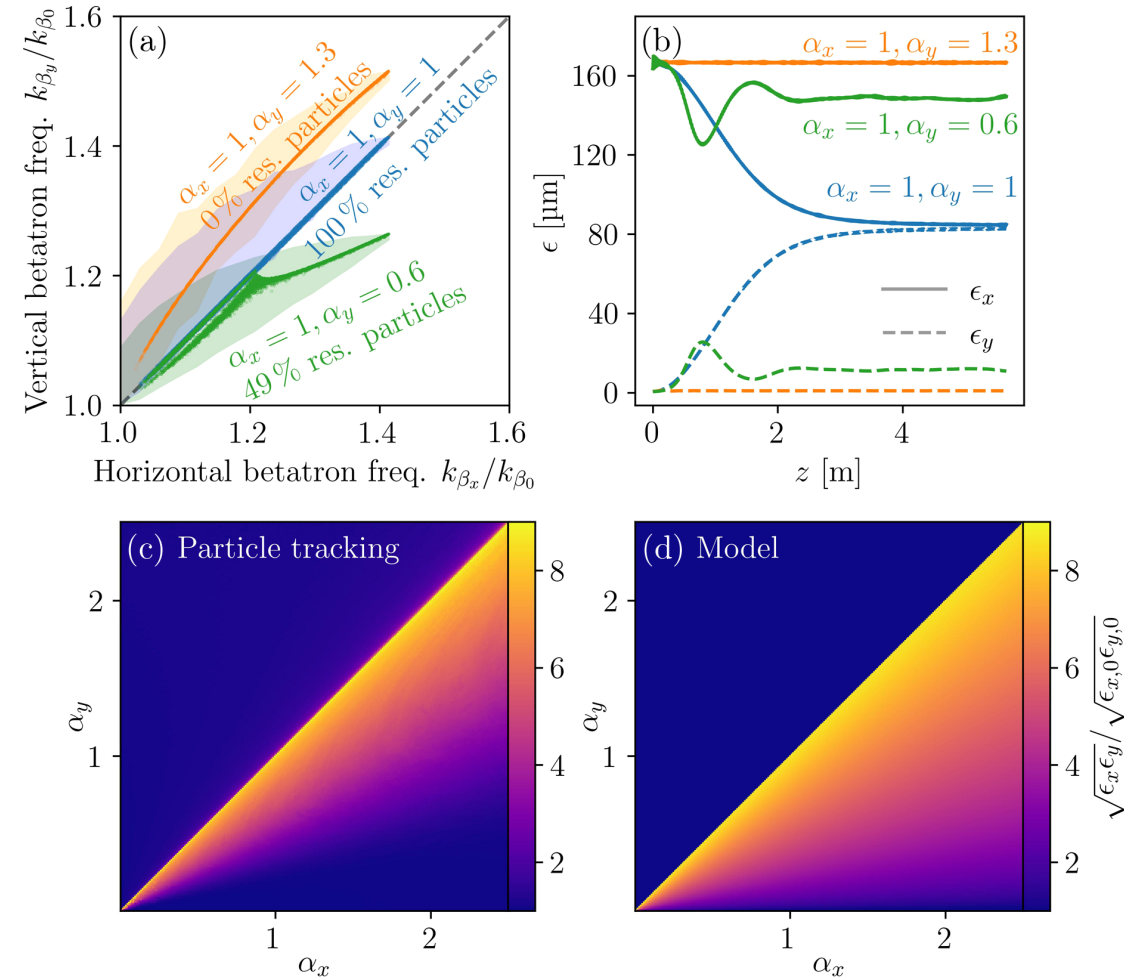


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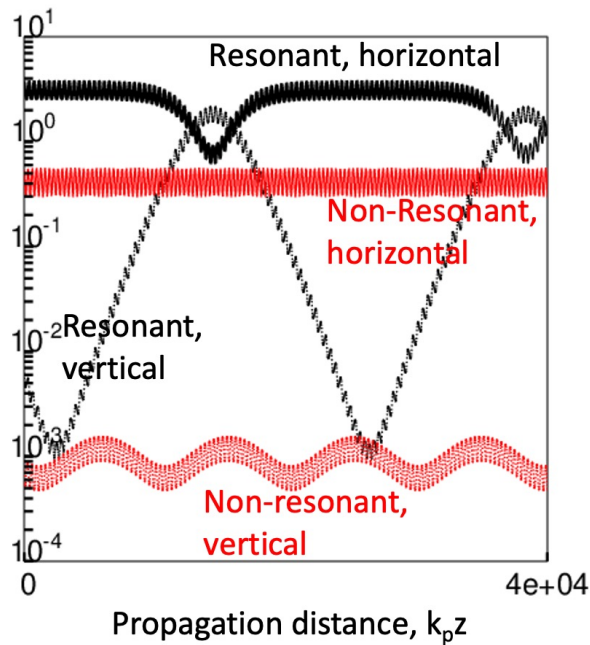
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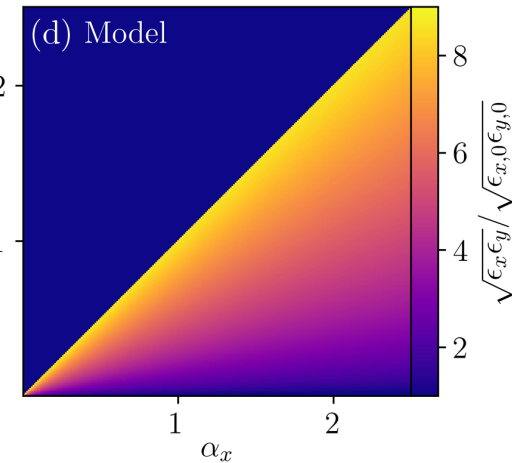
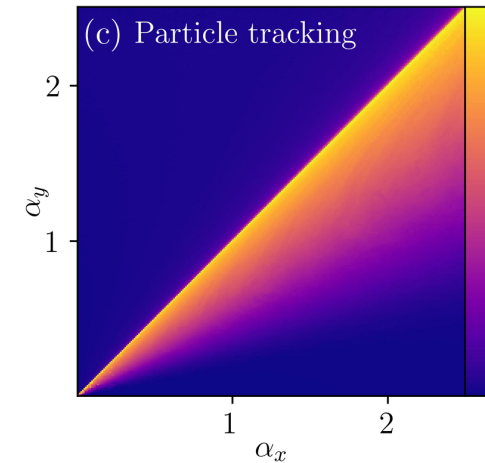
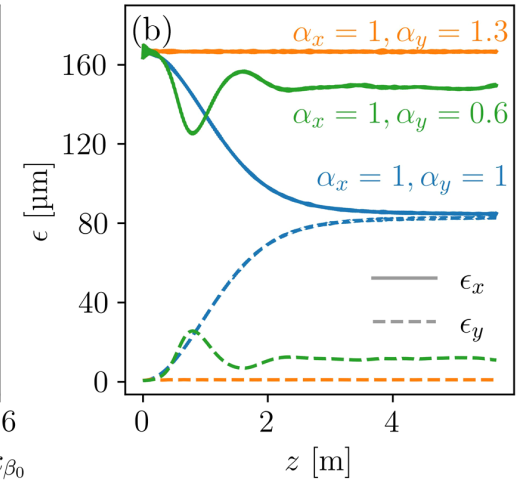
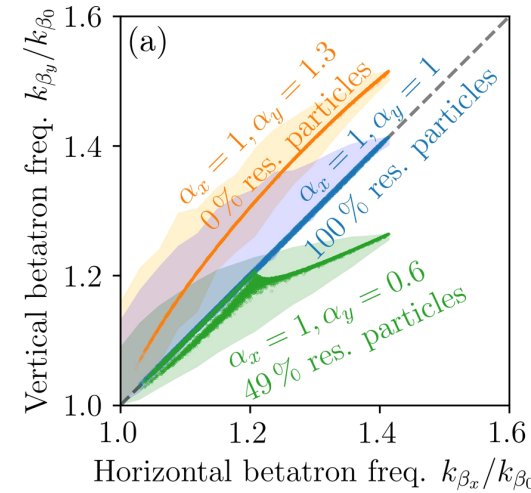


$$W_{[x,y]} = \frac{k_p[x,y]E_0}{2} \left[ 1 + \alpha_{[x,y]} H\left(\frac{r^2}{2L_{[x,y]}^2}\right) \right]$$

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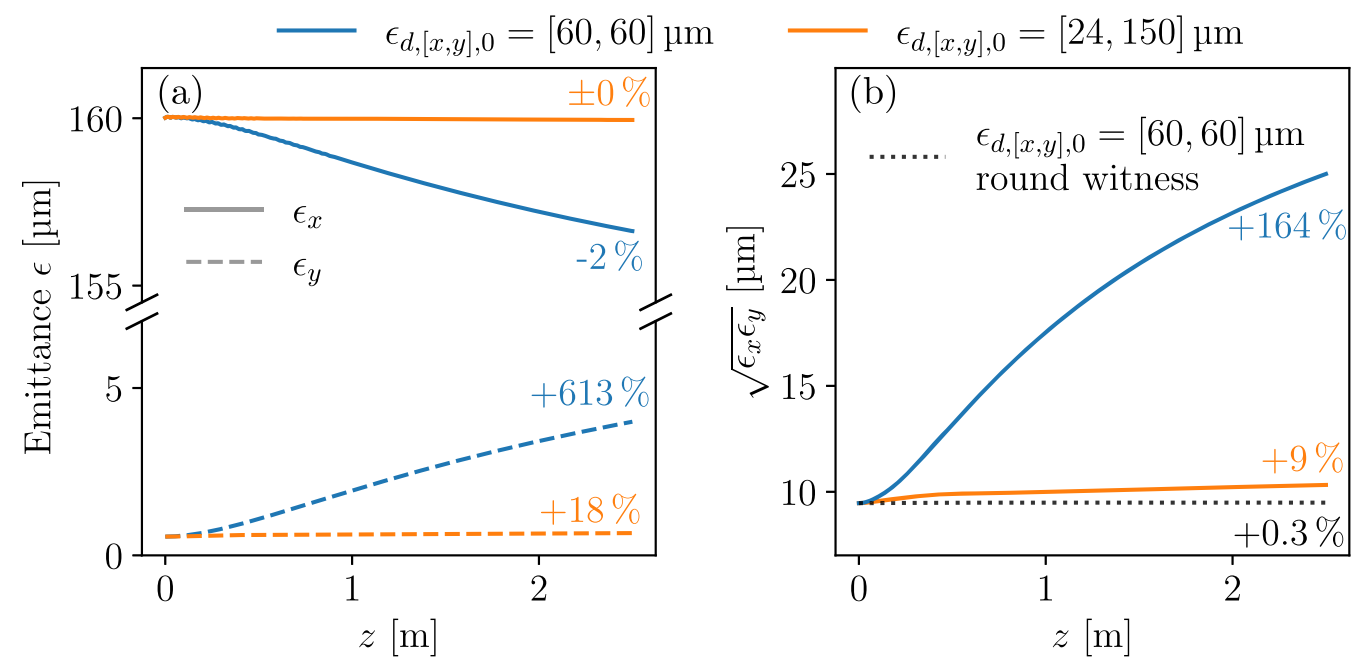
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- Well-known (complex) phenomenon in RF accelerators
- Axisymmetric + non-linear is the worst case  
all particles are resonant → **full emittance exchange**



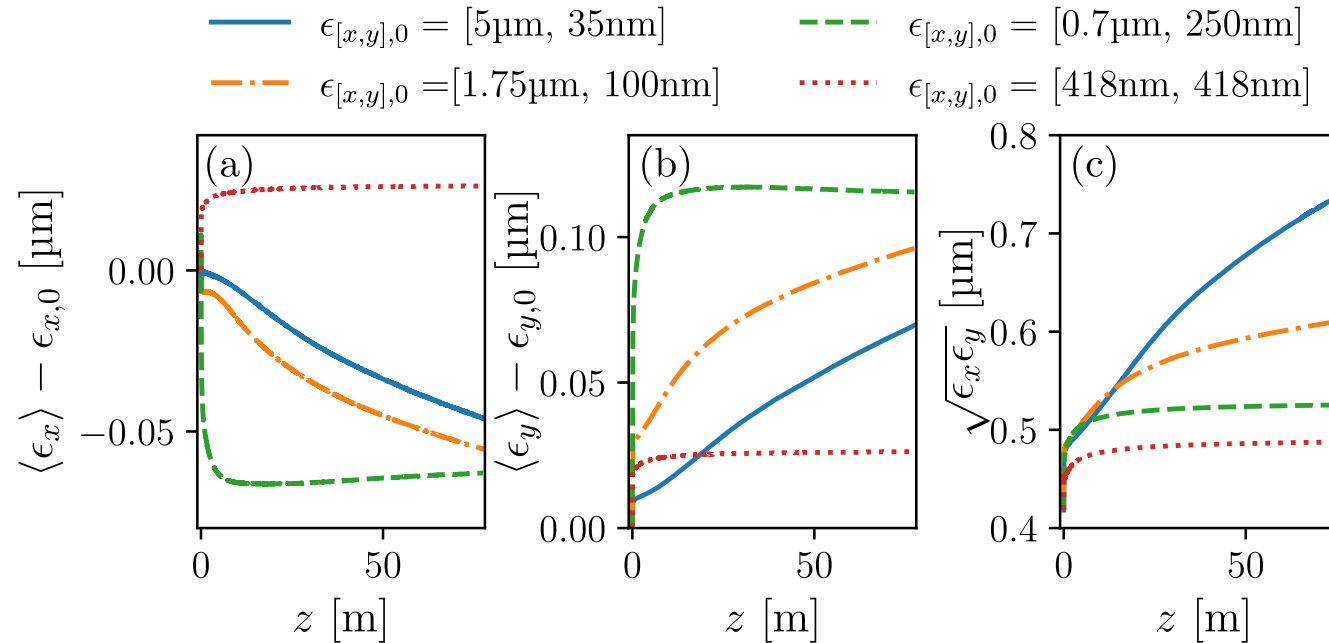
# Ion motion from the driver can be solved with a flat driver



Driver: 4.28 nC, 42  $\mu\text{m}$ ,  $\epsilon_{[x,y]} = [60, 60] \mu\text{m}, [24, 150] \mu\text{m}$   
Witness: 1.6 nC, 18  $\mu\text{m}$ ,  $\epsilon_{[x,y]} = [160, 0.54] \mu\text{m}$   
Plasma: Sodium 1+,  $n_0 = 7 \times 10^{15} \text{ cm}^{-3}$ , 2.5 + 5\*5 m.

→ A flat driver breaks the resonance and mitigates emittance exchange

# Ion motion from witness beam is not (yet) solved!



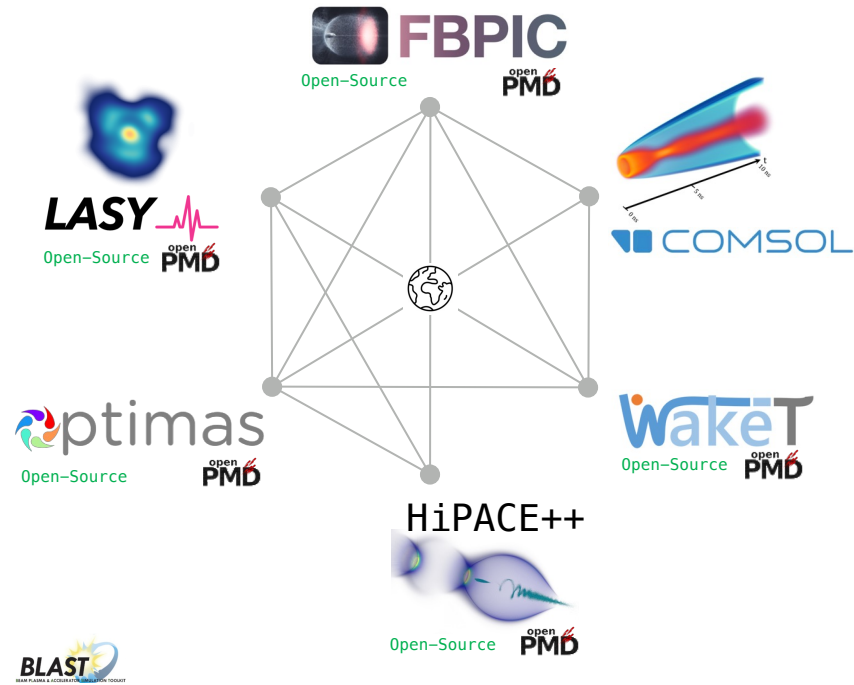
Driver: 4.28 nC, 42  $\mu\text{m}$ ,  $\epsilon_{[x,y]} = [2.8, 2.8]$  mm, rigid

Witness: 1.6 nC, 18  $\mu\text{m}$ ,  $\epsilon_{[x,y]} = [5, 0.035]$   $\mu\text{m}$

Plasma: Hydrogen,  $n_0 = 7 \times 10^{15} \text{ cm}^{-3}$ , length 77.5 m

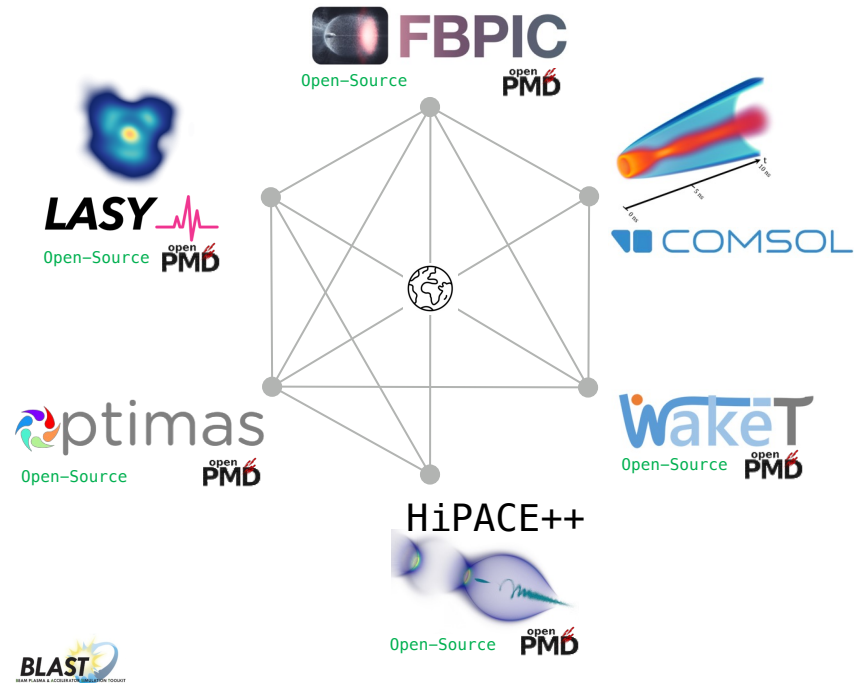
→ A flatter beam causes larger emittance exchange

# Studies made possible thanks to recent progress in simulations





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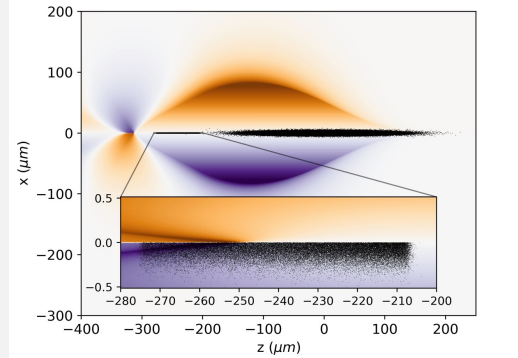


## ➤ HiPACE++

*Optimized for large 3D simulations*



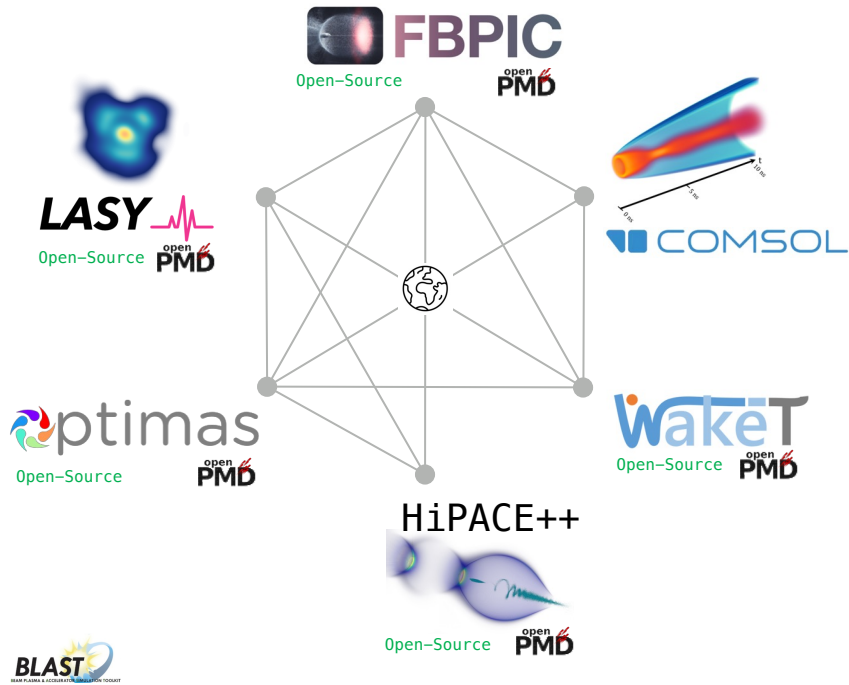
...



- 3D quasistatic particle-in-cell
- Multi-physics
- C++, on top supercomputers
- Mesh refinement

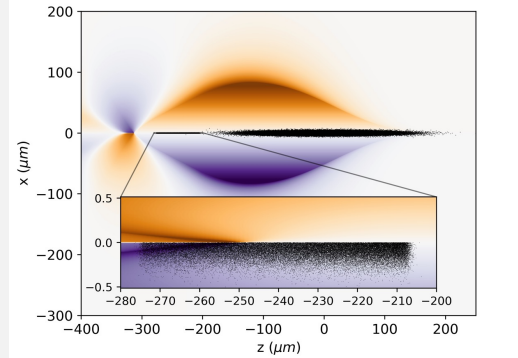
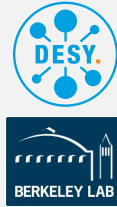
Open-source <https://github.com/Hi-PACE/hipace>

# Studies made possible thanks to recent progress in simulations



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## ➤ Wake-T

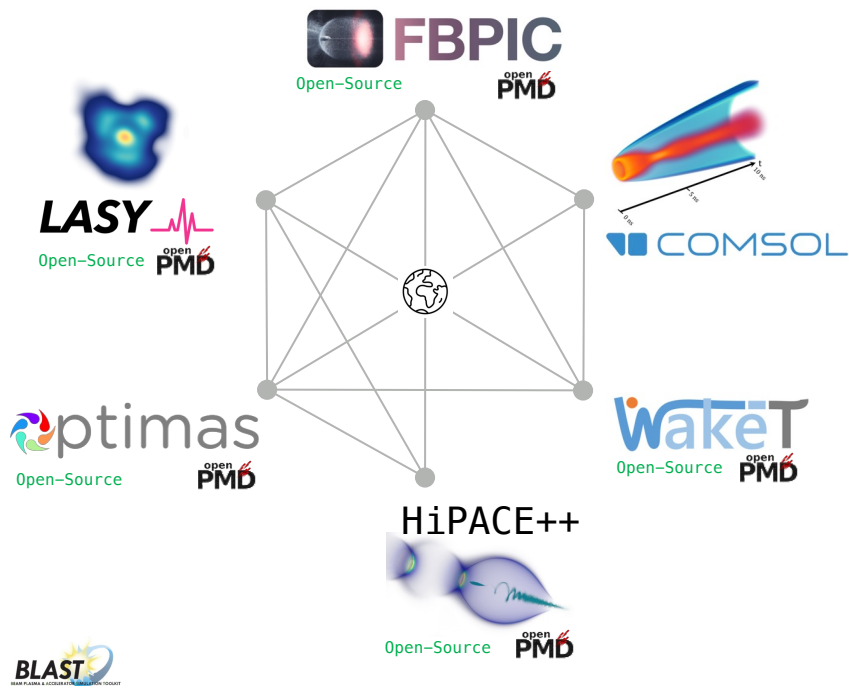
*Quick simulations (benchmarked vs. FBPIC)*



- 2D (axisymmetric) quasistatic
- Laser-driven or beam-driven
- Python, second/minutes on a laptop
- Adaptive grid & ion motion

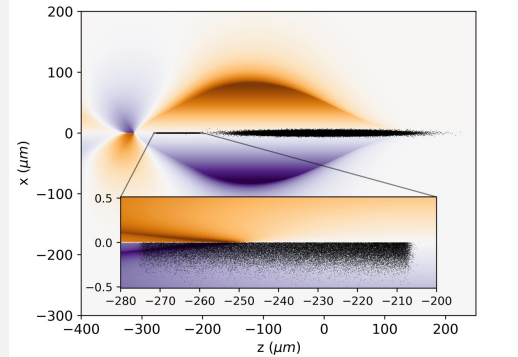
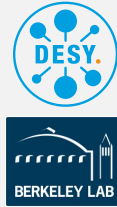
Open-source <https://github.com/AngelFP/Wake-T>

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Optimized for large 3D simulations



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Open-source <https://github.com/AngelFP/Wake-T>

- With mesh refinement, 3D simulations of a 20 GeV stage from 175 GeV, emittance 135 nm, are very affordable  
Numerical convergence with transverse resolution of **5 nanometers** to fully resolve ion motion effects
- Multi-stage simulation studies done routinely  
A. Ferran Pousa et al. Proc. IPAC'23 14: 1533-1536. ; S. Diederichs et al. arXiv:2403.05871 (2024).
- A collection of tools enables start-to-end multi-physics studies  
COMSOL-plasma for hydrodynamics simulations of HOFI, Optimas for scalable Bayesian optimization, LASY for laser manipulations

→ don't worry about simulations (anymore)



# The DESY plasma group MPL



**Andi Maier**  
group leader

**Wim Leemans**  
division director



**Paul Winkler**  
Coordinator  
Plasma Injector



**Manuel Kirchen**  
Team Leader  
High Average  
Power LPA



**Guido Palmer**  
Team Leader  
Laser  
Development



**Rob Shalloo**  
Team Leader  
High Energy LPA



**Maxence Thévenet**  
Team Leader  
Theory & Simulations



**Jon Wood**  
Team Leader  
Beam-Driven  
Plasma Acceleration



**Kris Pöder**  
Team Leader  
LPA Applications



**Lutz Winkelmann**  
Team Leader  
Scientific  
Engineering



**Andi Walker**  
Coordinator  
Scientific  
Infrastructure



# Conclusion

- Accelerating a flat beam in a plasma accelerator poses significant challenges.
- Emittance is transferred from the large to the small direction, degrading quality.
- This is caused by particles falling in a resonance.
- Avoiding the resonance might mitigate this effect.

Thank you for your attention