The Extreme Light Infrastructure

EAAC 2025

Single-mode guiding of intense laser pulses

Zsolt Lécz

Research Fellow, Particle Acceleration group

25.09.2025

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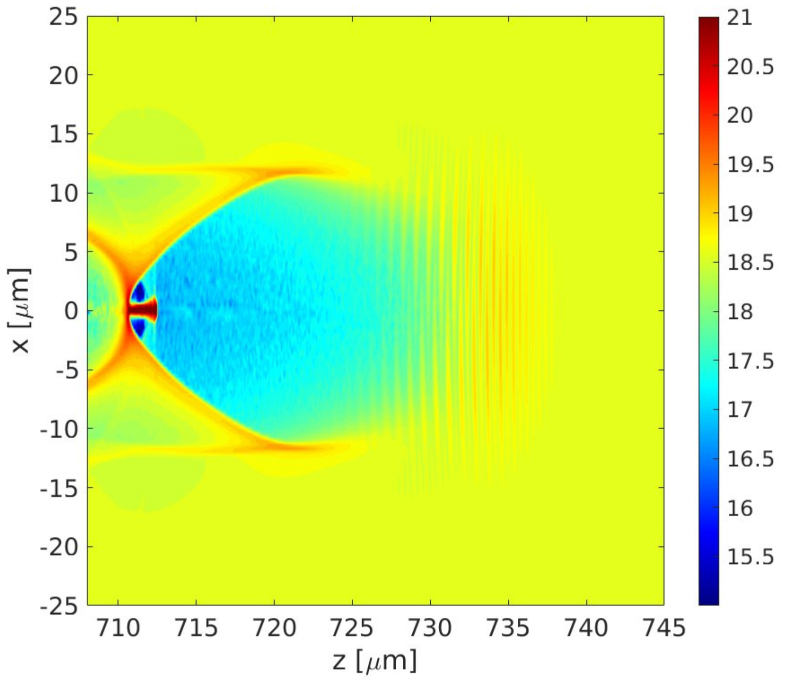
Scaling laws in LWFA



$$\lambda_p = \frac{c}{\omega_p} = c\sqrt{\frac{m_e \epsilon_0}{e^2 n_0}}$$

$$n_c = m_e \epsilon_0 \frac{\omega_0{}^2}{e^2}$$

Energy depletion length: $\frac{L_d}{\lambda_p} \propto \frac{n_c}{n_0}$

n_0 - plasma density

Rayleigh (diffraction) length:

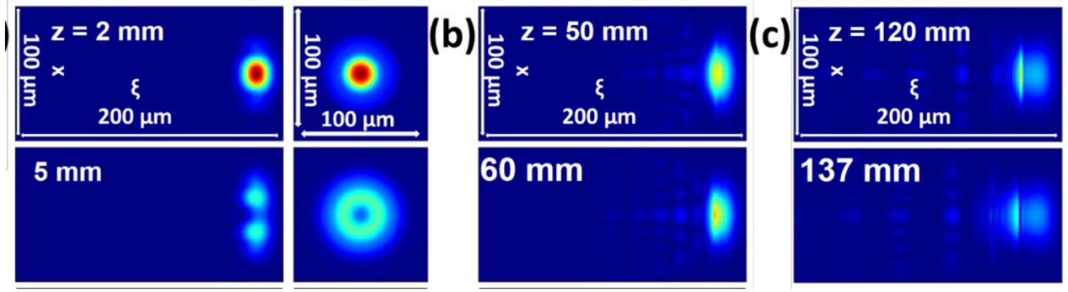
$$\frac{L_R}{\lambda_p} = \pi \left(\frac{w_0}{\lambda_p}\right)^2 \left(\frac{n_c}{n_0}\right)^{1/2}$$

In low density plasma ($n_o \ll n_c$) the depletion length is much longer than the Rayleigh length!

Guiding is needed!

Challenges

Mode dispersion

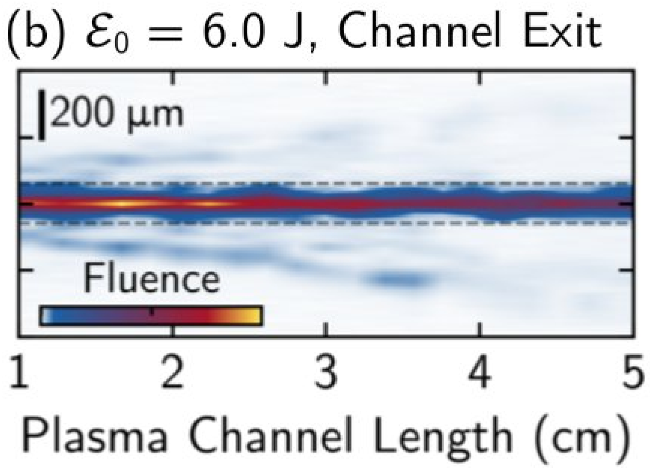


J. E. Shrock, et. al, Phys. Rev. Lett. **133**, 045002 (2024)

Each mode has different wave number (normalized to k_0):

$$\beta_{pm}^2 = 1 - \frac{4(2m+p+1)}{k^2 w^2} - \frac{n_0}{n_c}.$$

Energy leakage



A. Picksley et al., Phys. Rev. Lett. **133**, 255001 (2024)

High order modes have larger transverse wave number, they can propagate out radially.

Transverse evolution of a laser beam

Starting point:

$$\frac{\partial^2 A}{\partial t^2} - c^2 \nabla^2 A = \omega_p{}^2 \frac{n}{\gamma} A$$

$$A = a \exp[-i(\omega_0 t - k_0 z)]$$

$$2i \frac{n_c}{n_0} \frac{\partial a}{\partial \zeta} = -\nabla_\perp{}^2 a + \frac{n_e}{n_0 \gamma} a$$

$$\gamma = \sqrt{1 + a^2/2}$$

$$\zeta = k_0 z \qquad a = \frac{eA}{m_e c} \qquad n_e = n_0 + \delta n$$

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Applying the SDE method*.

$$\frac{\partial^2 w}{\partial z^2} = \frac{4}{k^2 w^3}(1 - K)$$

For Gaussian laser envelope!

*P. Sprangle et al., Phys Rev A 36, 2773 (1987)

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For Gaussian laser envelope!

$K \approx 1$ if $w_0 \approx \frac{\lambda_p}{\pi}\sqrt{a_0}$ and $a_0 \gg 1$

<u>Relativistic self-guiding</u> condition!
The laser spot size remains constant!

*P. Sprangle et al., Phys Rev A 36, 2773 (1987)

Why not self-guide?

1. The laser beam does not remain Gaussian
2. In the self-guiding model the density modulation is neglected!
3. The laser pulse has a temporal envelope, which changes in time:
   Self-phase modulation!

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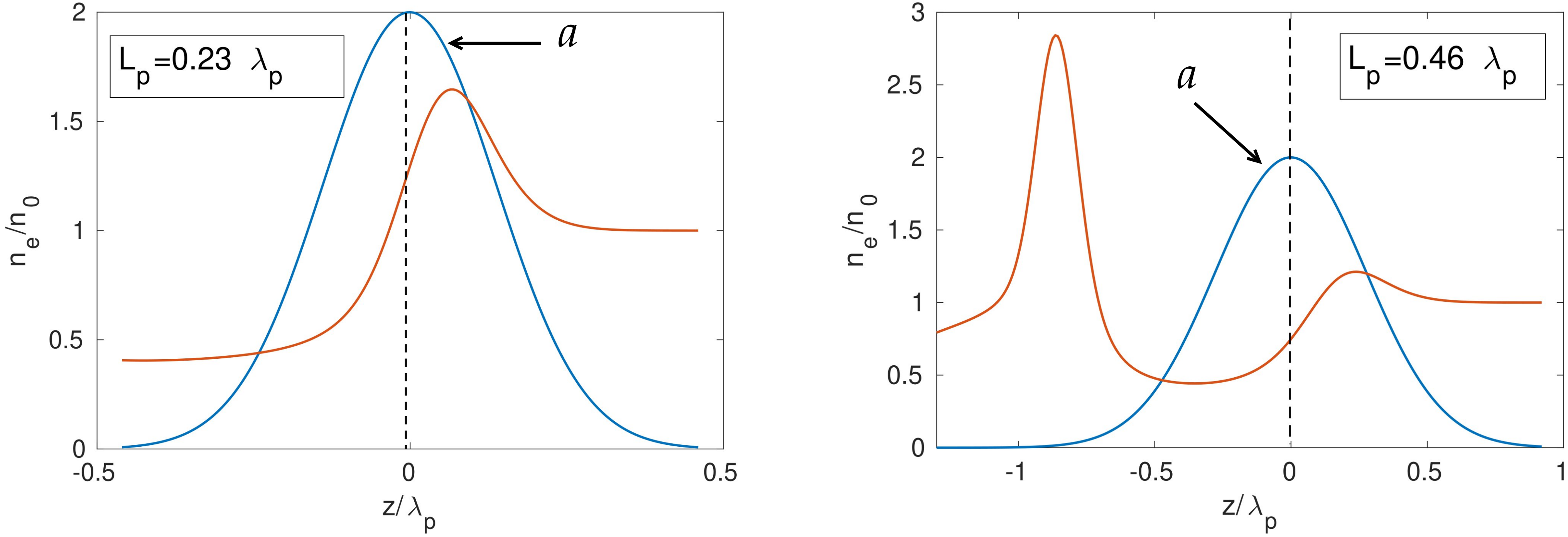
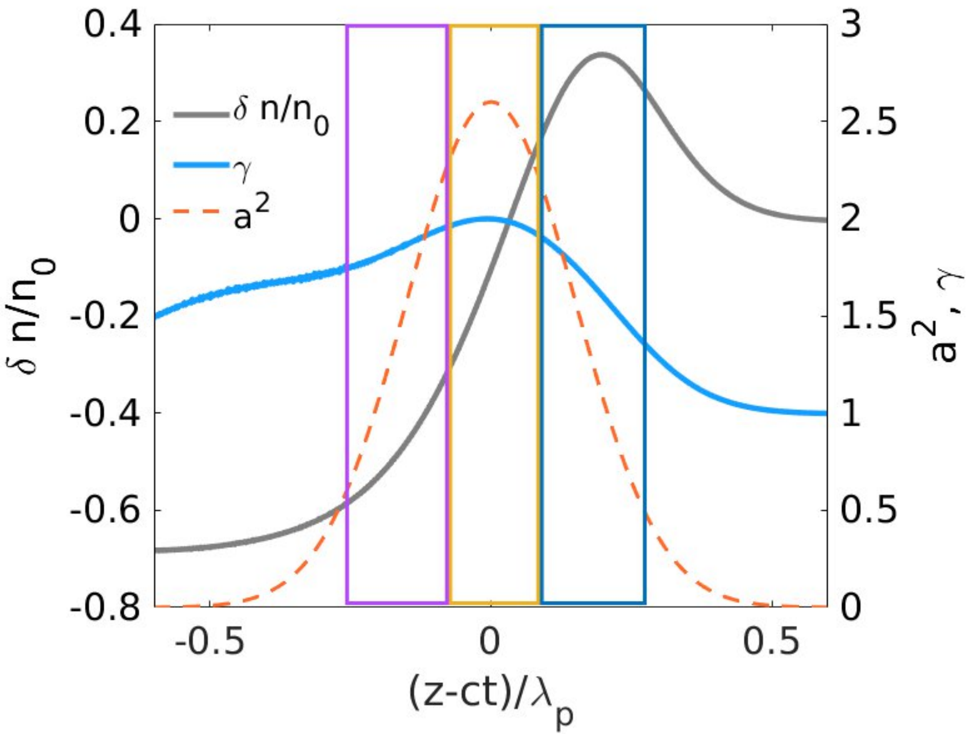


Non-uniform guiding

https://arxiv.org/abs/2412.14785v2



The front side and back sides of the laser pulse have different guiding conditions!

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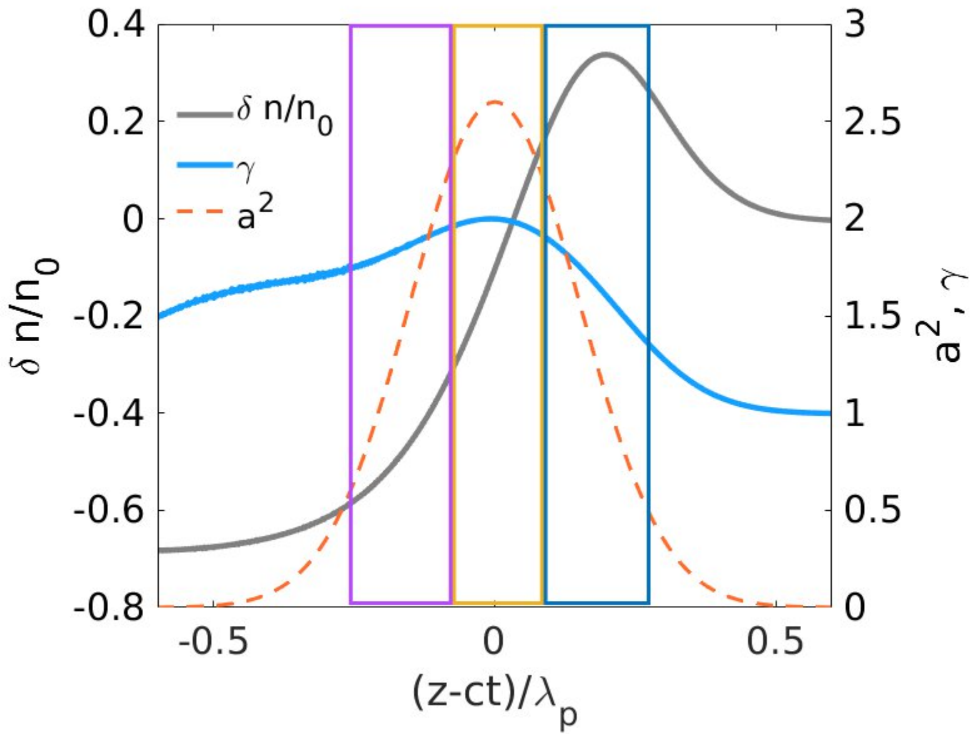
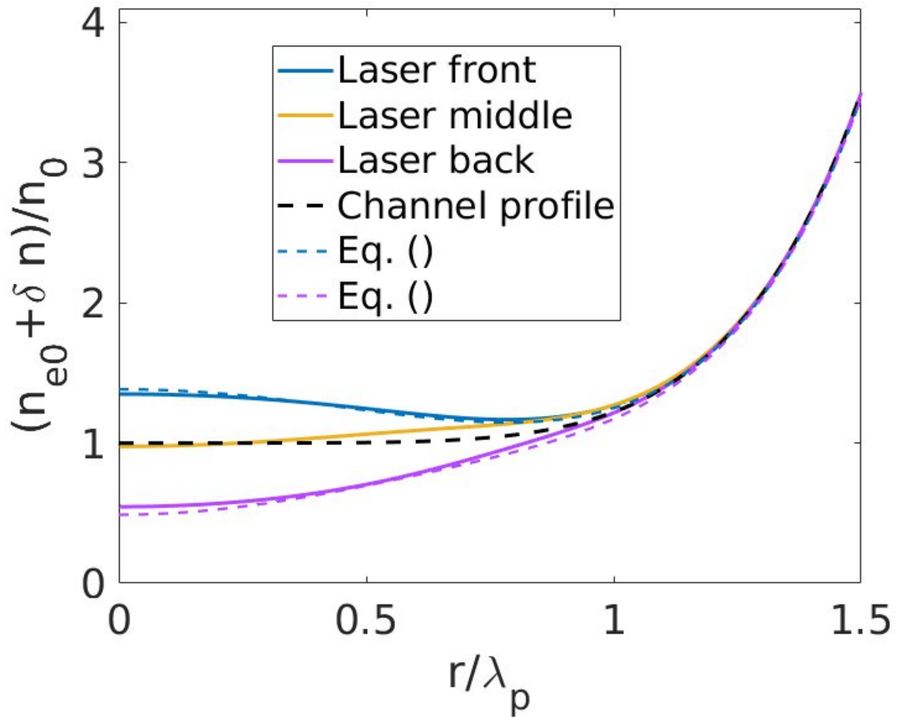


Radial density profiles within the pulse:



The front side and back sides of the laser pulse have different guiding conditions!

$$\delta n \approx -g(\xi)\frac{4a_0^2\exp(-2s^2)}{k_p^2 w_0^2\sqrt{1+(a_0^2/2)\exp(-2s^2)}}, \quad s = r/w_0.$$

Mode-preserving guiding

$$n_{e0} \approx n_0 \left(1 + \epsilon \frac{r^b}{r_c^b} \right), b > 2$$

$$n_e = F n_0 \approx n_0 \left(1 + \epsilon \frac{r^b}{r_c^b} + \delta n \right)$$

$$2i \frac{n_c}{n_0} \frac{\partial a_\xi}{\partial \tau} = -\nabla_\perp{}^2 a_\xi + \frac{n_e}{n_0 \gamma} a_\xi$$

$$\chi = \frac{F(r)}{\gamma}$$

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For single-mode guiding (mode matching) the **<u>susceptibility</u>** has to be a **<u>parabolic</u>** function of r:

$$\frac{F}{\gamma} \approx \frac{1+\epsilon\frac{r^b}{r_c^b}+\delta n}{\gamma\,(r)} = \frac{1}{\gamma_0} + \kappa\frac{4r^2}{k_p{}^2 w_0{}^4}$$

κ - focusing strength of the channel

When $\kappa = 1$ and $b = 2$, $\gamma \approx 1$ the „traditional“ perfect guiding is achieved, were the spot size remains constant.

Realistic plasma channel

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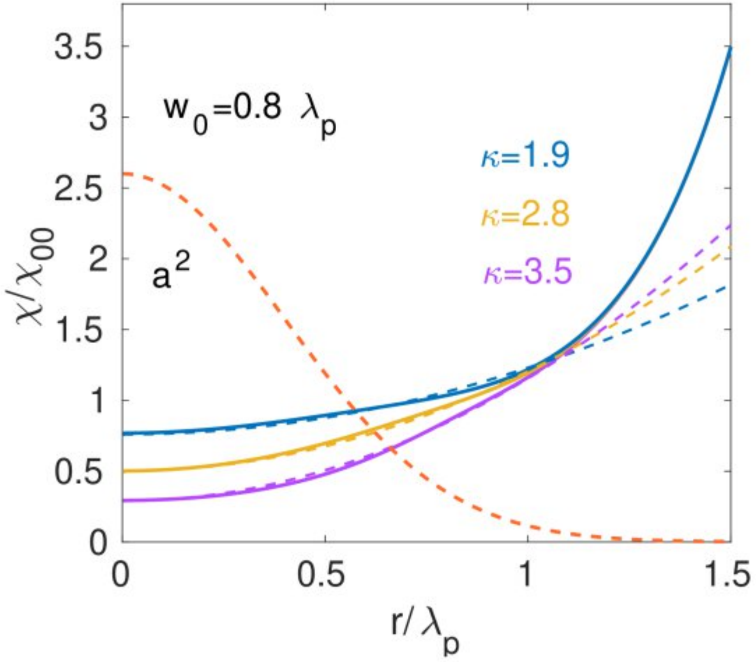
It is a curve fitting problem!



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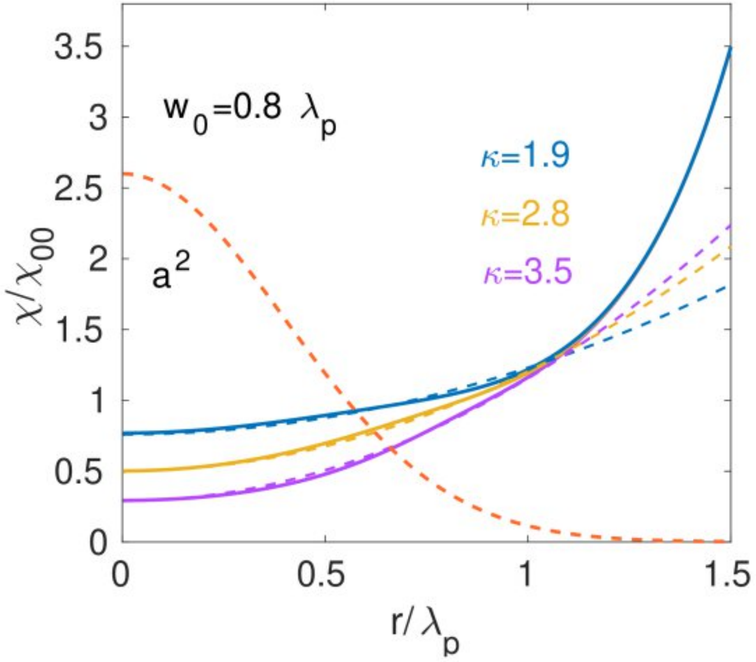

These two curves can (almost) overlap in a wide channel ($r_c > \lambda_p$)!

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For $\delta n = 0$, in the middle of the pulse:

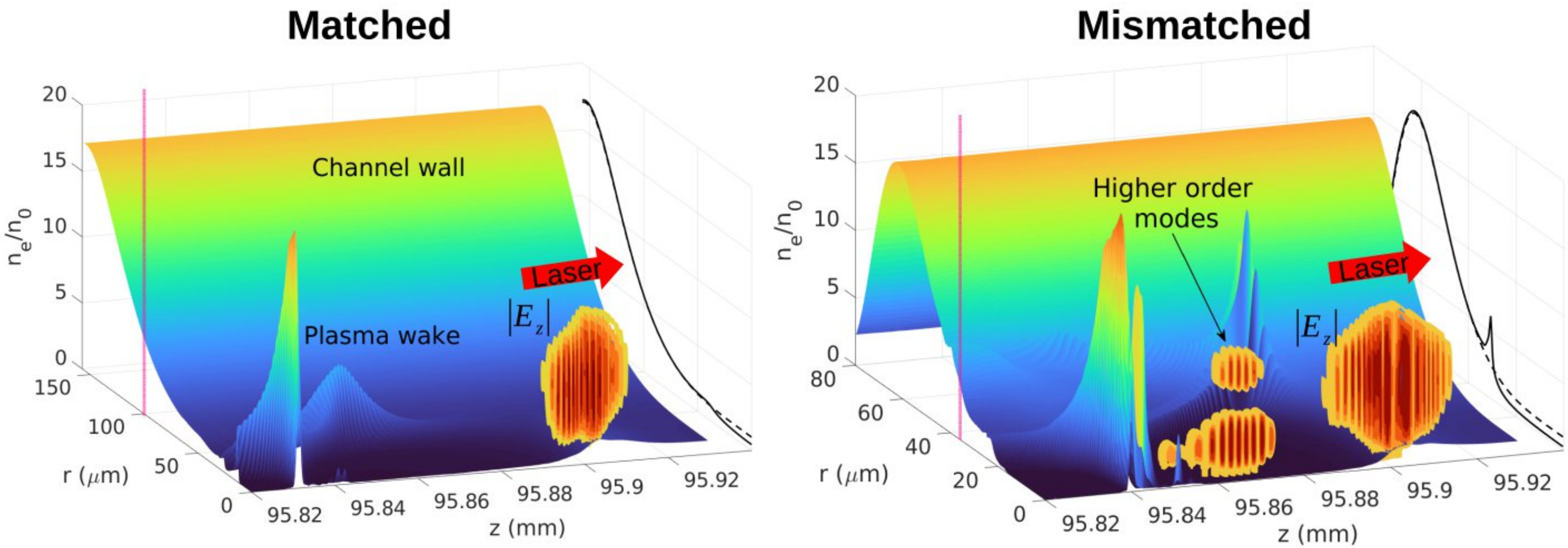
$$k_p^2 w_0^2 = \frac{4\gamma_0 {s^*}^2 \kappa (1 - 2/b)}{\gamma_0/\gamma^* - 1},$$

$$r_c = r^* \left(\frac{8\kappa\gamma^* {s^*}^2}{b\epsilon k_p^2 w_0^2} \right)^{-1/b}$$

$$\gamma_0 = \sqrt{1 + {a_0}^2/2}, \quad s^* = \frac{r^*}{w_0} \approx 1.5$$

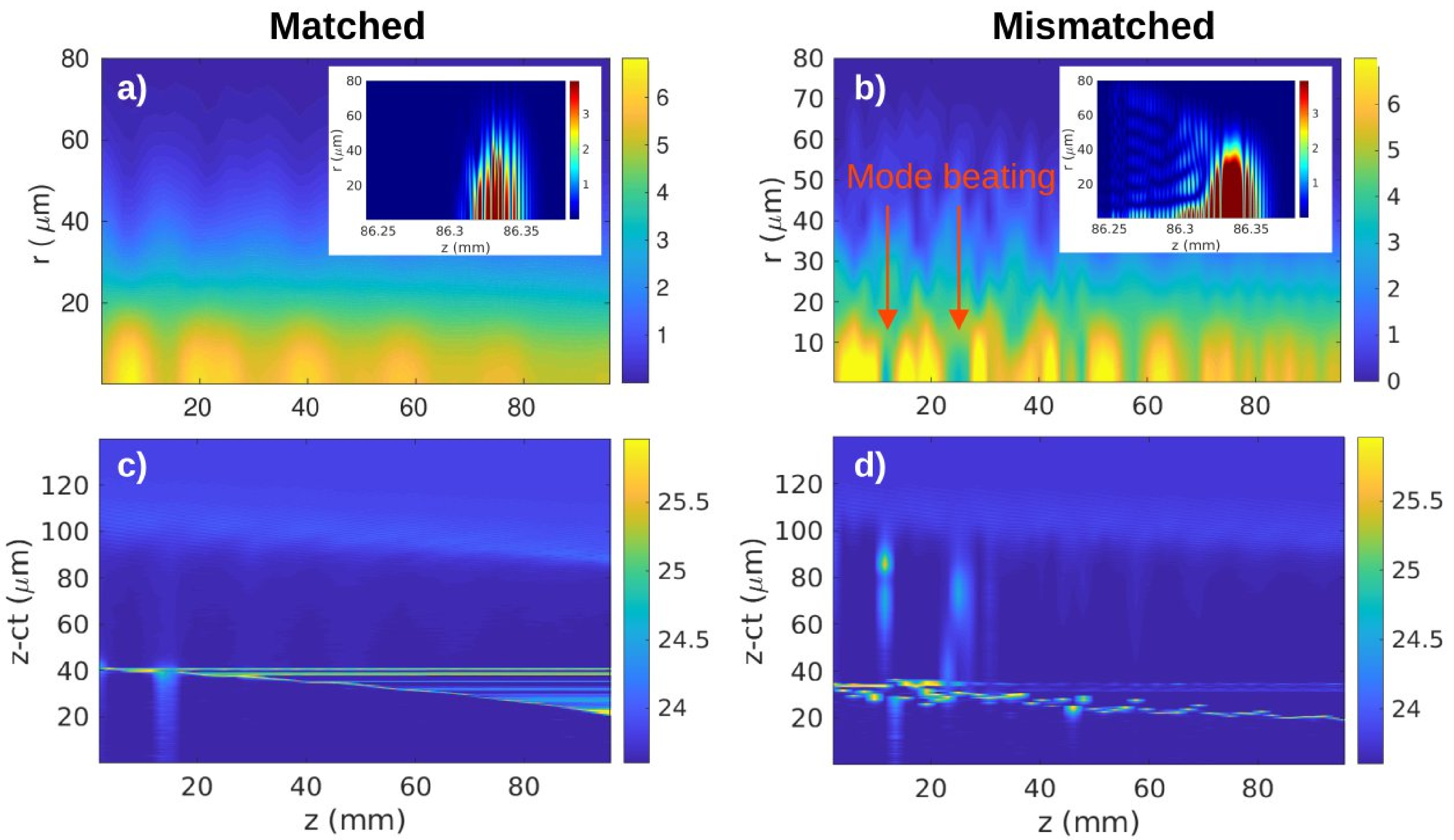
https://arxiv.org/abs/2412.14785v2

Particle-in-cell simulations (FBPIC)



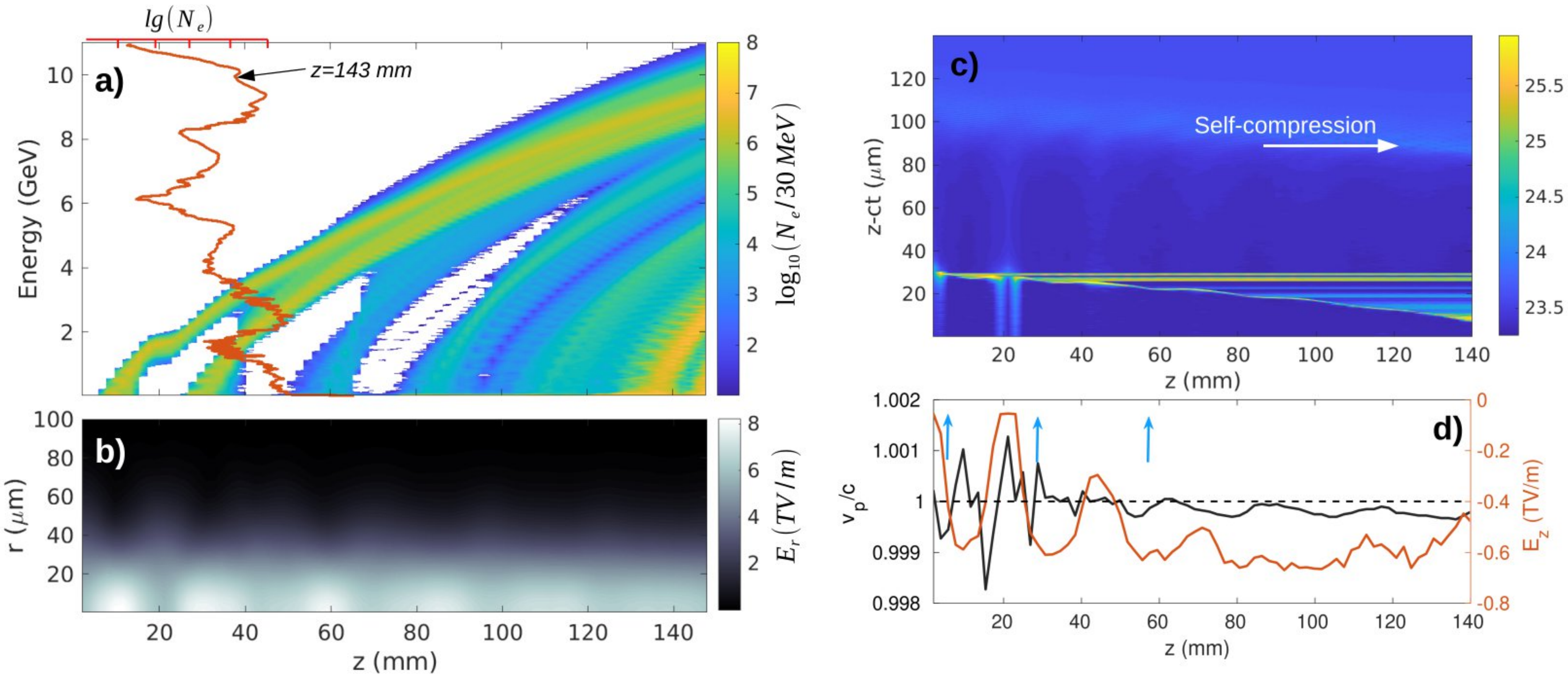
Very brief condition for mode matching: $r_c > \lambda_p > w_0$ and $a_0 > 2.$

Particle-in-cell simulations (FBPIC)



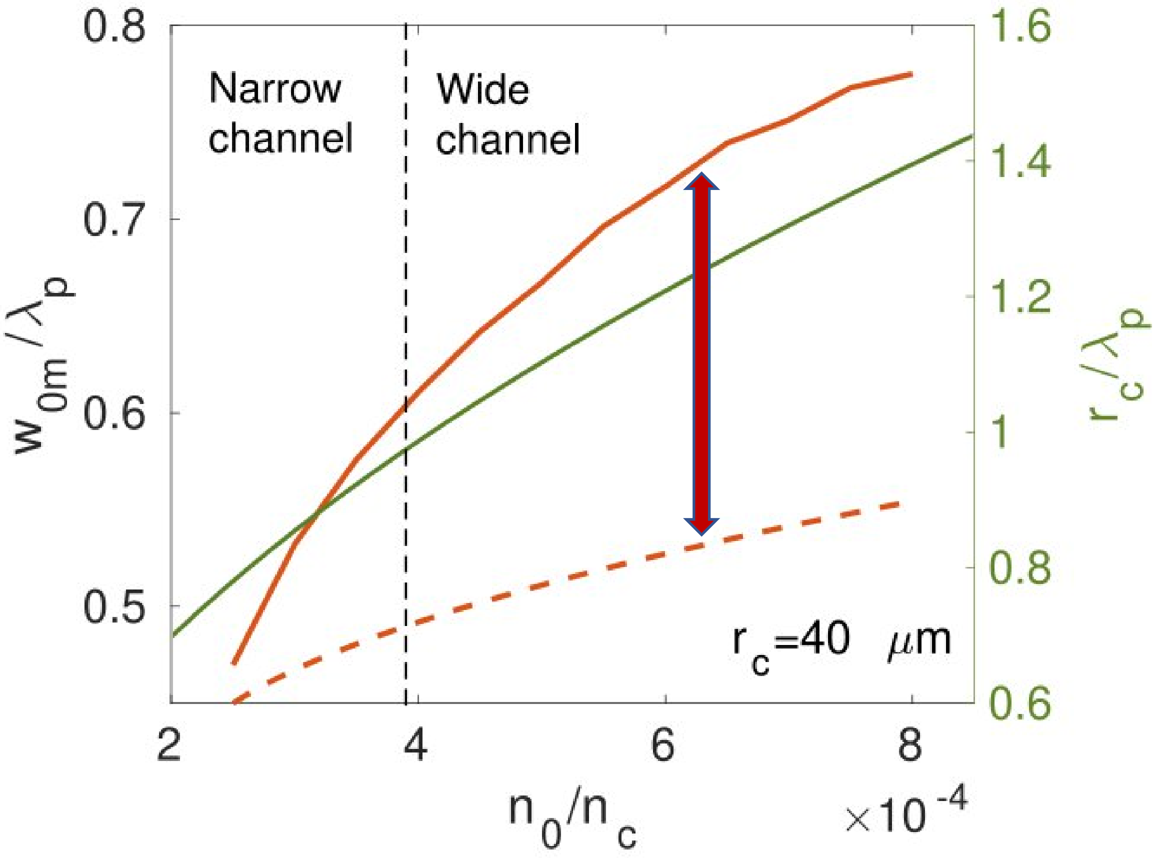
Mode dispersion is clearly visible!

Periodic self-injection of electrons



1D wave-breaking field: $E_{wb,rel} = E_{p,0}\sqrt{2(\gamma_p - 1)}.$

Mode evolution



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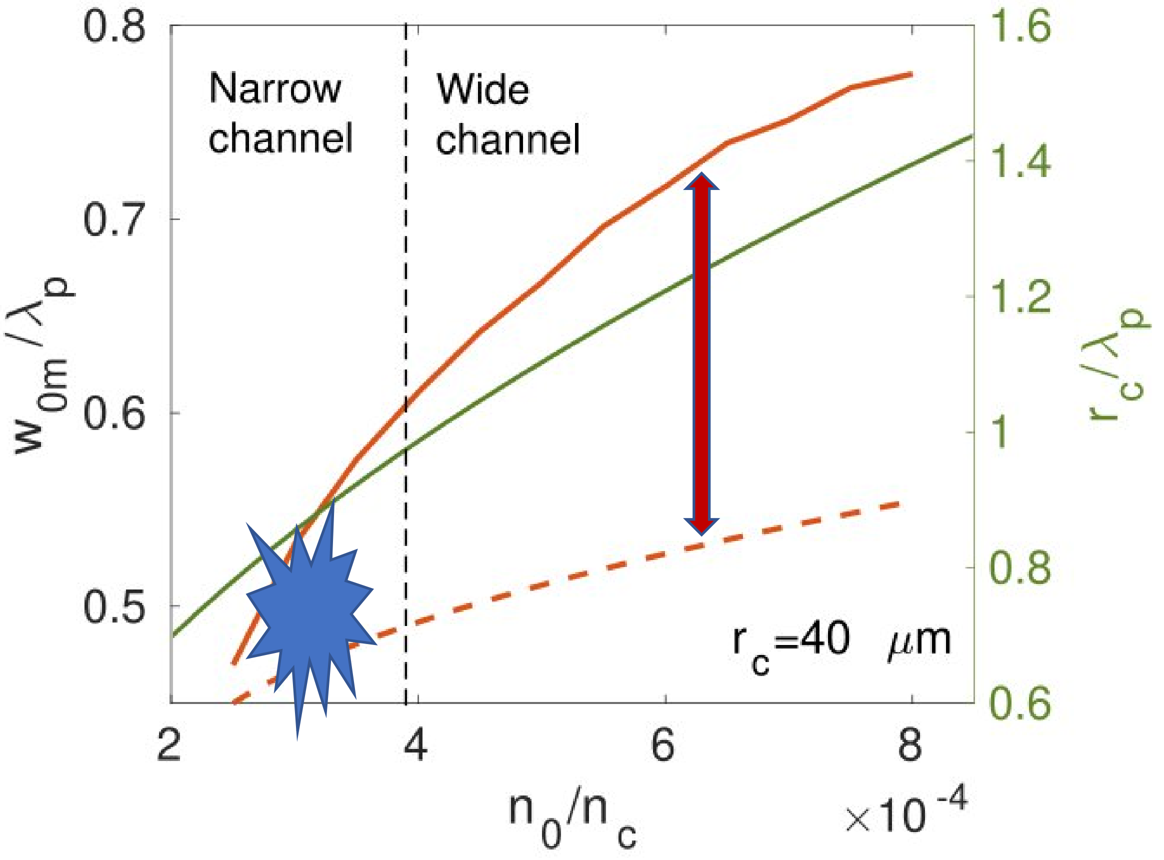

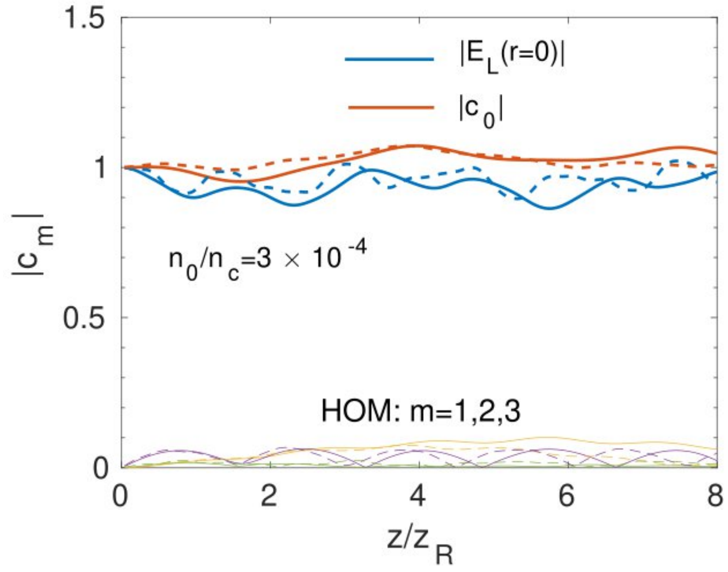

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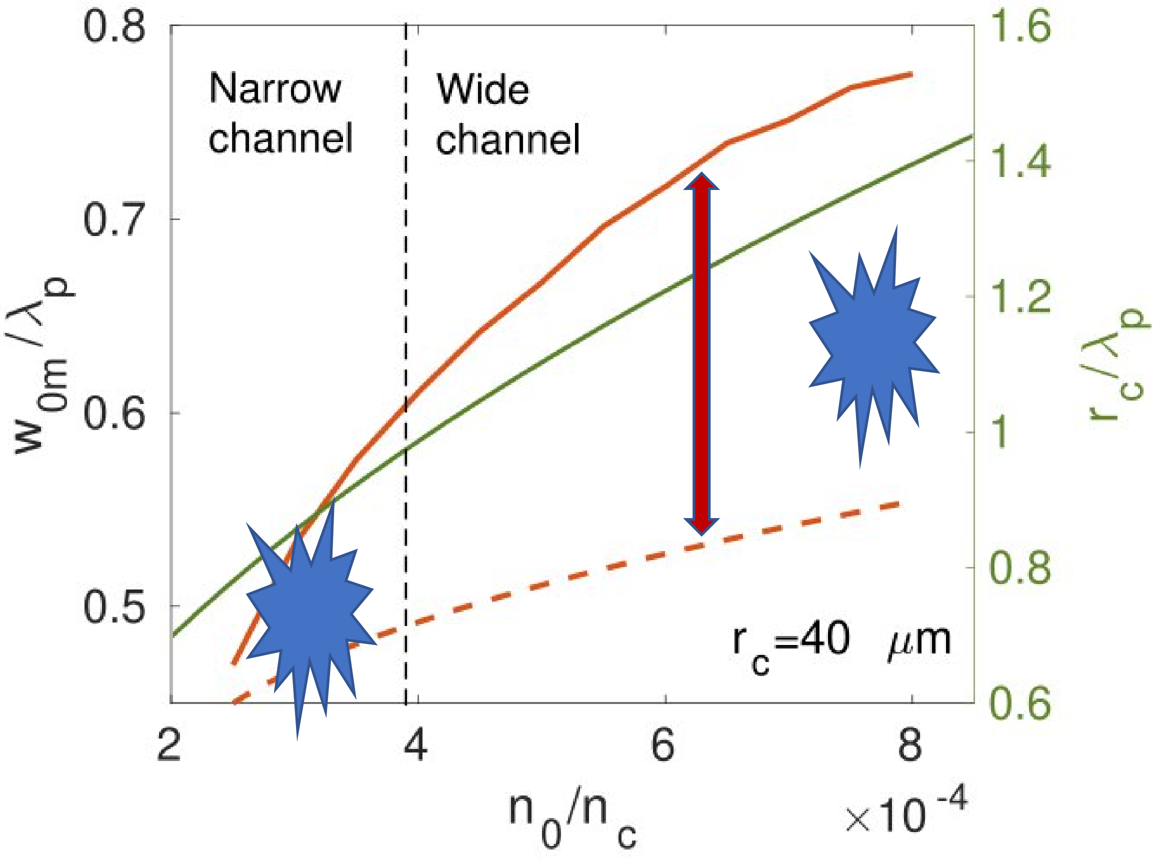
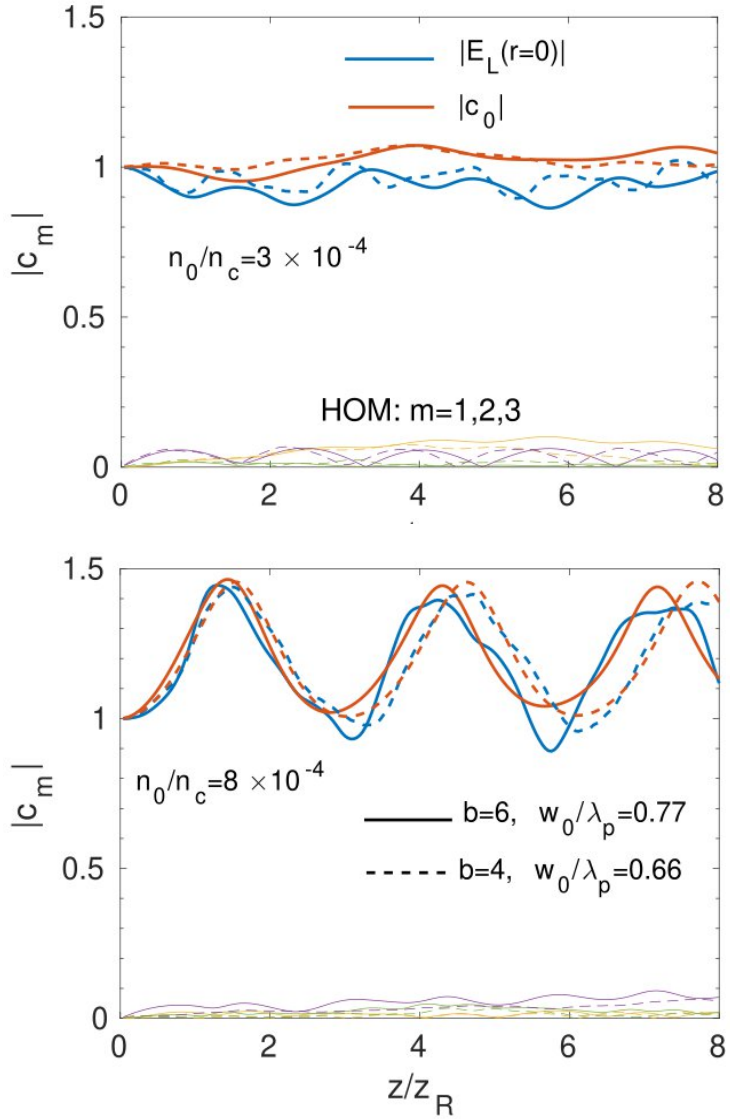
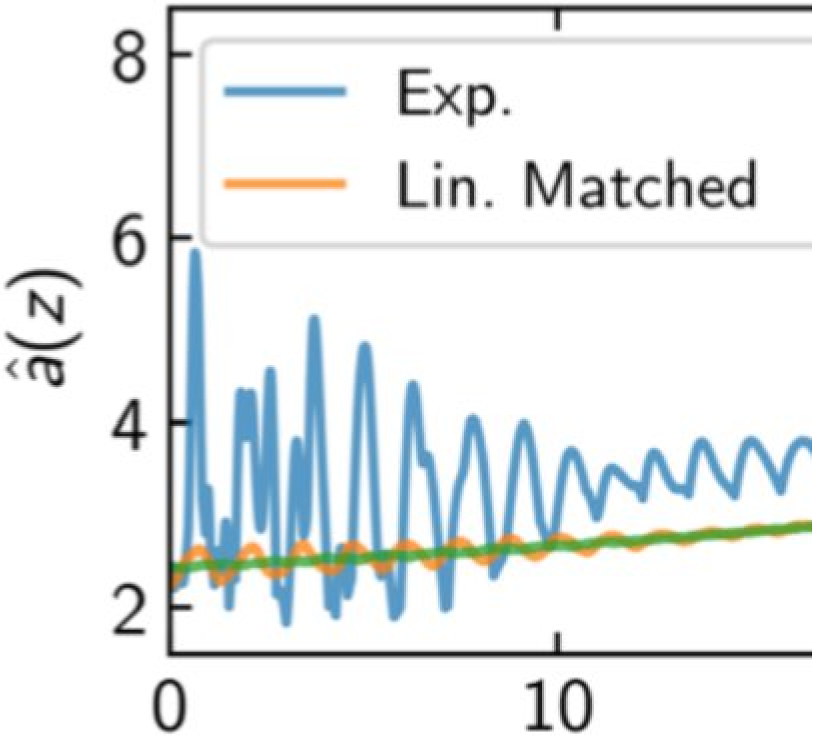
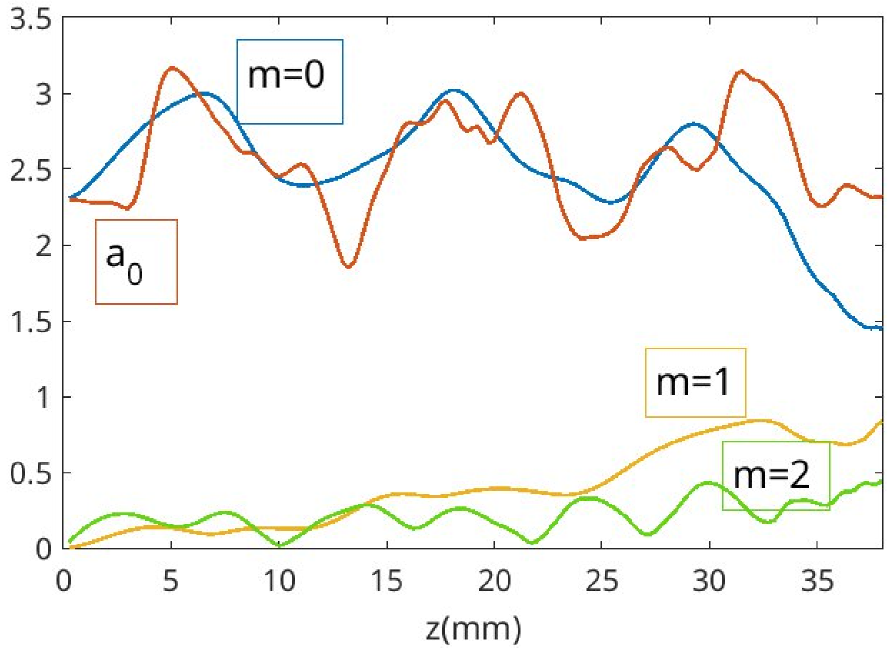




Coincidence?

A. Picksley et al., Phys. Rev. Lett. 133, 255001 (2024)





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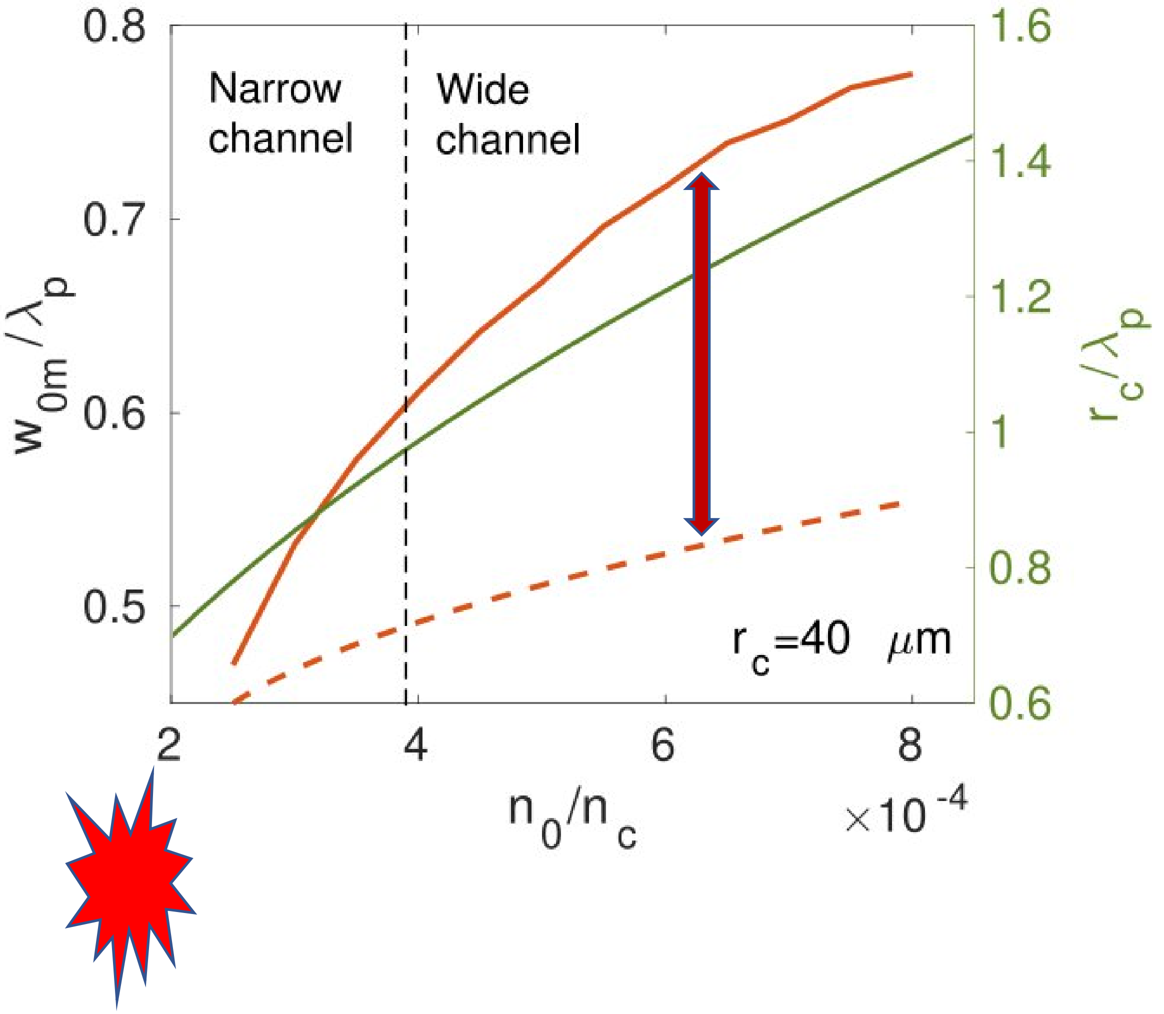
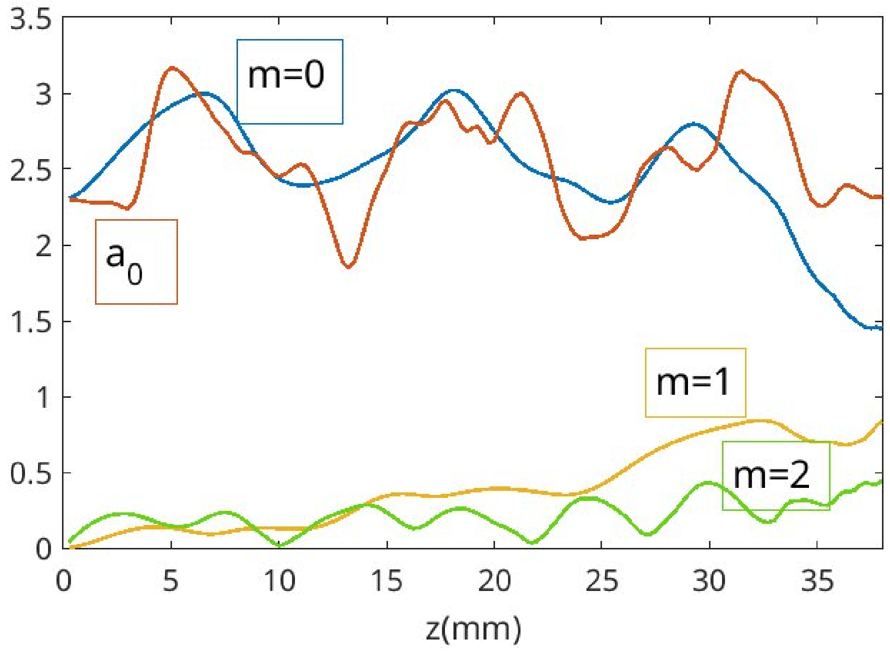




Conclusions

- HOFI channels are not infinite guiding structures, high-order radial modes can „leak out“.
- Only one (single) mode can be guided if the susceptibility is a second-degree polynomial in r.
- The fundamental (Gaussian) mode can modify the channel density to form a parabolic profile. Finding the optimal parameters leads to **mode matching**.
- With this new matching condition the LWFA can be more efficient.

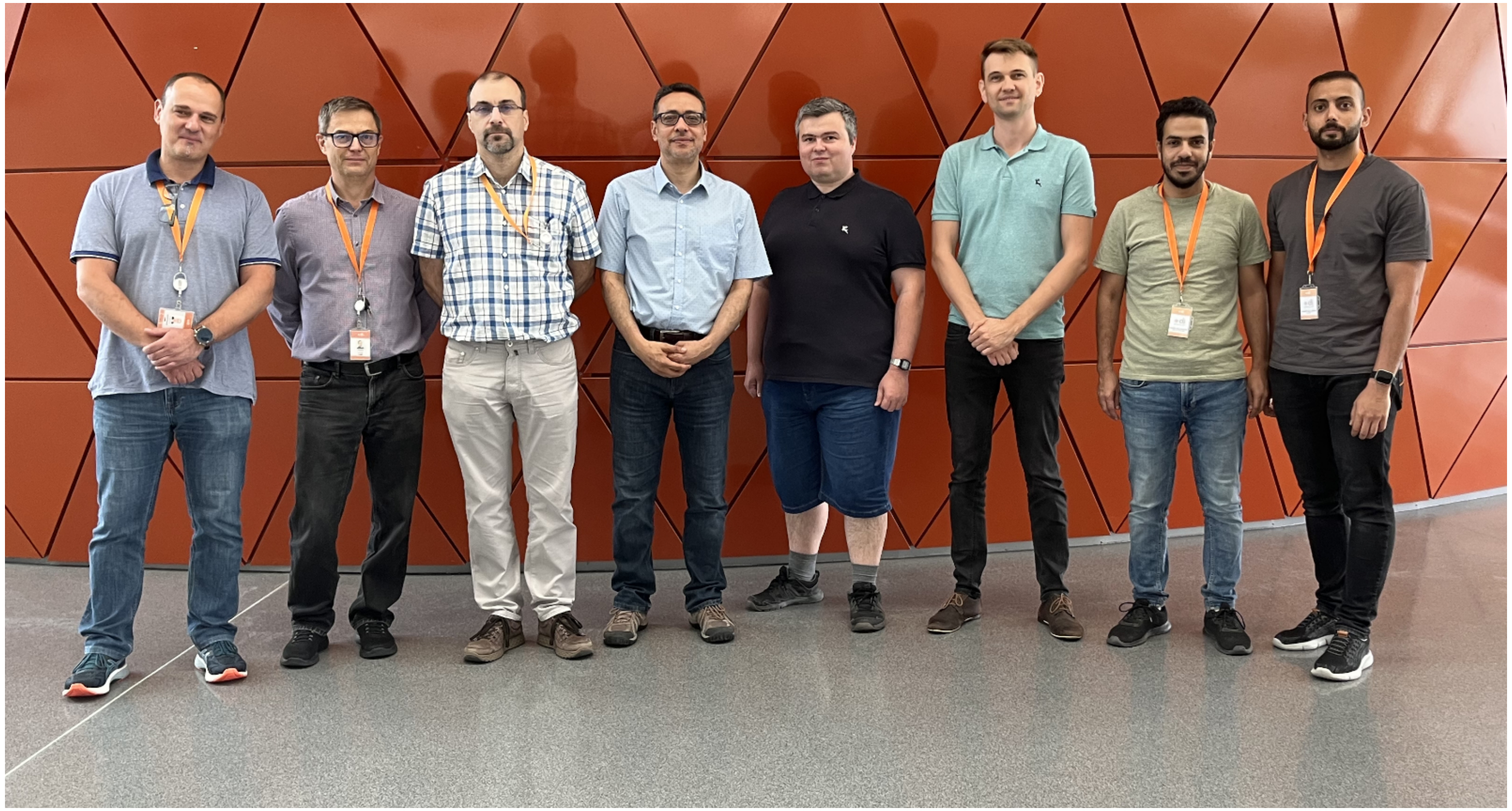
$$r_c > \lambda_p > w_0 \text{ and } a_0 > 2.$$

- Future plans: control over the self-injection and optimization of the energy gain.

7th User Call

https://up.eli-laser.eu/news/1739522201/pre-announcement-7th-eli-user-call-to-open-on-23rd-september-1739522201

Thank you for your attention !



Backup slides!

Wave equation for laser pulses

$$\nabla_\perp{}^2 A + 2\frac{v_g}{c^2}\frac{\partial^2 A}{\partial\tau\partial\xi} + \frac{1}{\gamma_g{}^2}\frac{\partial^2 A}{\partial^2\xi} = k_p^2\frac{n_e(\xi,r)}{n_0\gamma}A$$

$$A = a(\xi,r,\tau)\exp(-ik\xi)$$

$$\xi = z - v_g t$$

$$v_g = c\sqrt{1-\frac{n_0}{n_c}}$$

$$\frac{1}{\gamma_g{}^2} = 1 - \frac{v_g^2}{c^2} = \frac{n_0}{n_c}$$

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This term can be neglected if the plasma density is low!

$$2i\frac{n_c}{n_0}\frac{\partial a_\xi}{\partial\tau} = -\nabla_\perp{}^2 a_\xi + \frac{n_e}{n_0\gamma}a_\xi$$

$$A = a(\xi,r,\tau)\exp(-ik\xi)$$

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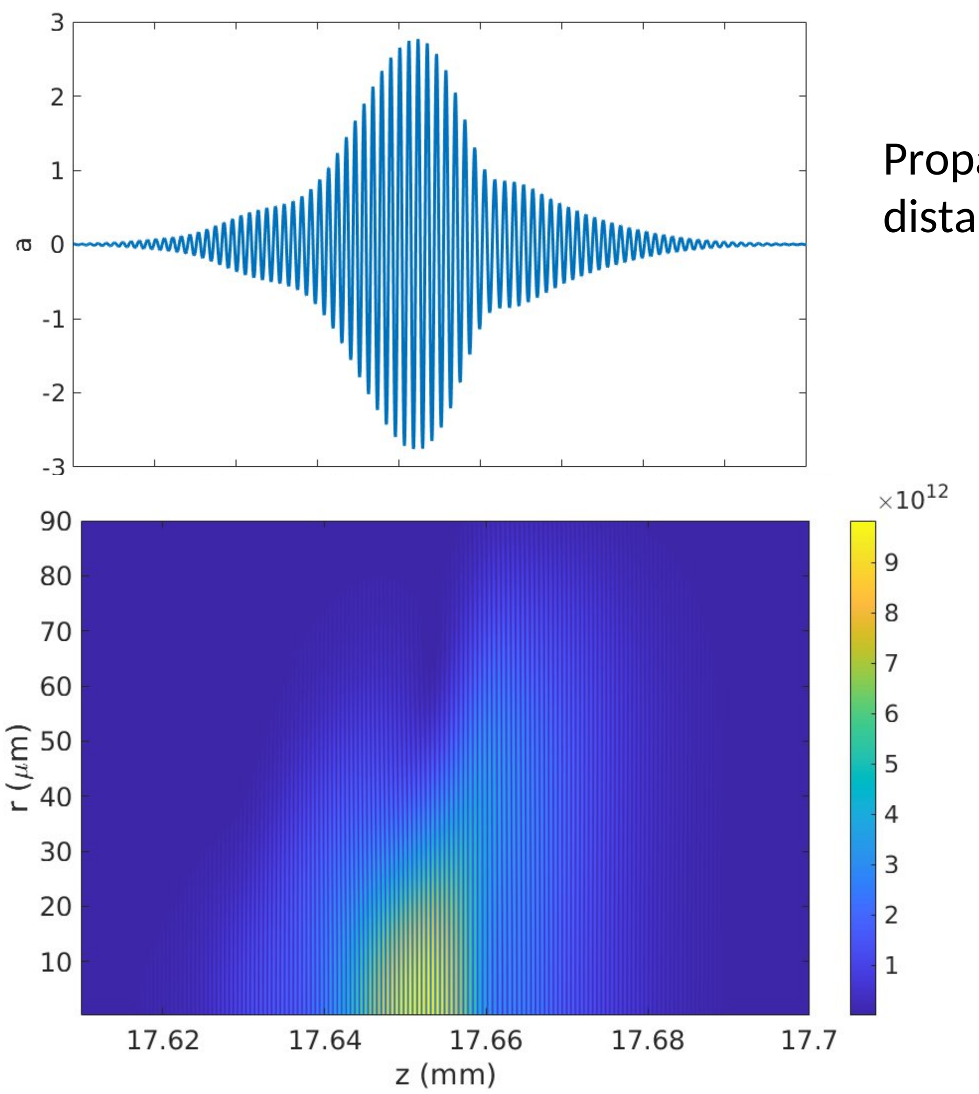
$$\frac{1}{\gamma_g{}^2} = 1 - \frac{v_g^2}{c^2} = \frac{n_0}{n_c}$$

If group velocity dispersion can be neglected ($n_0 \ll n_c$), one Helmholtz equation holds in each longitudinal (ξ) slice of the laser pulse.

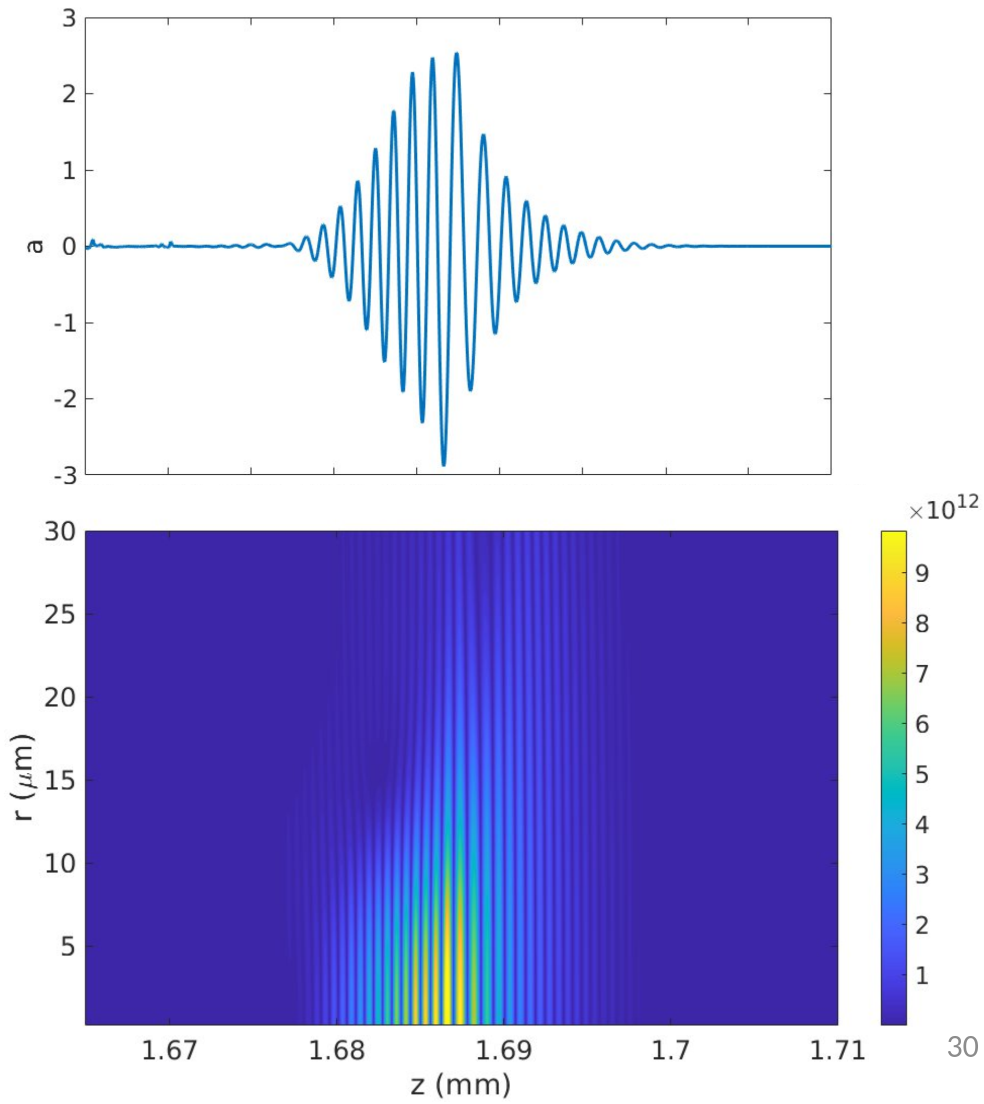
Pulse propagation is not self-similar

$n_0 = 0.4 \times 10^{18} cm^{-3}$

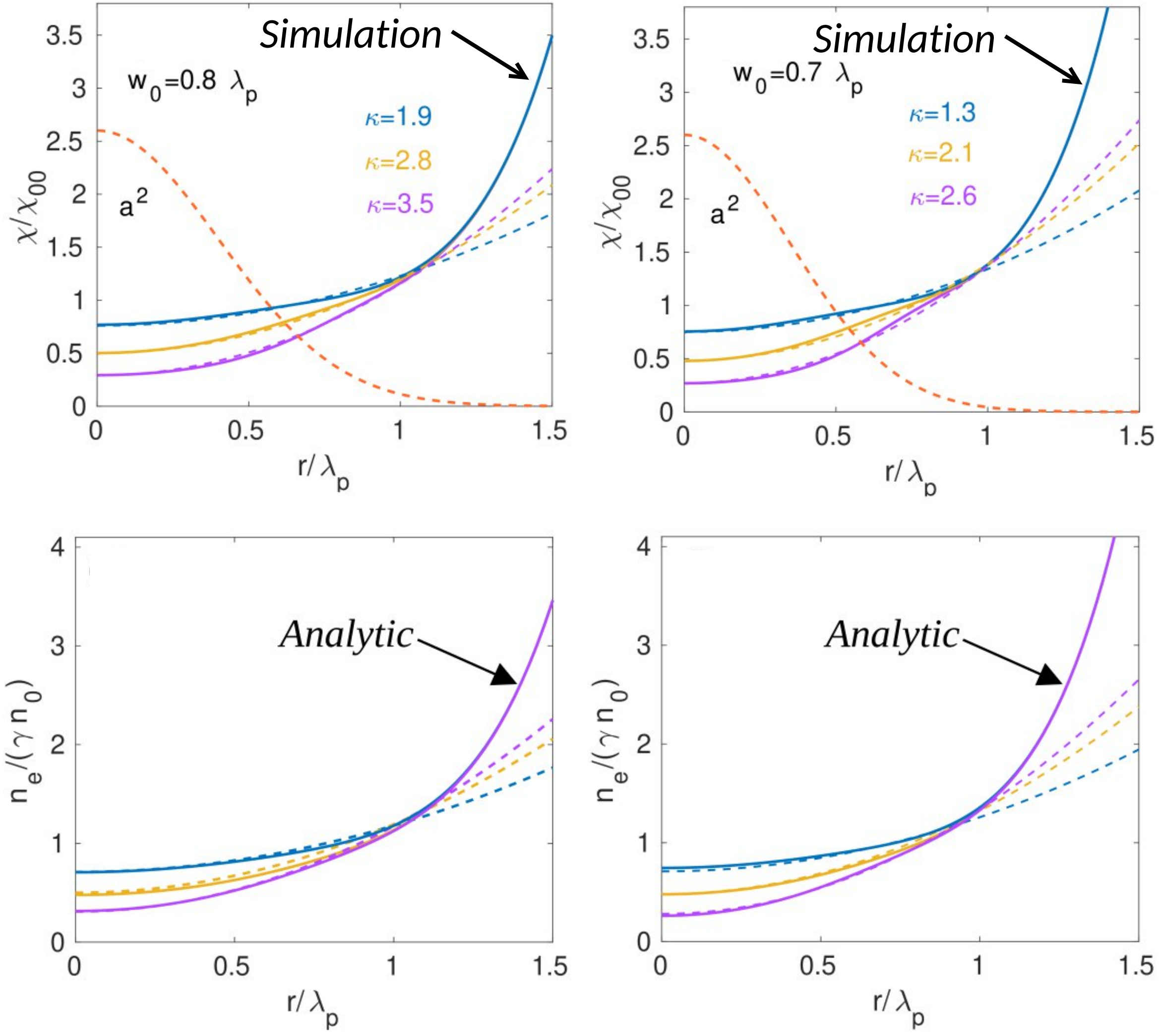


Propagation distance: $3.5L_R$

$n_0 = 3.6 \times 10^{18} cm^{-3}$



Self-generated parabolic channel



The laser pulse modifies the plasma density and generates a gamma distribution, which ensures a parabolic susceptibility in the radial direction!

Realistic (HOFI) plasma channel

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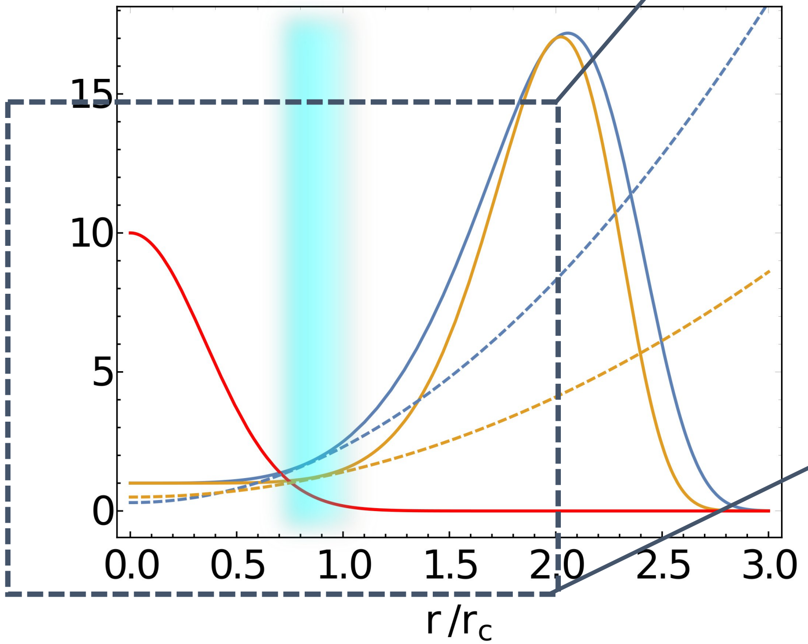
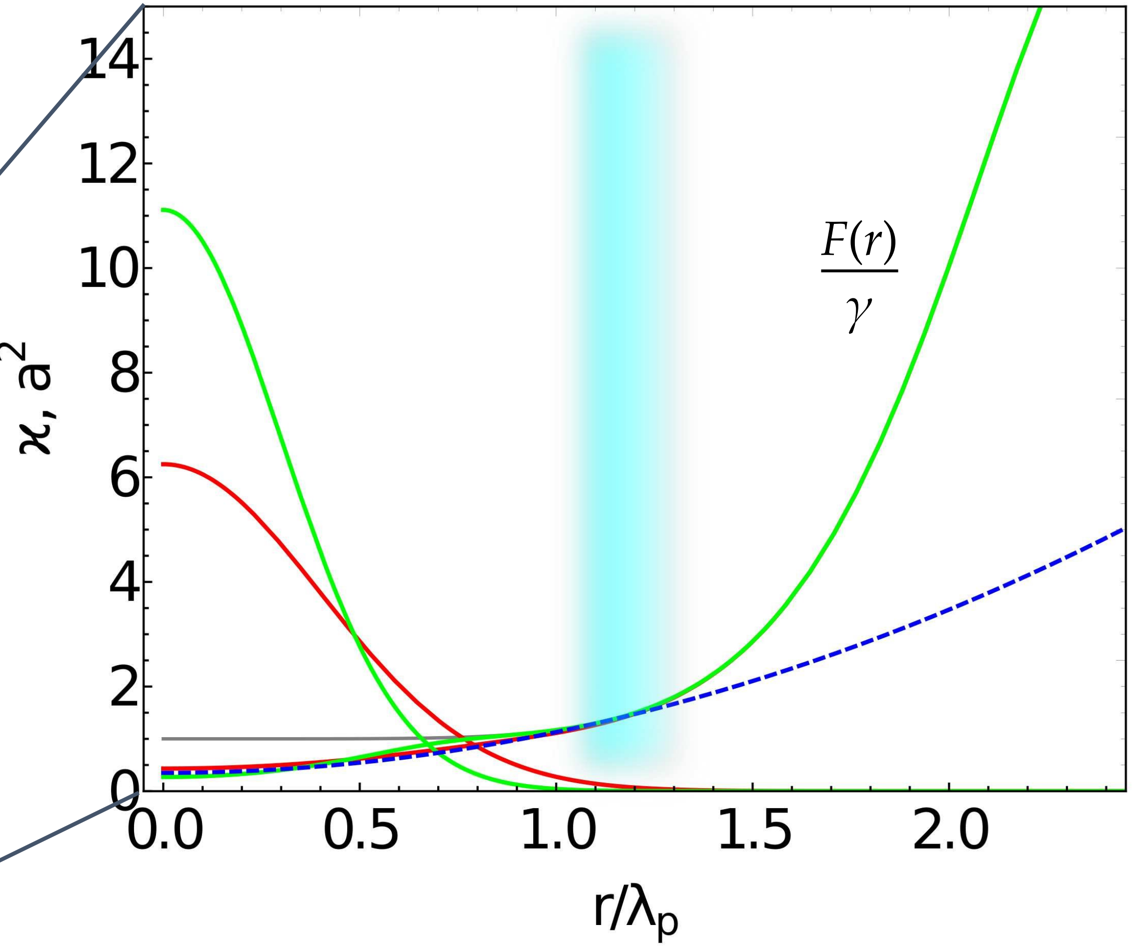




Numerical examples

$$2i\frac{n_c}{n_0}\frac{\partial a}{\partial \zeta} = -\nabla_\perp{}^2 a + \frac{n_e}{n_0\gamma}a$$

$$n_{e0} = n_0\left(1 + \epsilon\frac{r^b}{r_c^b}\right), b = 4$$

Main parameters:

$$\epsilon = 1.45$$

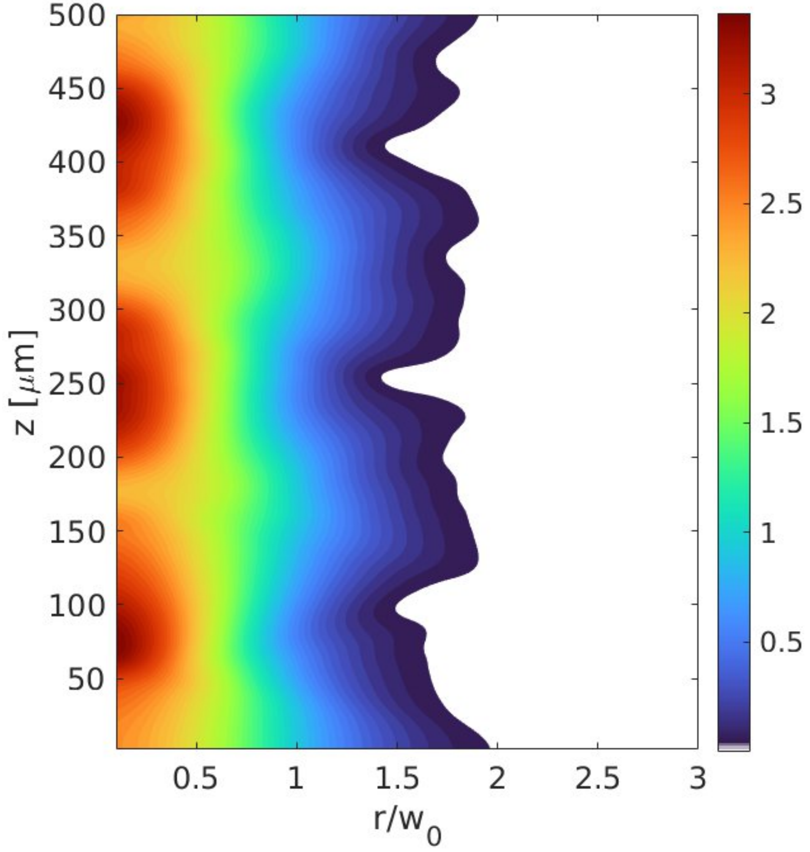
$$a_0 = 2.5$$

$$n_0 = 3\times 10^{17}\ cm^{-3}$$

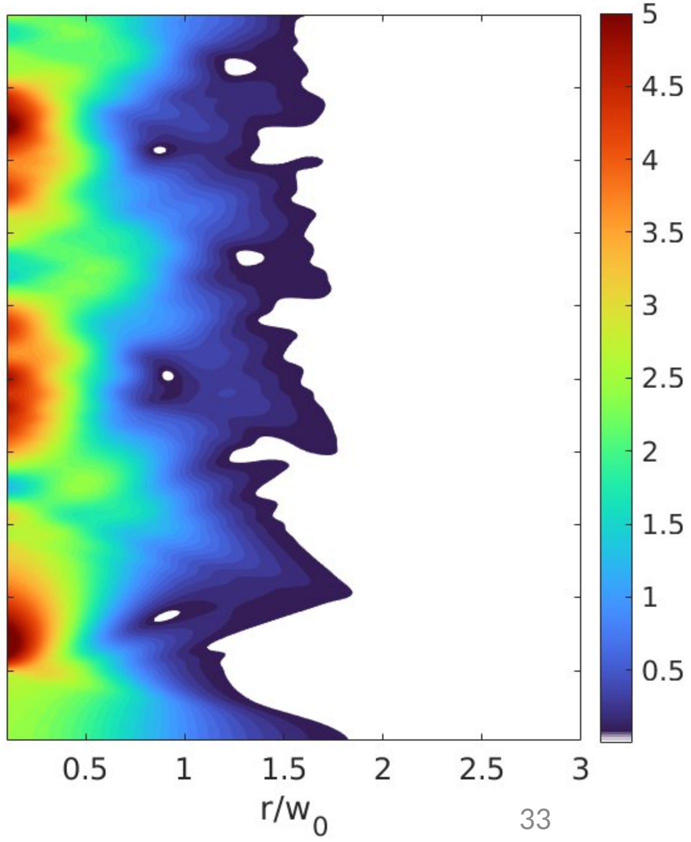
Solving it with the operator-splitting method:

$$\kappa = 1.5$$

Matched



Mismatched



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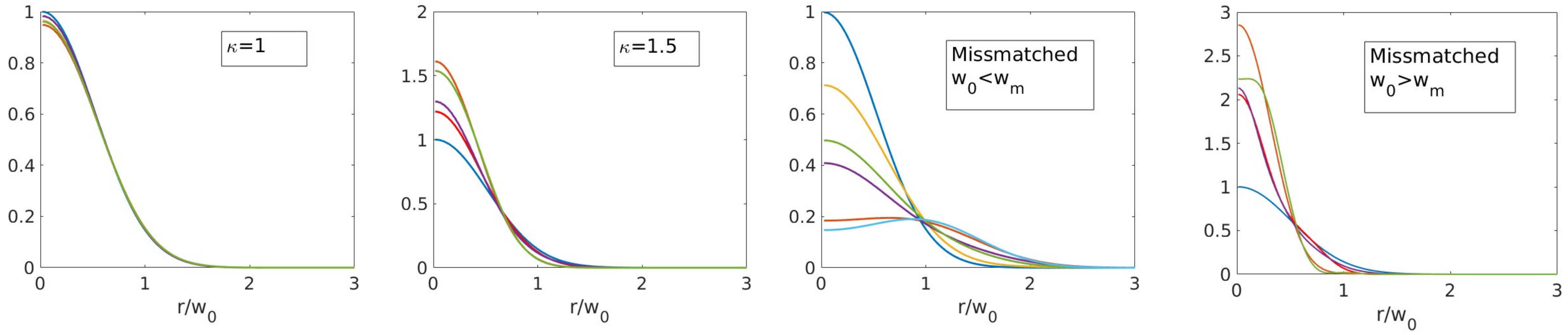
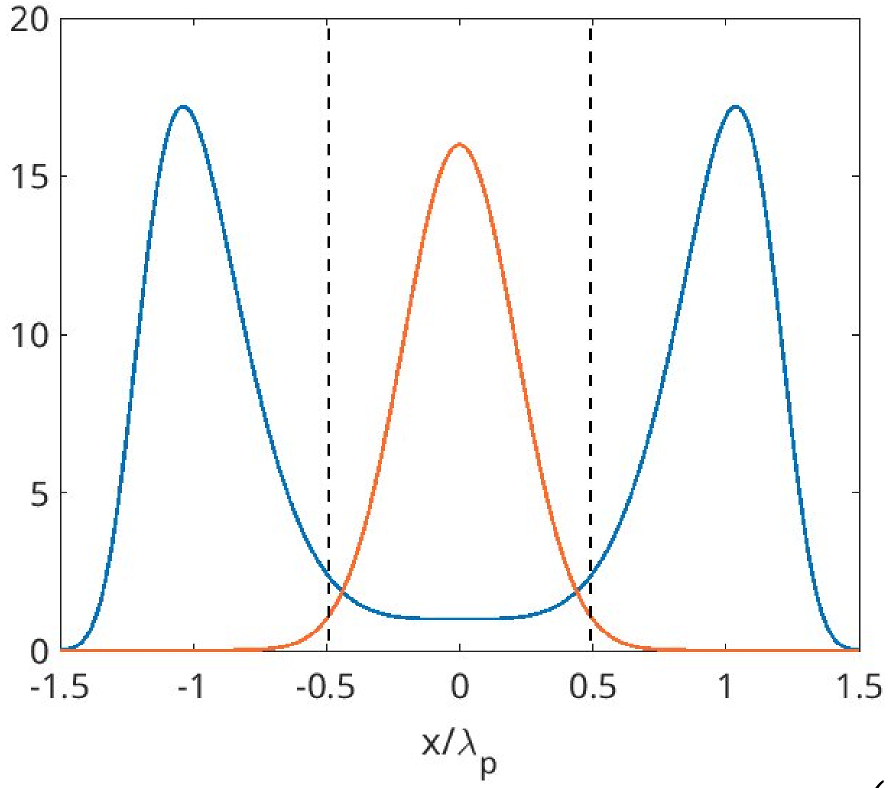
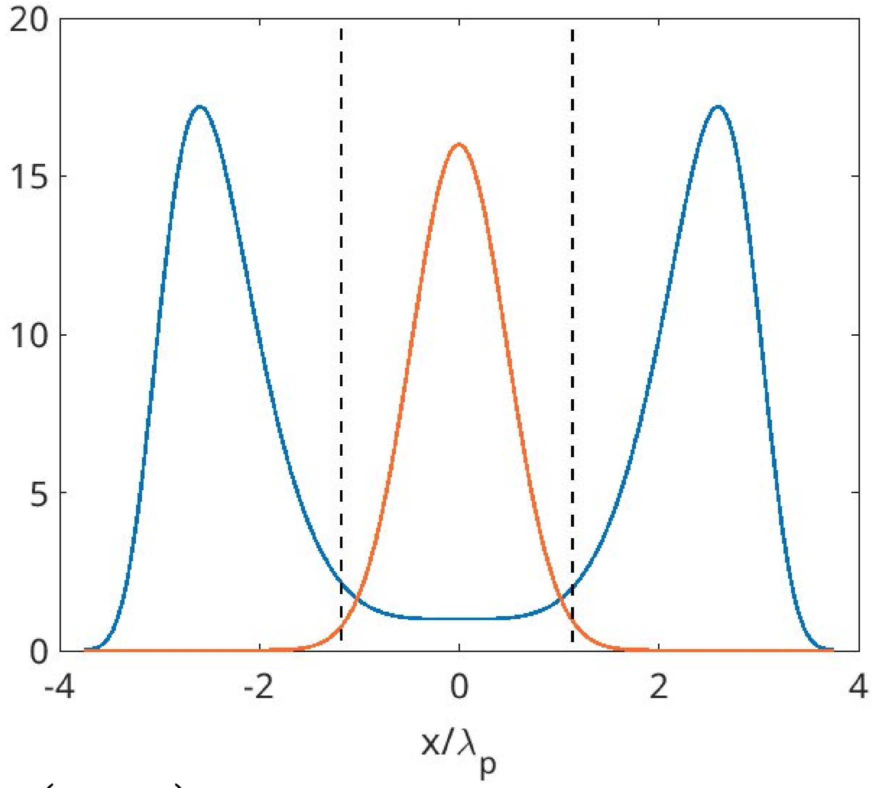


Narrow and wide channels





$$n_e = n_0\left(1 + \epsilon\frac{r^b}{{r_c}^b}\right)\exp\left(-\frac{\beta r^p}{{r_c}^p}\right)$$

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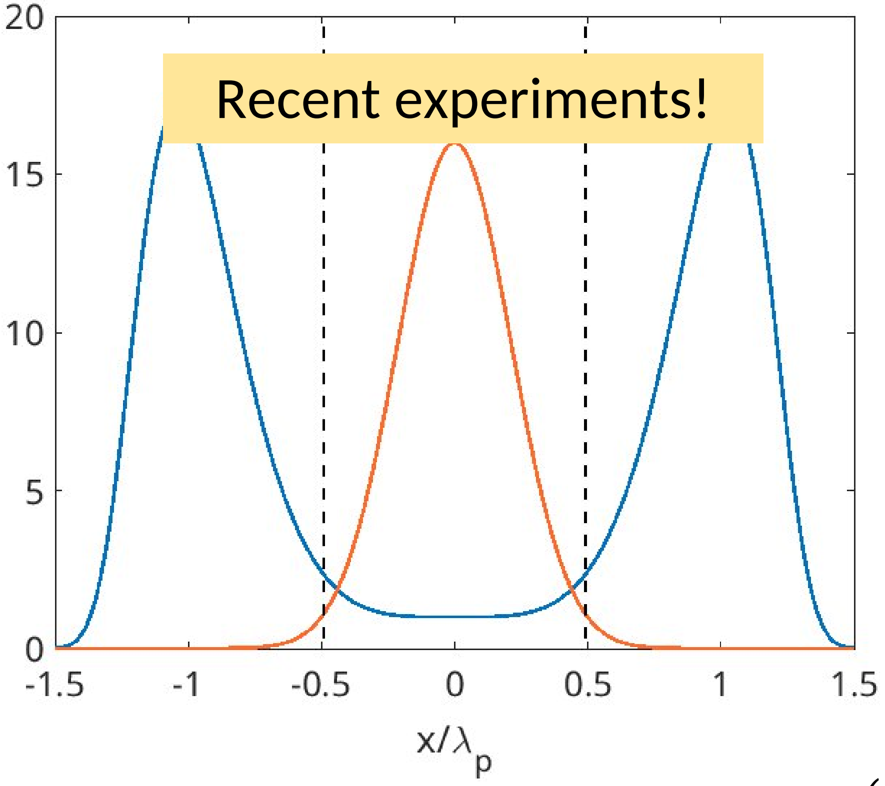

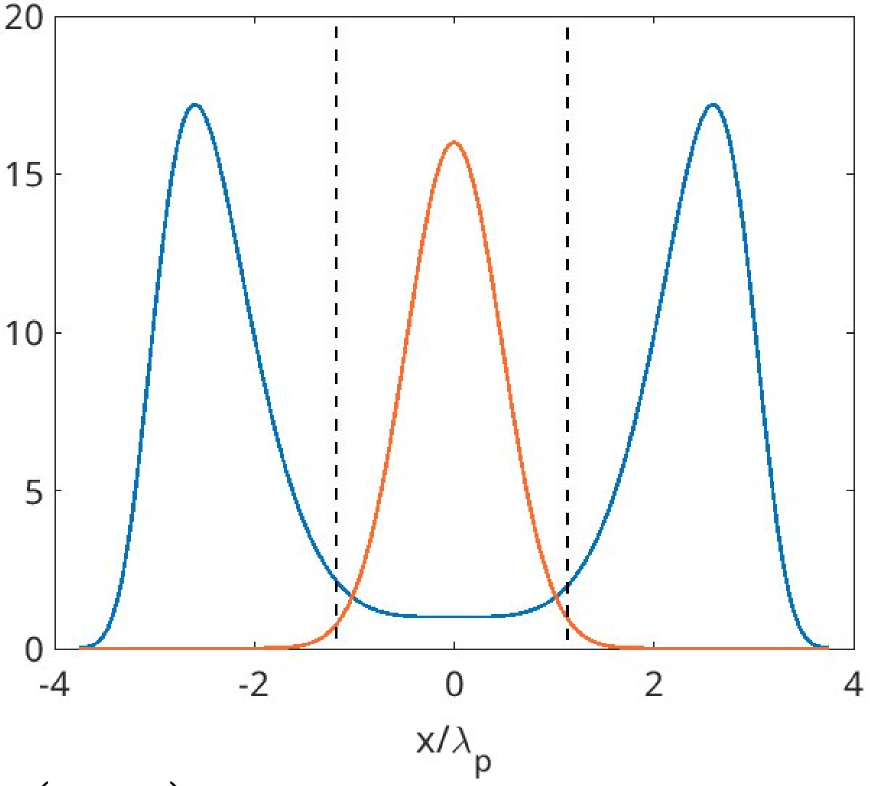

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