Simulation Studies of Temperature Effects for High-Repetition-Rate Plasma Wakefield Accelerators

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High-Repetition-Rate Plasma Acceleration

> **Goal:** Implement plasma accelerators into operational facilities - FEL's and Linear Colliders

Collider Luminosity: $L \propto \frac{Q_{bunch} f_{rep}}{\sigma_x \sigma_y}$ FEL Instantaneous Brightness: $dB/dt \propto \frac{Q_{bunch} f_{rep}}{\sigma_x \sigma_y \sigma_E}$

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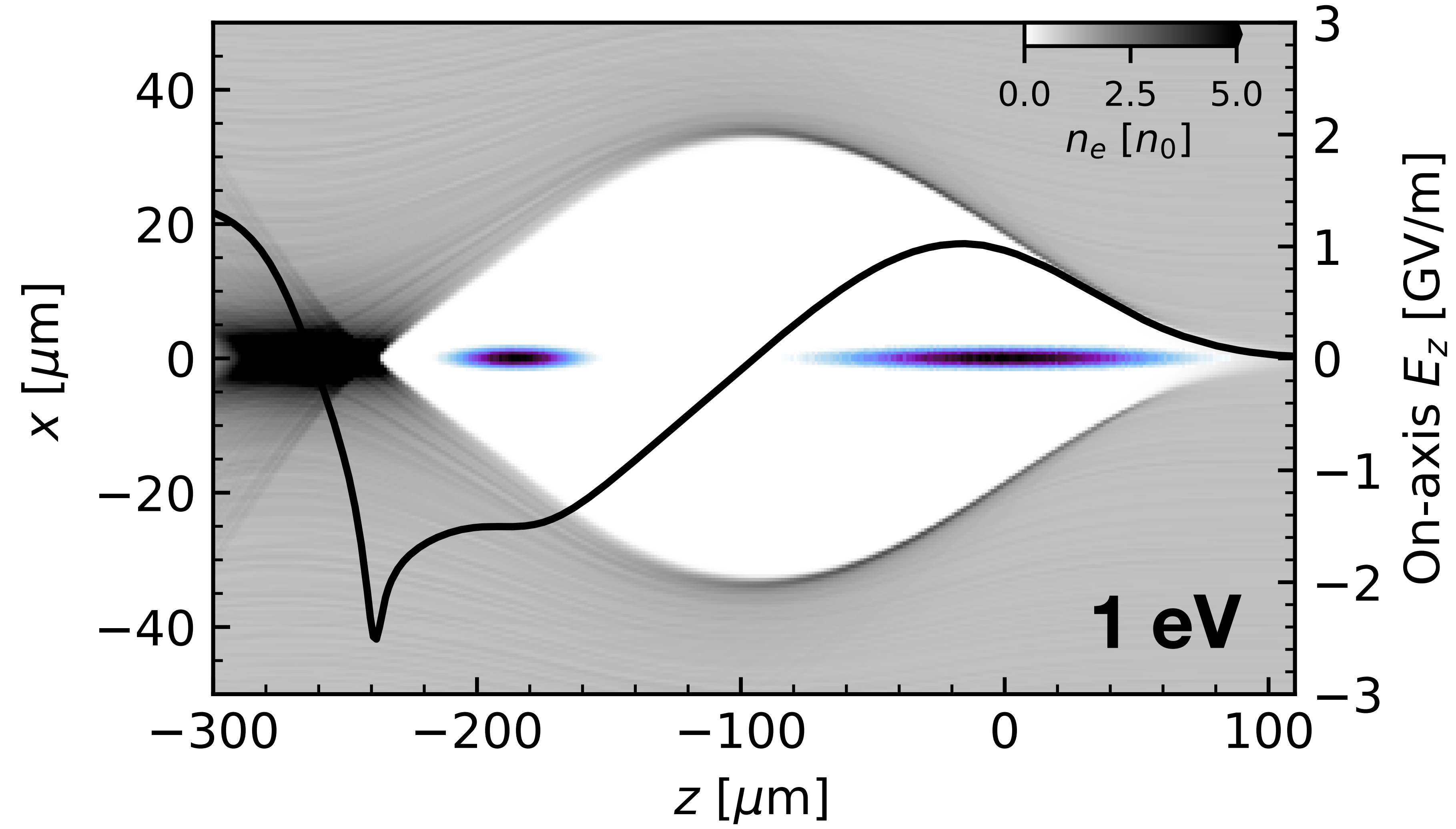
> **Requirement:** Demonstrate stable operation of plasma accelerator stages at competitively high repetition rates.

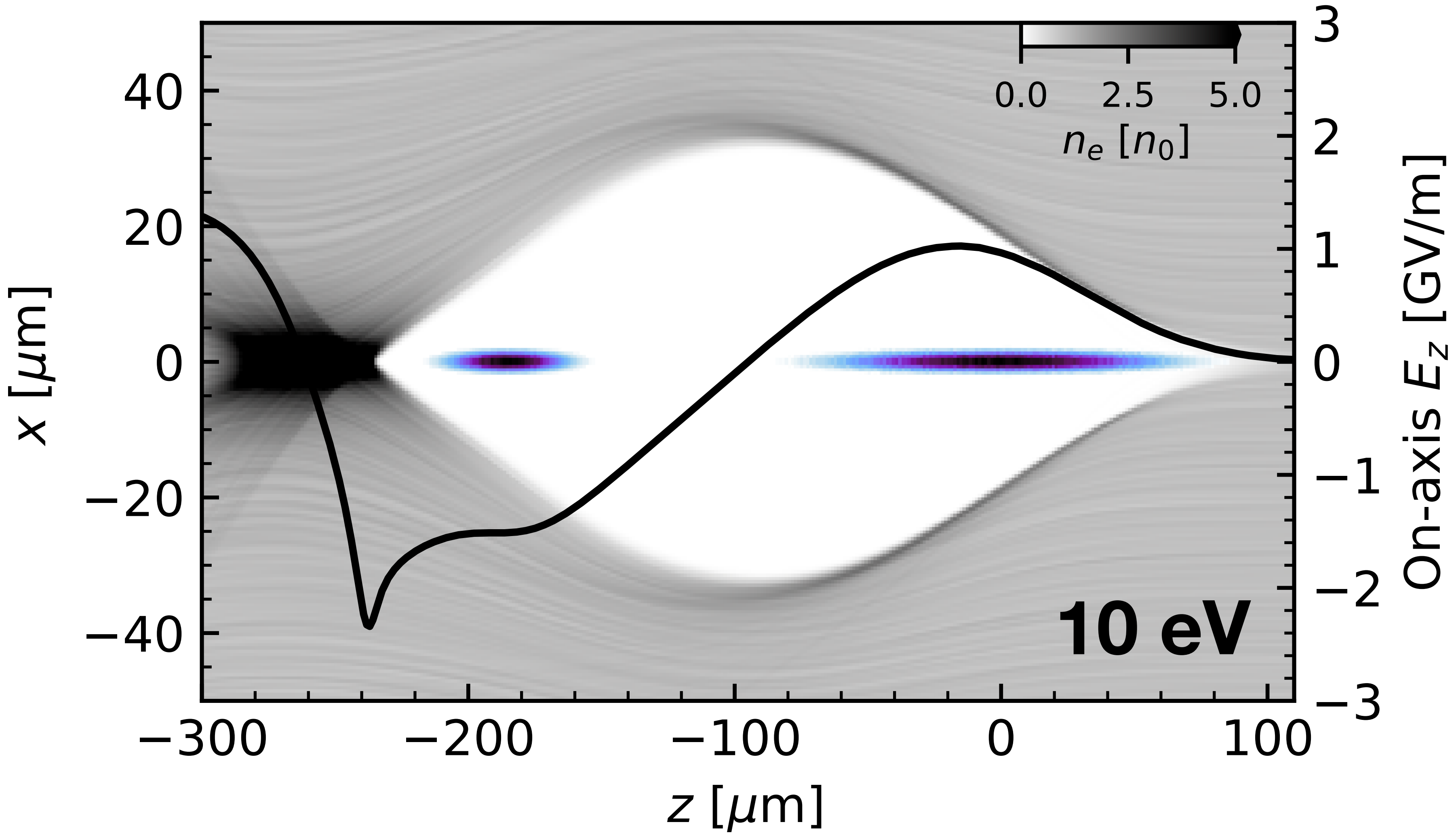
> **Challenges:** For high repetition rates, we need to reuse the plasma, so to maintain consistent quality of acceleration for many bunches, we need,
> > Consistent plasma densities / plasma profiles
> > Durable plasma sources / containers
> > To keep the **plasma temperature low (or consistent?)**

> **Why care about plasma temperature?** Plasma acceleration is not totally efficient, so any energy from the driver that is not extracted by the witness, makes it's way into and through the plasma. An increased plasma temperature then effectively changes the plasma frequency e.g via the Bohm-Gross dispersion relation:

$$\omega^2 = \omega_p^2 + \frac{3k_BT}{m_e}k^2$$

Effect of plasma temperature on non-linear wakefields





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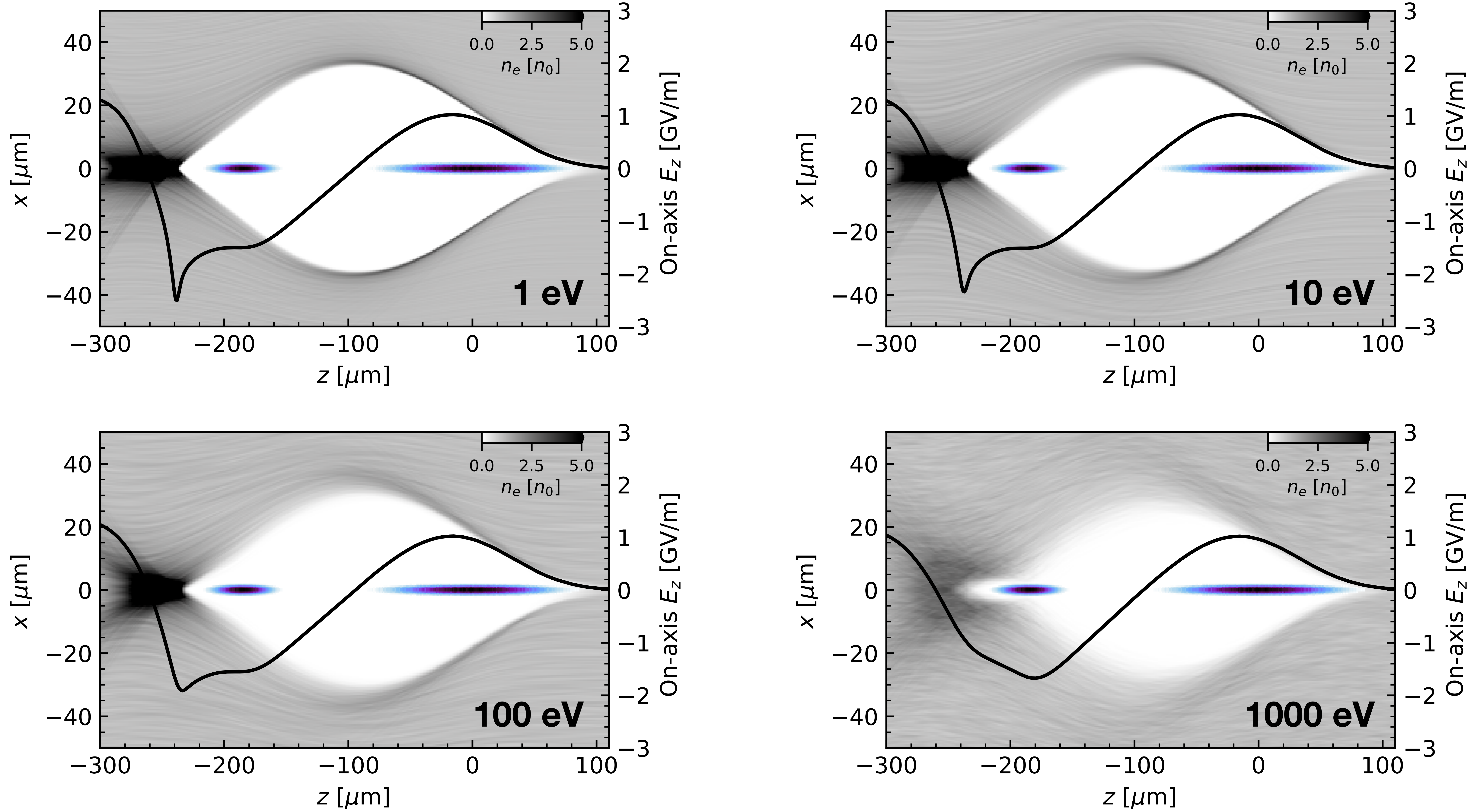
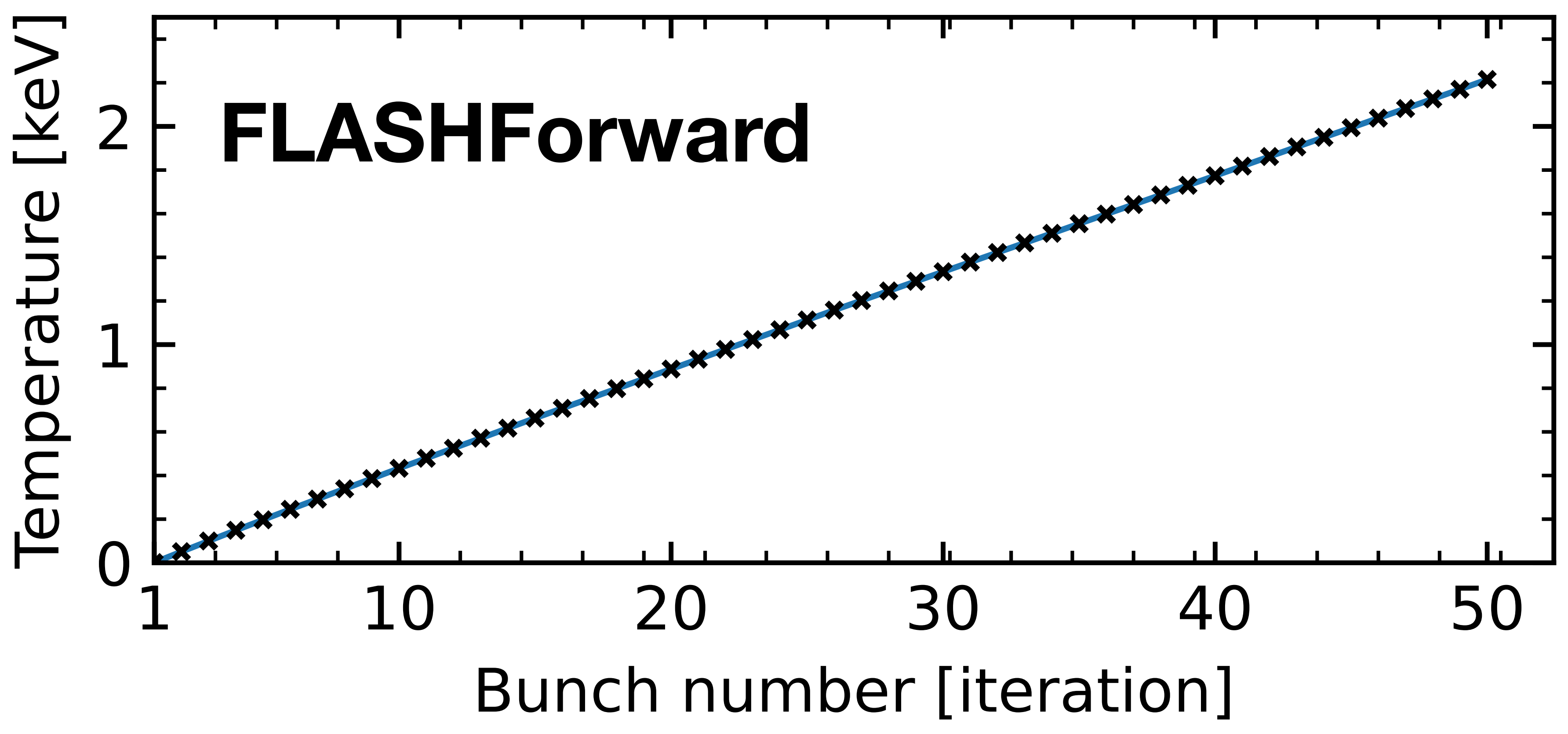
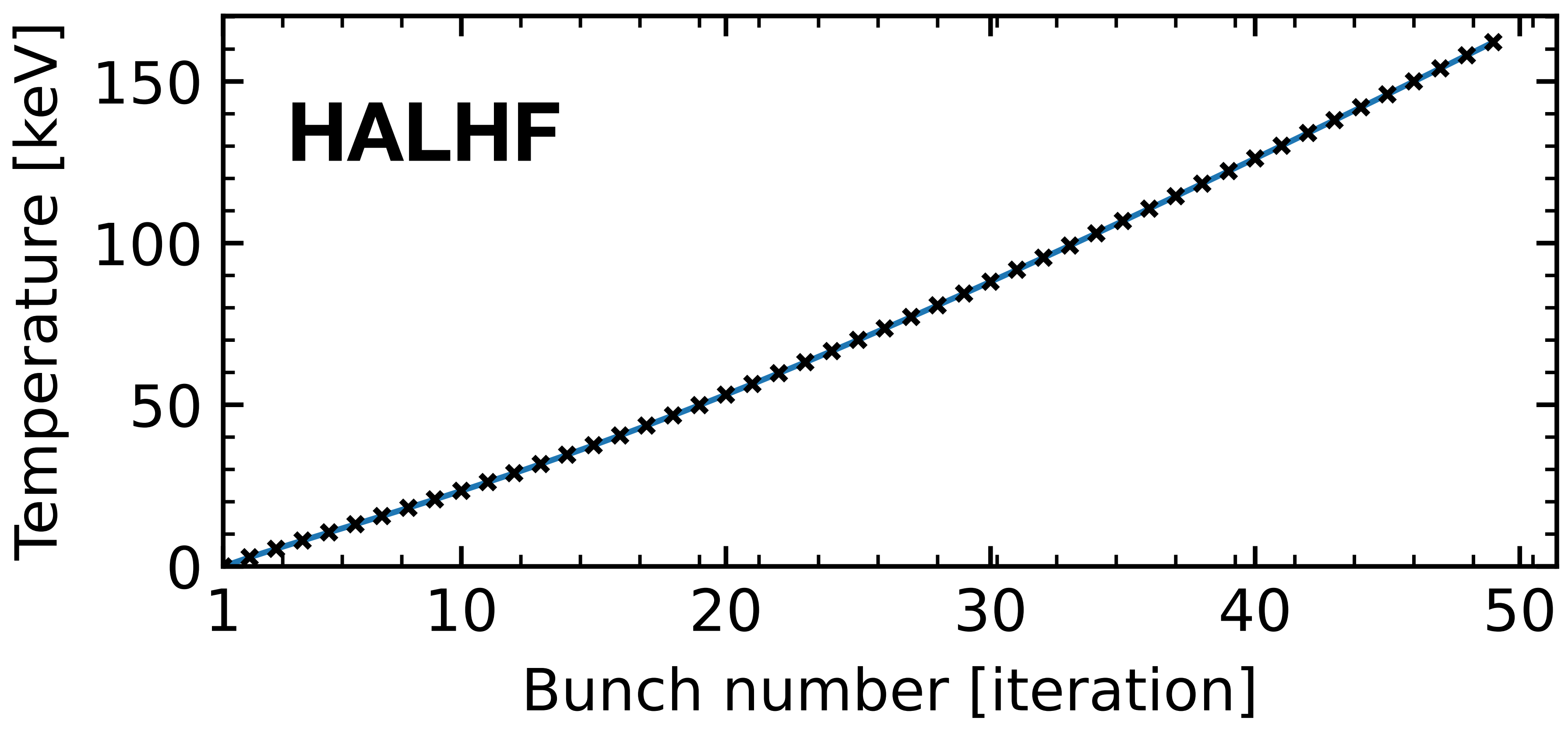


Naive assumptions to form an iterative PIC loop

> **What kind of temperatures are expected?**
> **Worst case scenario:** Plasma takes all deposited energy from the bunches and splits it evenly among the electrons and ions, leading to a corresponding rise in plasma temperature





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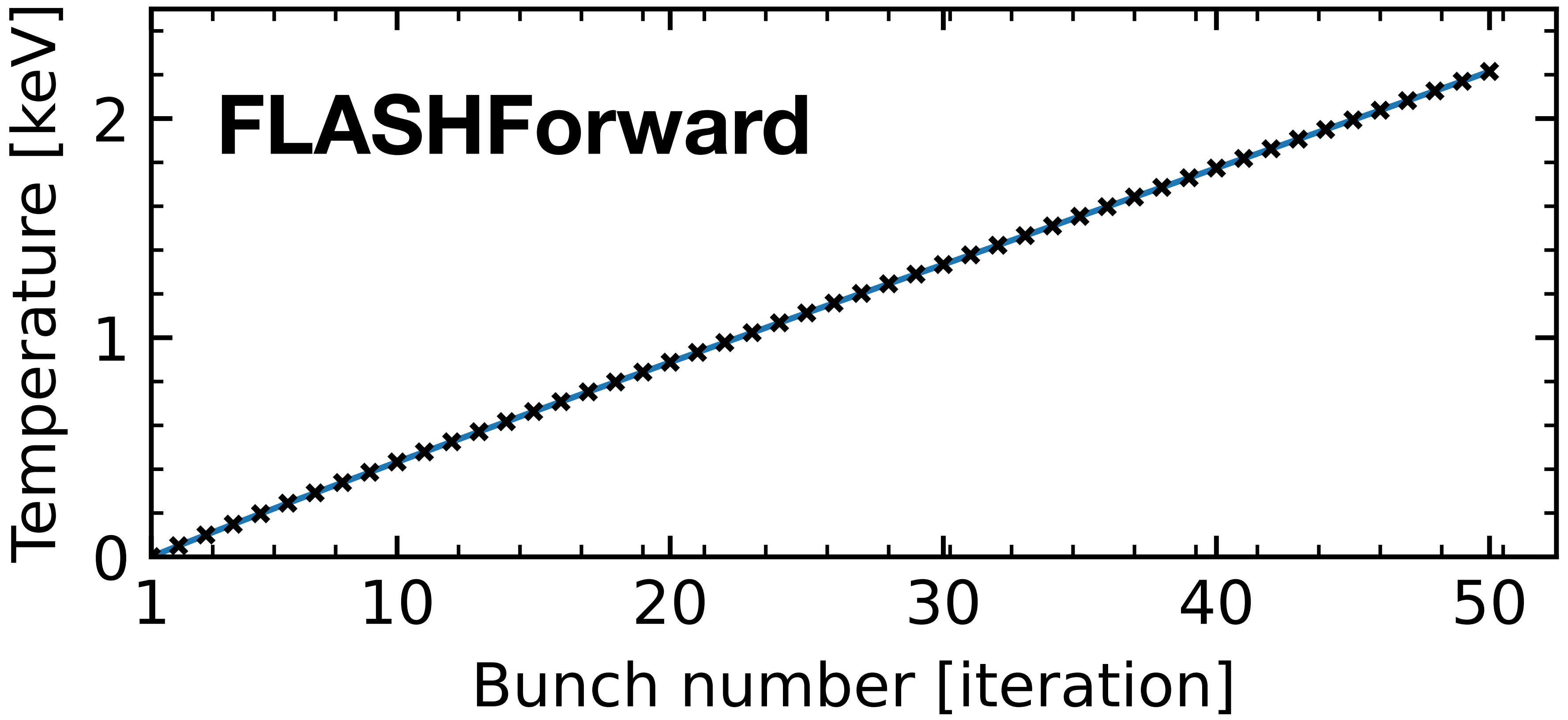
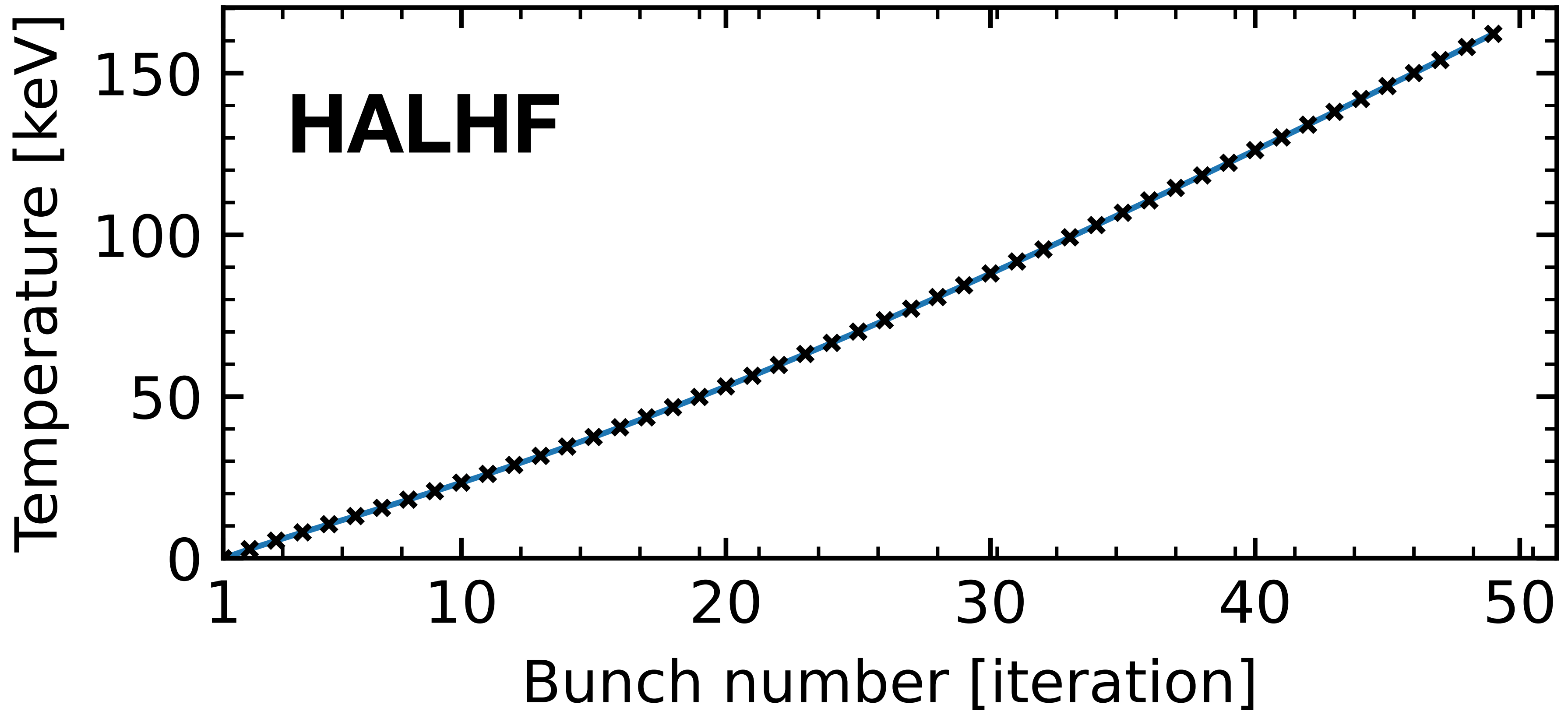


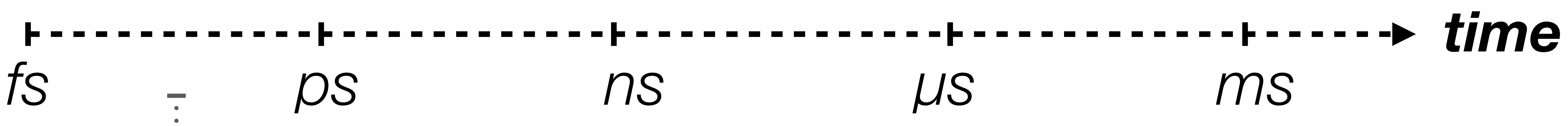


- **Runaway effect:** Becomes more inefficient with increasing plasma temperature
- **Obvious shortcomings:** No sinks in energy + no plasma evolution
  - *Hot and fast electrons* carry a large portion of the initial wakefield energy
  - *Heated ions* cause excitation and ionisation of background neutral atoms
  - *Plasma density and temperature distribution* for each next bunch may not be uniform

What are the effects of temperature on a plasma accelerator?

What are the relevant channels of energy transport that dictate plasma evolution between subsequent acceleration events?

What is known to happen after wakefield acceleration events?



Source: Gilljohann et al.
Phys. Rev. X. **9**, 011046
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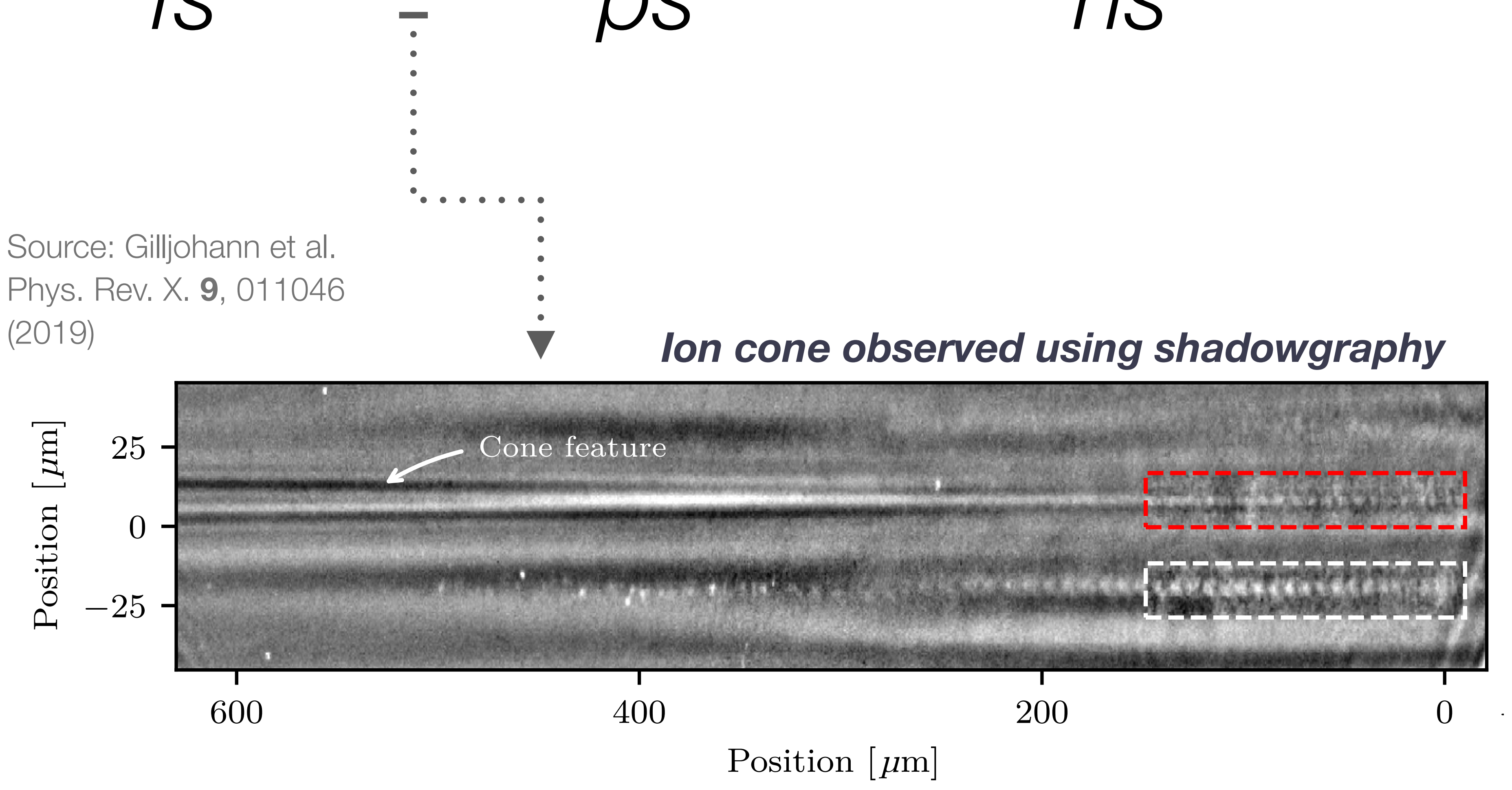
Ion cone observed using shadowgraphy



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time

fs – ps – ns µs ms

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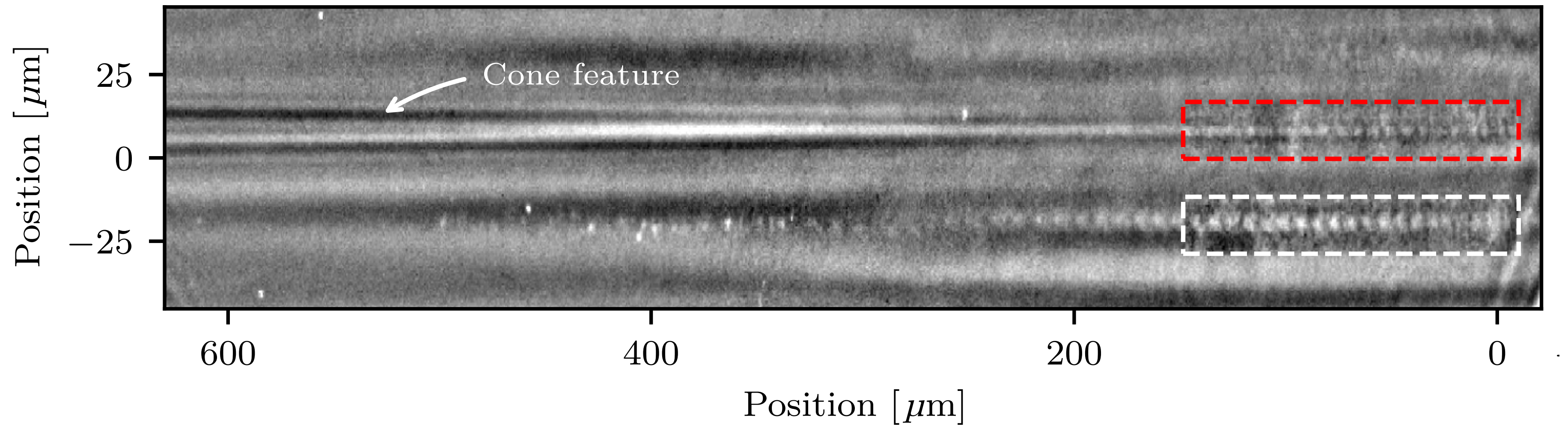
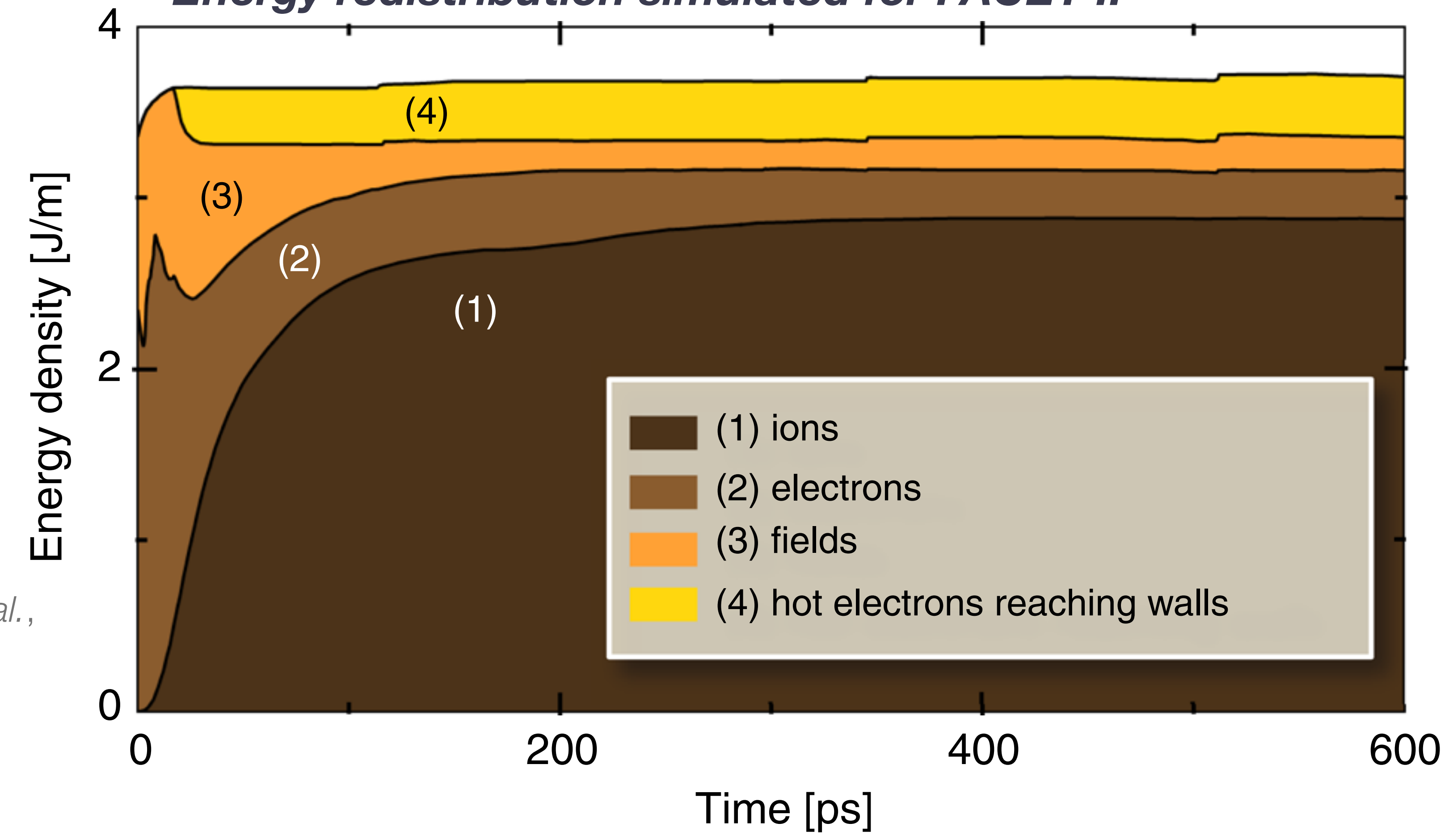


Energy redistribution simulated for FACET-II



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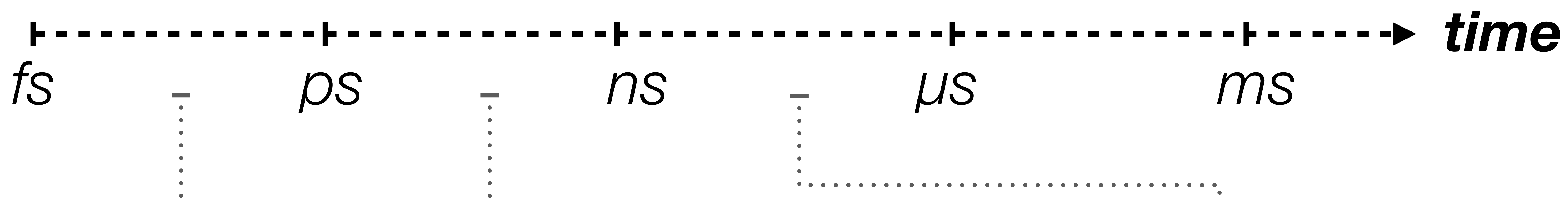

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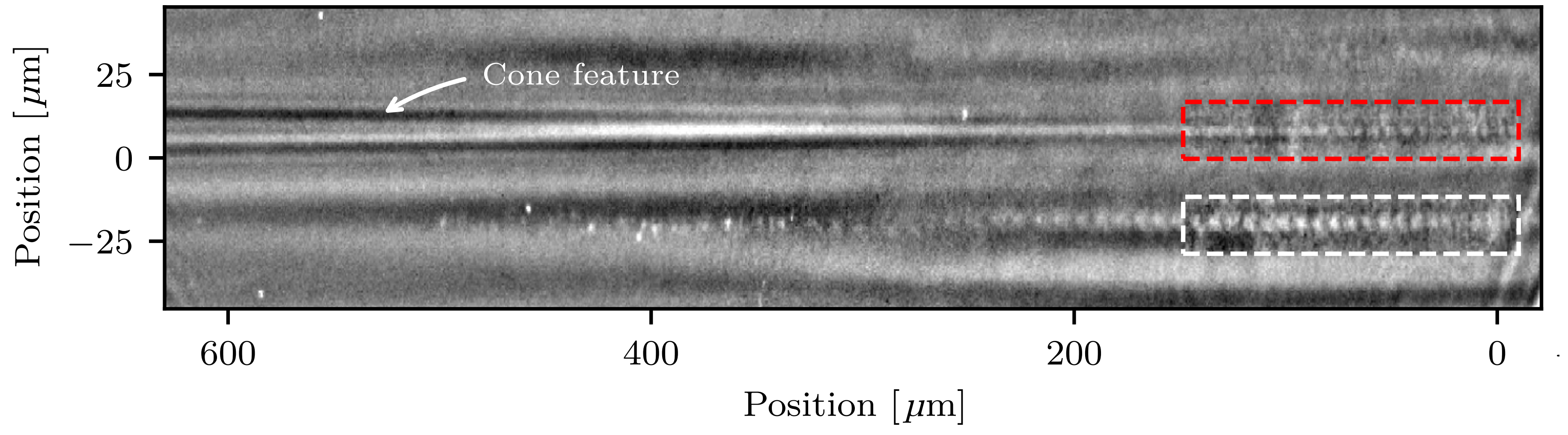

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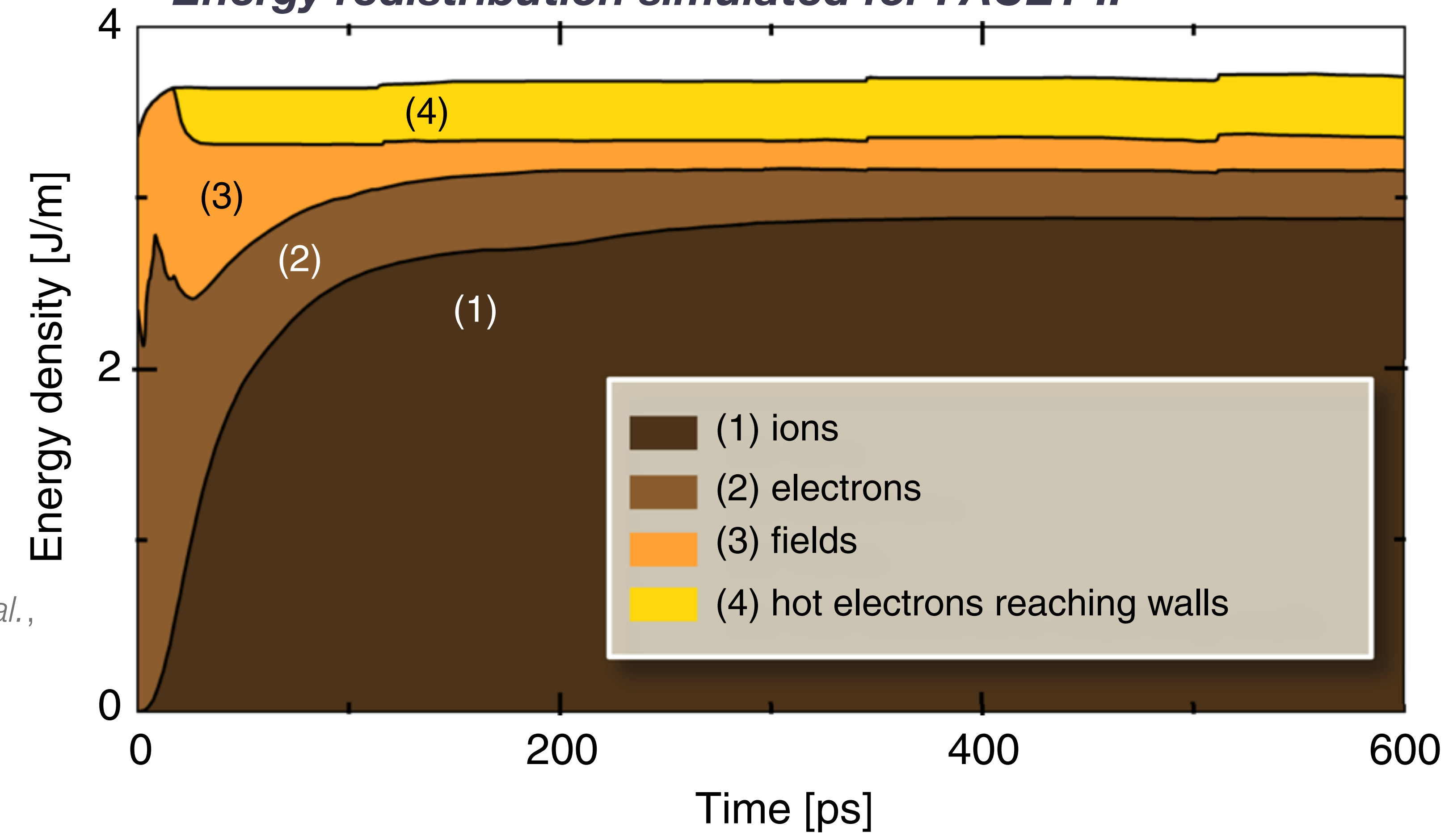

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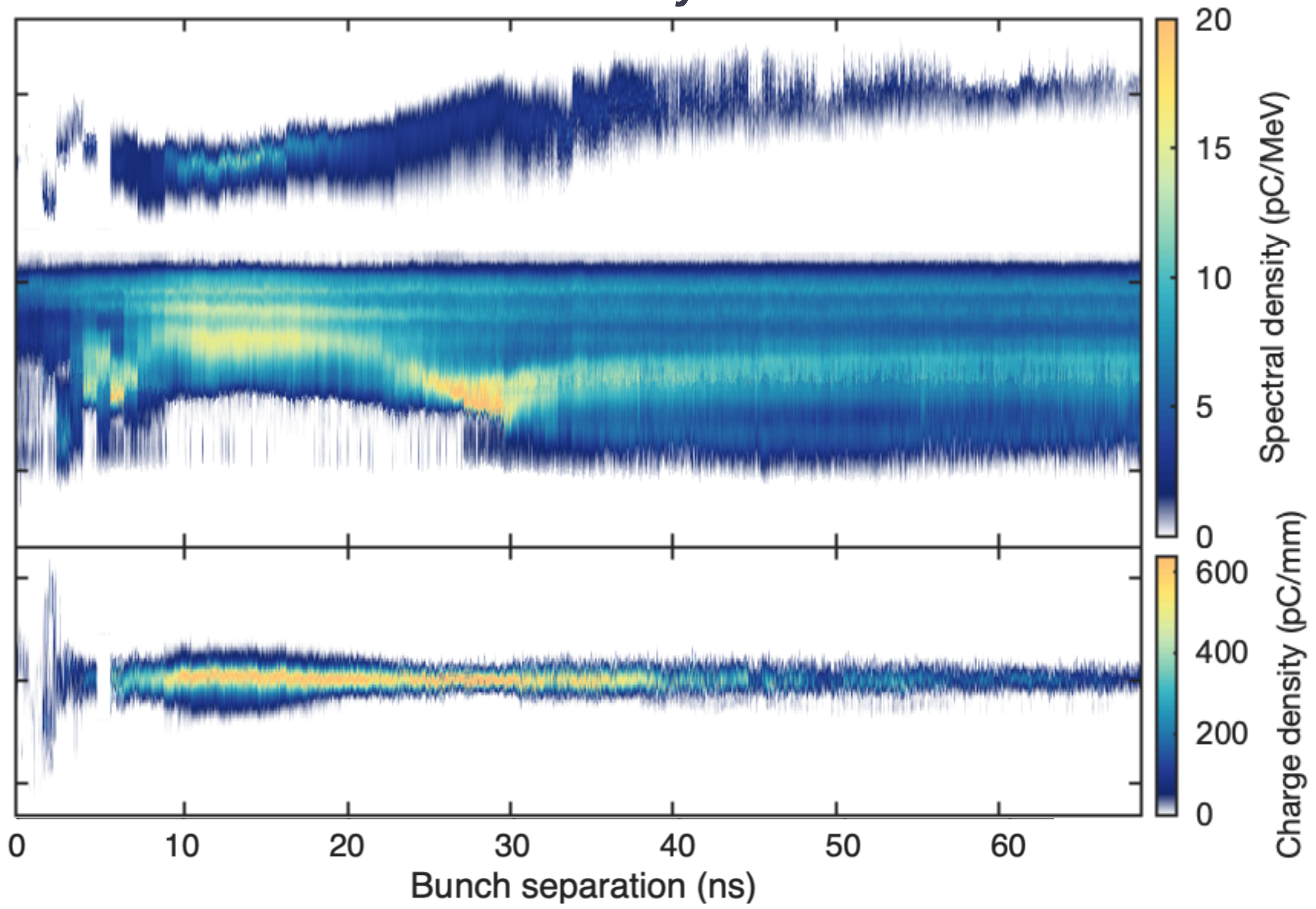

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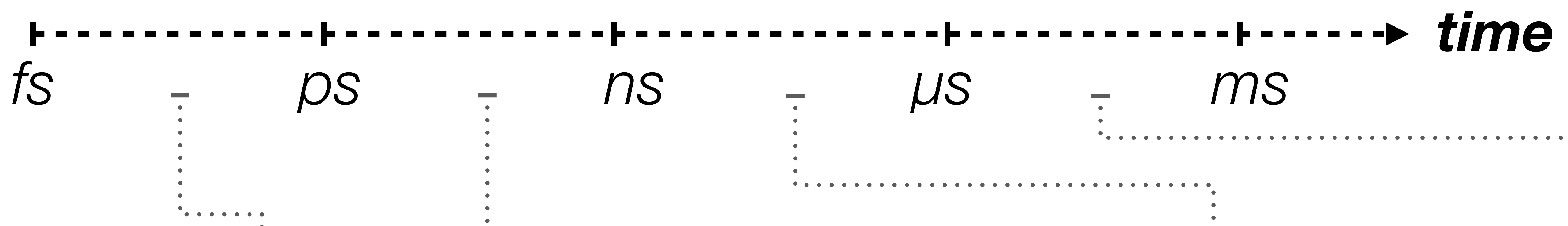

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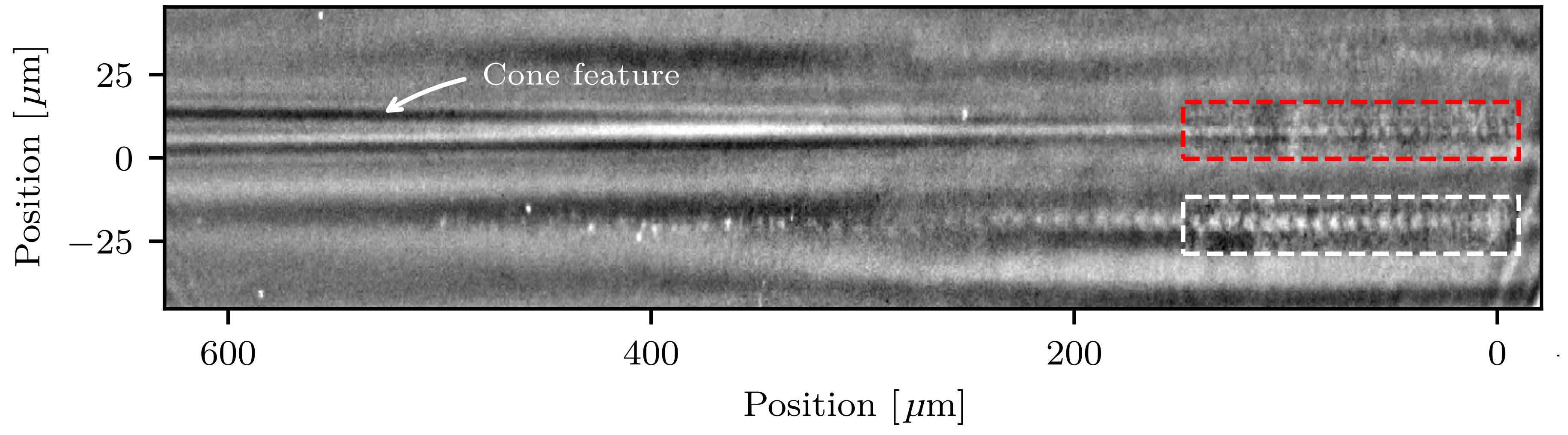
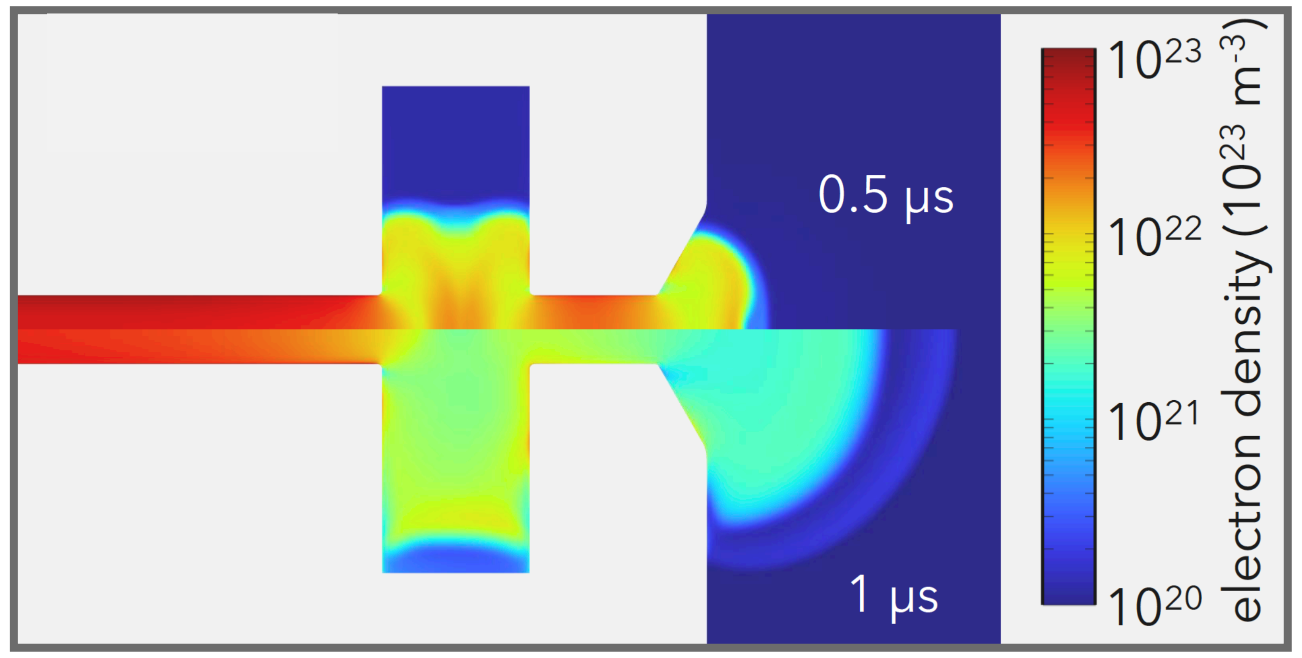


Source: Courtesy of M. Mewes

Simulation of plasma expulsion



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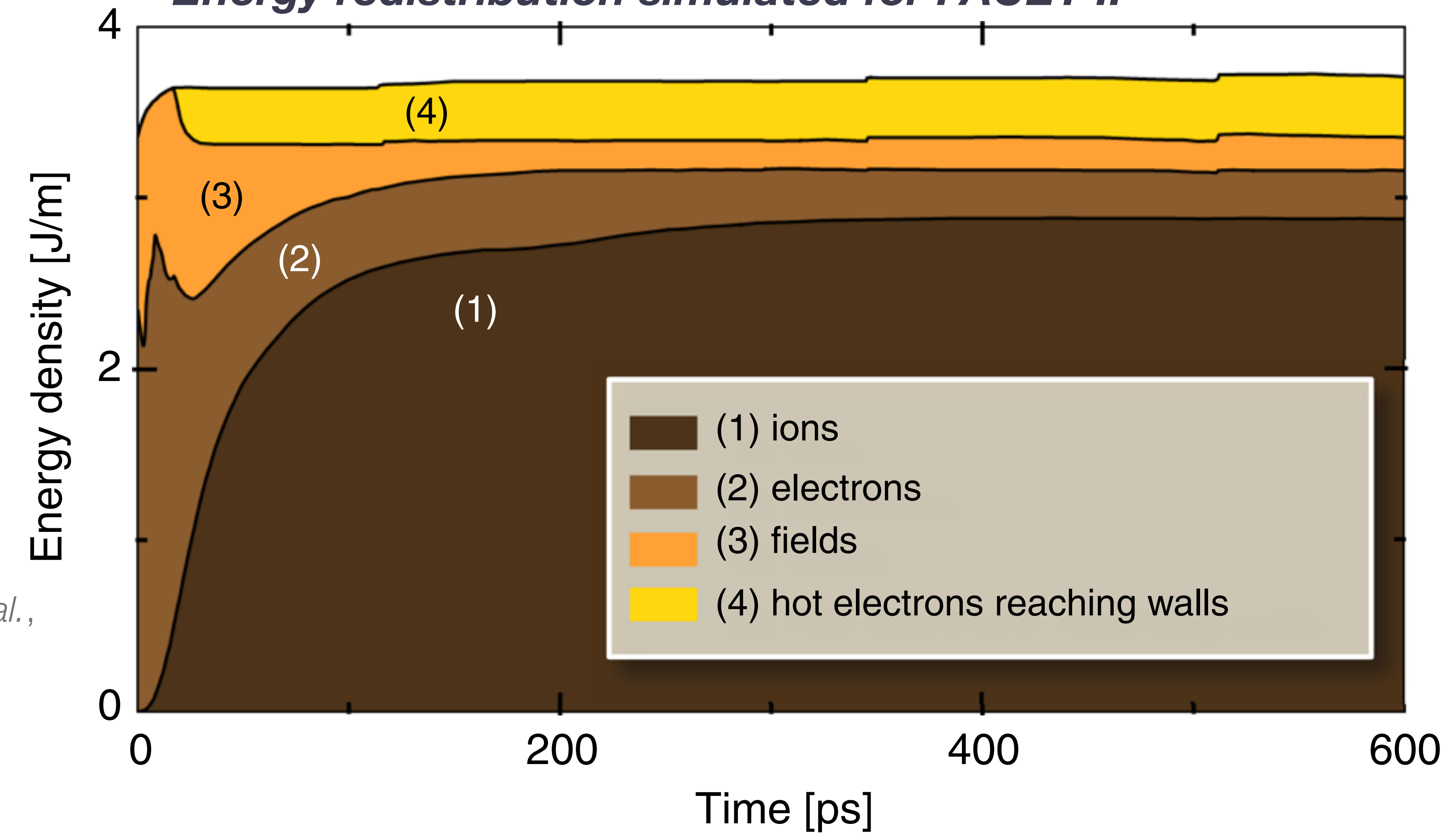

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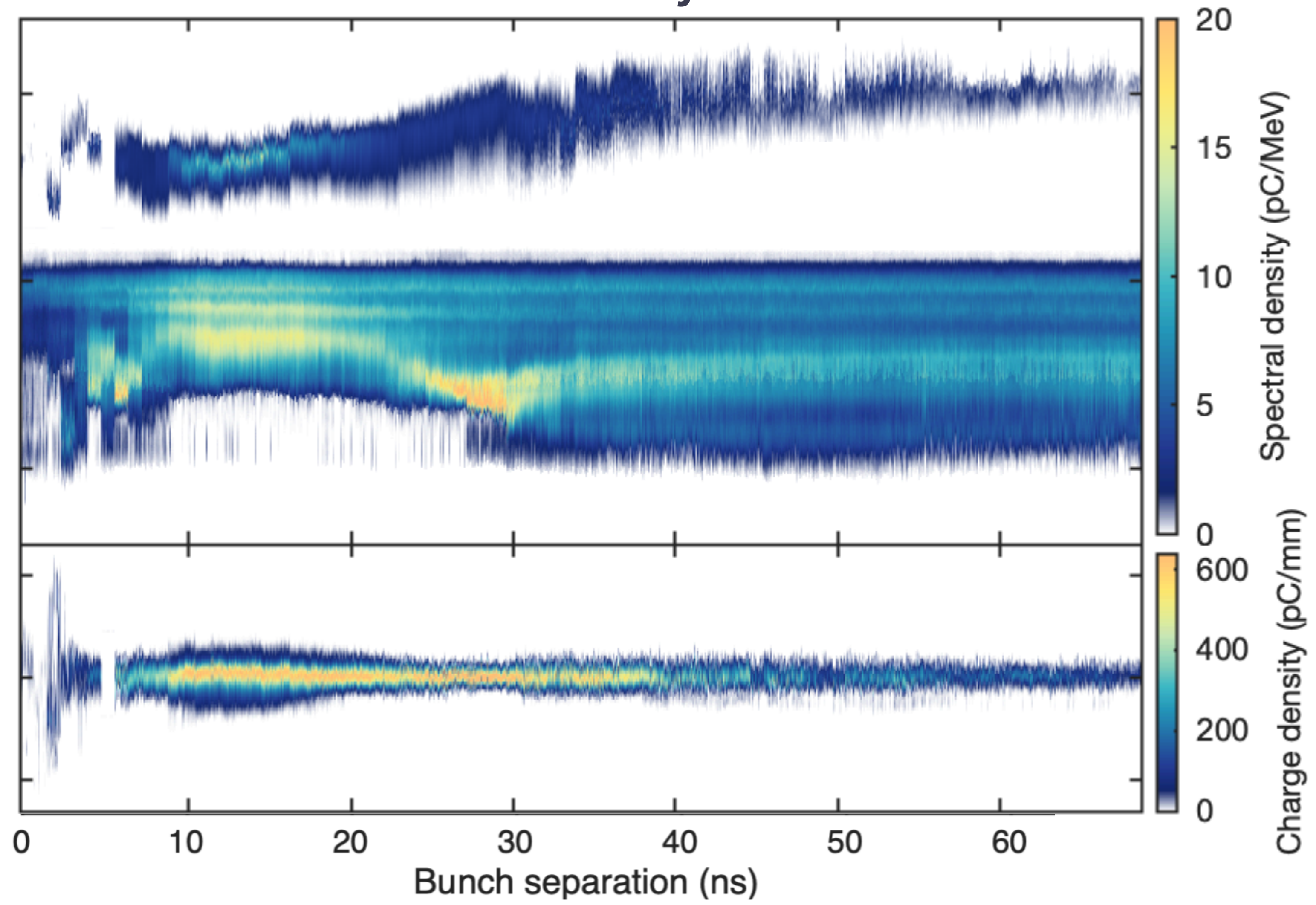

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- **Goal:** Simulate *long timescales* of plasma evolution after the intense & nonlinear perturbation from a driver
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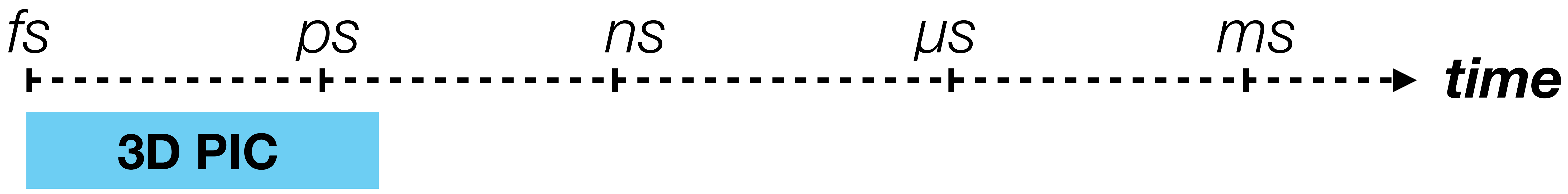

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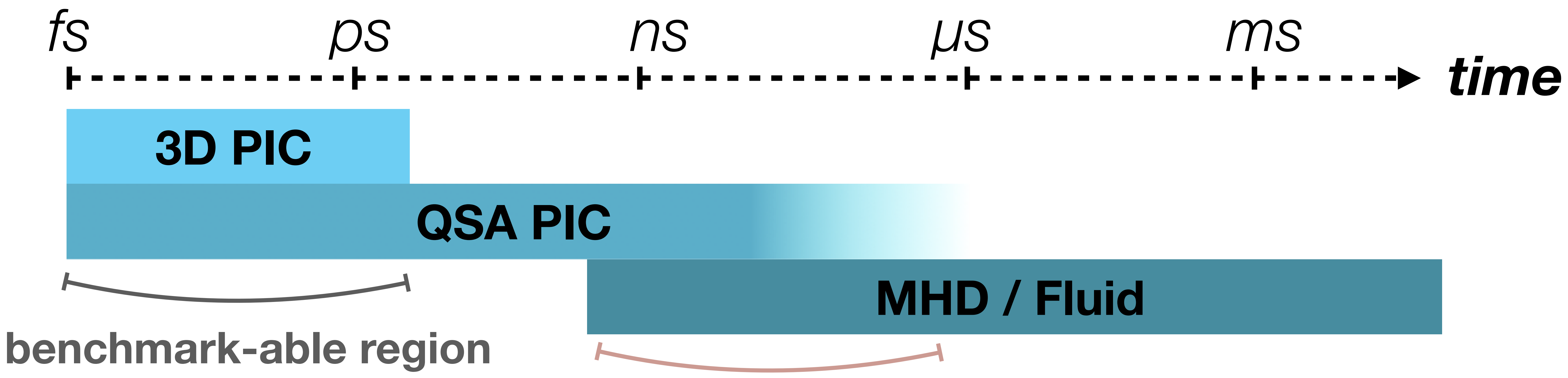
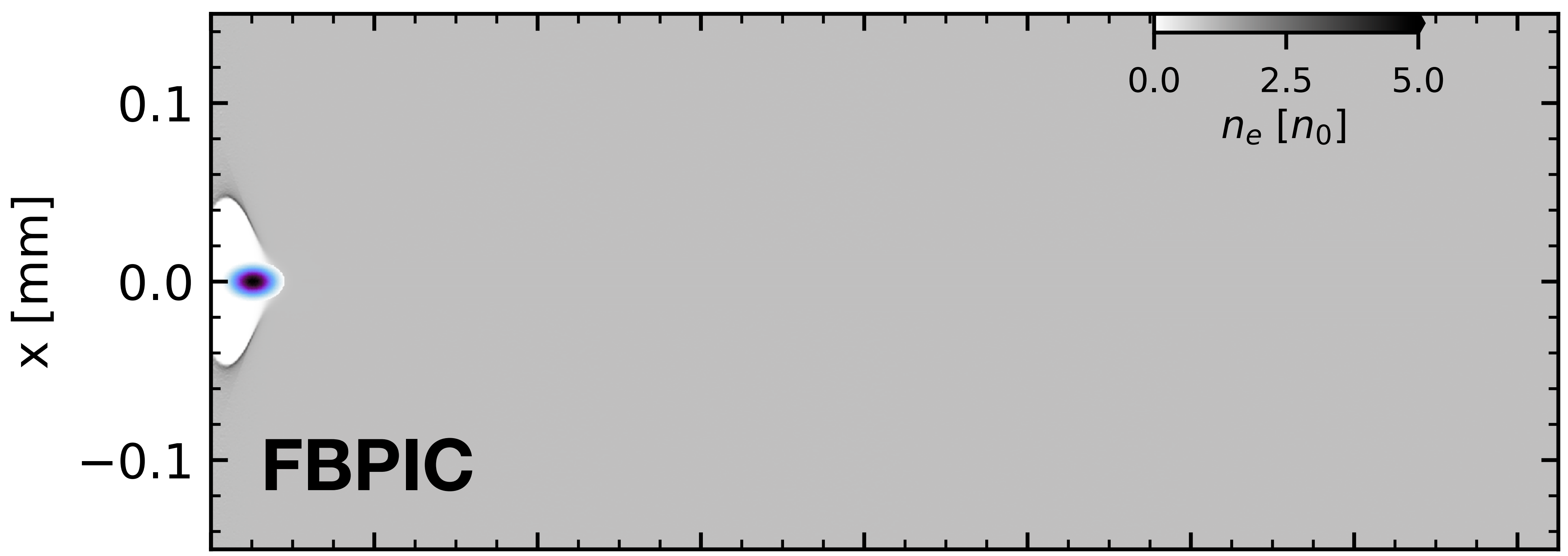
- **Middle Ground:** The Quasi-Static Approximation (QSA)
  - Used to separate the timescales involved in plasma evolution and driver evolution
  - A single timestep in driver evolution allows us to focus on the plasma evolution!



Can the QSA accurately describe extended timescales?



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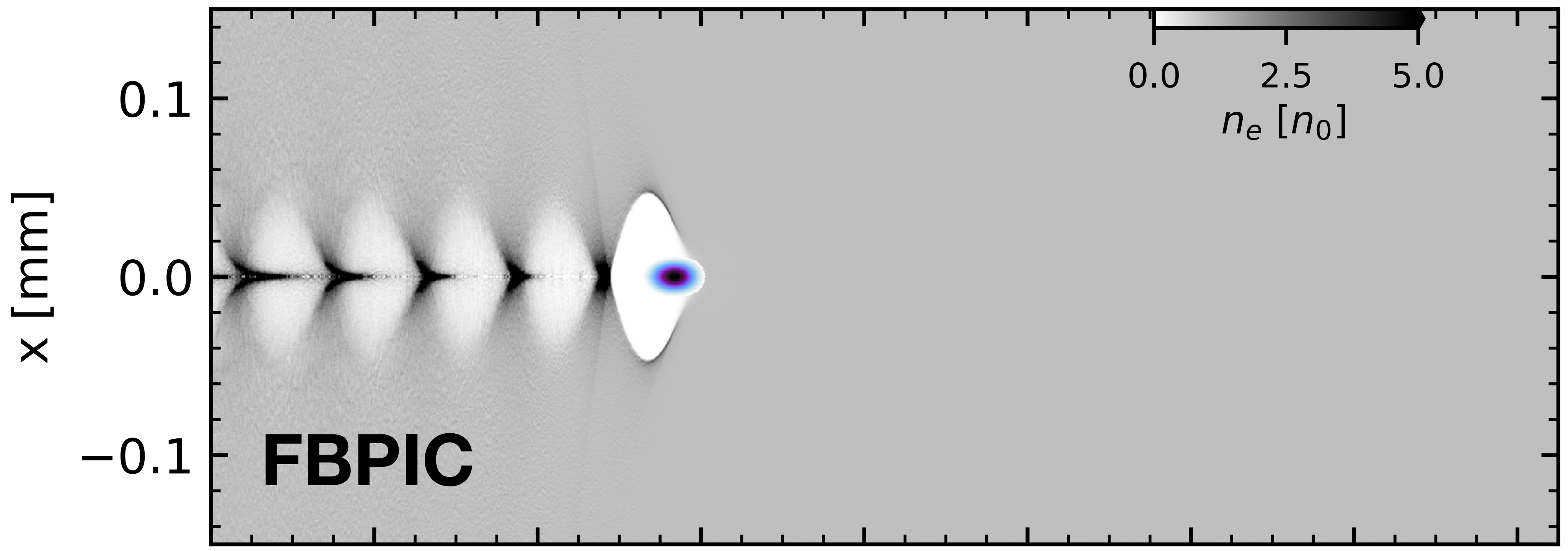

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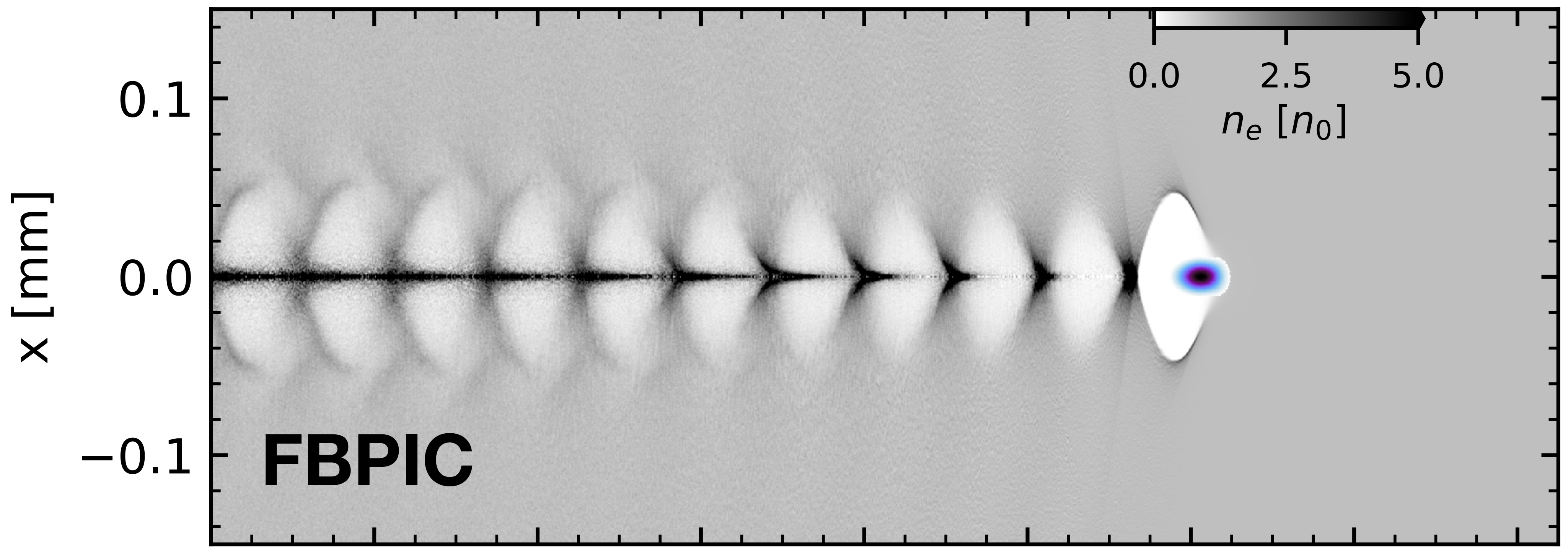

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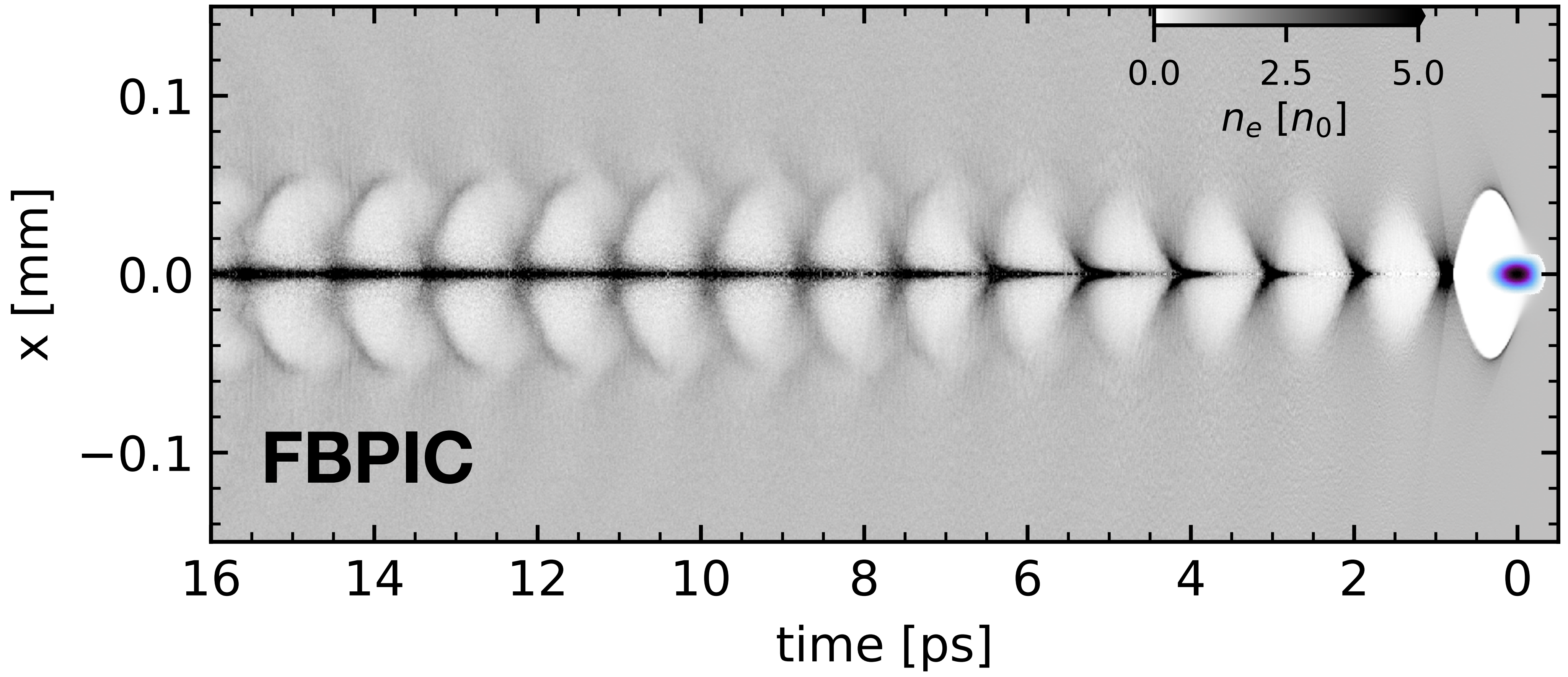

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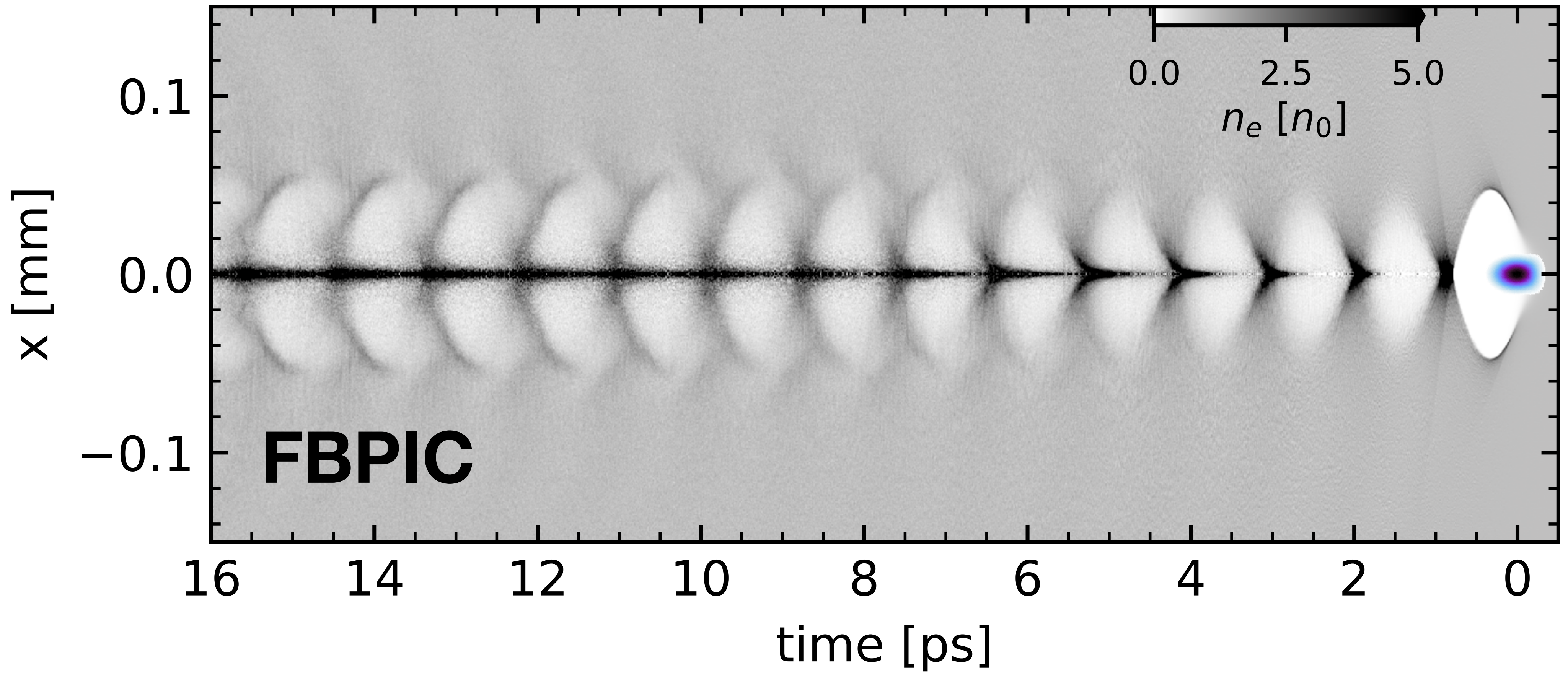

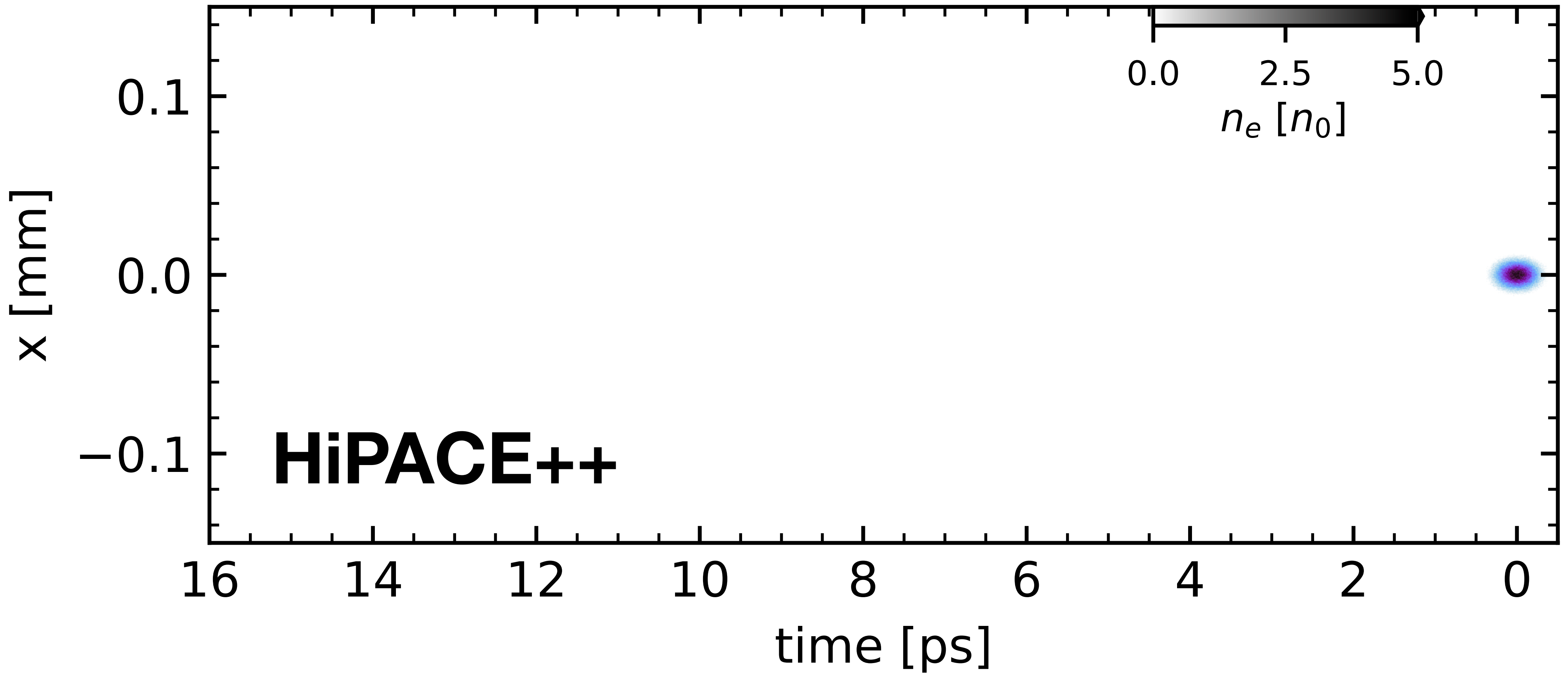

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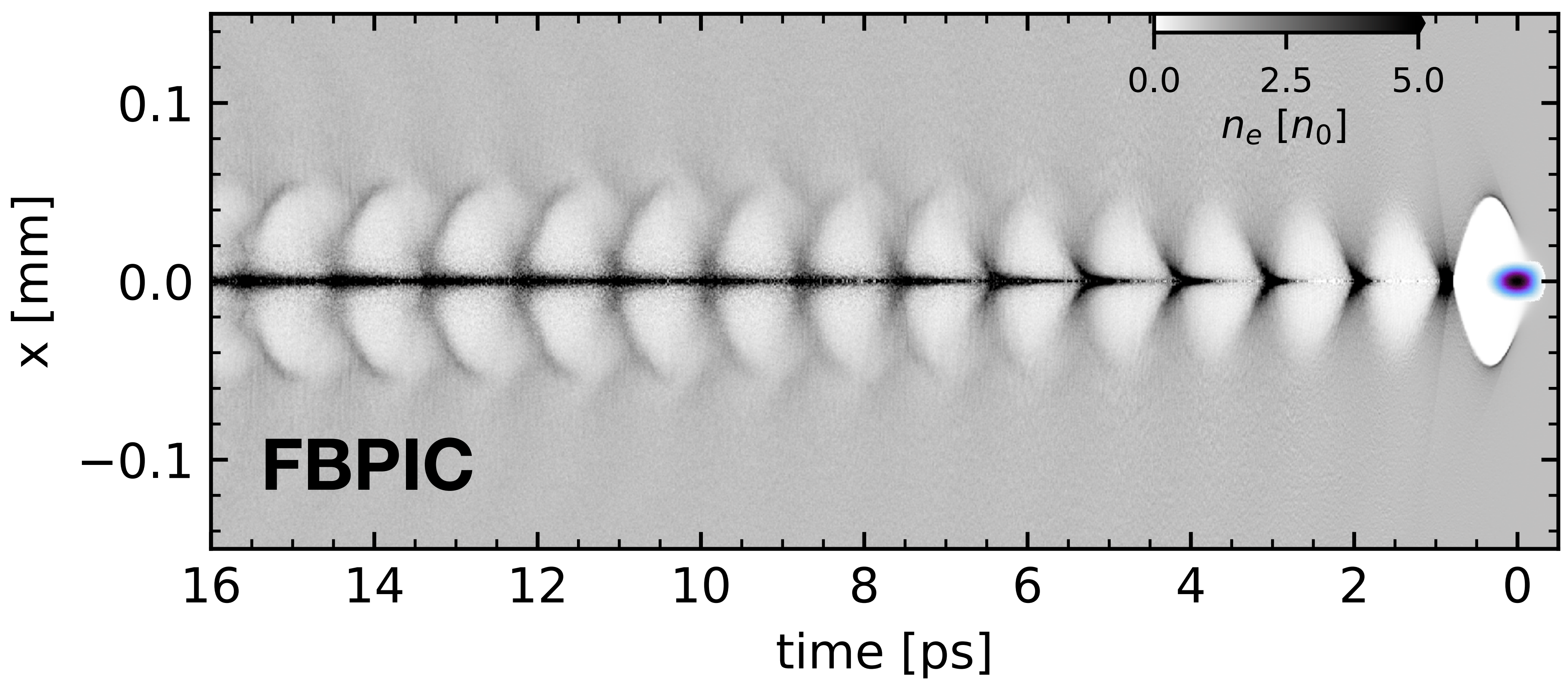

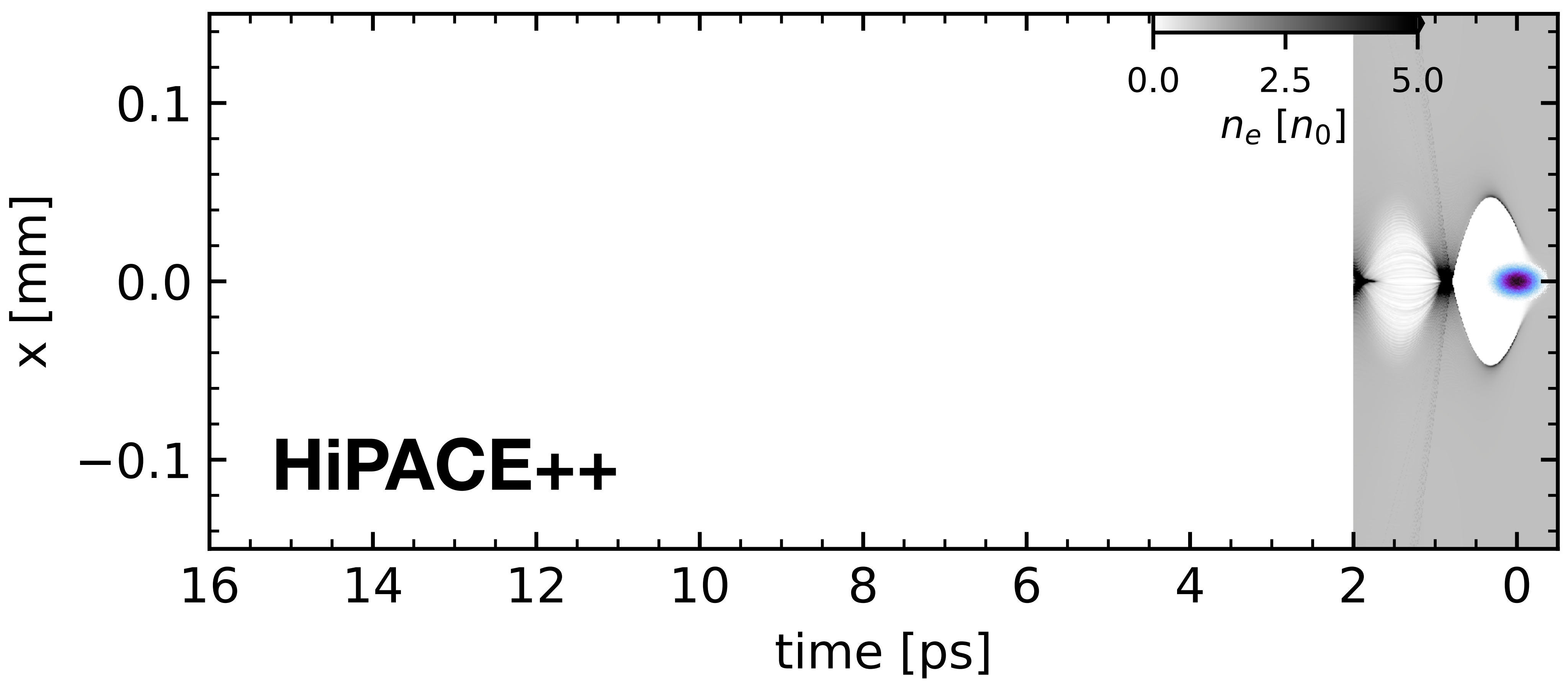

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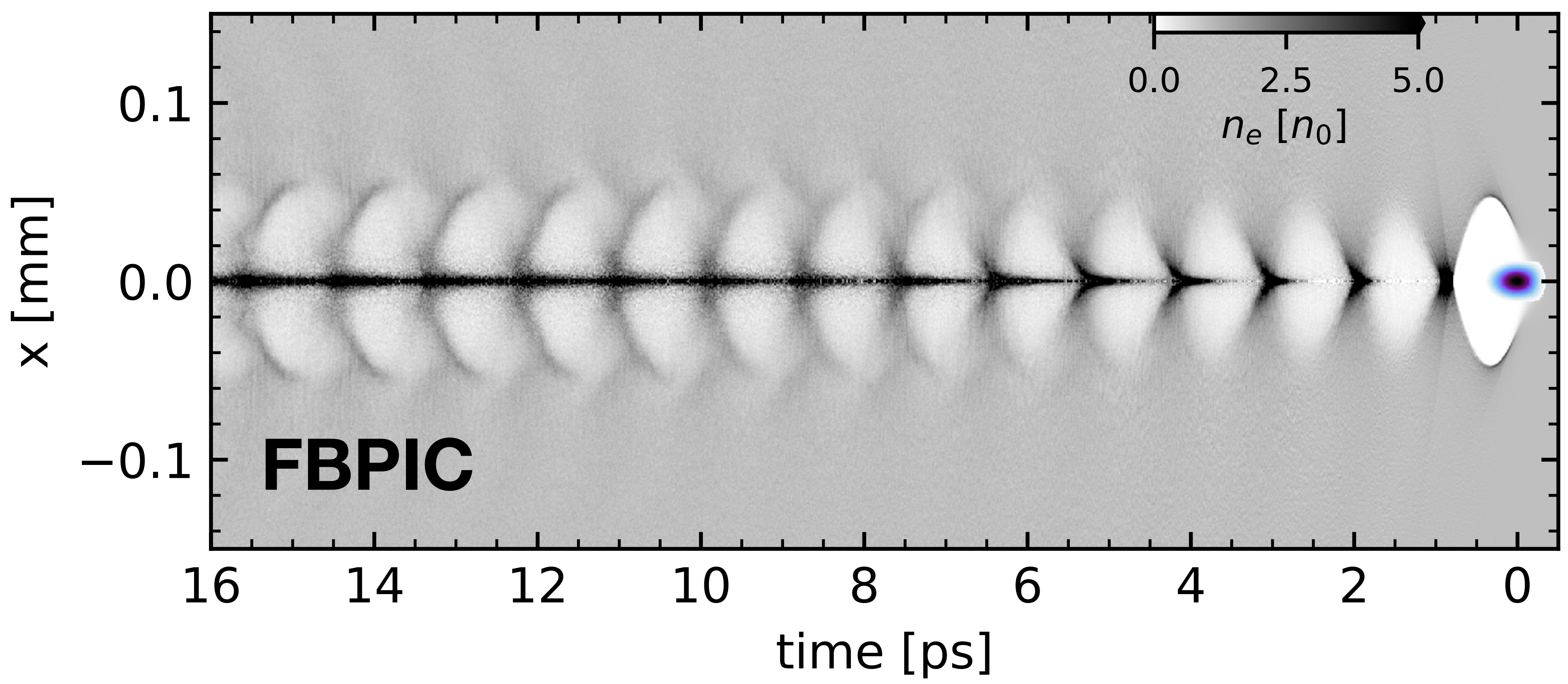

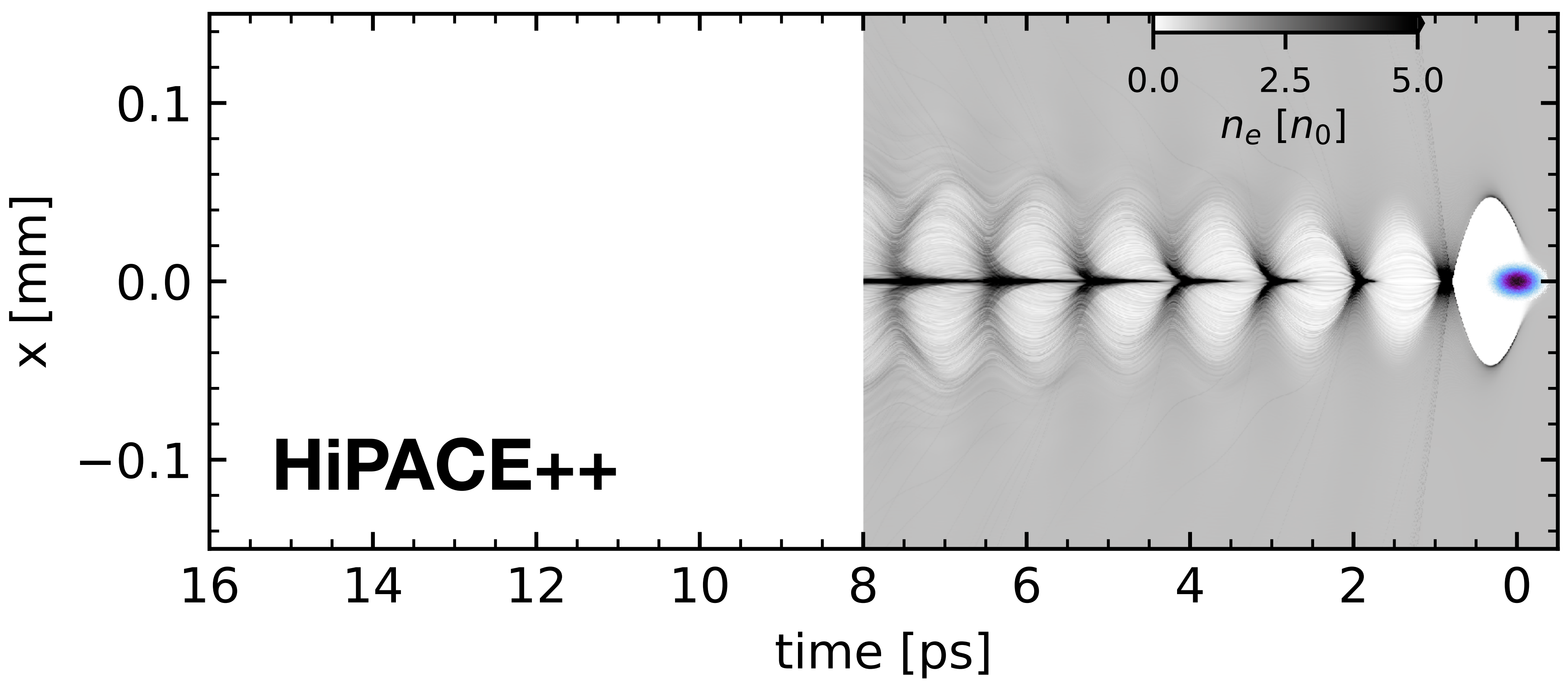

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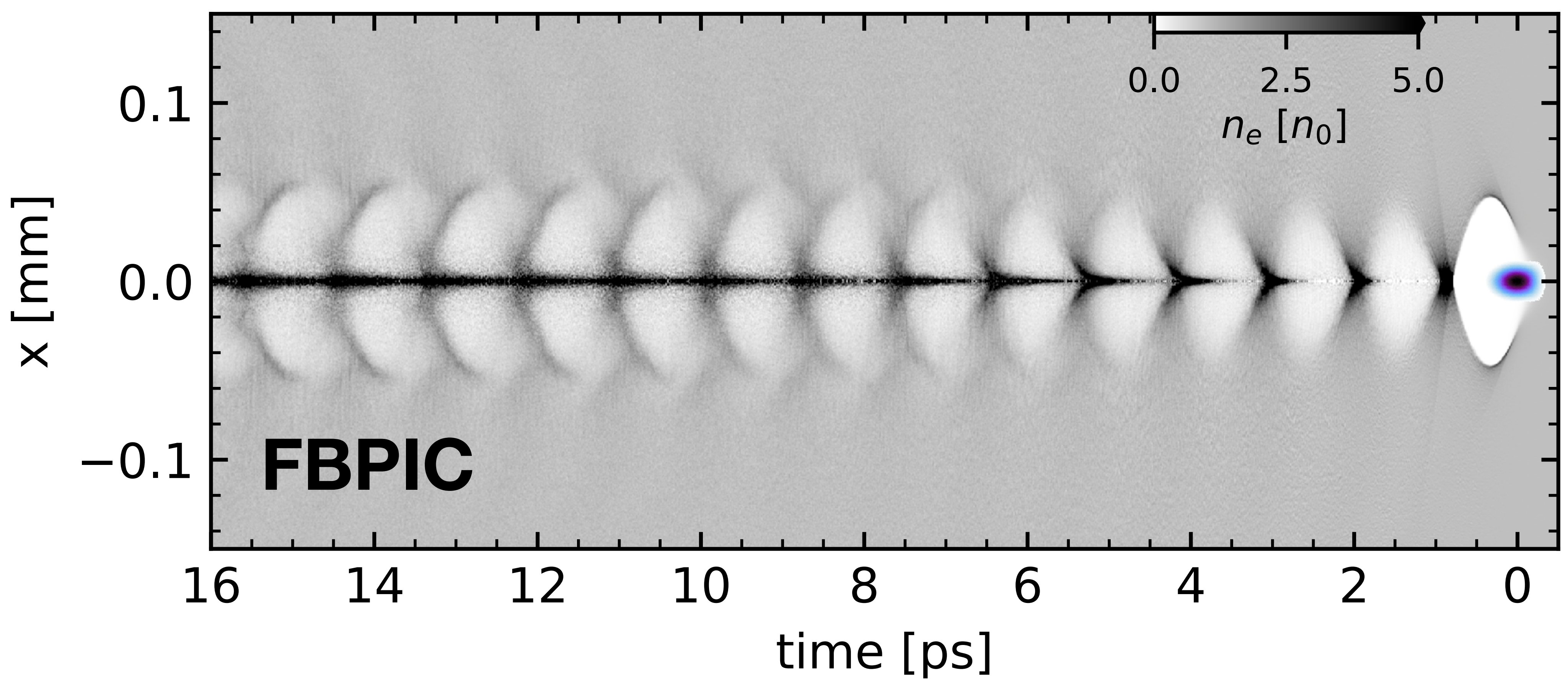

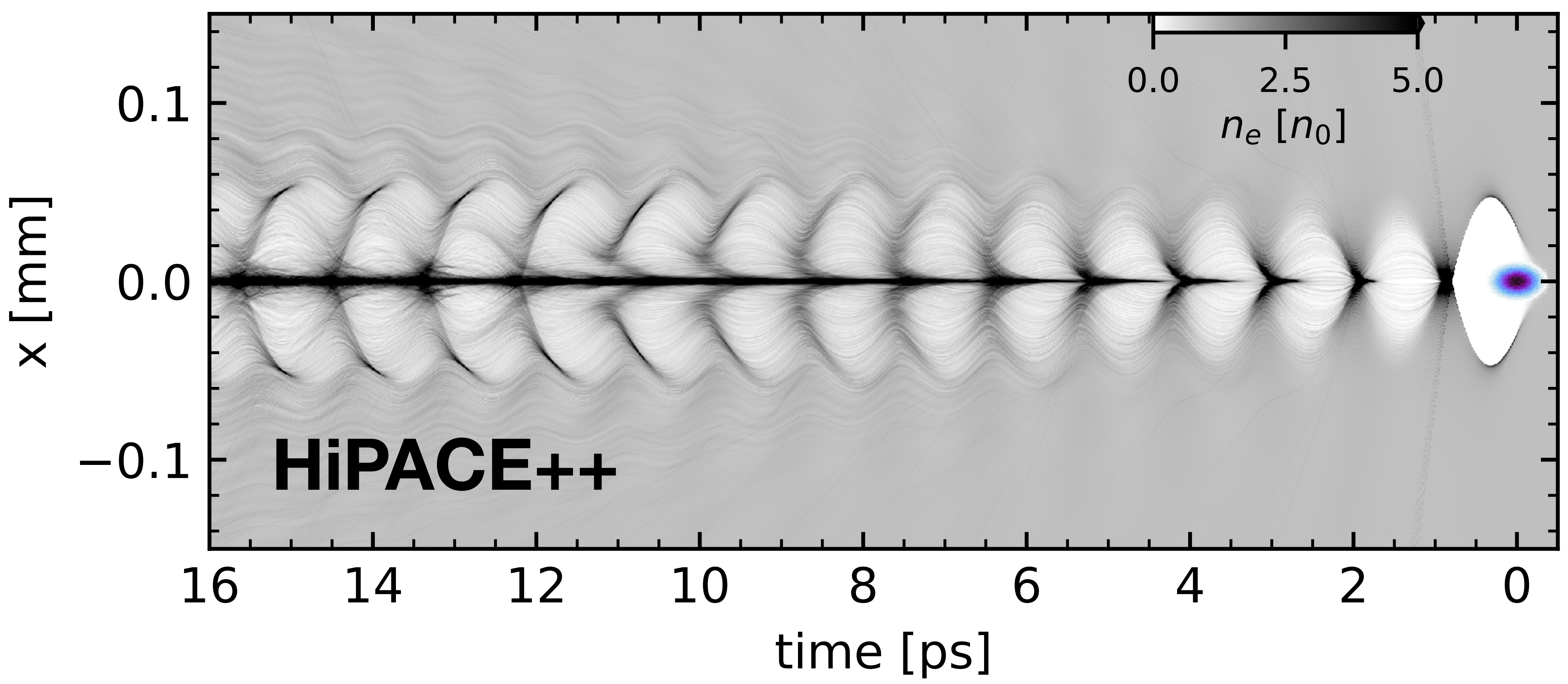

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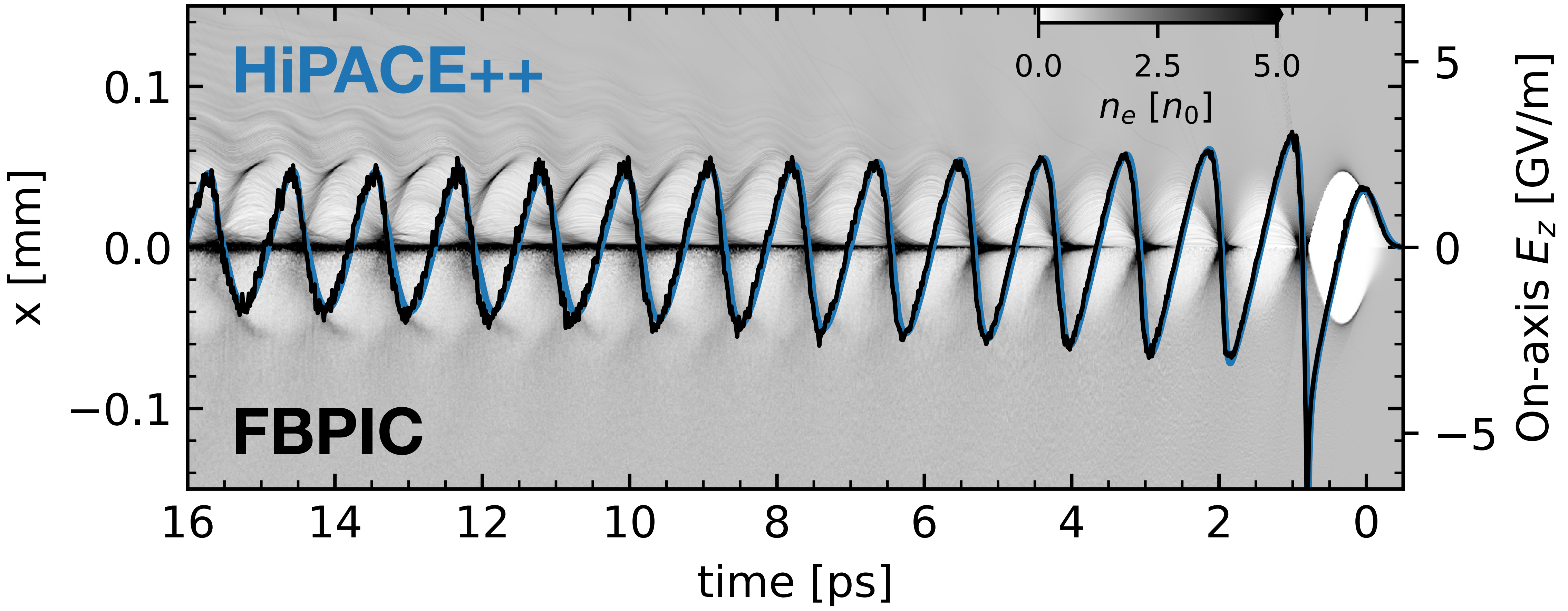

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> - Typically, comparisons of QSA-PIC codes ensure that correct accelerating fields are produced in the first bubble
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>   - Well-matched and/or slowly-evolving drivers are used — $\sigma_r^2 = \epsilon_n/(\gamma k_\beta)$
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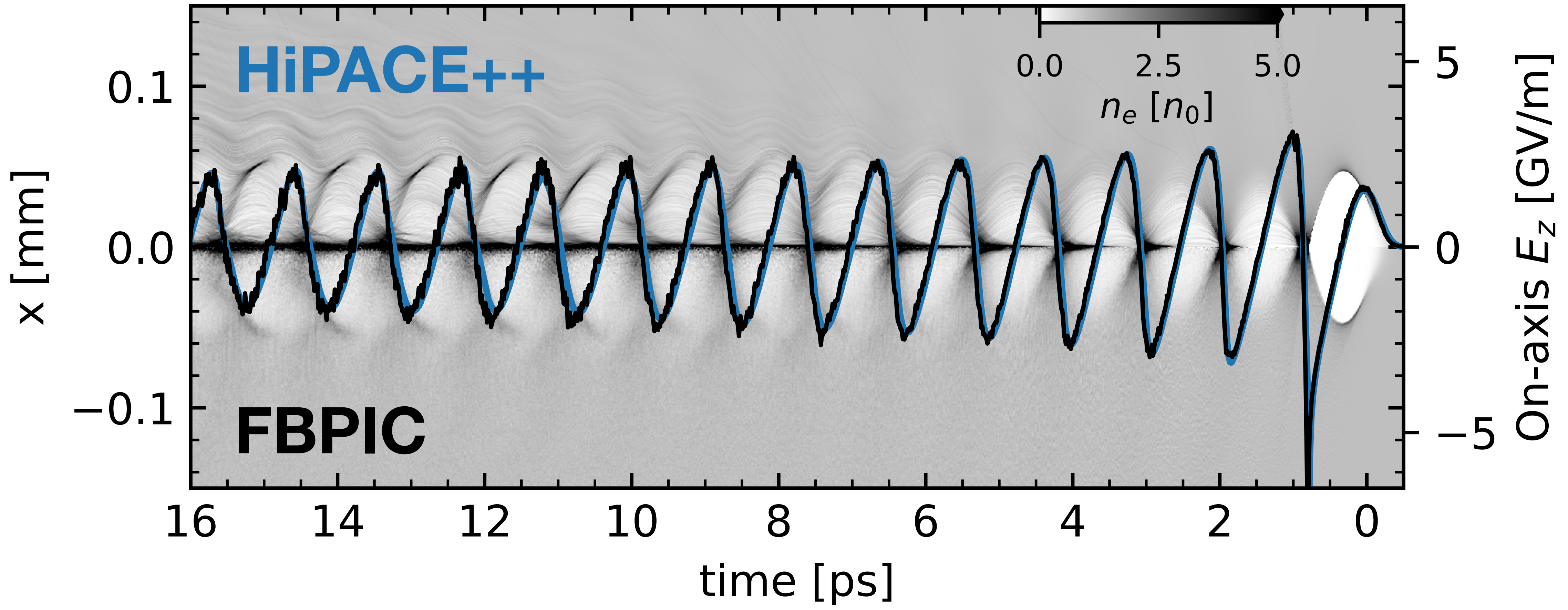

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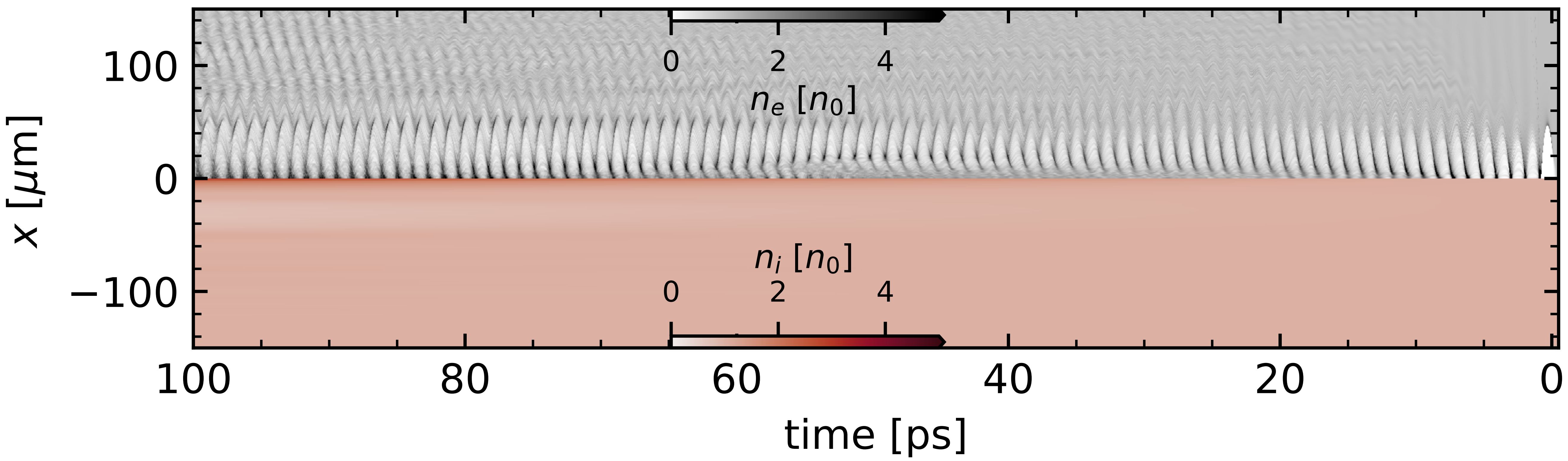
- **Advantages:**
  - 1. Allows long timescales to be simulated feasibly
  - 2. Allows fast parameter scans of plasma evolution at short-medium timescales

Extending to longer timescales with HiPACE++



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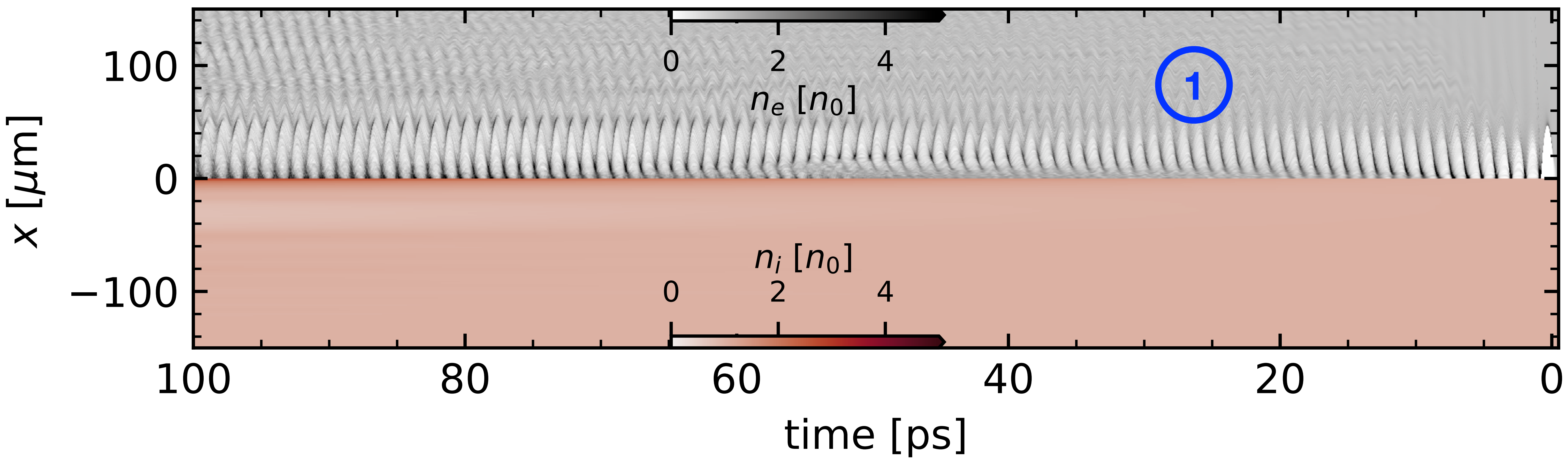

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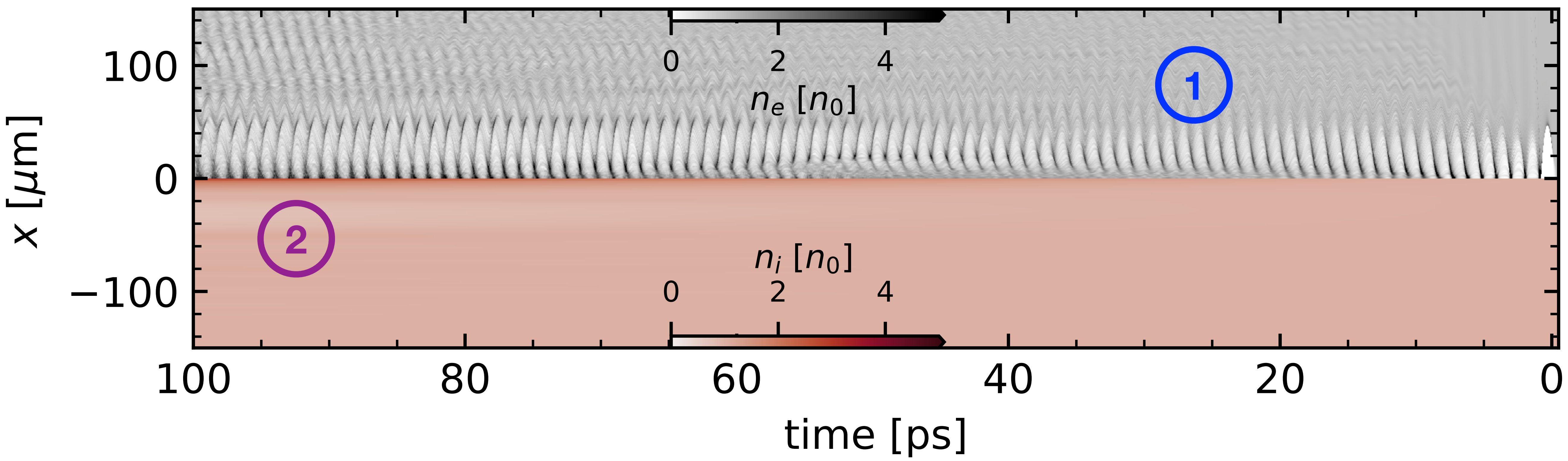

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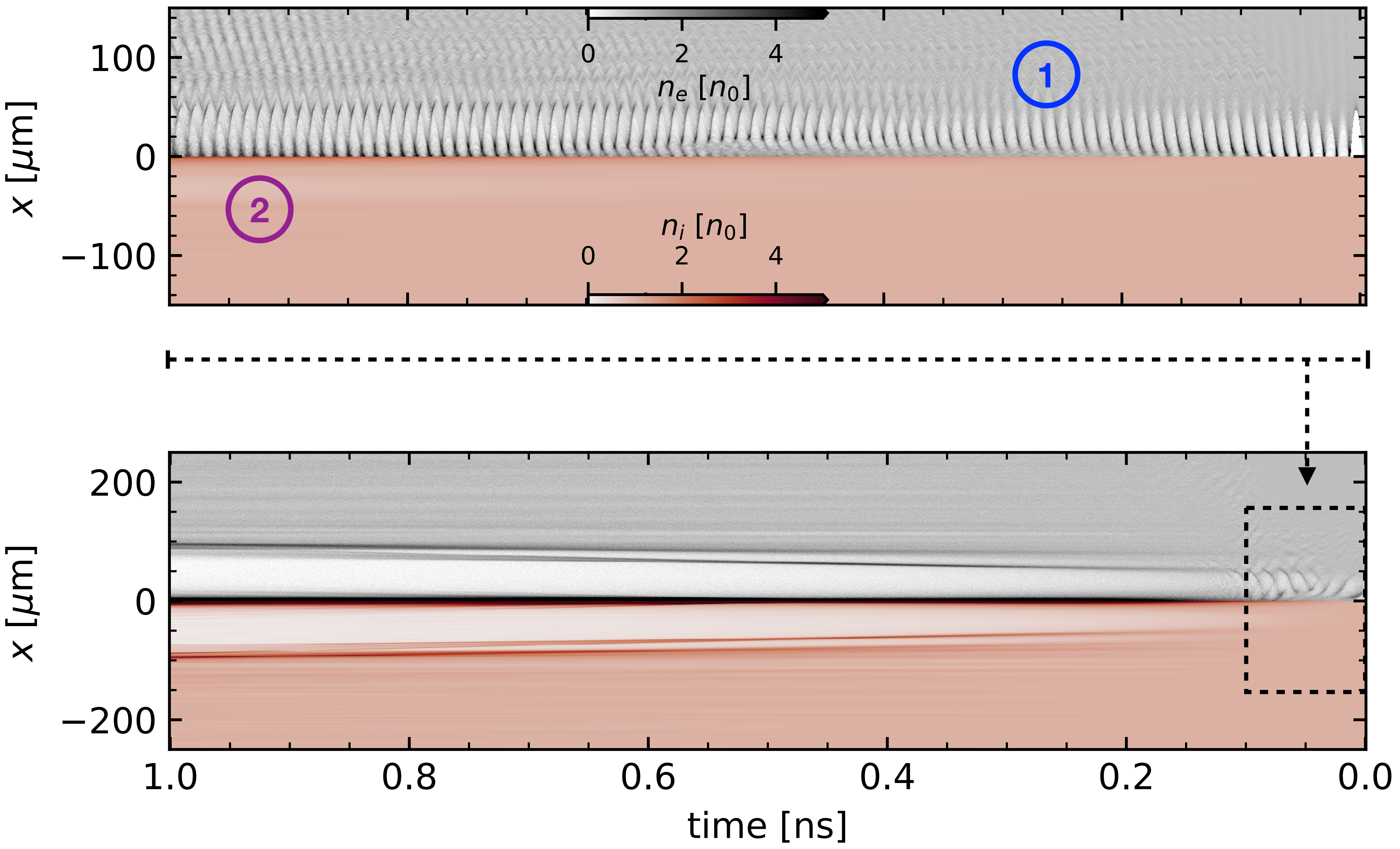

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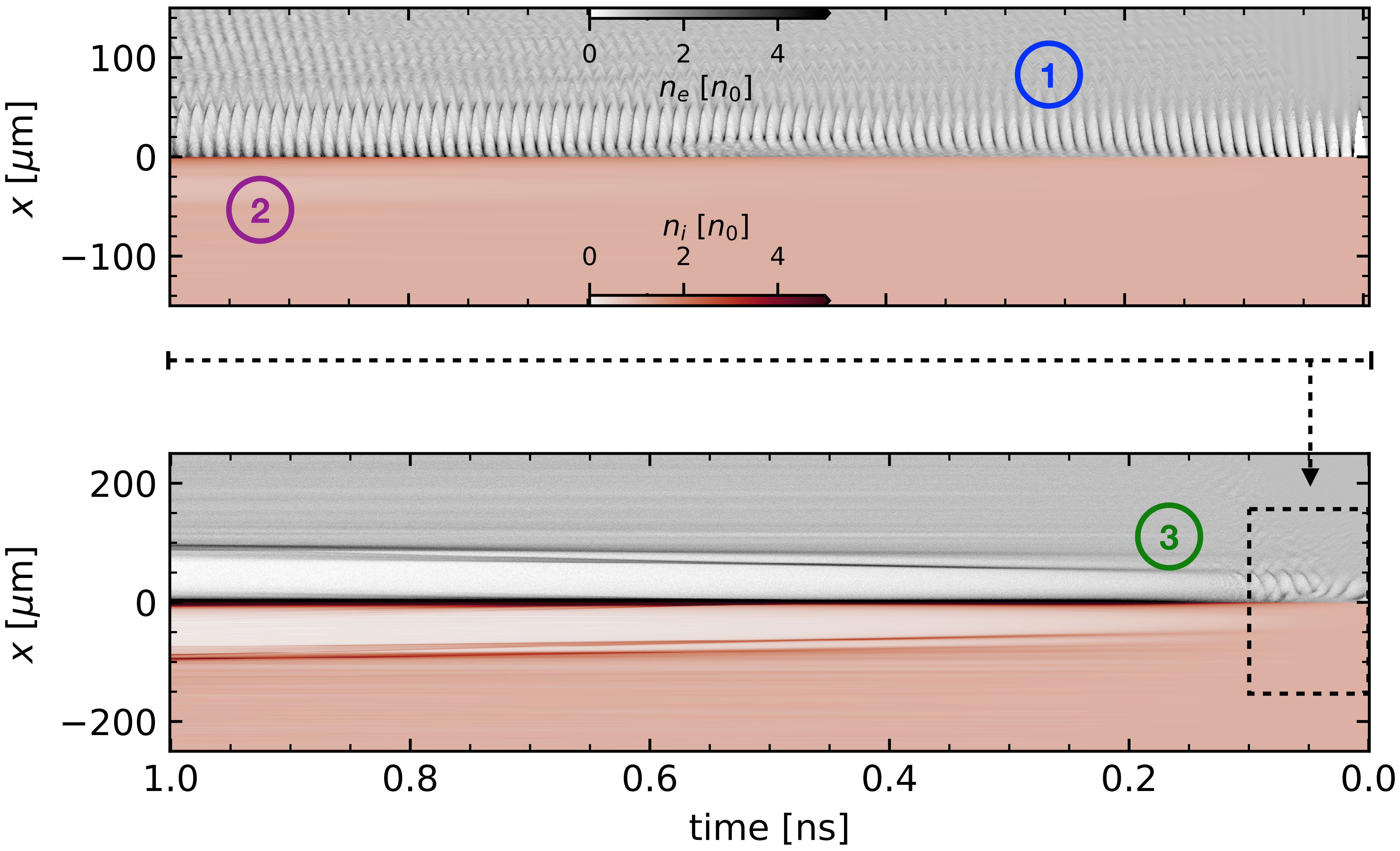

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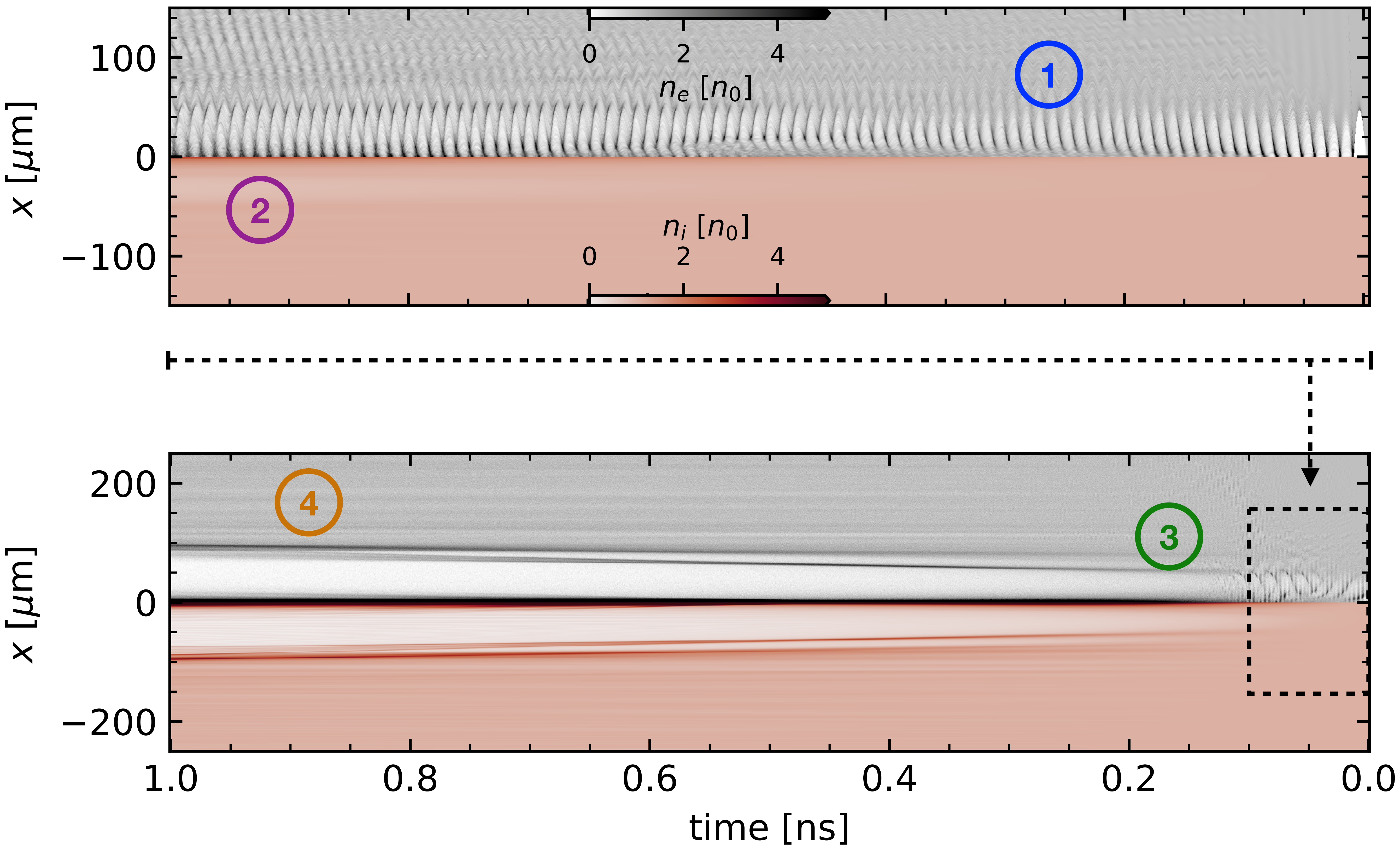

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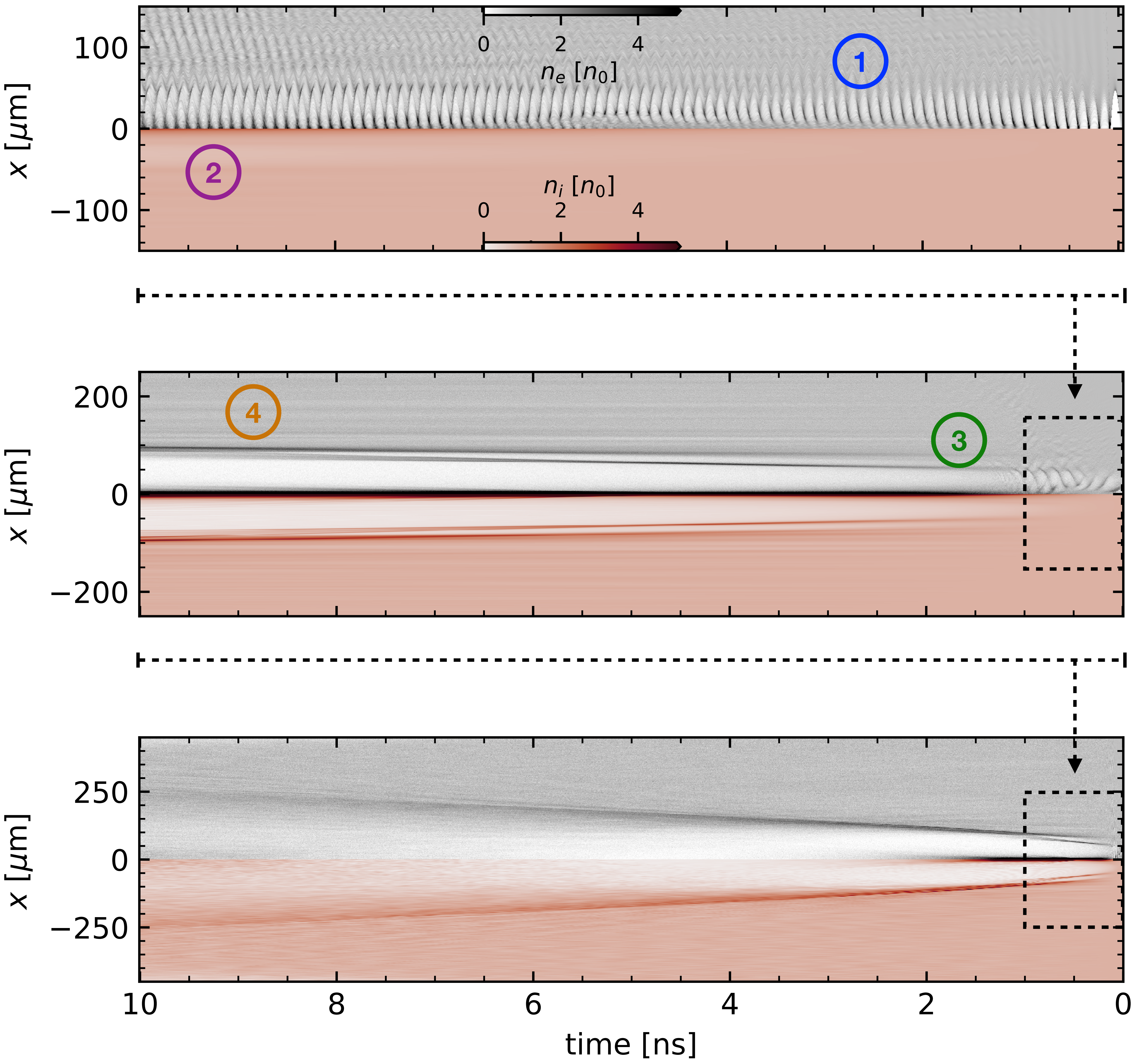

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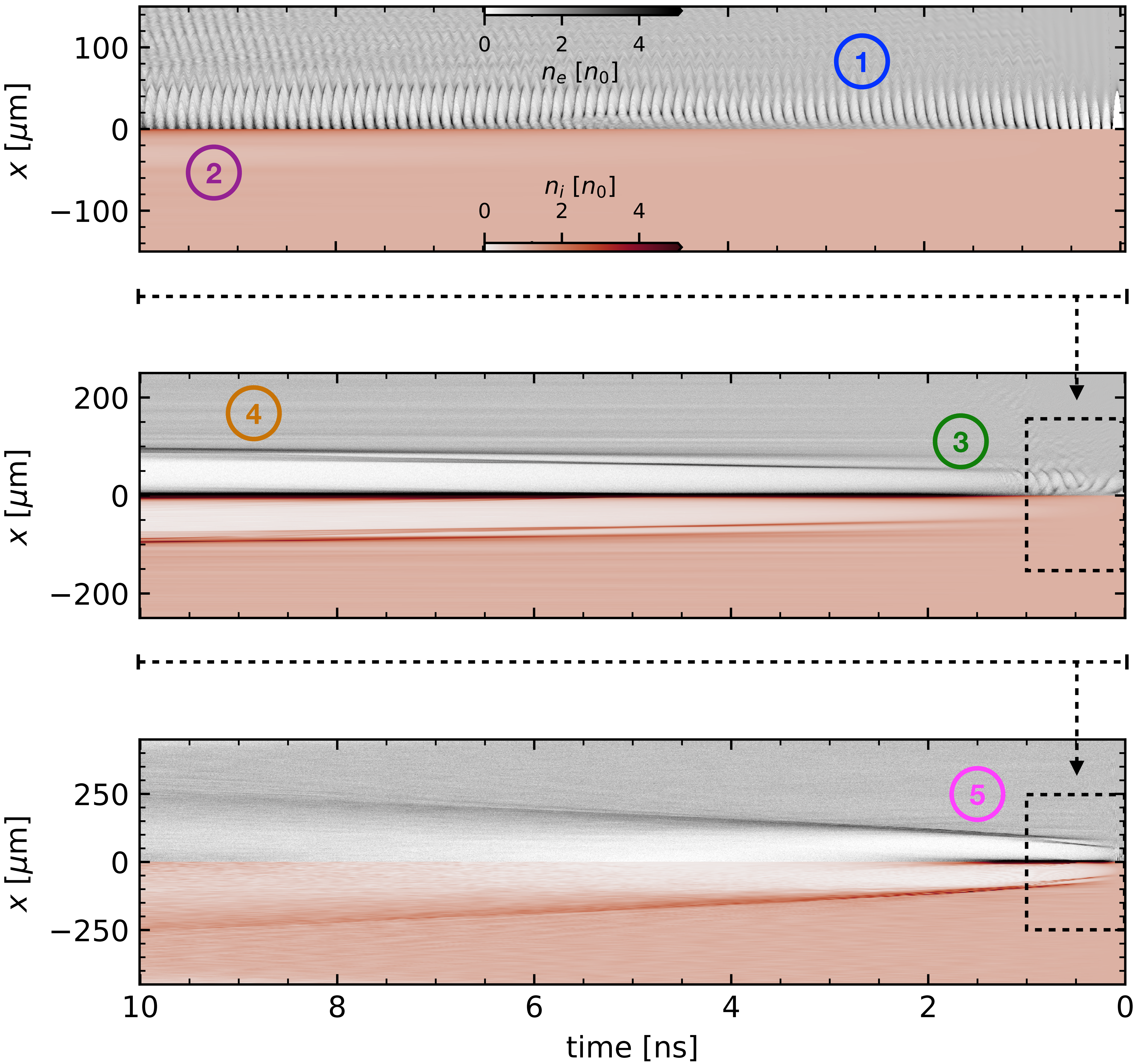

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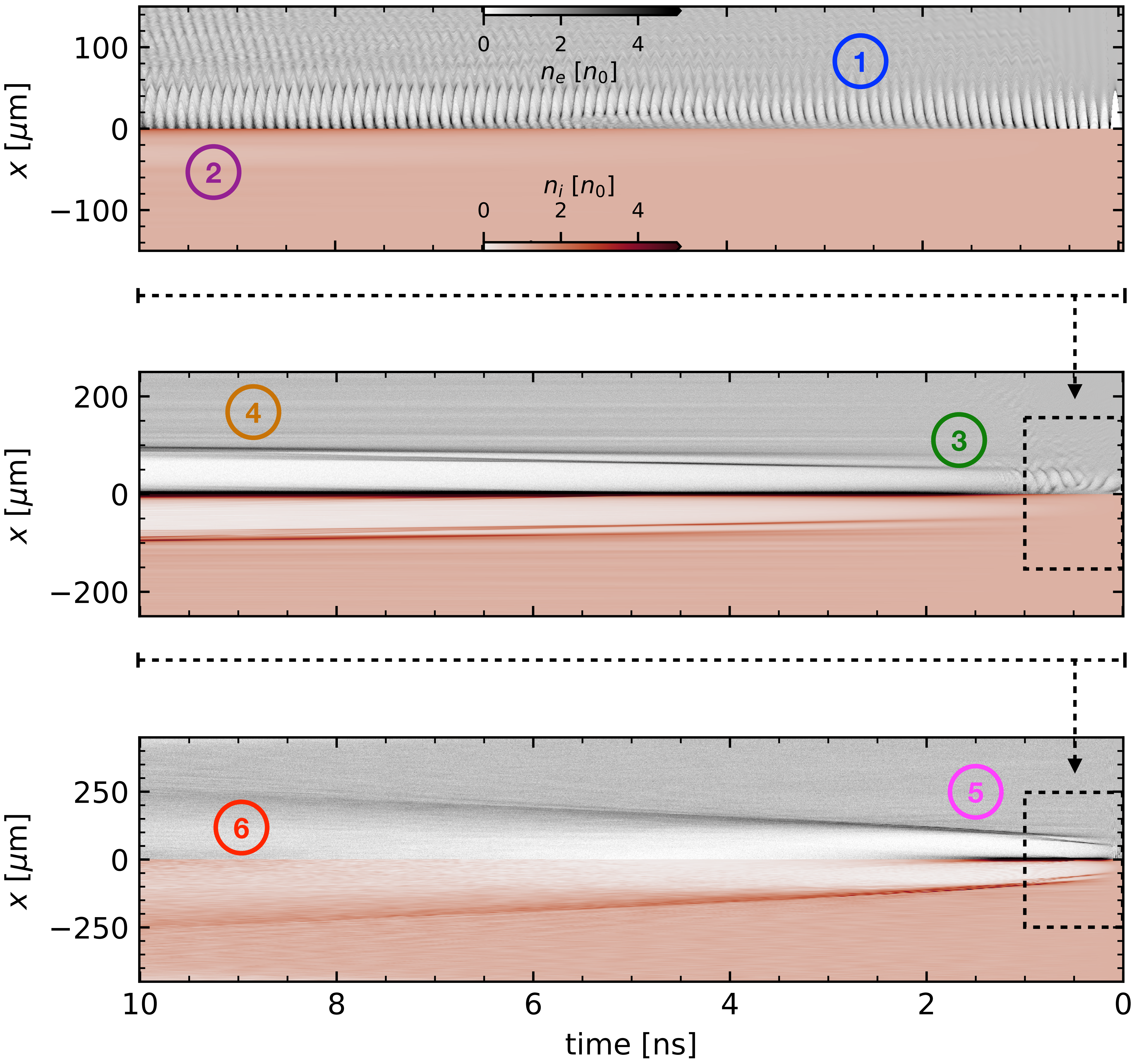
- > **6.** At very late timescales, expect plasma to begin to thermalise and equilibrate



Enabling qualitative scalings with ion species

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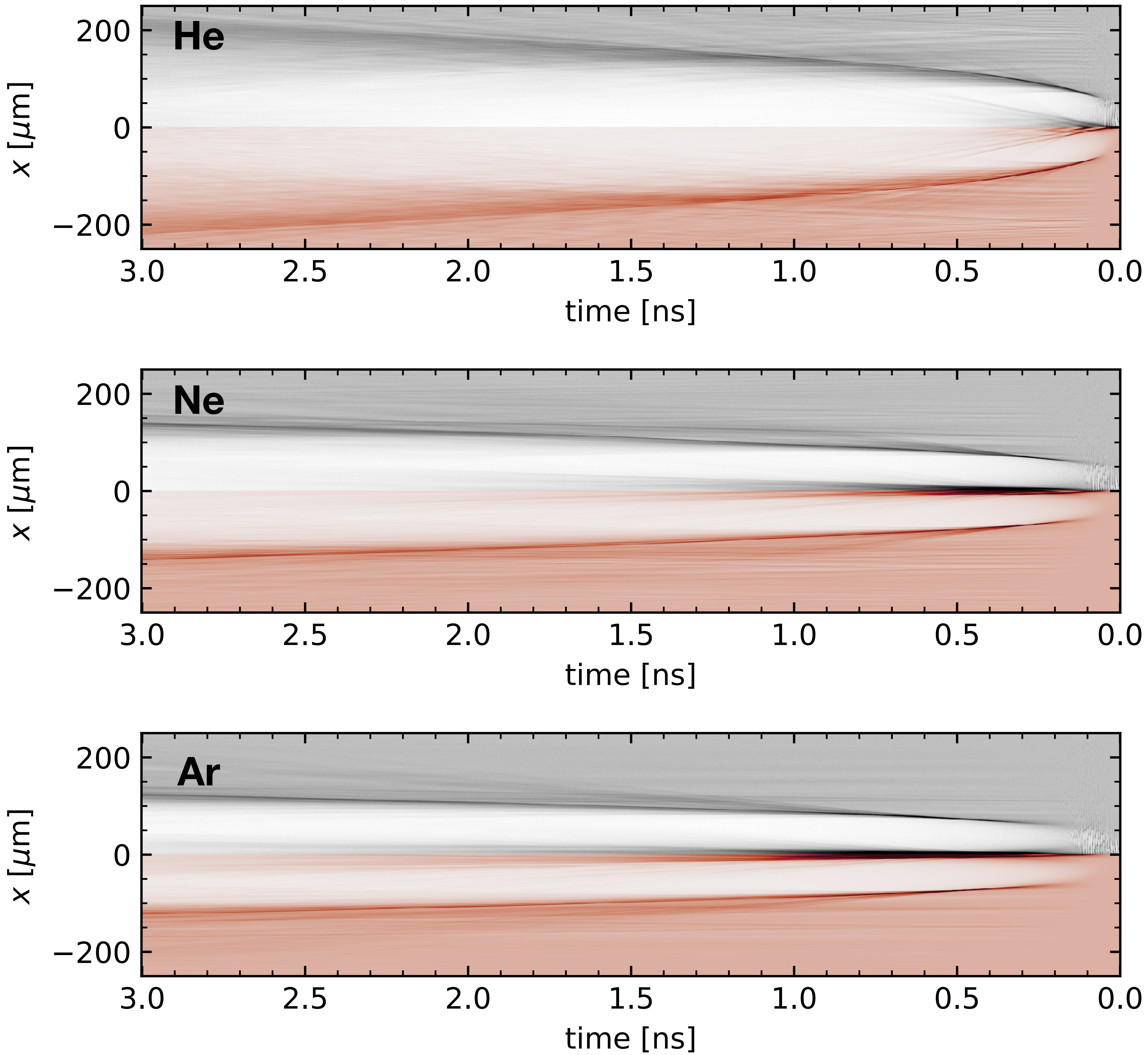

Enabling qualitative scalings with ion species

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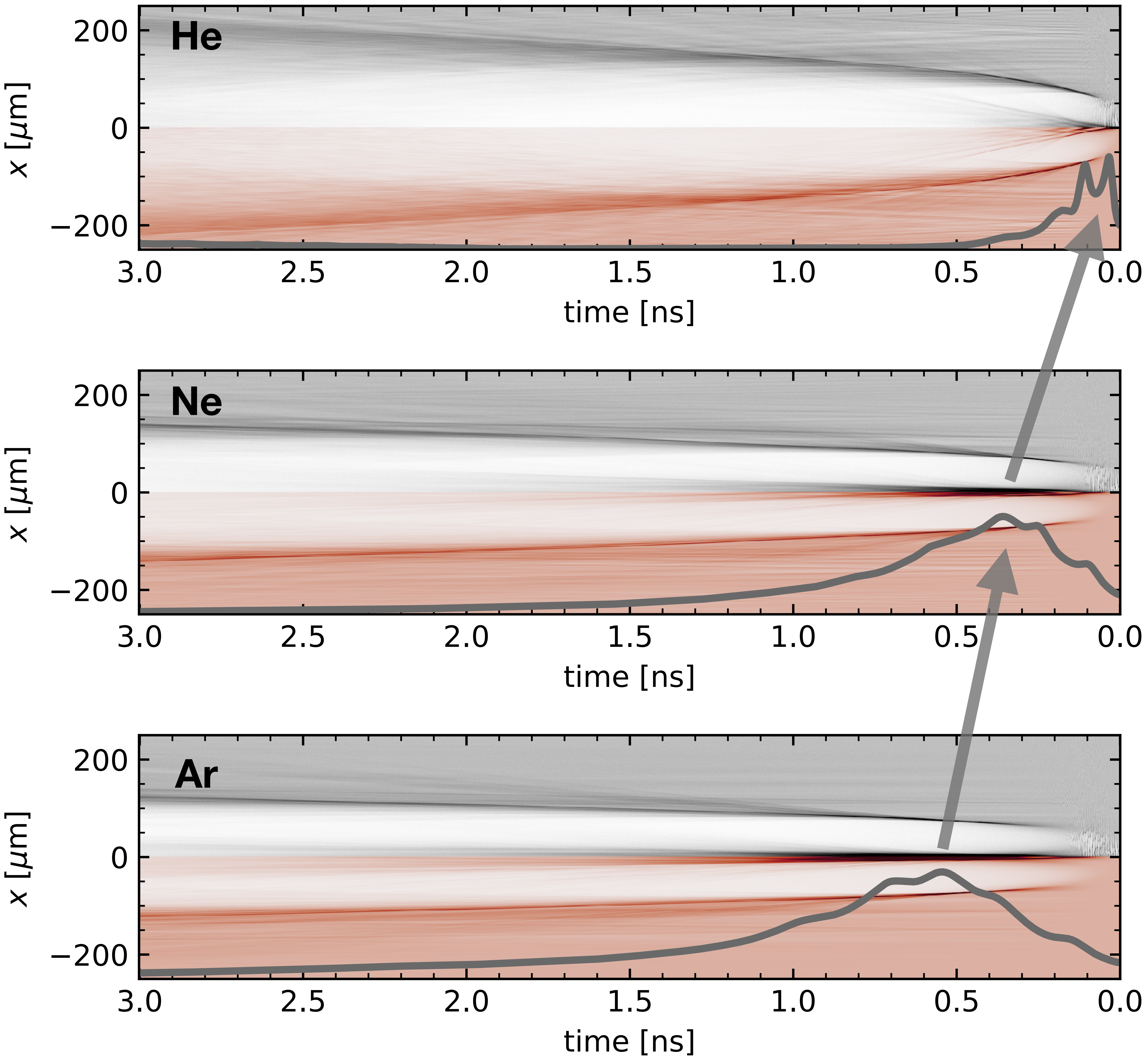

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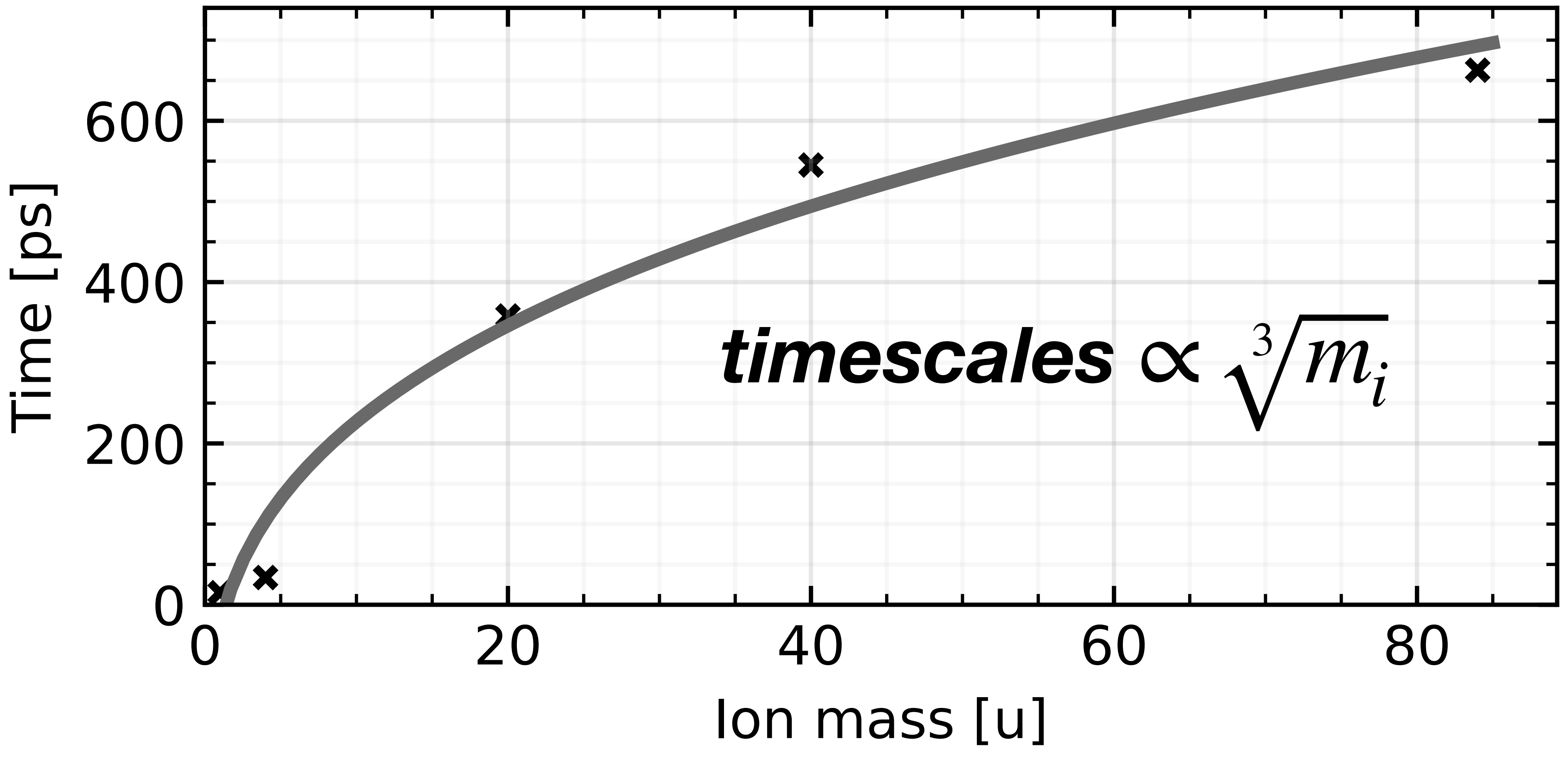
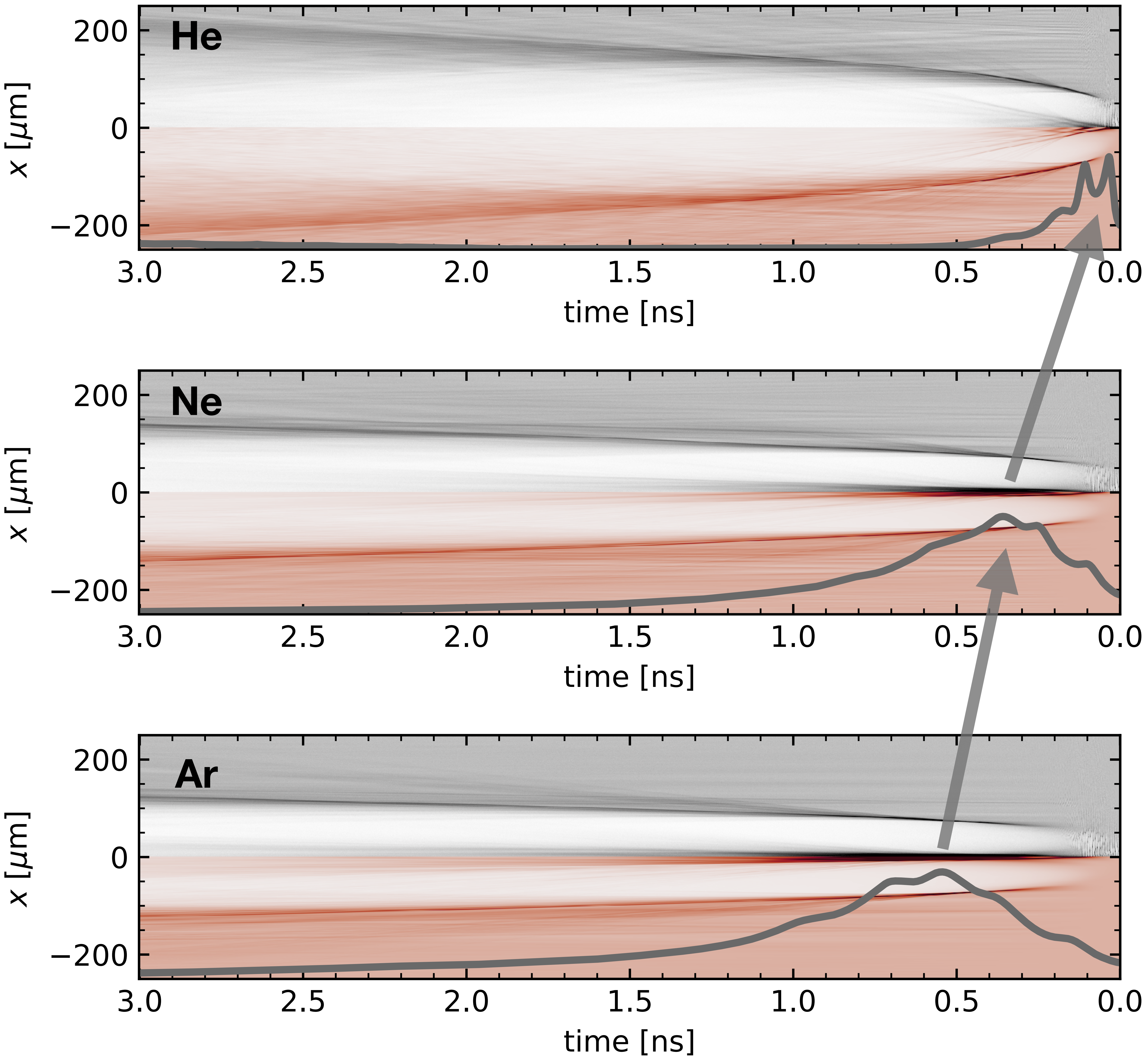
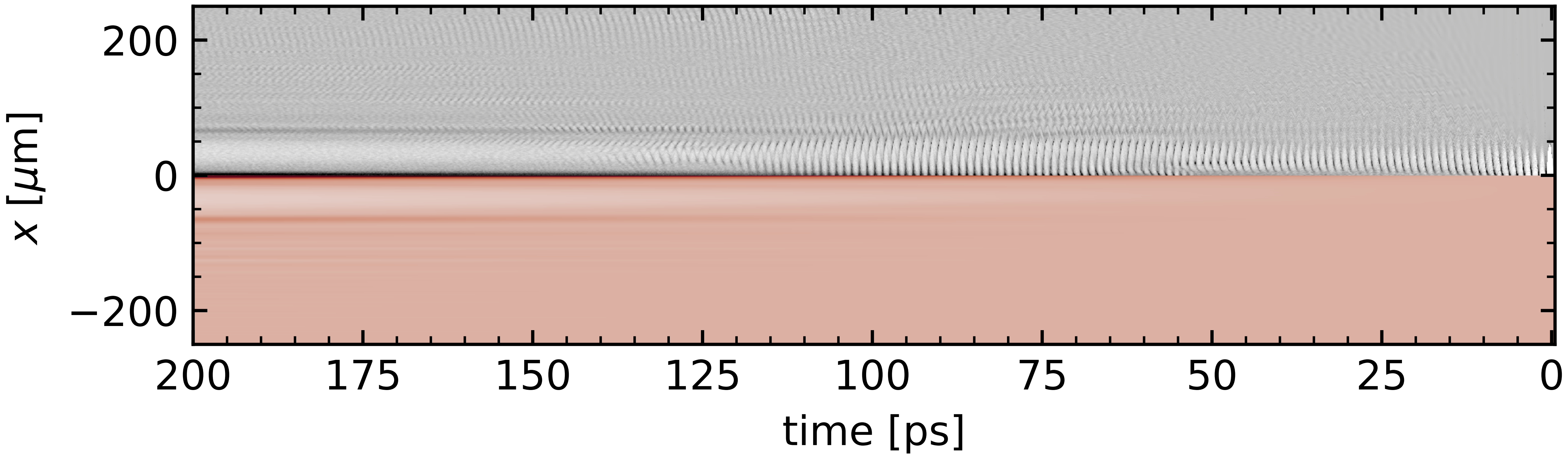




Understanding longer term effects by looking at the shorter term



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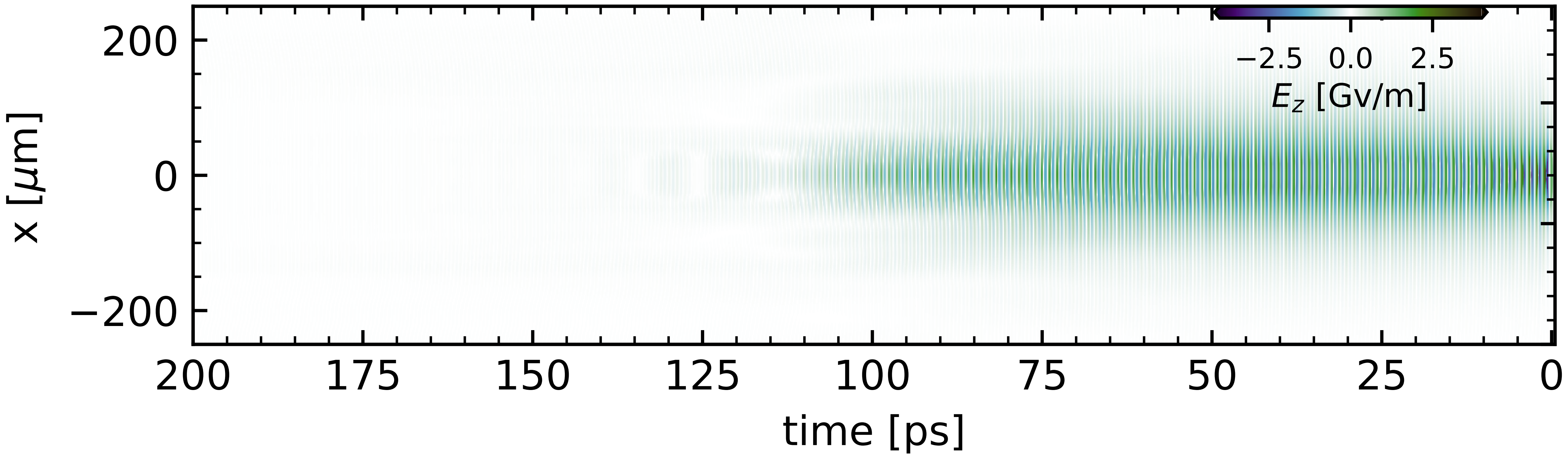

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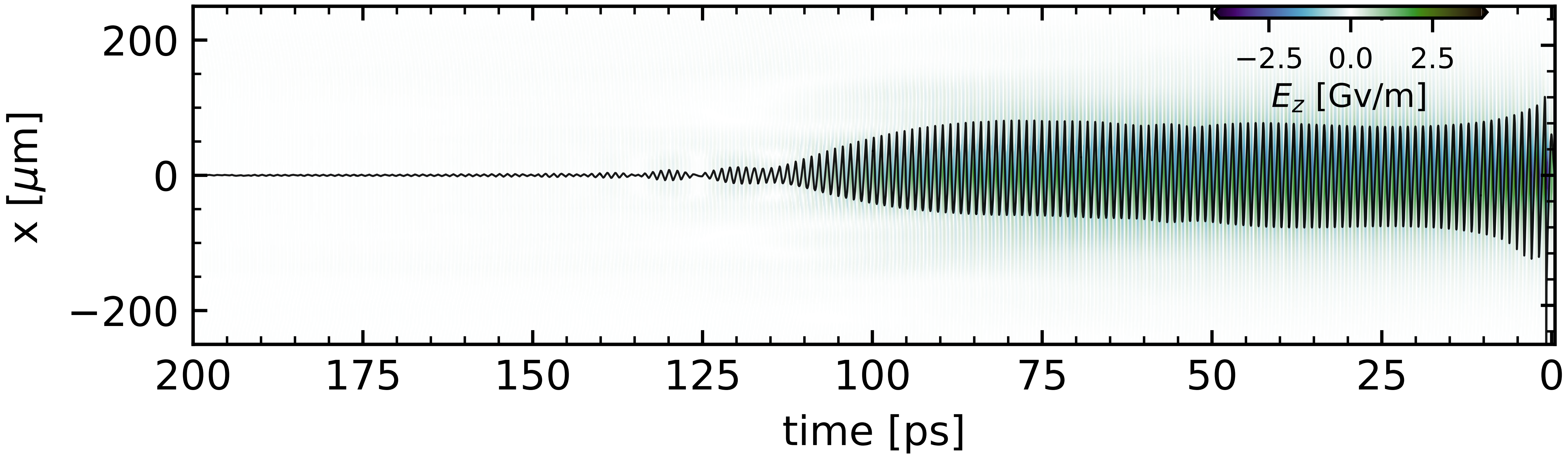
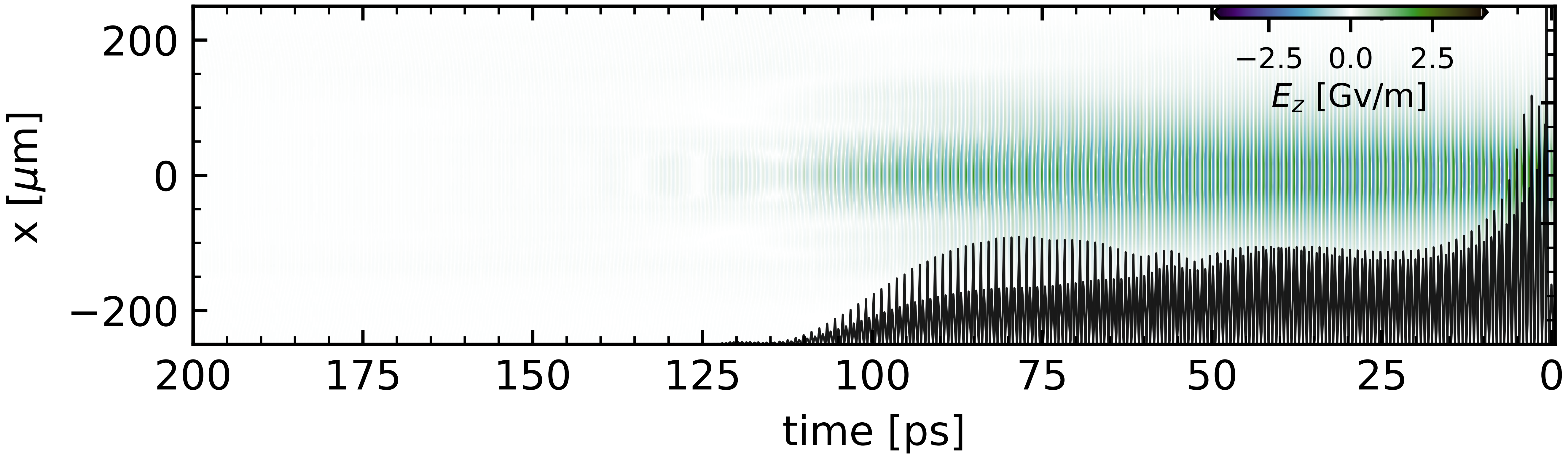


1. Look at axial longitudinal electric field

Understanding longer term effects by looking at the shorter term



1. *Look at axial longitudinal electric field*
2. *Look at axial field energy*

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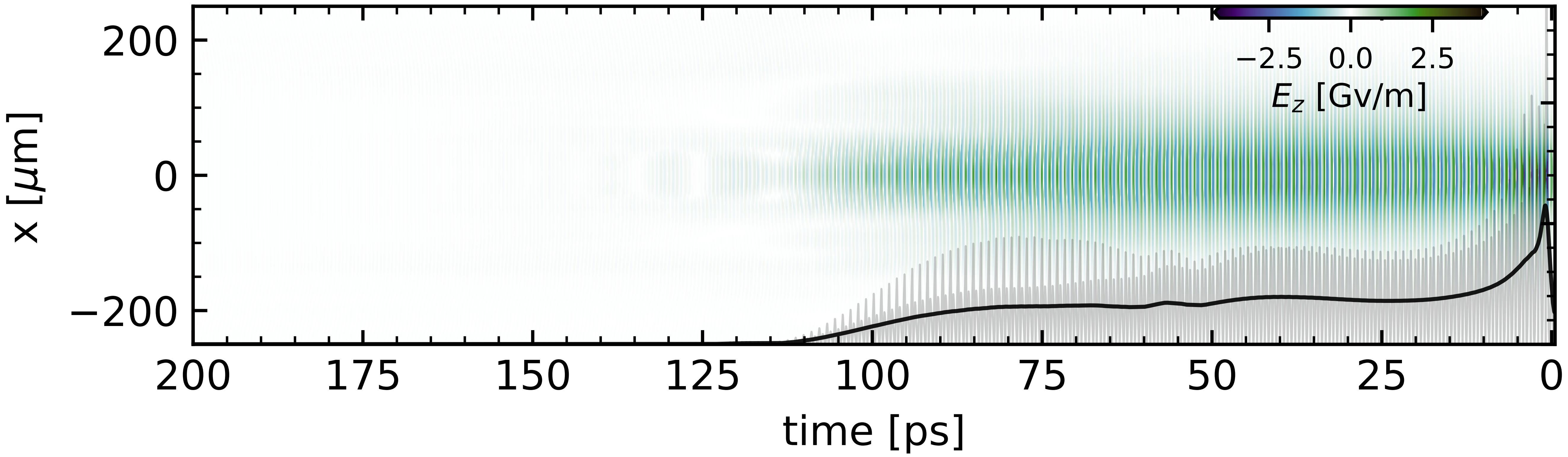
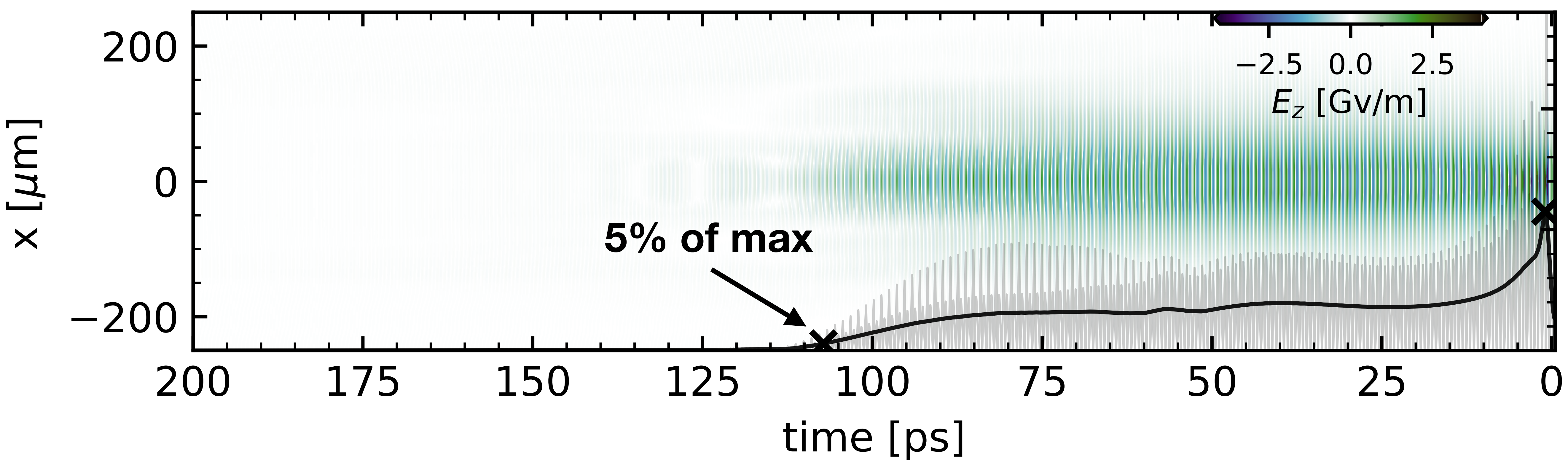


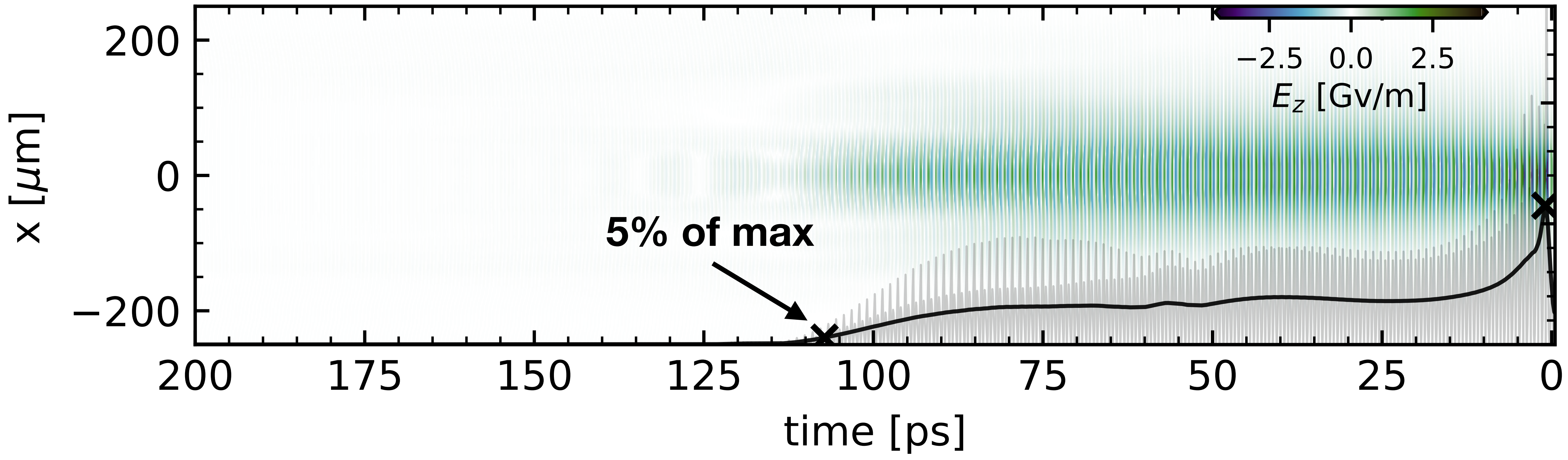
1. *Look at axial longitudinal electric field*
2. *Look at axial field energy*
3. *Gaussian smoothing — giving mean field energy*

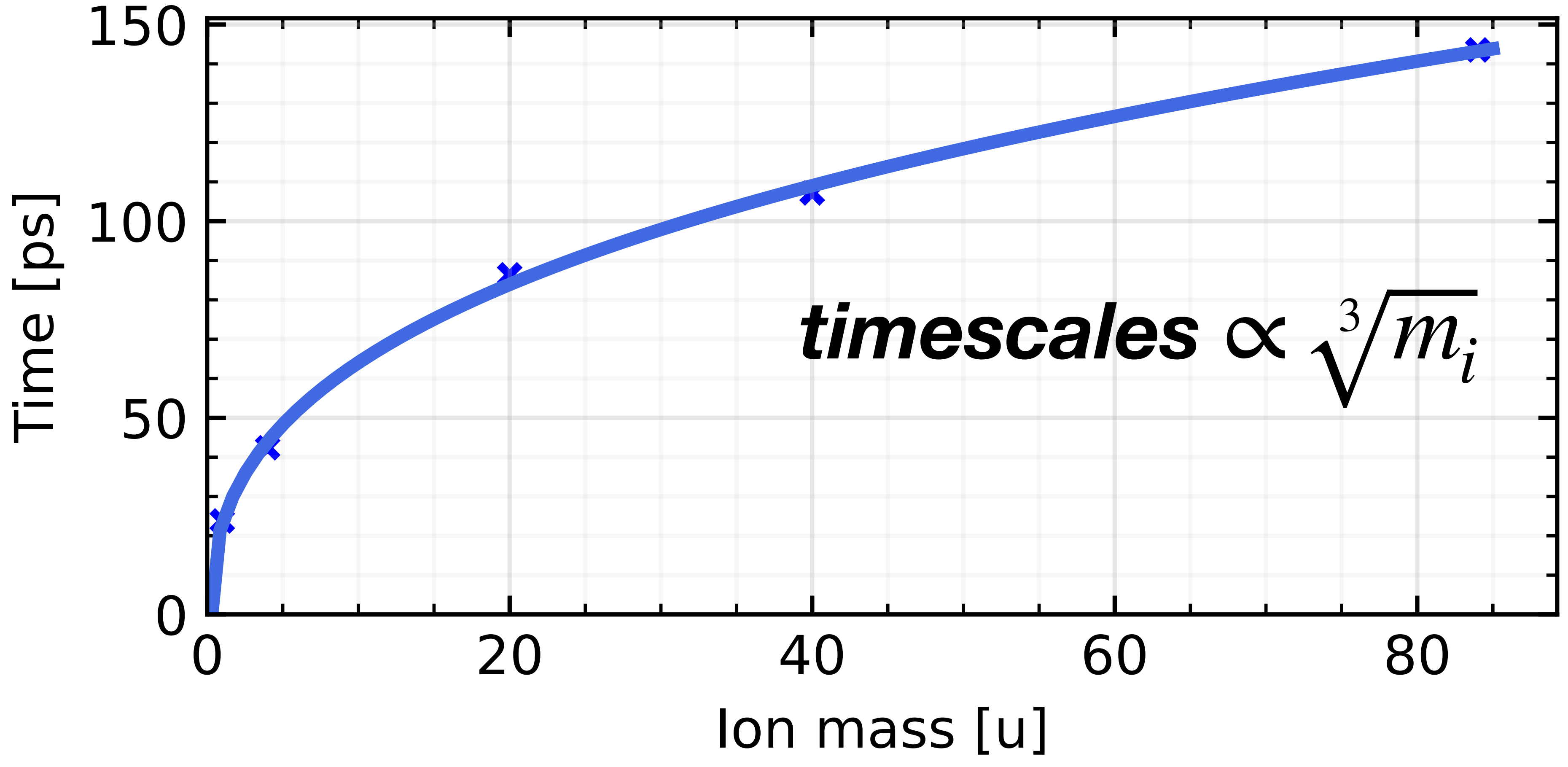
Understanding longer term effects by looking at the shorter term



1. *Look at axial longitudinal electric field*
2. *Look at axial field energy*
3. *Gaussian smoothing — giving mean field energy*
4. *Pick low threshold of field energy to signify wave decay time*

Understanding longer term effects by looking at the shorter term





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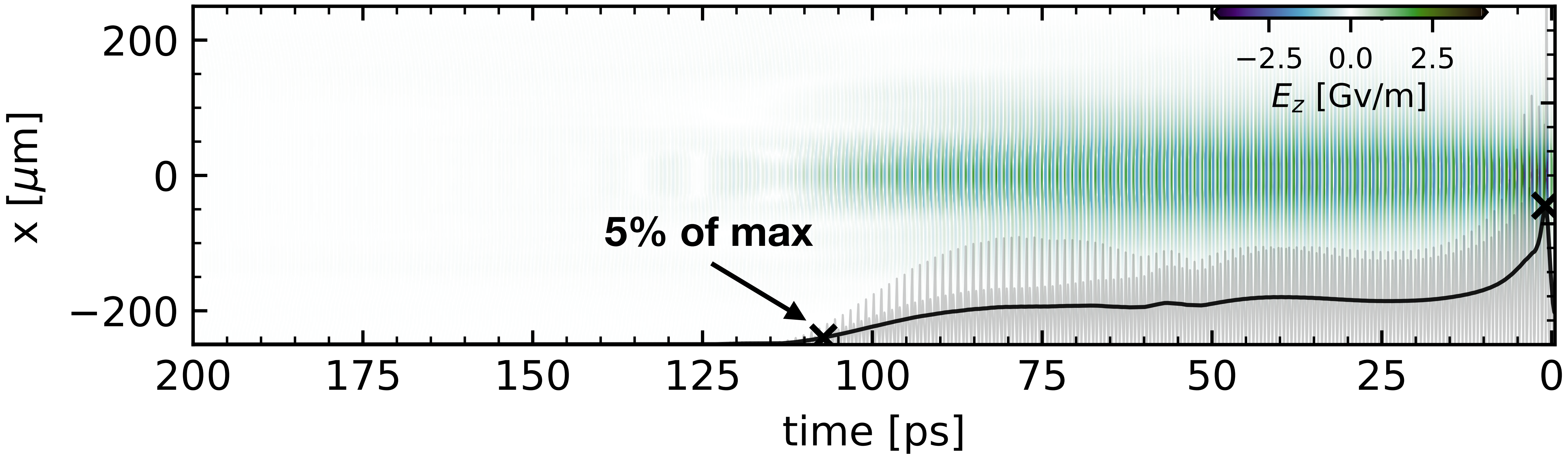

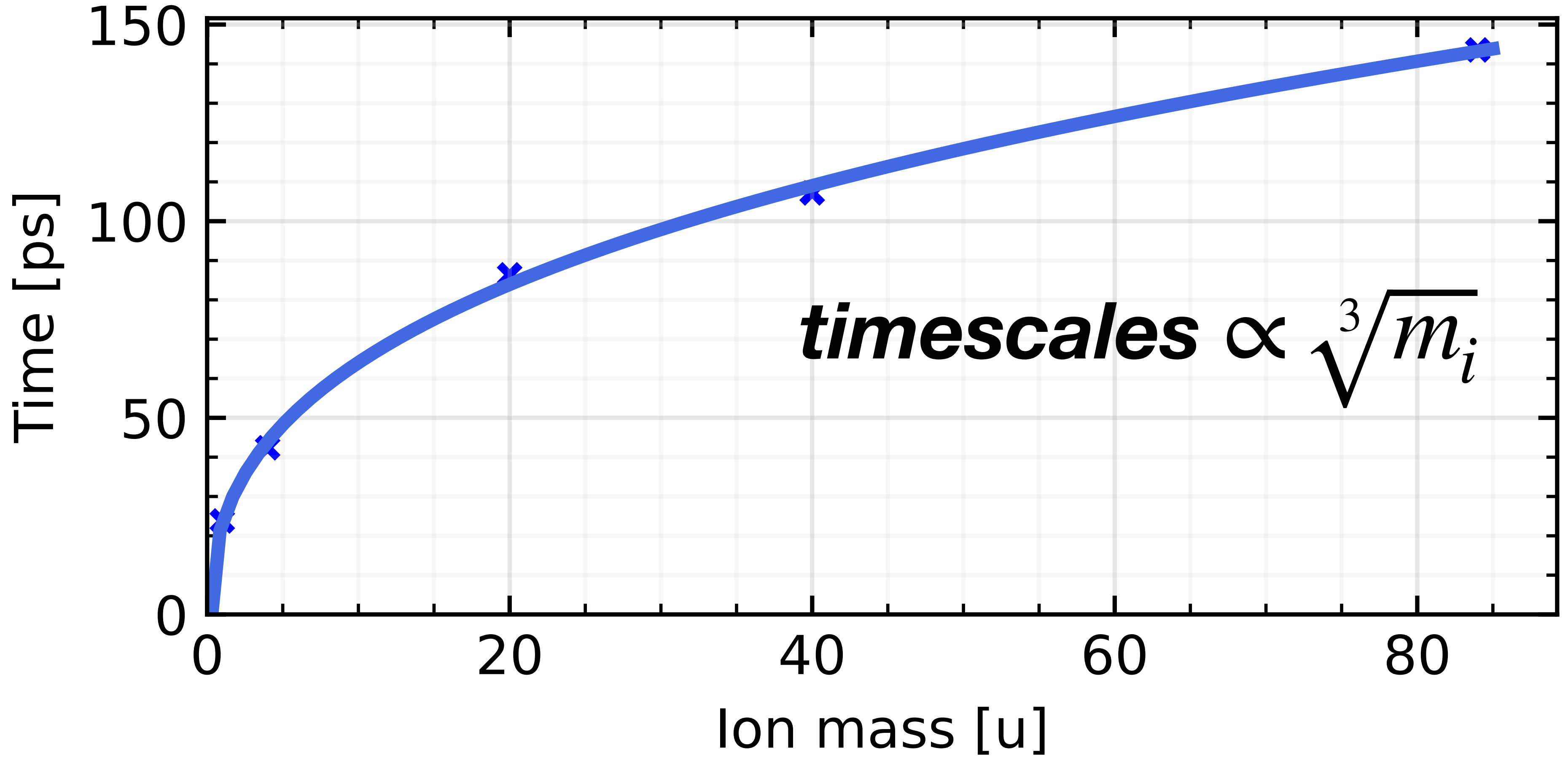

- **Plasma evolution is a series of causes and effects:**
  - Beam driver deposits energy into forming nonlinear plasma wave
  - Ions slowly quench energy of the wave
  - This depletes the wave with a $\sqrt[3]{m_i}$ dependence — which has been predicted and observed:
    - *Spitsyn et al. Phys. Plasmas 25, 103103 (2018) & M. Turner et al., Phys. Rev. Lett. 134, 155001 (2025)*
    - As a result, the ion spike peak ensues with the same $\sqrt[3]{m_i}$ dependence

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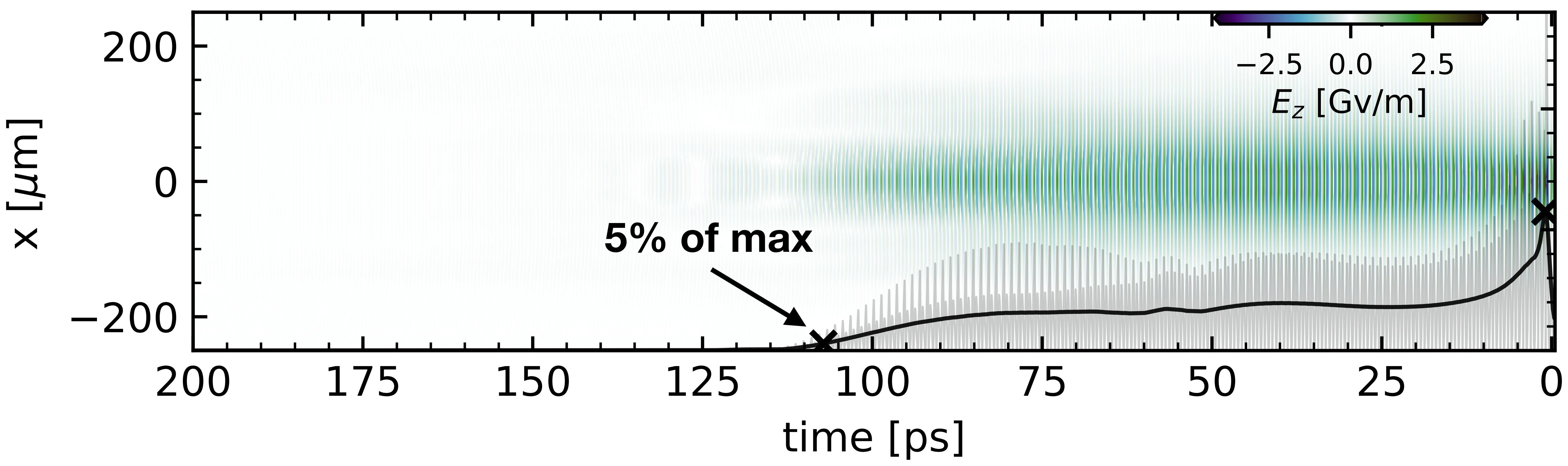

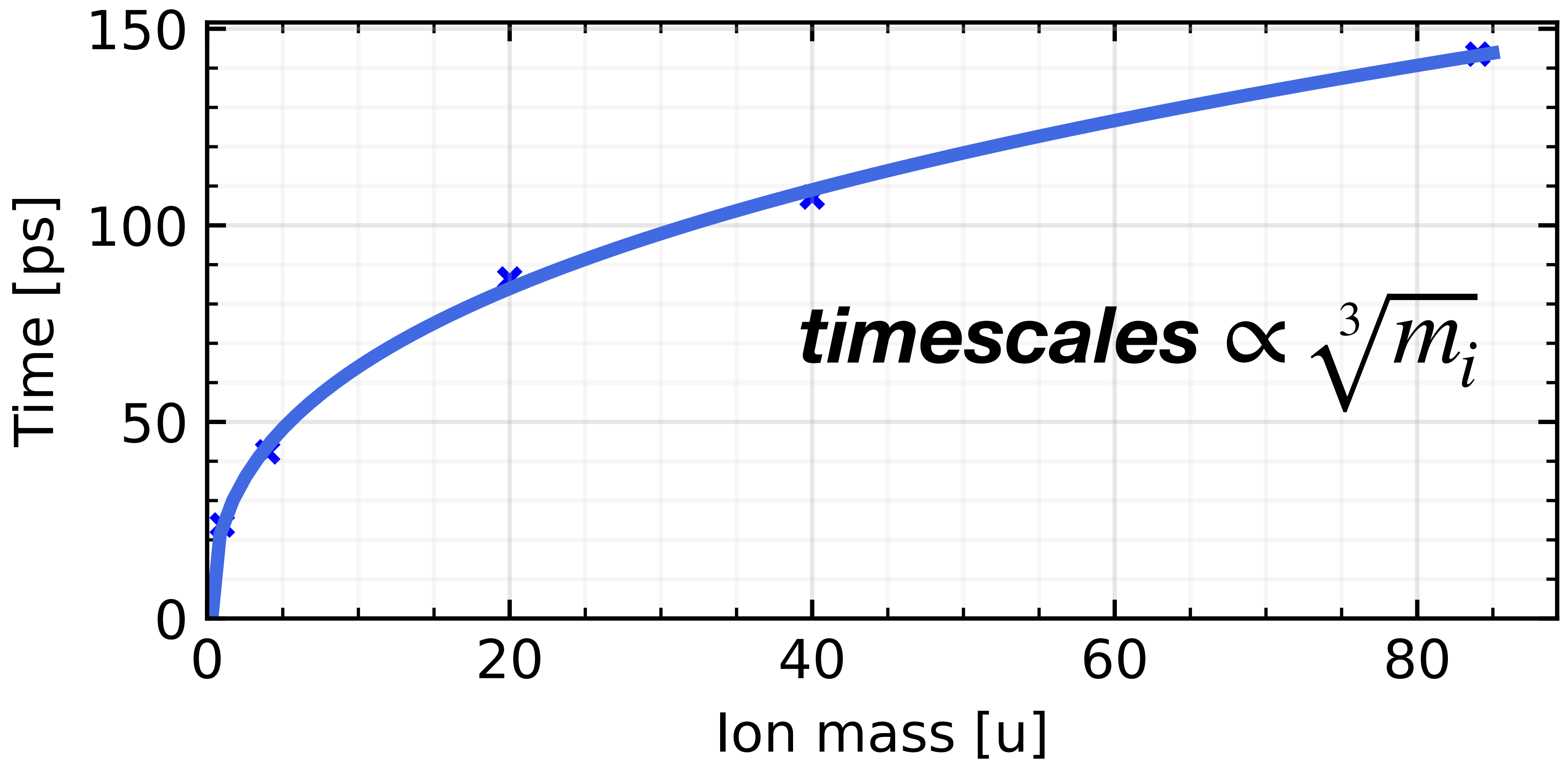

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- **Plasma evolution and recovery relies on the interdependence of many consecutive interlinked effects**
- **But so far, these were all only for a cold plasma…**

Extending to investigate temperature effects

- **Expectations and effects**
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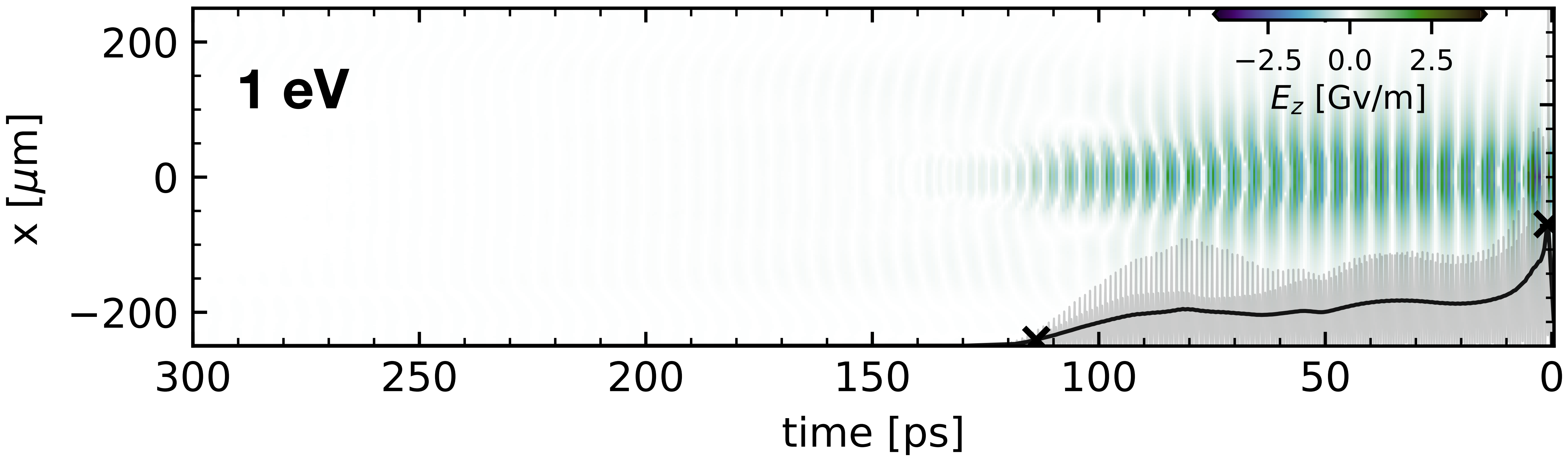

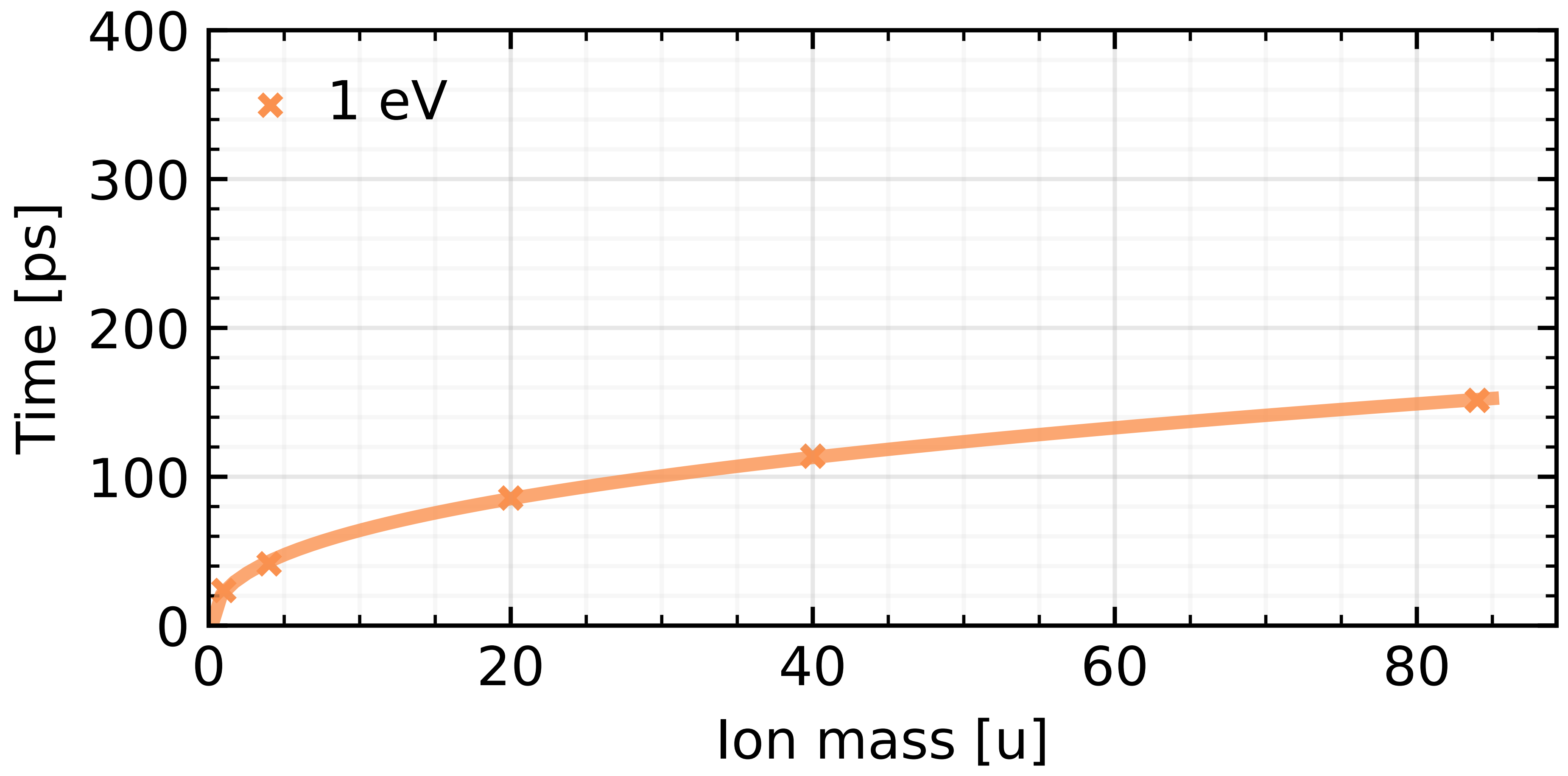

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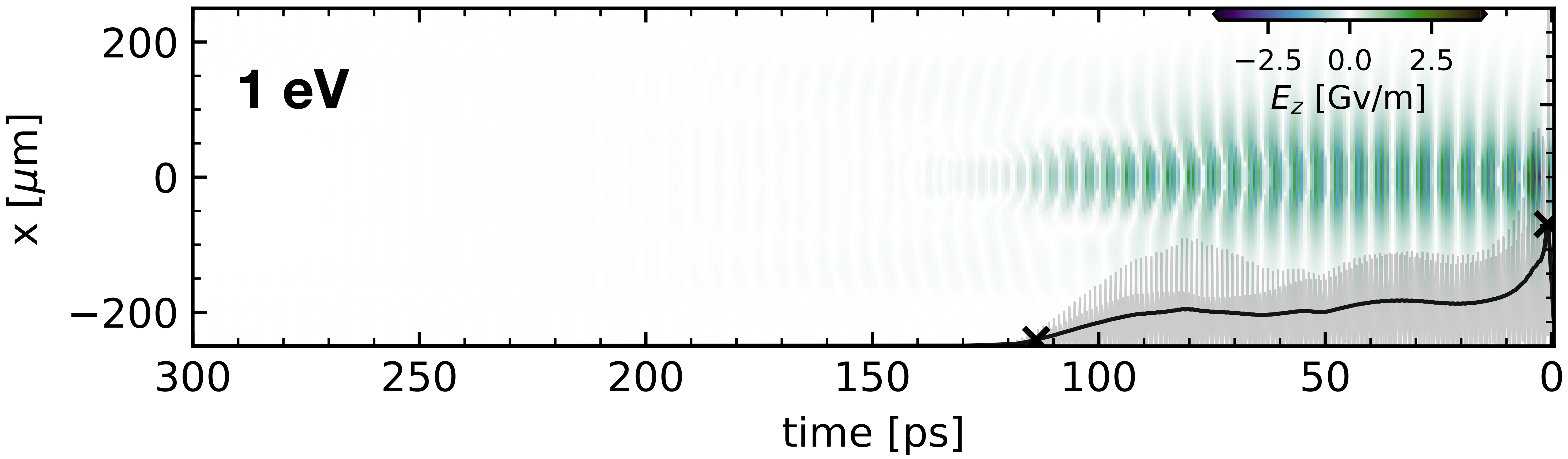

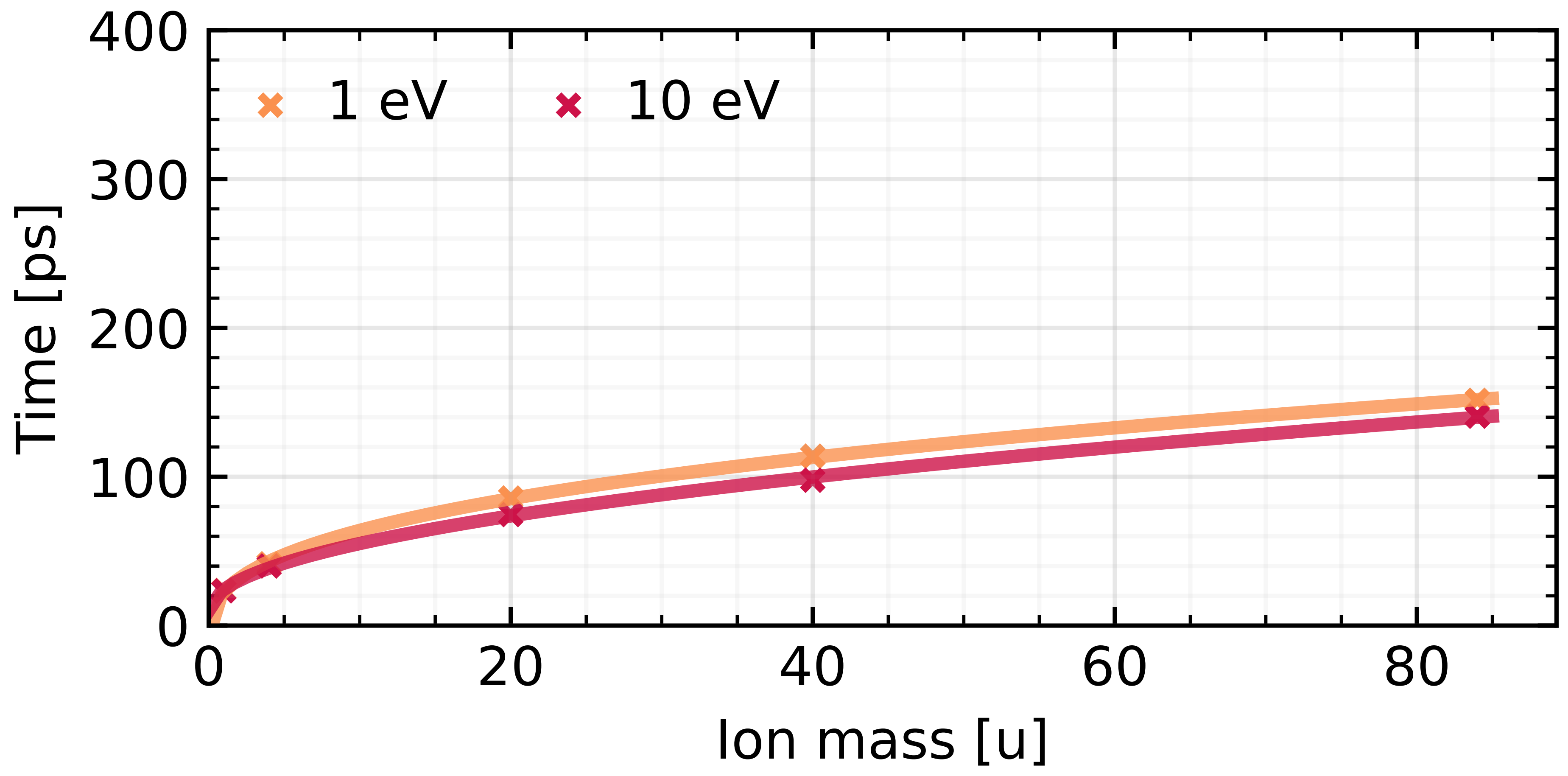

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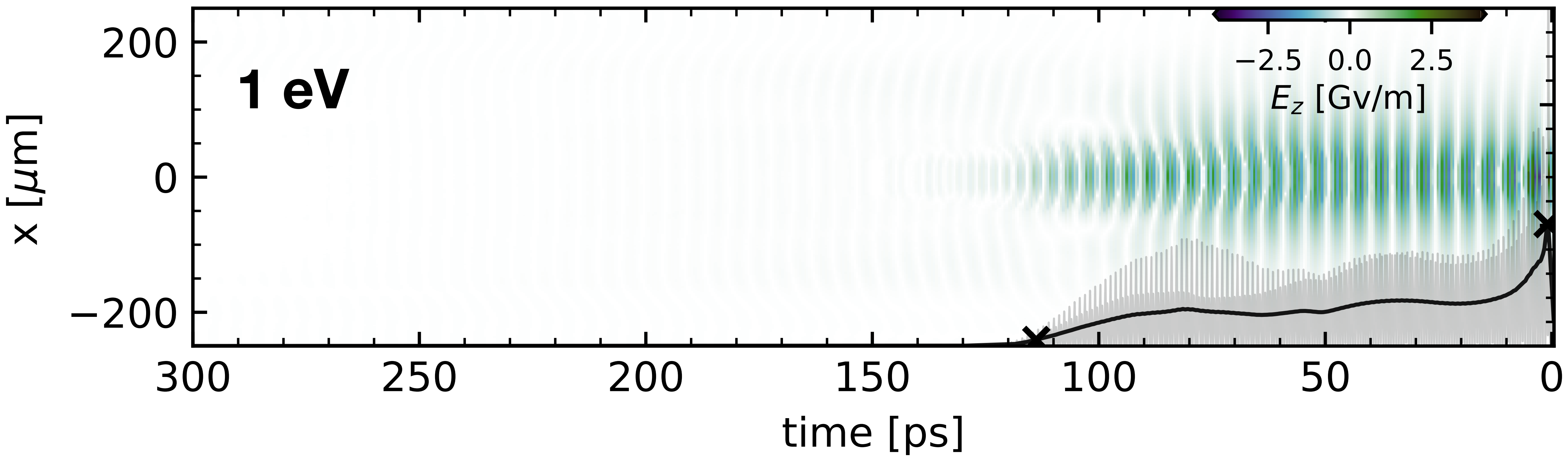

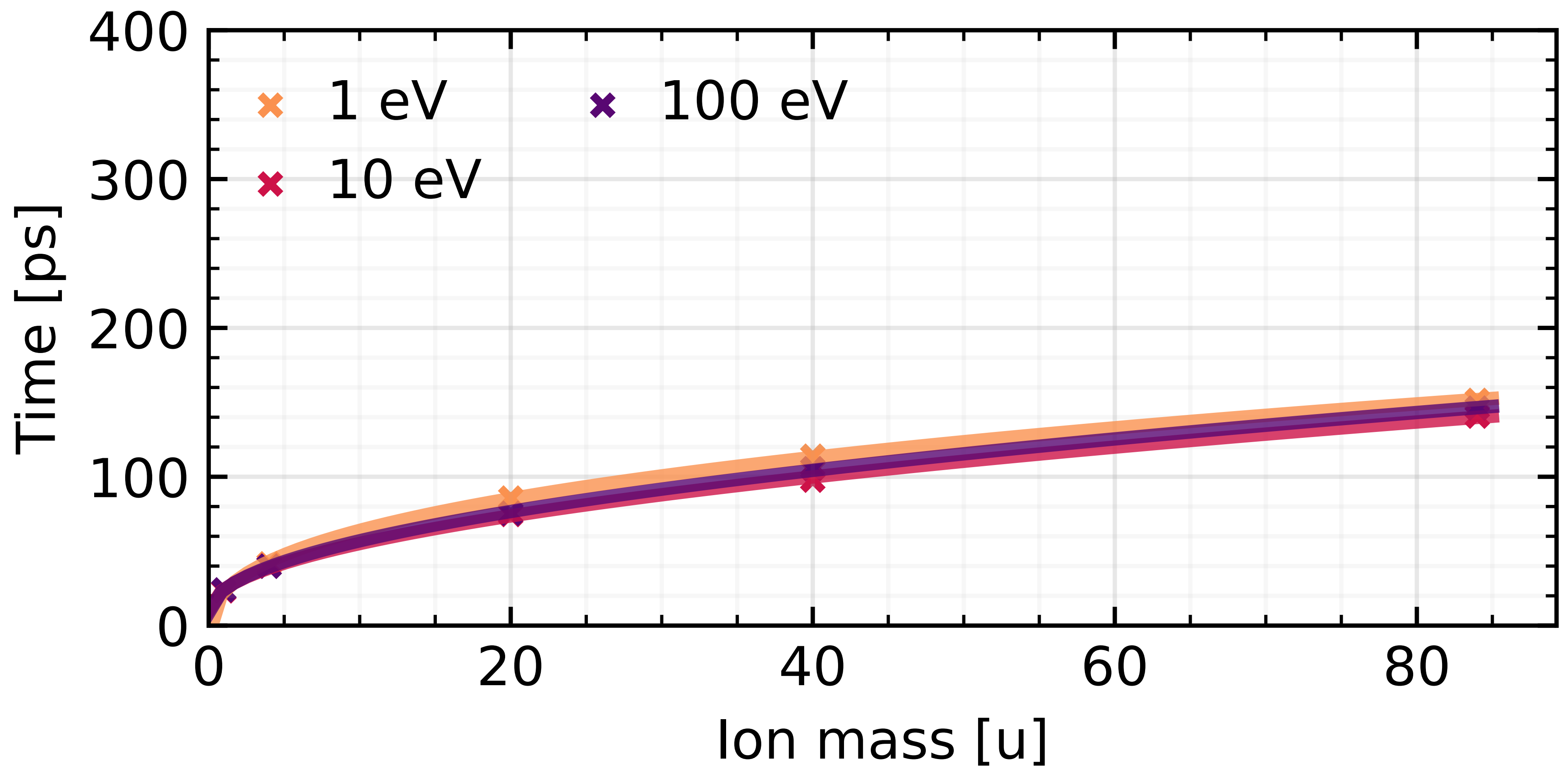

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- **Expectations and effects**
  - At hot temperatures, expect random thermal motion to damp the wave faster
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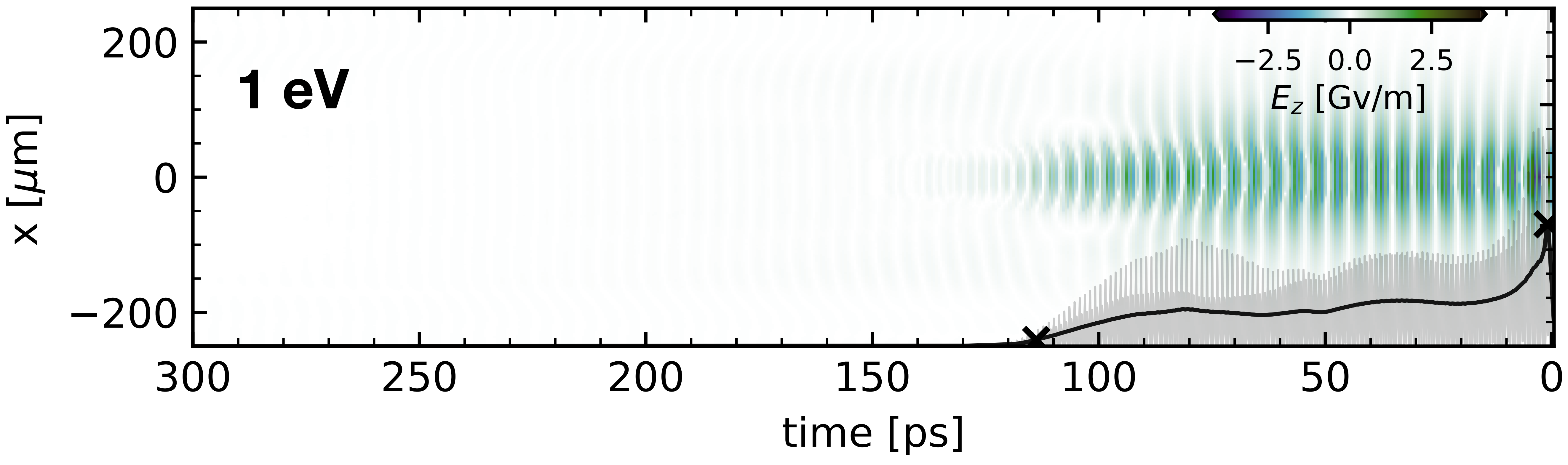
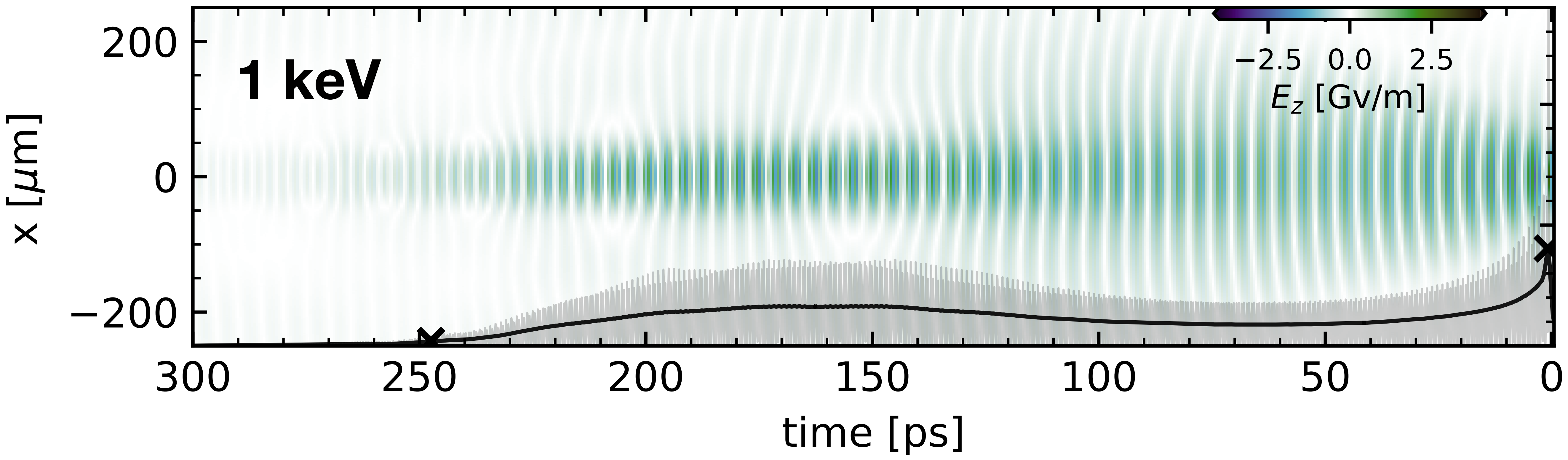
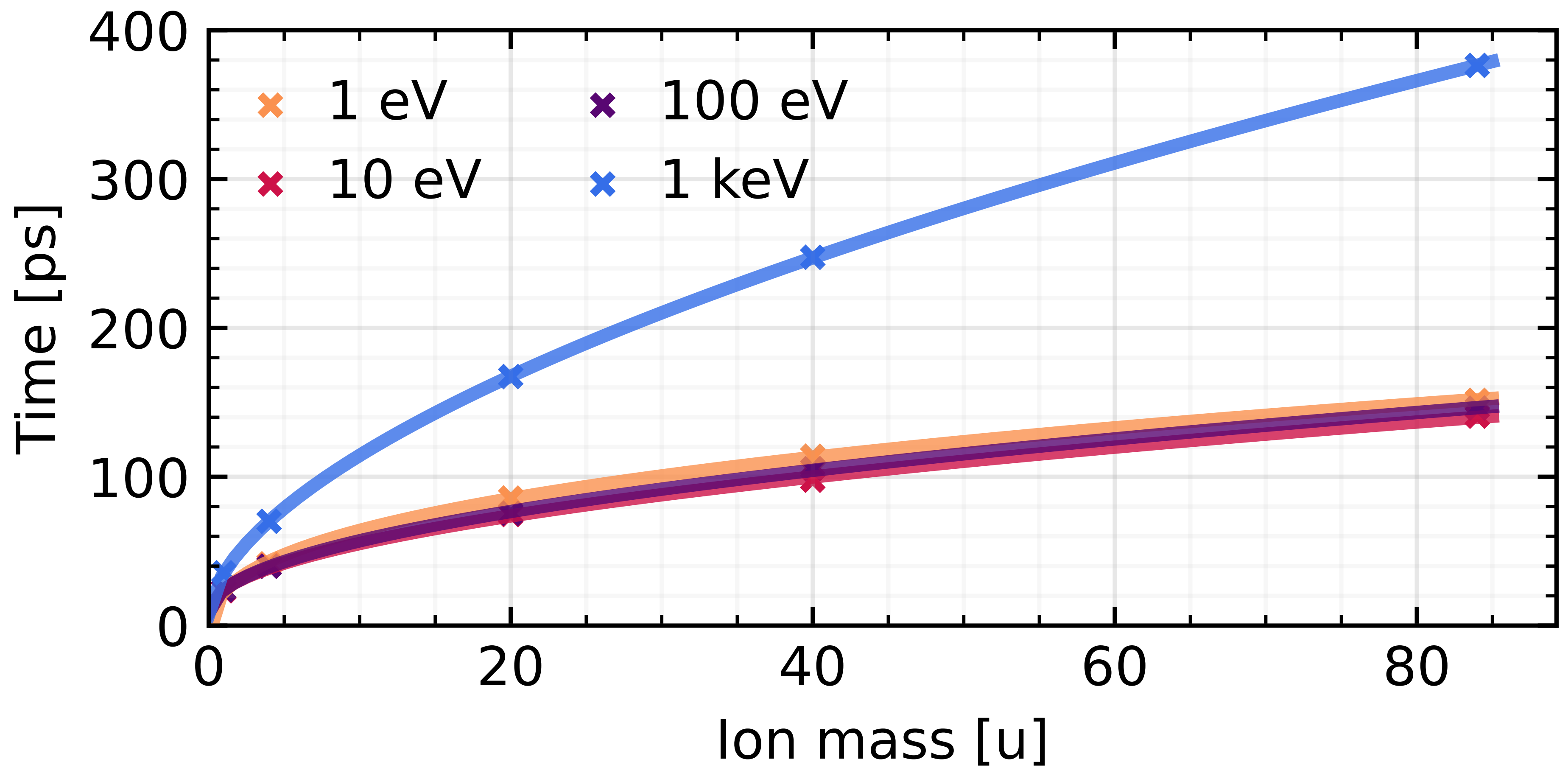
  - See region of high temperature which prolongs the plasma wave

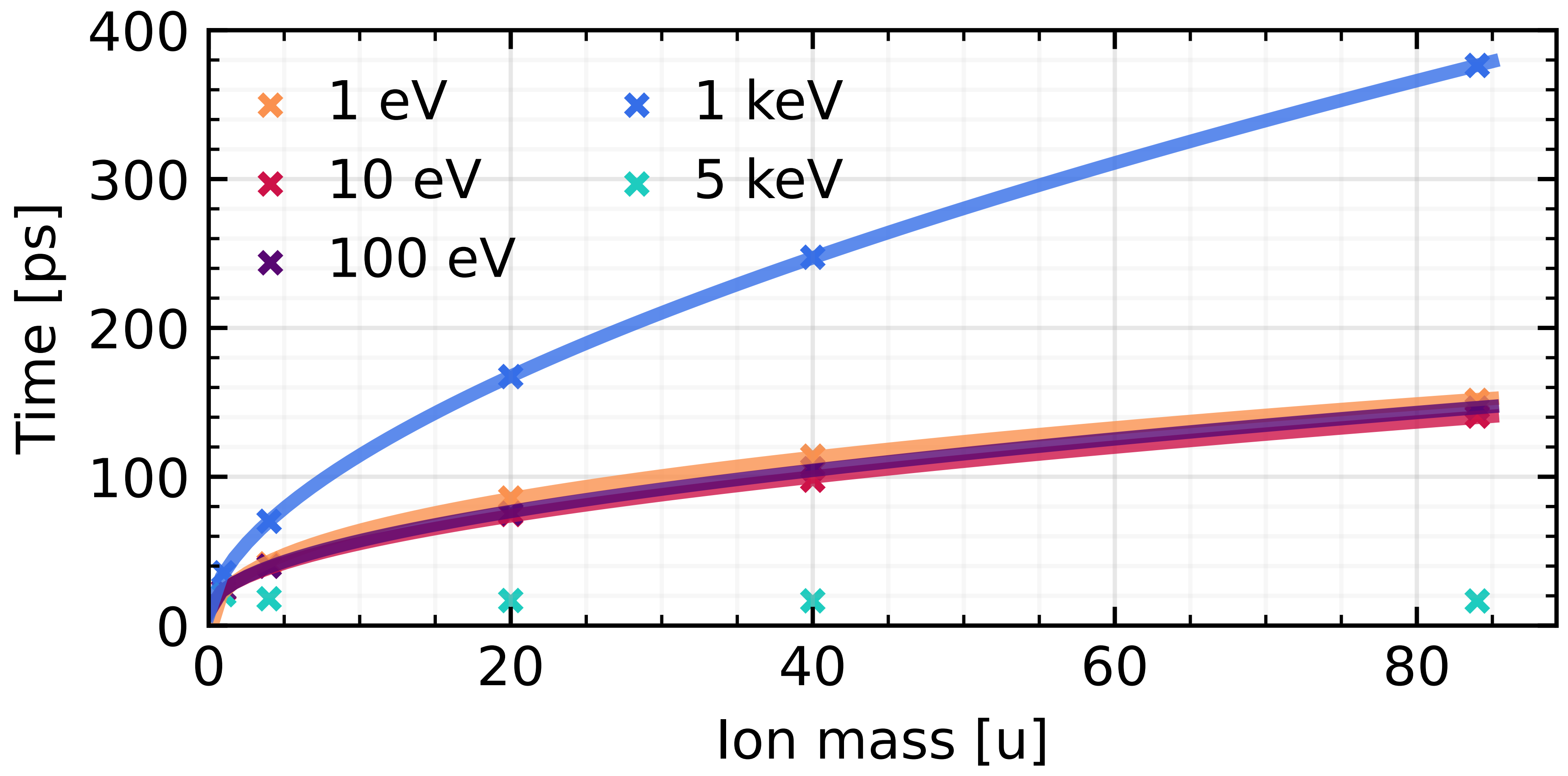


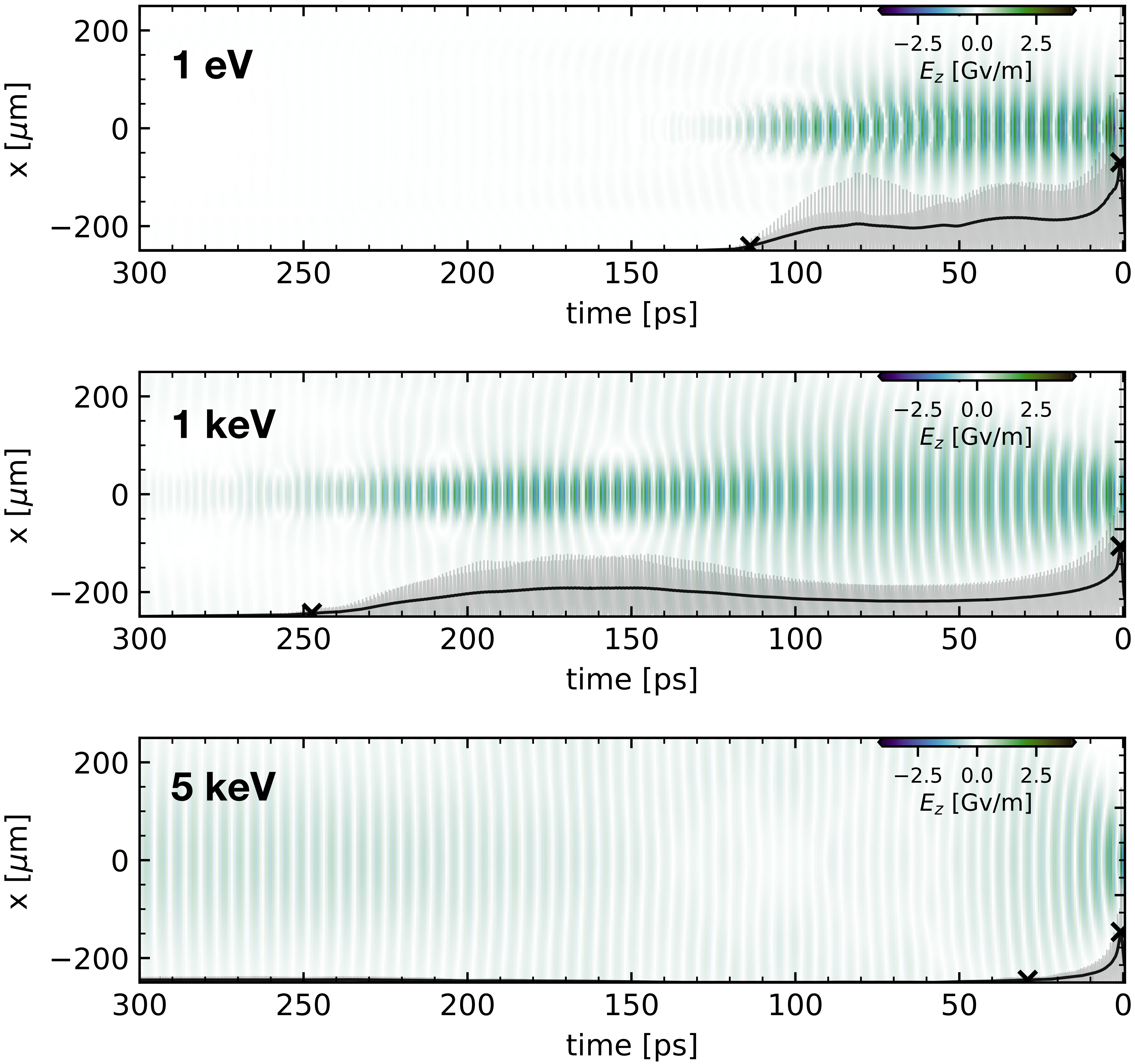




Extending to investigate temperature effects

- **Expectations and effects**
  - At hot temperatures, expect random thermal motion to damp the wave faster
  - See little changes at low temperatures
  - See region of high temperature which prolongs the plasma wave
  - See expected quick damping at extreme temperatures
  - Suggests use of extreme temperature plasma to be factored into designs?





Conclusions

- **Summary:**
  - Temperature effects are important for collider applications as they will deteriorate beam quality
  - Relevant timescales cannot be captured by PIC codes so introduced Quasi-Static Approximation
    - QSA allows multi-nanosecond simulations — extending beyond plasma quasi-neutrality
  - Demonstrated effects of ion species on plasma wave decay
    - He could be used to run high-repetition-rate machines like HALHF if a $\sqrt[3]{m_i}$ scaling is followed
  - But plasma recovery timescales found to be dependent on initial plasma temperature

- **Next steps:**
  - Address several limitations in HiPACE++ relevant to long timescales
    - Ionization, collisions, recombination etc.
  - Extend plasma evolution to longer timescales to reproduce experimental data of plasma recovery from FLASHForward
  - Apply these methods to inform on future experiments at FLASHForward
  - Apply these methods to inform on future baselines and modifications for the HALHF collider

Thank you!

Extra slide - simulation parameters & fits

Slide 2 simulations (HiPACE++)

Plasma: 1e16 /cm3, Singly ionised Ar

Driver parameters: 250 pC, 1 µm transverse rms, 40 µm longitudinal rms

Witness parameters: 65 pC, 1 µm transverse rms, 15 µm longitudinal rms

Slides 6-8 simulations (HiPACE++ & FBPIC)

Plasma density: 1e16 /cm3

Driver parameters: 600 pC, 5 µm transverse rms, 50 µm longitudinal rms

Slide 9-12 simulations (HiPACE++ varying 9. ion mass and 10. temperature in Ar)

Plasma density: 1e16 /cm3

Driver parameters: 567 pC (nb/n0=100), 5 µm transverse rms, 50 µm longitudinal rms

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All fits of t vs m_ion are to: t = a*m_ion**b + c

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Scan	b
Ion mass	0.31
Wave decay	0.33

T [eV]	b
1	0.37
10	0.49
100	0.49
1000	0.58