Modeling laser-wakefield accelerators using the time-averaged ponderomotive approximation in a Lorentz boosted frame

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Outline

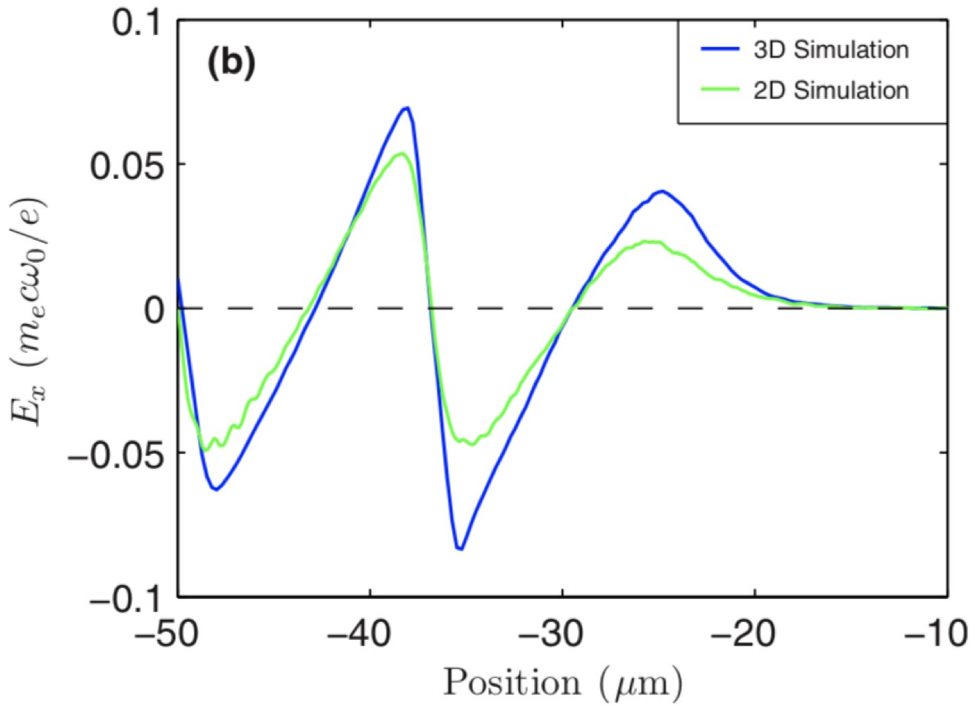
- Simulation of Laser Wakefield Acceleration (LWFA) and characteristic scales
- Lorentz boosted frame (LBF)
- Time-Averaged Ponderomotive Approximation (TPA)
- Combining the LBF and TPA

Reference publication:
F. Massimo, C. Benedetti, D. Terzani, A. Beck, B. Cros,
***Plasma Phys. Control. Fusion* 67, 065032 (2025)**

Simulation of LWFA and characteristic scales

- **Curse of dimensionality:**

3D-like description is needed

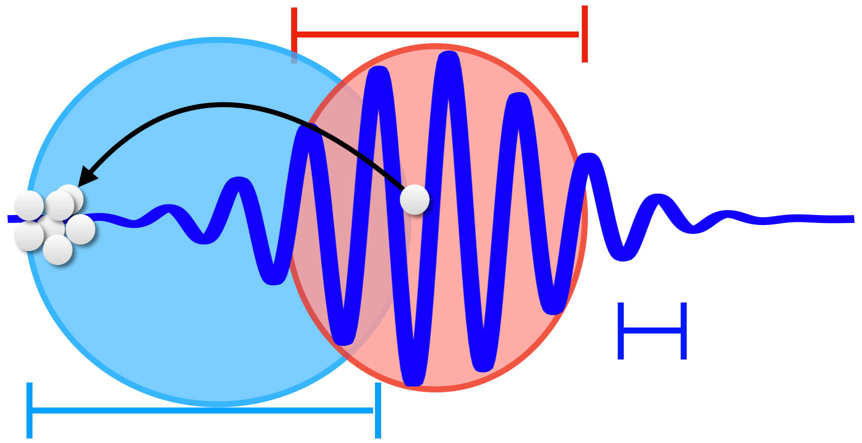


X. Davoine et al., Phys. Plasmas 15, 113102 (2008)

- **Minimum/Maximum scale disparity:**

Laser wavelength λ_0 ~1 μm
Plasma stage length L_p~ 10s mm, 10s cm, 1 m

Laser envelope ~ λ_p~10s-100s μm



Accelerating cavity λ_p ~10s-100s μm

Scale disparity in space and time with explicit solvers:

$\lambda_p/\lambda_0 = \omega_0/\omega_p = \Omega$

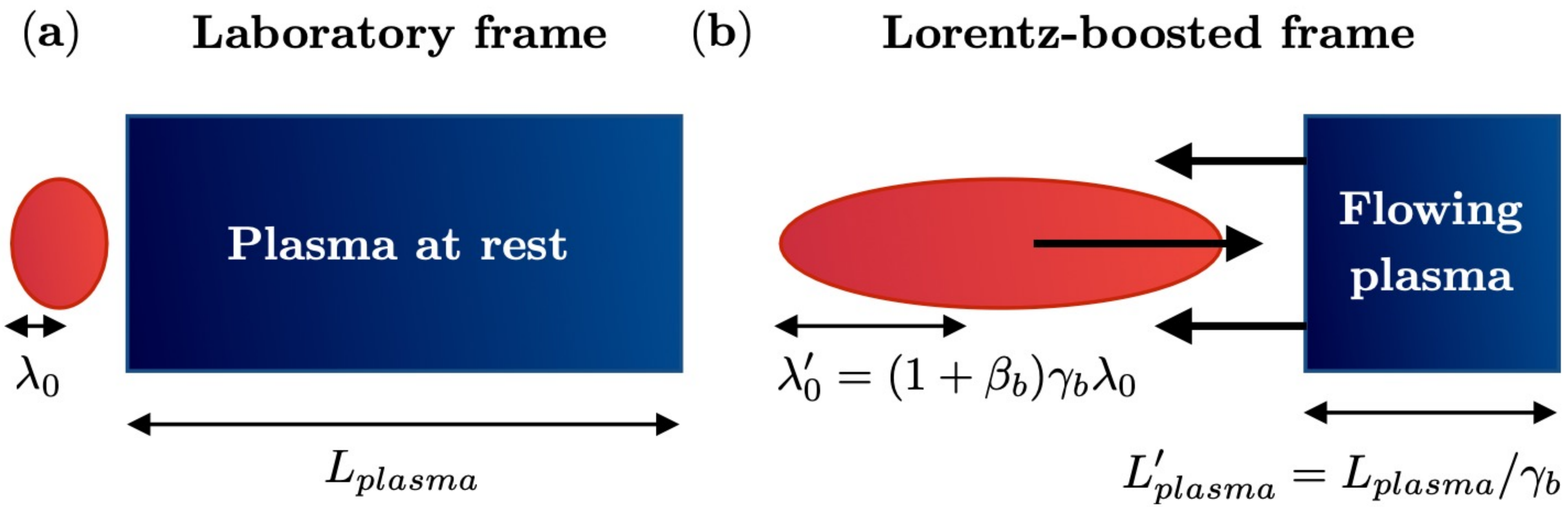
Full 3D simulations of ~1 m of propagation are currently too costly/unfeasible

"Problem size reduction" techniques (some can be combined too):
Cylindrical geometry with azimuthal Fourier decomposition, Quasi-static approximation, **time-averaged ponderomotive approximation**, **Lorentz boosted frame technique**, hybrid fluid/kinetic models

Lorentz boosted frame (LBF): concept

(J.-L. Vay PRL 2007, P. Yu et al. JCP 2016)

Image from P. Lee, PhD thesis (2017) https://theses.hal.science/tel-01581770

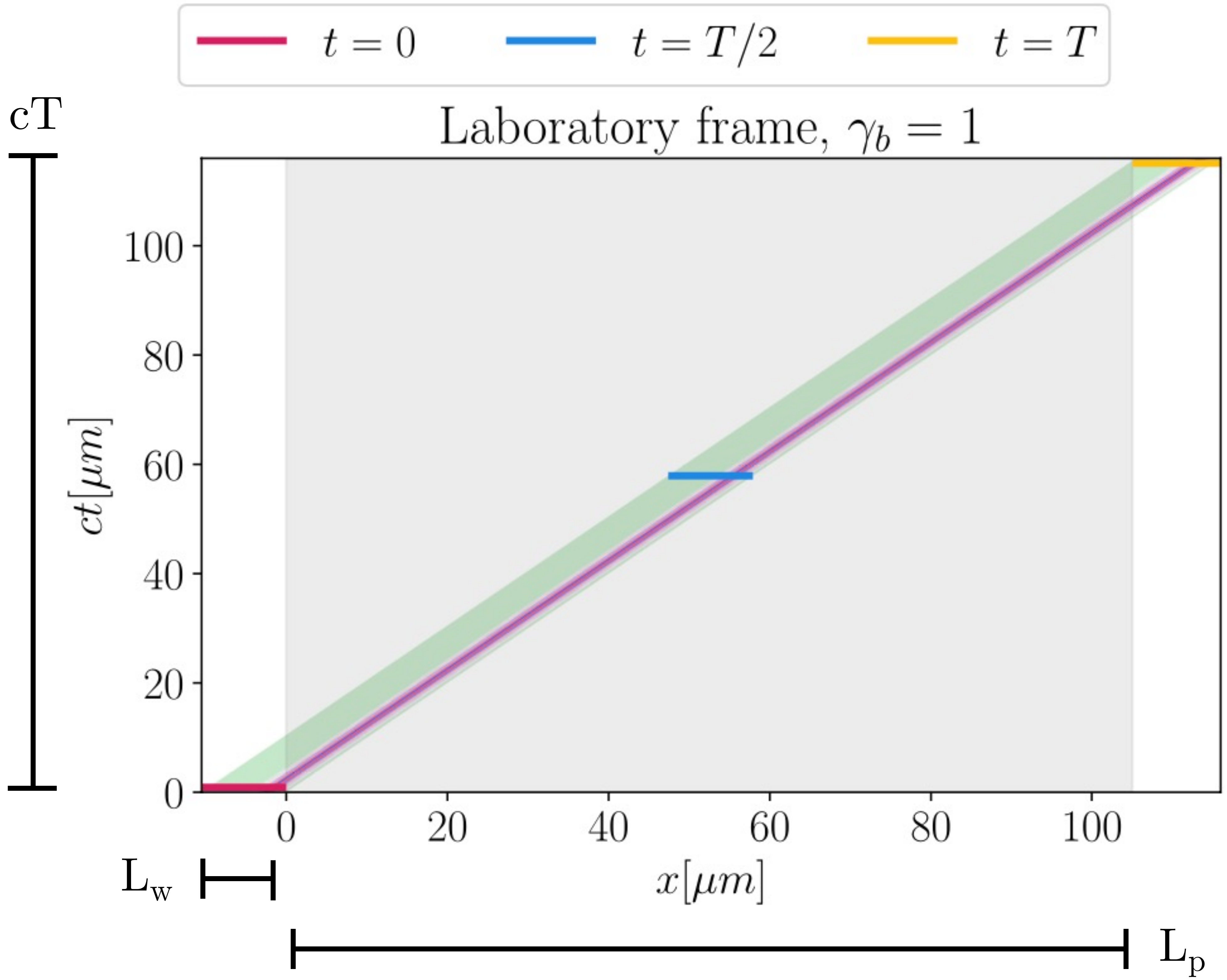


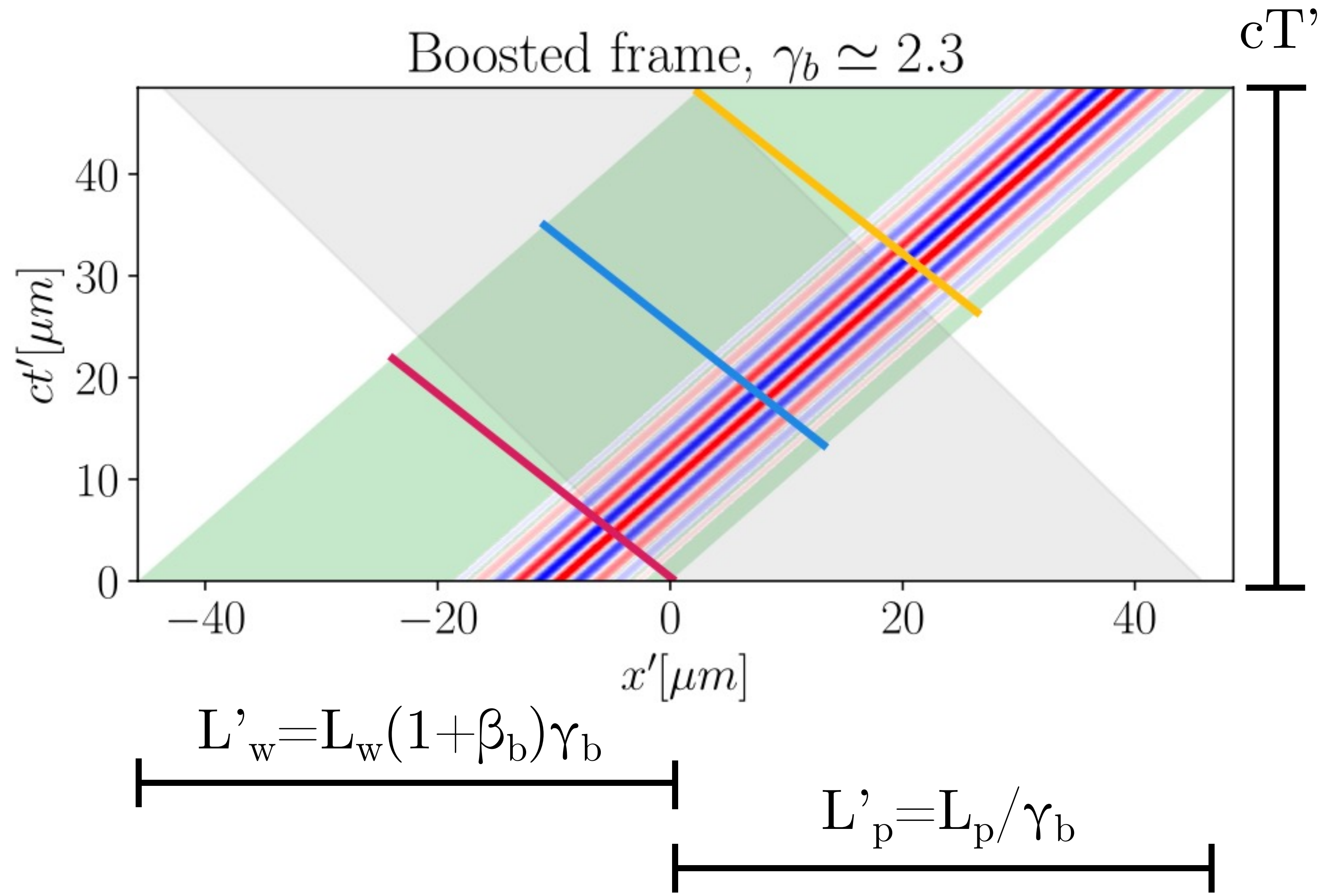
Same number of grid points along z: $\Delta z' = (1+\beta_b)\gamma_b \Delta z$

(Ideally) larger integration timestep: $\Delta t' = (1+\beta_b)\gamma_b \Delta t$

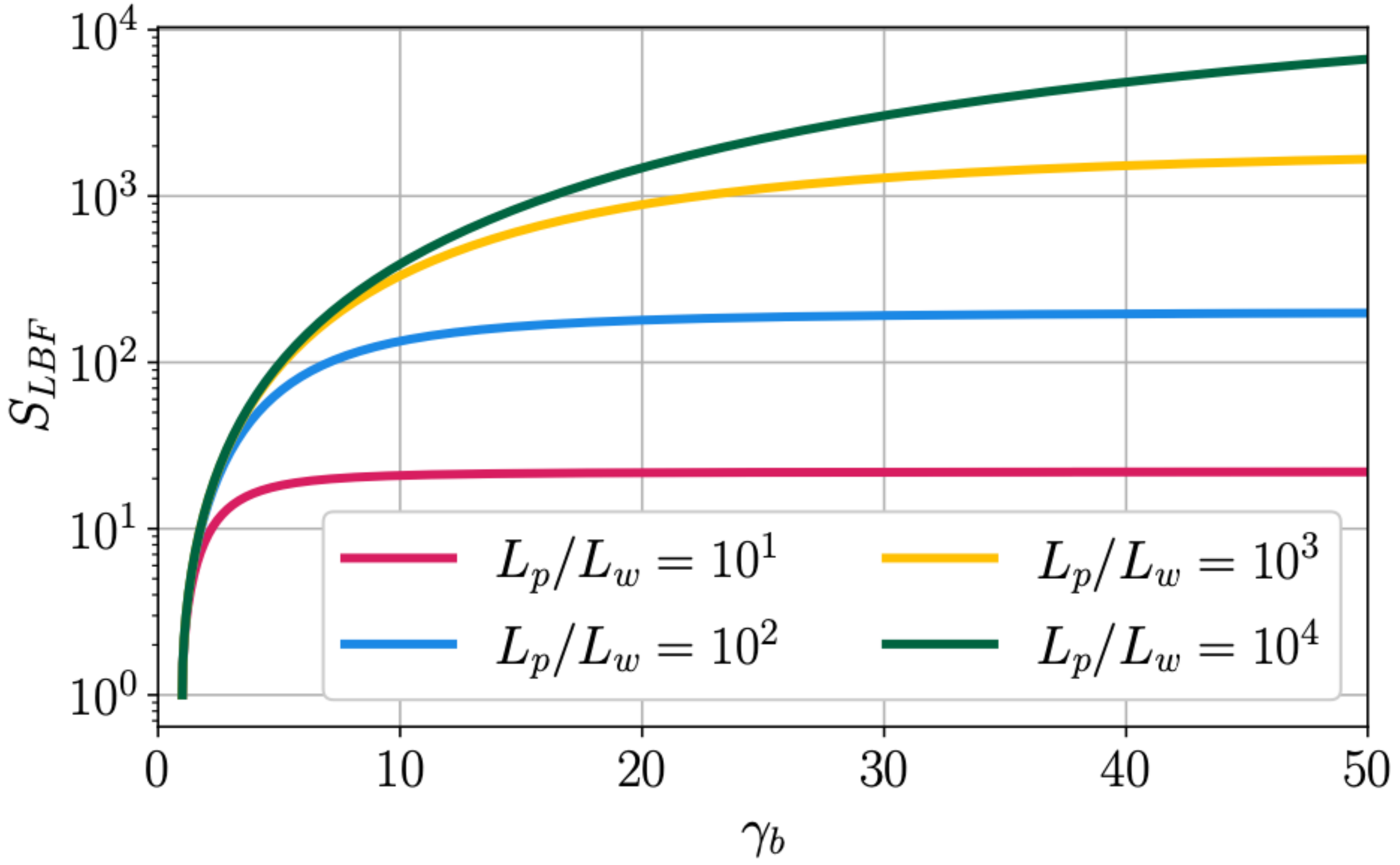
***Caveat:** Backward propagating waves are neglected

Lorentz boosted frame (LBF): space-time diagrams





Lorentz boosted frame (LBF): theoretical speed-up



$$S_{LBF} = \frac{N}{N'} = \frac{L_p/L_w + 1}{L_p/L_w + (1+\beta_b)\gamma_b^2}(1+\beta_b)^2\gamma_b^2$$

For $L_w = C\,\lambda_p$ ($C \sim 1$) and $L_p \sim \lambda^3_p/\lambda^2_0$ (dephasing length) the optimal Lorentz boost factor is

$\gamma_b{}^* \sim \lambda_p/\lambda_0 = \omega_0/\omega_p = \Omega$

(i.e. The Lorentz factor of the wakefield)

[See **J.-L. Vay et al., PoP 2011, F. Massimo et al., PPCF 2025**]

$\rightarrow S_{LBF} \sim 2\Omega^2 \sim 100$

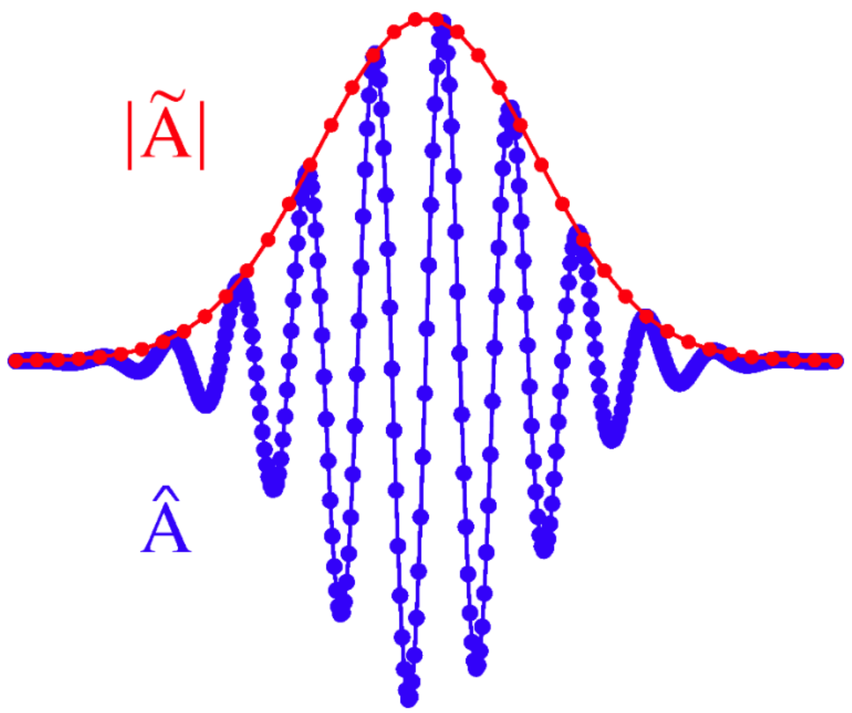
for $\lambda_0 = 1\ \mu m$, $n_0 = 10^{17}\ cm^{-3}$

***Caveat:**

Numerical constraints (e.g. CFL condition if present, numerical artefacts) and physical scales (e-bunch, laser, steep plasma gradients) may significantly reduce the optimum $\gamma_b{}^*$ and the speed-up

Time-averaged ponderomotive approximation (TPA)
(general theory in B. M. Cowan et al. JCP 2013, D. Terzani et al. Phys. Plasmas 2021)

Laser envelope



Regimes of validity
for GeV-class LWFA stages
(Assuming $\lambda_0 = 0.8\ \mu m$
see **D. Terzani et al. PoP 2021**)

$$T_{FWHM} \gtrsim 10\ fs$$
$$w_0 \gtrsim 10\ \mu m$$
$$a_0 \lesssim 10$$

See also A. Beck's presentation!

Resolution scale disparity with explicit solvers (space and time):

$$\lambda_p/\lambda_0 = \omega_0/\omega_p = \Omega$$

→ **Theoretical Speed-up:**

$$S_{TPA} \simeq \Omega^2$$

$$\sim 100$$

for $\lambda_0 = 1\ \mu m,\ n_0 = 10^{17}\ cm^{-3}$

***Caveat:**
Numerical constraints (e.g. CFL condition if present, numerical artefacts) and physical scales (e-bunch, laser, steep plasma gradients) may significantly reduce the optimum γ_b* and the speed-up

TPA Benchmark in lab frame: guided LWFA with external injection

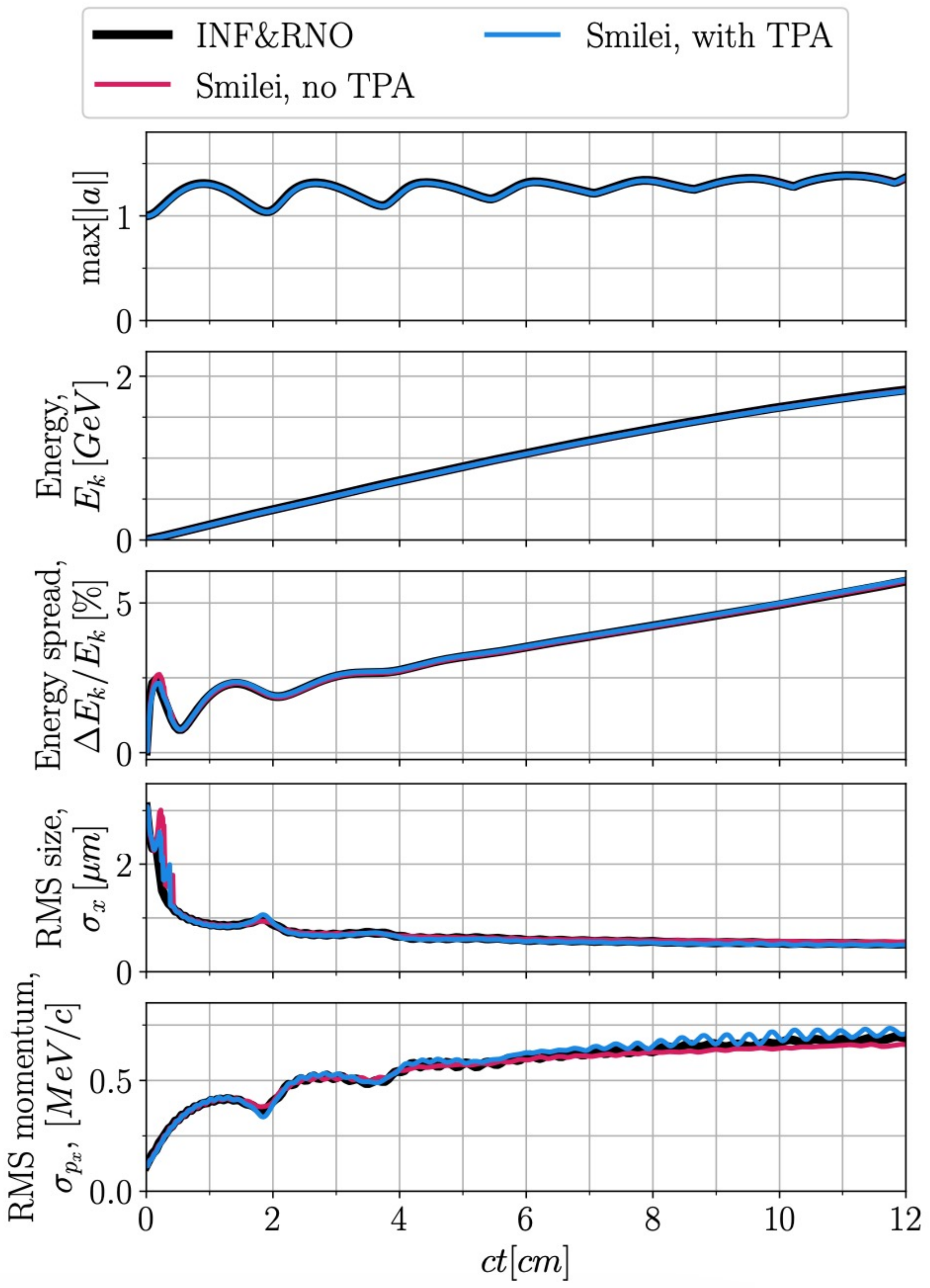
Gaussian Laser
λ_0=0.8 µm, τ=68 fs, w_0=41µm

Matched Plasma channel
n_0=2.7*10^{17} cm^{-3}→ λ_p=64 µm, R=w_0

Gaussian electron bunch
Q=0.65 pC, σ_z=1 µm, σ_r = 3 µm,
σ_{py} = 0.23 m_ec, ε_x=0.67 mm-mrad, γ_0 = 10, $\Delta\gamma$~0

INF&RNO sim.:
cylindrical symmetry, hybrid PIC-fluid, with TPA

Smilei sim.:
cylindrical symmetry and 2 modes, full PIC, with TPA and not



Coupling LBF and TPA

Initialization and outputs:

- as in "classic" LBF, but add laser envelope quantities (check paraxiality if analytical formulas!)

Laser envelope solver:

- Doppler-shift the laser frequency in the envelope equation
- use susceptibility in LBF (background plasma is moving towards the laser, density is higher)
- **Use a Lorentz covariant formulation to derive the envelope equation**, e.g.:

$$\left(\nabla_\perp^2 + 2i\frac{k_0}{k_p}\frac{\partial}{\partial\tau} + 2\frac{\partial^2}{\partial\zeta\partial\tau} - \frac{\partial^2}{\partial\tau^2}\right)\hat{a} = \chi\hat{a}$$

In comoving coordinates $\zeta = z - ct,\ \tau = t$

C. Benedetti et al. ICAP Proceedings 2012,
C. Benedetti et al., PPCF 2018

$$\nabla^2\tilde{A} + 2ik_0\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\tilde{A} - \frac{1}{c^2}\frac{\partial^2\tilde{A}}{\partial t^2} = \chi\tilde{A},$$

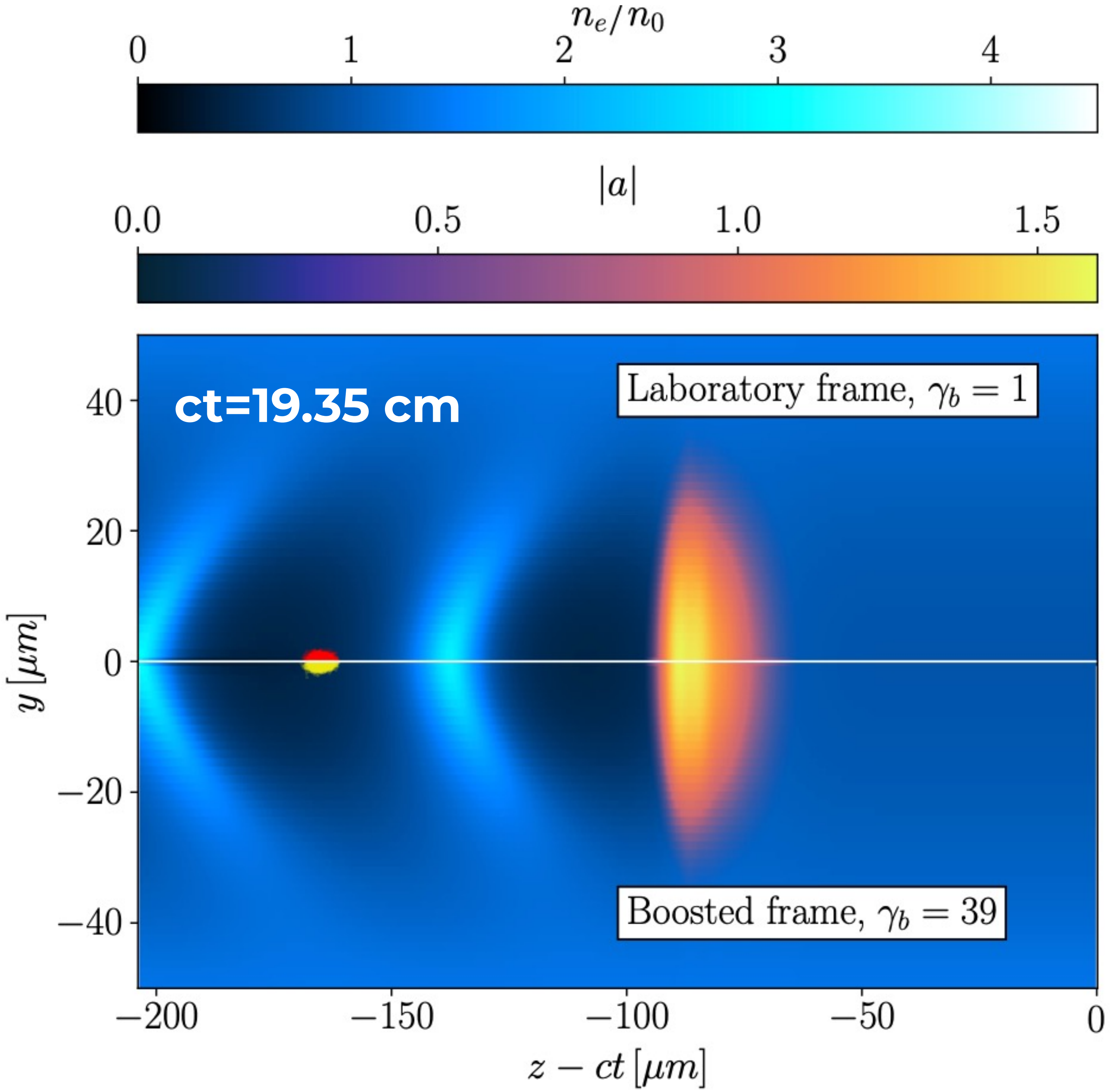
D. Terzani et al, Com. Phys. Comm. 2019

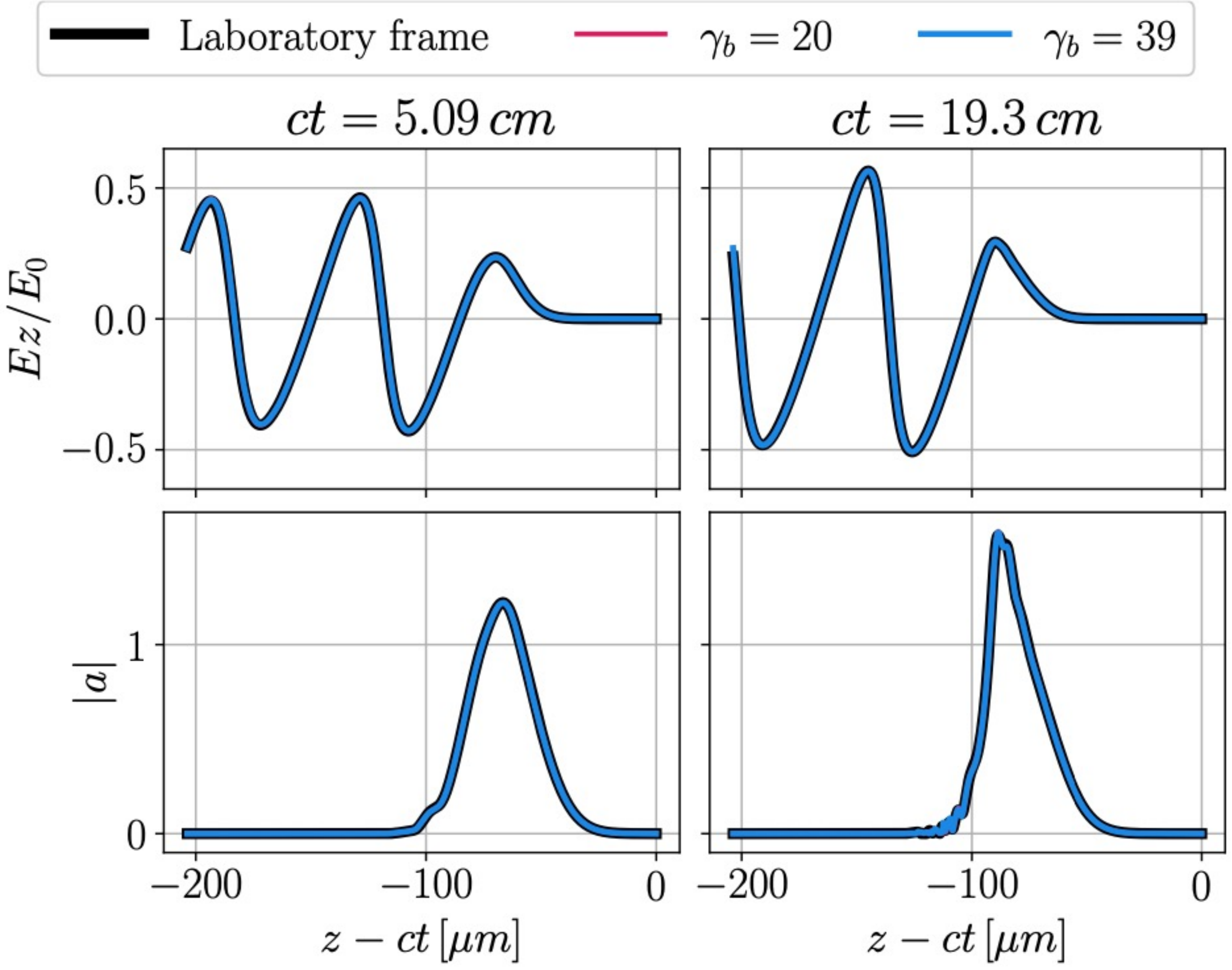
Theoretical Speed-up: $S_{TPA+LBF} \approx (1+\beta_b)^2\gamma_b^2\Omega^2$

The caveats for LBF and TPA speedups are even more restrictive when they are coupled!

Benchmark LBF+TPA:
guided LWFA with external injection

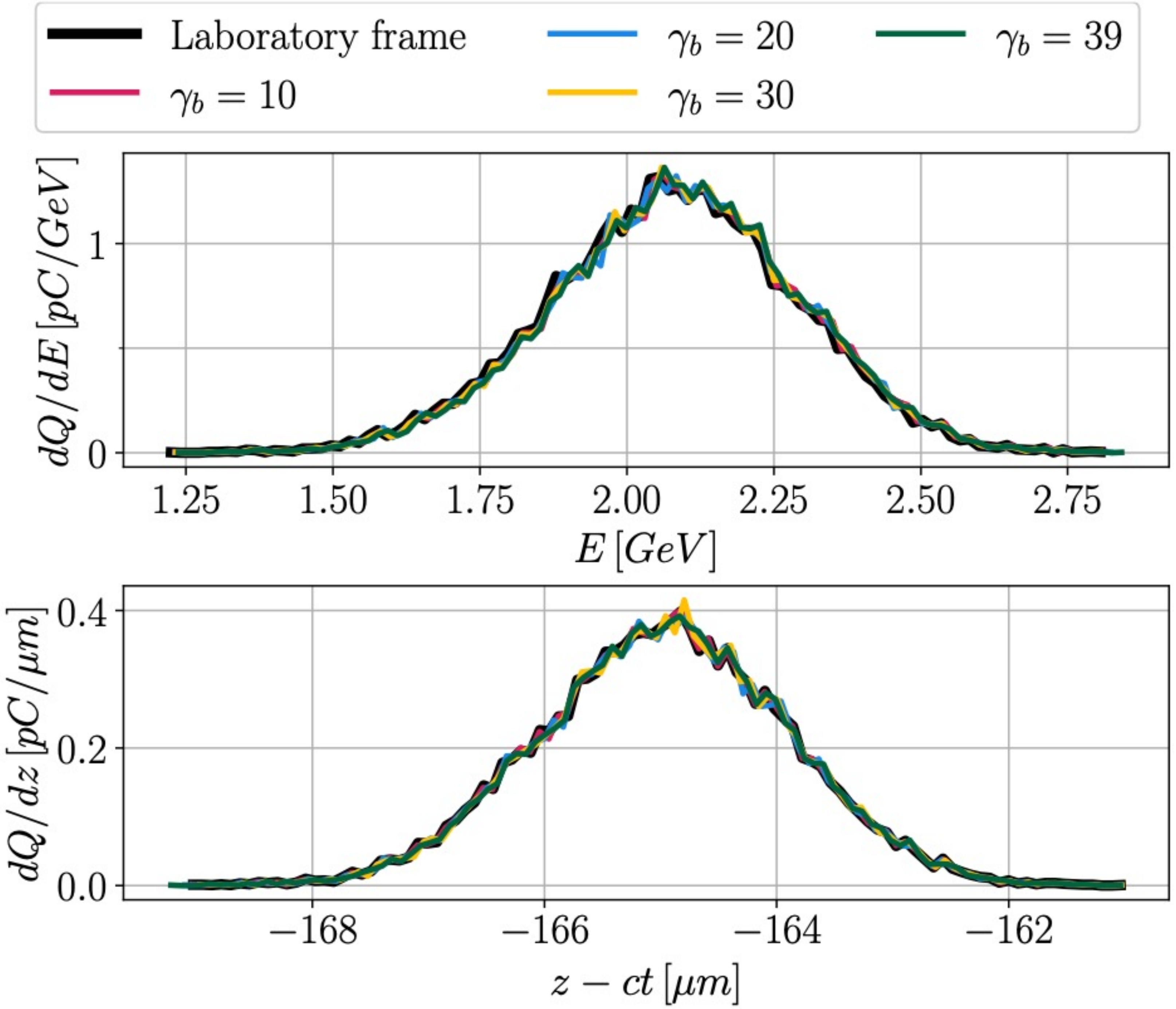
Theoretical $L_{dephasing}$ ~ 20.5 cm

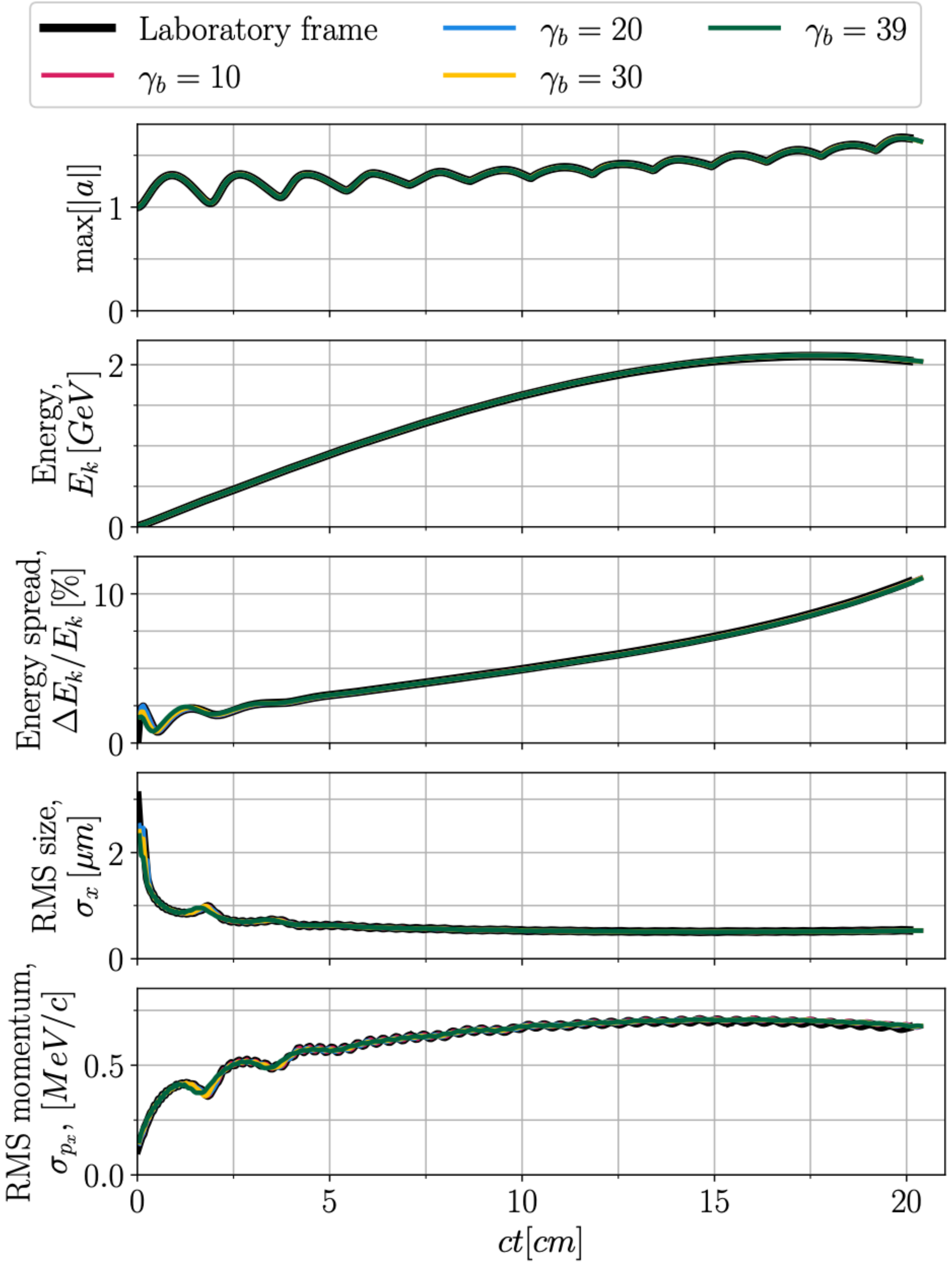




Benchmark LBF+TPA:
guided LWFA with external injection

Spectrum at ct=19.35 cm





LBF+TPA: Speed-up

	$k_p\Delta z$	$\eta = c\Delta t/\Delta z$	$k_p\Delta r$	Time [hours]	Measured speed-up	Theoretical speed-up
laboratory frame	1/80	0.24	1/8	99.5	–	–
$\gamma_b = 2$	1/80	0.24	1/8	7.02	14	14
$\gamma_b = 4$	1/80	0.24	1/8	1.60	62	60
$\gamma_b = 6$	1/80	0.24	1/8	0.73	136	133
$\gamma_b = 8$	1/80	0.24	1/8	0.42	237	225
$\gamma_b = 10$	1/120	0.24	1/8	0.63	158	148
$\gamma_b = 20$	1/120	0.10	1/8	0.57	175	165
$\gamma_b = 30$	1/120	0.05	1/8	0.72	138	119
$\gamma_b = 39$	1/120	0.05	1/8	0.60	166	139

For $\gamma_b \lesssim (\Delta x_\perp/\Delta z)/2$, in theory: $S_{TPA+LBF} \approx (1+\beta_b)^2\gamma_b^2\Omega^2$

But with the Courant Friedrichs Lévy limitation: $S_{LBF}^{Effective} = \frac{\eta}{\eta_{ref}}\left(\frac{\Delta z}{\Delta z_{ref}}\right)^2 S_{LBF}$

Conclusions

- Demonstrated coupling of **Time-averaged Ponderomotive Approximation (TPA)** and **Lorentz Boosted Frame (LBF)** techniques for GeV-class LWFA stage

- **Excellent agreement found between lab-frame TPA and combined TPA+LBF results** (no significant differences across Lorentz factors up to $\gamma_b = 39$)

- Even small Lorentz boosts ($\gamma_b \lesssim 10$) may yield **two orders of magnitude speed-up** over lab-frame TPA for long-distance, high-energy LWFA stages:
  →**Up to eight orders of magnitude speed-up** vs. 3D lab-frame simulations without TPA

- Speed-up comparable to quasi-static approximation, but retains kinetic physics effects (e.g., electron injection from plasma)

Acknowledgments

This work was supported by the Director, Office of Science, Office of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC02- 05CH11231, and used the computational facilities at the National Energy Research Scientific Computing Center (NERSC).

This work was granted access to the HPC resources of TGCC under the allocation 2023- A0150510062 (Virtual Laplace) made by GENCI. Most of the development of Smilei relative to this article was made on the meso-scale HPC “3Lab Computing” hosted at École polytechnique and administrated by the Laboratoire Leprince-Ringuet, Laboratoire des Solides Irradiés et Laboratoire pour l’Utilisation des Lasers Intenses. Most of the simulation results obtained with Smilei were obtained on the same cluster.

Extra slides