Acceleration Induced Self Interactions of Ultra Short Compact Bunches

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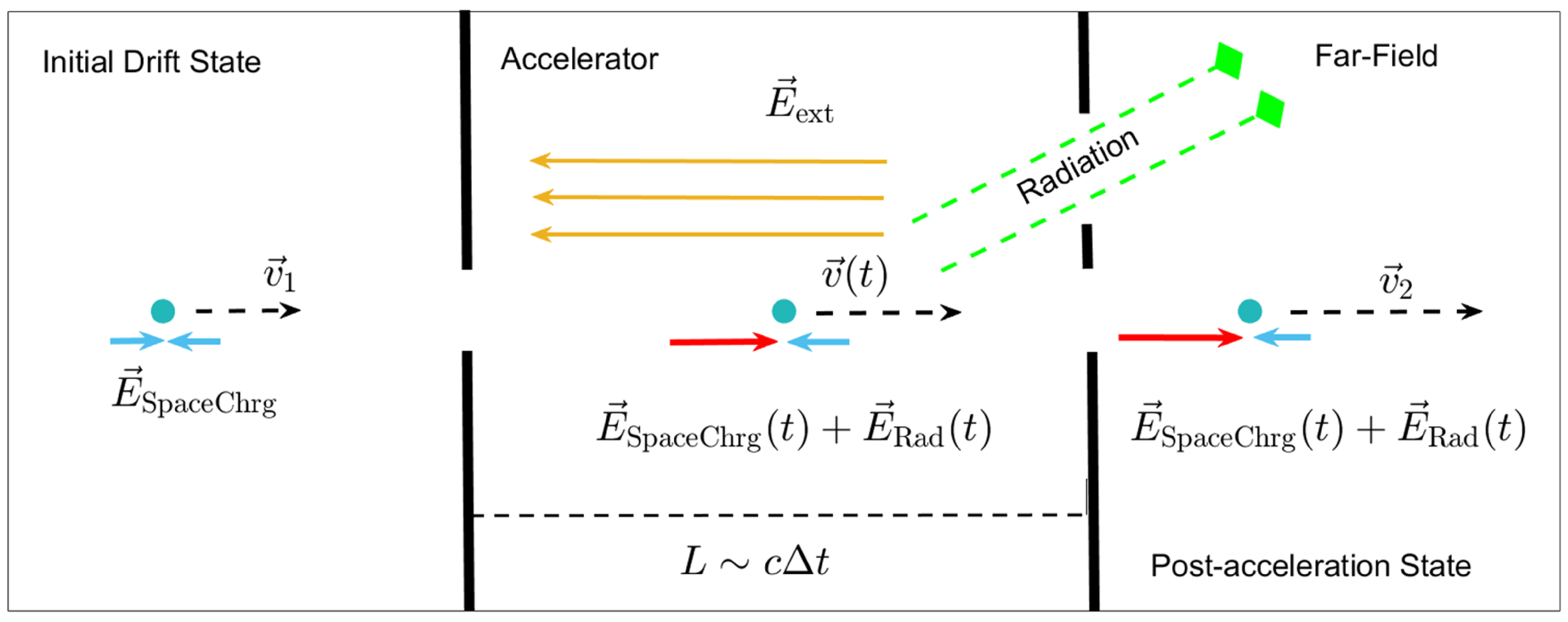
Introduction

Novel high-field gradient particle acceleration methods must meet strict performance requirements, e.g.. Successful demonstration of an FEL requires, low-energy-spread (< 0.5%), low-emittance (< 1μm) electron beams with high charge (> 30 pC)[1]. In previous work, it was found that **an accelerated compact bunch emits non-neglible amounts of coherent radiation**[2] (power scales as N^2, N:number of charges) into the far-field, constituting a loss of energy of the bunch. **We investigated the interaction induced by the acceleration/coherent emission process and identify effects degrading the quality of the bunch and limiting performance.**

Fig1. Schematic of motion and interaction.



Model

A negative charge distribution of longitudinal length σ_z, transverse length σ_x, is accelerated over distance L for duration Δt by an externally applied field E_{ext}, gaining energy $\Delta\gamma mc^2$ and emitting radiation. The charge distribution is considered to be rigid with motion defined by a particular trajectory, the influence of radiation reaction on the motion of the charge is neglected. The interaction begins upon onset of acceleration and continues down the beamline, during and post-acceleration.

2. Retardation of point charges and distributions

The electromagnetic fields of a point charge moving with trajectory $r_0(t)$ are described by the **"Lienard Weichert"** Fields [3], containing velocity and acceleration dependent terms. The fields are **retarded**, that is they appear to emanate from a charged source at a previous or "retarded" time, t^{ret}. For a distribution, we promote the single particle LW fields to an integral over a **retarded charge distribution.** $\mathbf{n}$ is vector from source to observer, c is speed of light, ε_0 is permittivity of free space.

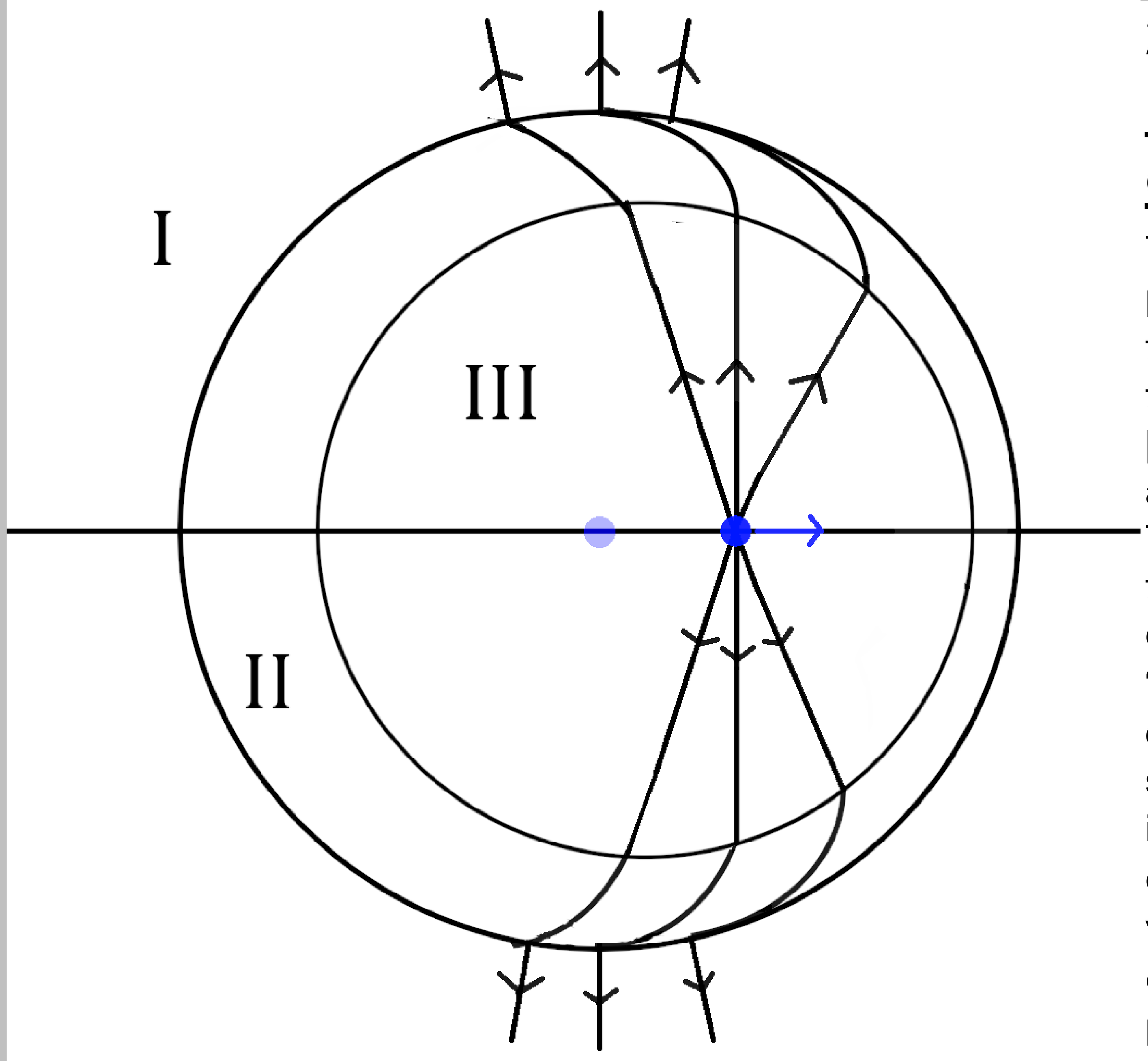


Fig. 2. Fields of charge accelerated for finite time. An observer sees **I**: Coulomb Fields from before acceleration, **II**: Fields induced by acceleration (radiation), **III**: Coulomb Fields after acceleration (charge energy has increased). Radiation can be viewed as a "shearing" of the fields due to the acceleration.

"Acceleration" term

$$\mathbf{E}_{\rm LW}(\mathbf{r}_0(t),\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\left[\frac{(1-\dot{\mathbf{r}}_0\cdot\dot{\mathbf{r}}_0/c^2)(\hat{\mathbf{n}}-\dot{\mathbf{r}}_0/c)}{(1-\hat{\mathbf{n}}\cdot\dot{\mathbf{r}}_0/c)^3|\mathbf{r}_0-\mathbf{x}|^2} + \frac{1}{4\pi\epsilon_0}\frac{\hat{\mathbf{n}}\times(\hat{\mathbf{n}}-\dot{\mathbf{r}}_0/c)\times\ddot{\mathbf{r}}_0}{c^2(1-\hat{\mathbf{n}}\cdot\dot{\mathbf{r}}_0/c)^3|\mathbf{r}_0-\mathbf{x}|}\right]^{\rm ret}$$

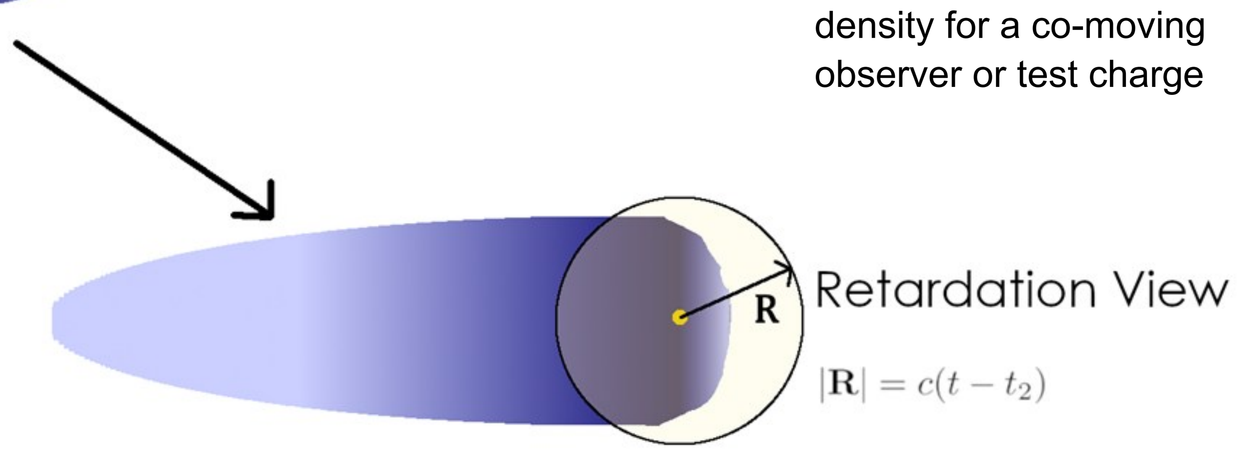
"Velocity" term

Integrate over single particle kernel . . .

$$\mathbf{E}(\mathbf{s},t) = \int_0^\infty dR\int_0^\pi d\theta\sin\theta\int_0^{2\pi}d\phi\rho\left(\mathbf{s}+\mathbf{r}_0(t^{\rm ret})-\mathbf{r}_0(t^{\rm ret})-\mathbf{R}\right)\mathbf{K}(R,\theta,\phi,\mathbf{r}_0(t))$$

Encodes retardation such that co-moving test charge experiences radiation and acceleration induced field from charge within particular neighborhood. **s** denotes position of test charge relative to moving bunch.

Fig. 3. Retardation of an accelerated charge density for a co-moving observer or test charge



Charge density becomes assymmetrically stretched from the perspective of observer. Transparent yellow region indicates the causal region that contributes radiative forces.

Charge outside of this region hasn't started accelerating yet and contributes coulomb-like "drift" fields.

KEY

- Observer s
- Charge Density
- Causal region

3a. Results: Fields during Acceleration

We calculate the longitudinal fields of a compact electron bunch

- Q=100pC
- $\sigma_{x,z}$=1μm
- E_{ext}=20GeV/m
- Δt =100ps (L~3cm)
- γ_1=100 ($\gamma_1 mc^2$ ~50MeV)

where γ_1 is the initial energy (Lorentz factor). Acceleration induced fields are visibly **asymmetric** for $\Delta\gamma mc^2 > \gamma_1 mc^2$ such that middle and back of the bunch experience strong positive fields.

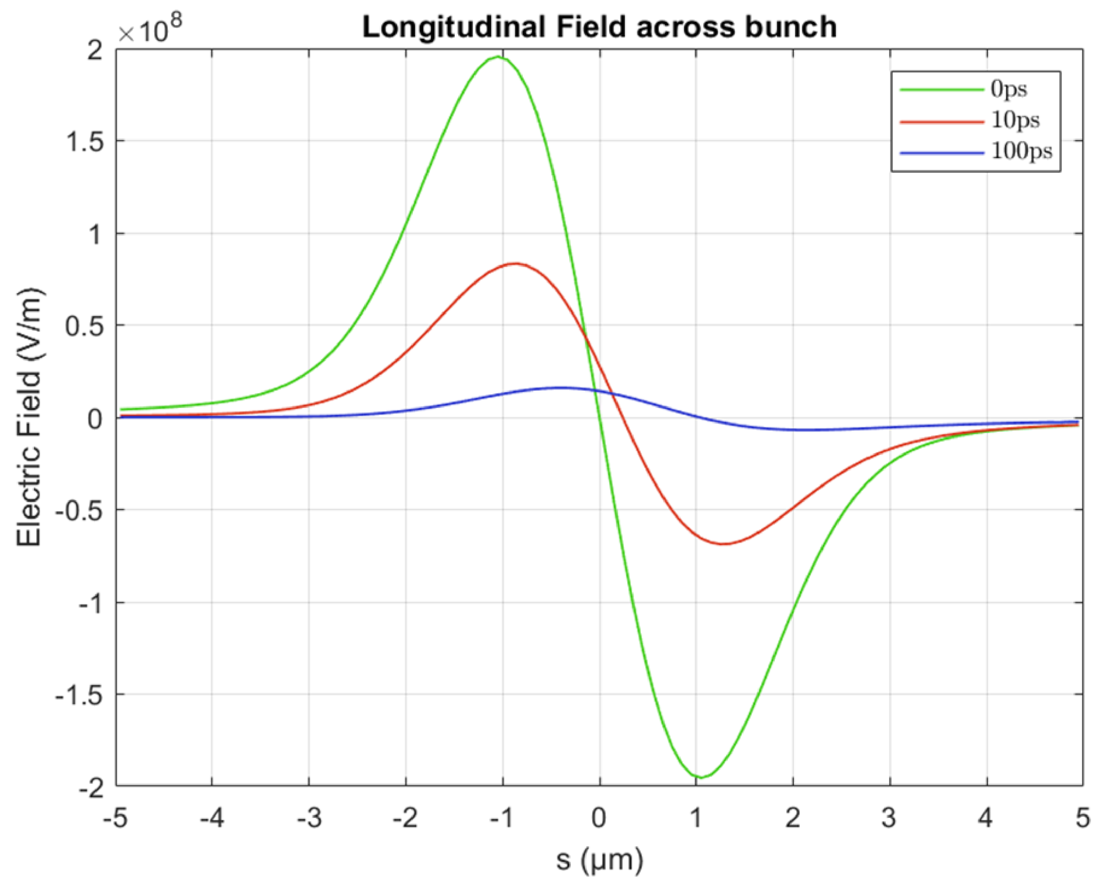


Fig4: Longitudinal fields during acceleration. Overall magnitude decreases as energy increases.

3 .b. Results: Fields Post-Acceleration

Transient fields **continue to interact** with bunch up-to formation length $2\sigma_z\gamma_2{}^2$ i.e. the length over which the radiation decouples from the bunch[4]. Asymmetry of field/interaction reflects asymmetry in retardation point of view.

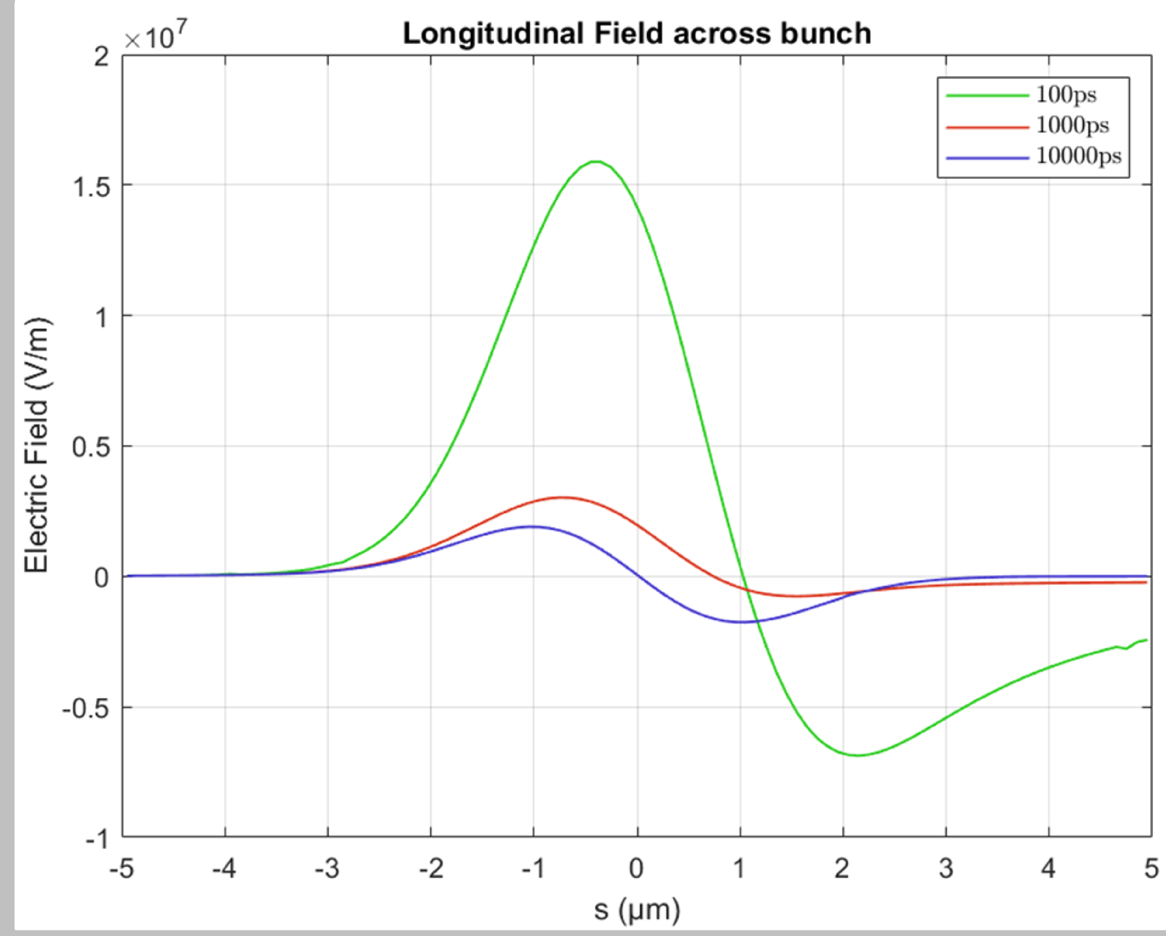


Fig. 5: Longitudinal fields after the acceleration process, decays into regular coulomb field after formation length ($2\sigma_z\gamma_2{}^2/c$=1ns).

4. Results: Collective Effects

A combination of space-charge and radiation field alters the previously monochromatic energy spectrum of the bunch; **a negative chirp in the back and centre of the bunch of order O(0.1-1%) is induced**. Net loss of energy/momentum due to self-fields (fig. 7). Longer bunches lose energy for longer times post-acceleration, however **shorter bunches lose more energy overall and during the acceleration, such that the exact mechanism of energy loss depends on the nature of the acceleration.** Net losses of 100pC, $\sigma_{x,z}$=1μm bunch on order of 1%, and scales as N^2; near-field losses correspond to energy of radiation emitted into far-field.

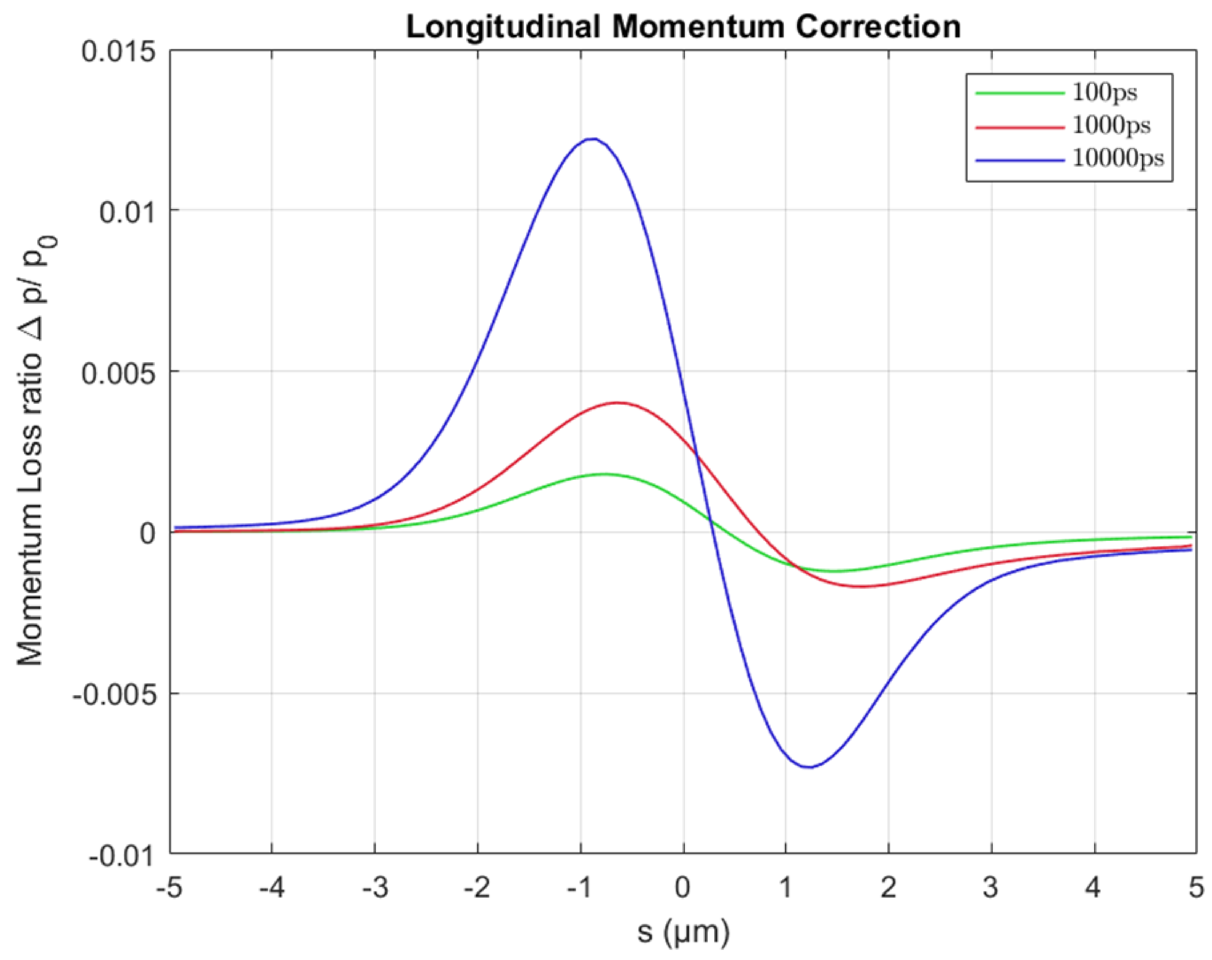


Fig. 6. Longitudinal momentum chirp across bunch as a ratio of total momentum

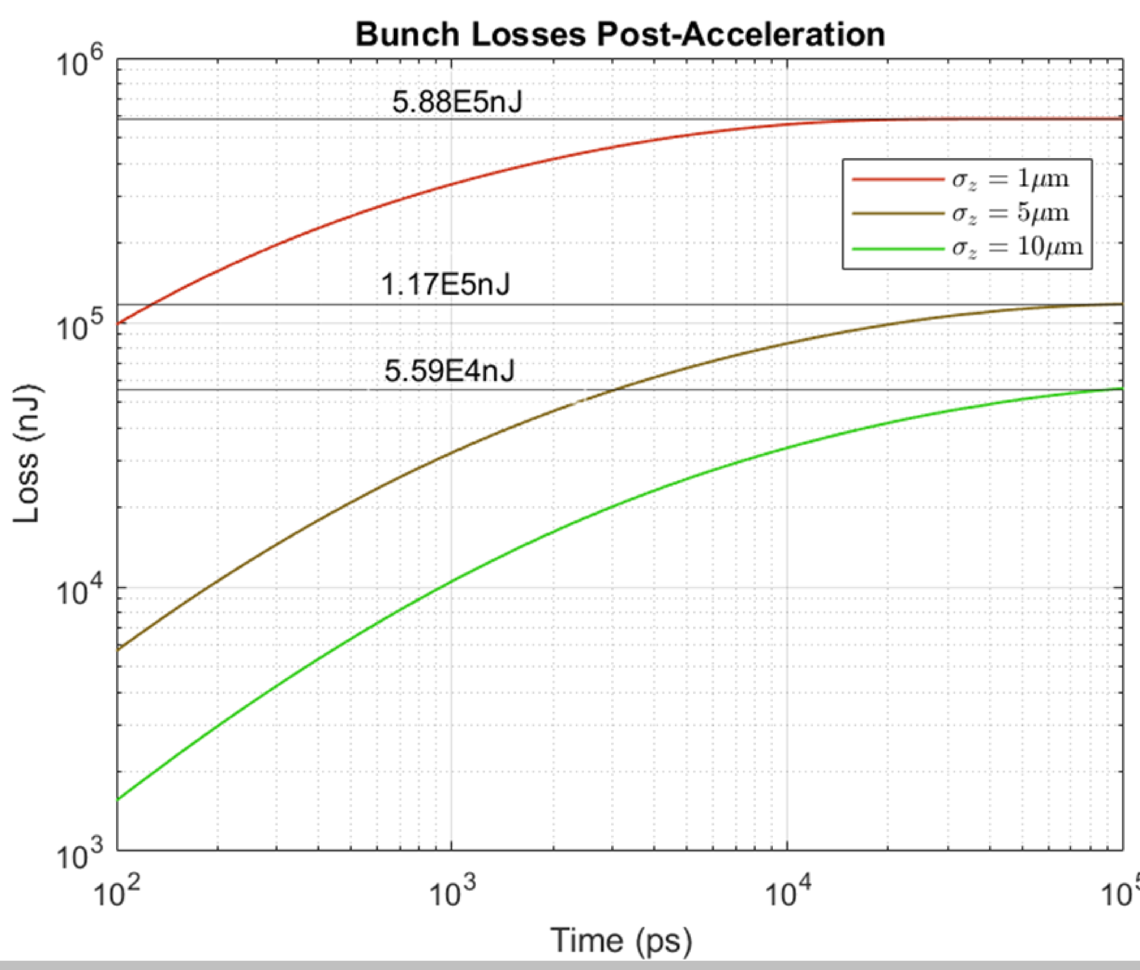


Fig. 7: Logarithmic plot of losses as a function of time post-acceleration for various bunch lengths.

Summary

This model points to an inherent vacuum beam-loading process that is exacerbated rather than compensated by higher-gradient accelerating fields. Novel high-field accelerators of compact bunches are at risk of suffering from coherent emission losses and significant momentum spread.

References

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[2] R.J. McGuigan, S.P. Jamison, Coherent Radiation Of Ultra-Short Bunches from Linear Acceleration Processes (unpublished)

[3] A. Zangwill, Modern Electrodynamics, C. U.Press (2012)

[4]. G. Geloni, Acceleration-induced self-interactions within a relativistic electron bunch : an analytical study, Quality and Rel. Eng. Int., 01 (2003)