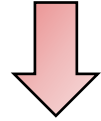


Physics of transverse dynamics in a laser-plasma accelerator

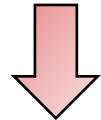
L. Batista, S. Marini, N. Chauvin, D. Uriot, A. Chancé, P.A.P. Nghiem



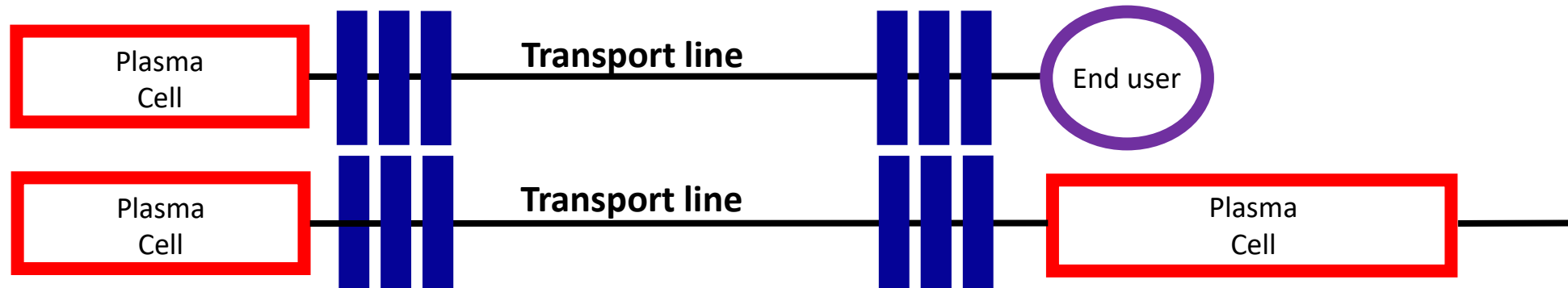
Using the accelerated electron beam



- Extracting the accelerated beam from the plasma
- Sending this beam to the next user or plasma
- Plasma - transport line coupling

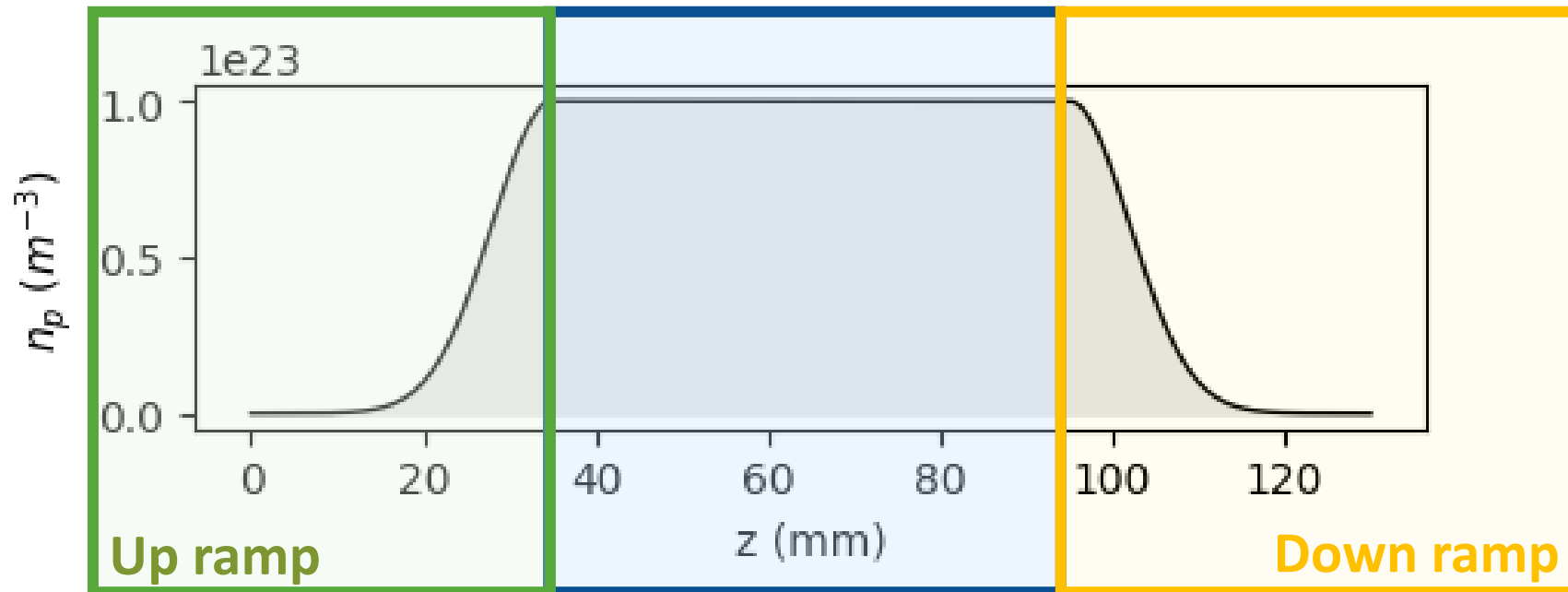


- **Transverse beam physics through the plasma**



Outline

- 1) Context & Background
- 2) Transport through the plasma **density plateau**
- 3) Transport through the **density down ramp** & **up ramp**
- 4) Example of coupling and conclusion



RMS quantities and Twiss parameters

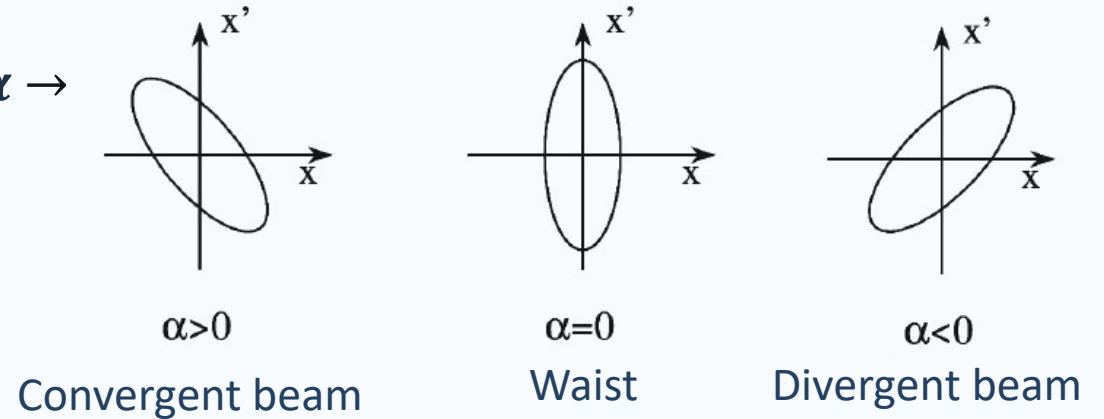
The electron beam can be described by:

- ❖ RMS Size: $\sigma_u = \sqrt{\langle u^2 \rangle}$
- ❖ RMS divergence: $\sigma_{u'} = \sqrt{\langle u'^2 \rangle}$
- ❖ **Twiss parameters: α, β, γ**
- ❖ Emittance ε

Twiss $\beta \rightarrow$ Related to the **RMS size** : $\beta = \frac{\sigma_u^2}{\varepsilon}$

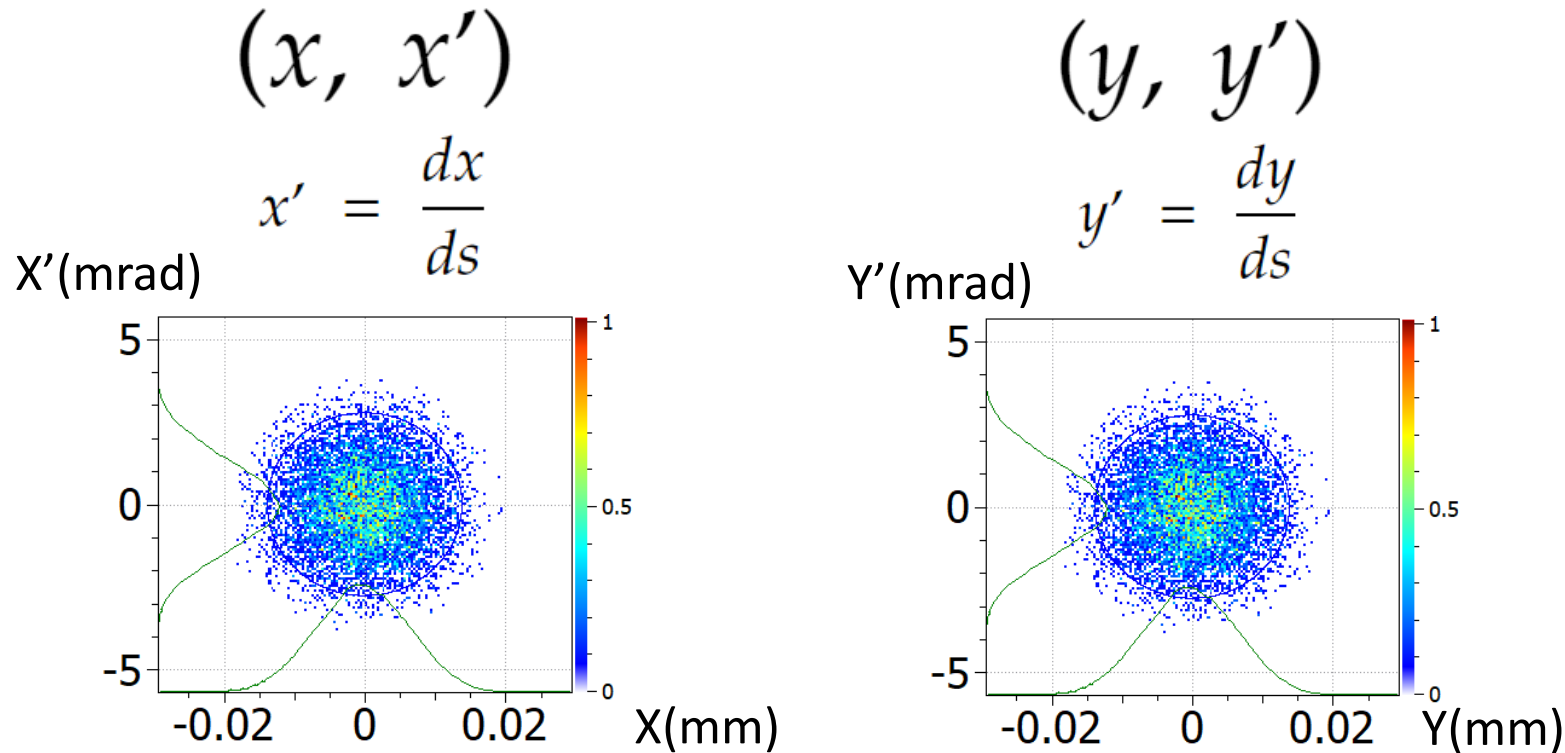
Twiss $\gamma \rightarrow$ Related to the **RMS divergence** : $\gamma = \frac{\sigma_{u'}^2}{\varepsilon}$

Twiss $\alpha \rightarrow$

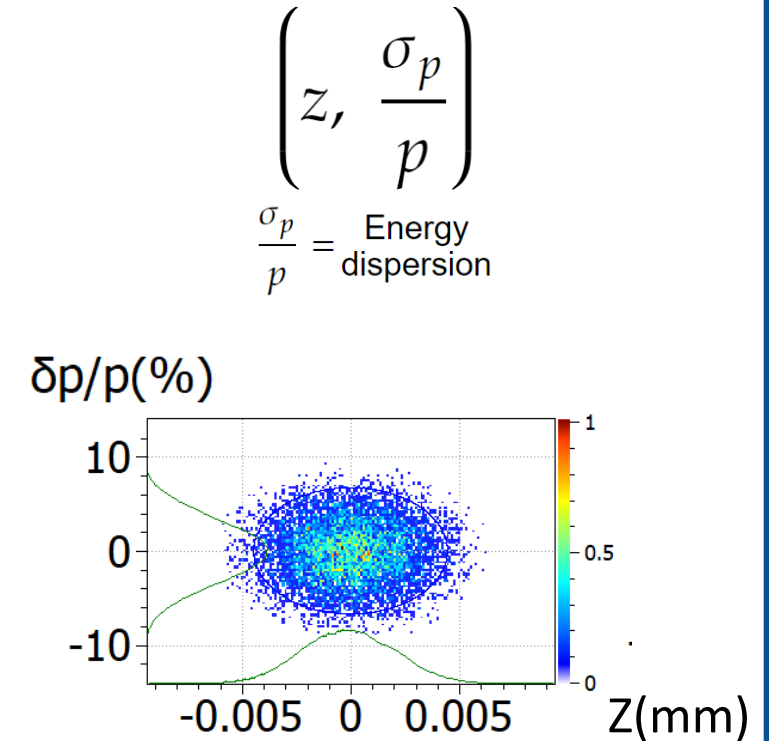


Beam distribution (6D) and emittance

Transverse



Longitudinal



Emittance ε $\rightarrow$ Surface occupied by the beam in the phase space

Focusing gradient K (m^{-2}):

$$K = \frac{e}{\gamma_{\text{rel}} m_e c^2} \left(\frac{\partial E_r}{\partial r} - \beta_{\text{rel}} c \frac{\partial B_\theta}{\partial r} \right)_{r=0}$$

γ_{rel}

Lorentz factor
→ Related to the beam energy

Gradient of the radial electric field

Gradient of the azimuthal magnetic field

USUAL QUADRUPOLES IN TRANSPORT LINES

Model with a sequence of “special” quadrupoles {up-ramp + plateau}

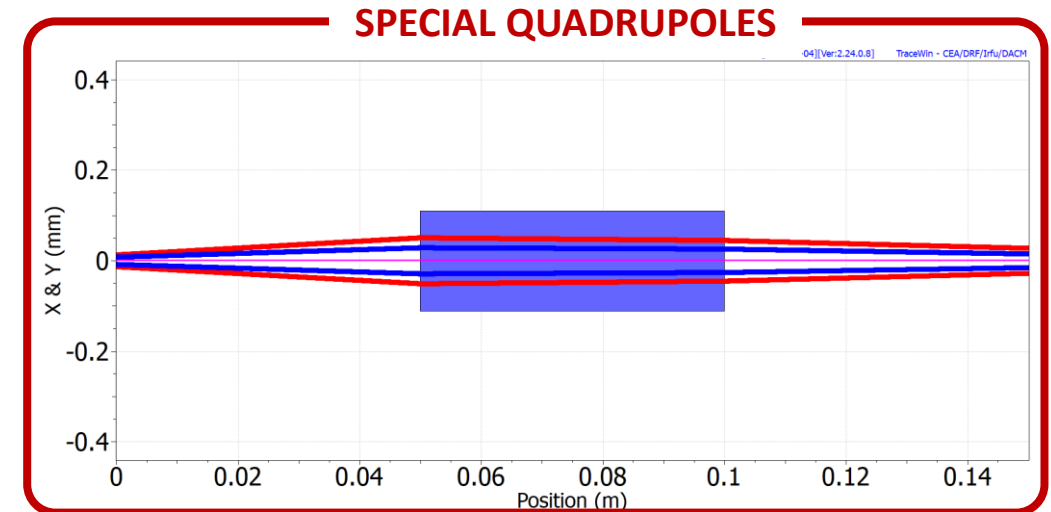
CPU time: a few seconds

SPECIAL QUADRUPOLES

$$K = \frac{e}{\gamma_{\text{rel}} m_e c^2} \left(\frac{\partial E_r}{\partial r} - \beta_{\text{rel}} c \frac{\partial B_\theta}{\partial r} \right)_{r=0}$$

Gradient of
the radial
electric
field

CPU time: a few seconds



Nominal parameters

Laser

Strength a_0	2
Pulse waist w_0	50 μm
Pulse duration (field) τ_0	50 fs

Plasma

Density n_0	10^{23} m^{-3}
Plateau length	up to 45 mm

Injected electron beam

Mean energy E	200 MeV
Energy spread σ_E/E	3%
Normalized emittance ε_{xN}	3 mm mrad
Normalized emittance ε_{yN}	1 mm mrad
Beam length σ_z	3 μm
Laser-electron bunch distance ξ	50 μm

Use of:

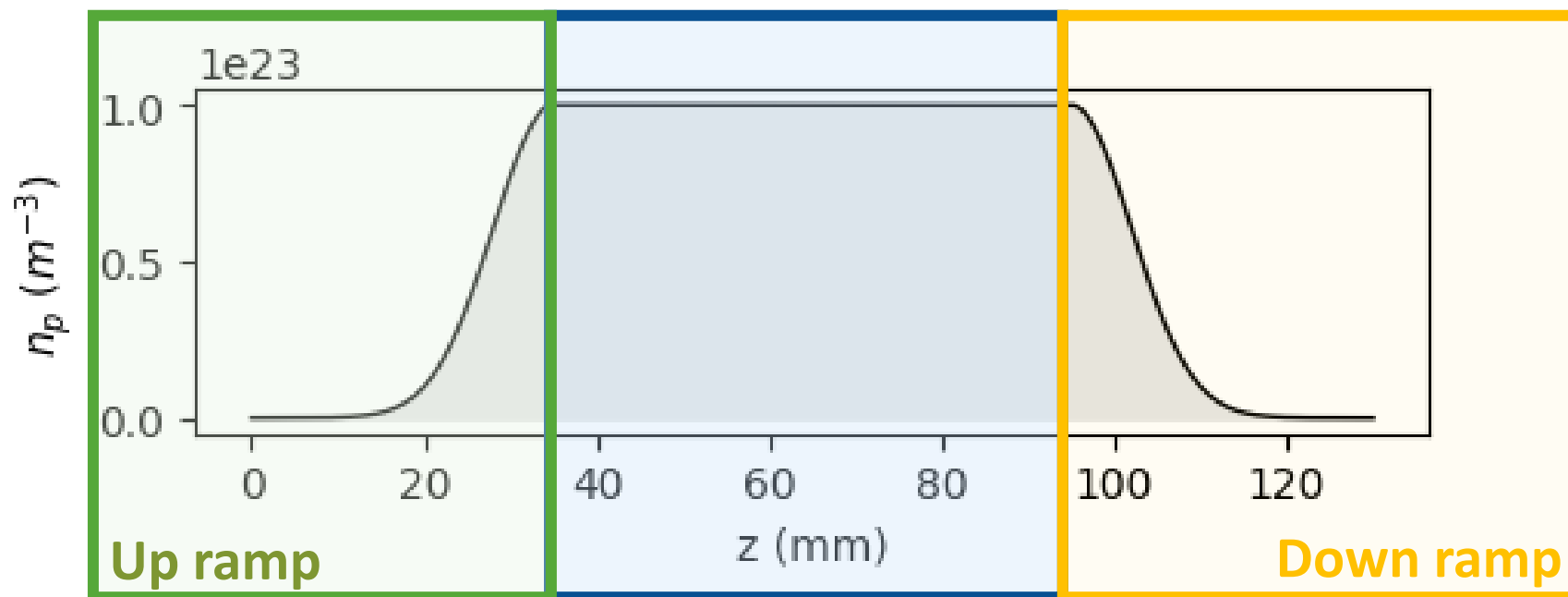
FBPIC: A laser-plasma PIC code

Wake-T: A laser-plasma tracking code
(quasistatic approach)

TraceWin: Beam Dynamics tracking code

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Transport through the plasma density plateau

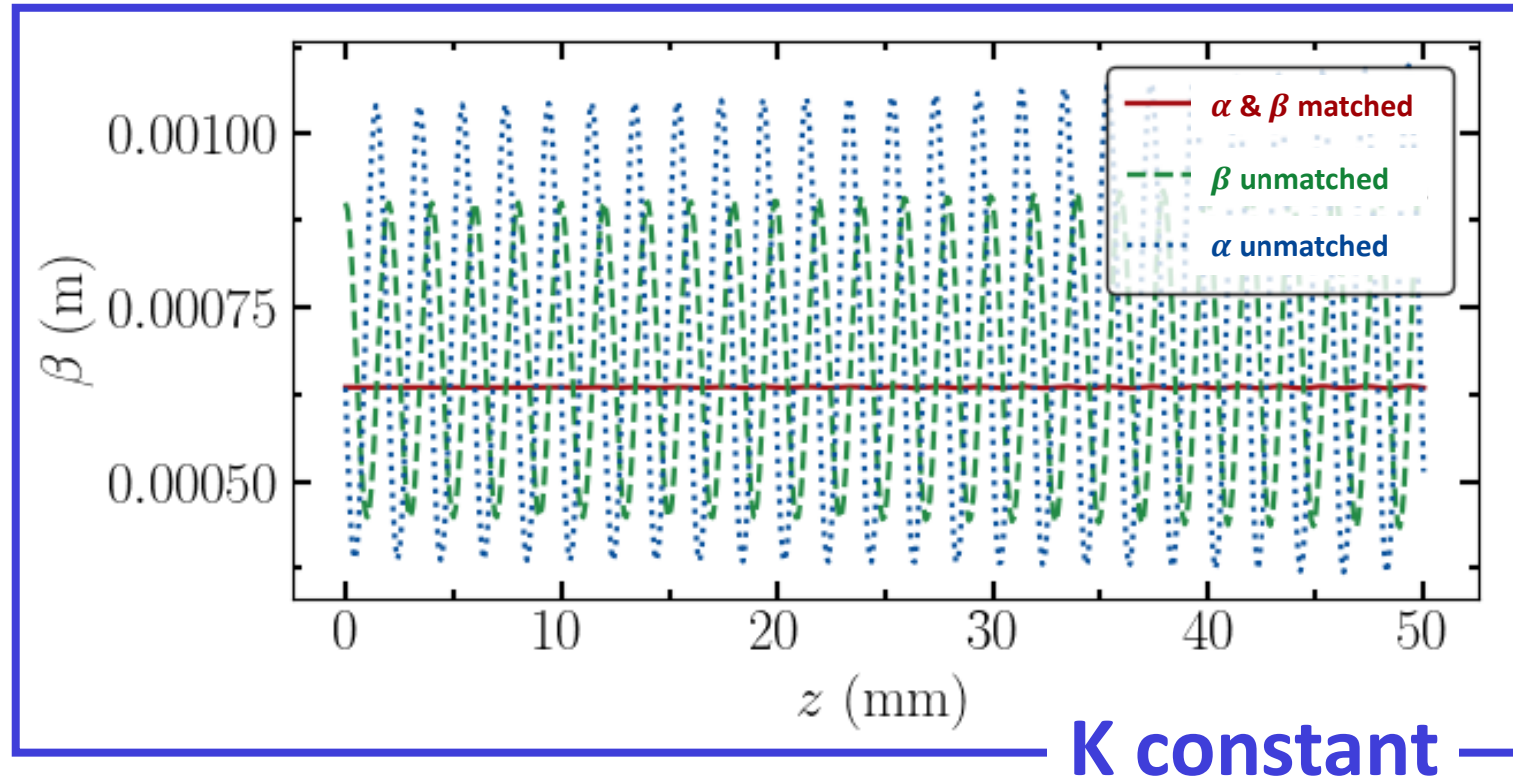
Objectives

Study the evolution of **Twiss parameters and emittance** across the plateau for all initial conditions (α_0 , β_0 and δ)

Characterize the influence of **energy spread**

Compare analytical results with numerical simulations

Transport through the plasma density plateau ($\delta = 0$)



For any input $\alpha_0, \beta_0 \rightarrow \beta$ oscillates : Frequency $2\sqrt{K}$

$$\beta(z) = \frac{b_+}{\sqrt{K}} - A \sin(2z\sqrt{K} + \phi)$$

$$b_+(\alpha_0, \beta_0, K)$$

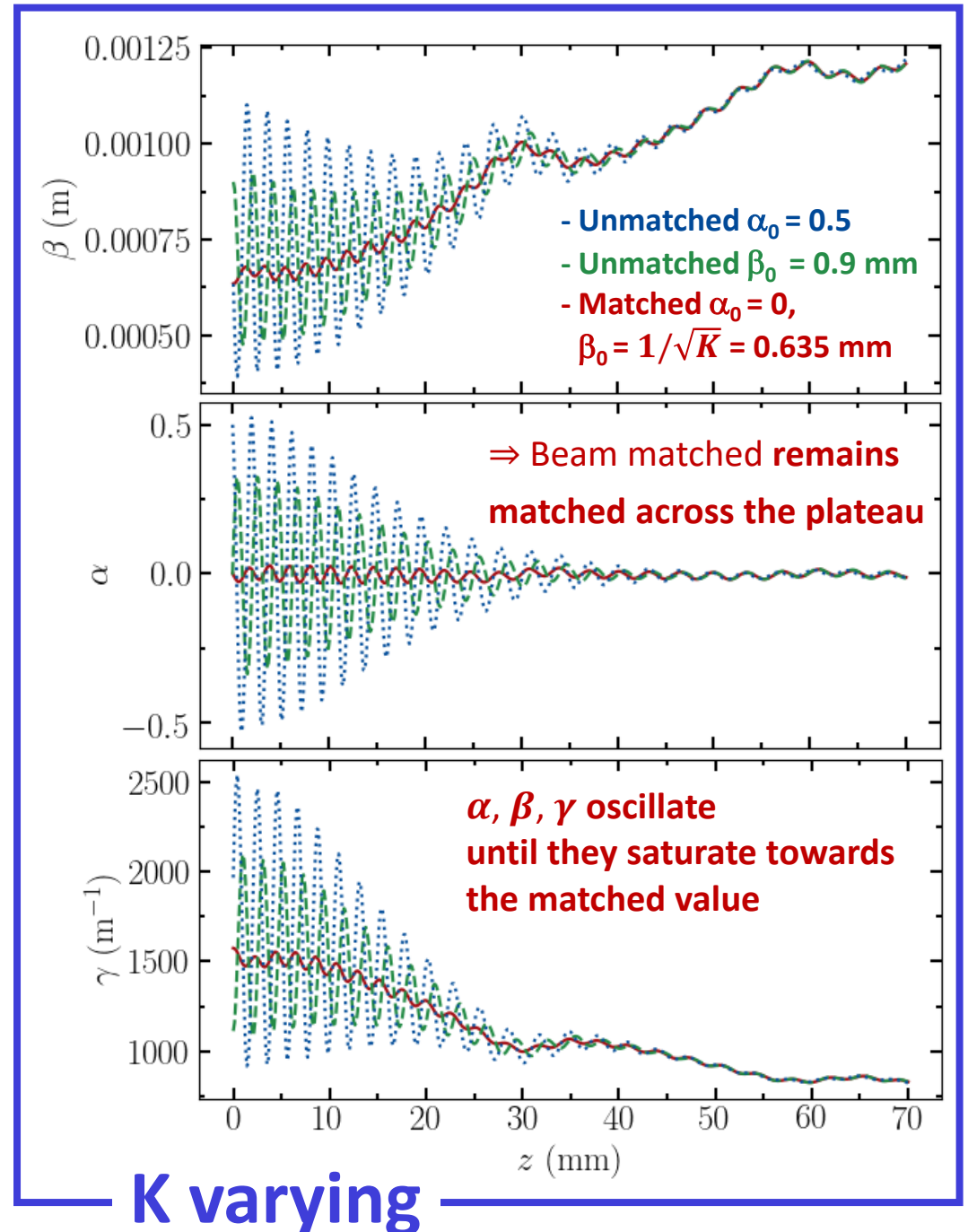
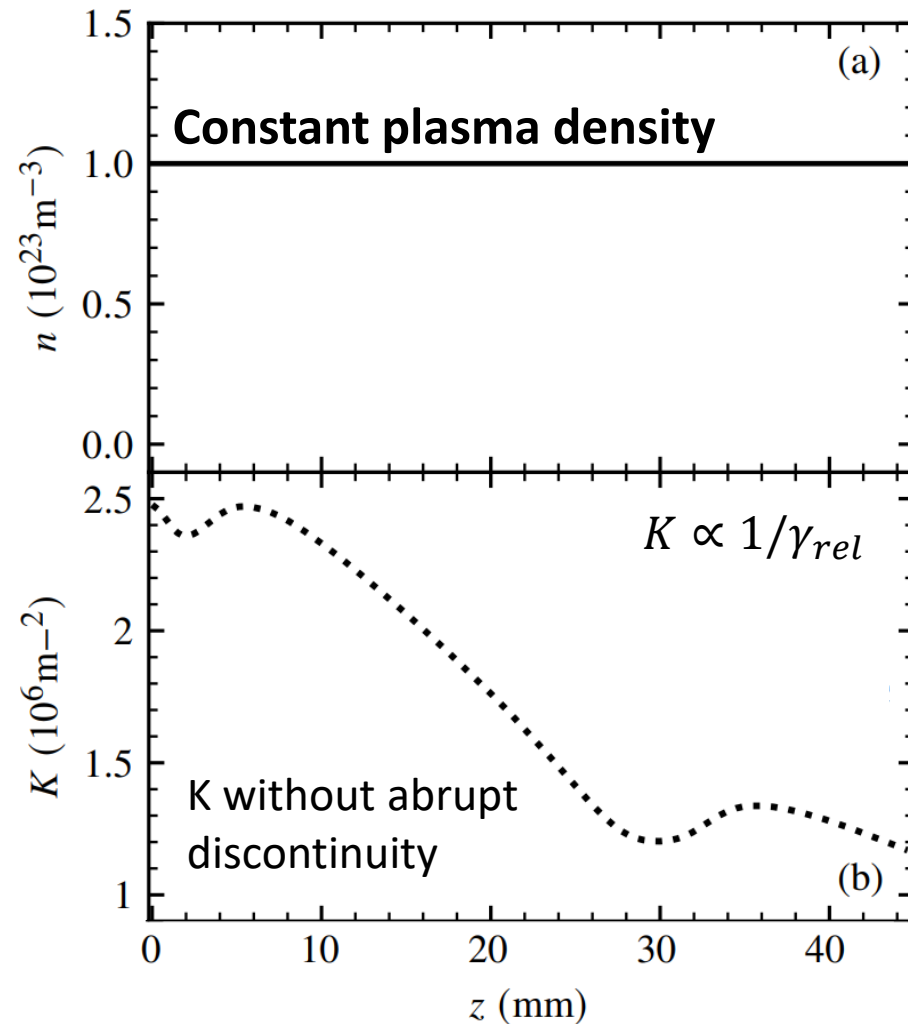
$$A(\alpha_0, \beta_0, K)$$

For $\alpha_0 = 0, \beta_0 = 1/\sqrt{K} \rightarrow \beta$ does not oscillate $\Leftrightarrow \beta_{min} = \beta_{max} = \beta_0$

The Twiss parameters are matched

Transport on the plasma density plateau ($\delta \neq 0$)

Energy spread : $\sigma_\delta = 5\%$



Transport through the plasma density plateau ($\delta \neq 0$)

Particles of different energies

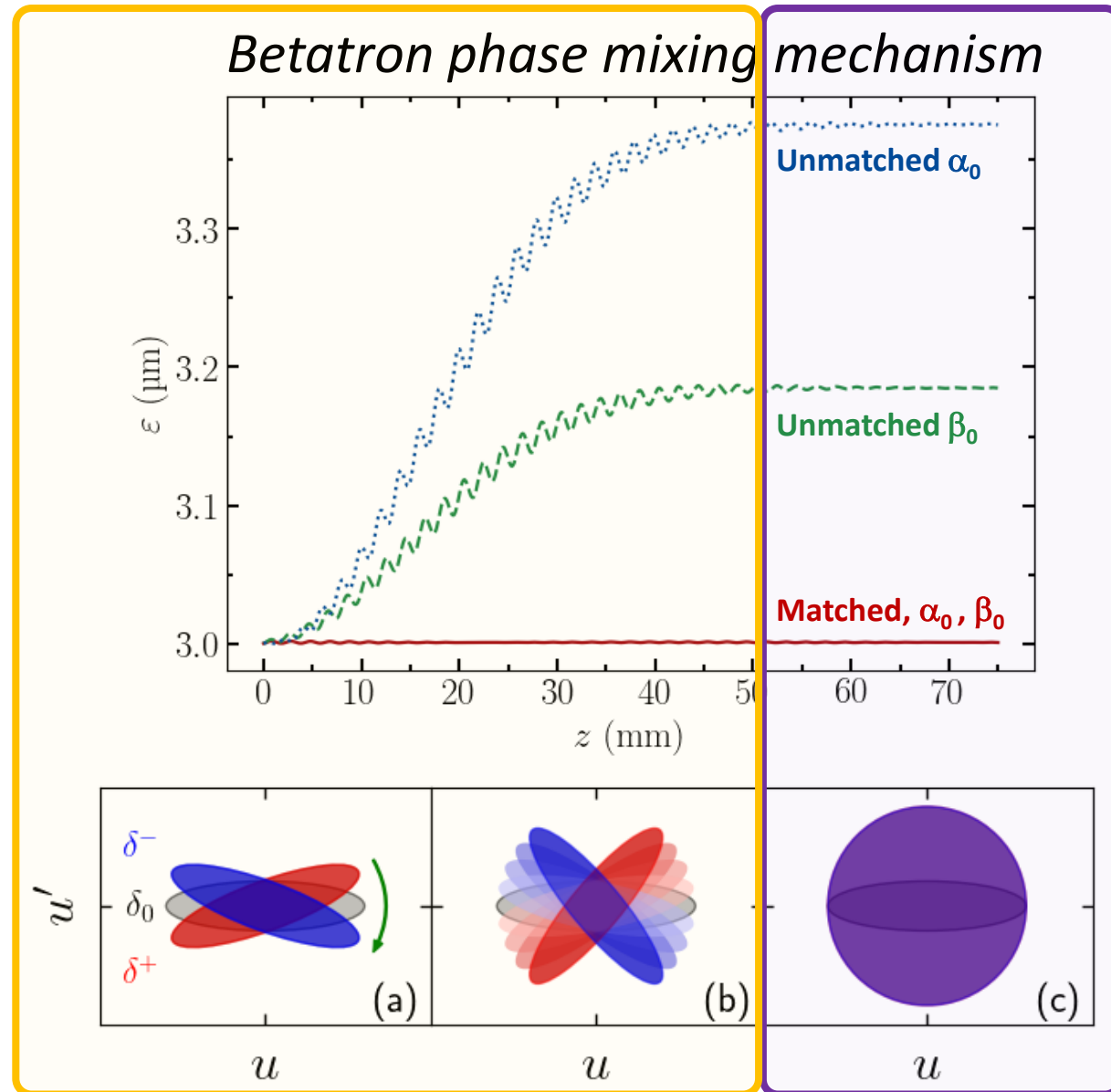
$$K \propto 1/\gamma_{rel}$$

→ focused differently

Oscillation frequency $2\sqrt{K}$

→ Oscillate at different frequencies

Growth and oscillation of emittance



Valid for constant or variable K

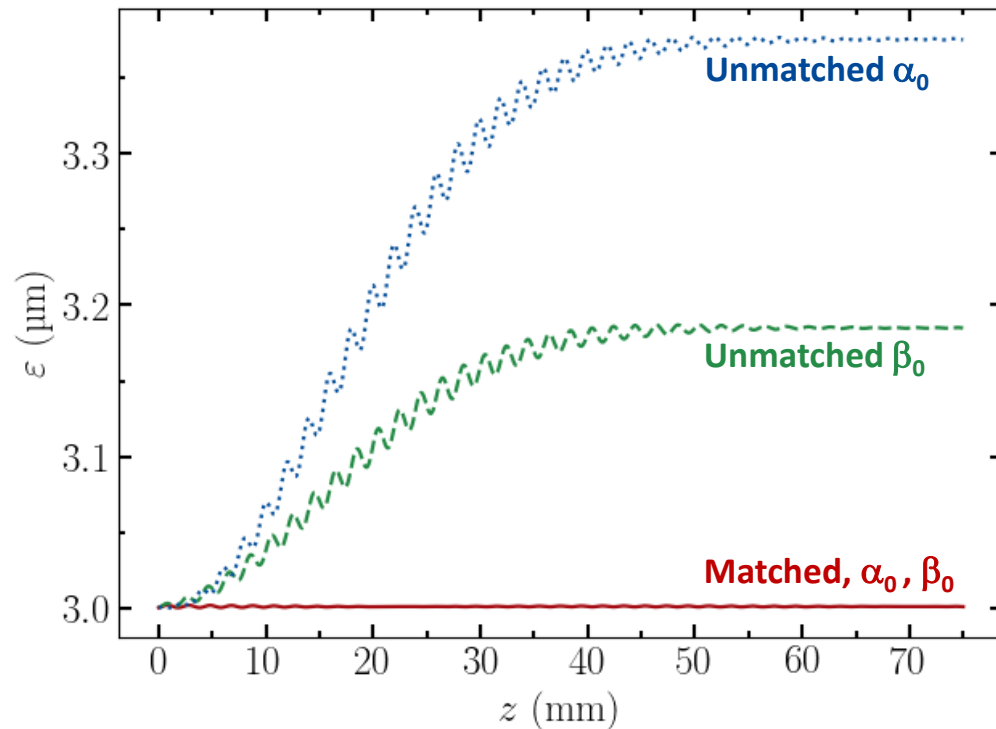
Mehrling, T et. al. (2012). *PRSTAB*, 15(11), 111303.

Ariniello, R, et.al. (2019). *PRAB*, 22(4), 041304.

Saturated emittance value

Transport through the plasma density plateau ($\delta \neq 0$)

Emittance evolution
from TraceWin
with plasma modeled by
special quadrupoles



Analytical model

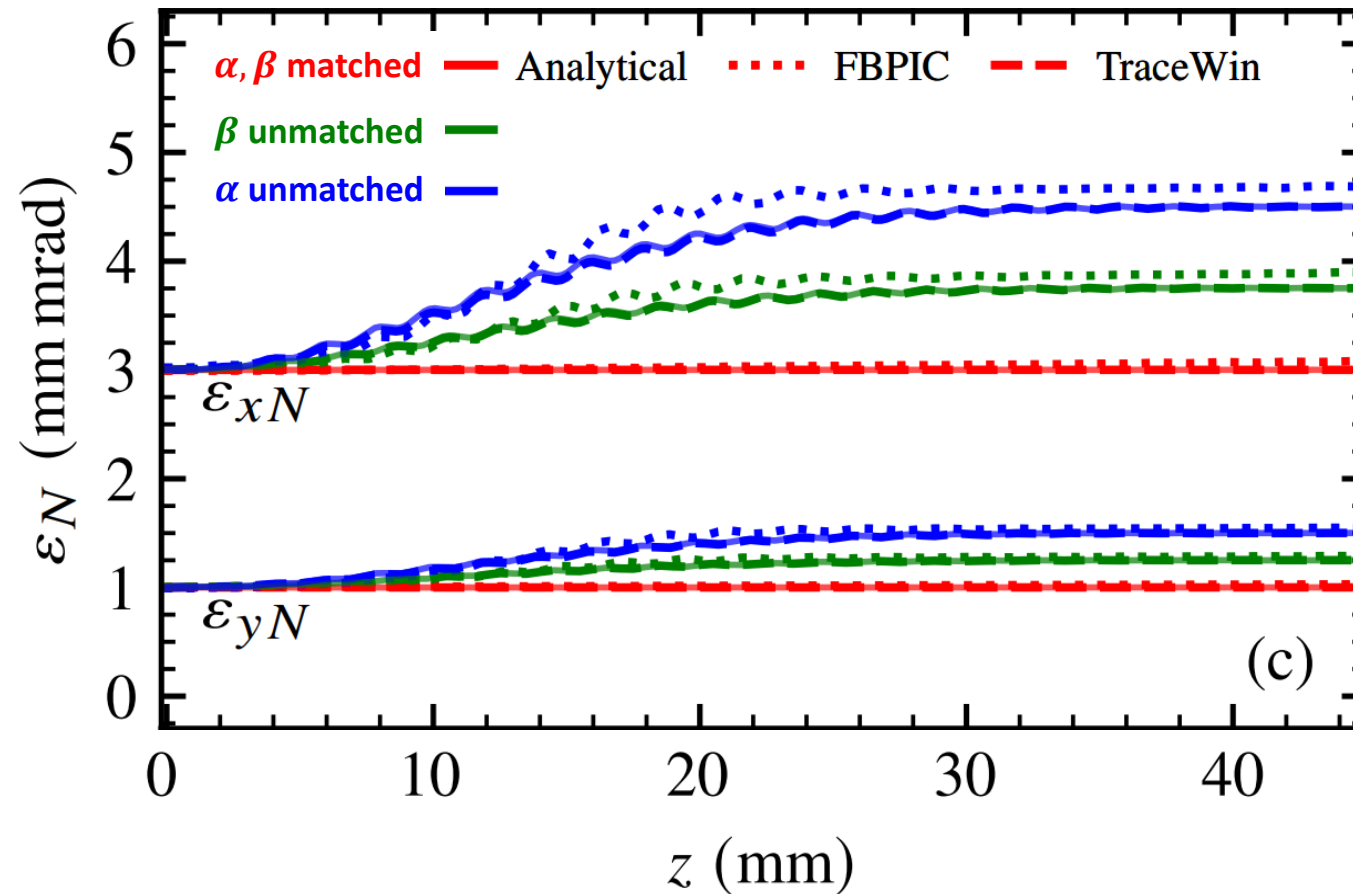
$$\frac{\varepsilon^2}{\varepsilon_0^2} = \underbrace{b_+^2}_{\text{Static term}} + \underbrace{\sigma_\delta^2 \frac{\sqrt{K} \beta_0}{2} b_+}_{\text{Chromatic offset}} + \underbrace{e^{-\sigma_\delta^2 z^2 K} (1 - b_+^2)}_{\text{Term of decoherence and oscillations}}$$

$$- \frac{\sigma_\delta^2}{8} e^{-\sigma_\delta^2 z^2 K} \left[\alpha_0^2 + 8z\sqrt{K}\alpha_0 b_+ - (3\sqrt{K}\beta_0 + b_+) b_- \right. \\ \left. + 2\alpha_0 b_- \sin[4z\sqrt{K}] - (\alpha_0^2 - b_-^2) \cos[4z\sqrt{K}] \right] \\ + \frac{\sigma_\delta^2}{2} e^{-\frac{1}{2}\sigma_\delta^2 z^2 K} \left[(-\beta_0\gamma_0 + 2z\sqrt{K}\alpha_0 b_+) \cos[2z\sqrt{K}] \right. \\ \left. + b_- (\alpha_0 - 2z\sqrt{K}b_+) \sin[2z\sqrt{K}] \right], \quad (24)$$

$$\text{With : } b_{\pm} = \frac{1 + \alpha_0^2}{2\beta_0\sqrt{K}} \pm \frac{\beta_0\sqrt{K}}{2}$$

Transport through the plasma density plateau ($\delta \neq 0$)

Comparison of the three approaches



To be compared with the semi-analytical approach of :
Aschikhin et. al. J. (2018). *NIMA*, 909, 414-418.

Impact of the energy spread

Expression of the decoherence length

$$z_D \propto \frac{1}{\sigma_\delta \sqrt{K}}$$

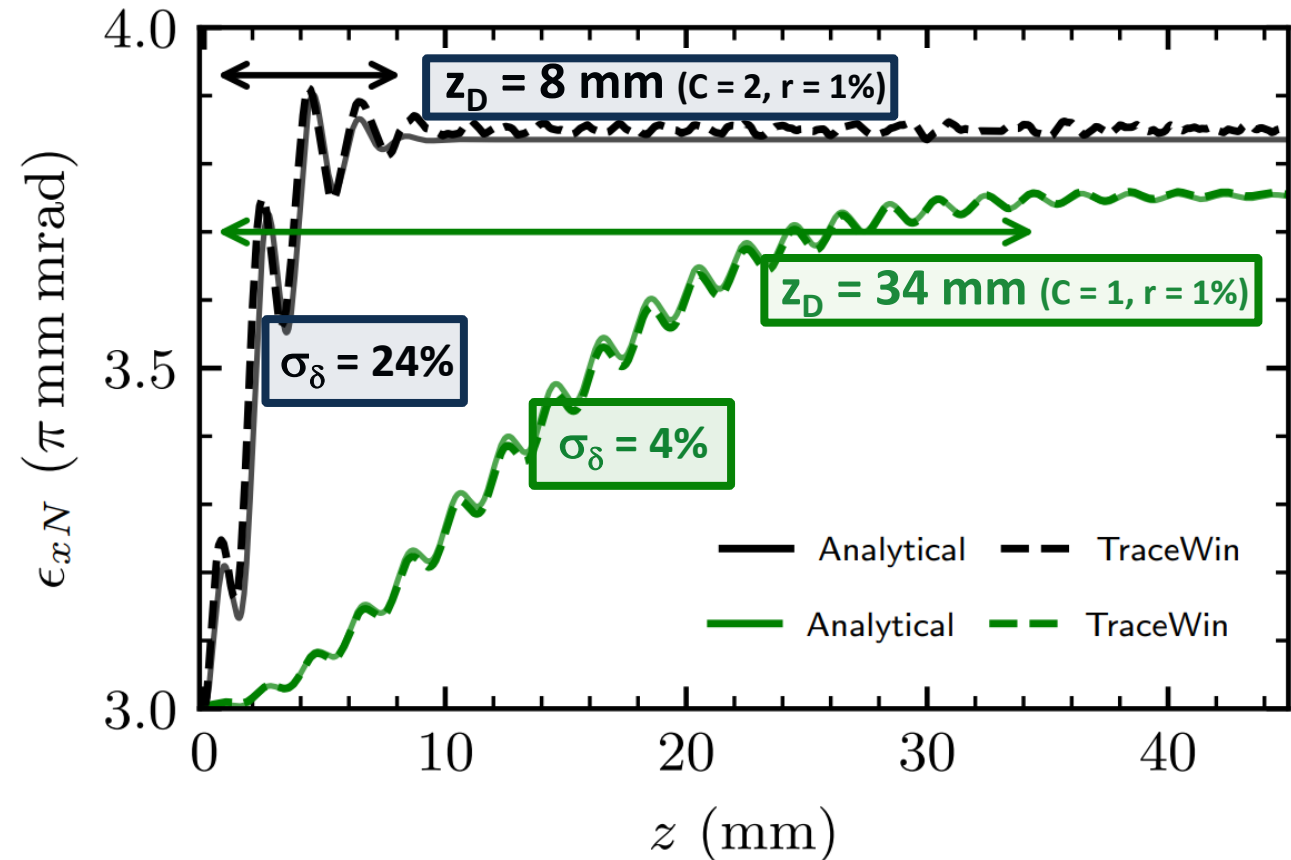
Expression of the emittance saturation value ($z \gg z_D$)

$$\varepsilon \approx \varepsilon_0 \left(\frac{1 + \alpha_0^2}{2\beta_0 \sqrt{K}} + \frac{\beta_0 \sqrt{K}}{2} \right) + \boxed{\varepsilon_0 \sigma_\delta^2 \frac{\sqrt{K} \beta_0}{4}}$$

Second
order term

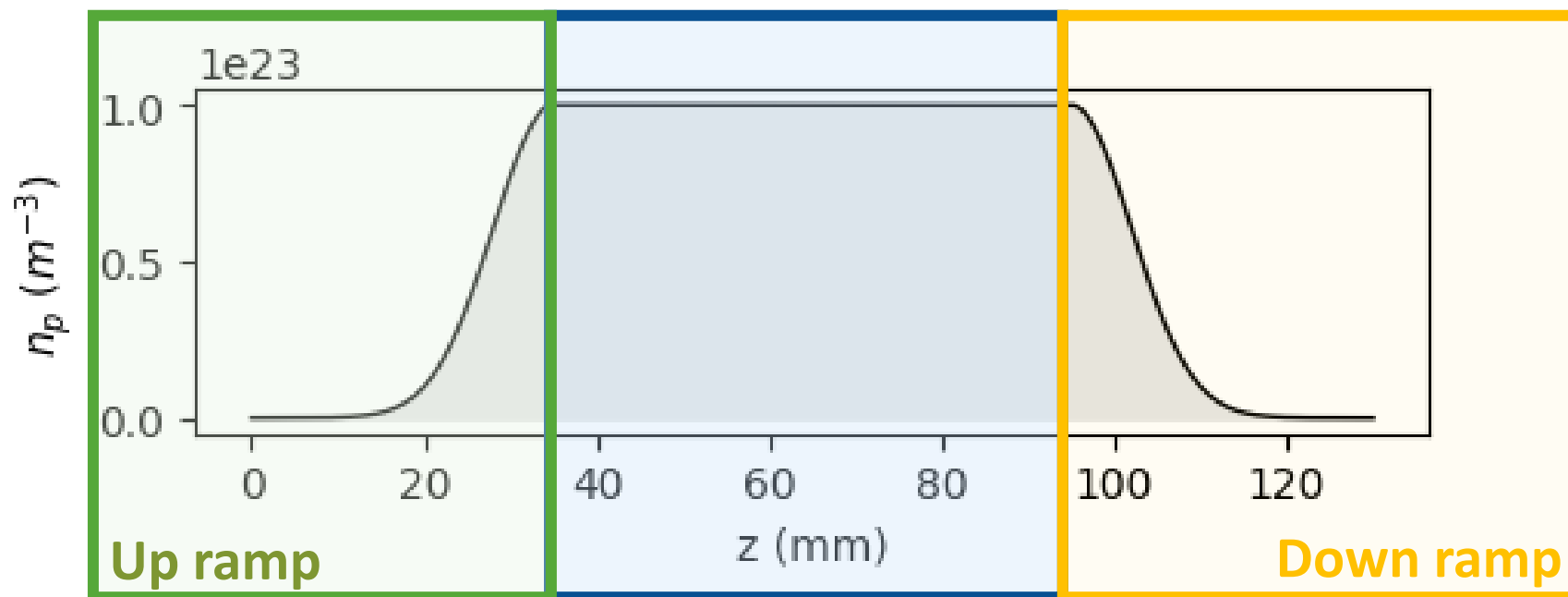
Consistent with:

Mehrling, T., et. al. (2012). *PRSTAB*, 15(11), 111303.

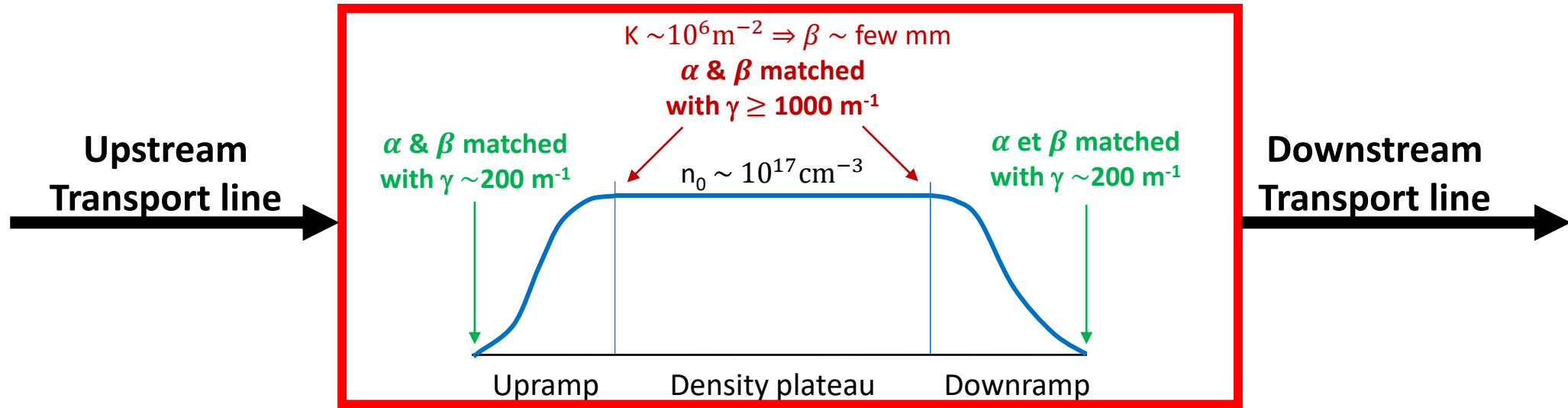


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Use of a density ramp



Symmetrical behavior upstream and downstream of the density plateau

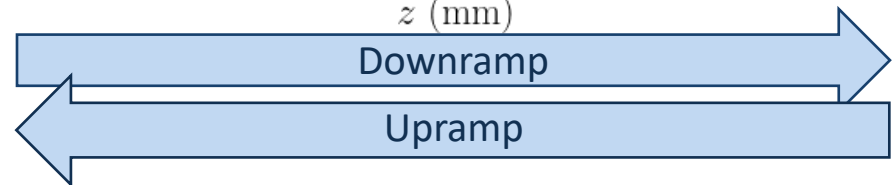
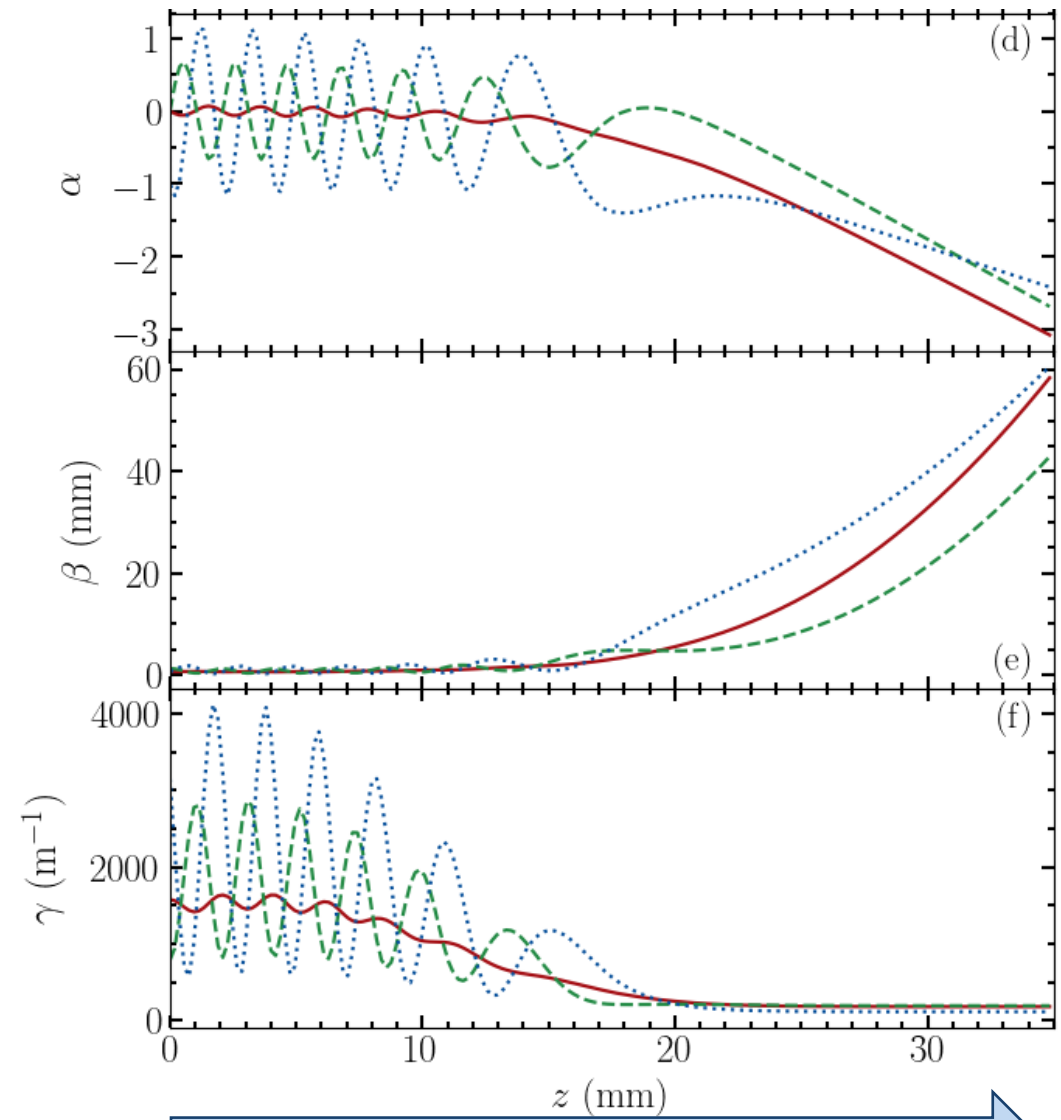
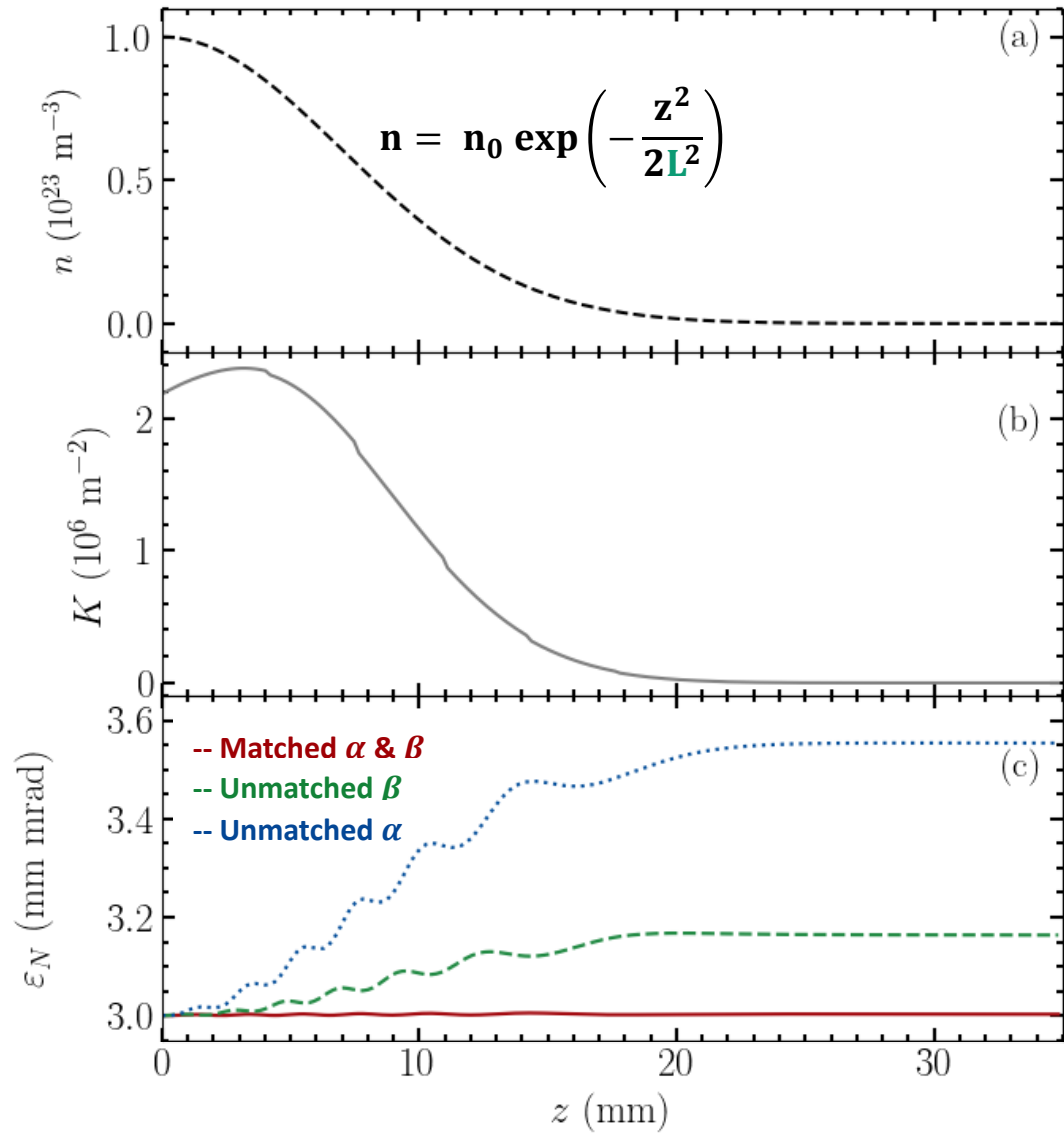
Our approach:

Any ramp profile can be used if the **length is sufficient to reduce γ sufficiently**.

Otherwise, the emittance will increase considerably in the upstream or downstream transport line.

Li, X. et. al. (2019) *PRAB*, 22(2), 021304.

Example : Down ramp of Gaussian profile ($L = 7$ mm)

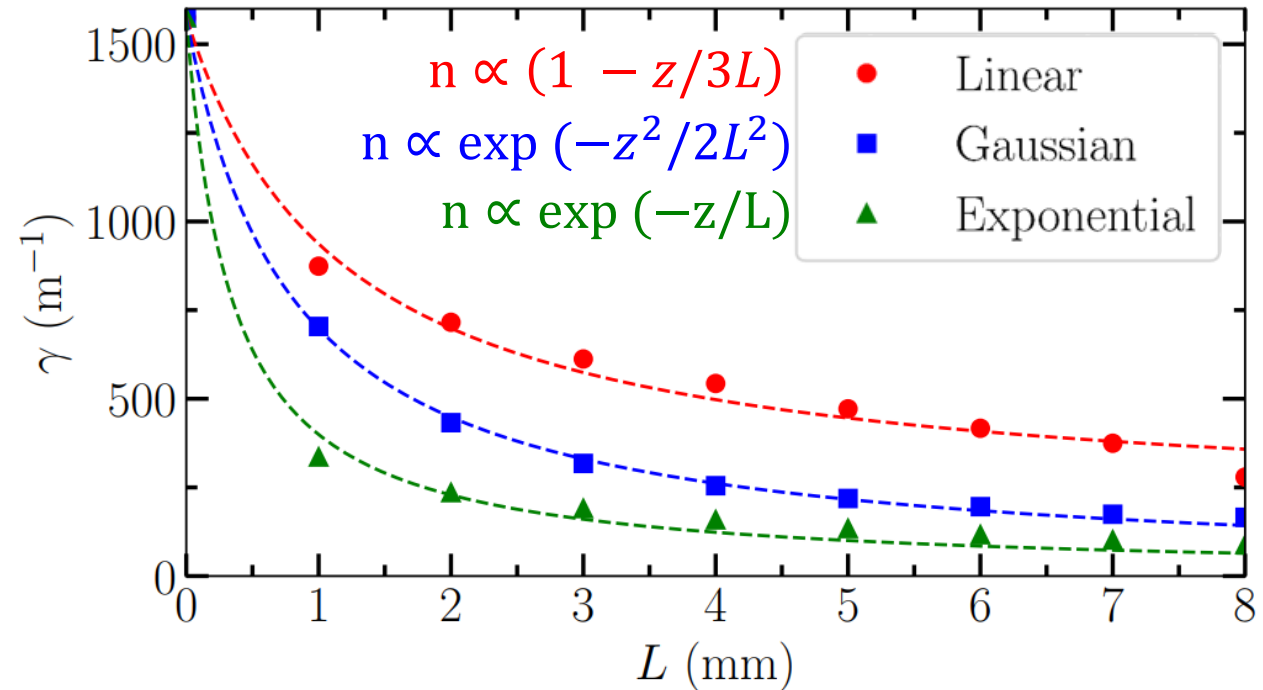
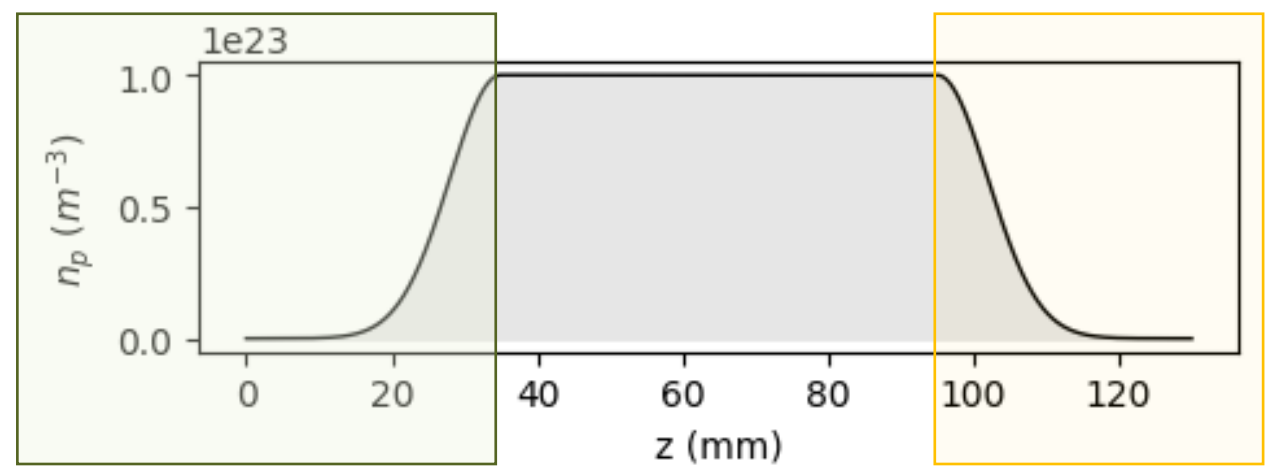


Study of linear, exponential, and Gaussian ramps

For each characteristic length L
→ Ramp simulation
→ Twiss γ at output

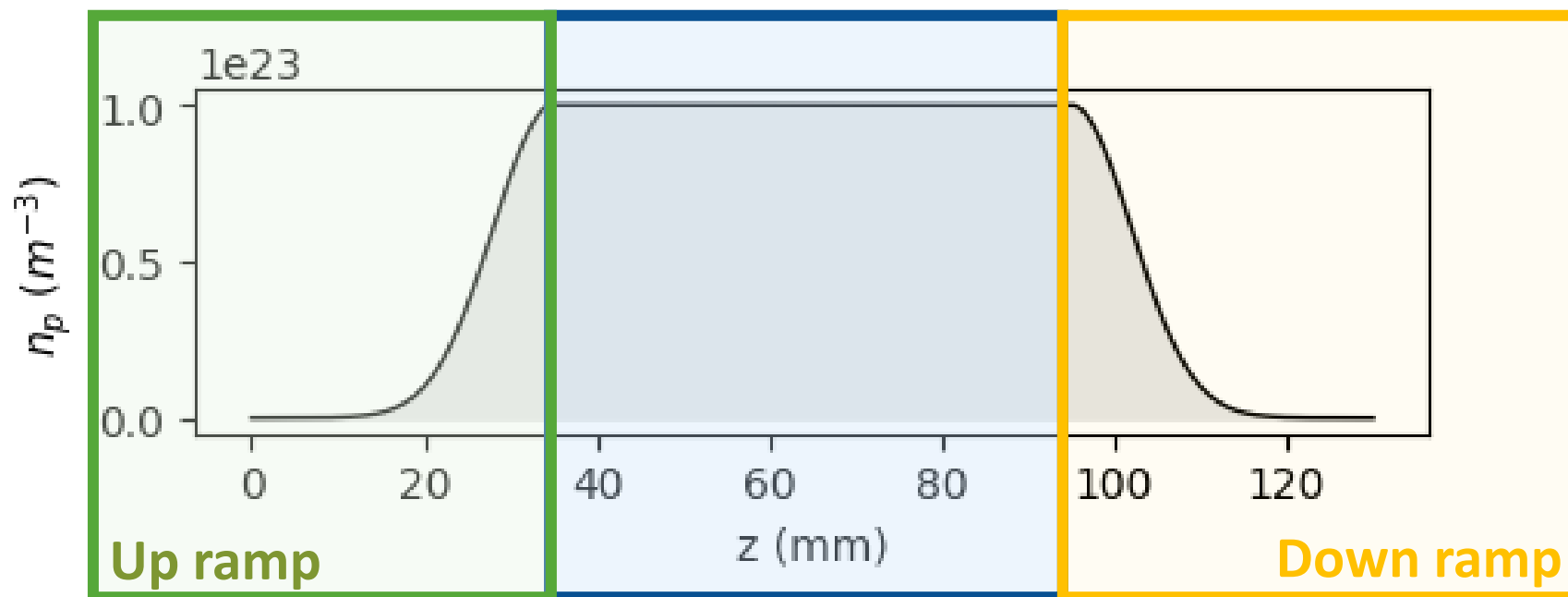
Key parameter = Ramp length

Whatever the ramp shape,
the greater L is, the lower γ is

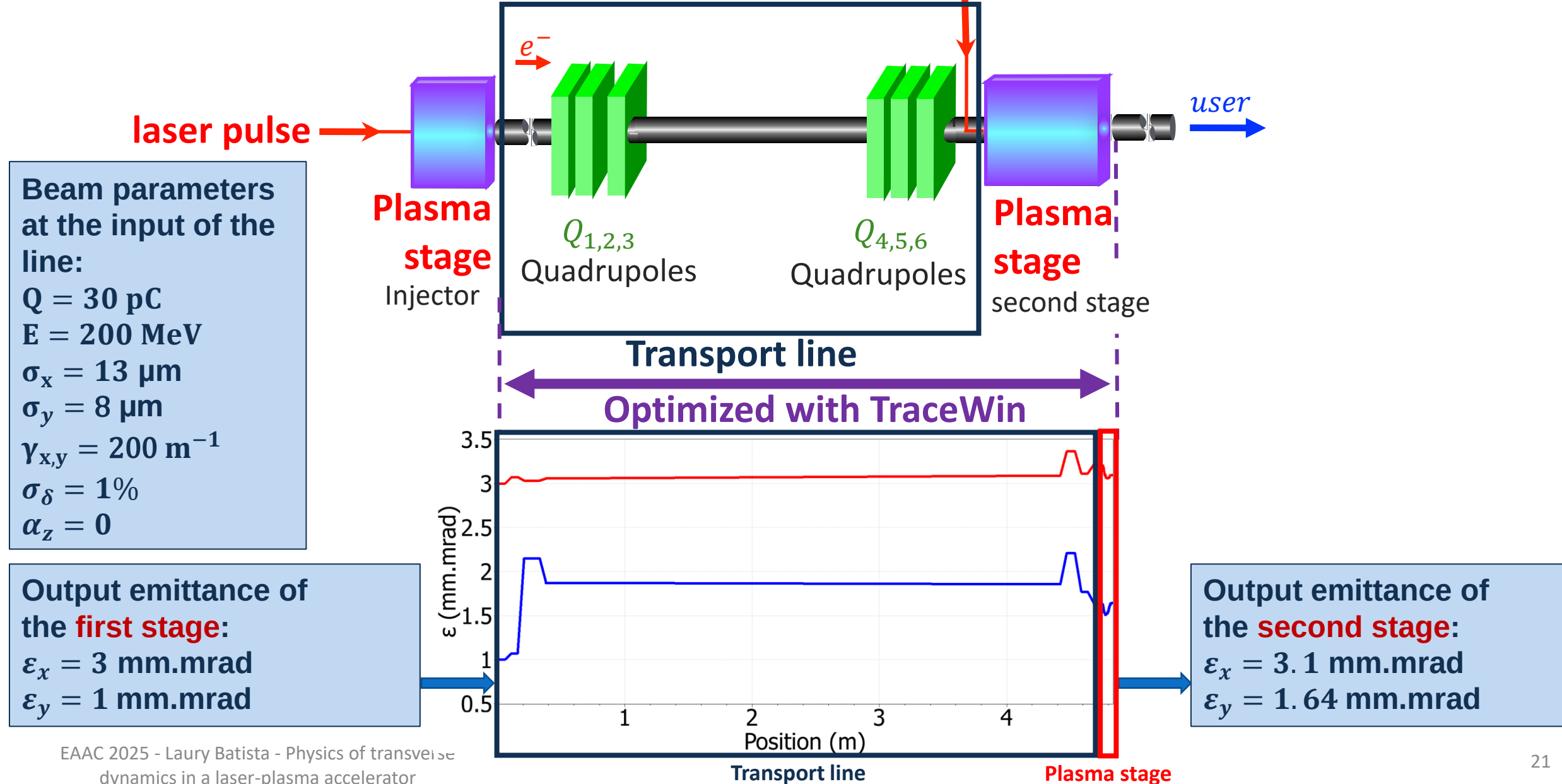


Outline

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Example 2-stage design



Conclusion

Key parameters:

- Twiss gamma parameter
- The focusing gradient K
- The ramp length

Modeling the plasma with equivalent transverse magnetic elements proved to be an efficient method for **optimizing the coupling between the transport line and the plasma stage**

2025 IPAC Proceeding : Batista, L., et. al (2025, September). Simulations of transverse dynamics in a laser-plasma accelerator. In *Journal of Physics: Conference Series* (Vol. 3094, No. 1, p. 012018). IOP Publishing.

Thank you for your attention!