Physics of transverse dynamics in a laser-plasma accelerator

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EUROPEAN NETWORK FOR NOVEL ACCELERATORS

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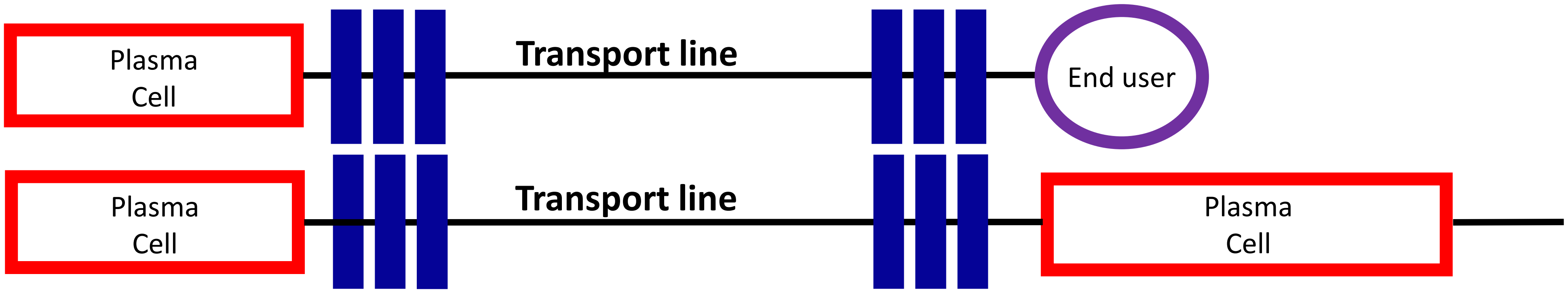
Using the accelerated electron beam

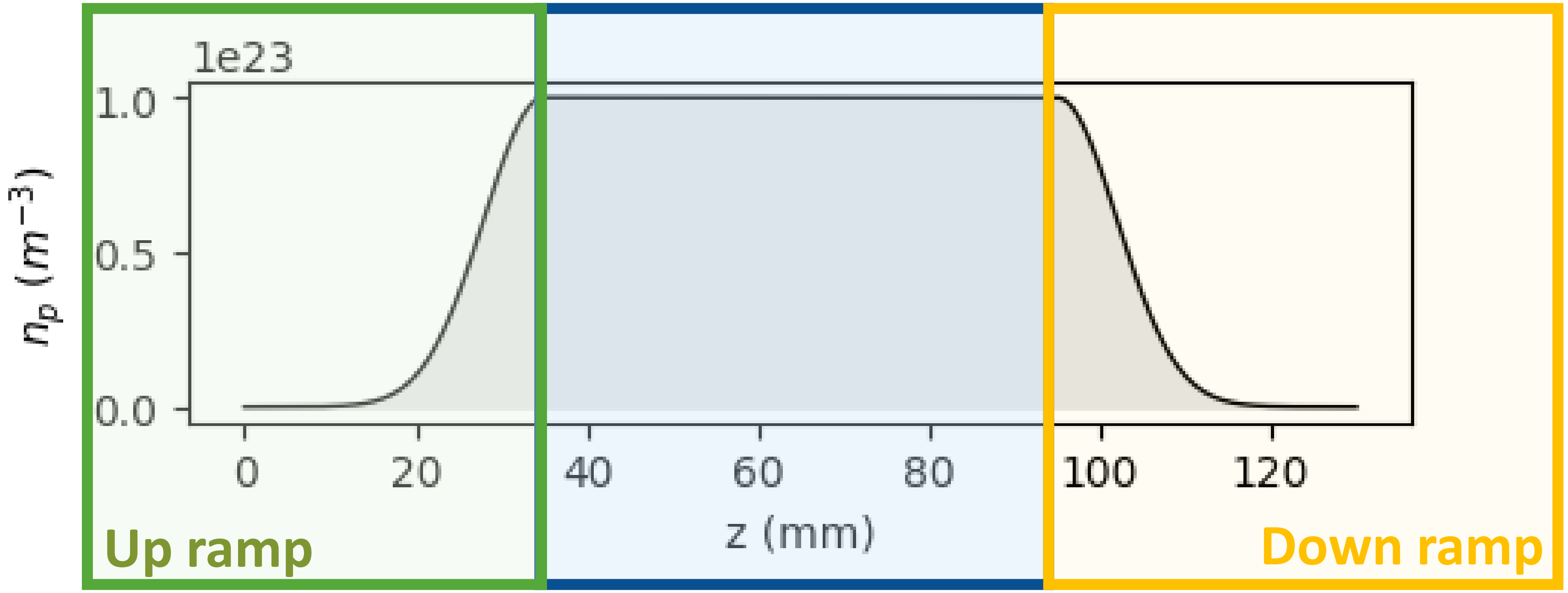
→ Extracting the accelerated beam from the plasma
→ Sending this beam to the next user or plasma
→ Plasma - transport line coupling

→ **Transverse beam physics through the plasma**



Outline

1) Context & Background
2) Transport through the plasma **density plateau**
3) Transport through the **density down ramp & up ramp**
4) Example of coupling and conclusion



RMS quantities and Twiss parameters

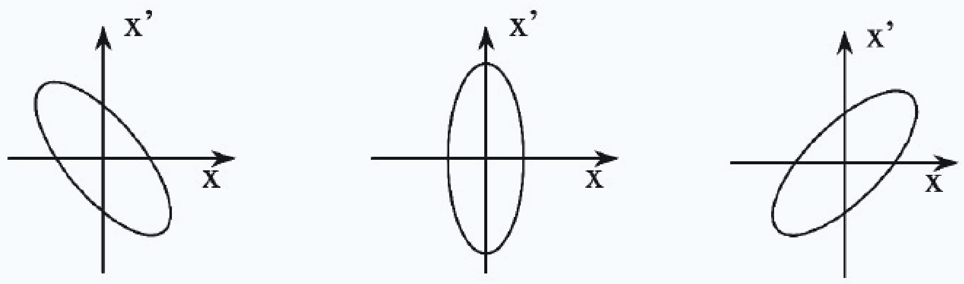
The electron beam can be described by:

- RMS Size: $\sigma_u = \sqrt{\langle u^2 \rangle}$
- RMS divergence: $\sigma_{u'} = \sqrt{\langle u'^2 \rangle}$
- **Twiss parameters: α, β, γ**
- Emittance ε

Twiss β → Related to the **RMS size : $\beta = \frac{\sigma_u^2}{\varepsilon}$**

Twiss γ → Related to the **RMS divergence : $\gamma = \frac{\sigma_{u'}^2}{\varepsilon}$**

Twiss α →



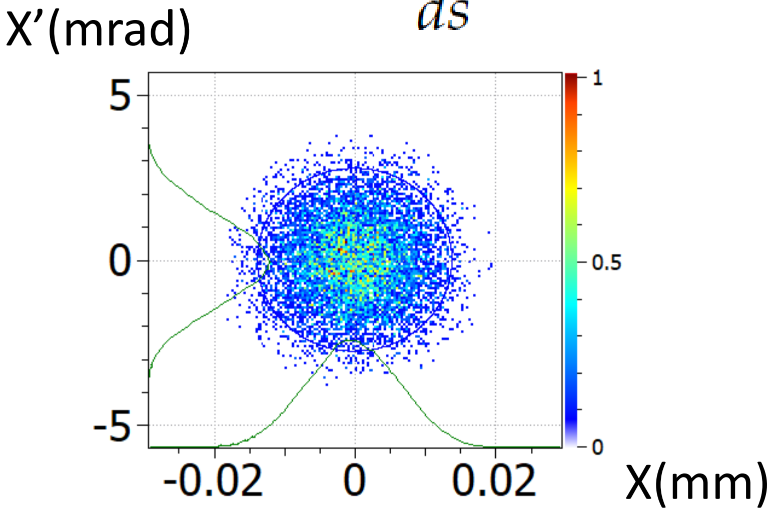
α>0 Convergent beam | α=0 Waist | α<0 Divergent beam

Beam distribution (6D) and emittance

Transverse

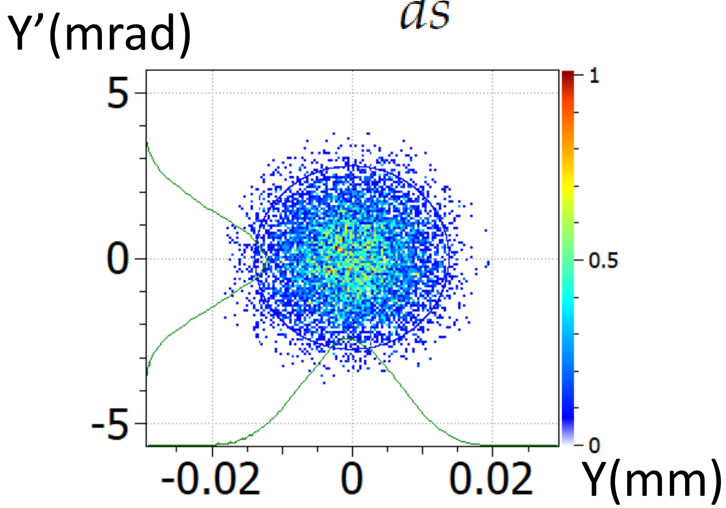
$$(x,\ x')$$

$$x' = \frac{dx}{ds}$$



$$(y,\ y')$$

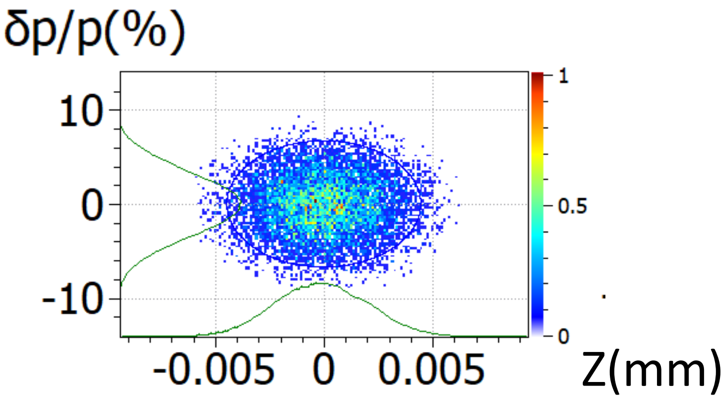
$$y' = \frac{dy}{ds}$$



Longitudinal

$$\left(z,\ \frac{\sigma_p}{p}\right)$$

$\frac{\sigma_p}{p}$ = Energy dispersion



Emittance ε → Surface occupied by the beam in the phase space

New approach : Transverse beam focusing model

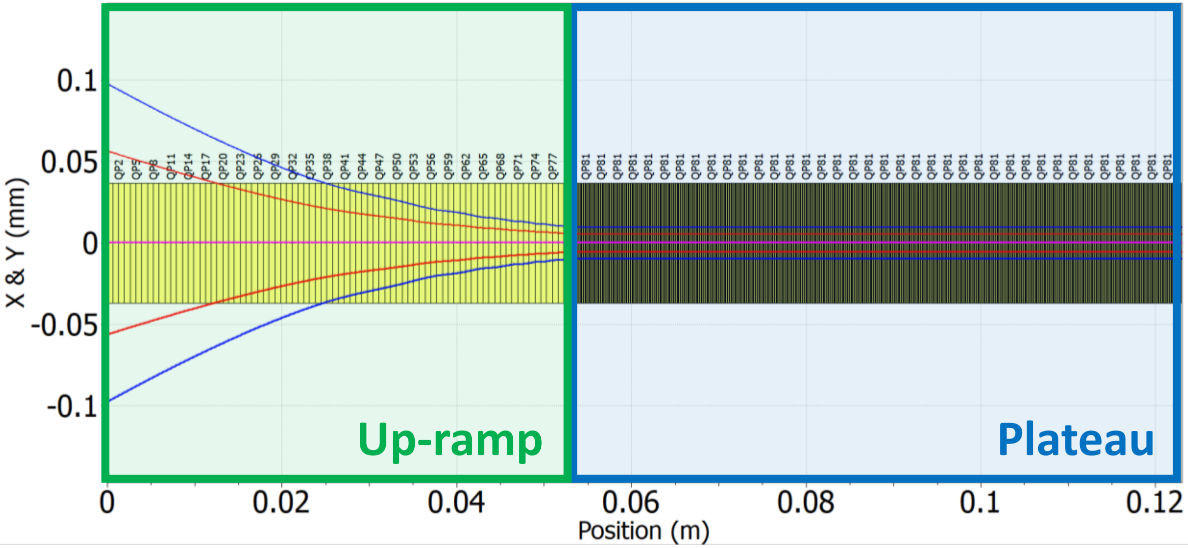
Focusing gradient K ($\mathbf{m^{-2}}$):

$$K = \frac{e}{\gamma_{rel} m_e c^2} \left(\frac{\partial E_r}{\partial r} - \beta_{rel} c \frac{\partial B_\theta}{\partial r} \right)_{r=0}$$

Lorentz factor → Related to the beam energy

Gradient of the radial electric field

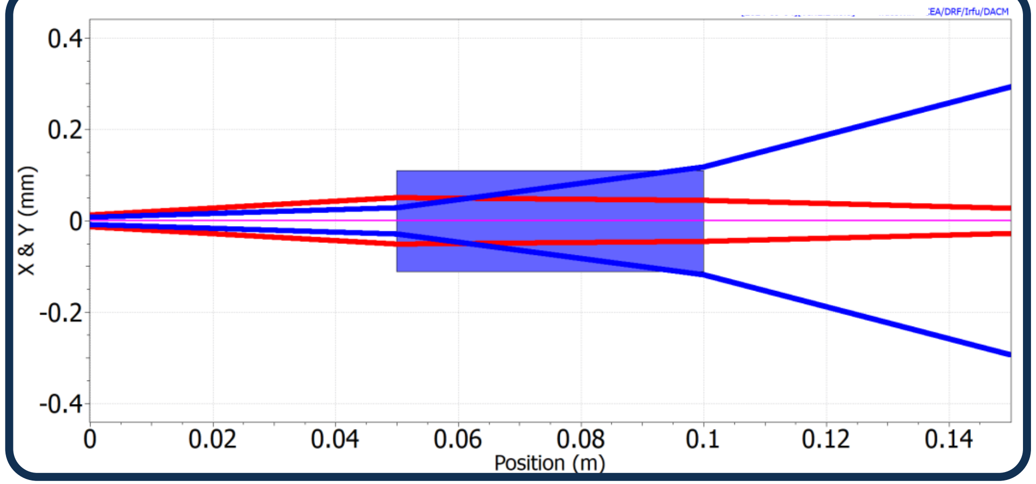
Gradient of the azimuthal magnetic field



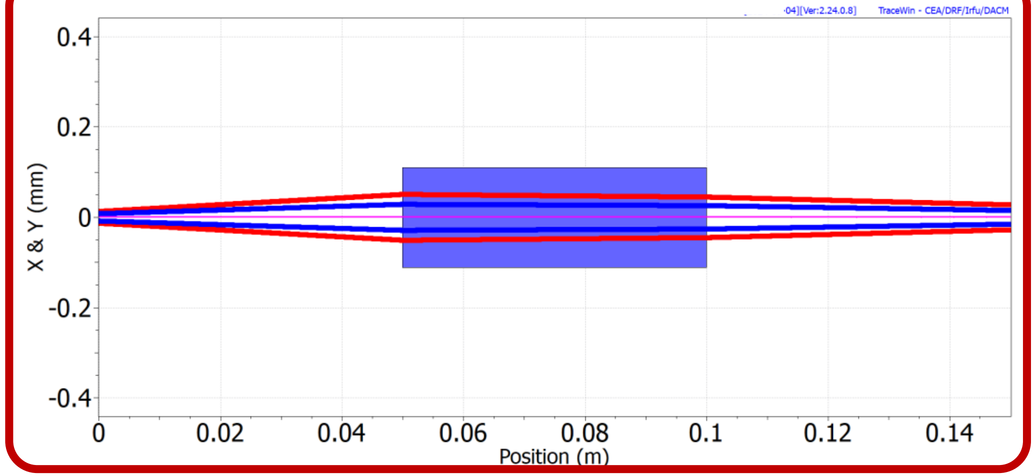
Model with a sequence of "special" quadrupoles {up-ramp + plateau}

CPU time: a few seconds

USUAL QUADRUPOLES IN TRANSPORT LINES



SPECIAL QUADRUPOLES



Nominal parameters

Laser	
Strength a_0	2
Pulse waist w_0	50 µm
Pulse duration (field) τ_0	50 fs
Plasma	
Density n_0	10^{23} m^{-3}
Plateau length	up to 45 mm
Injected electron beam	
Mean energy E	200 MeV
Energy spread σ_E/E	3%
Normalized emittance ε_{xN}	3 mm mrad
Normalized emittance ε_{yN}	1 mm mrad
Beam length σ_z	3 µm
Laser-electron bunch distance ξ	50 µm

Use of:

FBPIC: A laser-plasma PIC code

Wake-T: A laser-plasma tracking code (quasistatic approach)

TraceWin: Beam Dynamics tracking code

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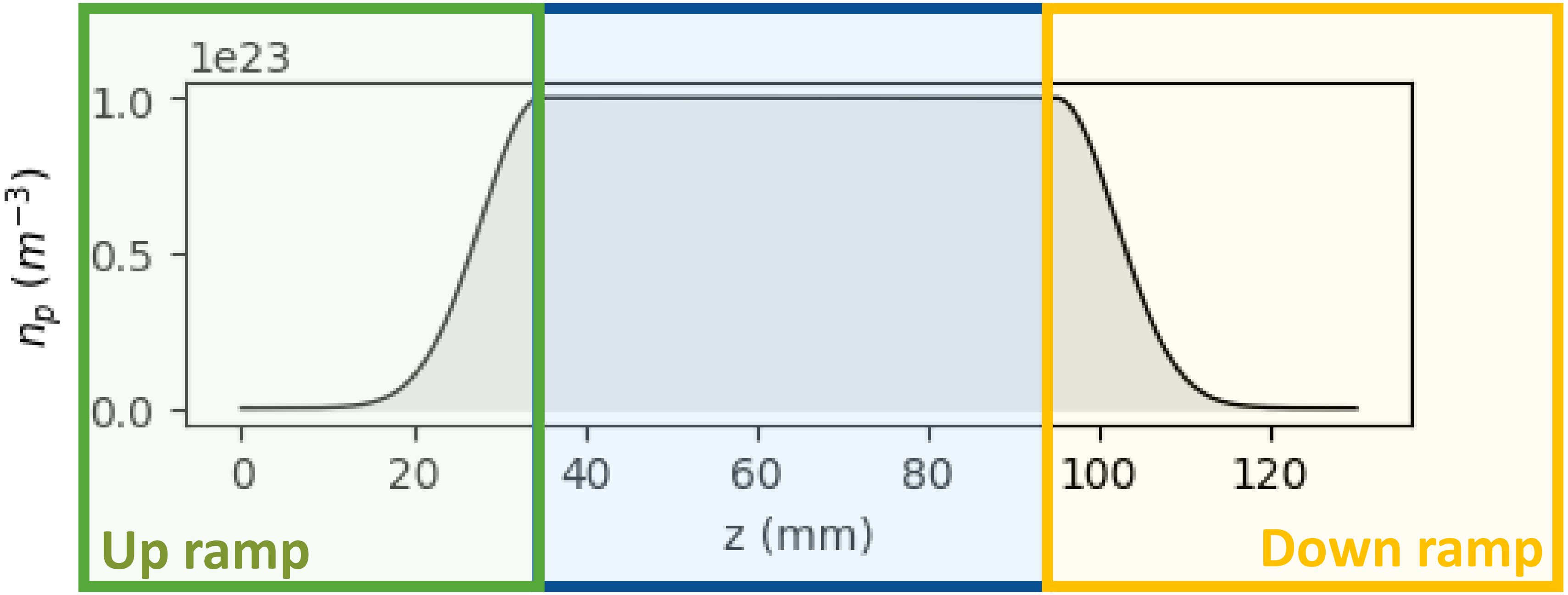


Transport through the plasma density plateau

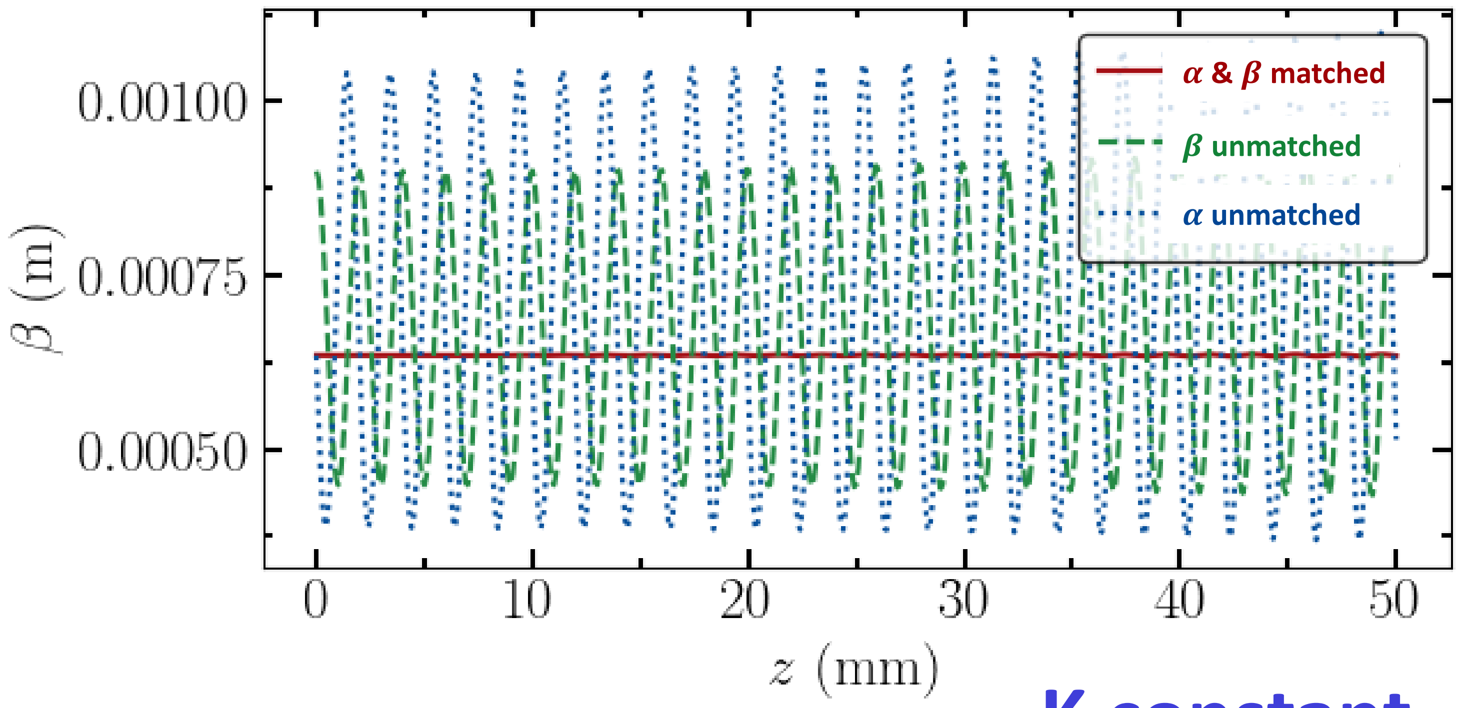
Objectives

Study the evolution of **Twiss parameters and emittance** across the plateau for all initial conditions **(α_0, β_0 and δ)**

Characterize the influence of **energy spread**

Compare analytical results with **numerical simulations**

Transport through the plasma density plateau ($\delta = 0$)



K constant

For any input $\boldsymbol{\alpha_0}, \boldsymbol{\beta_0} \rightarrow \boldsymbol{\beta}$ **oscillates : Frequency** $2\sqrt{K}$

$$\beta(z) = \frac{b_+}{\sqrt{K}} - A \sin\left(2z\sqrt{K} + \phi\right)$$

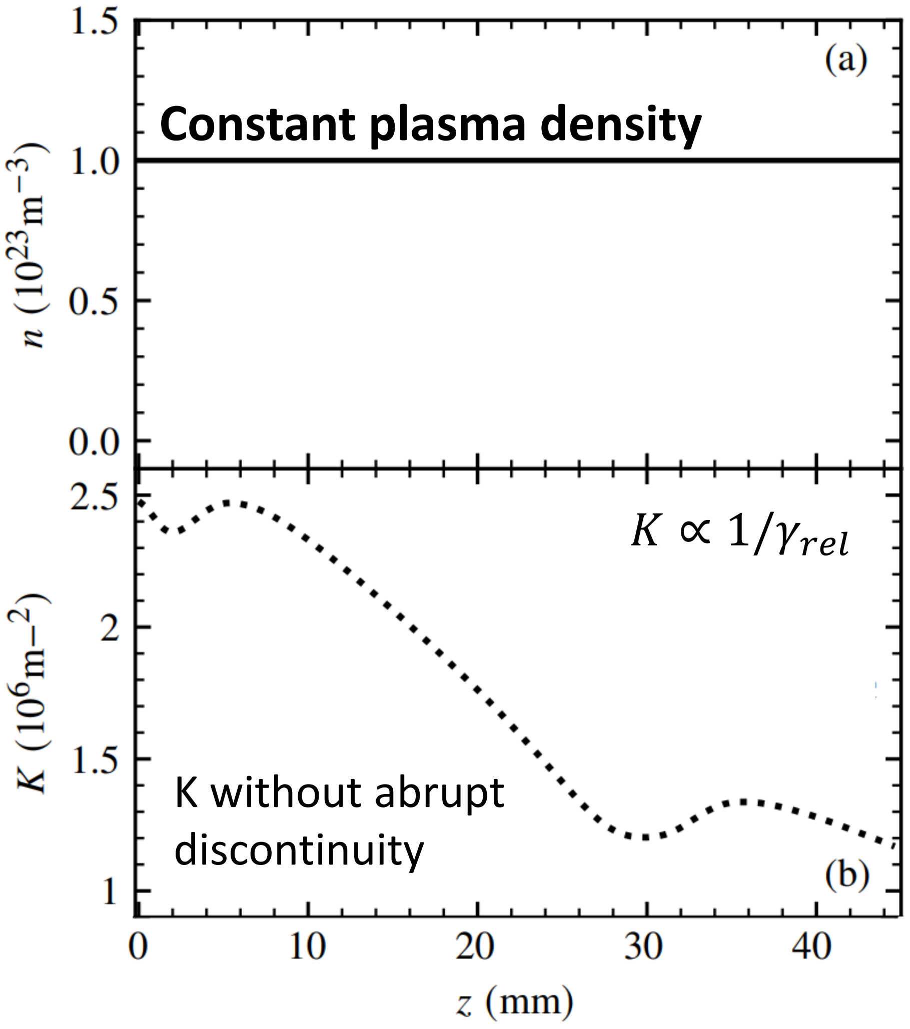
$b_+(\alpha_0, \beta_0, K)$

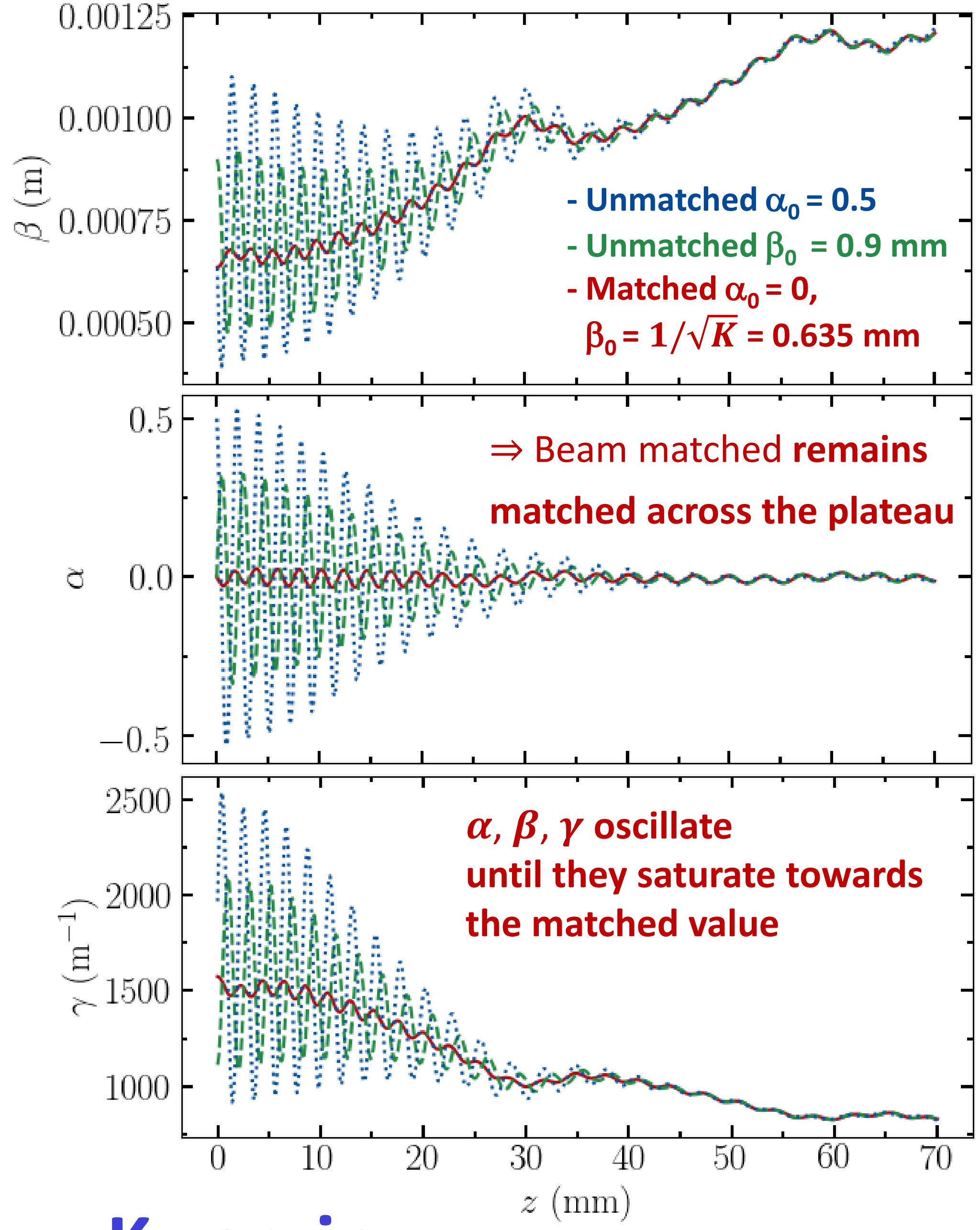
$A(\alpha_0, \beta_0, K)$

For $\boldsymbol{\alpha_0 = 0, \beta_0 = 1/\sqrt{K}}$ $\rightarrow$ $\boldsymbol{\beta}$ **does not oscillate** $\Leftrightarrow \beta_{min} = \beta_{max} = \beta_0$ **The Twiss parameters are matched**

Transport on the plasma density plateau ($\delta \neq 0$)

Energy spread : $\sigma_\delta = 5\%$





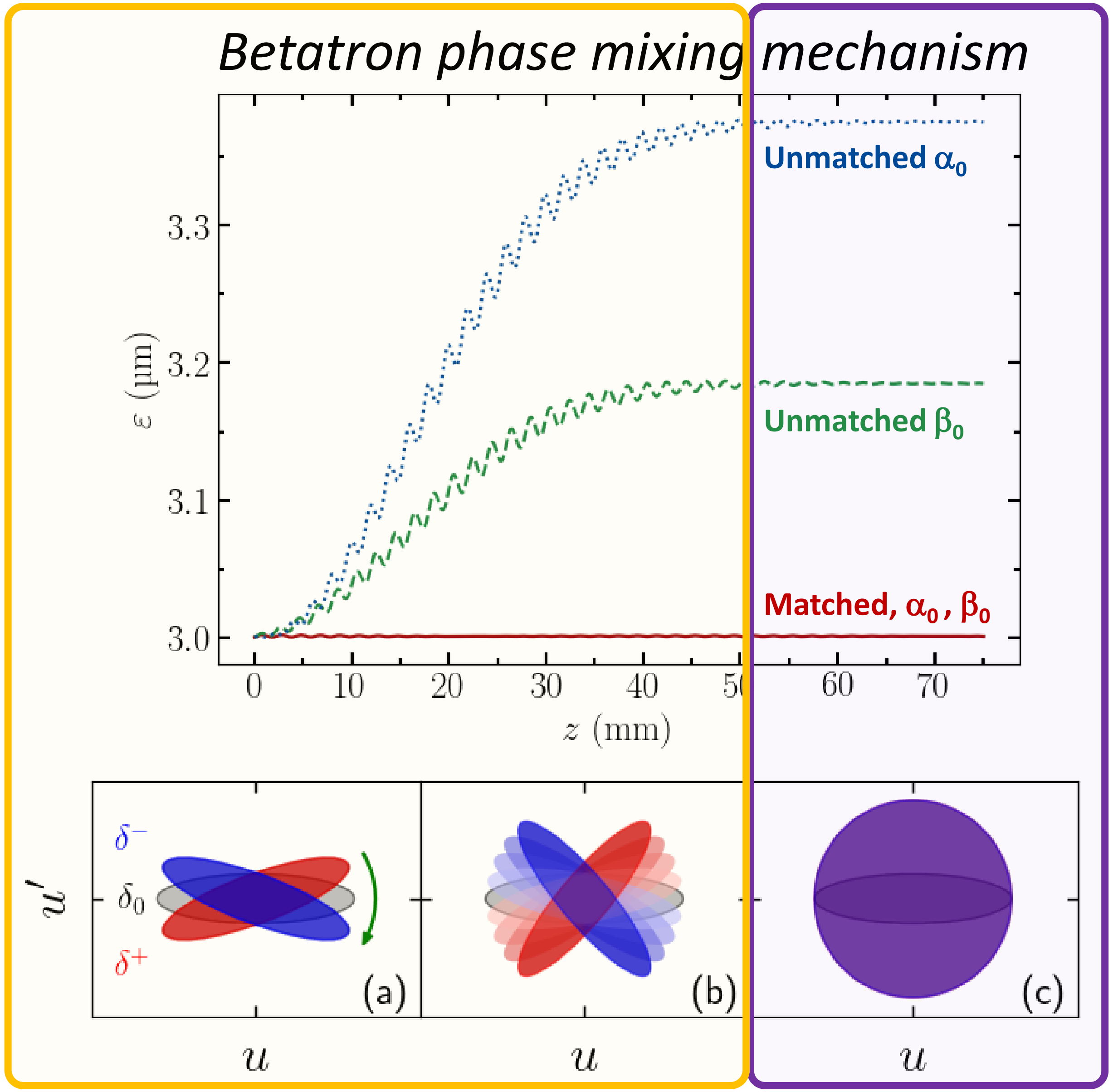
K varying

Transport through the plasma density plateau ($\delta \neq 0$)

Particles of different energies
$K \propto 1/\gamma_{rel}$
→ focused differently

Oscillation frequency $2\sqrt{K}$
→ Oscillate at different frequencies

Growth and oscillation of emittance



Saturated emittance value

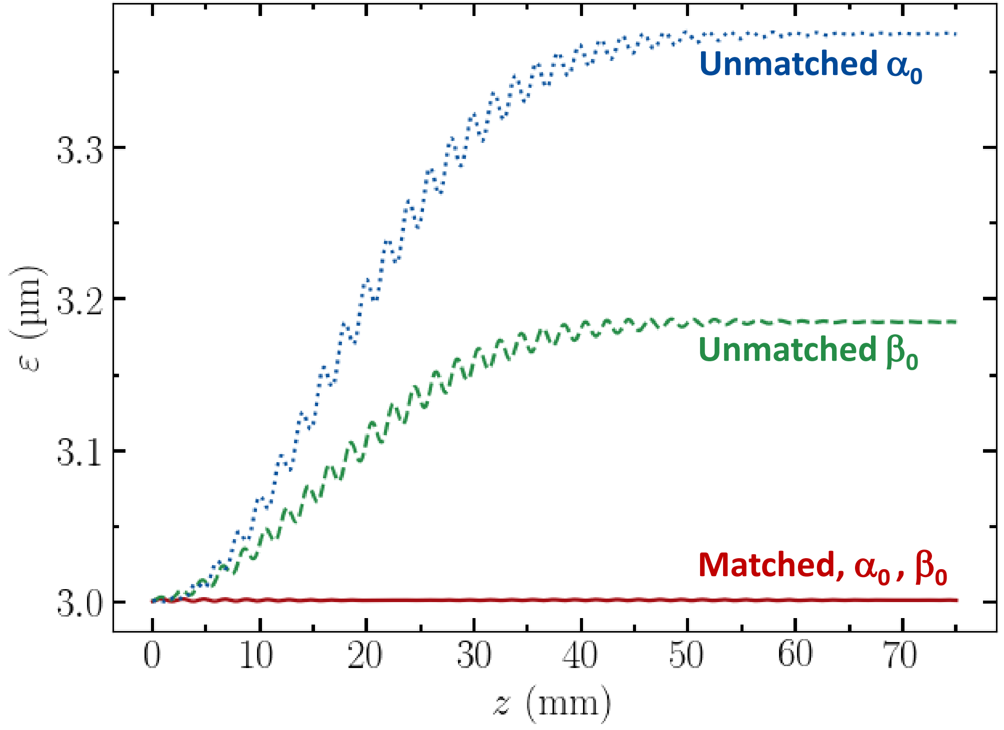
Valid for constant or variable K

Mehrling, T et. al. (2012). *PRSTAB,15*(11), 111303.

Ariniello, R, et.al. (2019). *PRAB*, *22*(4), 041304.

Transport through the plasma density plateau ($\delta \neq 0$)

Emittance evolution
from TraceWin
with plasma modeled by
special quadrupoles



Analytical model

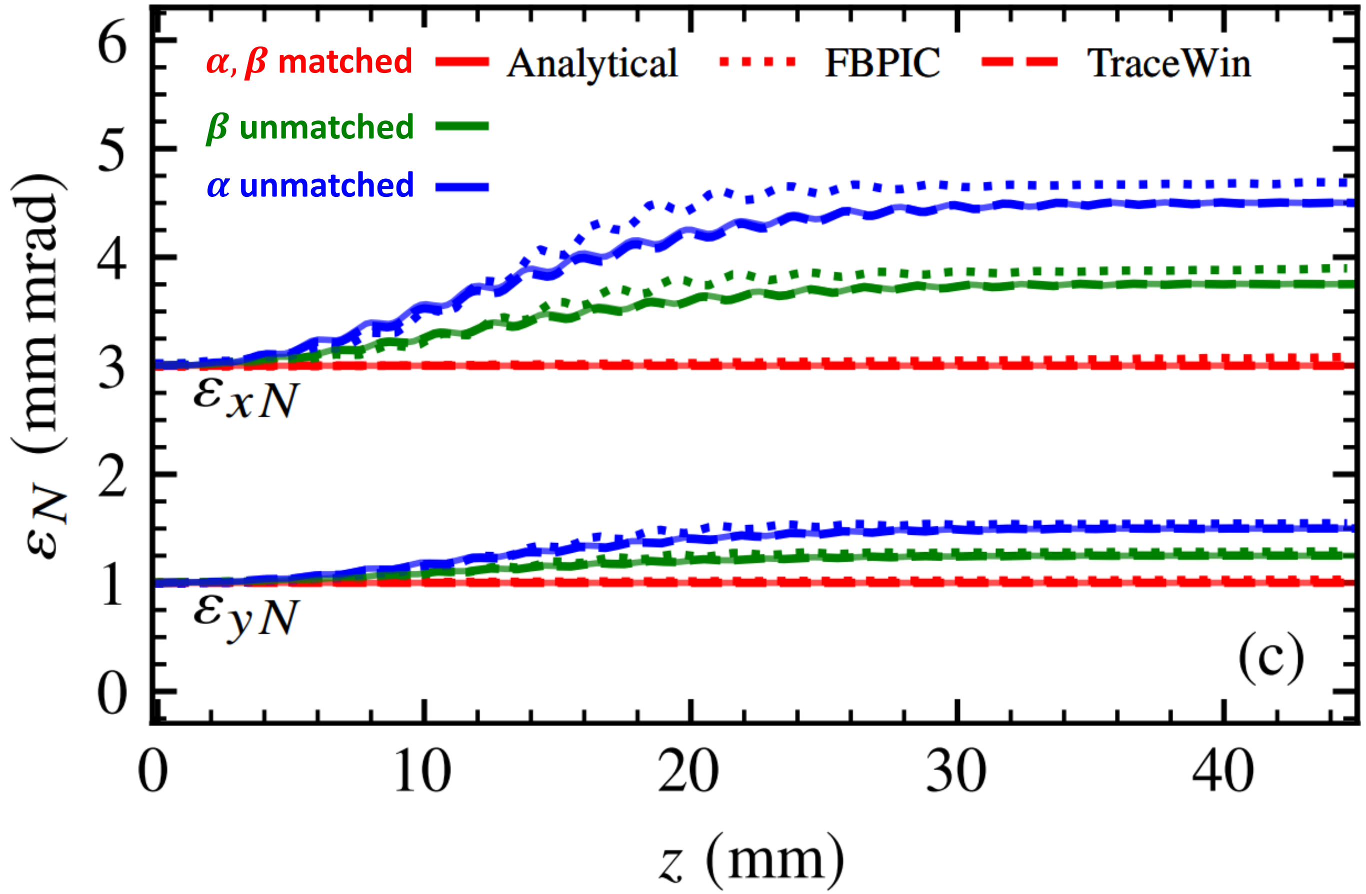
Static term | Chromatic offset | Term of decoherence and oscillations

$$\frac{\varepsilon^2}{\varepsilon_0^2} = b_+^2 + \sigma_\delta^2 \frac{\sqrt{K}\,\beta_0}{2} b_+ + e^{-\sigma_\delta^2 z^2 K}\left(1 - b_+^2\right)$$
$$- \frac{\sigma_\delta^2}{8} e^{-\sigma_\delta^2 z^2 K}\Big[\alpha_0^2 + 8z\sqrt{K}\alpha_0 b_+ - \left(3\sqrt{K}\beta_0 + b_+\right) b_-$$
$$+ 2\alpha_0 b_- \sin[4z\sqrt{K}] - \left(\alpha_0^2 - b_-^2\right)\cos[4z\sqrt{K}]\Big]$$
$$+ \frac{\sigma_\delta^2}{2} e^{-\frac{1}{2}\sigma_\delta^2 z^2 K}\Big[\left(-\beta_0\gamma_0 + 2z\sqrt{K}\alpha_0 b_+\right)\cos[2z\sqrt{K}]$$
$$+ b_-\left(\alpha_0 - 2z\sqrt{K} b_+\right)\sin[2z\sqrt{K}]\Big], \qquad (24)$$

With : $b_\pm = \frac{1+\alpha_0^2}{2\beta_0\sqrt{K}} \pm \frac{\beta_0\sqrt{K}}{2}$

Transport through the plasma density plateau ($\delta \neq 0$)

Comparison of the three approaches



To be compared with the semi-analytical approach of :
Aschikhin et. al. J. (2018). *NIMA*, *909*, 414-418.

Impact of the energy spread

Expression of the decoherence length

$$z_D \propto \frac{1}{\sigma_\delta \sqrt{K}}$$

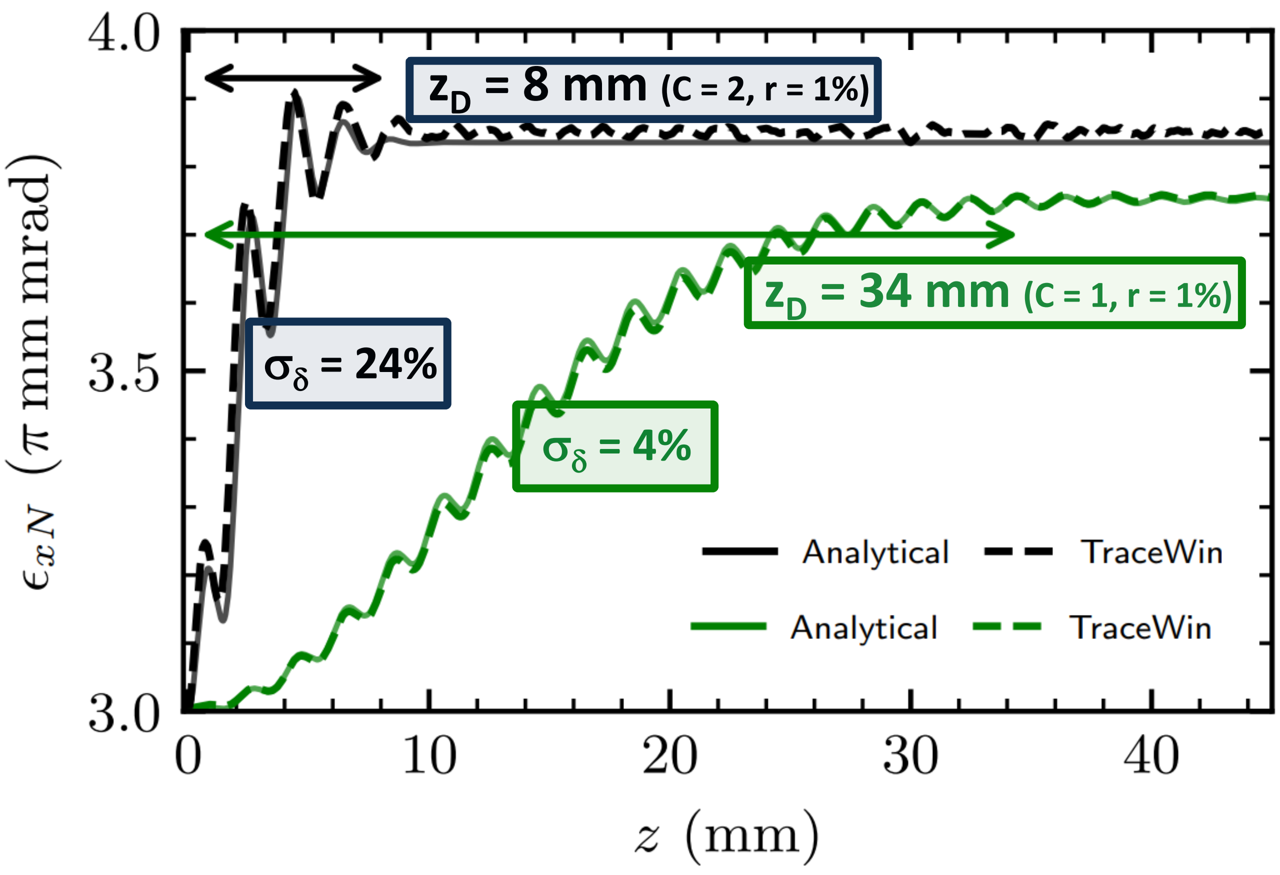
Expression of the emittance saturation value $(z \gg z_D)$

$$\varepsilon \approx \varepsilon_0 \left(\frac{1+\alpha_0^2}{2\beta_0\sqrt{K}} + \frac{\beta_0\sqrt{K}}{2} \right) + \varepsilon_0 \sigma_\delta^2 \frac{\sqrt{K}\beta_0}{4}$$

Static term

Second order term

Consistent with:
Mehrling, T., et. al. (2012). *PRSTAB*, *15*(11), 111303.



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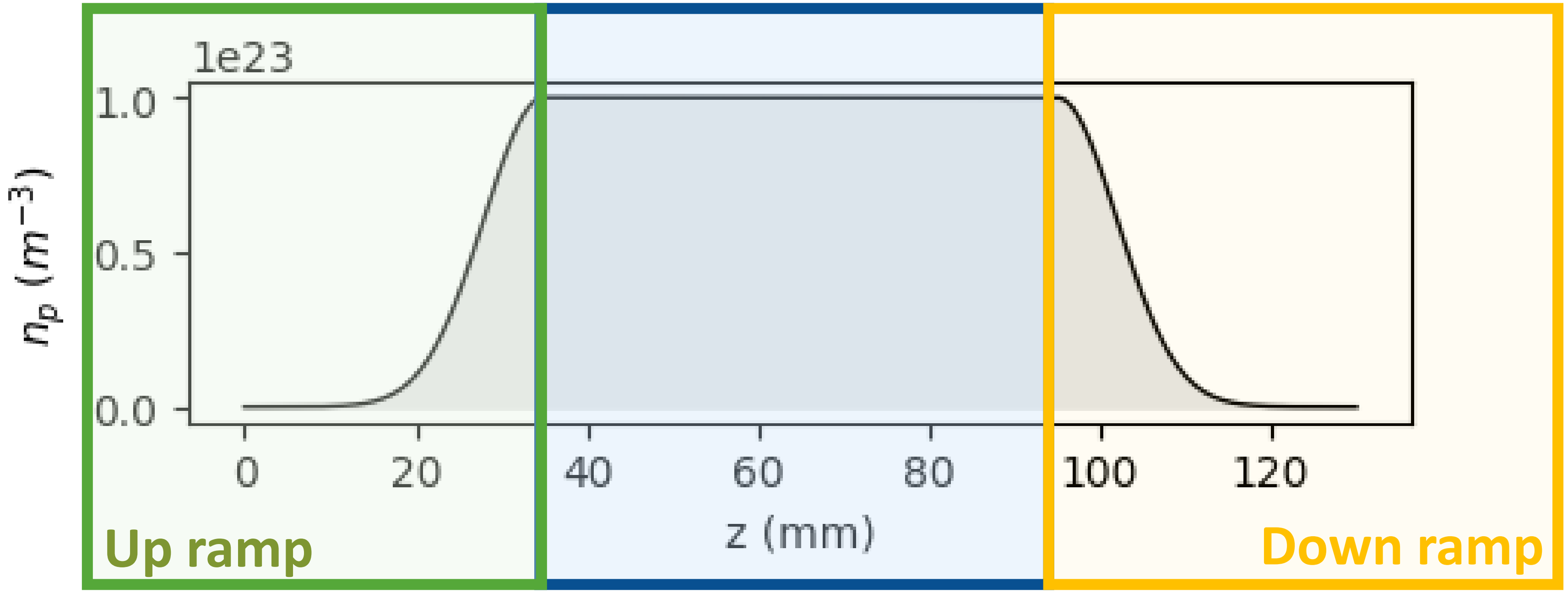
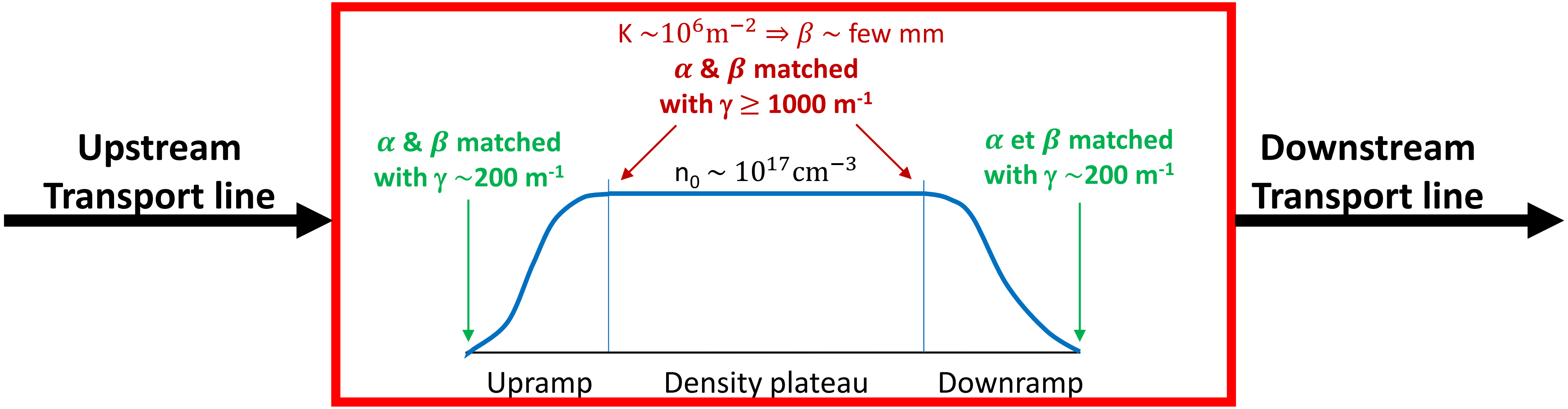


Use of a density ramp



Symmetrical behavior upstream and downstream of the density plateau

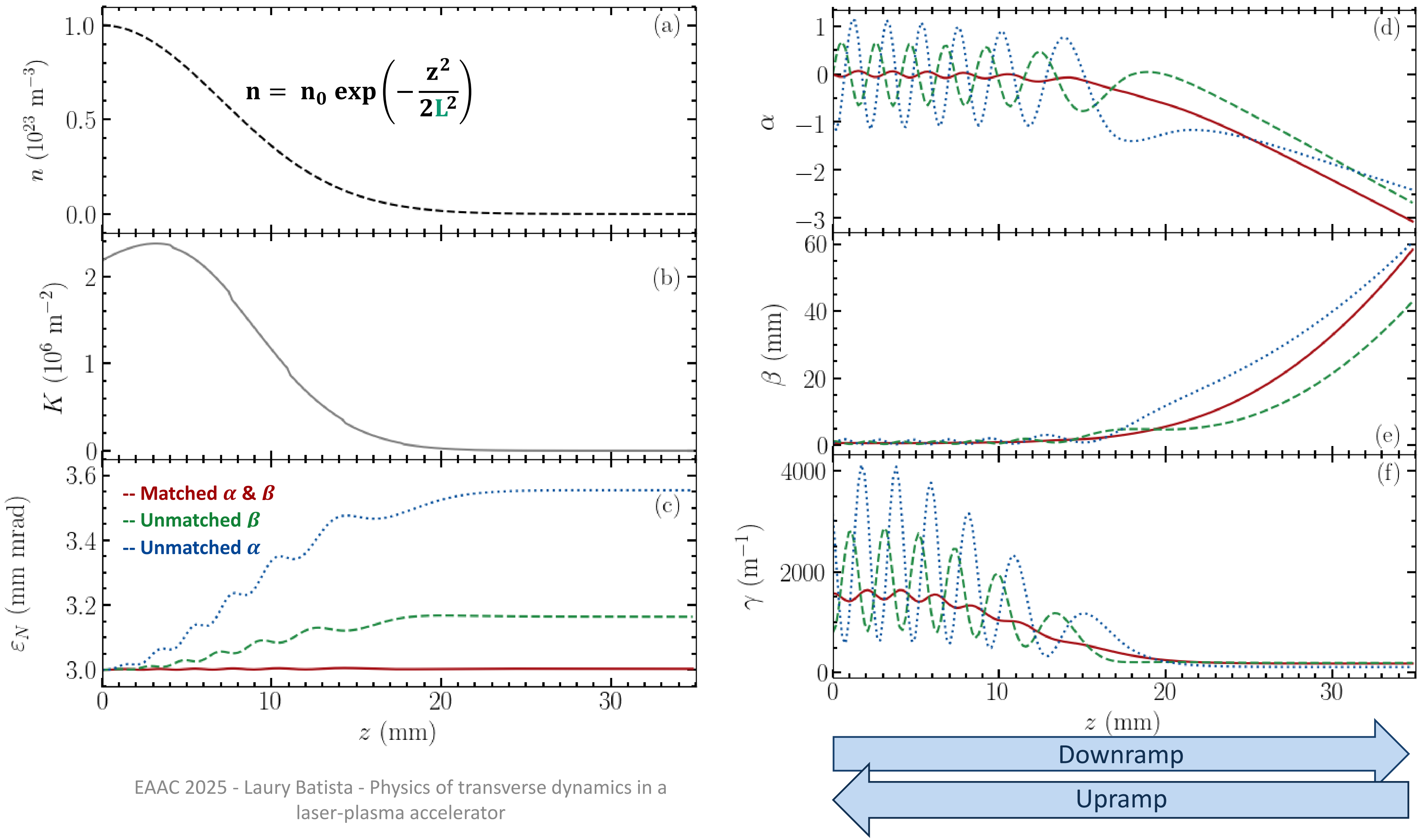
Our approach:

Any ramp profile can be used if the length is sufficient to reduce γ sufficiently.

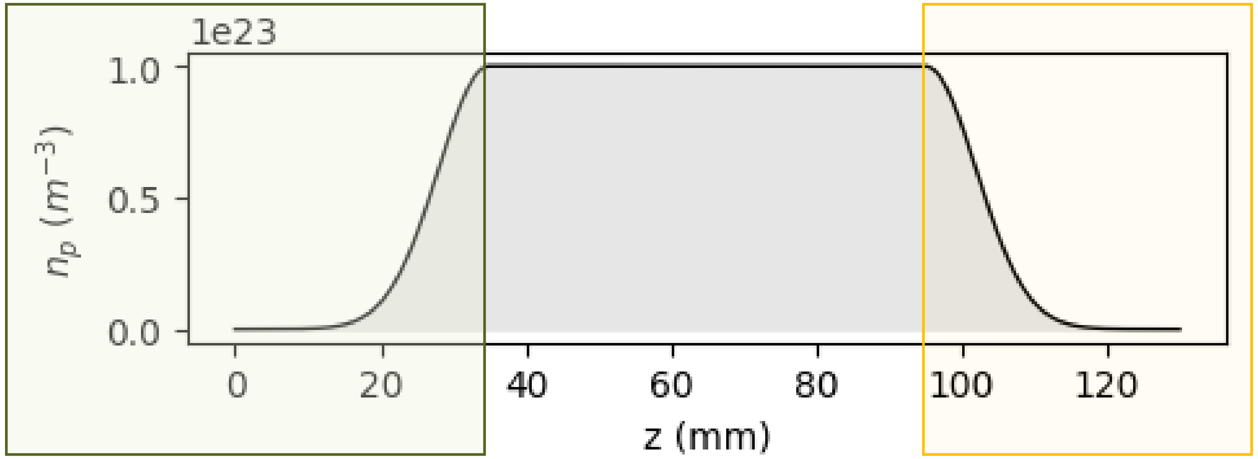
Otherwise, the emittance will increase considerably in the upstream or downstream transport line.

Li, X. et. al. (2019) *PRAB*, *22*(2), 021304.

Example : Down ramp of Gaussian profile (L = 7 mm)

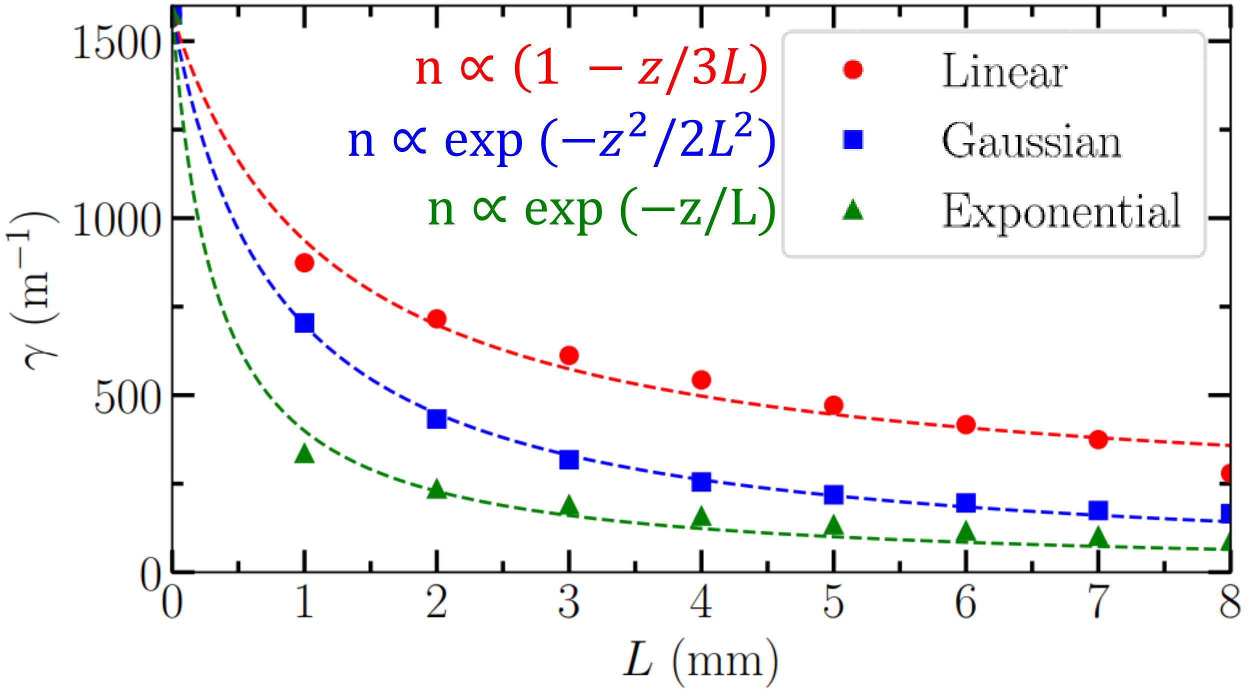


Study of linear, exponential, and Gaussian ramps



For each characteristic length L
→ Ramp simulation
→ Twiss γ at output

Key parameter = Ramp length



Whatever the ramp shape,
the greater L is, the lower γ is

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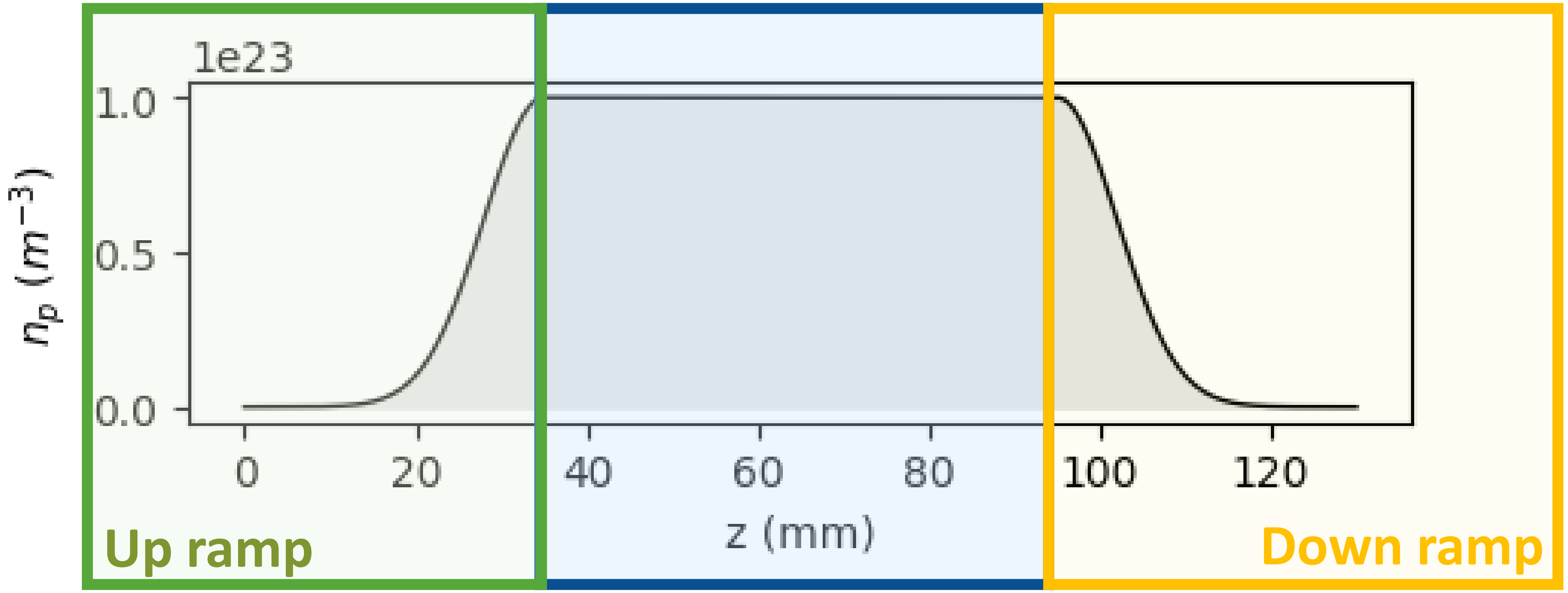
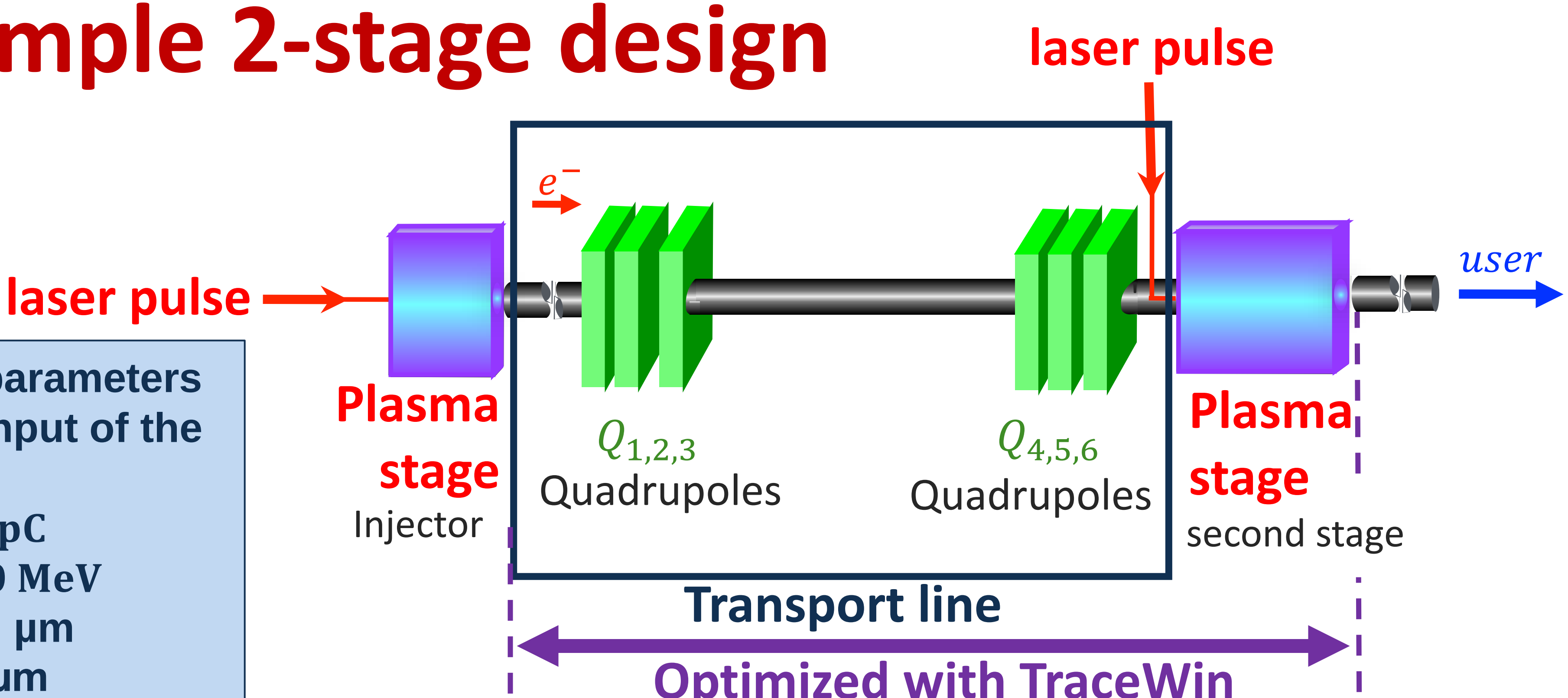


Example 2-stage design



Beam parameters at the input of the line:

$\mathbf{Q = 30\ pC}$

$\mathbf{E = 200\ MeV}$

$\sigma_x = 13\ \mu m$

$\sigma_y = 8\ \mu m$

$\gamma_{x,y} = 200\ m^{-1}$

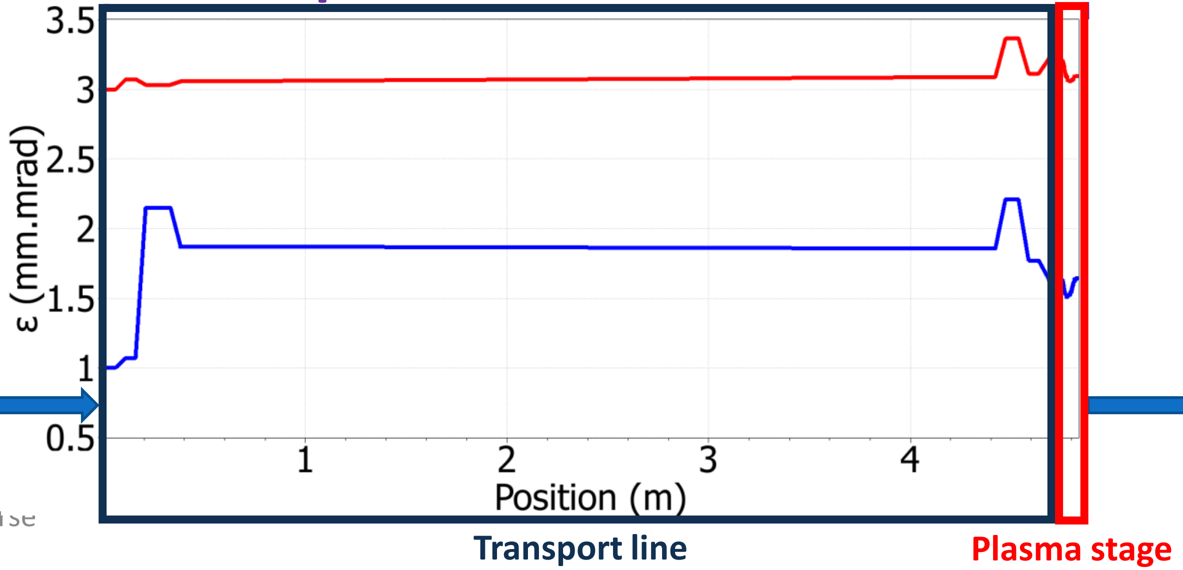
$\sigma_\delta = 1\%$

$\alpha_z = 0$

Output emittance of the first stage:

$\varepsilon_x = 3$ mm.mrad

$\varepsilon_y = 1$ mm.mrad



Output emittance of the second stage:

$\varepsilon_x = 3.1$ mm.mrad

$\varepsilon_y = 1.64$ mm.mrad

Conclusion

Key parameters:

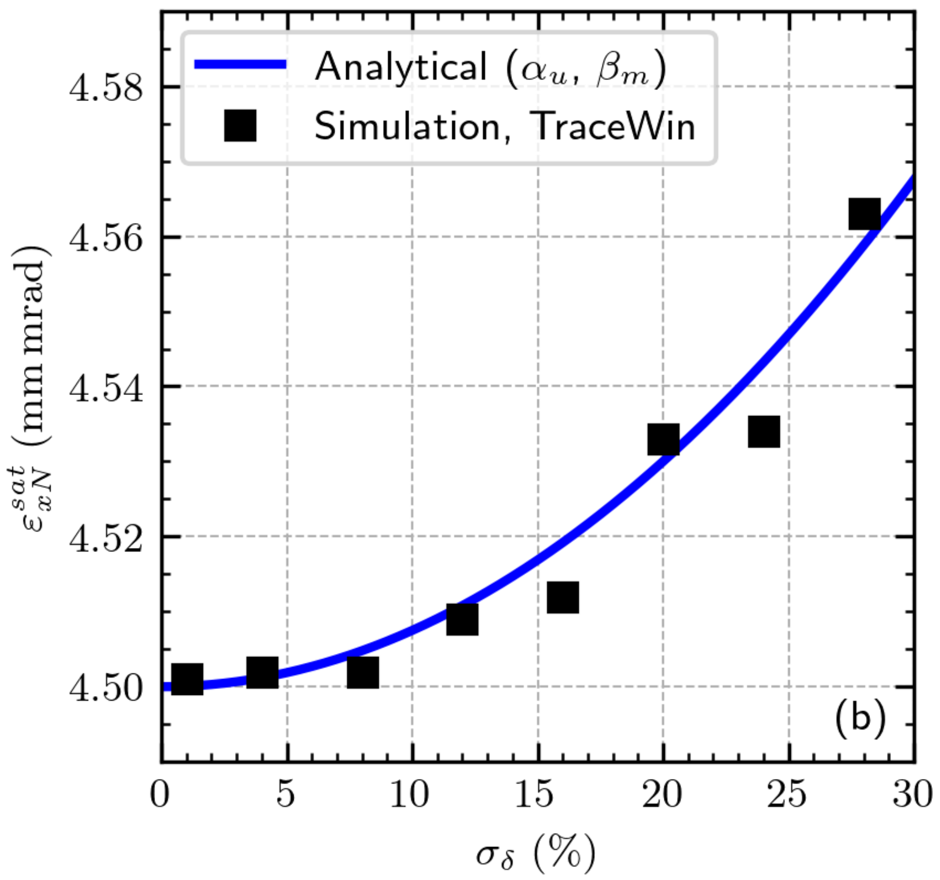
- **Twiss gamma parameter**
- **The focusing gradient K**
- **The ramp length**

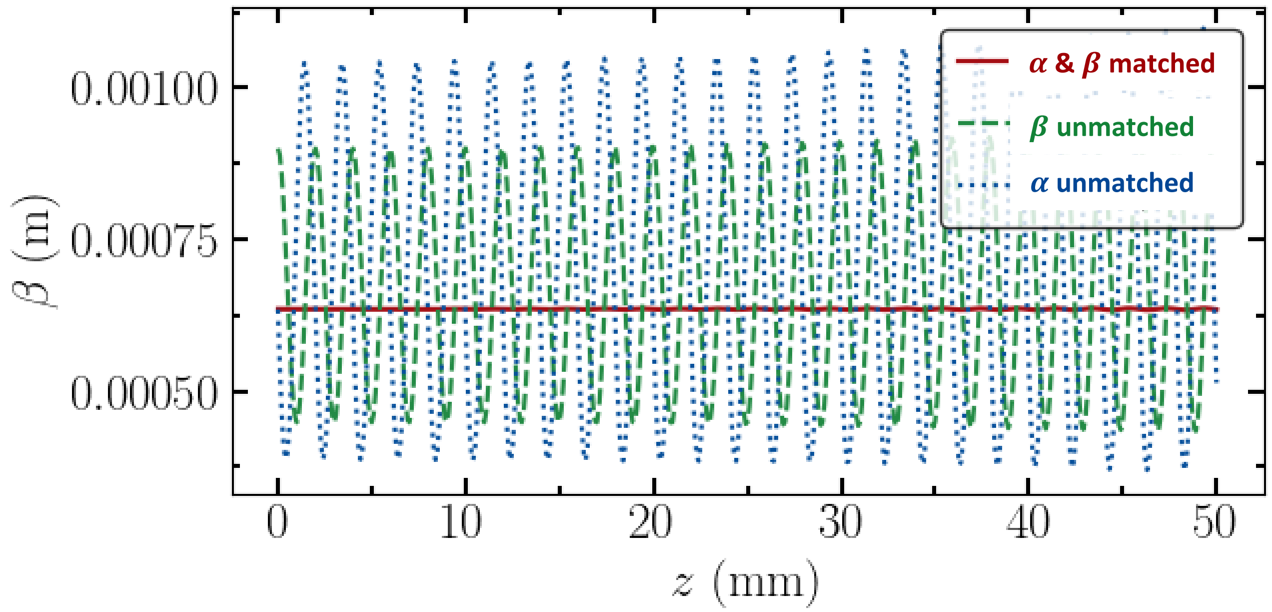
Modeling the plasma with equivalent transverse magnetic elements proved to be an efficient method for **optimizing the coupling between the transport line and the plasma stage**

2025 IPAC Proceeding : Batista, L., et. al (2025, September). Simulations of transverse dynamics in a laser-plasma accelerator. In *Journal of Physics: Conference Series* (Vol. 3094, No. 1, p. 012018). IOP Publishing.

Thank you for your attention!

Emittance saturation value for different energy dispersion





Transport through the plasma density plateau ($\delta = 0$)

Initial values β_0, α_0	For K = constant
For any α_0, β_0	$\beta(z) = \frac{b_+}{\sqrt{K}} - A \sin\left(2z\sqrt{K} + \phi\right)$ $A = \frac{\alpha_0}{\sqrt{K}}\sqrt{1 + \left(\frac{b_-}{\alpha_0}\right)^2}$ $\quad b_\pm = \frac{1+\alpha_0^2}{2\beta_0\sqrt{K}} \pm \frac{\beta_0\sqrt{K}}{2}$ **β oscillates : frequency** $2\sqrt{K}$ $\quad \beta_{\max / \min} = b_+/\sqrt{K} \pm \lvert A \rvert$
$\alpha_0 = 0,$ $\beta_0 = 1/\sqrt{K}$	**β does not oscillate** $\Leftrightarrow \beta_{min} = \beta_{max} = \beta_0$ The Twiss parameters are matched

Transport through the plasma density plateau ($\delta \neq 0$)

Analytical model

Calculation of $\langle u \rangle$, $\langle u' \rangle$ & $\langle uu' \rangle$ over a normalized Gaussian distribution

Assumption : No correlation between the longitudinal and transverse plans

Use of the emittance definition : $\varepsilon^2 = \langle u^2 \rangle \langle u'^2 \rangle - \langle uu' \rangle^2$

Static term | **Chromatic offset** | **Term of decoherence and oscillations**

$$\frac{\varepsilon^2}{\varepsilon_0^2} = b_+^2 + \sigma_\delta^2 \frac{\sqrt{K}\,\beta_0}{2} b_+ + e^{-\sigma_\delta^2 z^2 K}\left(1 - b_+^2\right)$$

$$- \frac{\sigma_\delta^2}{8} e^{-\sigma_\delta^2 z^2 K}\left[\alpha_0^2 + 8z\sqrt{K}\alpha_0 b_+ - \left(3\sqrt{K}\beta_0 + b_+\right) b_- + 2\alpha_0 b_- \sin[4z\sqrt{K}] - \left(\alpha_0^2 - b_-^2\right)\cos[4z\sqrt{K}]\right]$$

$$+ \frac{\sigma_\delta^2}{2} e^{-\frac{1}{2}\sigma_\delta^2 z^2 K}\left[\left(-\beta_0\gamma_0 + 2z\sqrt{K}\alpha_0 b_+\right)\cos[2z\sqrt{K}] + b_-\left(\alpha_0 - 2z\sqrt{K} b_+\right)\sin[2z\sqrt{K}]\right],$$

$$b_\pm = \frac{1+\alpha_0^2}{2\beta_0\sqrt{K}} \pm \frac{\beta_0\sqrt{K}}{2}$$

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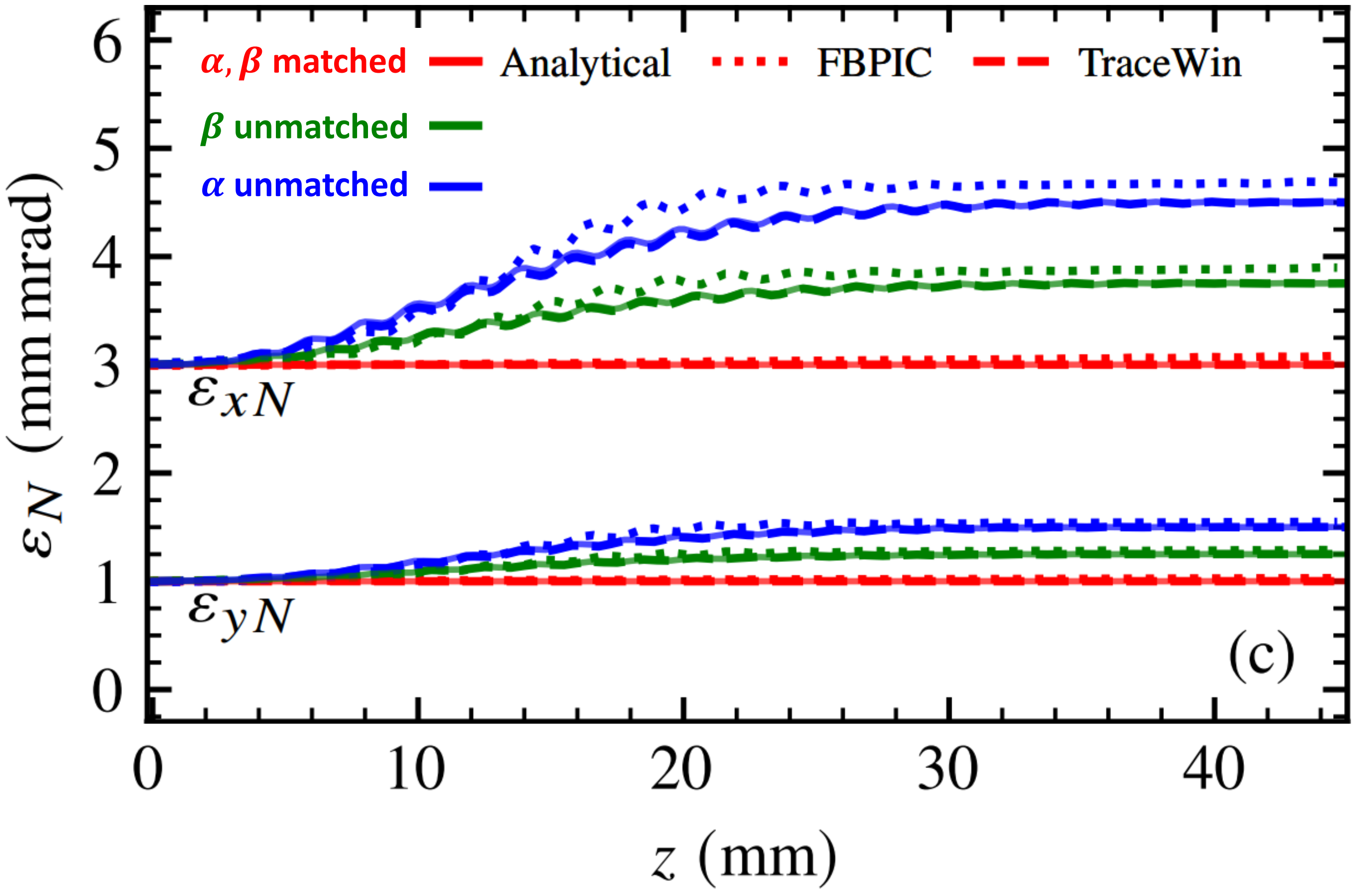

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