Acceleration Induced Self Interactions of Ultra Short Compact Bunches

R. J. McGuigan, S. P. Jamison

r.mcguigan1@lancaster.ac.uk

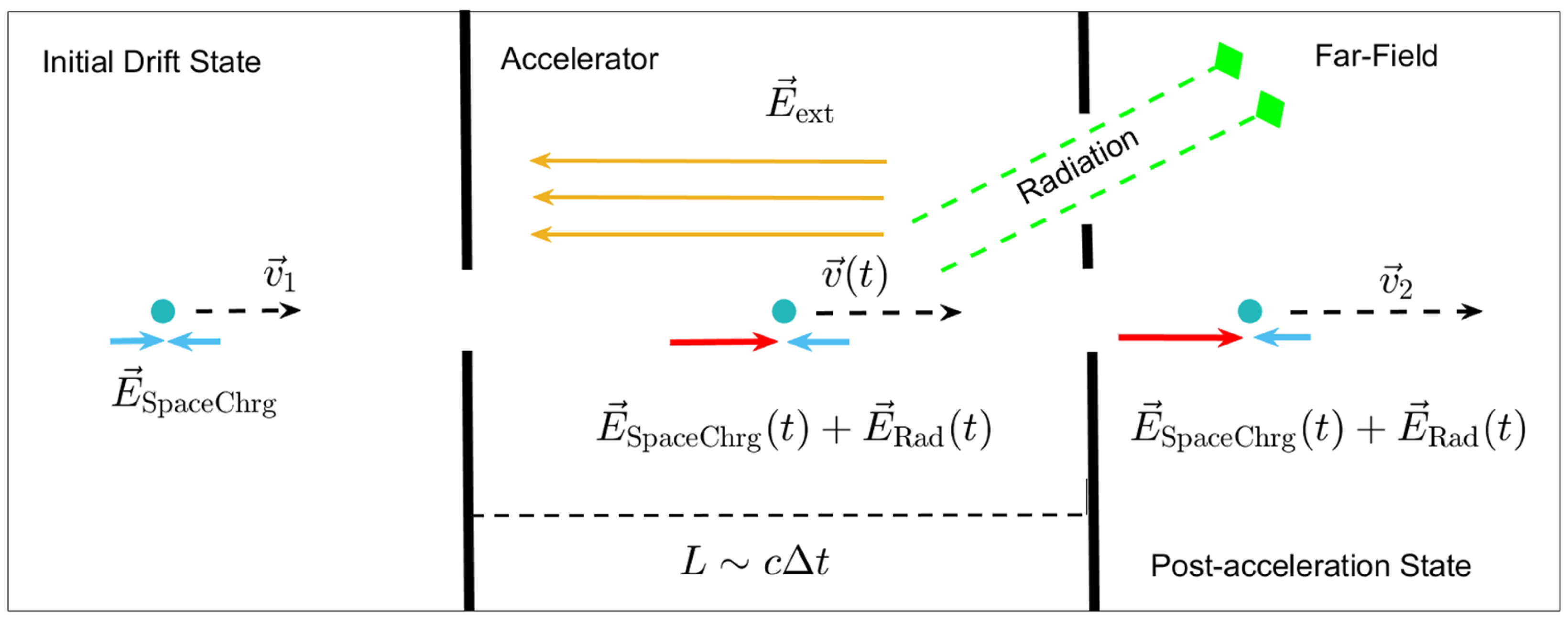
Bailrigg, Lancaster LA1 4YW

ACKNOWLEDGEMENT - This poster presentation has received support from the European Union's Horizon 2020 Research and Innovation programme under Grant Agreement No 101004730.

Introduction

Novel high-field gradient particle acceleration methods must meet strict performance requirements, e.g.. Successful demonstration of an FEL requires, low-energy-spread (< 0.5%), low-emittance (< 1μm) electron beams with high charge (> 30 pC)[1]. In previous work, it was found that **an accelerated compact bunch emits non-neglible amounts of coherent radiation**[2] (power scales as N^2, N:number of charges) into the far-field, constituting a loss of energy of the bunch. **We investigated the interaction induced by the acceleration/coherent emission process and identify effects degrading the quality of the bunch and limiting performance.**

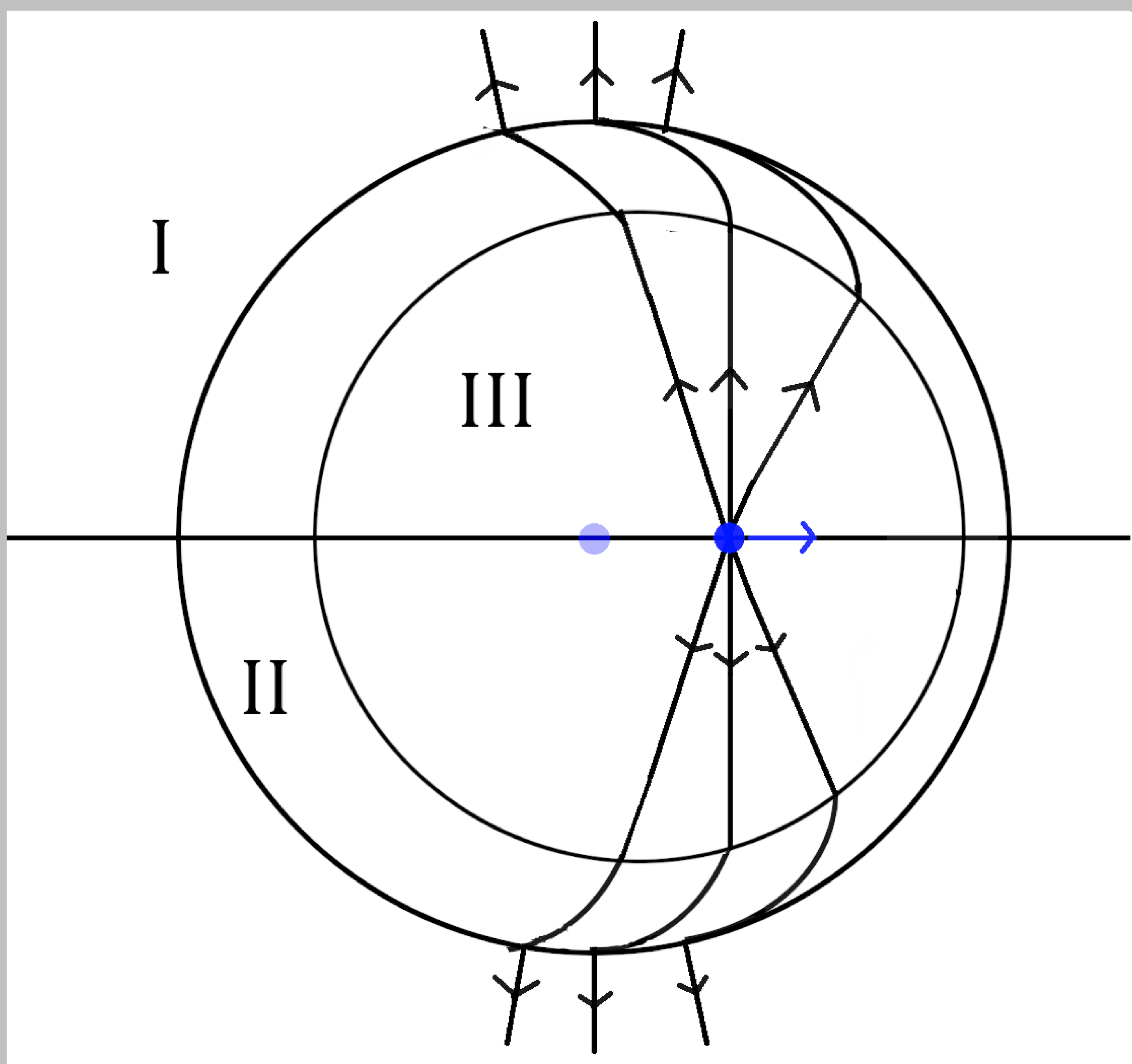
Fig1. Schematic of motion and interaction.



Model

A negative charge distribution of longitudinal length σ_z, transverse length σ_x, is accelerated over distance L for duration Δt by an externally applied field E_{ext}, gaining energy $\Delta\gamma mc^2$ and emitting radiation. The charge distribution is considered to be rigid with motion defined by a particular trajectory, the influence of radiation reaction on the motion of the charge is neglected. The interaction begins upon onset of acceleration and continues down the beamline, during and post-acceleration.

2. Retardation of point charges and distributions



The electromagnetic fields of a point charge moving with trajectory $r_0(t)$ are described by the **"Lienard Weichert"** Fields [3], containing velocity and acceleration dependent terms. The fields are **retarded**, that is they appear to emanate from a charged source at a previous or "retarded" time, t^{ret}. For a distribution, we promote the single particle LW fields to an integral over a **retarded charge distribution.** n is vector from source to observer, c is speed of light, ε_0 is permittivity of free space.

Fig. 2. Fields of charge accelerated for finite time. An observer sees **I**: Coulomb Fields from before acceleration, **II**: Fields induced by acceleration (radiation), **III**: Coulomb Fields after acceleration (charge energy has increased). Radiation can be viewed as a "shearing" of the fields due to the acceleration.

"Acceleration" term

$$\mathbf{E}_{\text{LW}}(\mathbf{r}_0(t), \mathbf{x}) = \frac{1}{4\pi\epsilon_0}\left[\frac{(1-\dot{\mathbf{r}}_0\cdot\dot{\mathbf{r}}_0/c^2)(\hat{\mathbf{n}}-\dot{\mathbf{r}}_0/c)}{(1-\hat{\mathbf{n}}\cdot\dot{\mathbf{r}}_0/c)^3|\mathbf{r}_0-\mathbf{x}|^2} + \frac{1}{4\pi\epsilon_0}\frac{\hat{\mathbf{n}}\times(\hat{\mathbf{n}}-\dot{\mathbf{r}}_0/c)\times\ddot{\mathbf{r}}_0}{c^2(1-\hat{\mathbf{n}}\cdot\dot{\mathbf{r}}_0/c)^3|\mathbf{r}_0-\mathbf{x}|}\right]^{\text{ret}}$$

"Velocity" term

Integrate over single particle kernel . . .

$$\mathbf{E}(\mathbf{s},t) = \int_0^\infty dR \int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi \rho\left(\mathbf{s}+\mathbf{r}_0(t^{\text{ret}})-\mathbf{r}_0(t^{\text{ret}})-\mathbf{R}\right)\mathbf{K}(R,\theta,\phi,\mathbf{r}_0(t))$$

Encodes retardation such that co-moving test charge experiences radiation and acceleration induced field from charge within particular neighborhood. **s** denotes position of test charge relative to moving bunch.

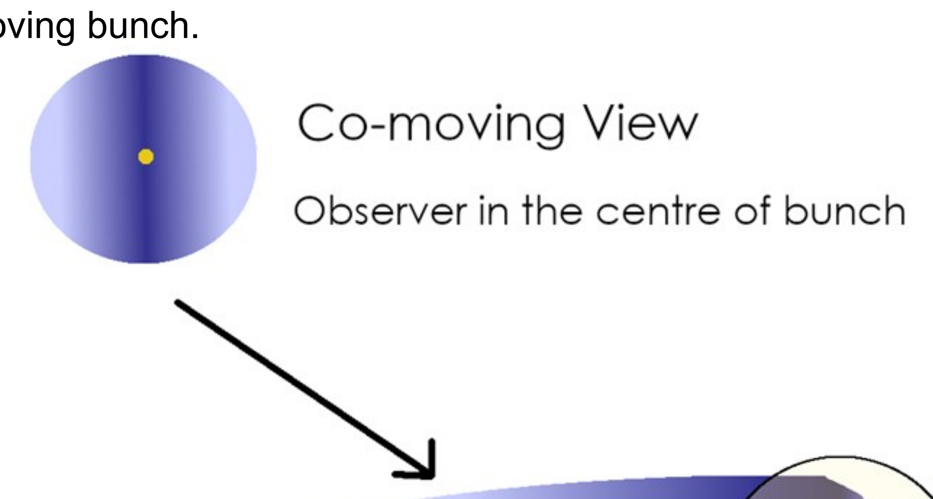
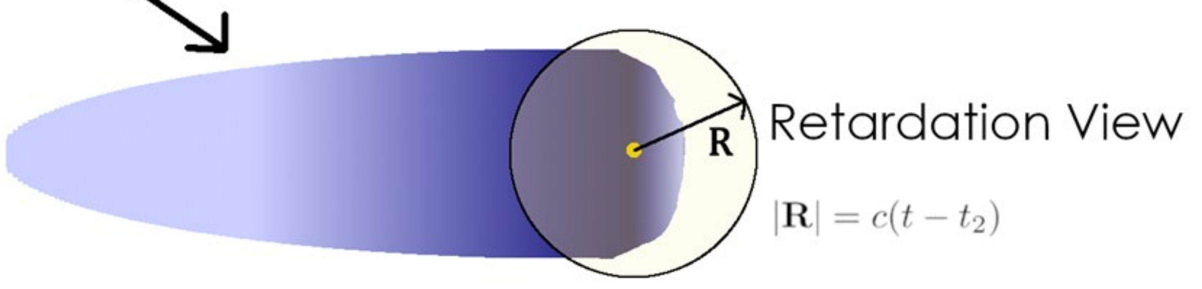


Fig. 3. Retardation of an accelerated charge density for a co-moving observer or test charge



Charge density becomes assymmetrically stretched from the perspective of observer. Transparent yellow region indicates the causal region that contributes radiative forces.

KEY

- Observer **s**
- Charge Density
- Causal region

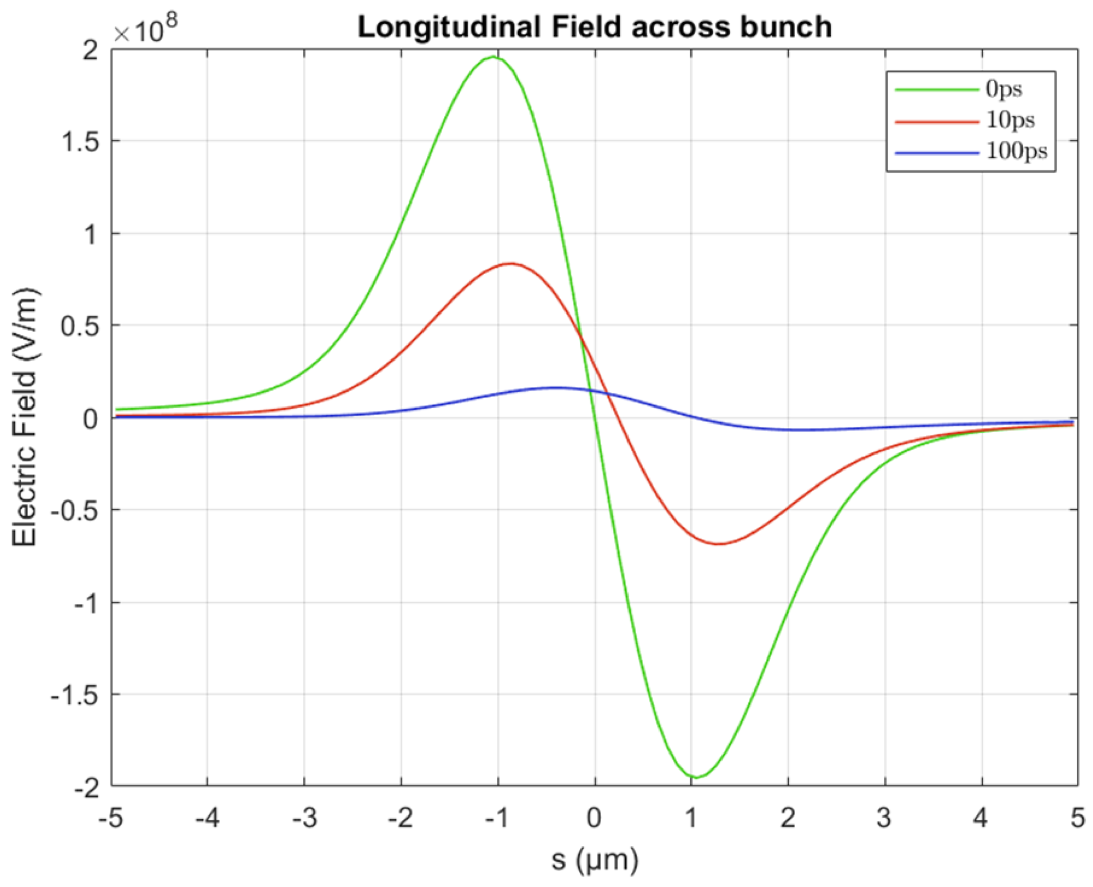
Charge outside of this region hasn't started accelerating yet and contributes coulomb-like "drift" fields.

3a. Results: Fields during Acceleration

We calculate the longitudinal fields of a compact electron bunch

- Q=100pC
- $\sigma_{x,z}$=1μm
- E_{ext}=20GeV/m
- Δt=100ps (L~3cm)
- γ_1=100 ($\gamma_1 mc^2$ ~50MeV)

where γ_1 is the initial energy (Lorentz factor). Acceleration induced fields are visibly **asymmetric** for $\Delta\gamma mc^2 > \gamma_1 mc^2$ such that middle and back of the bunch experience strong positive fields.



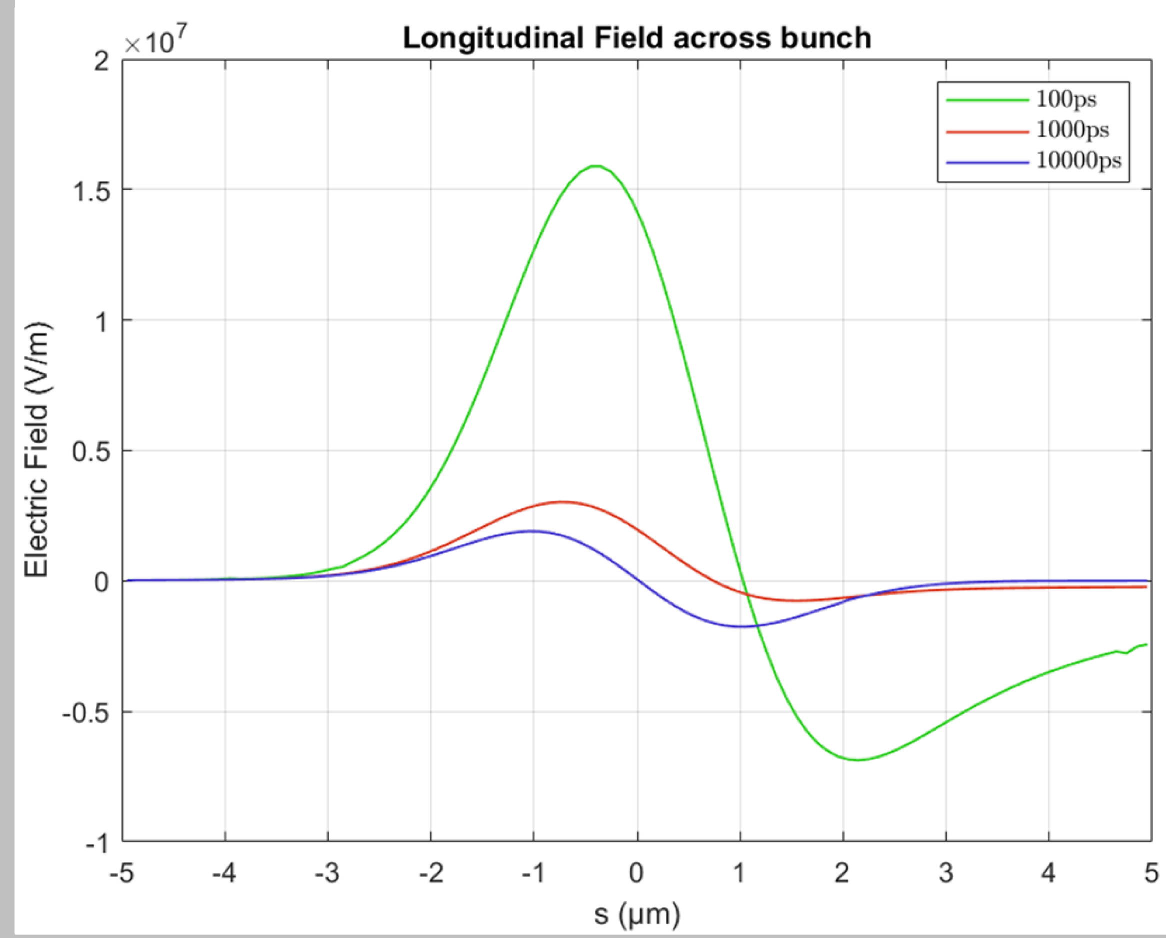


Fig4: Longitudinal fields during acceleration. Overall magnitude decreases as energy increases.

3 .b. Results: Fields Post-Acceleration

Transient fields **continue to interact** with bunch up-to formation length $2\sigma_z\gamma_2^2$ i.e. the length over which the radiation decouples from the bunch[4]. Asymmetry of field/interaction reflects asymmetry in retardation point of view.

Fig. 5: Longitudinal fields after the acceleration process, decays into regular coulomb field after formation length ($2\sigma_z\gamma_2^2/c$=1ns).

4. Results: Collective Effects

A combination of space-charge and radiation field alters the previously monochromatic energy spectrum of the bunch; **a negative chirp in the back and centre of the bunch of order O(0.1-1%) is induced.** Net loss of energy/momentum due to self-fields (fig. 7). Longer bunches lose energy for longer times post-acceleration, however **shorter bunches lose more energy overall and during the acceleration, such that the exact mechanism of energy loss depends on the nature of the acceleration.** Net losses of 100pC, $\sigma_{x,z}$=1μm bunch on order of 1%, and scales as N^2; near-field losses correspond to energy of radiation emitted into far-field.

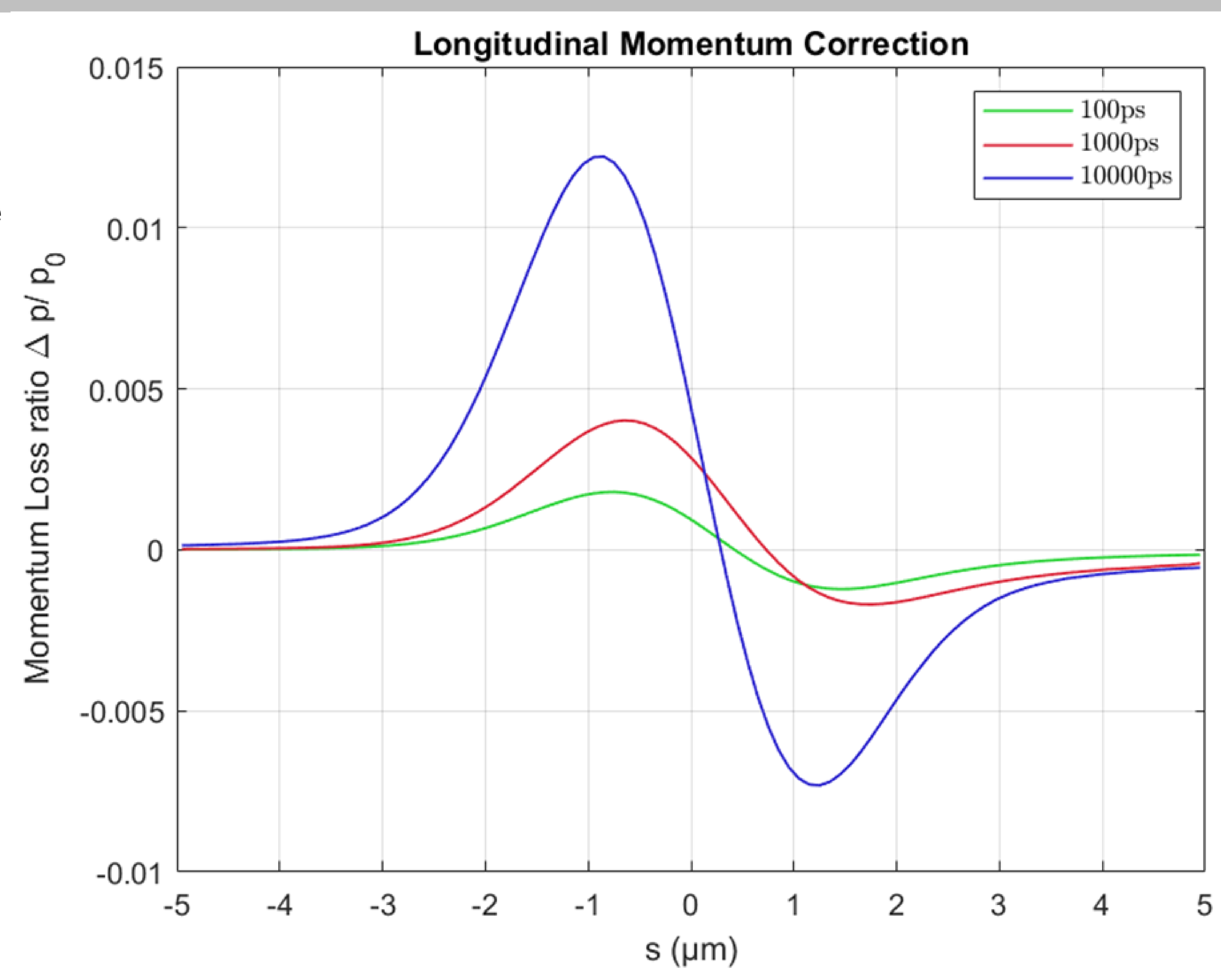


Fig. 6. Longitudinal momentum chirp across bunch as a ratio of total momentum

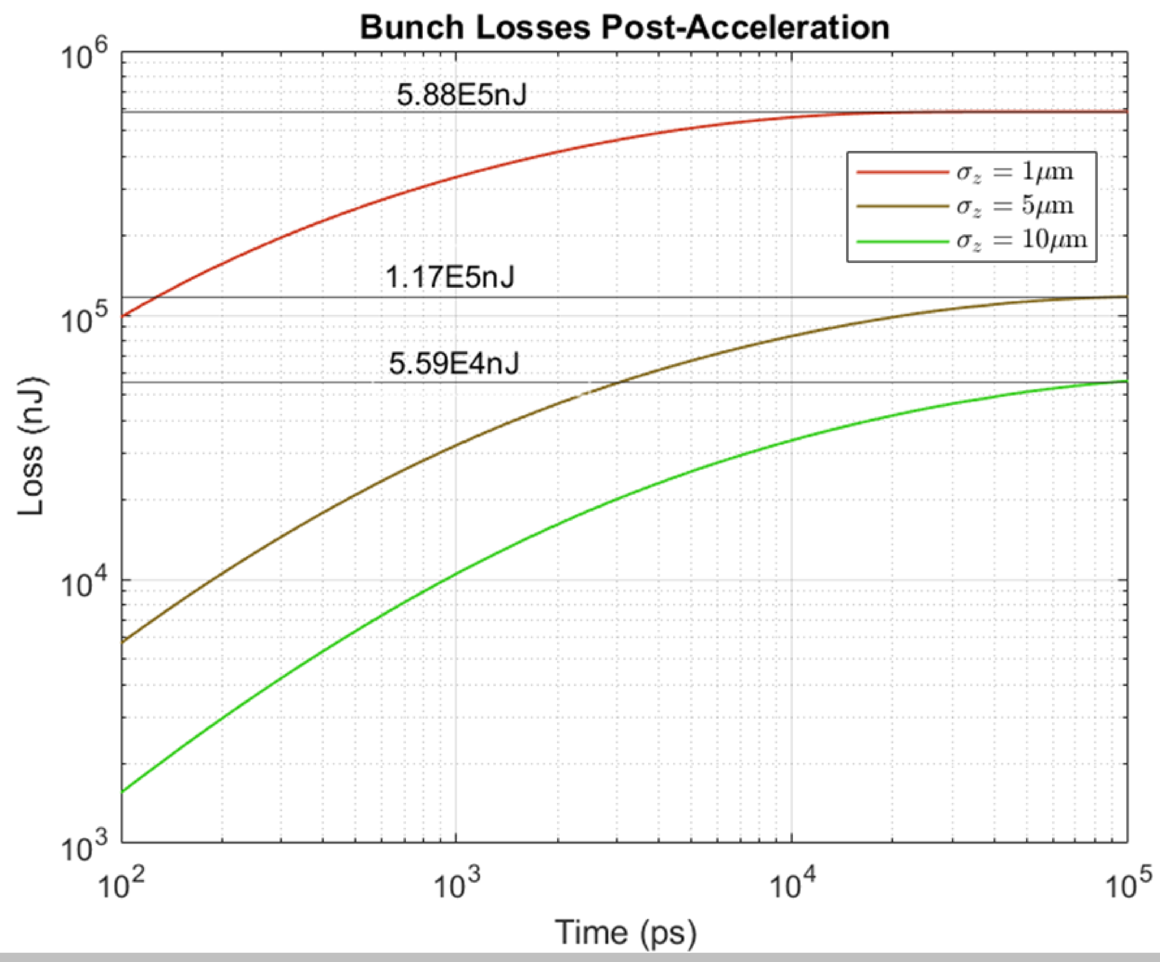


Fig. 7: Logarithmic plot of losses as a function of time post-acceleration for various bunch lengths.

Summary

This model points to an inherent vacuum beam-loading process that is exacerbated rather than compensated by higher-gradient accelerating fields. Novel high-field accelerators of compact bunches are at risk of suffering from coherent emission losses and significant momentum spread.

References

[1] Félicie Albert et al, (2020) roadmap on plasma accelerators, 2021 New J. Phys. 23 031101

[2] R.J. McGuigan, S.P. Jamison, Coherent Radiation Of Ultra-Short Bunches from Linear Acceleration Processes (unpublished)

[3] A. Zangwill, Modern Electrodynamics, C. U.Press (2012)

[4]. G. Geloni, Acceleration-induced self-interactions within a relativistic electron bunch : an analytical study, Quality and Rel. Eng. Int.,