An explicit algorithm in the quasi-static approximate PIC program QPAD

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Introduction

- Plasma wakefield acceleration is a method that uses a driving particle beam or an intense laser to excite a wakefield in the plasma and uses the wakefield to accelerate another bunch of particles.
- In response to the need to simulate the plasma wakefield acceleration, large-scale parallel computing programs such as QuickPIC [1] and QPAD [2] have been developed. These programs use the quasi-static approximation PIC algorithm, while QPAD introduces the azimuthal Fourier decomposition on top of QuickPIC.
- Currently, both QuickPIC and QPAD use a predictor-corrector method to solve the field equations. In certain cases, multiple iterations are required to achieve convergence. Recently, T. Wang et al. [3] proposed a new explicit algorithm in the quasi-static approximation PIC code, thereby avoiding the need for iterative computation.
- We first developed an explicit algorithm within the azimuthal Fourier decomposition framework by analytically extracting implicit transverse magnetic terms from current derivatives using conserved quantities. In the explicit equation, the coefficients dynamically change during plasma solving, and their product with the transverse magnetic field quantity causes a modal nesting issue. The algorithm solves all-mode transverse magnetic equations via global matrix construction, enabling explicit quasi-static PIC with azimuthal Fourier decomposition.

Method

transverse magnetic field equations with the azimuthal Fourier decomposition

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{(m\pm1)^2}{r^2}\right)B_\pm^m=S_\pm^m,\ S_\pm^m=-\frac{\partial J_\phi^m}{\partial\xi}-\frac{im}{r}J_z^m\pm i\left(\frac{\partial J_r^m}{\partial\xi}+\frac{\partial J_z^m}{\partial r}\right)$$

predictor-corrector

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{(m\pm1)^2}{r^2}-1\right)B_\pm^{m,l+1}=S_\pm^{m,l}-B_\pm^{m,l}$$

m: the mode number, l: the iteration number

The procedure begins with the computation of the magnetic filed using a predicted Lorentz factor,current, the ξ derivative of the transverse current.

explicit solver

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{(m\pm1)^2}{r^2}\right)B_\pm^m+X^m=S_\pm^m$$

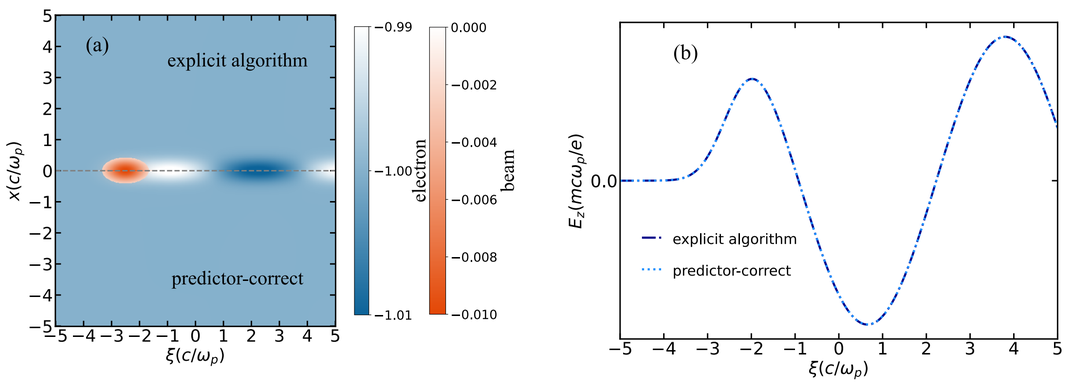
$$S_\pm^m=-T_\phi^m-\frac{im}{r}J_z^m\pm i\left(T_r^m+\frac{\partial J_z^m}{\partial r}\right)$$

$$\frac{\frac{q}{m}(\rho-J_z)}{1-\frac{q}{m}\psi}B_\perp=\frac{\sum_{n_1=-m}^{m}a_{n_1}e^{-in_1\phi}}{\sum_{n_2=-m}^{m}b_{n_2}e^{-in_2\phi}}=\sum_{n_3=-m}^{m}u_{n_3}e^{-in_3\phi}\sum_{n_4=-m}^{m}B_{\perp,n_4}e^{-in_4\phi}=\sum_{n_5=-m}^{m}X_{\perp,n_5}e^{-in_5\phi}$$

Using the conserved quantity $\gamma-p_z=1-\frac{q}{m}\psi$, the derivative of the momentum with respect to ξ is obtained, and then deposit the source term $\frac{\partial J_\perp}{\partial\xi}$

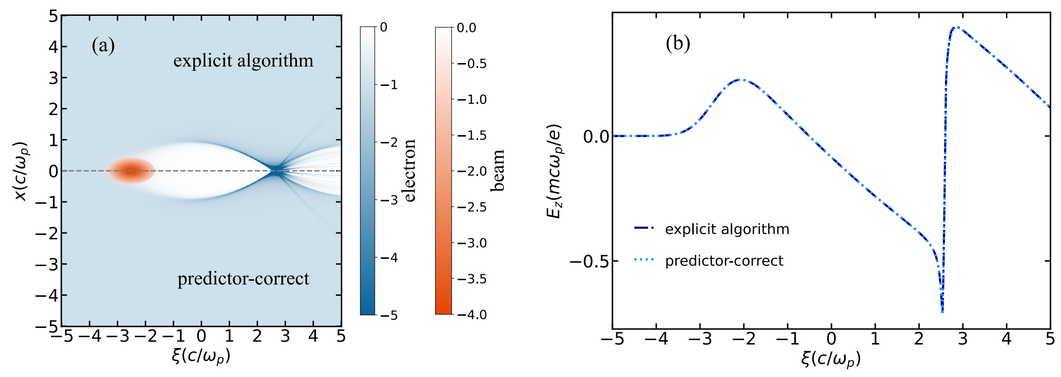
Results

- **Linear wakefield**

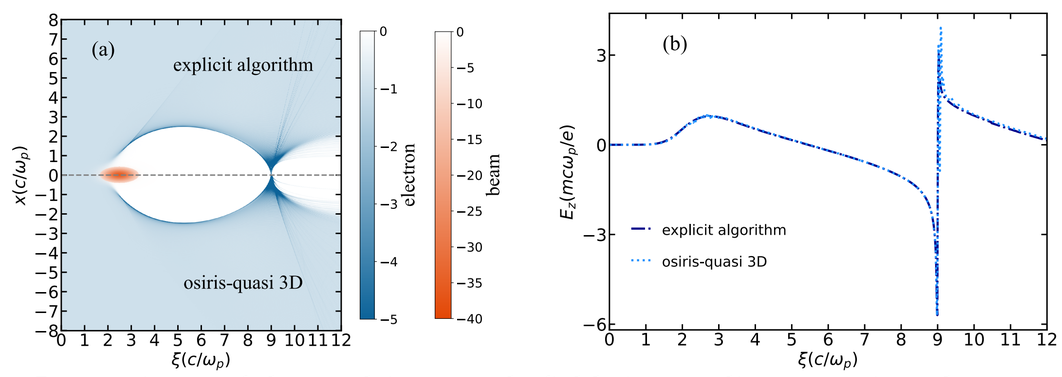


Comparison of beam-driven wakefield in linear/nonlinear regime between explicit algorithm and predictor-correct method. (a) Beam density and background electron density. (b) On-axis lineouts of Ez fields

- **Nonlinear wakefield**



We increased the peak density of the drive bunch. Figures demonstrates stable operation of the explicit algorithm, while the single iteration predictor algorithm diverges. Even when the number of iterations is increased to 10, the simulation still fails at the rear of the bubble.



Comparison of beam-driven wakefield in nonlinear regime between explicit algorithm and Osiris-quasi 3D. (a) Beam density and background electron density. (b) On-axis lineouts of Ez fields

Conclusion

- We successfully developed a new explicit algorithm for quasi-static PIC under azimuthal Fourier decomposition that eliminates the need for iterative computation.
- After implementing the explicit algorithm, we compared the results of both algorithms and found excellent agreement. The explicit algorithm also allows computations in more nonlinear wakefield regions.
- In future work, we aim to further optimize the modes nesting problem in the explicit algorithm to improve computational efficiency.

Reference

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