Istituto Nazionale di Fisica Nucleare
SEZIONE DI TORINO

Particle identification at colliders

(With a strong Cherenkov bias.)

XXXIII Bonaudi-Chiavassa International School on particle detectors
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Particle identification

What do we mean by "particle identification"?

→ **Determine the mass of "stable" particles**

→ Really stable particles:
e, p, γ, Fe, d...

- Lightest <u>lepton</u>,
- Lightest <u>baryon</u>
- Stable nuclei
- $\tau >>$ universe life

Particle identification

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→ Almost stable particles:
μ, π, K, n (Ξ, Ω...)

- Lightest particle of each flavour content (uu+dd, us, uds, uss, sss)
- neutron (almost degenerate with the proton)
- **$\tau = 10^{-10} - 10^{3}$ s**
  Travel cm~m in the detectors

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Here: Charged particles only!

Particle identification

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How can we do that?

Particle identification

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$\rightarrow$ Determine the mass of "stable" particles

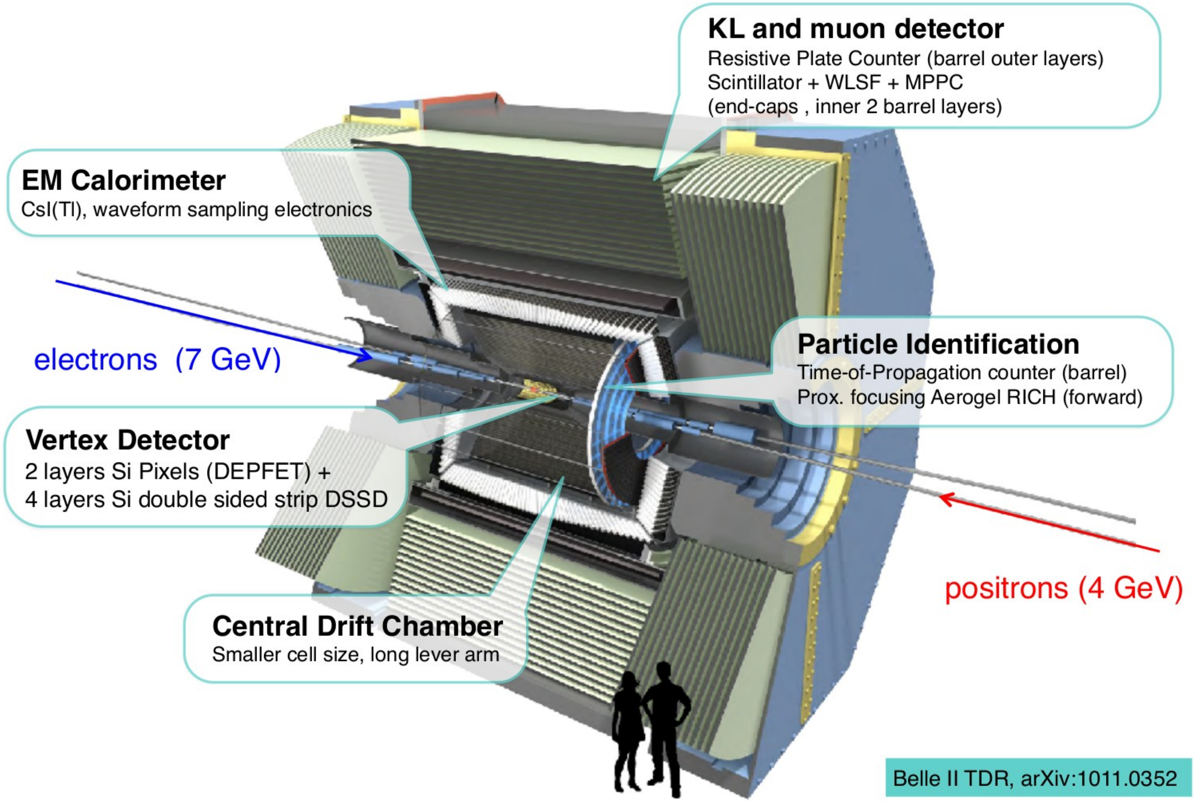
How can we do that?

$\rightarrow$ Measure simultaneously (and independently!) β and p

$$\beta\gamma = \frac{p}{m}$$

$\rightarrow$ Observe the interaction with a material (CsI, Iron...)

The Belle II detector



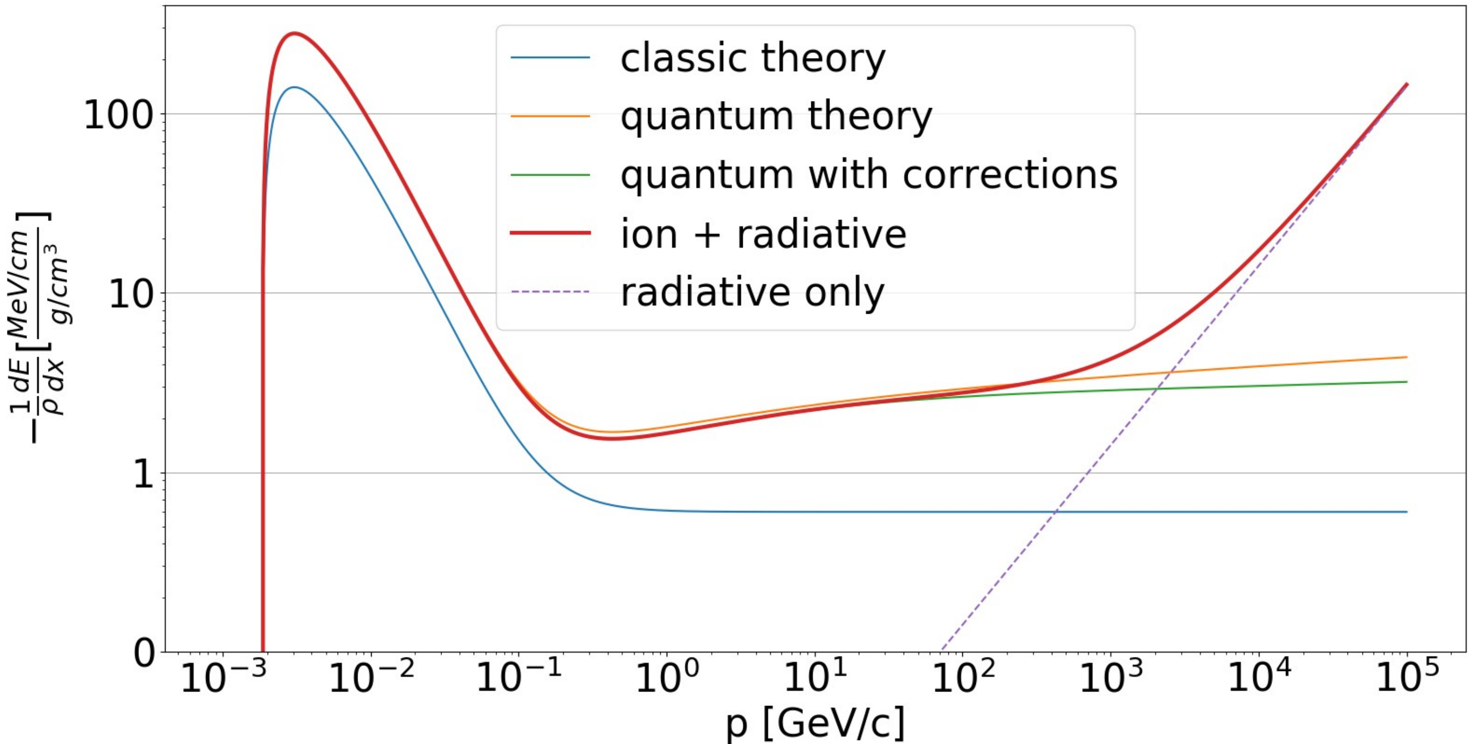
→ CP violation

→ **Every sub-detector provides PID information**

- → dE/dx from Drift Chamber and Vertex detector
- → **Cherenkov signal from TOP and ARICH**
- → Shower shapes from the calorimeter
- → Penetration depth from the muon system

PID by interaction

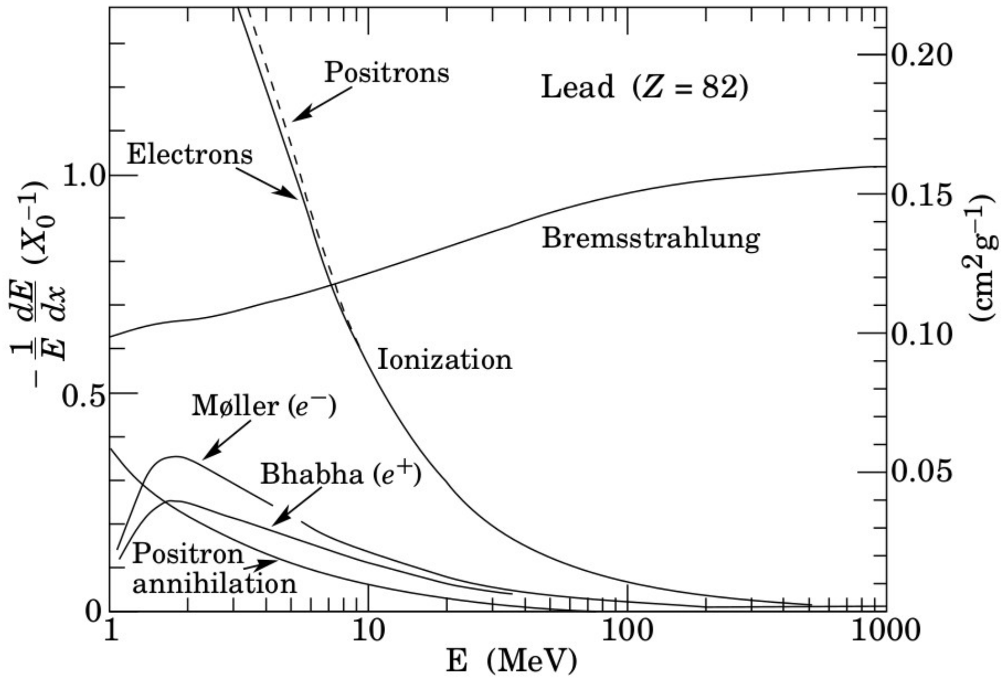
$$-\left(\frac{1}{\rho}\right)\frac{dE}{dx}\bigg|_{\mathrm{TOT}} = Kz^2\frac{1}{\beta^2}\frac{Z}{A}\left[\frac{1}{2}\ln\frac{2c^2\beta^2 m_e\gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{C}{Z}\right] + \frac{dE}{dx}\bigg|_{\mathrm{rad}}$$



Muons almost only loose energy by dE/dx

PID by interaction

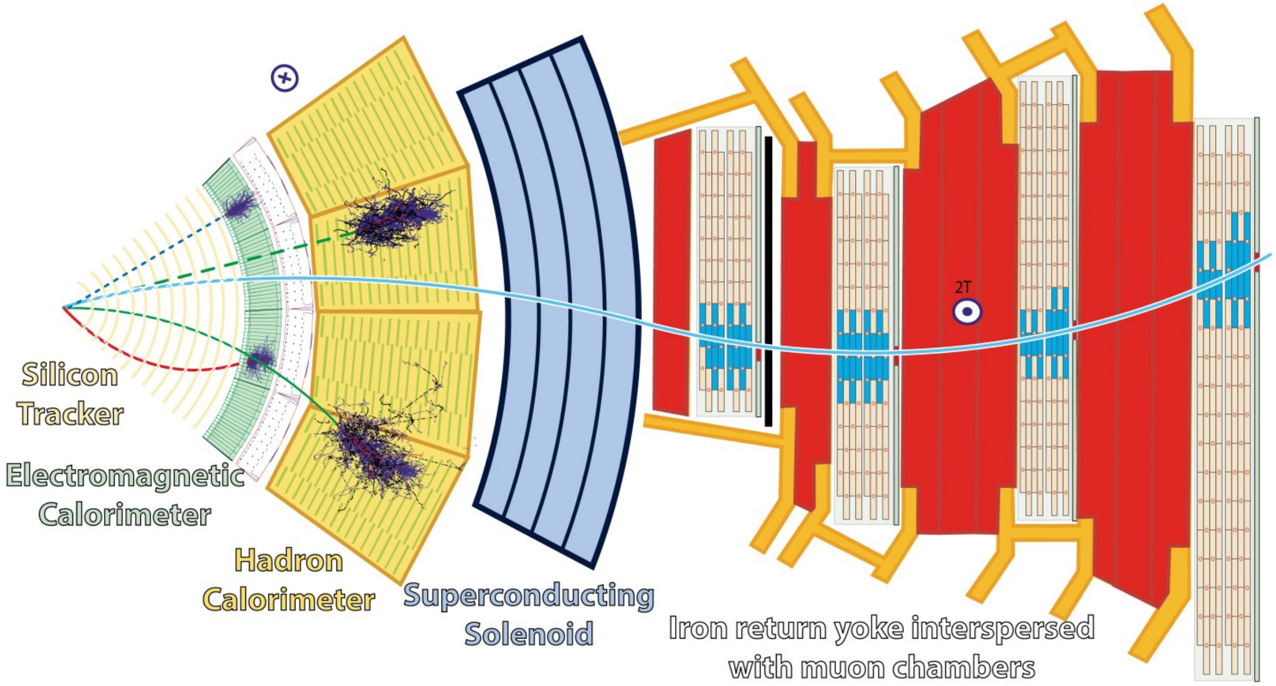
$$-\left(\frac{1}{\rho}\right)\frac{dE}{dx}\bigg|_{\mathrm{TOT}} = Kz^2\frac{1}{\beta^2}\frac{Z}{A}\left[\frac{1}{2}\ln\frac{2c^2\beta^2 m_e\gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{C}{Z}\right] + \frac{dE}{dx}\bigg|_{\mathrm{rad}}$$



Electrons radiate above 10 MeV!

PID by interactions

Which particles can you identify by interaction?



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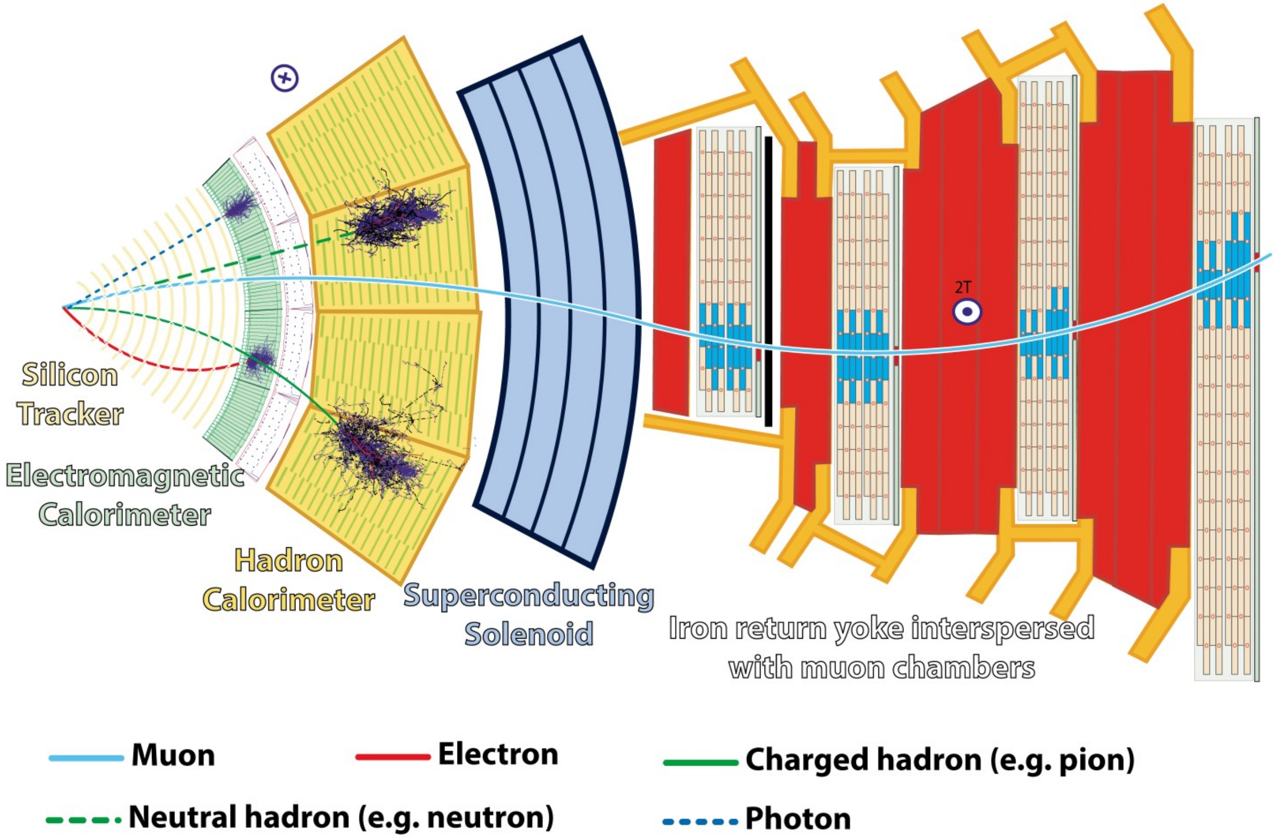
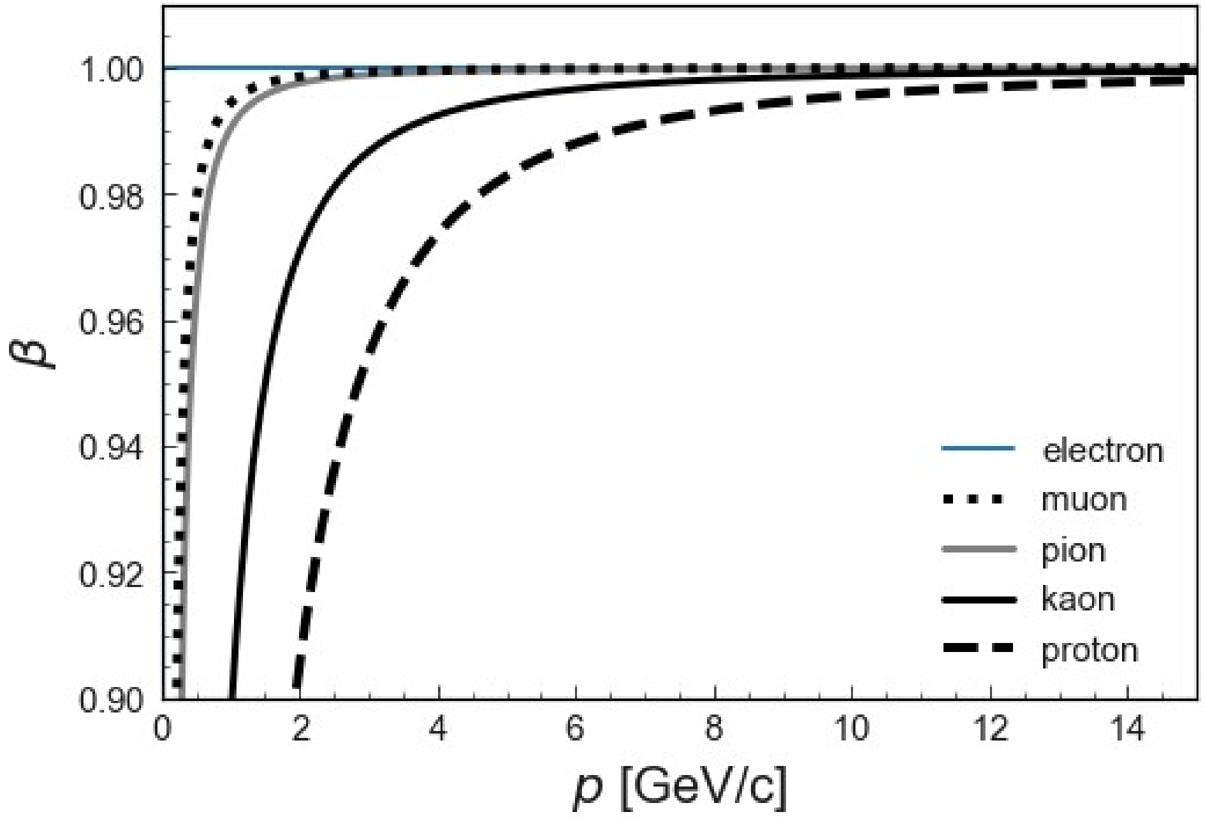


PID by velocity

How do you measure the velocity of a charged particle?



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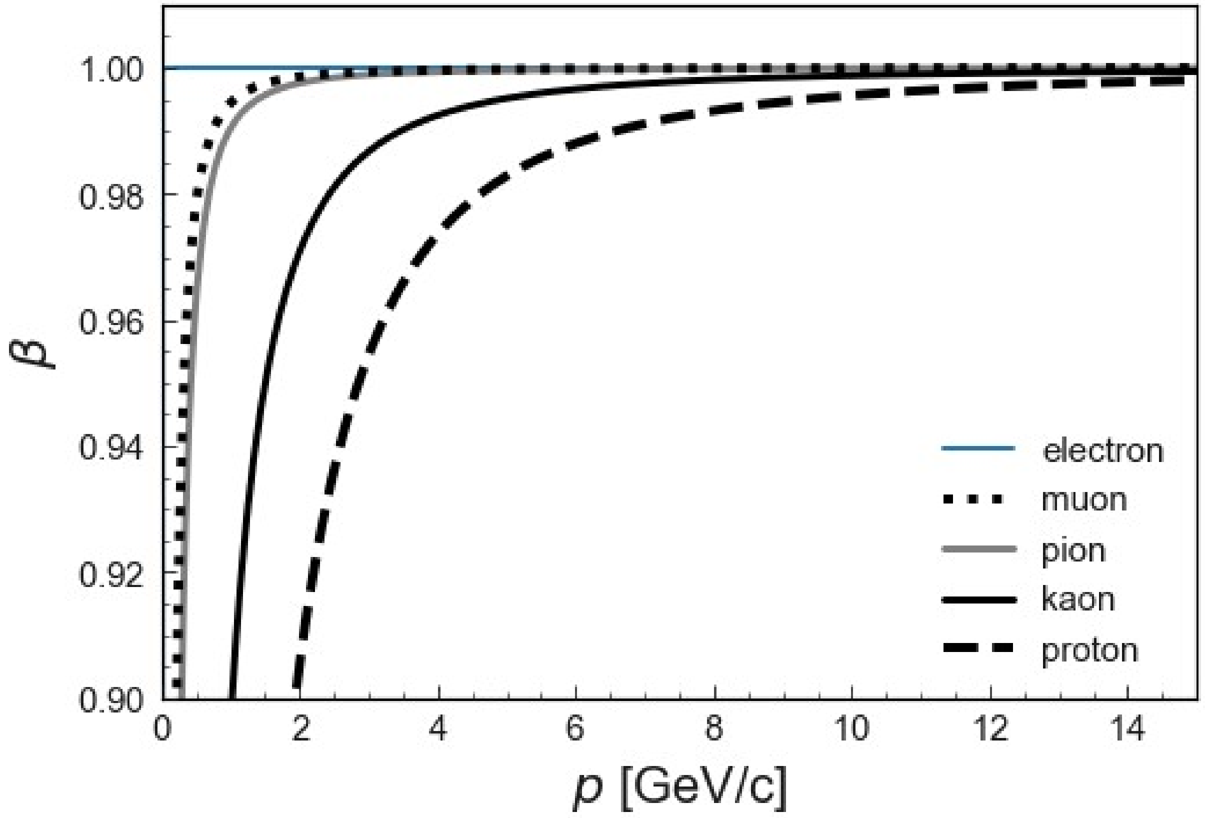
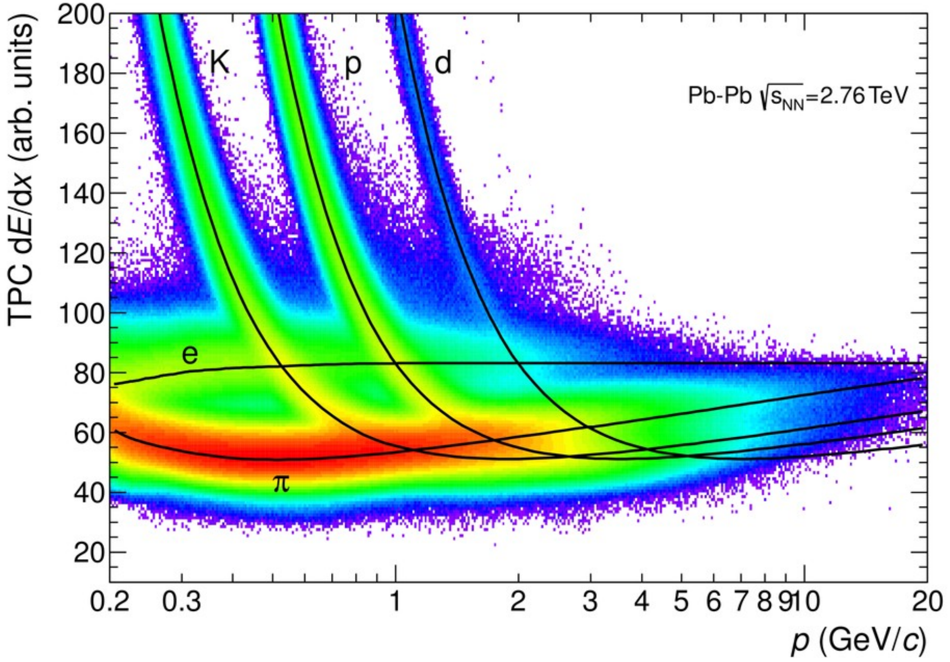
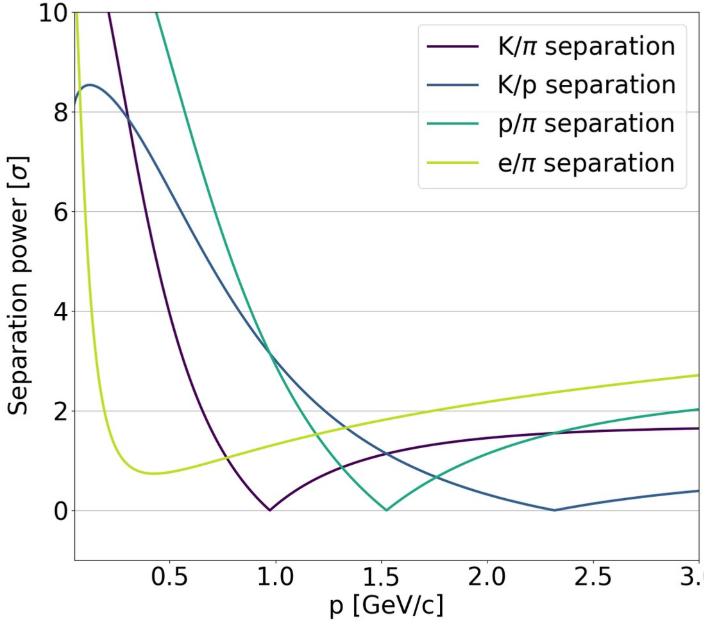


- → dE/dx
- → Time-Of-Flight
- → Cherenkov effect

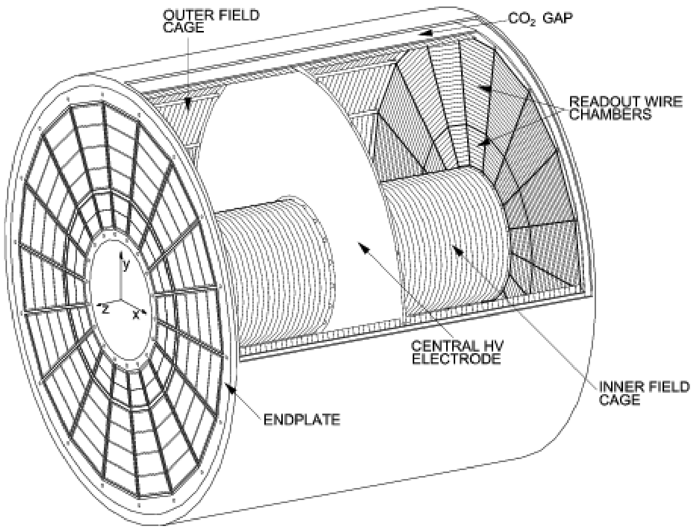
Specific ionization

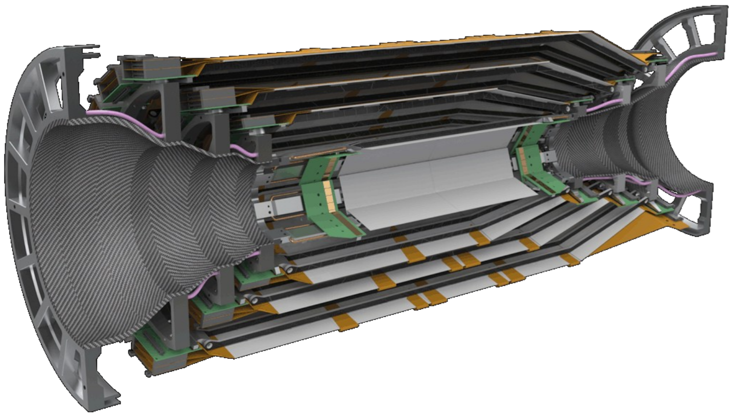
$$-\left(\frac{1}{\rho}\right)\frac{dE}{dx}\bigg|_{\mathrm{TOT}} = Kz^2\frac{1}{\beta^2}\frac{Z}{A}\left[\frac{1}{2}\ln\frac{2c^2\beta^2 m_e\gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{C}{Z}\right] + \frac{dE}{dx}\bigg|_{\mathrm{rad}}$$

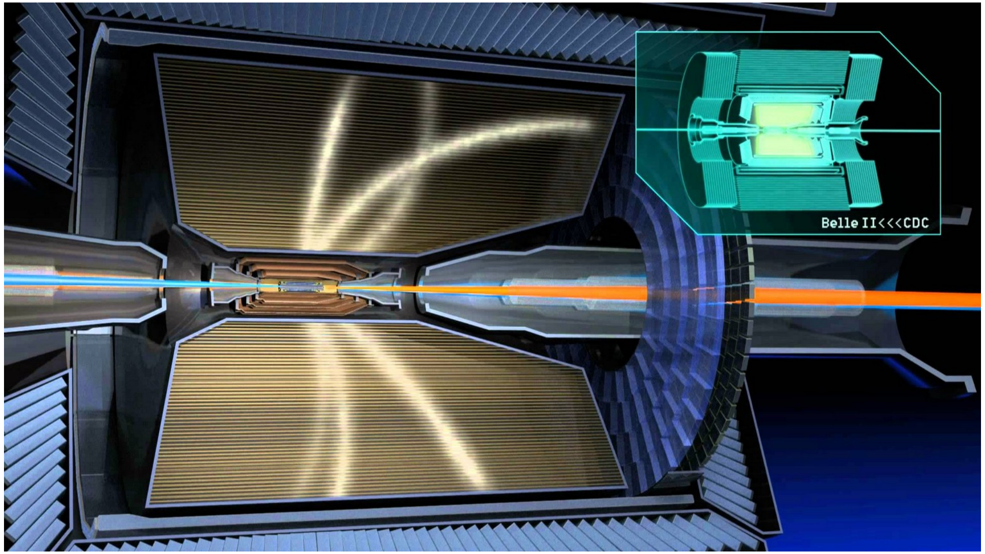




Specific ionization







Time of flight counters

- Time of flight

Measure signal time difference between two detectors with good time resolution [start and stop counter]

- Typical detectors:

  Scintillation counter + photodetector

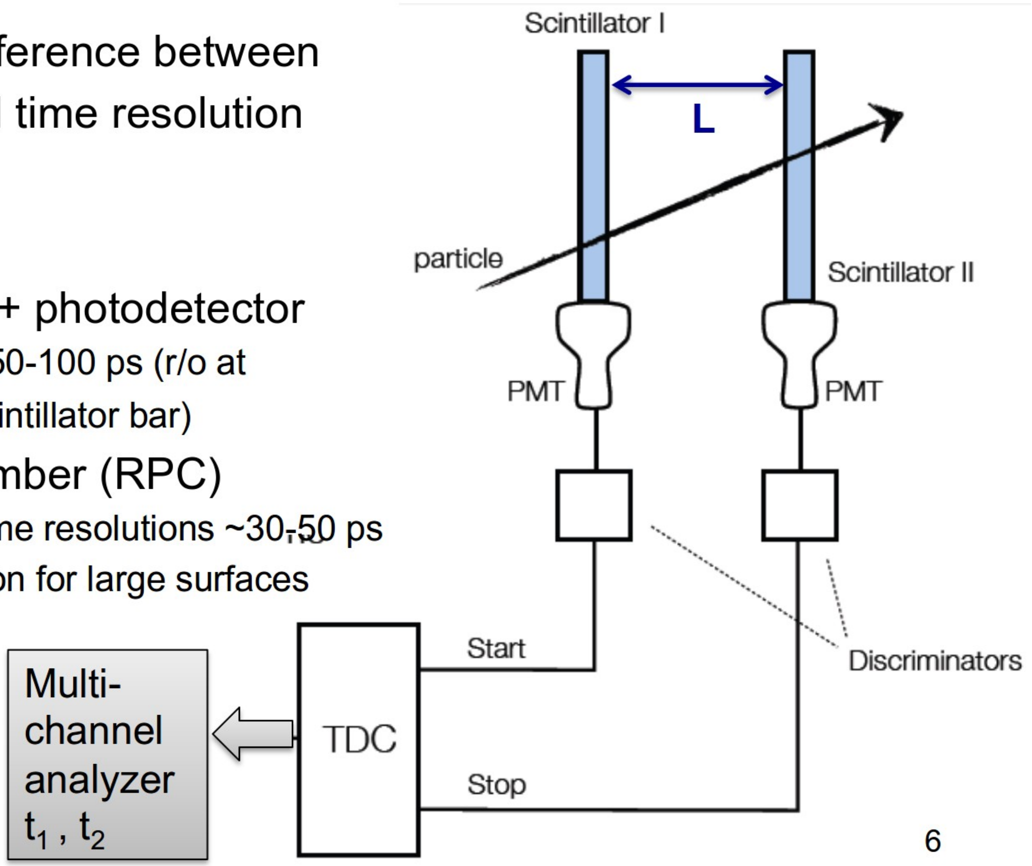
  time resolutions ~50-100 ps (r/o at both ends of the scintillator bar)

  Resistive Plate Chamber (RPC)

  not sensitive to B, time resolutions ~30-50 ps

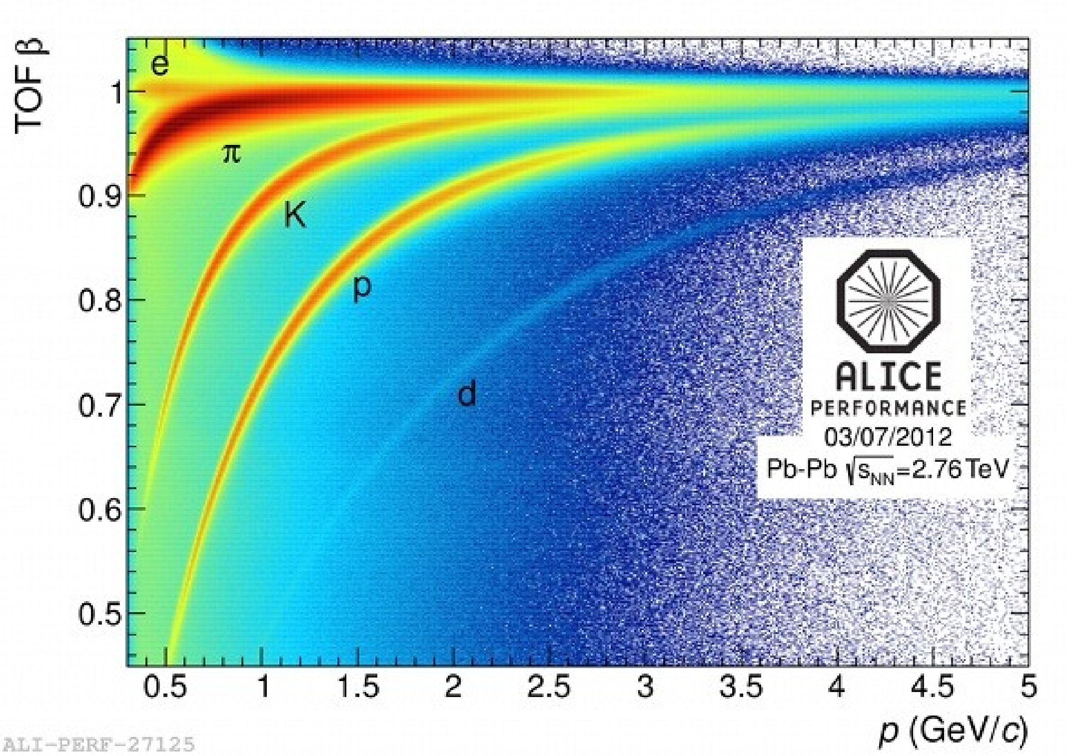
  cost effective solution for large surfaces

$$\Delta t = t_2 - t_1 = \frac{L}{c\beta}$$



Time of flight counters

Alice RPC



Belle scintillators

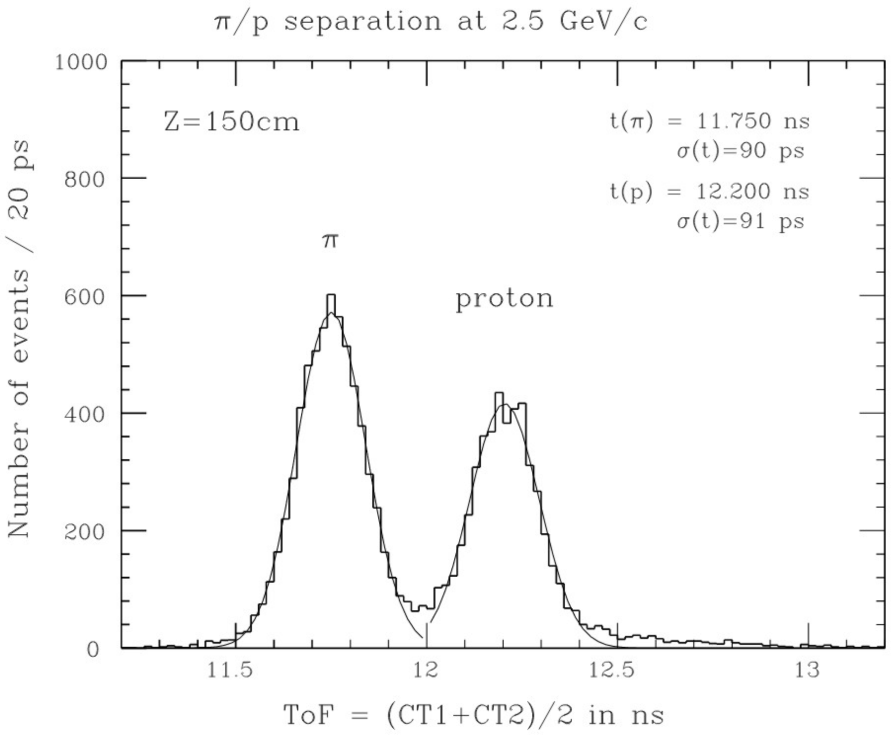
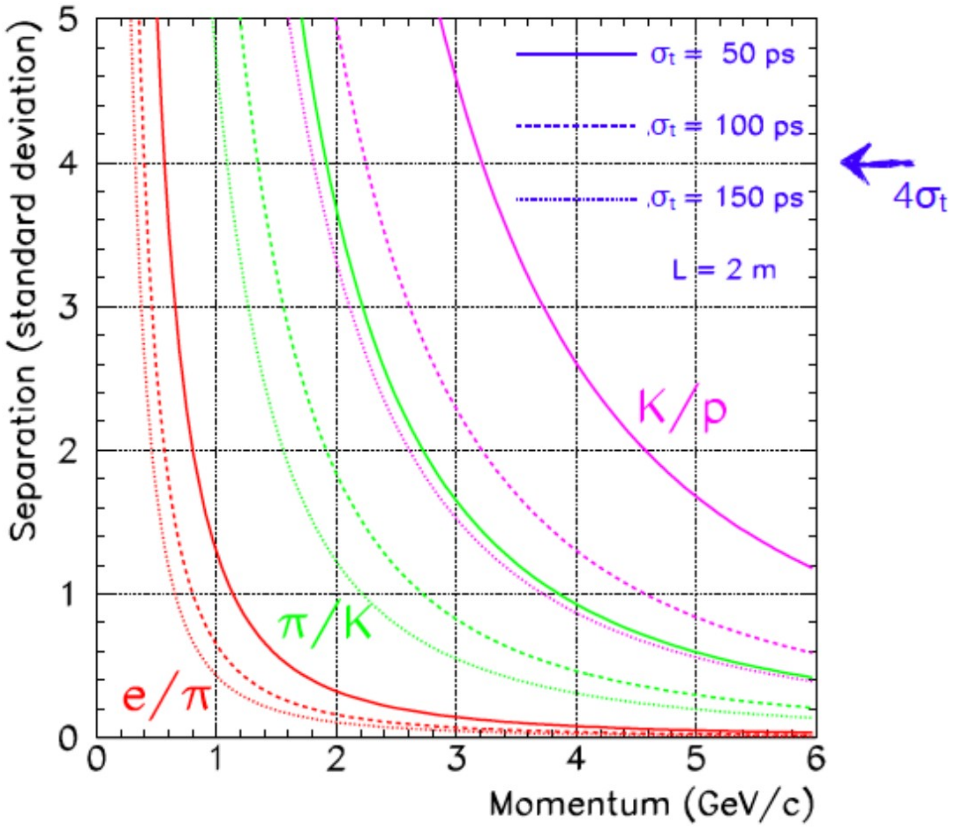
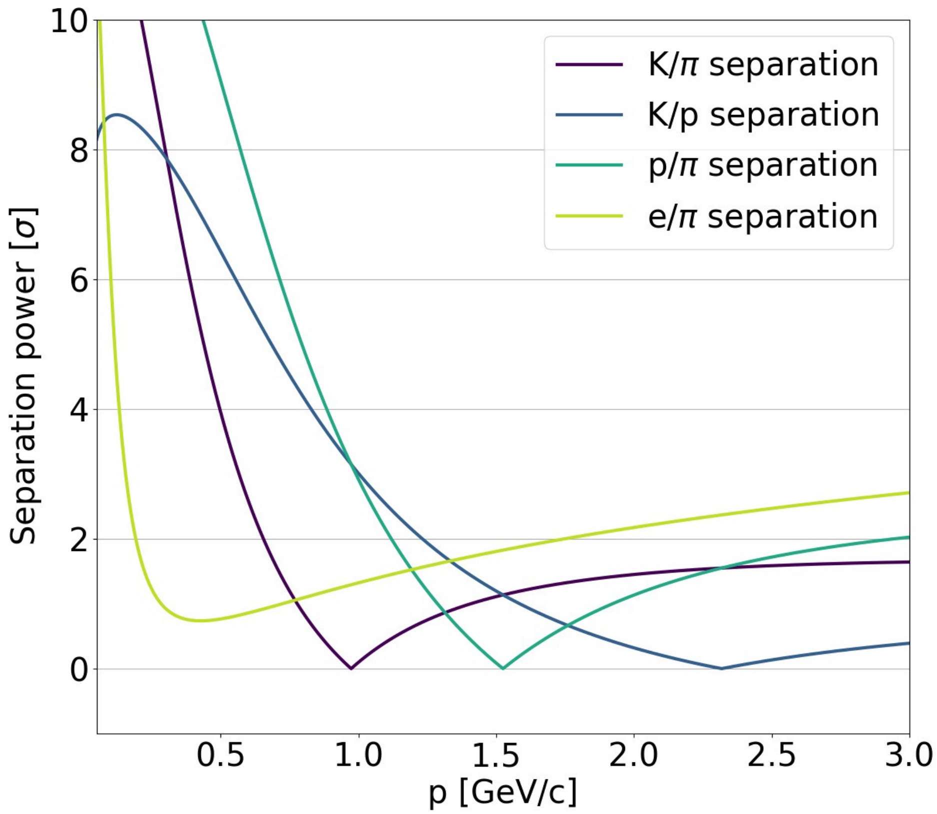


Fig. 62. π/p separation at 2.5 GeV/c.

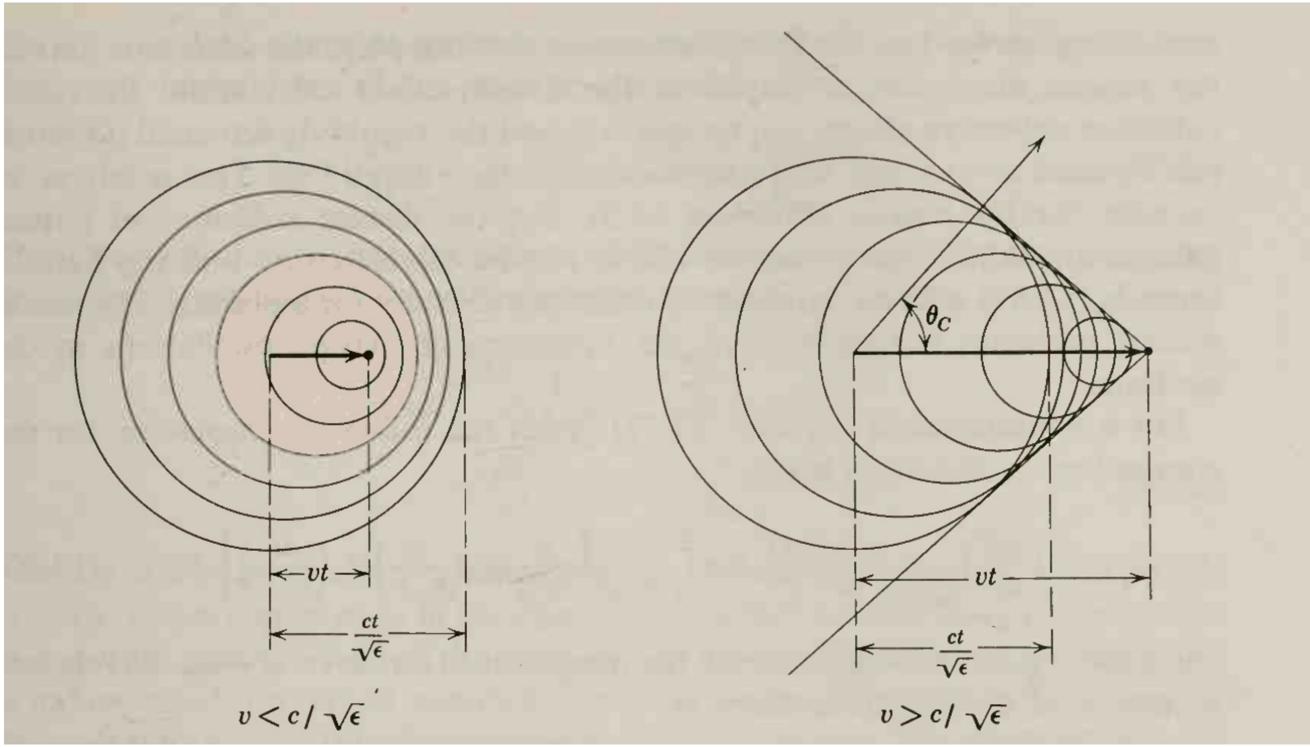
Time of flight performaces





Threshold and Imaging Cherenkov counters

Cherenkov detectors

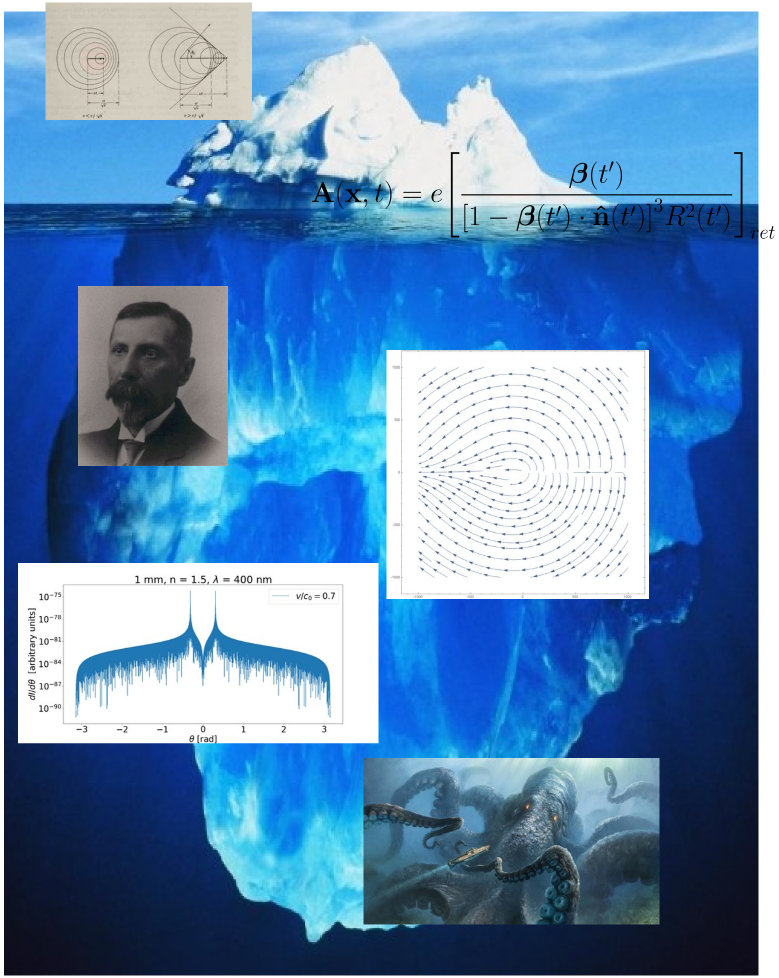


The theory: a disclaimer

For this course I wanted to go more deeply into the theory, and I found myself in a deep, dark rabbit hole...

It turns out that the exact theory of the Cherenkov effect is neither easy, not immediate, nor straightforward.

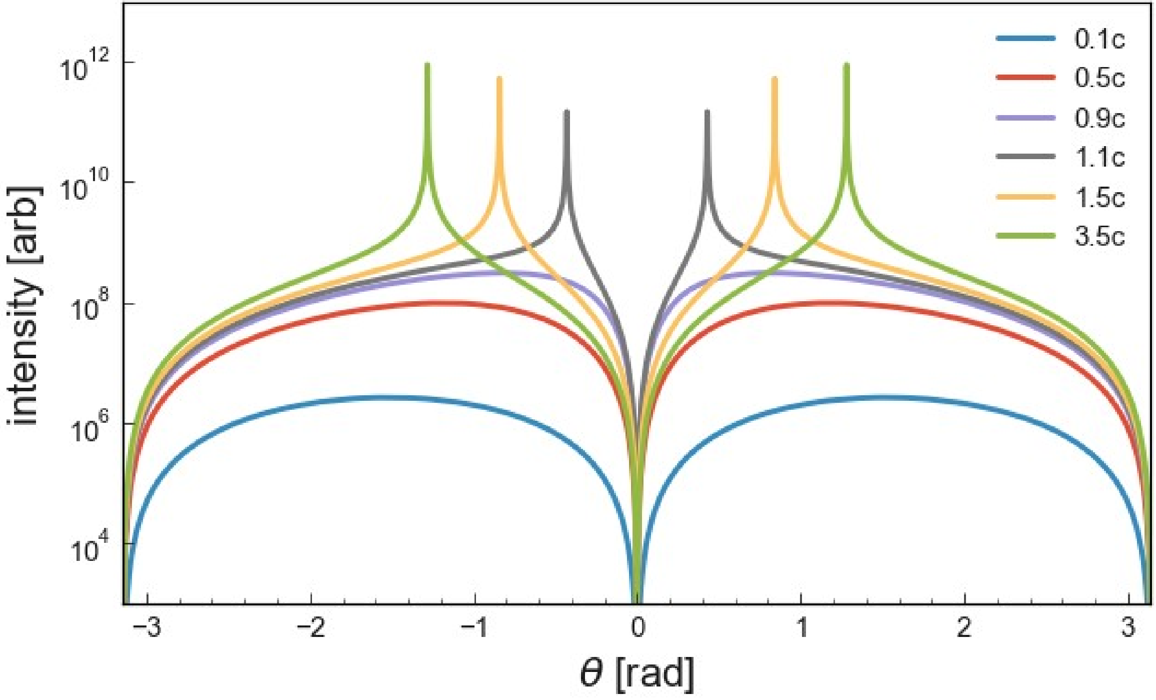
Also, I don't have time so...
$\rightarrow$ backup!



Cherenkov radiation in a nutshell

Master formula nr. 1 : **The radiation lays in a cone, whose aperture depends on β**

$$\cos\theta = \frac{1}{\beta n(\lambda)}$$



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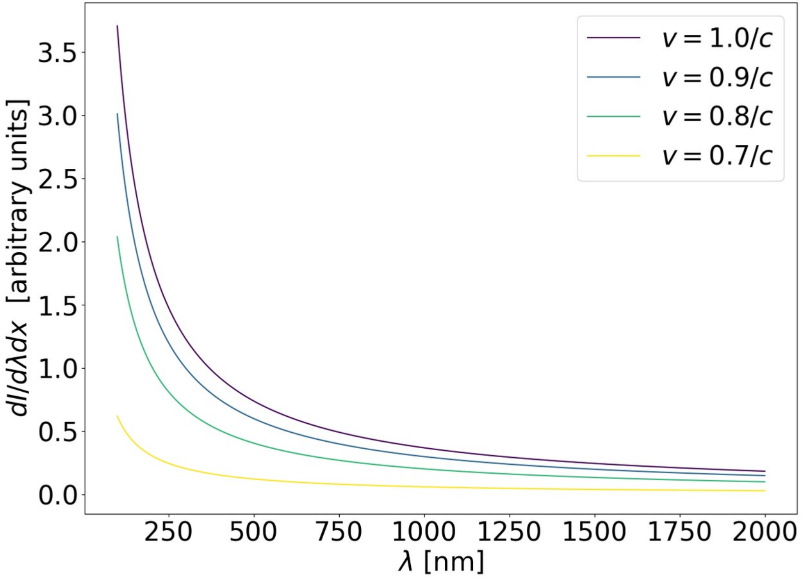
Master formula nr. 1 : **The radiation lays in a cone, whose aperture depends on β**

$$\cos\theta = \frac{1}{\beta n(\lambda)}$$

Master formula nr. 2: **The Spectrum is smooth, in the visible but peaked in the violet**

$$\frac{dI}{dx d\lambda} = e^2 \frac{n(\lambda)}{c_0}\left(1 - \frac{1}{\beta n(\lambda)}\right)$$



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Master formula nr. 3: The radiation faint: O(100) photons per cm of water!!

$$N = 2\pi\alpha z^2\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\left(1 - \frac{1}{\beta^2 n^2}\right) = 2\pi\alpha z^2\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)(1 - \cos^2\theta)$$

Let's built a Cherenkov detector

What do you need to build a Cherenkov detector for PID?

Let’s built a Cherenkov detector

A Cherenkov detector (for PID) has 4 components:

$\rightarrow$ Radiator, where the Cherenkov effect happens

$\rightarrow$ An optics that collects and convey the Cherenkov photon to detection

$\rightarrow$ A photomultiplier

$\rightarrow$ A readout electronics

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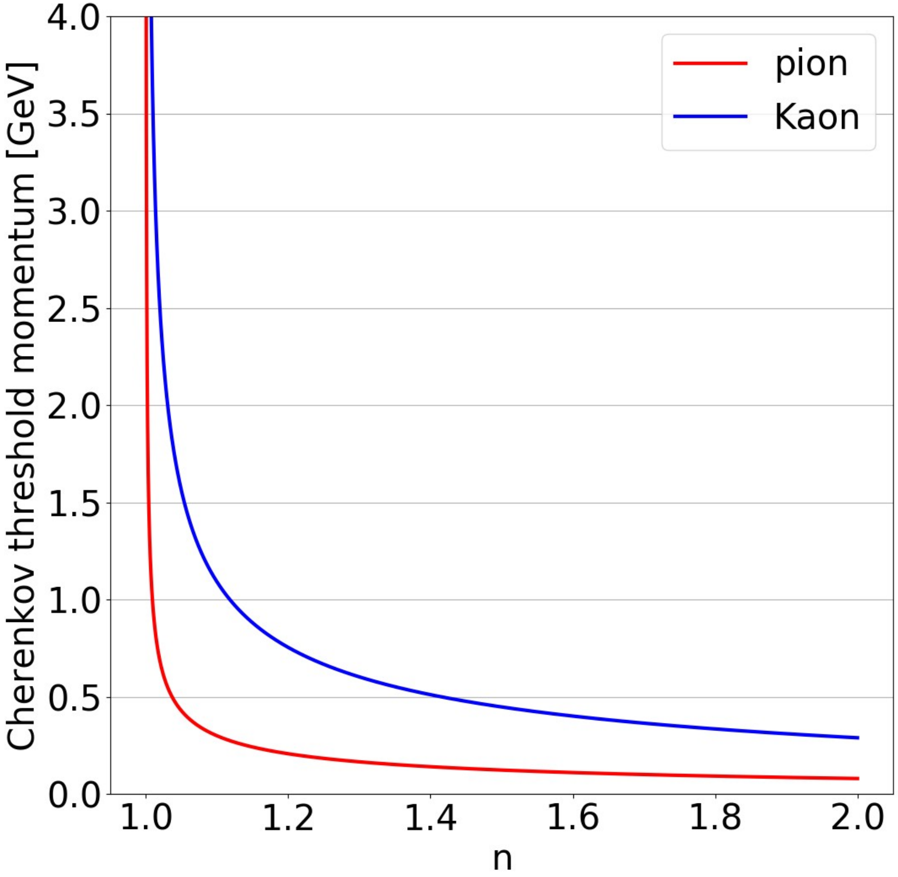
Let's assume you have:
$\rightarrow$ **one** photomultiplier
$\rightarrow$ a beam of particles of $p = 3$ GeV.
Some of them are pions, some of them are kaons. How do you decide when you have one and when you have the other?

Choose your radiator

The easiest Cherenkov counter is a **threshold counter**

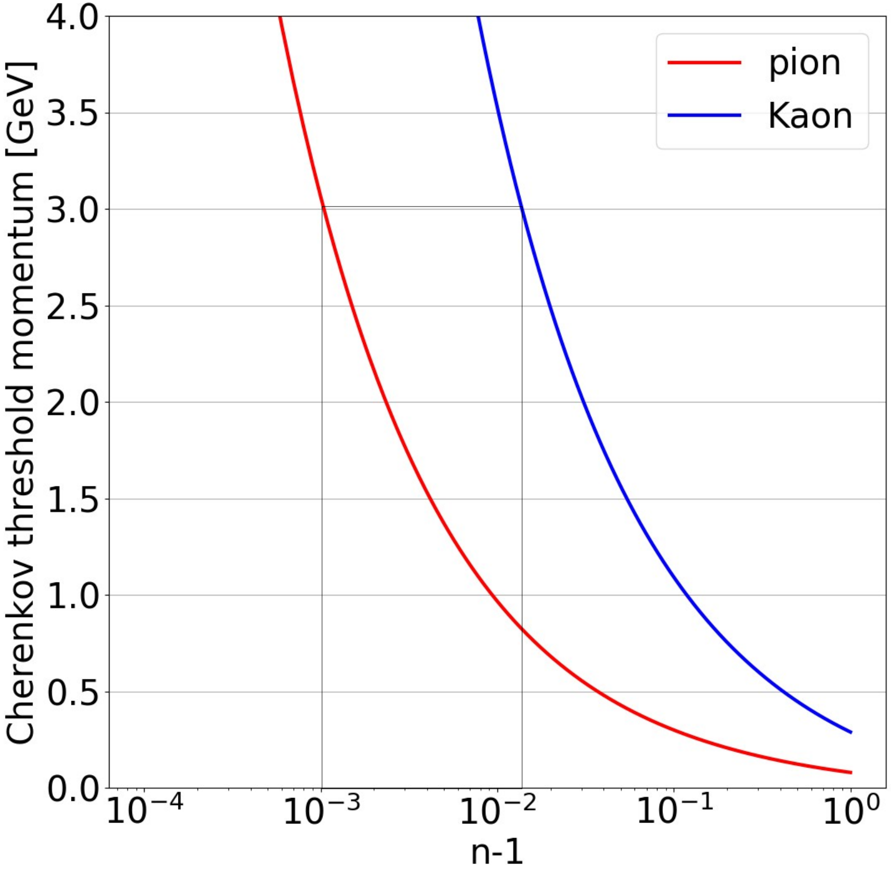


Gases (NTP)	$(n\text{-}1)10^6$ (at 7eV)	N_{PH} /cm
He	33	0.005
Ne	67.3	0.01
Ar	300	0.05
CH_4	510	0.08
CF_4	488	0.08
C_2F_6	793	0.13
i-C_4H_{10}	1500	0.24
Liquids (NTP)	***n* (at 6.5 eV)**	
C_5F_{12}	1.262	30
C_6F_{14}	1.278	31
Solids	***n* (5.5 eV)**	
LiF	1.42	43
MgF_2	1.41	40
CaF_2	1.47	43
Fused silica	1.52	45

Aerogel: an value between 1. and 1.1

Choose your radiator

Isobuthane seems to be our guy



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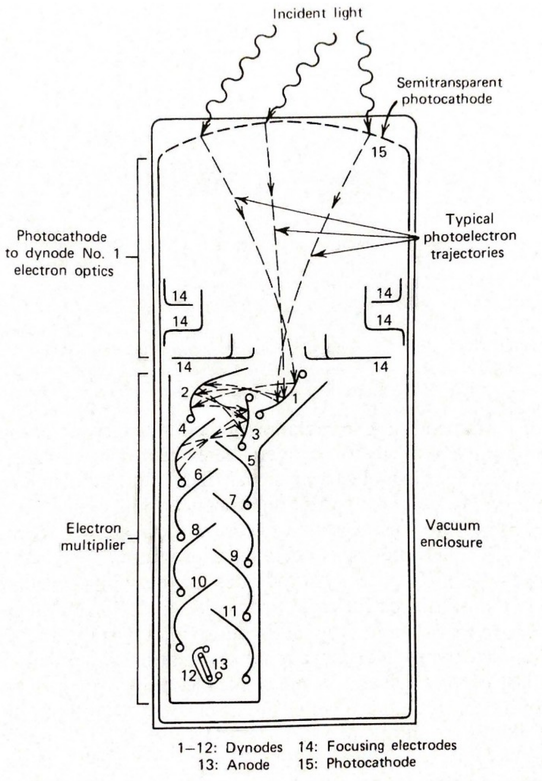
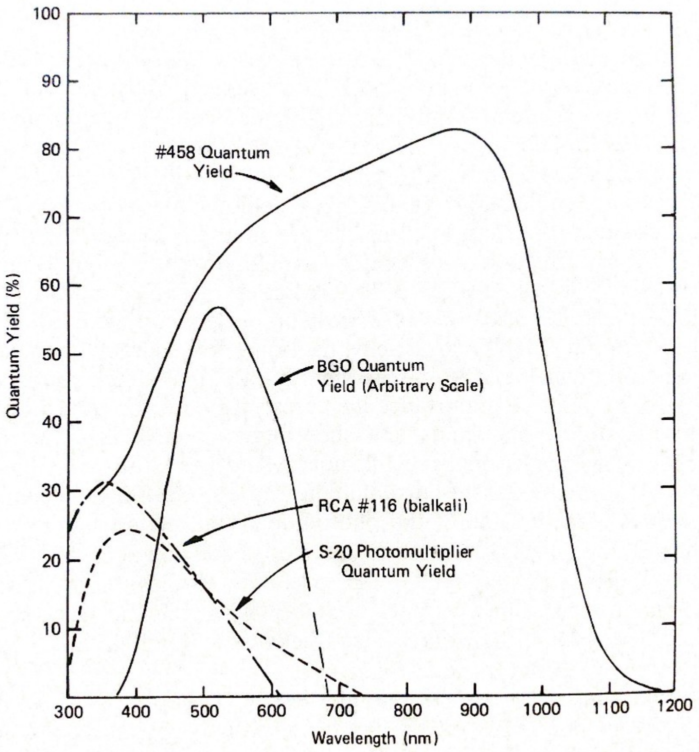
Choose your PMT

PMTs are a very broad topic

→ No time to really dig into it here (*see Knoll, 4th edition, pp. 275 - 318*)

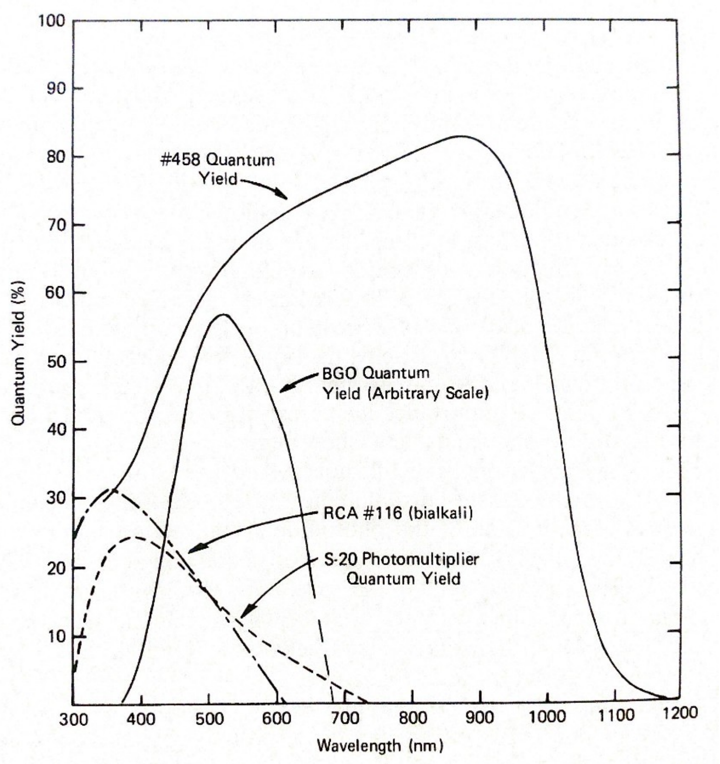
→ For the moment, let's assume we have a rather **conventional PMT**





Dimension your detector

Assume we have a 15% average Q.E. between 300 nm and 500 nm



$$N = 2\pi\alpha z^2\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)\left(1-\frac{1}{\beta^2 n^2}\right) = 2\pi\alpha z^2\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)(1-\cos^2\theta)$$

In our case:

z = 1

$\cos\theta$ = 0.999542865

N = 0.6 photons / cm

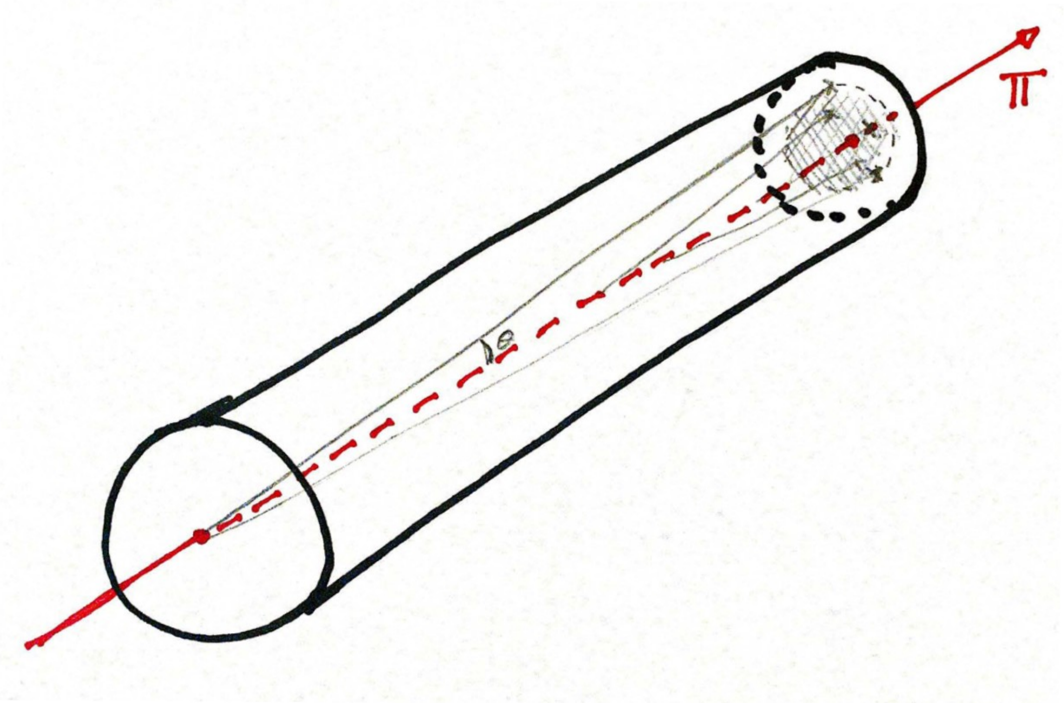
signal ~ 0.1 photoelectrons / cm

→ your detector must be ~ 1 m long.

From now on, let's assume 1 m

Choose the PMT location

The Cherenkov light emitted by our particle project within a disk of $\sim$ 9cm of radius at the end of your pipe full of isobuthane.
$\rightarrow$ Where do we put our PMT?

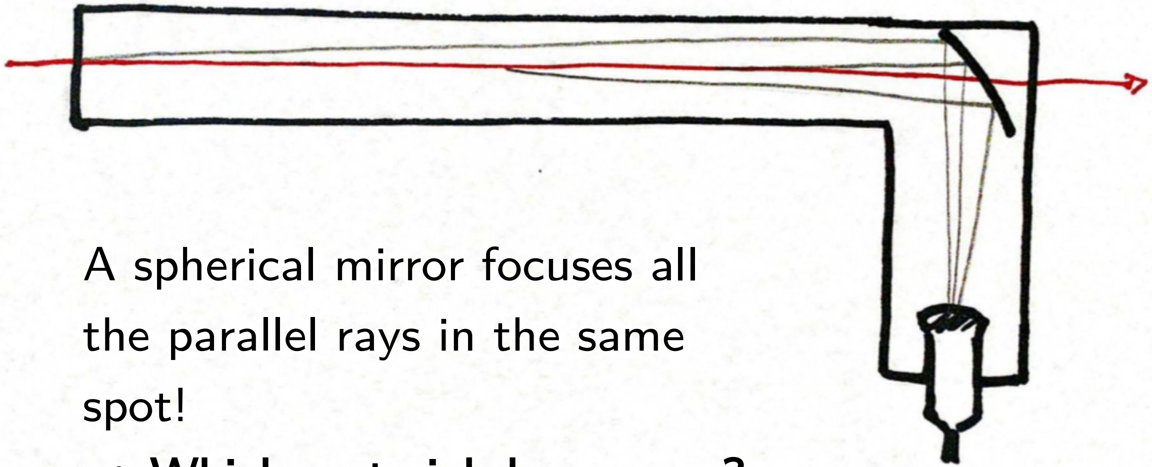


- $\rightarrow$ We probably do not have a 6in PMT
- $\rightarrow$ We don't want the beam to drill a hole through the PMT itself

Choose the optics

The Cherenkov light emitted by our particle project within a disk of $\sim$ 9cm of radius at the end of your pipe full of isobuthane.

$\rightarrow$ Where do we put our PMT?



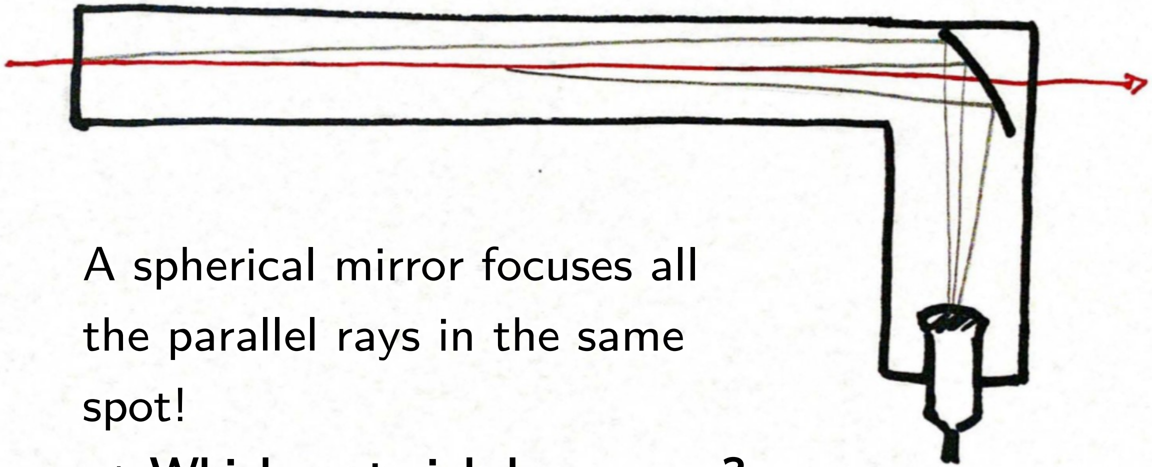
A spherical mirror focuses all the parallel rays in the same spot!

$\rightarrow$ **Which material do you use?**

Choose the optics

The Cherenkov light emitted by our particle project within a disk of ~ 9cm of radius at the end of your pipe full of isobuthane.

→ Where do we put our PMT?



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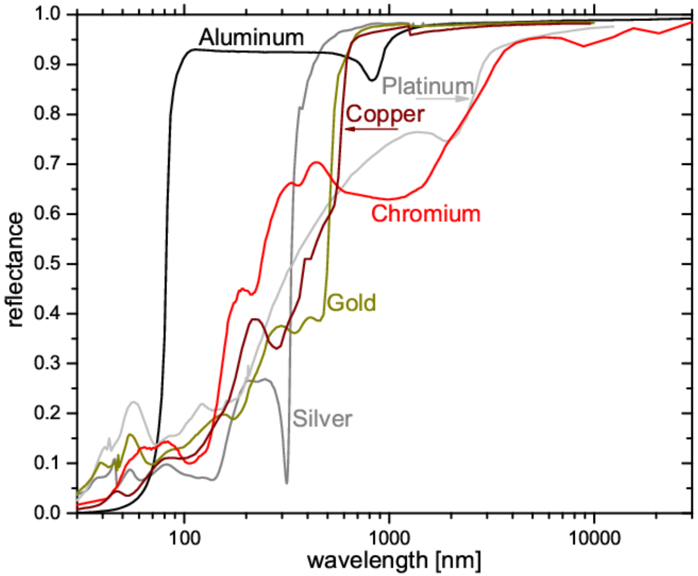

Performances

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Few metrics are used to characterize the performances of a PID detector

$\rightarrow$ **Efficiency: ability to correctly assign the ID**

$\varepsilon(\mathrm{K}) = \mathsf{N}(\mathrm{K}\ \text{identified as}\ \mathrm{K})/\mathsf{N}(\text{real}\ \mathrm{K})$

Equal, by definition, to the "probability of a kaon to be called kaon"

$\rightarrow$ **Mis-ID: ability not to assign the incorrect ID**

$\text{Mis-ID}(\mathrm{K}) = \mathsf{N}(\text{non-}\mathrm{K}\ \text{identified as}\ \mathrm{K})/\mathsf{N}(\text{non}\ \mathrm{K})$

Equal, by definition, to the "probability for a non-kaon to be called kaon"

$\rightarrow$ **Fake rate: fraction of particles with the wrong ID**

$\mathsf{F}(\mathrm{K}) = \mathsf{N}(\text{non-}\mathrm{K}\ \text{identified as}\ \mathrm{K})/\mathsf{N}(\text{identified as}\ \mathrm{K})$

Equal, by definition, to the "fraction of non-kaons in my collection of kaons"

Performances

In our case:

→ There is Cherenkov light: pion

→ There isn't Cherenkov light: kaon

→ We expect 10 p.e./pion on average

Performances

In our case:

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The efficiency for pion selection is determined by the Poisson statistics.

Let's require to have at least 3 p.e. to declare to have a signal:

→ $P(n < 3) \sim 1$ %

→ There is 1% of chances to miss a pion (i.e. a 1% of chance to call kaon a pion)

Mis-ID probability (K) ~ 1%

→ There is no chance of missing a kaon (no signal) [Question: is it really true?]

ε(K) ~ 100%

Fake rates

The fake rate is (to a certain extent) a Bayesian idea

Given that I have something that looks like a kaon, what are the chances for this to really be a kaon and not a pion?

Let's assume to have 2% of kaons and 98% pions in the beam

Bayes theorem:

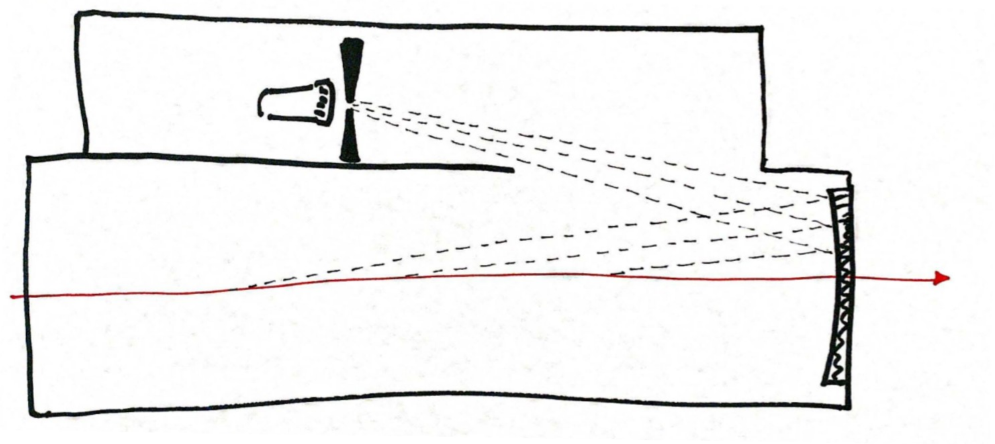
P(my "kaon" is kaon) = 1 x 0.02 / (1 x 0.02 + 0.01 x 0.98) ~ 67%

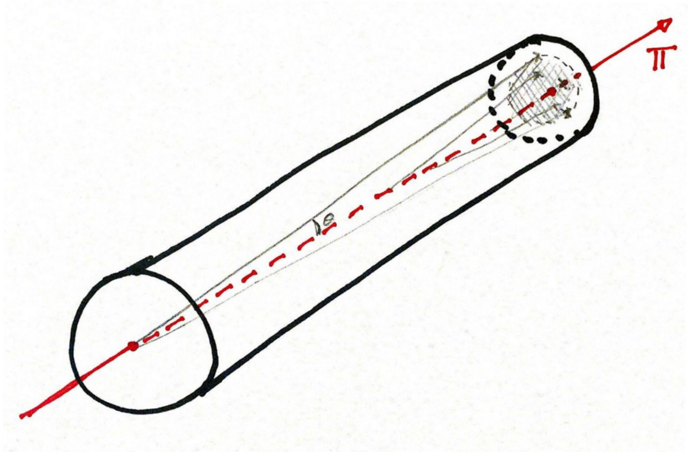
Fake rate = 33%

Priors matter!

From angles to radii

Let's go back to the differential counter





The spherical mirror focalizes the photons according to their angle.

→ A given angle value becomes a length in the focal plane

→ The DISC of Cherenkov photons becomes a RING in the focal plane!

→ **If only we could reconstruct the ring...**

The RICH idea

To reconstruct a ring we must do imaging

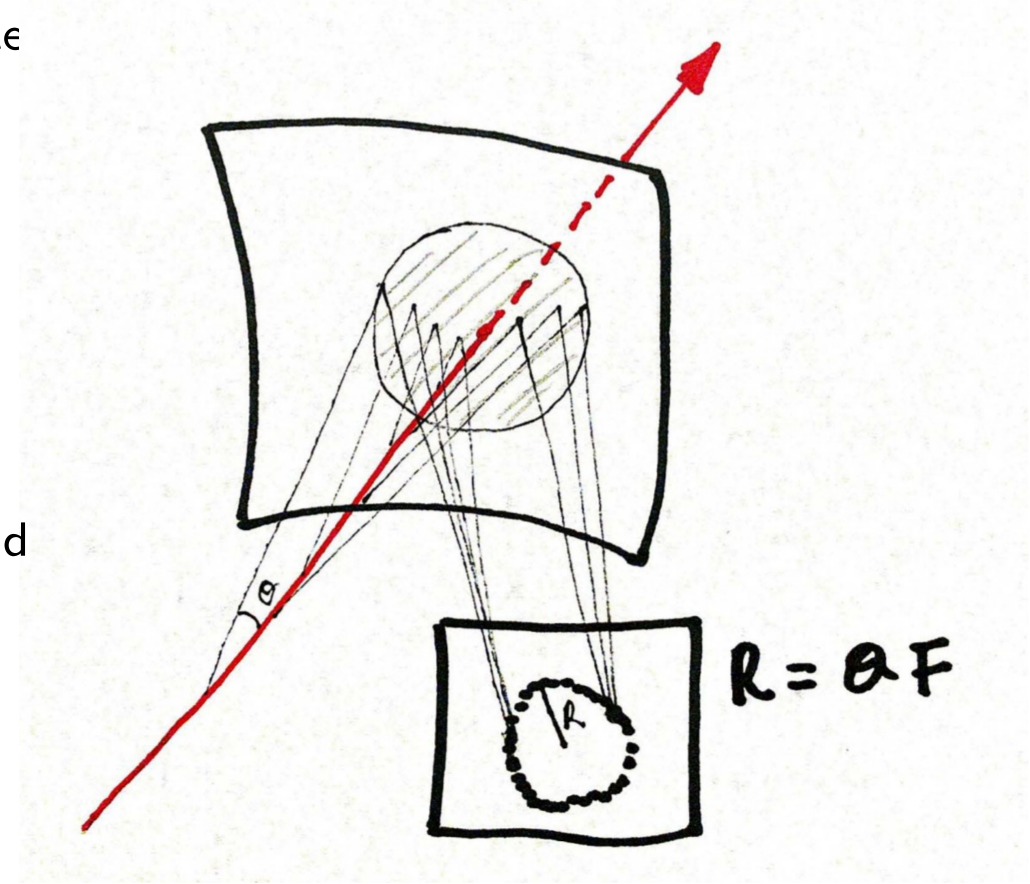
One big PMT (i.e. one big pixel) is not enough

PMT $\rightarrow$ PMT array, Gas photon dete

Resolution

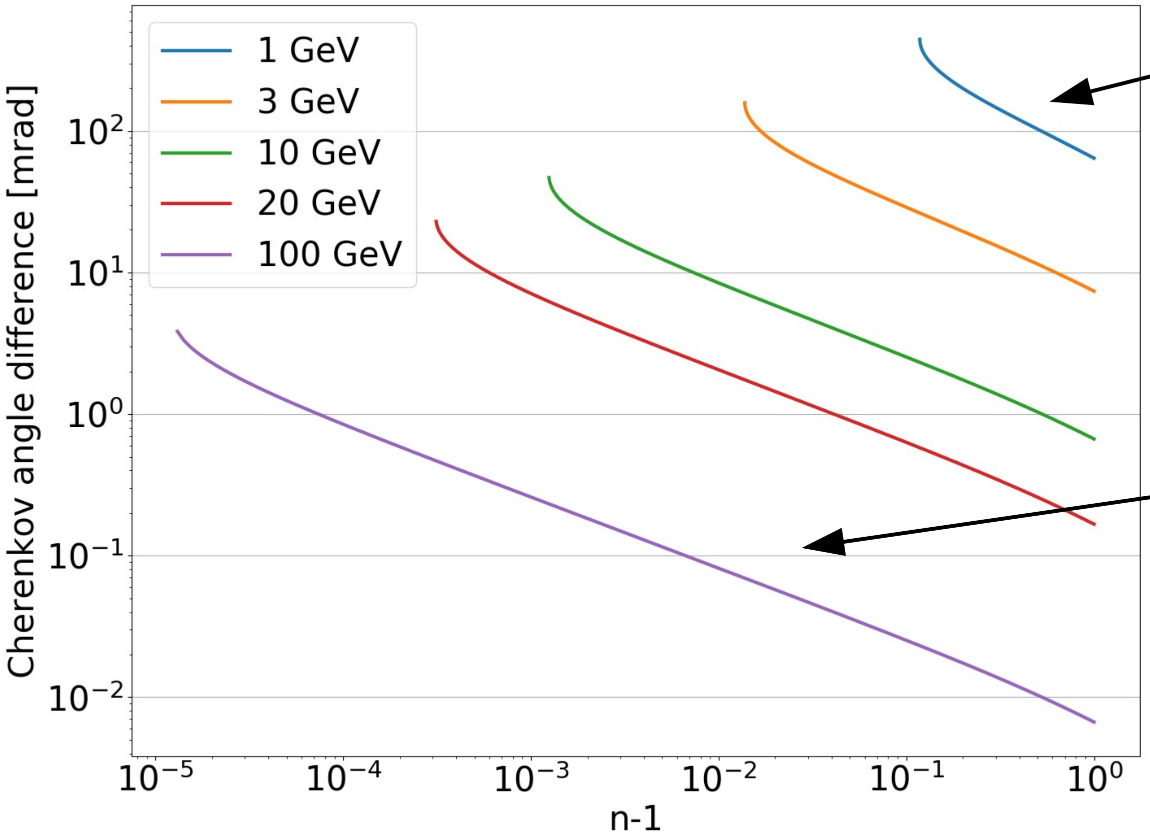
$$\frac{\Delta\beta}{\beta} = \tan\theta\Delta\theta = \tan\theta\frac{\overline{\Delta\theta}}{\sqrt{N}}$$

Typical angular resolution: ~10-30 mrad



Back to the radiators

Give that now we really measure the Cherenkov angle, we should perhaps reconsider the choice of the radiator



Low momenta

Large n $\rightarrow$ solids

Many photons

Small detector

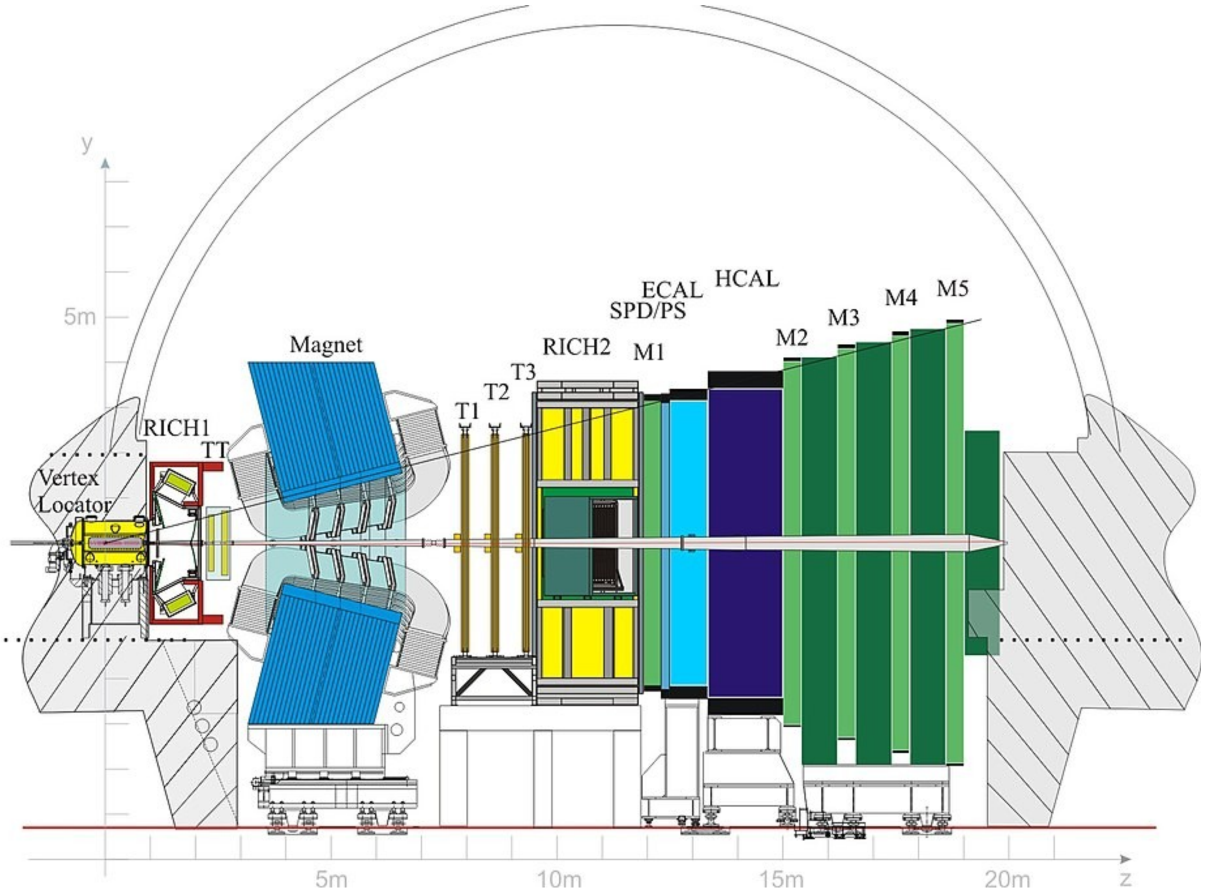
High momenta

Low n $\rightarrow$ gas

Few photons

Huge detector

RICH examples: LHCb



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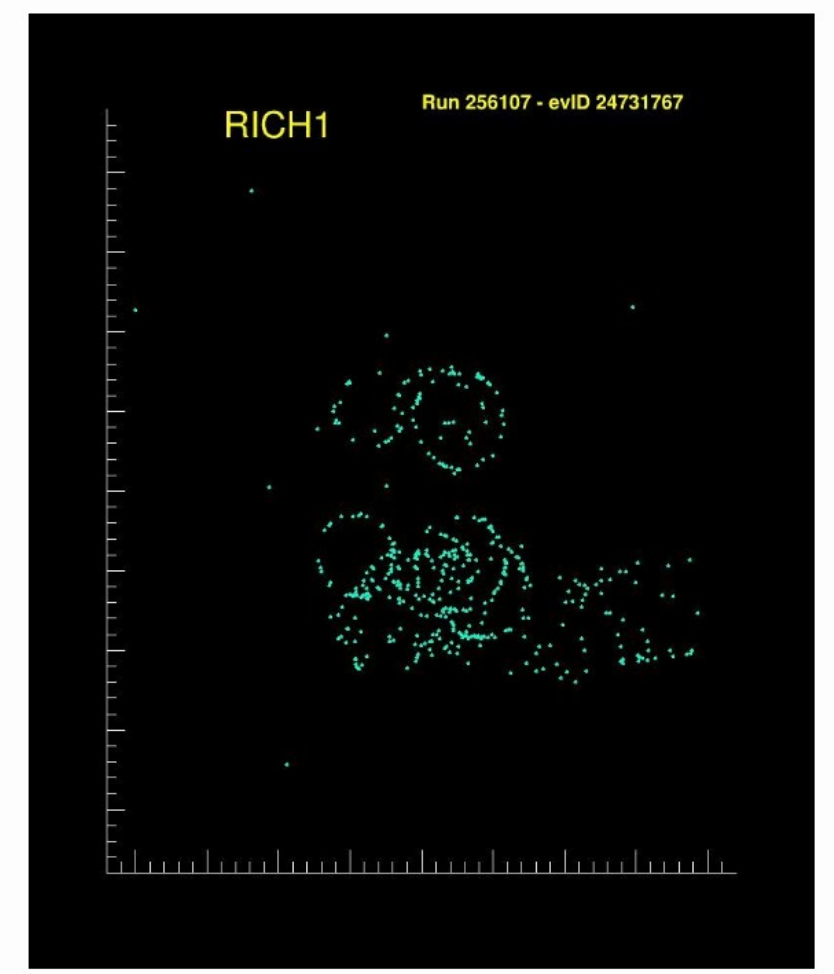
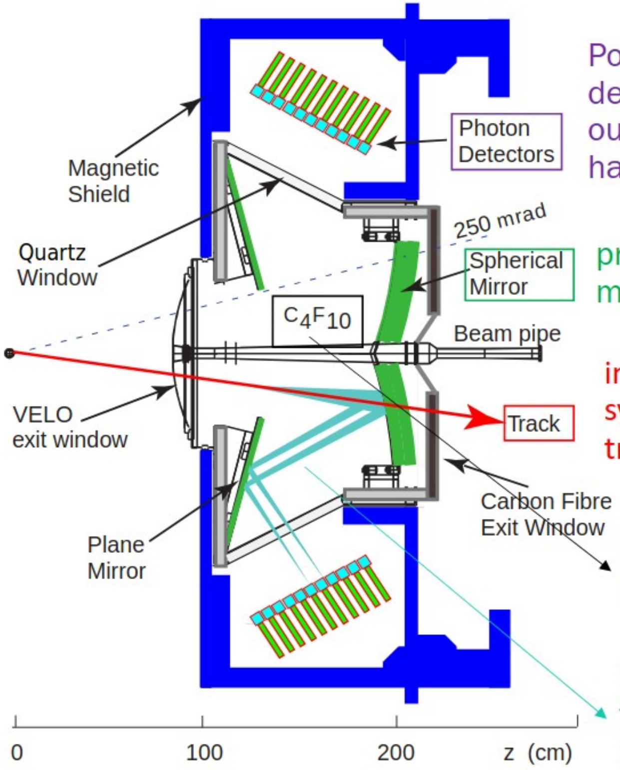




Position sensitive single-photon detectors and frontend electronics outside the acceptance, radiation hardness

provide focussing with minimal material budget

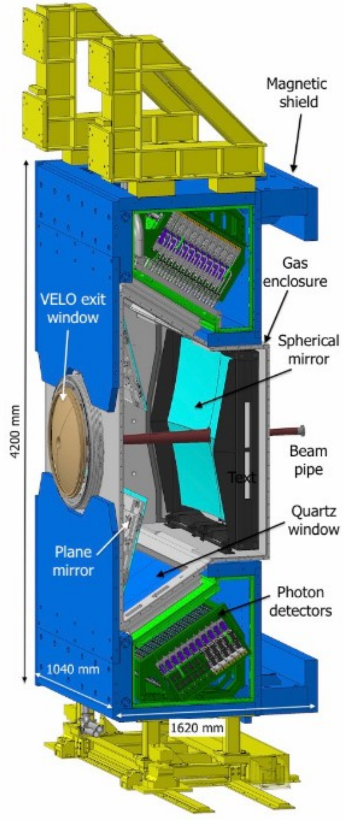
information from the tracking system: trajectory/curvature of track and momentum estimate

Radiator refractive index tuned to match momentum range

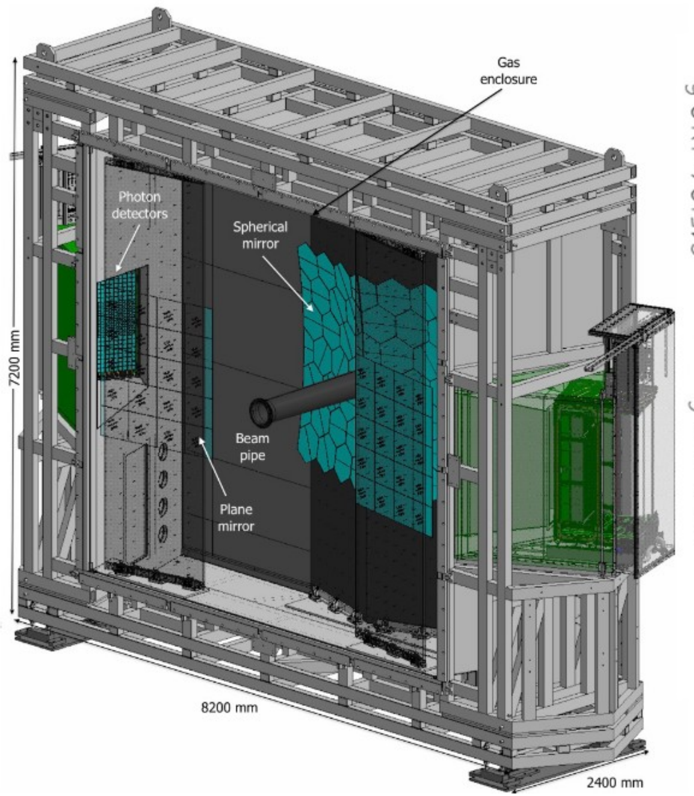
Large radiator volume to maximise track optical paths and therefore number of Cherenkov photons (N_{ph})

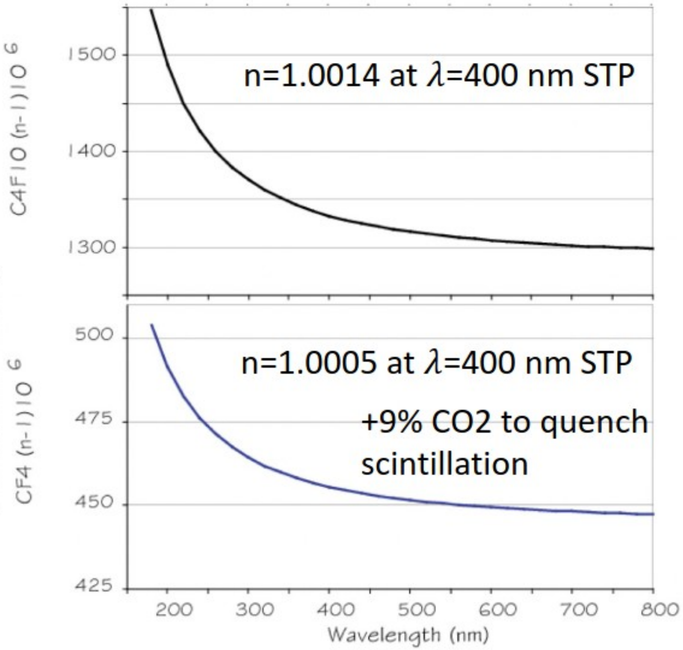
RICH examples: LHCb

RICH1, ~ 4 m^3, upstream, 8% X0
3-40 GeV over 25-300 mrad



RICH2, ~ 100 m^3, downstream, 15% X0
15-100 GeV over 15-120 mrad

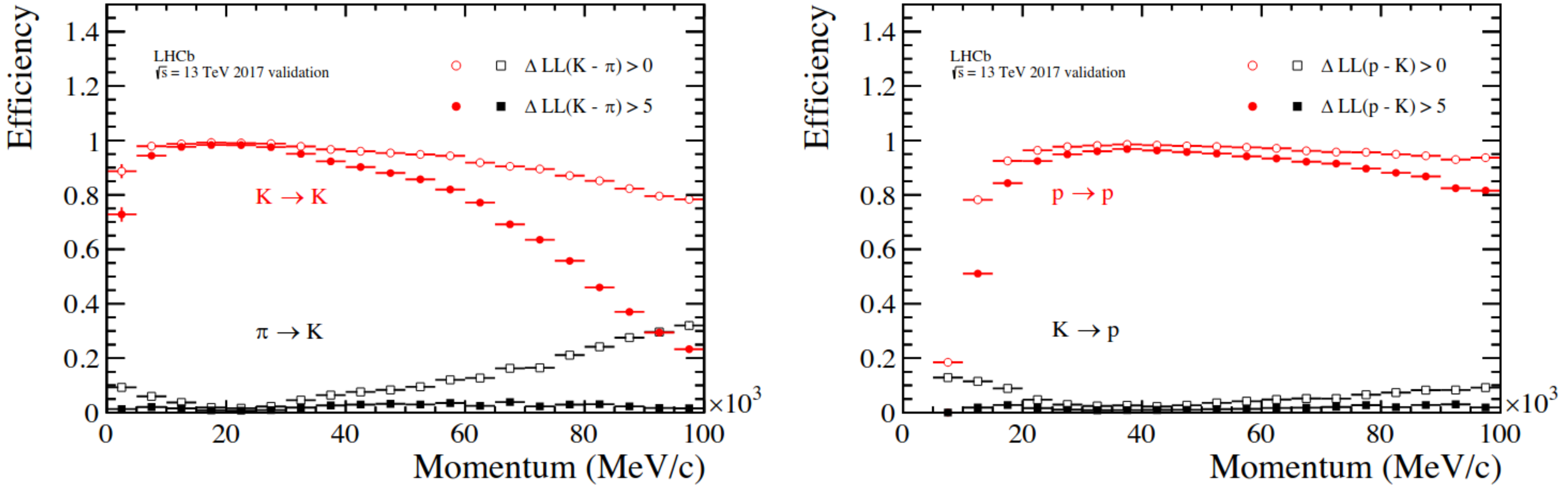




RICH examples: LHCb

- The first generation of the LHCb RICH detectors operated in Run 1 and 2 with an excellent performance
- RICH1: $\sigma_c = 1.662 \pm 0.023$ **mrad,** $N_{ph} = 30 \pm 2$
- RICH2: $\sigma_c = 0.621 \pm 0.012$ **mrad,** $N_{ph} = 18.5 \pm 1.2$

[JINST 17 (2022) 07, P07013]



RICHs for low momenta

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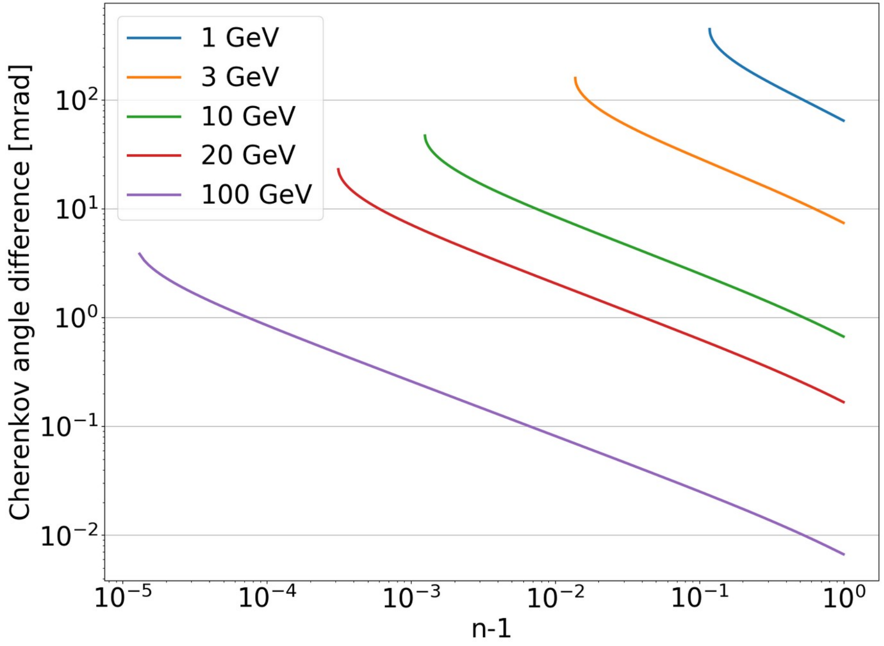
Let's go back to the beginning. So far we saw detectors for

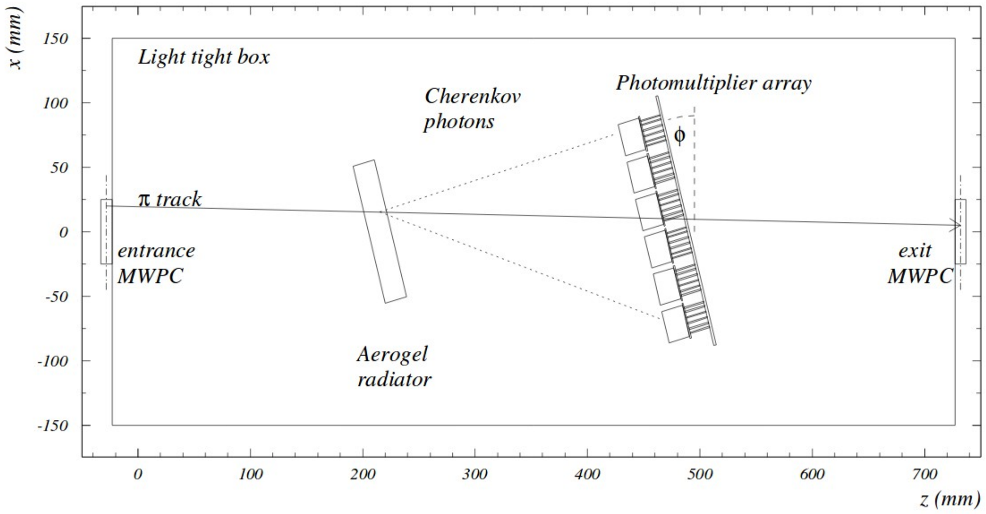
→ High momentum particles

→ Single arm, forward experiments (lots of space!)

→ What if we want to have a RICH in a barrel geometry (little space) and for low momenta (~1- 4 GeV)?



Proximity focusing!



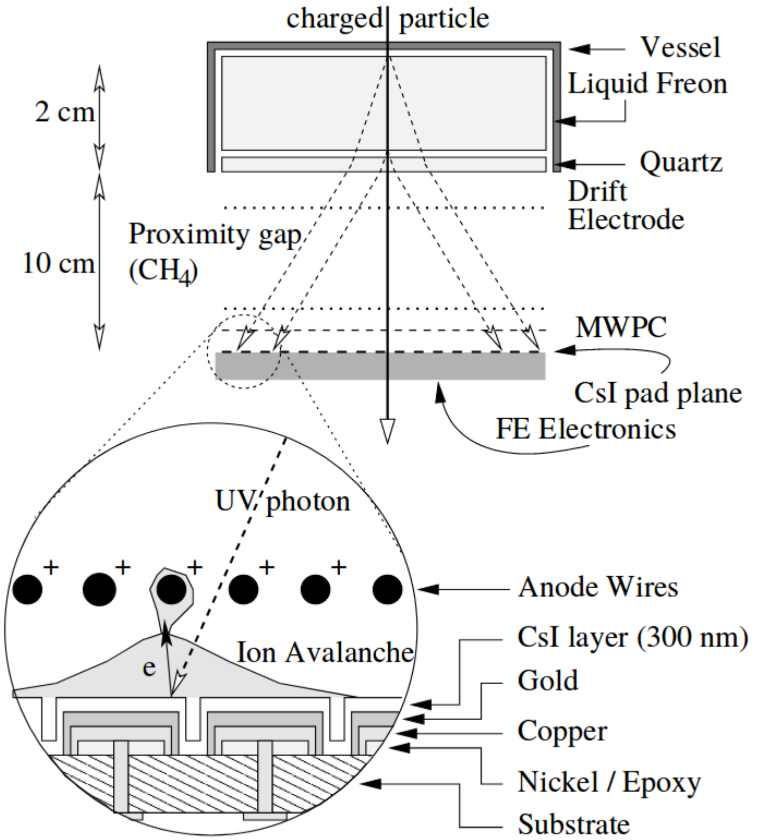
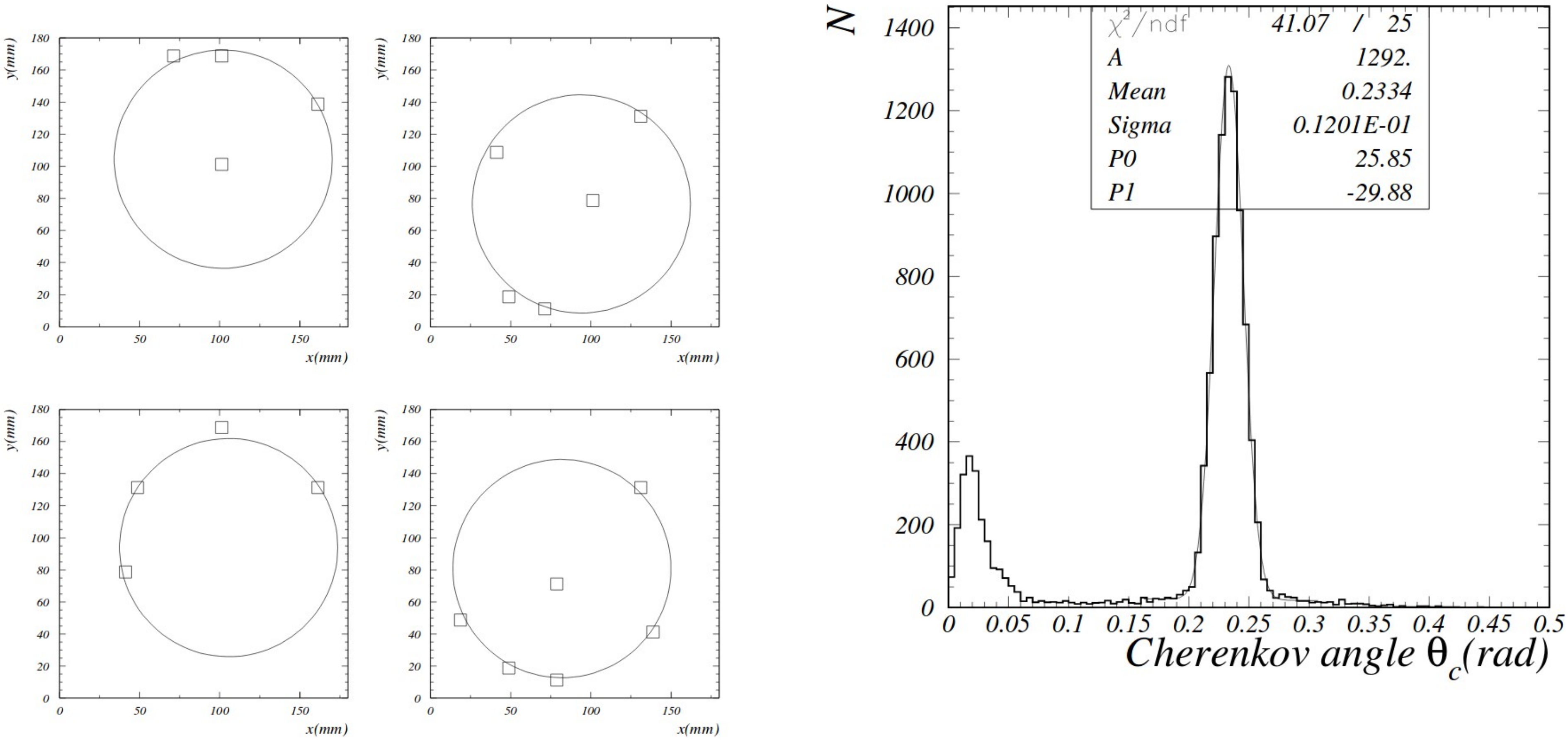


Fig. 1. Pictorial view of the JLab RICH: Cherenkov emission, propagation, photon conversion and charge collection.

Rings in a proximity focusing RICH



Few words about resolution

In general, the resolution of any Cherenkov device is factorized:

Total angular resolution

$$\sigma_\theta^{tot} = \frac{\sigma_\theta^{stat}}{\sqrt{N_\gamma}} \oplus \sigma_\theta^{corr}$$

Everything related to the photons (detection, production, propagation...)

Correlated part: tracking, alignment...

Few words about resolution

In general, the resolution of any Cherenkov device is factorized:

Total angular resolution

Everything related to the photons (detection, production, propagation...)

$$\sigma_\theta^{tot} = \frac{\sigma_\theta^{stat}}{\sqrt{N_\gamma}} \oplus \sigma_\theta^{corr}$$

Photon production

Photon detection

Chromatic dispersion

$$\sigma_\theta^{tot} = \frac{\sigma_\theta^{radiator} \oplus \sigma_\theta^{detector} \oplus \sigma_\theta^{chrom}}{\sqrt{N_\gamma}} \oplus \sigma_\theta^{corr}$$

Chromatic dispersion

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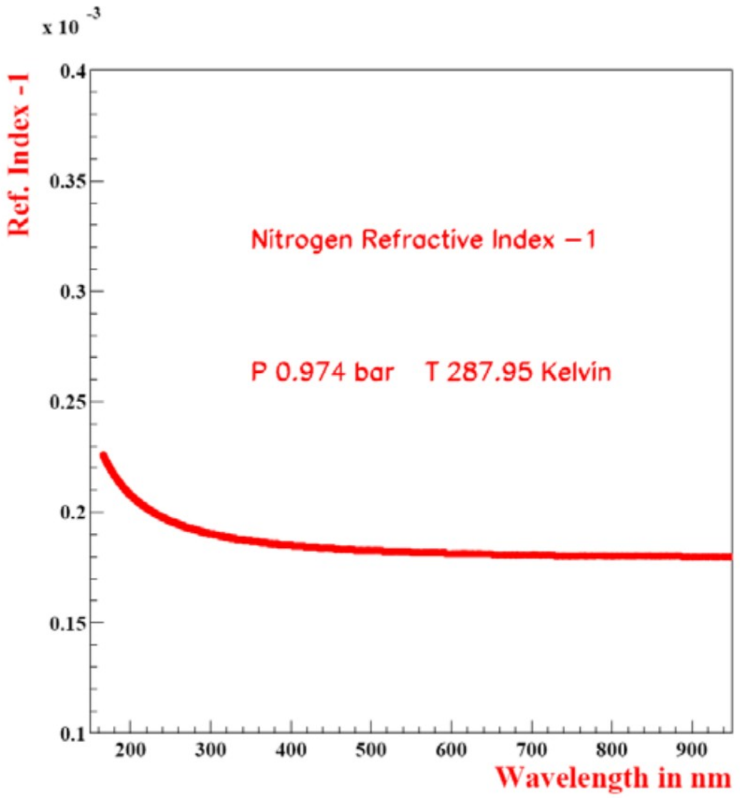
Chromatic dispersion quickly becomes an issue when you do imaging

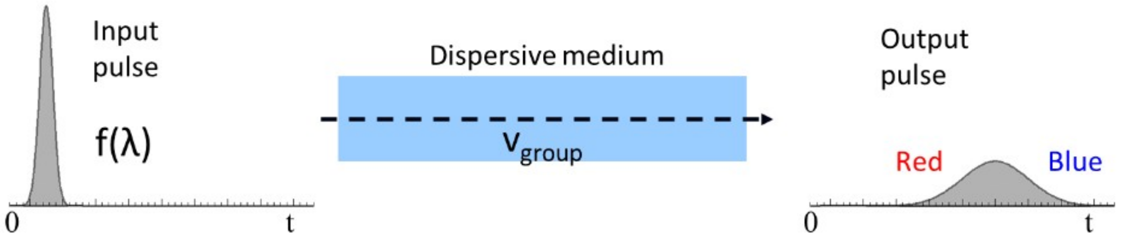


Particles with different velocities can emit photons of different wavelengths at the very same angle

→ Wavelenght filters

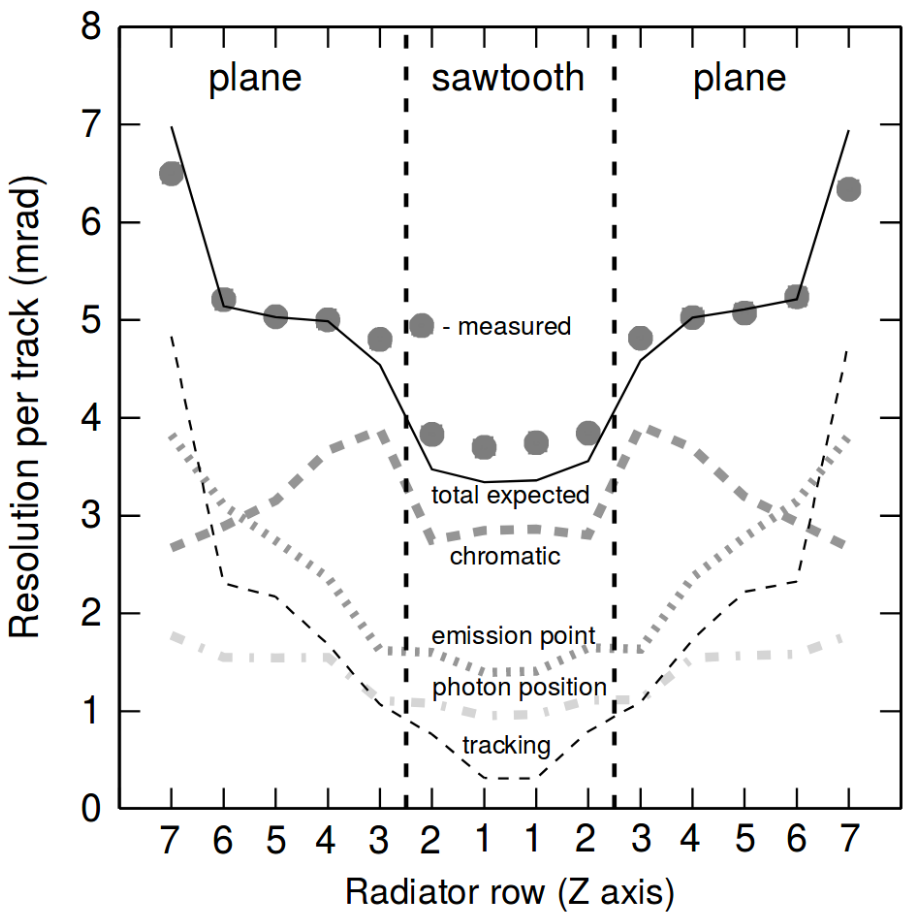
→ Corrective optics

→ (more recently) photon time!



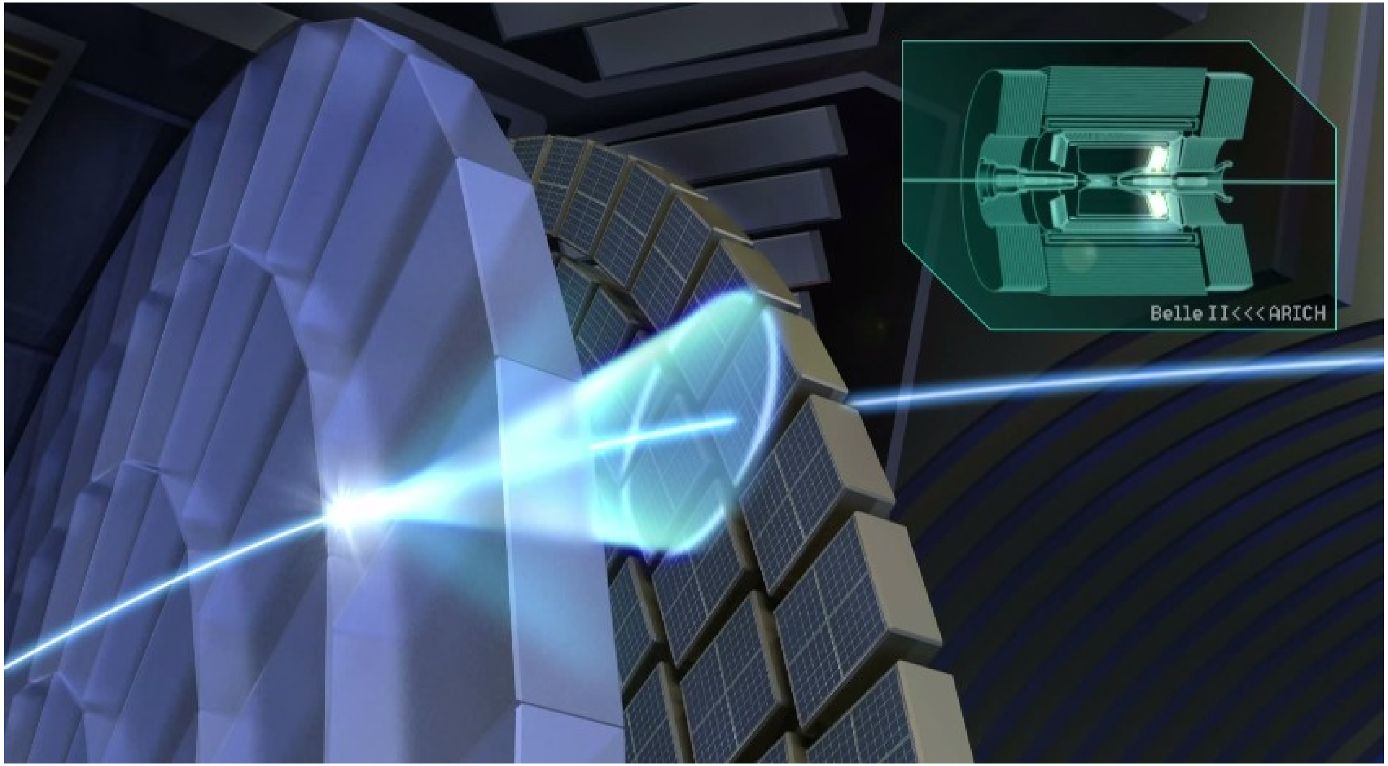
An error budget example: CLEO

The error budget of a proximity focusing RICH is rather different from the one of a focusing RICH



Drawbacks of proximity focusing

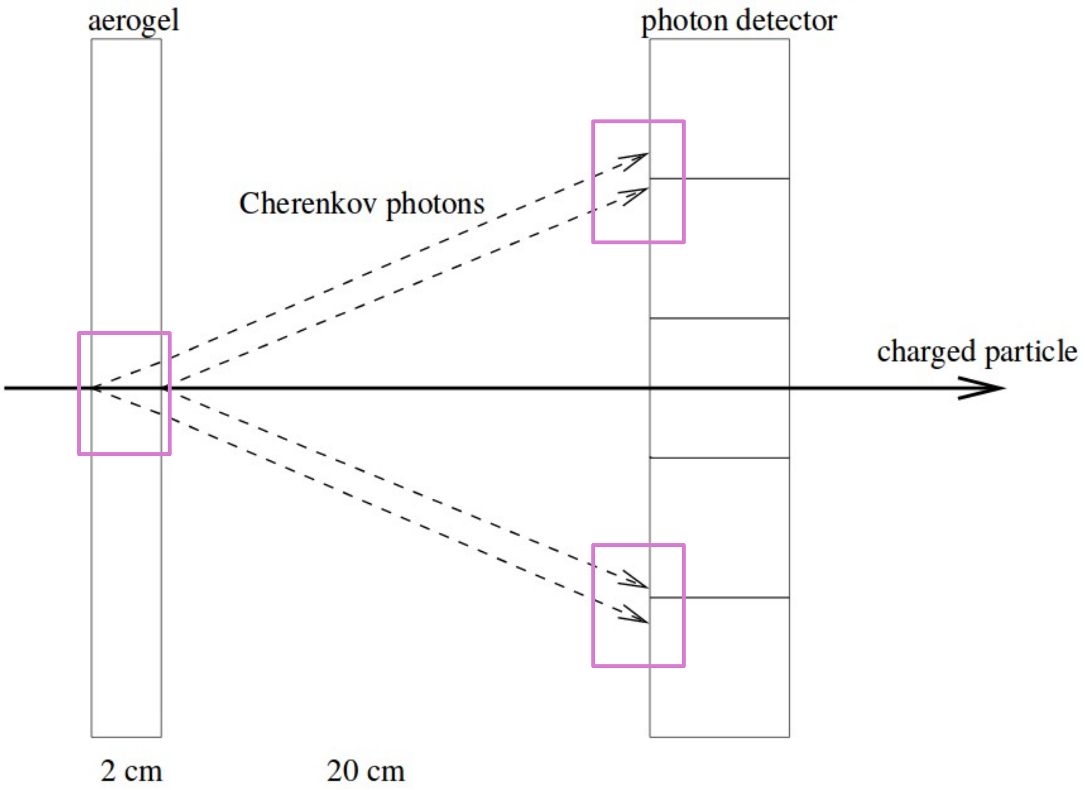
1) The rings are not always circles
   → The incident track angle project and ellipse rather than a circumference



Drawbacks of proximity focusing

1) The rings are not always circles
   → The incident track angle project and ellipse rather than a circumference

1) Thick radiators blur the image
   → How can you mitigate this?

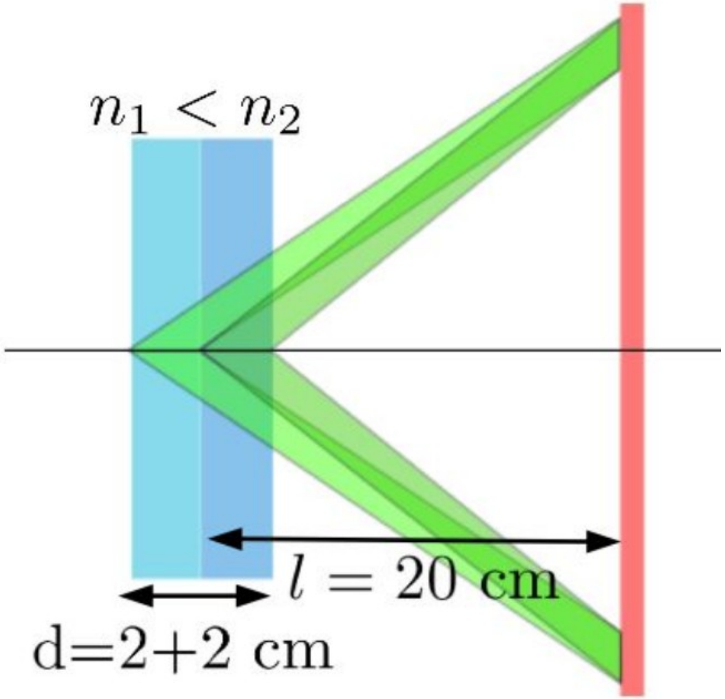


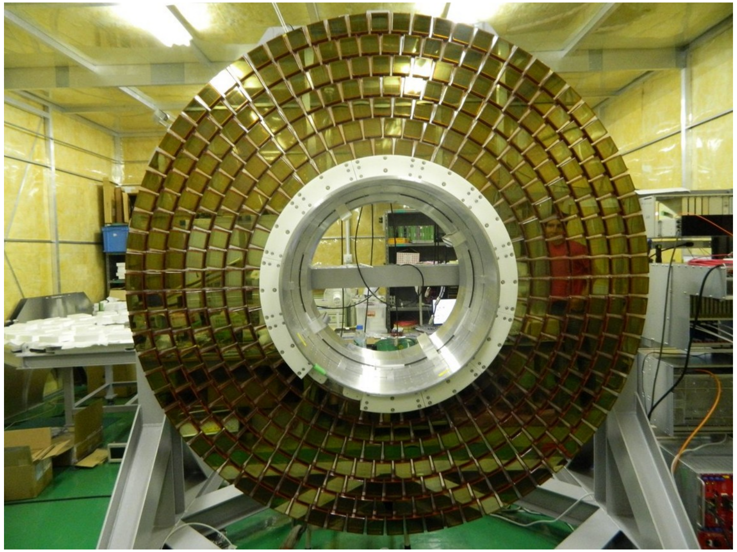
Dual radiators: the Belle II ARICH

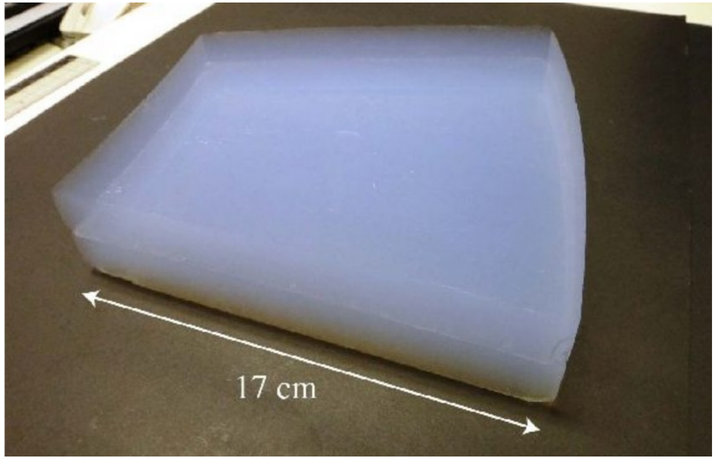
Dual radiator design

$\rightarrow$ Twice the photons, $\sim$ same ring size

$\rightarrow$ endcap PID for the Belle II experiment





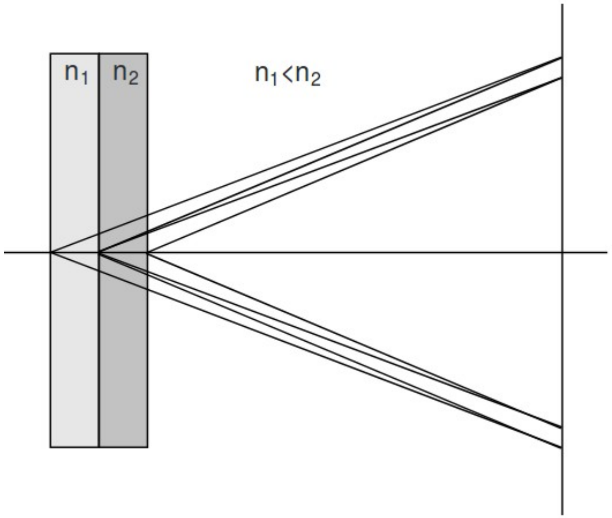


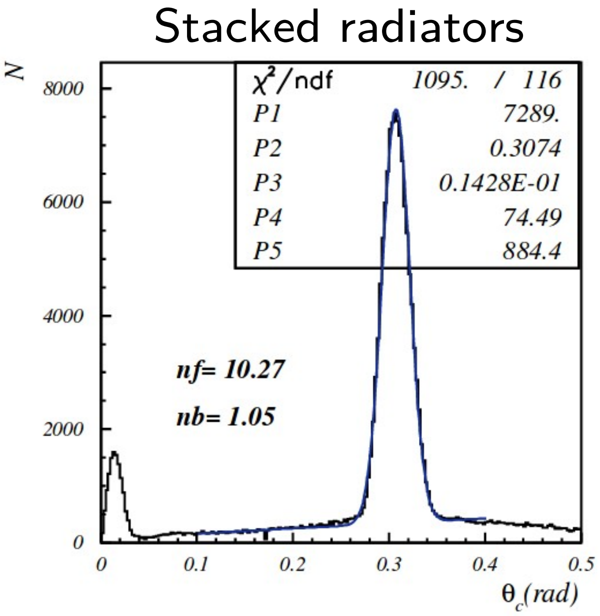
Dual radiators: the Belle II ARICH

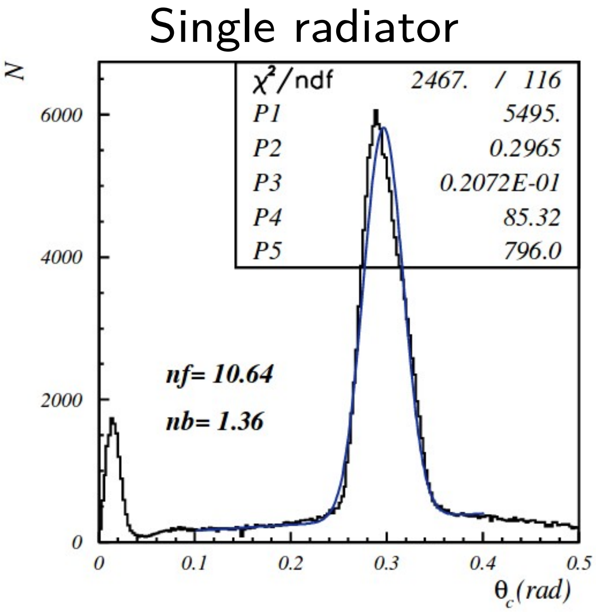
Dual radiator design

→ Twice the photons, ∼ same ring size

→ it actually works!



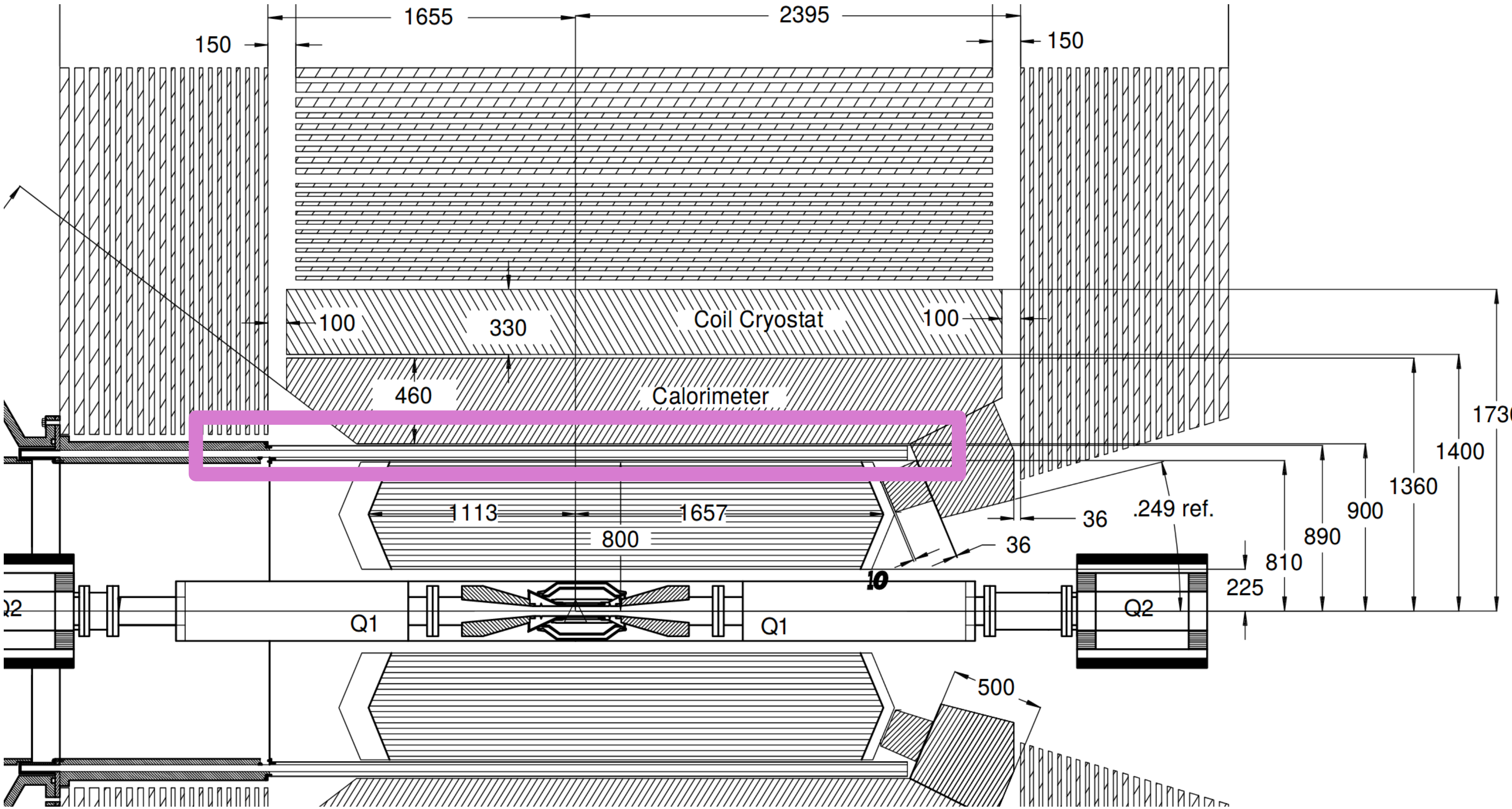




Total internal reflection counters

Solve the problem

This is your detector.



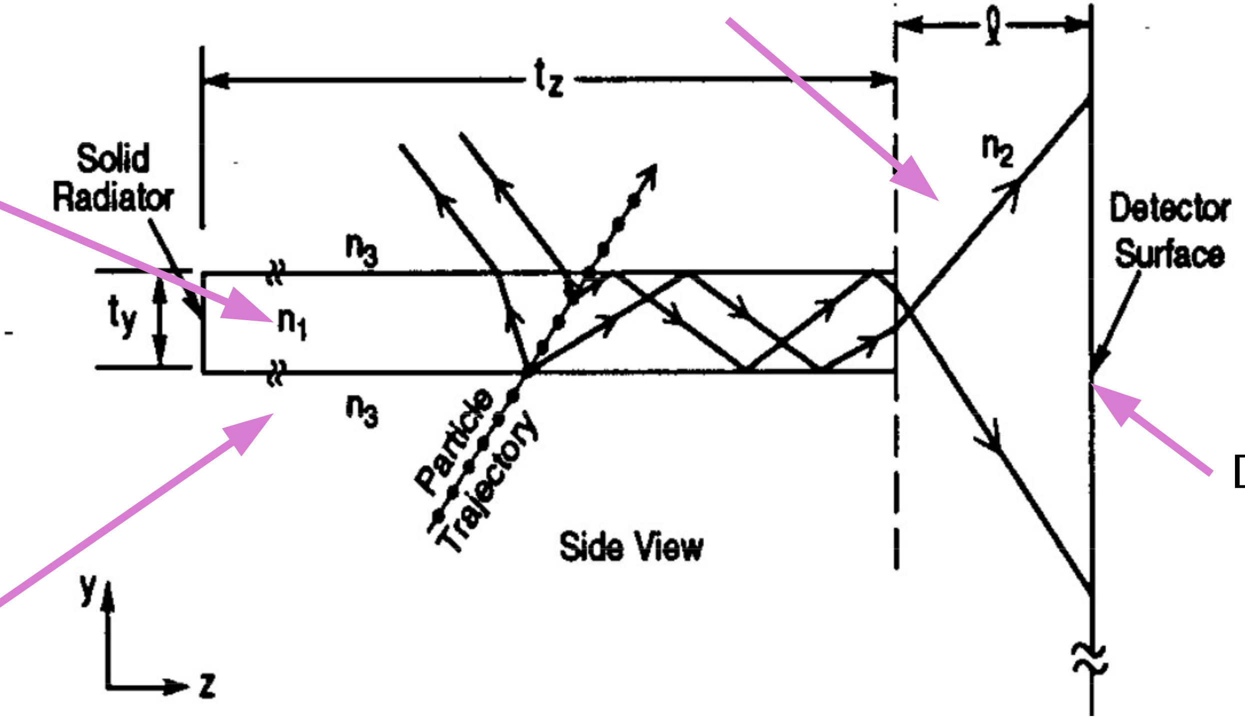
→ You have 7 cm to fit your PID detector for K/pion separation up to 3 GeV

→ We saw that we need ~ 5 cm of solid radiator and ~ 1 m of focusing length

General structure of a DIRC

Standoff area where the light path expands

Radiator/light guide. Must be made of a high-n material

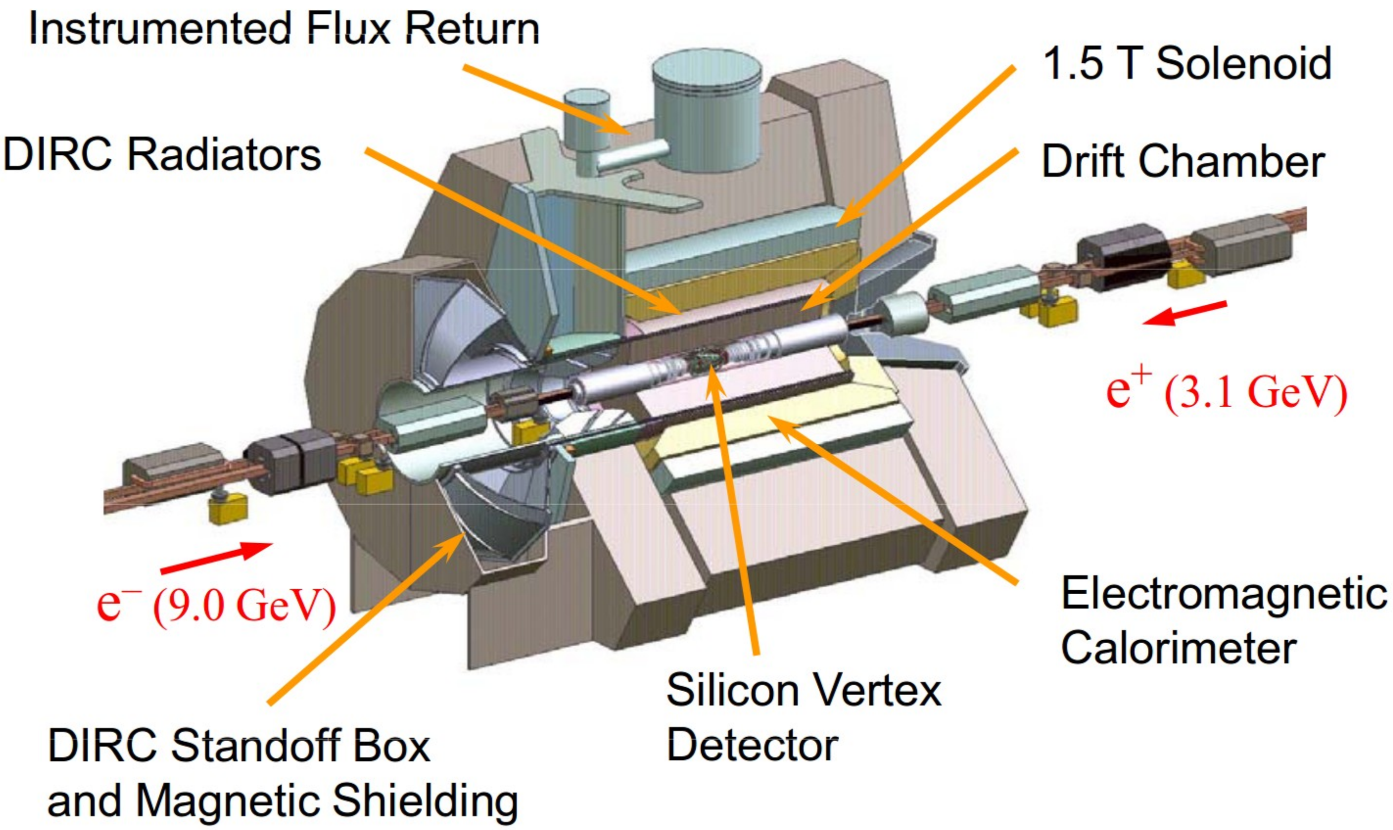


Detector plane

Air (or gas) to provide total reflection in the bar

A real-life DIRC

The only DIRC ever built was installed in the BaBar experiment



A real-life DIRC

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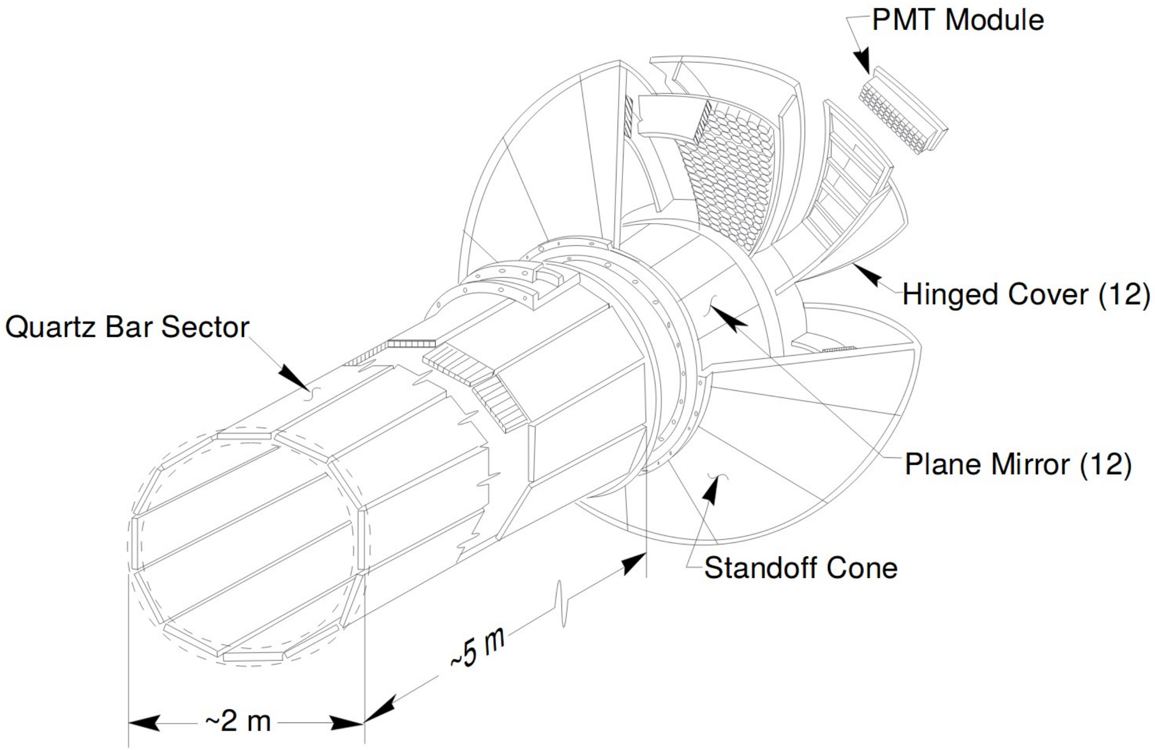
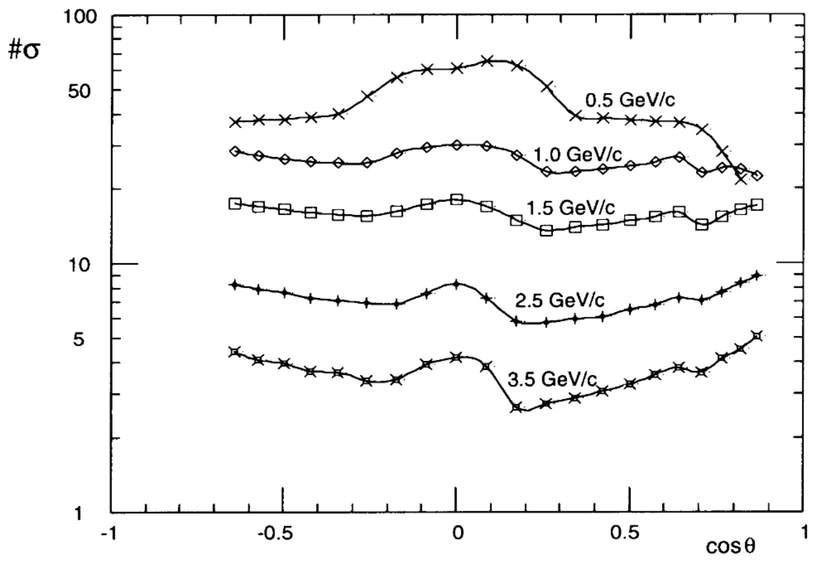
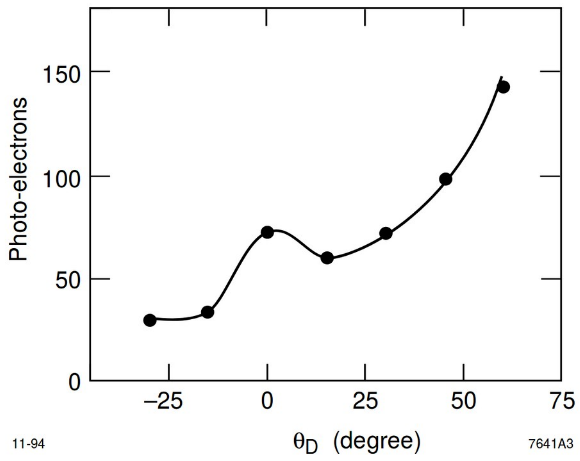
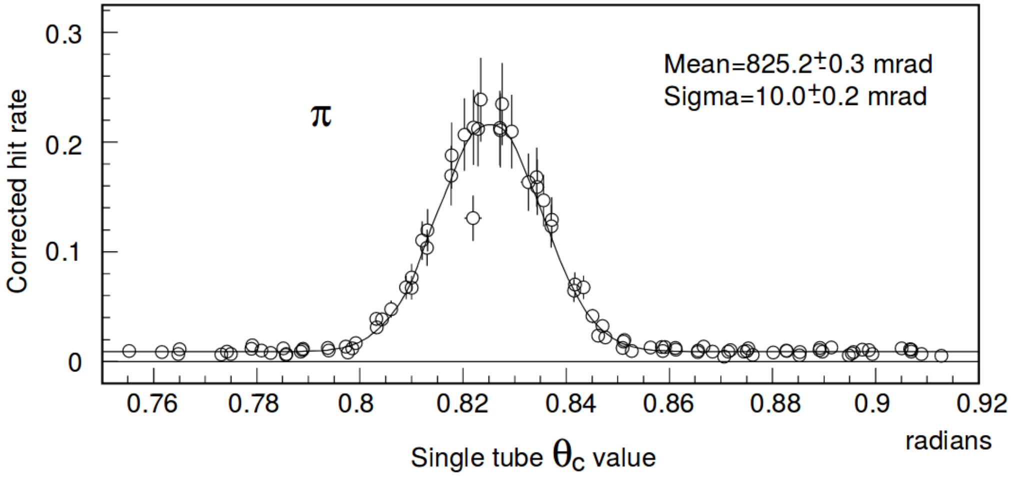


A real-life DIRC



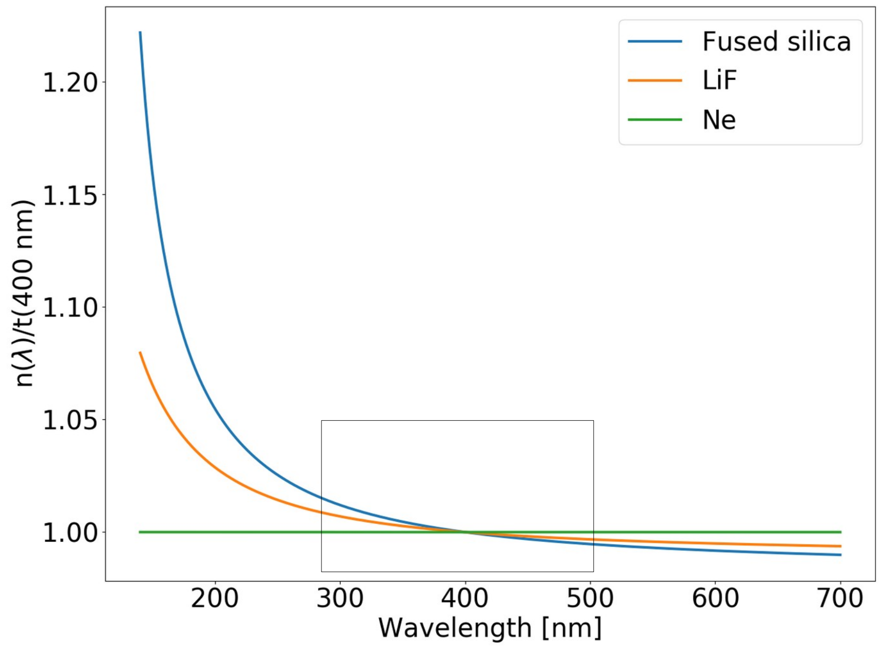




Limitations

$$\sigma_\alpha \approx \sqrt{\frac{t_x^2/12 + \sigma_x^2(\text{detector})}{L^2}}$$

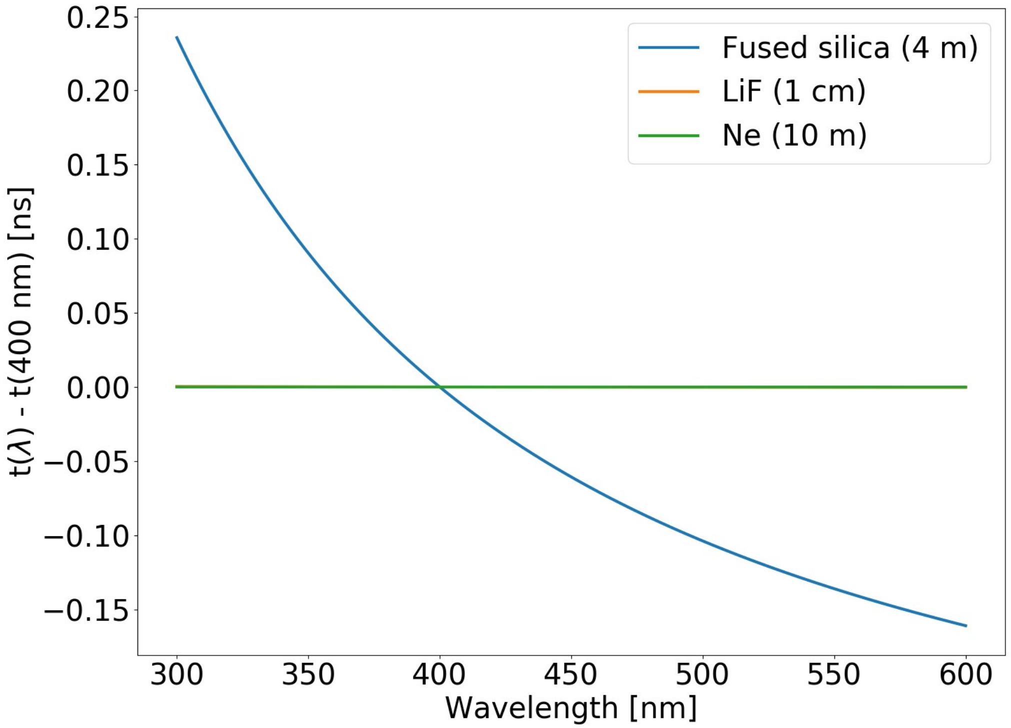
The resolution is limited by the focal length (L), the detector pitch and the bar thickness.

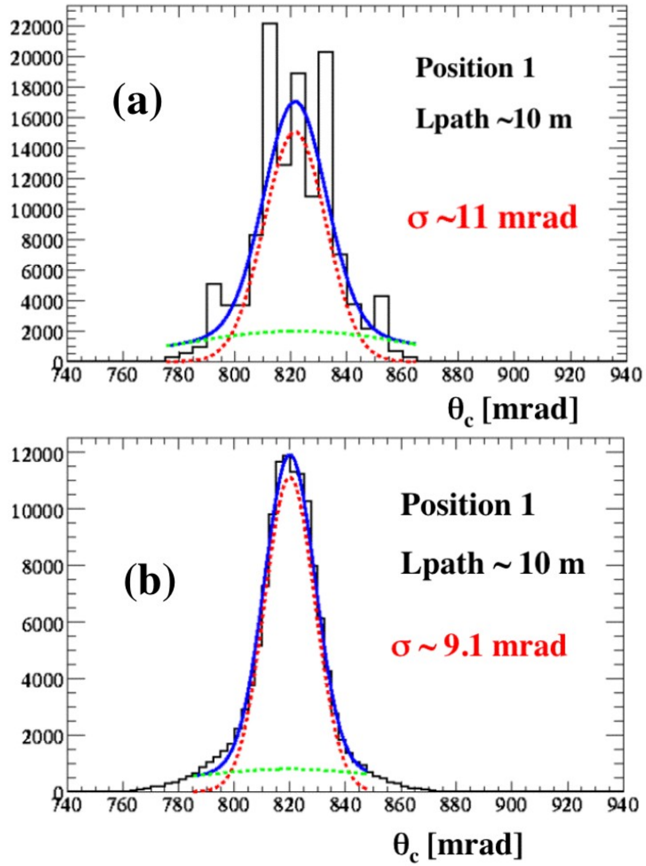


As usual, the second term is the chromatic dispersion.
This is mitigated using PMTs and looking the the near UV/visible

Limitations

A way to reduce the chromatic dispersion is to measure the timing of the photons in addition to the ring!

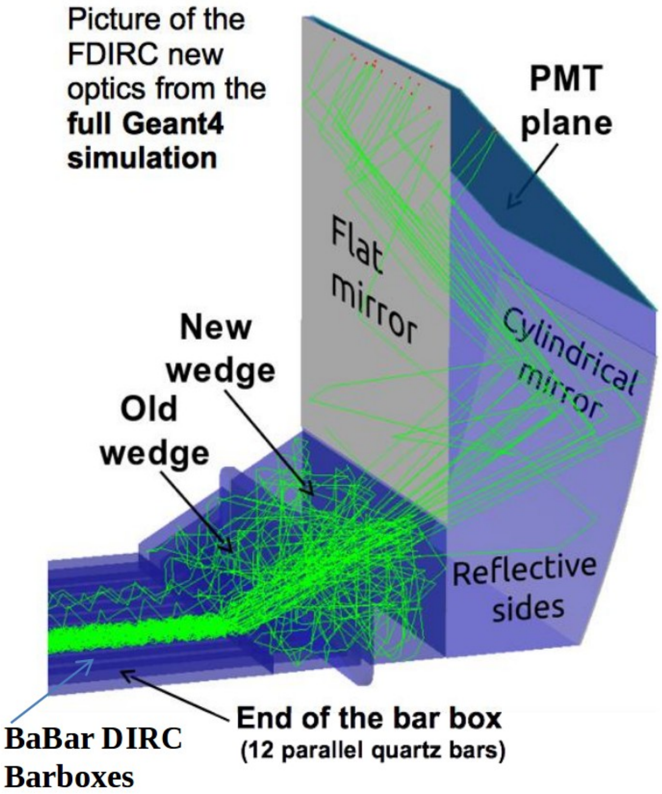


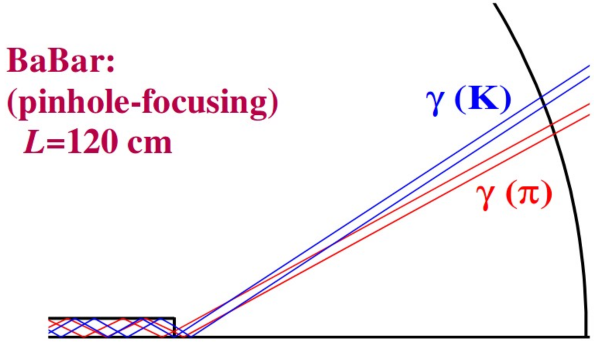


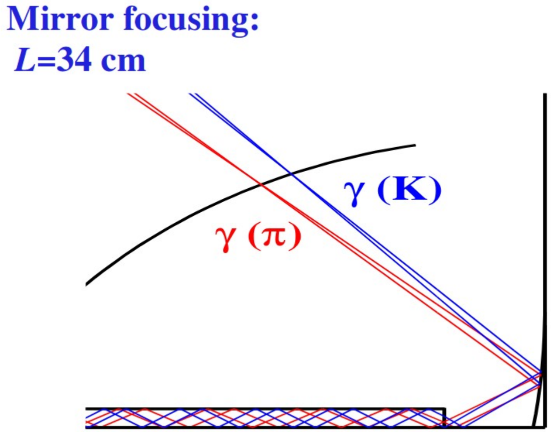
The FDIRC concept

Add a focusing mirror to the DIRC

$\rightarrow$ Eliminate the bar thickness term from the resolution





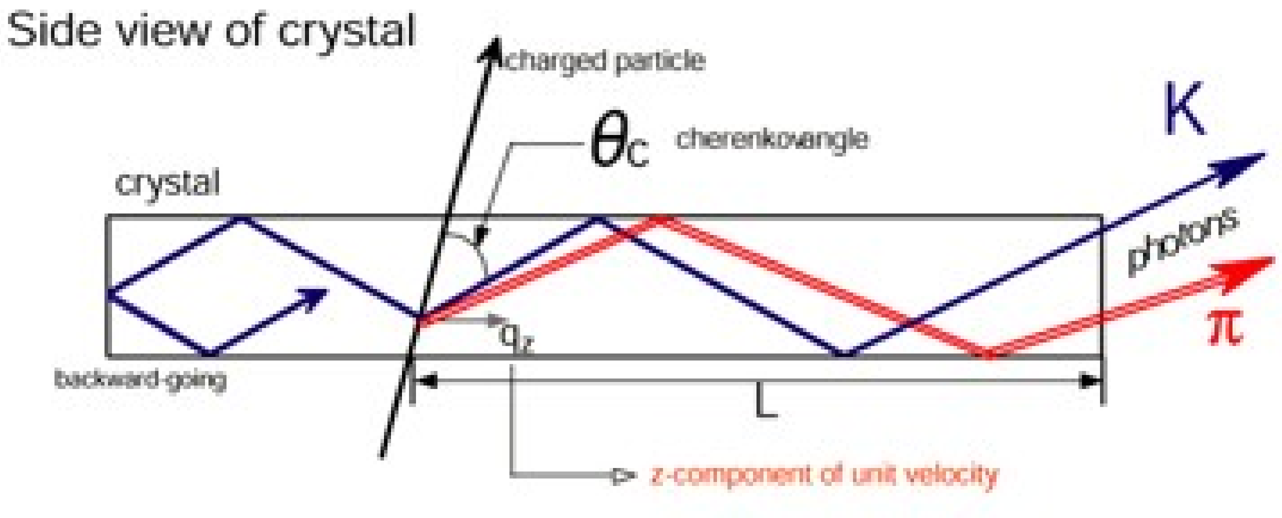


The TOP counter principle

The TOP is a "*DIRC in the time domain*"

$\rightarrow$ Cherenkov light trapped and propagated to the readout in a wide bar of fused silica

$\rightarrow$ The Cherenkov angle is measured by the **time of propagation** rather than the ring image on the PMT surface



The TOP counter at Belle II

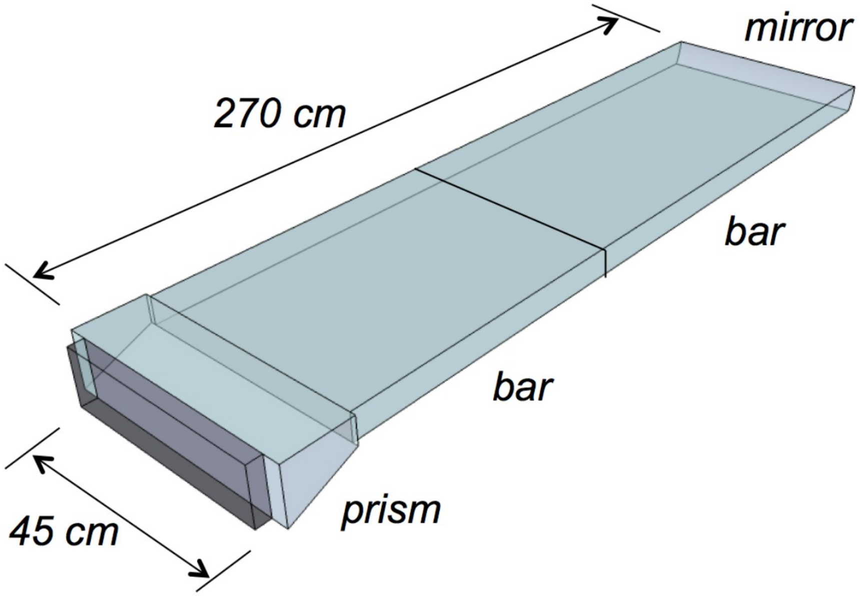
TOP implementation in Belle II:

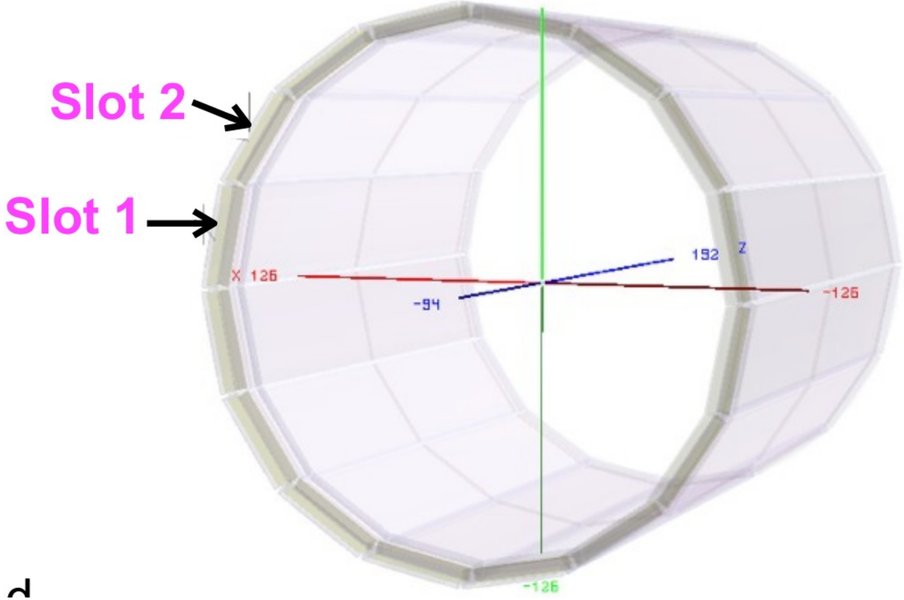
→ 16 modules (or slots) arranged around the interaction point

→ Each module is made of two identical bars of fused silica glued together

→ Backward side: expansion prism, PMTs and readout

→ Forward side: spherical mirror





The chromatic dispersion

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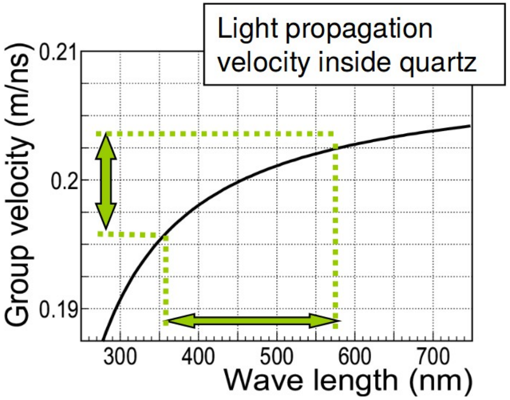
The chromatic dispersion is expected to contributed for ~50 ps/m in the DIRC

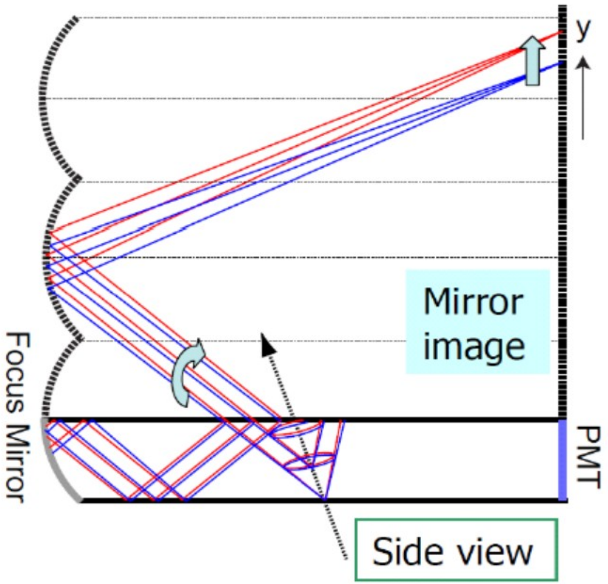
→ **Wavelenght filter to cut the spectrum**

→ **Make the TOP as short as possible (~ 3 m VS 5 m)**

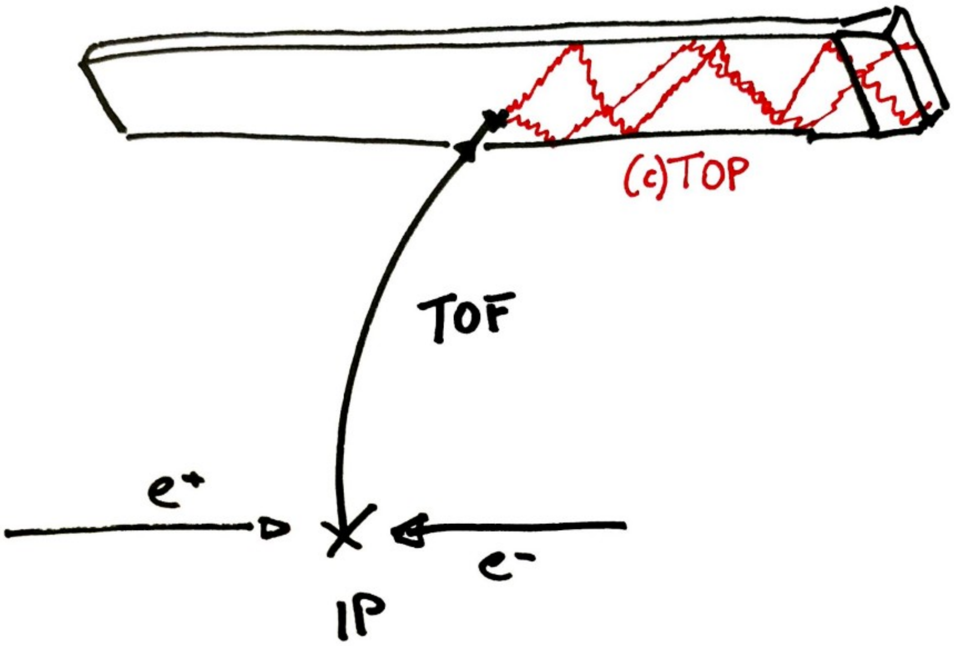
→ **Add a focusing mirror at the end of the bar**

- Long focusing length enlarges y difference.
  - $\Delta\theta_c$~5mrad → Δy~14mm for 2.5m length





What does the TOP measure?



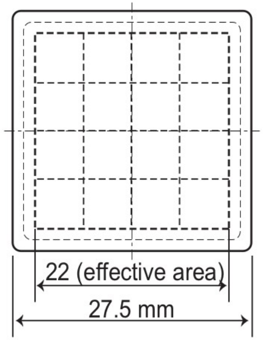
At a collider machine, we can combine the **ToF** and the **Cherenkov angle** in one single measurement

Key ingredients:
→ Impact point on the detector
→ Single p.e. time resolution (PMT + readout only)$<$ 100 ps
→ RF locking resolution $<$ 10 ps

Readout: The PMTs

The single photoelectron time resolution is the key parameter for the TOP

Our target is σ(1 p.e.) < 100 ps



Hamamatsu MCP-MPTs

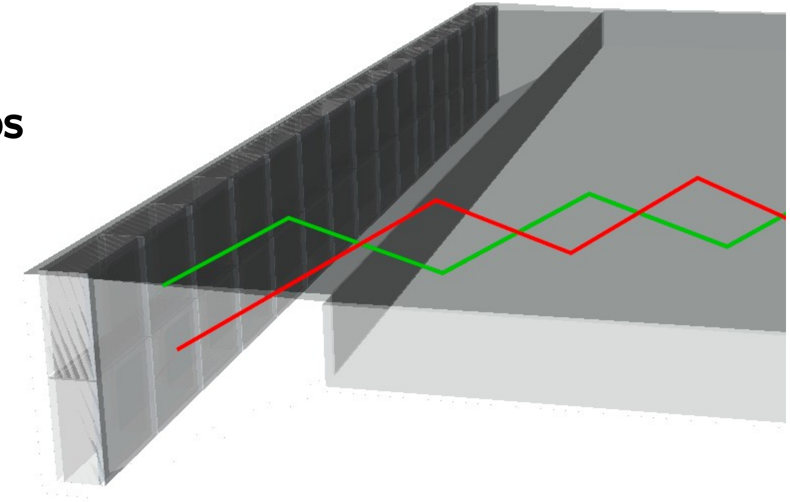
→ (1 x 1) in, ~70% active area

→ NaKSbCs photocathode; QE ≥ 24% (28% on average) at 380 nm

→ 55% collection efficiency

→ Gain = $10^5 - 10^6$

→ Transient time spread < 40 ps



Readout: The electronics

TOP front end electronics is based on the ***IRSX chip*** developed by Hawaii University
arXiv:1804.10782

Scope-on-a-chip

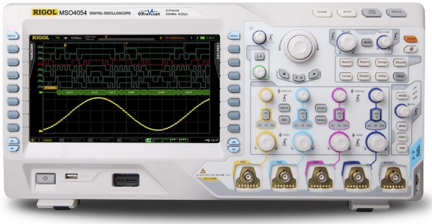
→ 8 channel waveform digitizer

→ 500 MHz Bandwidth

→ 2.7 GSa/s

→ 11.6 μs storage buffer

→ ***Full waveform output***

2 x  = 

Controlled by Xilinx Zinq FPGAs

→ Online pedestal subtraction

→ Online waveform analysis

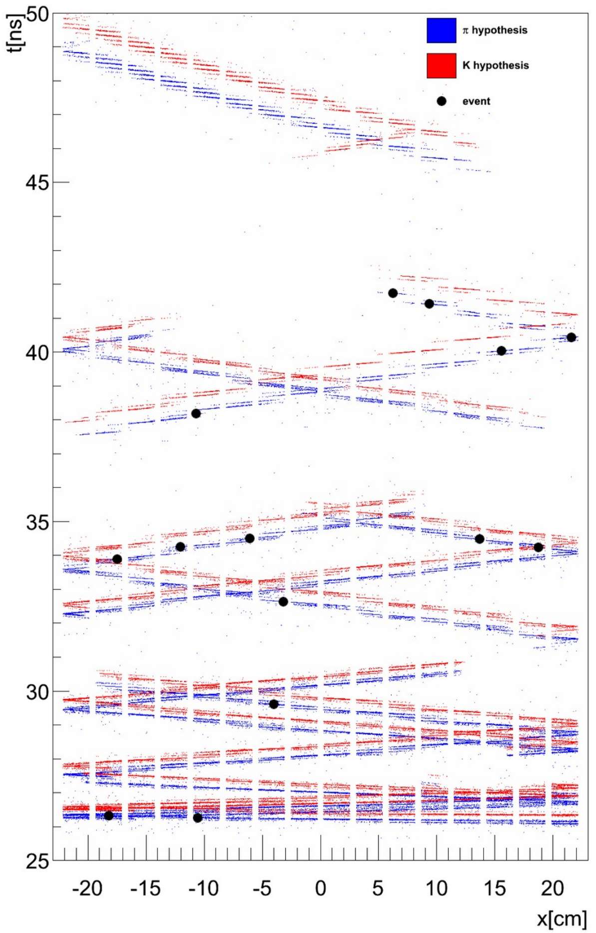
See Maeda-san's poster for more information!

The TOP rings

With such a detector, the Cherenkov rings are only a distant memory...

In such a detector, the PID is done exclusively comparing the expected PDFs with the photon time and position

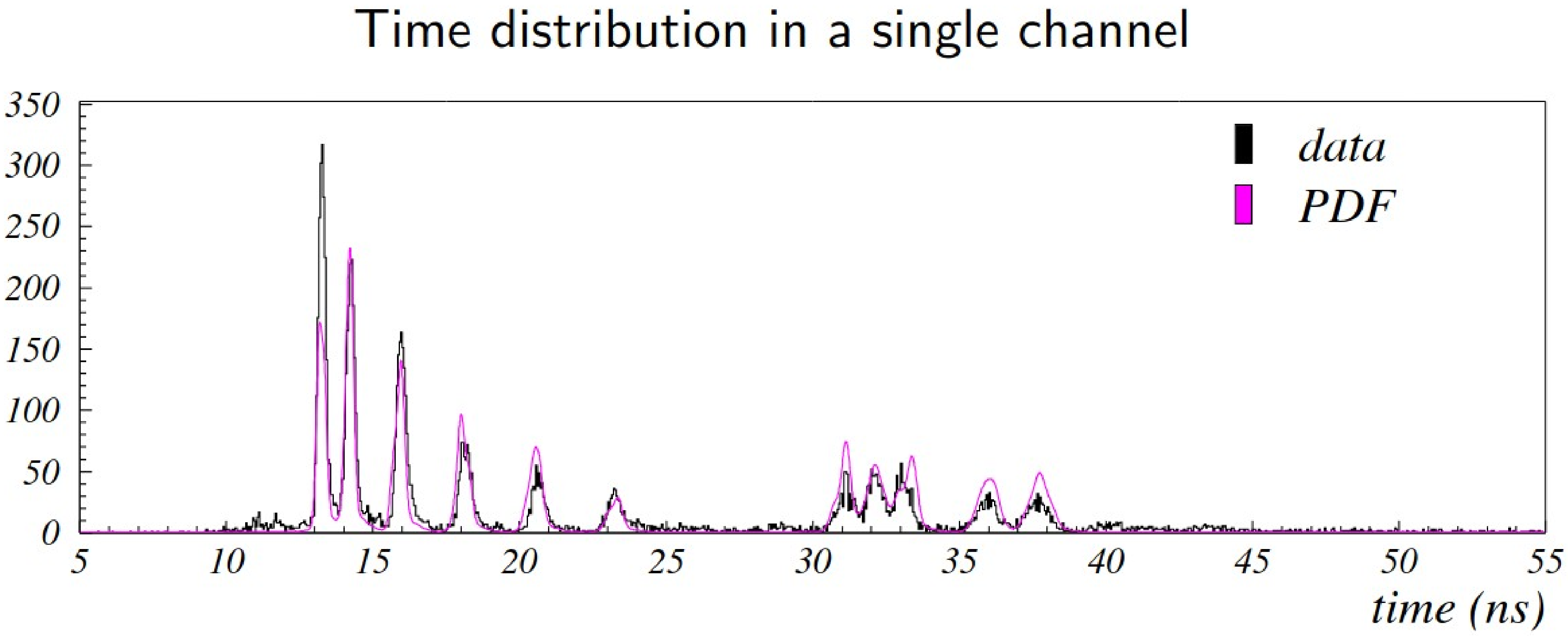
This is how a pion looks like



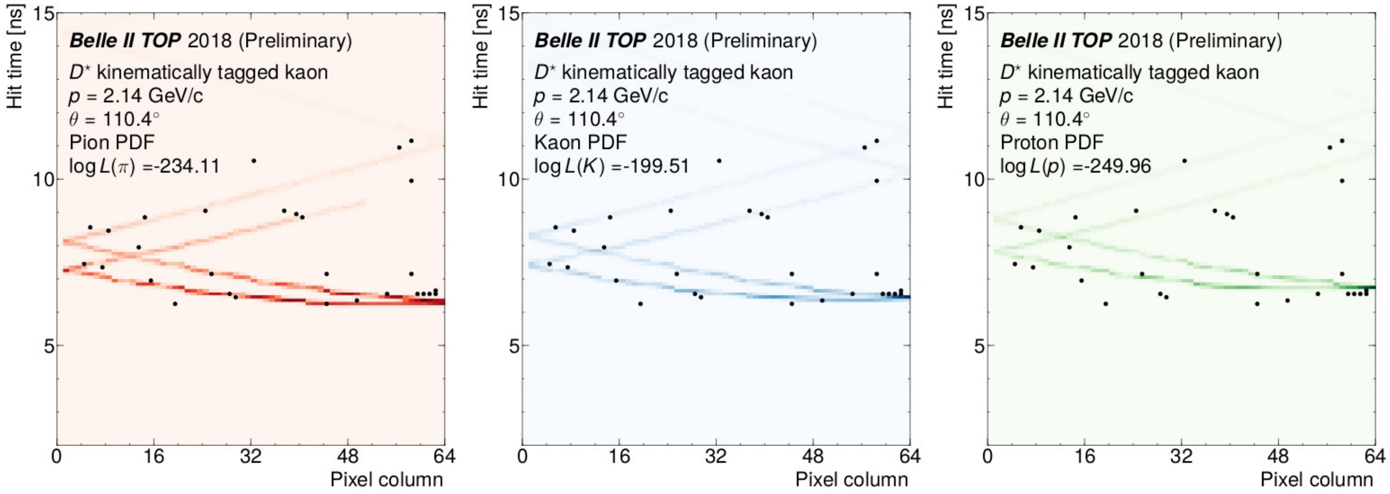
Single photon time resolution must be (much) better than 100 ps

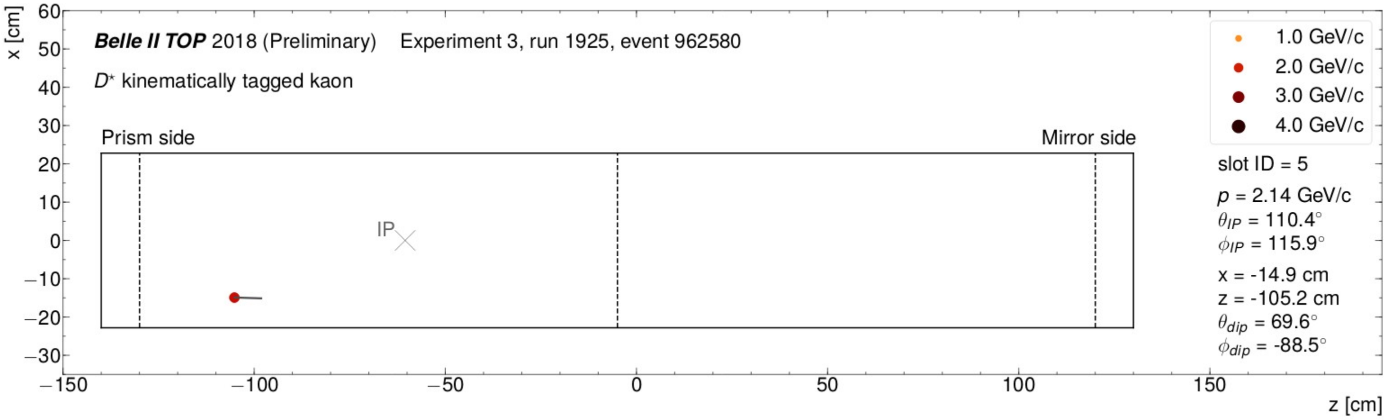
The TOP reconstruction

NIM A 595 (2008) 252–255, slides by Marko Staric (IJS)



Visualizing the Cherenkov rings





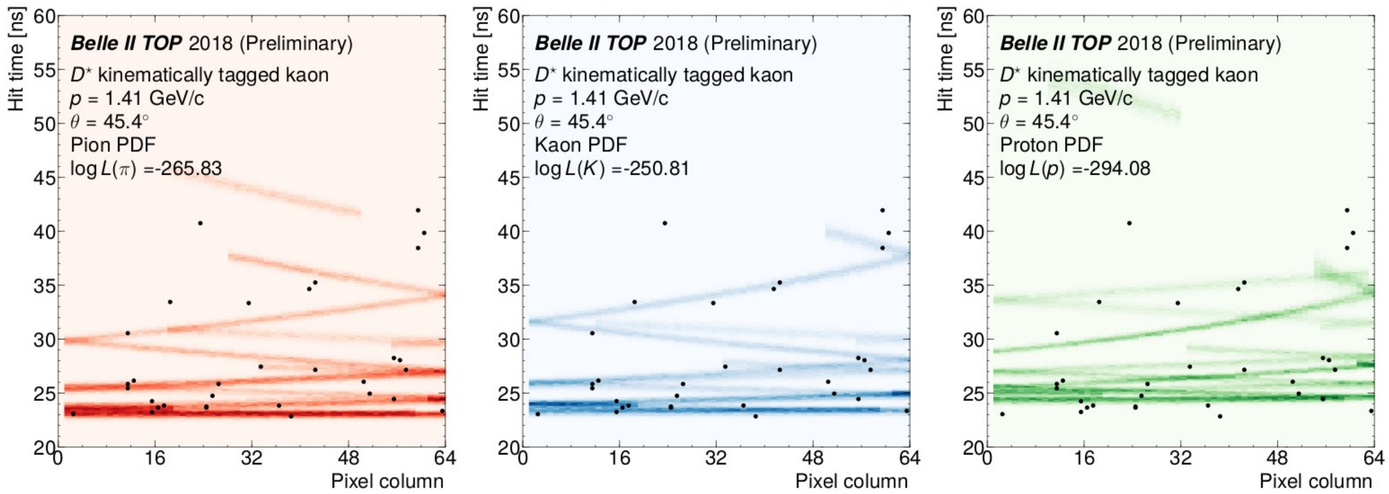
2.14 GeV

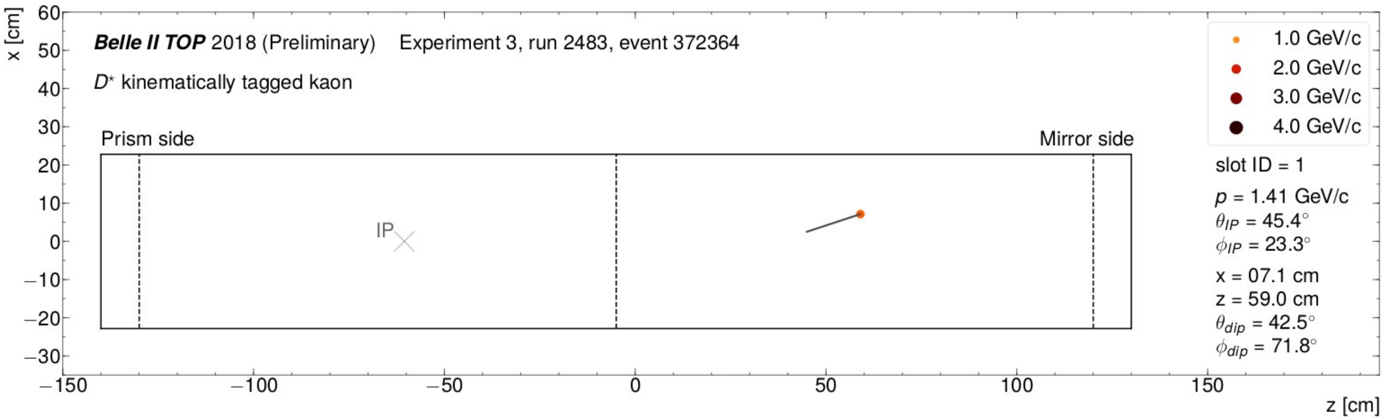
prism-facing event

Little room for the Cherenkov cone to open up

ID is dominated by the PDF shift (i.e. ToF) rather than the shape difference

Visualizing the Cherenkov rings





1.41 GeV

mirror-facing event

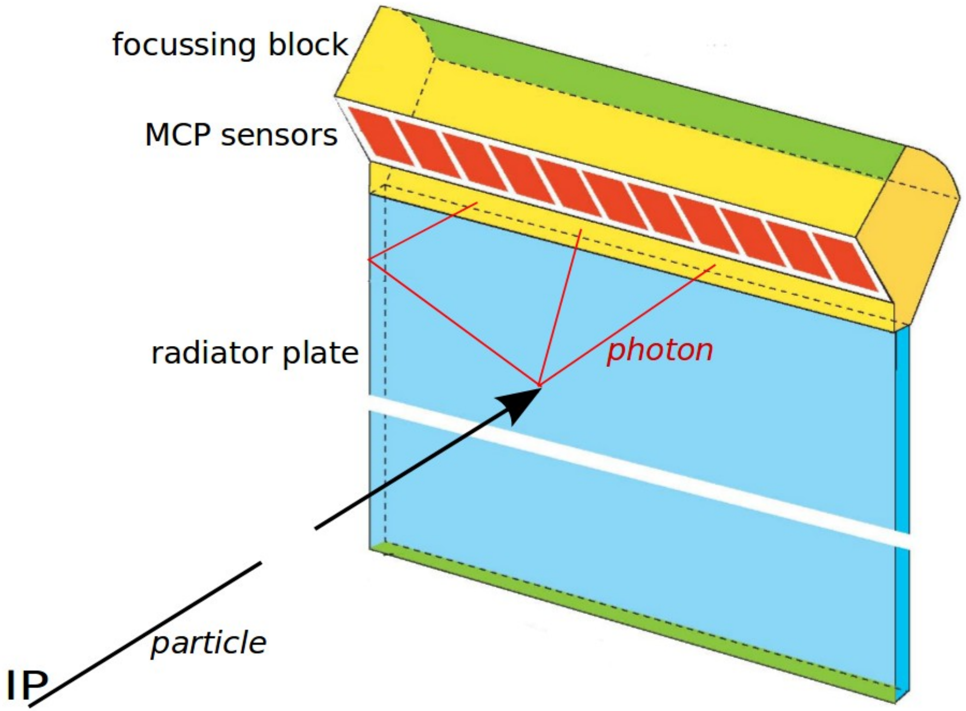
ID is dominated by the PDF shape (i.e. Cherenkov ring) rather than the global offset

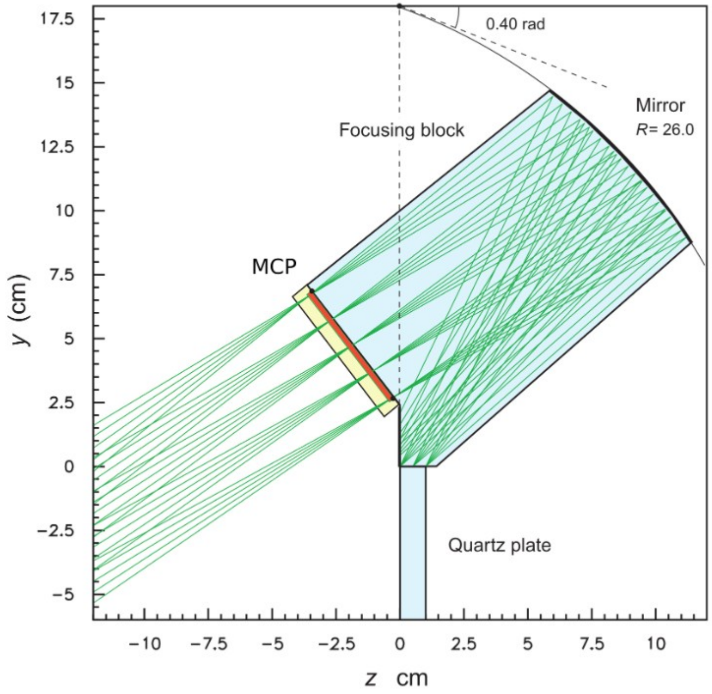
TORCH

Designed for the LHCb upgrade

→ Basically a TOP counter + focusing

→ (mostly) a timing device (15 ps resolution!) plus a bit of Cherenkov imaging





Putting everything together

Belle II

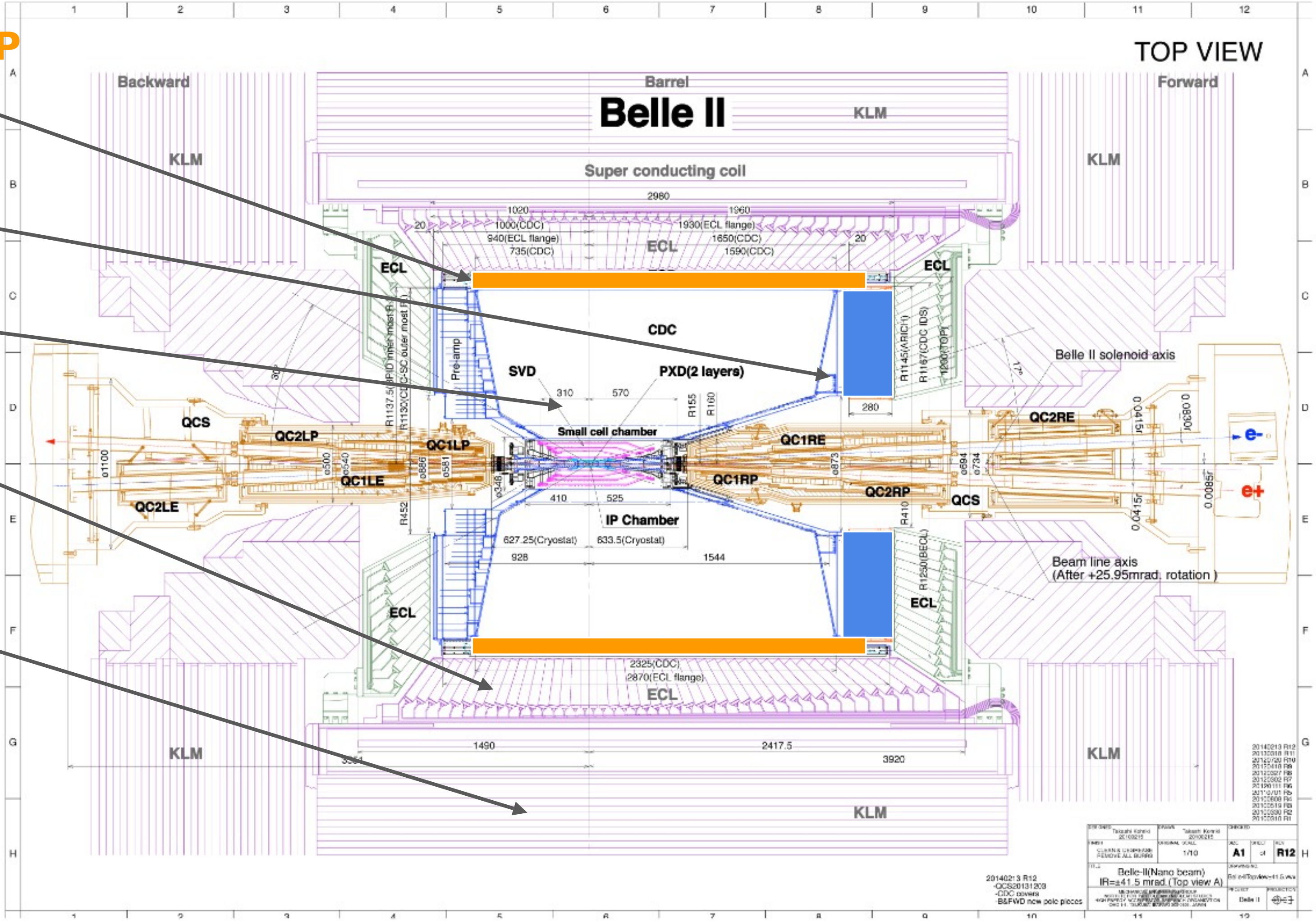
Barrel cherenkov: TOP

Endcap Cherenkov: ARICH

dEdx: SVD + CDC

Shower shape: ECL

Pen. depth: KLM



Belle II

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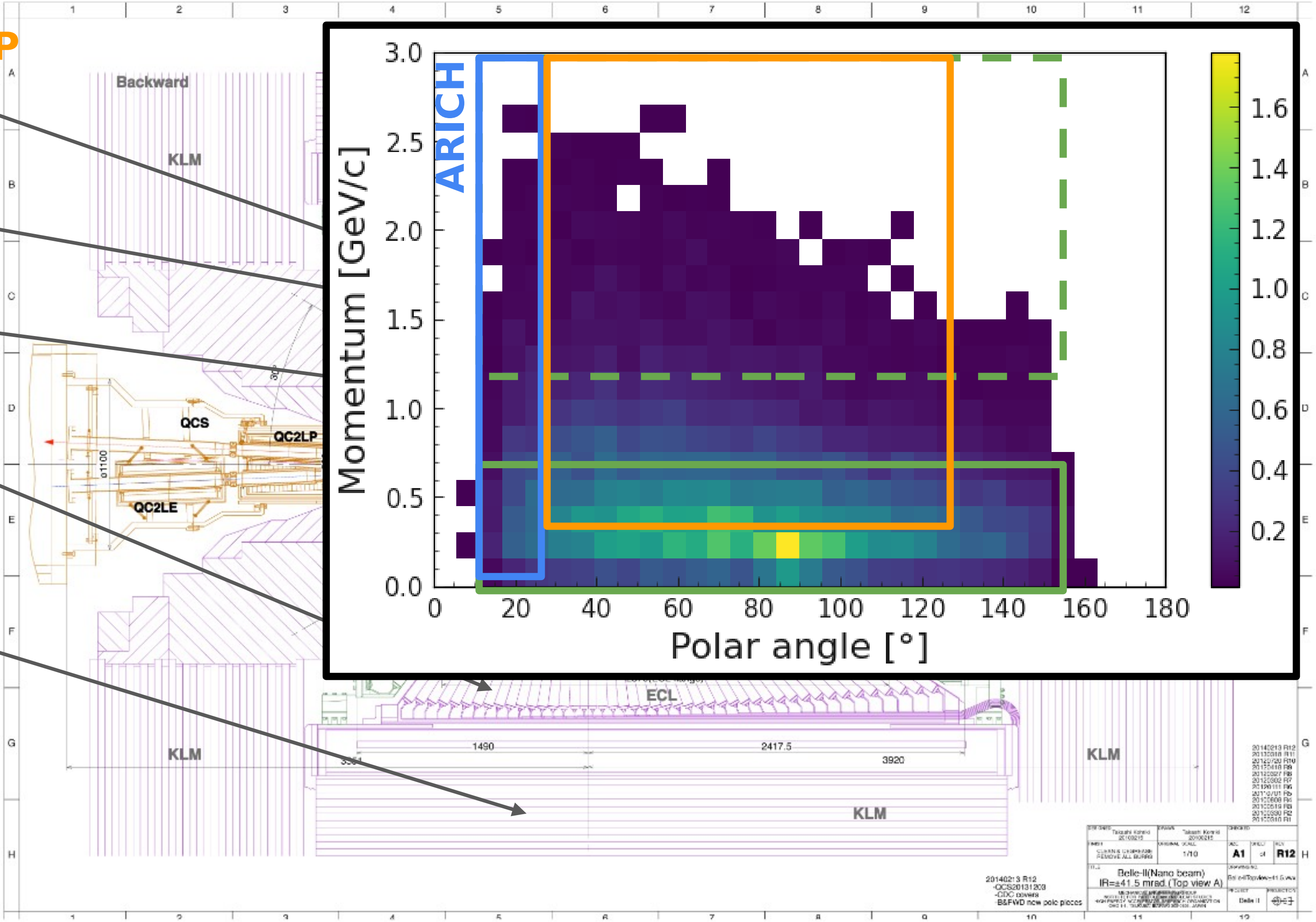


Combining information

Each sub-detector provides a likelihood value for 6 possible PID hypotheses:

→ electron, muon, pion, kaon, proton, deuteron

→ The likelihood values are calculated comparing the observed signal with the expectation for each particle hypothesis (based in MC, data template, or analytic models)

→ If particle is out-of-acceptance, LogL = 0 for all hypotheses

$$\mathcal{L}_\alpha^d = \mathcal{L}^d(\mathbf{x}|\alpha)$$

Likelihood for hypothesis α from detector *d* that observed **x** hits

$$\mathcal{L}(\mathbf{x}|i) = \exp\left(\sum_d^{d\in D} \log\mathcal{L}^d(\mathbf{x}|i)\right)$$

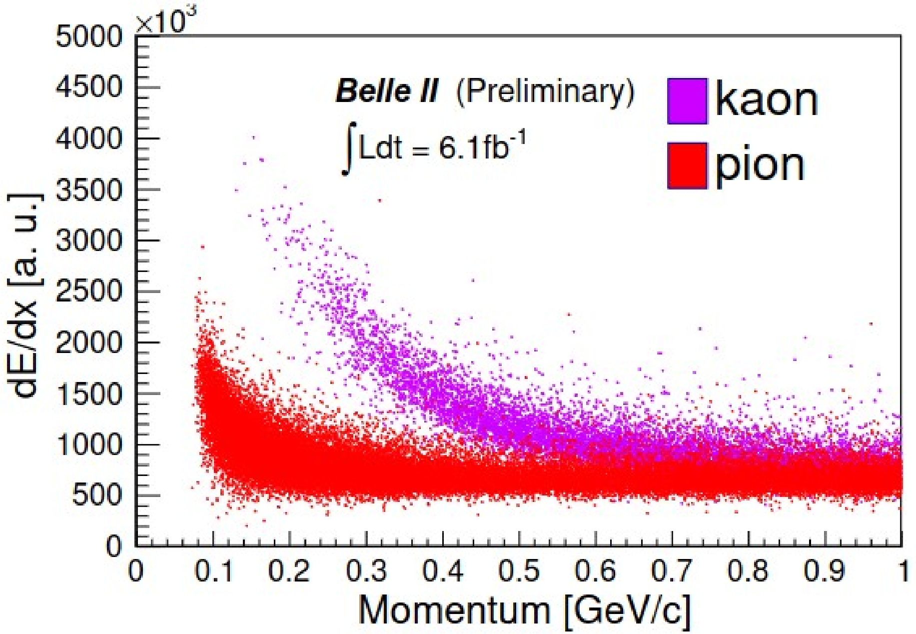
Likelihood for hypothesis α from all detectors

$$P(A_i|\mathbf{x}) = \frac{P(\mathbf{x}|A_i)\cdot P(A_i)}{\sum_j P(\mathbf{x}|A_j)P(A_j)} \quad \Rightarrow P(i|\mathbf{x}) = \frac{\mathcal{L}_i}{\sum_j \mathcal{L}_j}$$

PID probability

Silicon tracker

→ PDF is templated directly from data using tagged p, K, protons



$$\mathcal{L}_{\alpha}^{\mathrm{SVD}}(\mathrm{d}E/\mathrm{d}x,\ p) = \prod_i \mathcal{P}_{\alpha}[(\mathrm{d}E/\mathrm{d}x)_i, p]$$

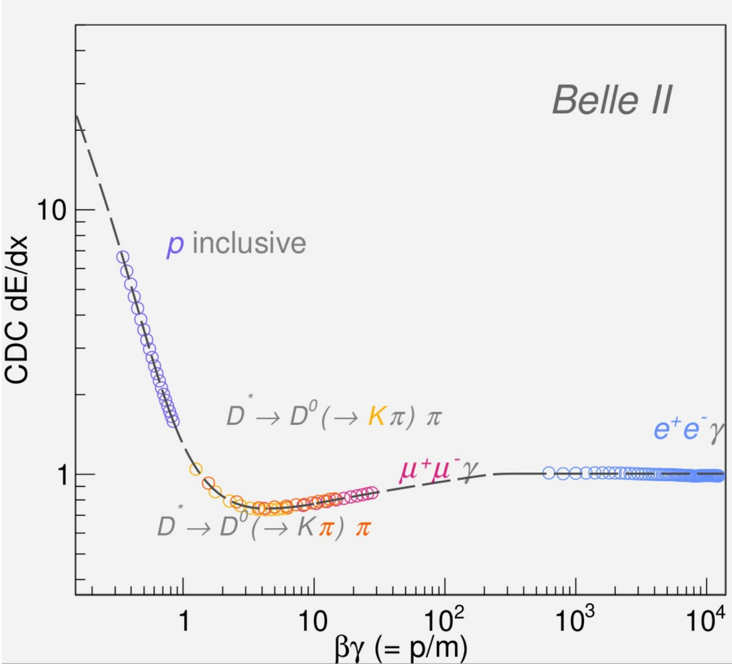
dE/dx

Silicon tracker

→ PDF is templated directly from data using tagged p, K, protons

Drift chamber

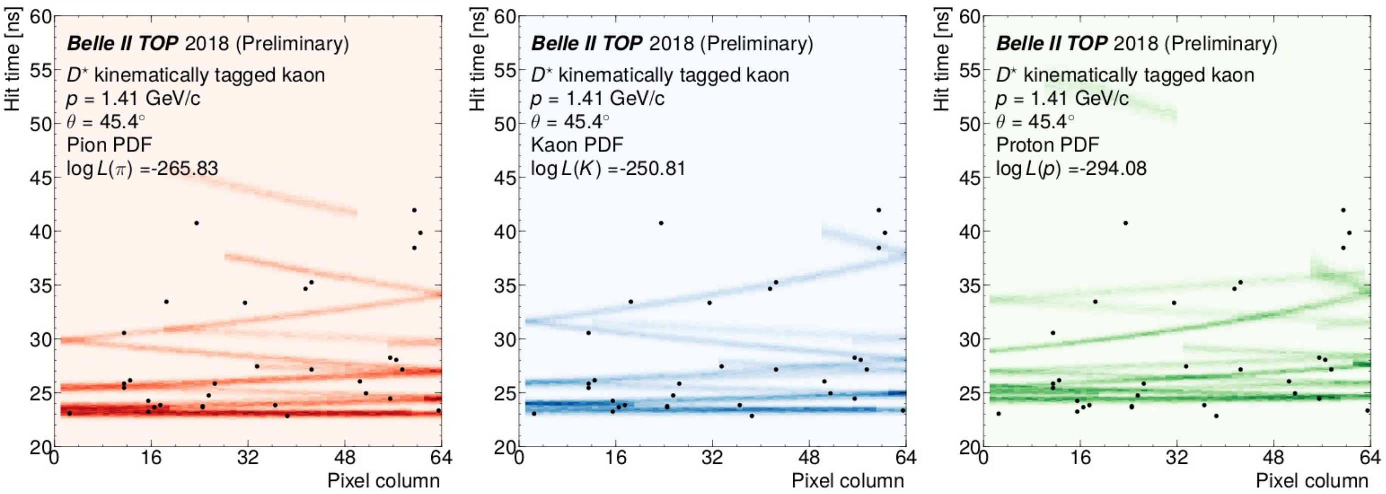
→ Calculate the expected dE/dx after running several data-driven calibrations



$$\chi_\alpha = \frac{(\mathrm{d}E/\mathrm{d}x)_{\mathrm{meas}} - (\mathrm{d}E/\mathrm{d}x)_{\mathrm{pred}}}{\sigma_{\mathrm{pred}}}$$

$$\mathcal{L}_\alpha^{\mathrm{CDC}} = \exp\left(-\frac{\chi_\alpha^2}{2}\right)$$

Time-of-Propagation



1.41 GeV

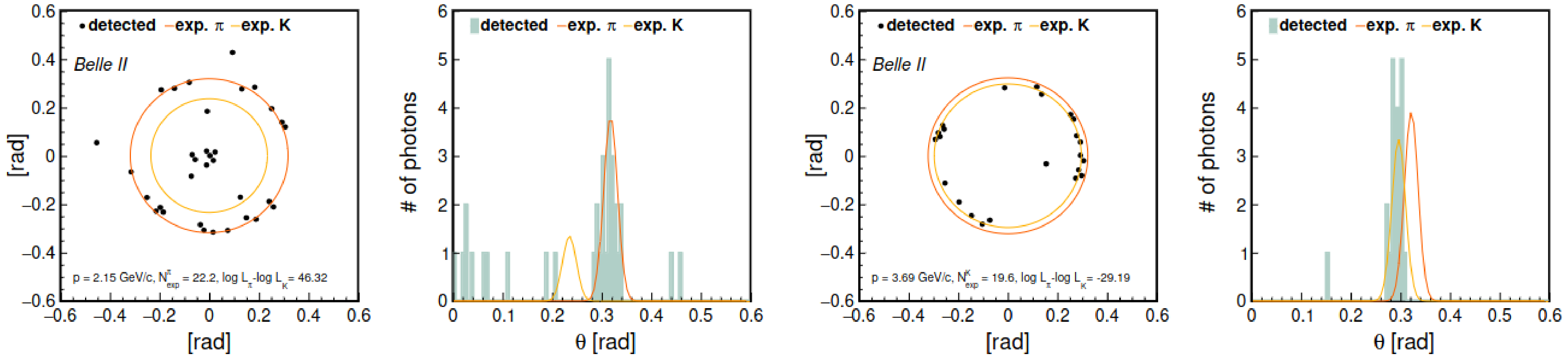
mirror-facing event

$$\mathcal{L}_\alpha^{\mathrm{TOP}} = \exp\left[\sum_{i=1}^{N} \log\left(\frac{N_\alpha S_\alpha(c_i,t_i) + N_B B(c_i,t_i)}{N_\alpha + N_B}\right) + \log P_N(N_\alpha + N_B)\right]$$

Dual aerogel proximity RICH

Dual radiator (but another kind of)

→ Two thin (2 cm) layers with different refractive index
→ Tuned to have **overlapping rings**
→ Reconstruction: count the number of hints in the expected ring



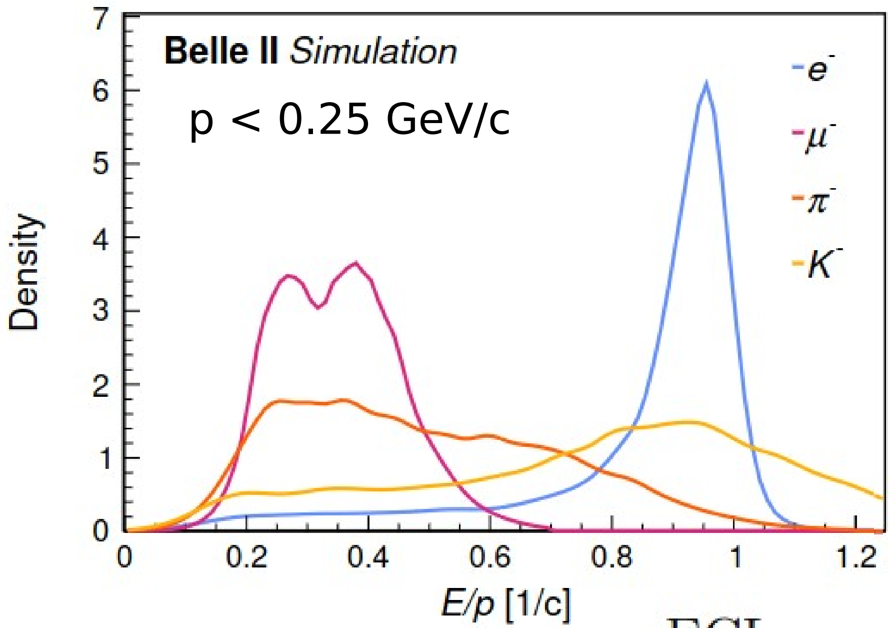
$$\mathcal{L}_{\alpha}^{\mathrm{ARICH}} = \exp\left[-N_{\alpha} + \sum_{k=1}^{N}\left(n_{\alpha,k} + \log(1 - e^{-n_{\alpha,k}})\right)\right]$$

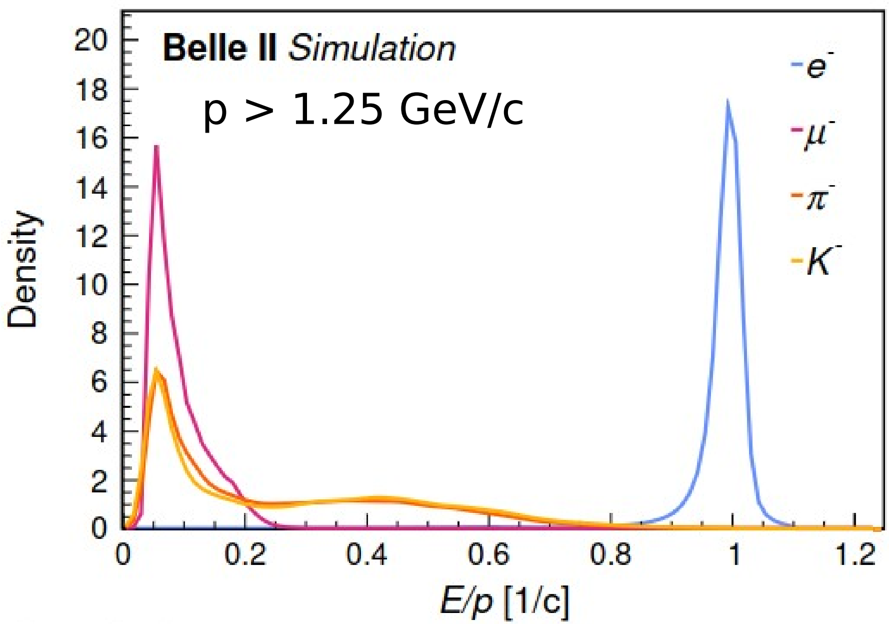
Calorimeter and K_L system

Electromagnetic calorimeter

→ Use the E/p ratio, PDF templated from MC.

→ More recently: combine all shower shape variables into a BDT

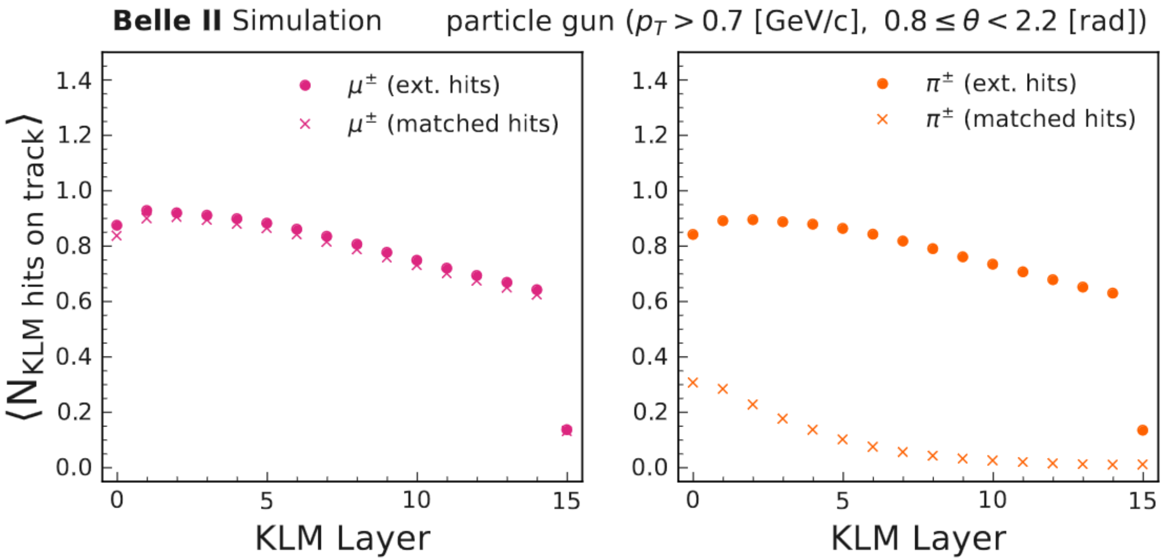




$$\mathcal{L}_{\alpha}^{\mathrm{ECL}} = \mathcal{P}_{\alpha}(E/p)$$

KLM (instrumented return yoke)

→ use the penetration depth in the iron plates, accounting for the scintillator efficiency



$$\mathcal{L}_{\alpha}^{\mathrm{KLM,L}} = \prod_{k=1}^{N_{\mathrm{ext}}} \mathcal{L}_{\alpha,k}^{\mathrm{L}}$$

$$\mathcal{L}_{\alpha,k}^{\mathrm{L}} = p_{\alpha,k}$$ if hit

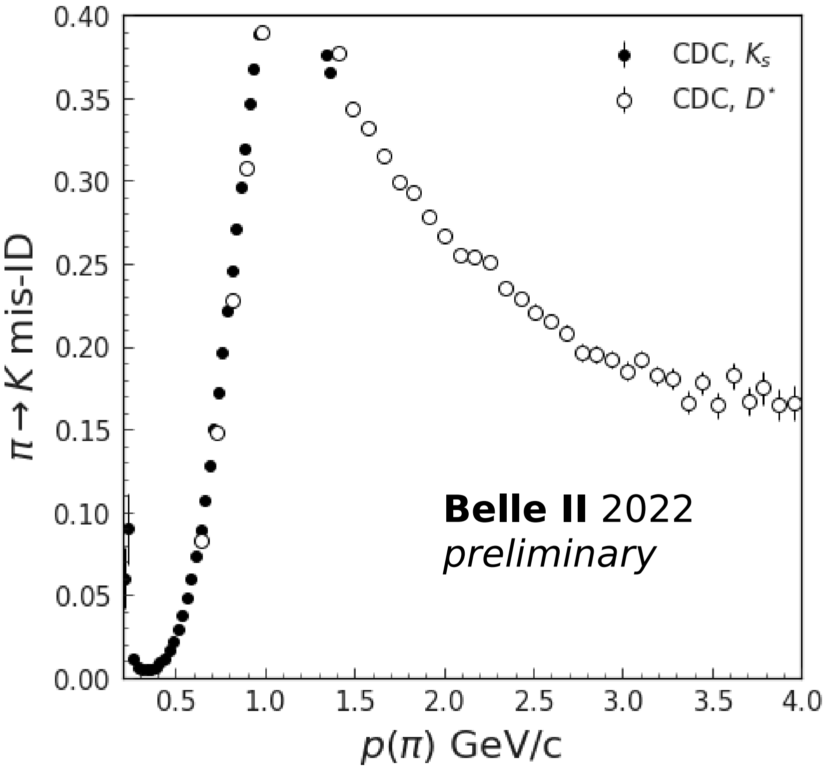
$$\mathcal{L}_{\alpha,k}^{\mathrm{L}} = (1 - p_{\alpha,k}\varepsilon_k)$$ if not hit

The impact of TOP and ARICH

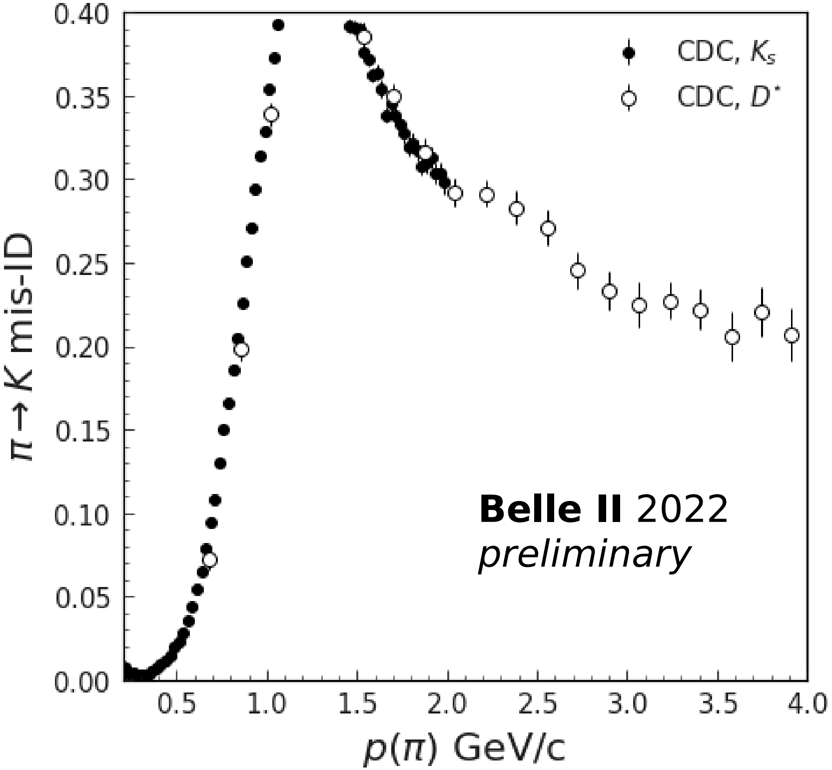
π → K mis-identification probability in collision data

- True pions tagged in D and K_s decays
- Ask for LL(K) > LL(π)

in TOP acceptance



in ARICH acceptance



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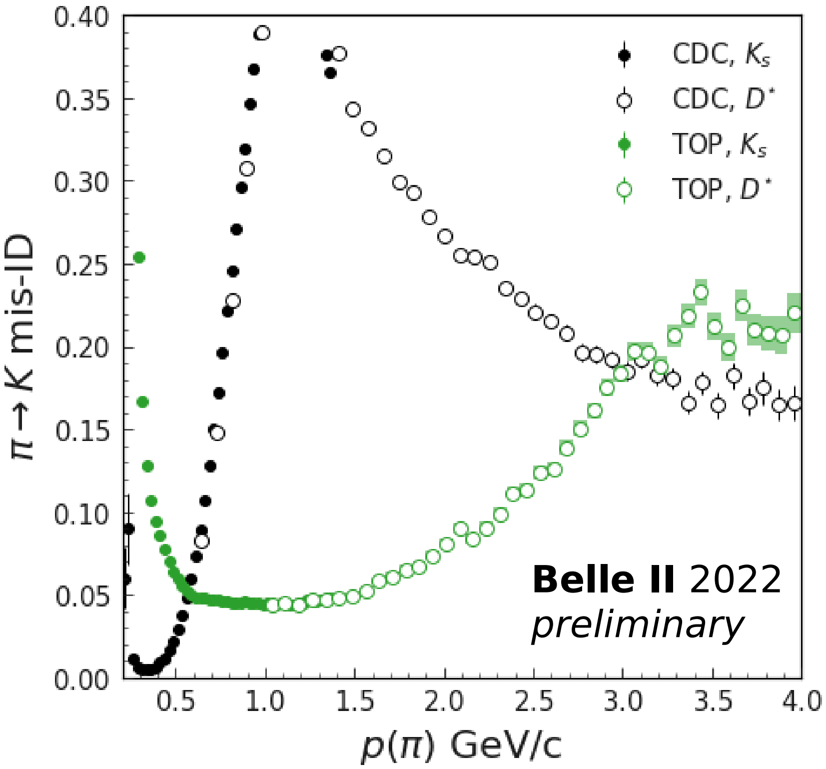
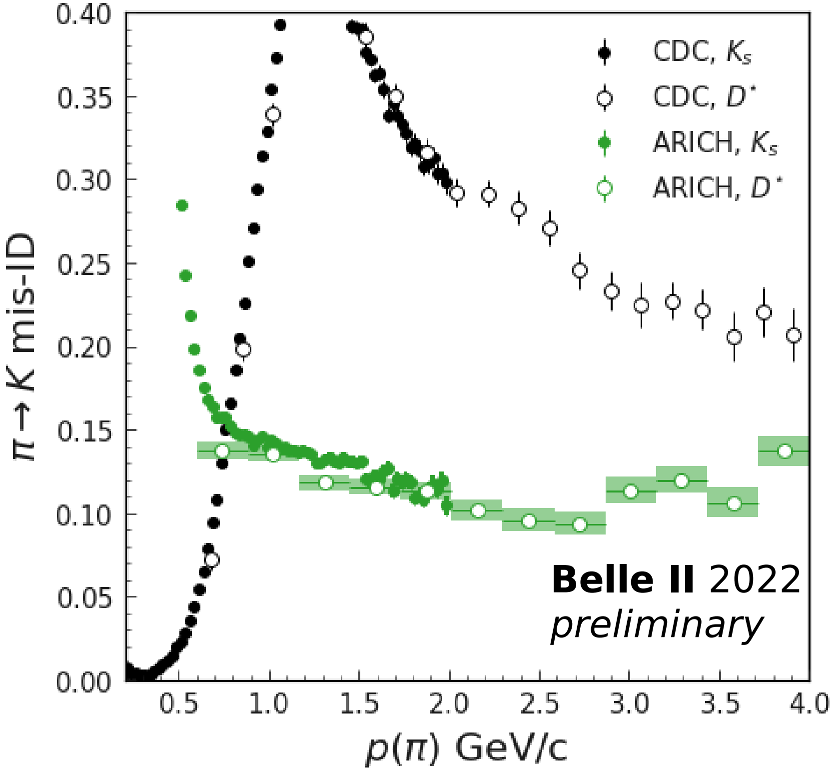
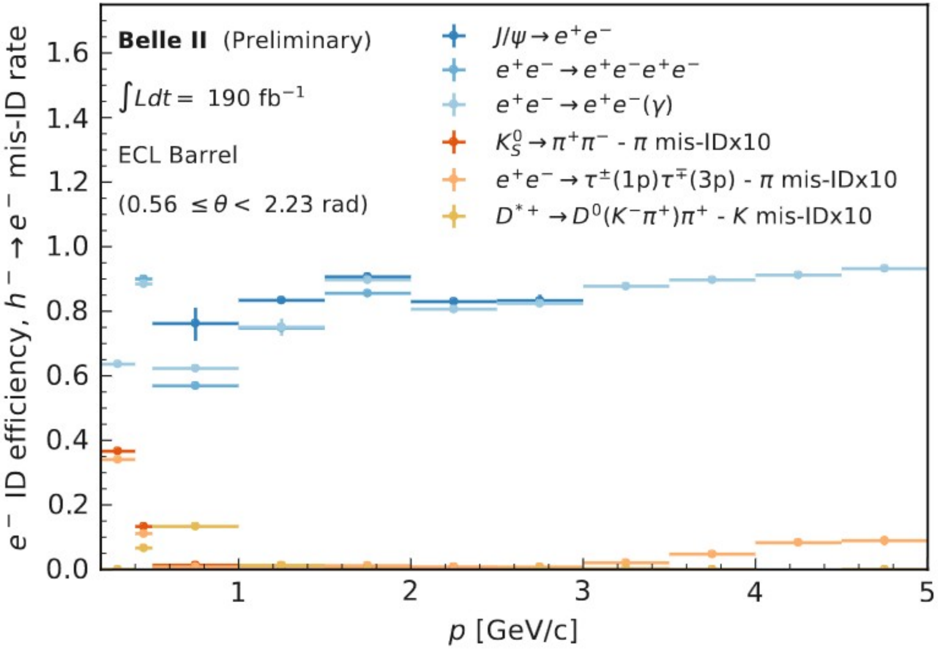


in ARICH acceptance



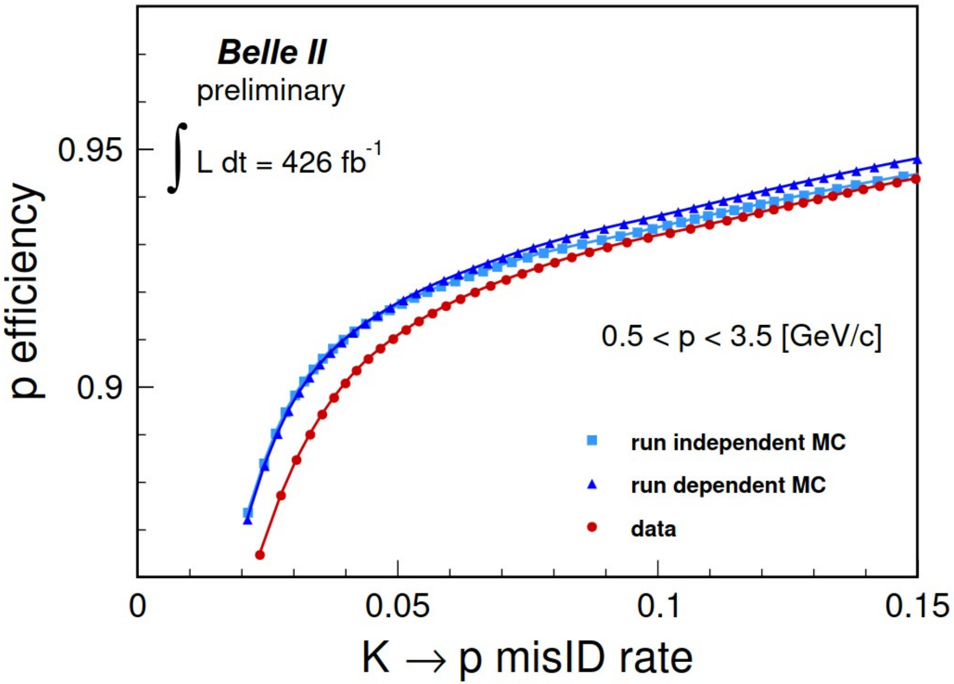
Global performance

Electron-pion separation



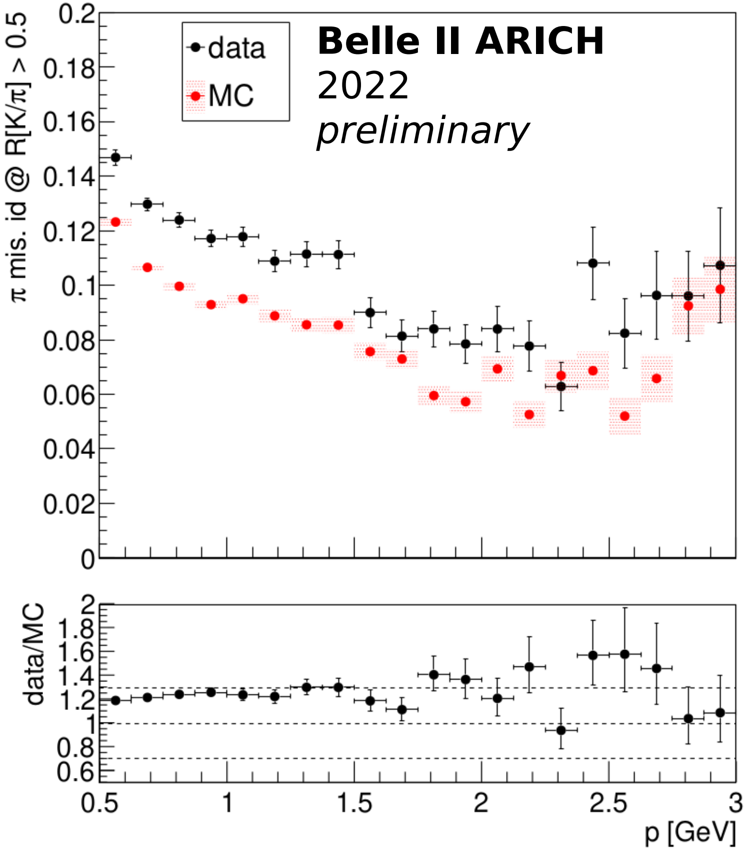
(c) eID > 0.9 selection.

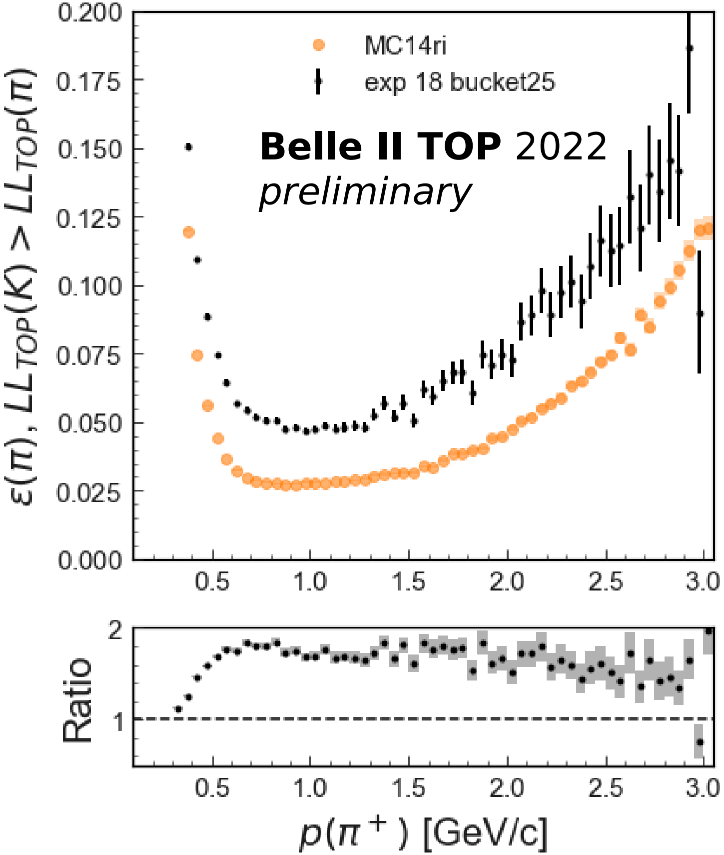
Kaon-proton separation



Expectations VS reality

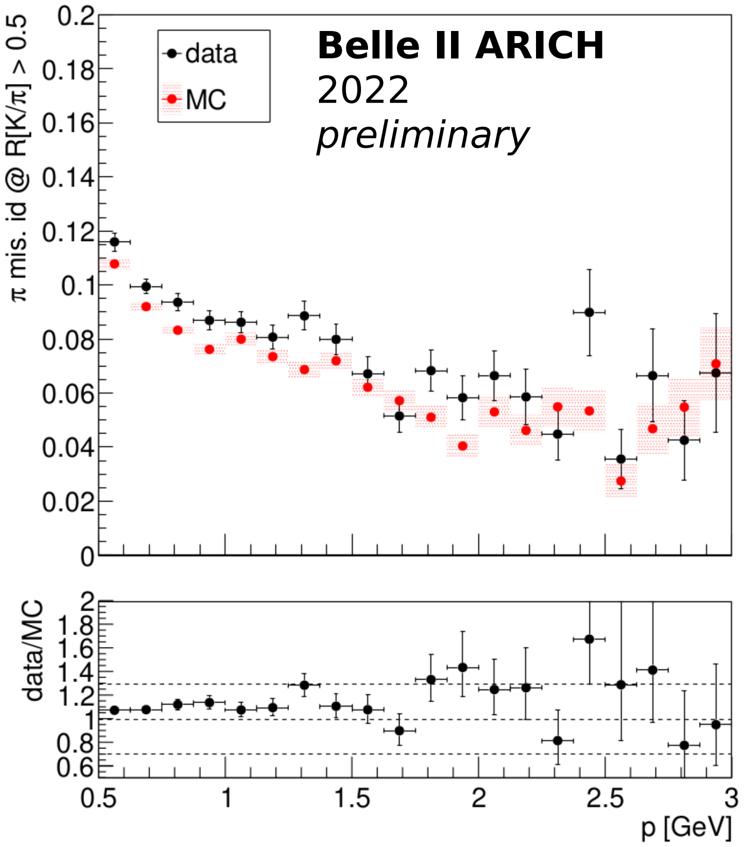
Performance observed in data still don't match with (optimistic) MC
- Many lessons learned so far!



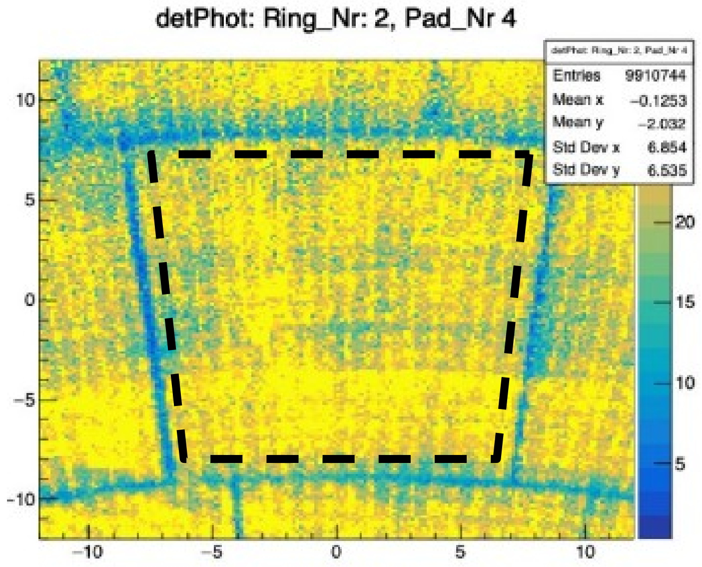


Lessons learned: ARICH tile alignment

Aerogel tile edges are responsible for most of the disagreement in ARICH

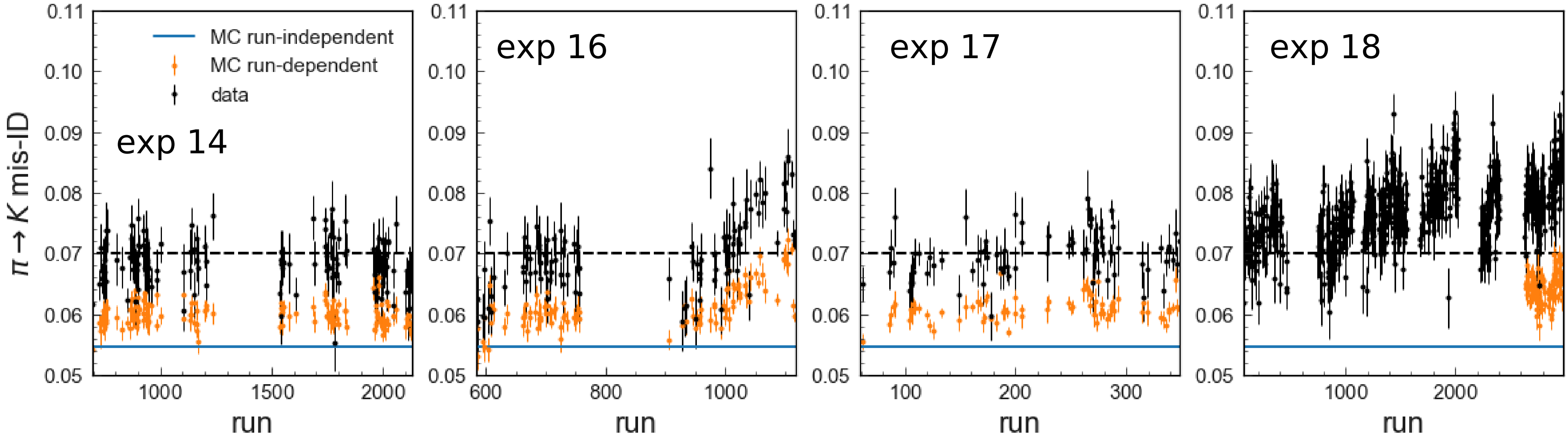


Removing tracks extrapolated in the edges
- Improves PID (expected) reducing acceptance
- Improves data/MC (not expected)
  - Work towards better tile alignment



Lessons learned: background effects on TOP

For TOP, half of the data/MC disagreement is recovered with more realistic simulation
→ Actual dead/hot channel maps form data
→ Backgrounds from random triggers instead of simulation

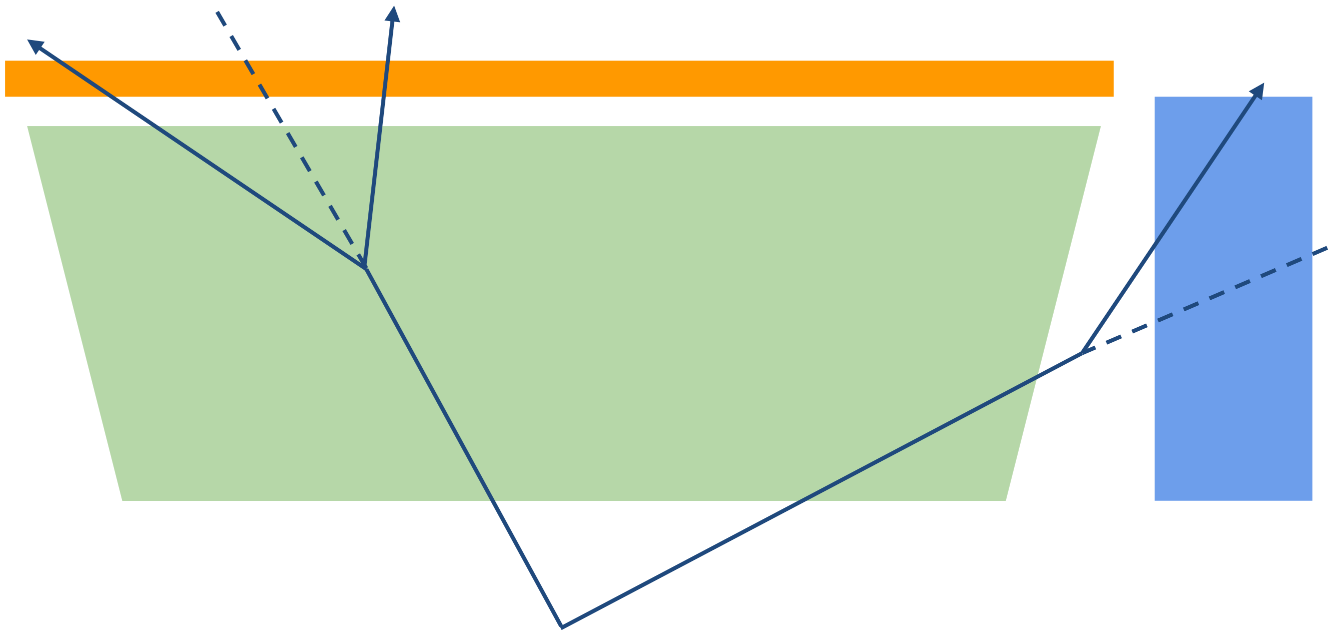


Residual discrepancy is under investigation.

Lessons learned: extrapolating is dangerous

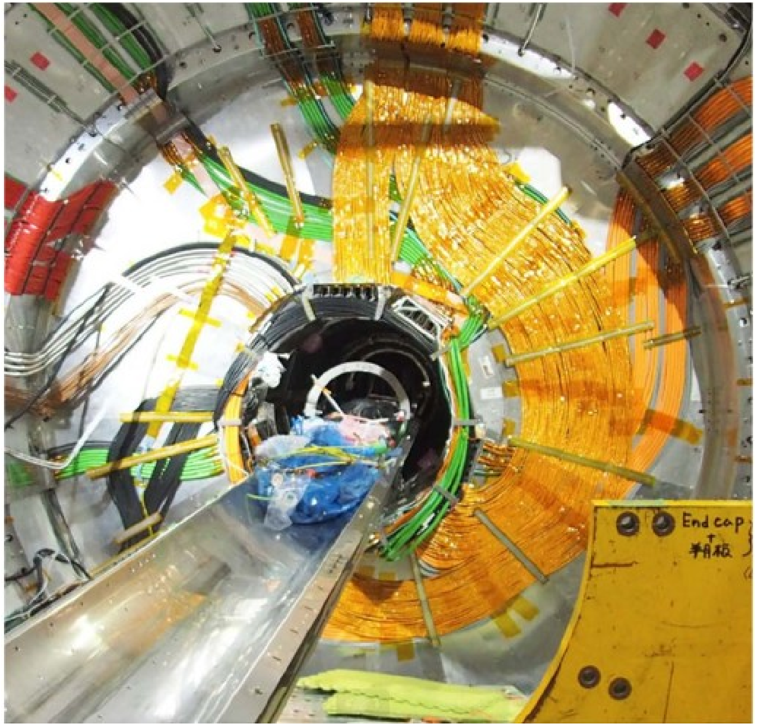
Both TOP and ARICH are outside the tracking volume

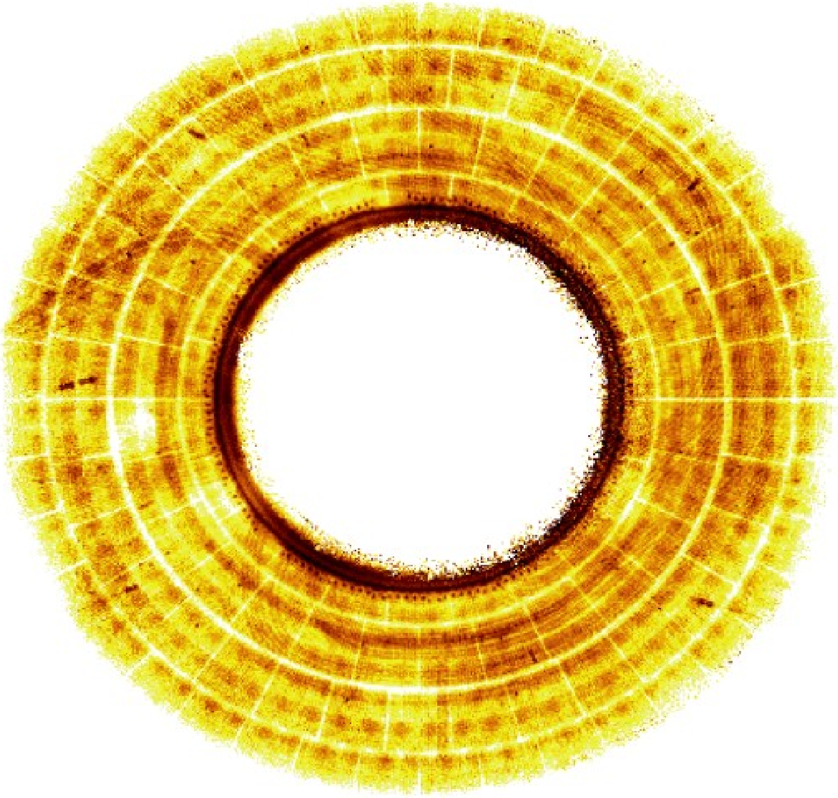
- Rely on track extrapolation
- Decays-in-flight and hard scattering lead to wrong extrapolation
- **Significant PID degradation from hard-scattering**



Lessons learned: hard scattering in ARICH

Sizable material budget in front of ARICH
→ CDC backplane, inner tracker cables…
→ Clearly seen mapping the impact points of electrons with associated photons

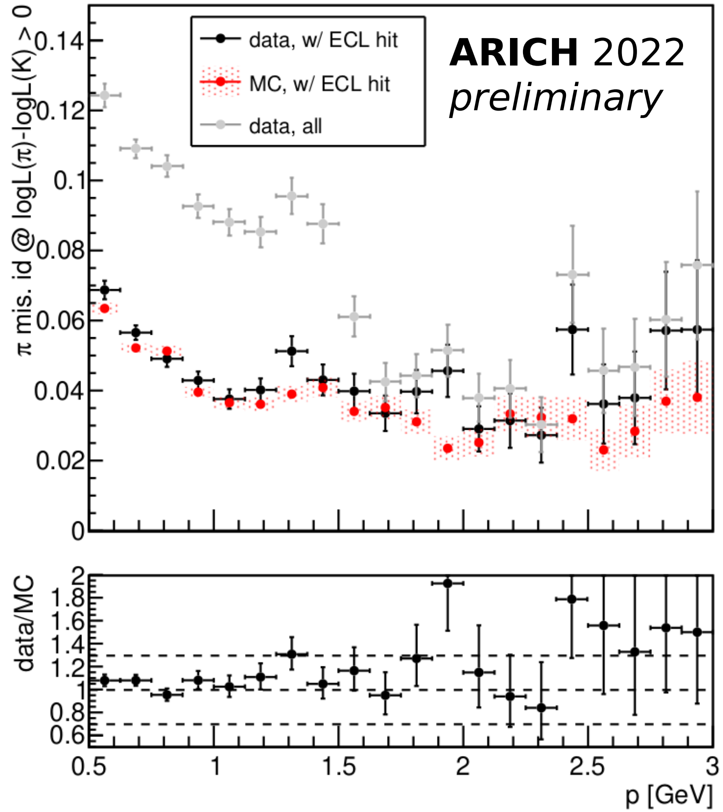


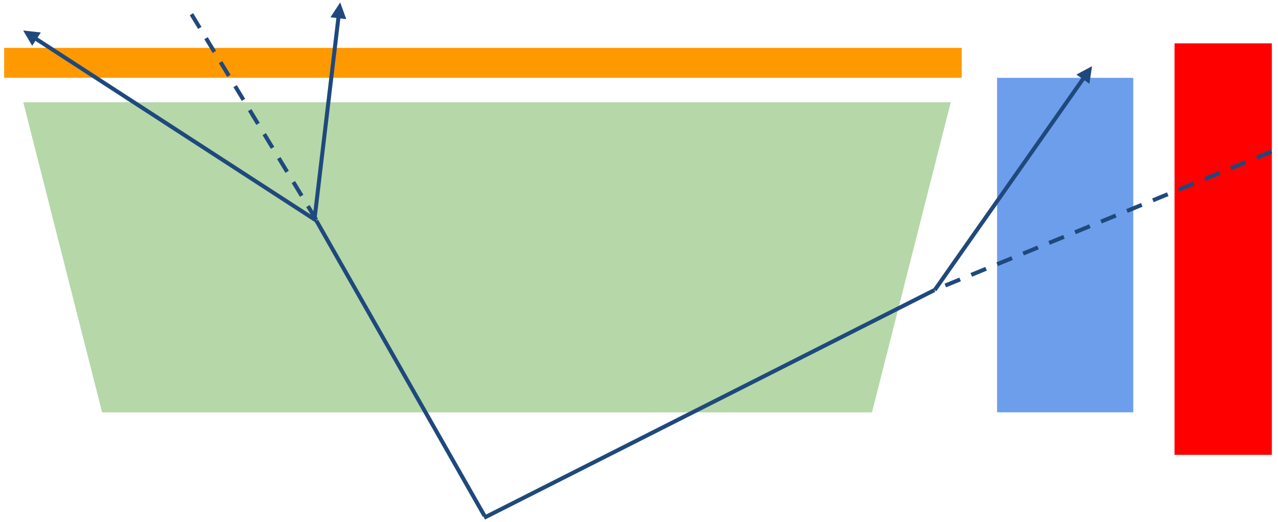


Mitigating material scattering

Use the Calorimenter behind ARICH and TOP to remove bad extrapolations

- Require a cluster matched with the track
- Powerful tool, but introduced correlation between subdetectors…





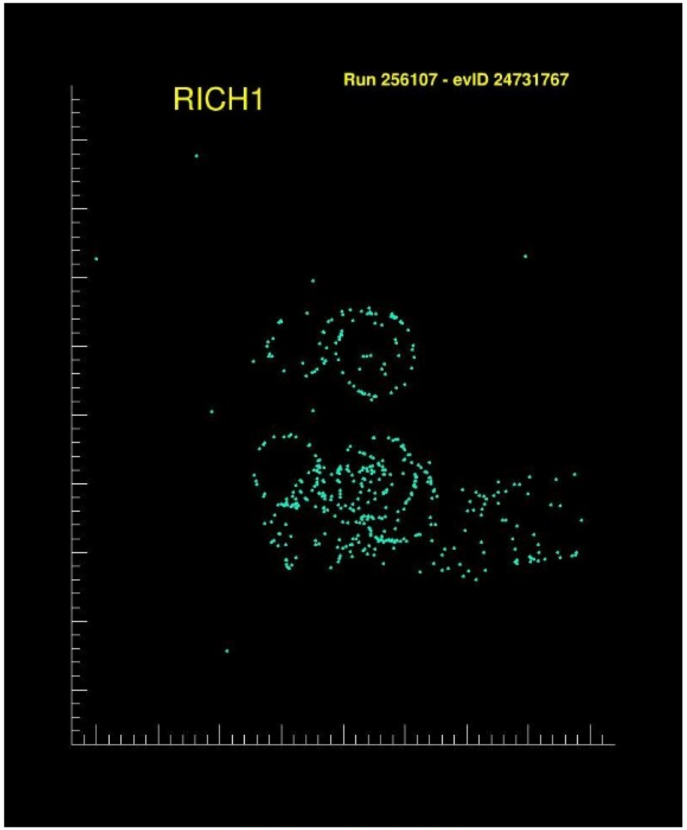
Looking ahead

Problems

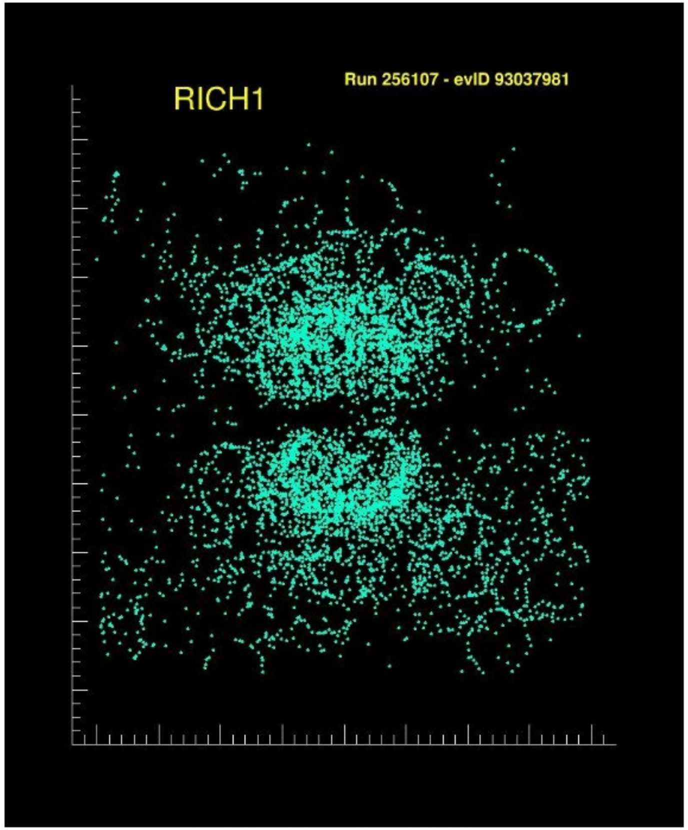
What do you think are the main challenges ahead?

Problems

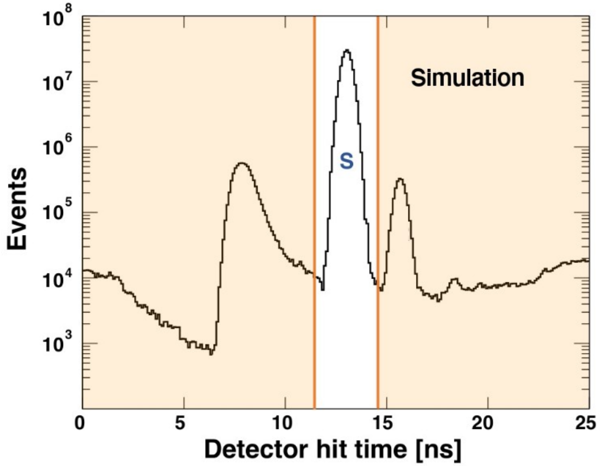
→ Rate



483 hits in RICH1



5720 hits in RICH1



Need timing and Granularity

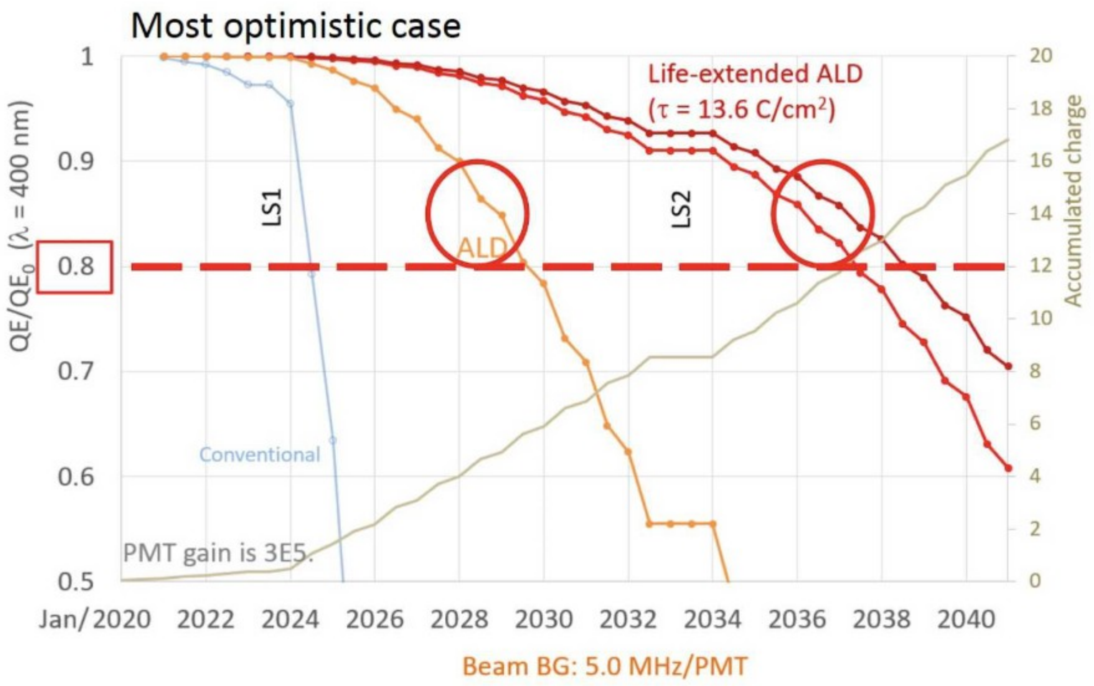
→ MPCPMT

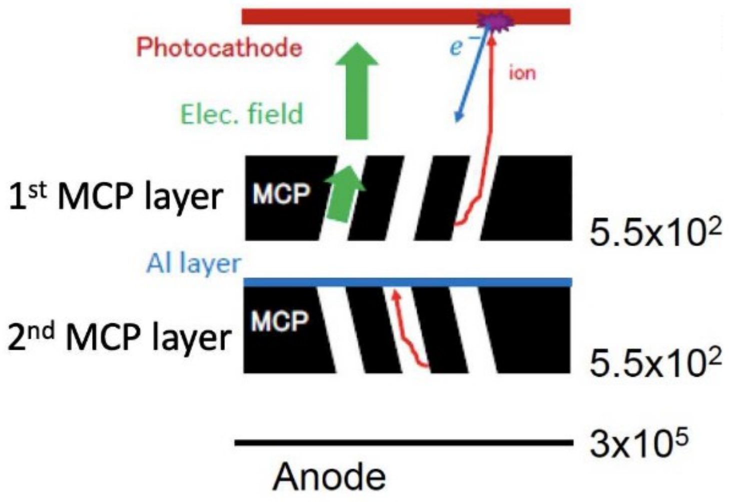
→ SiPM

→ Focussing

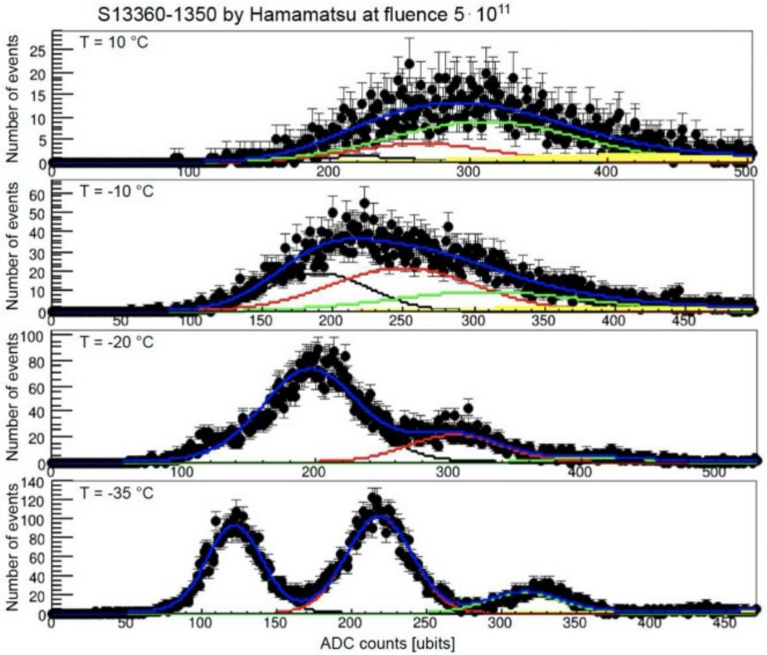
Problems

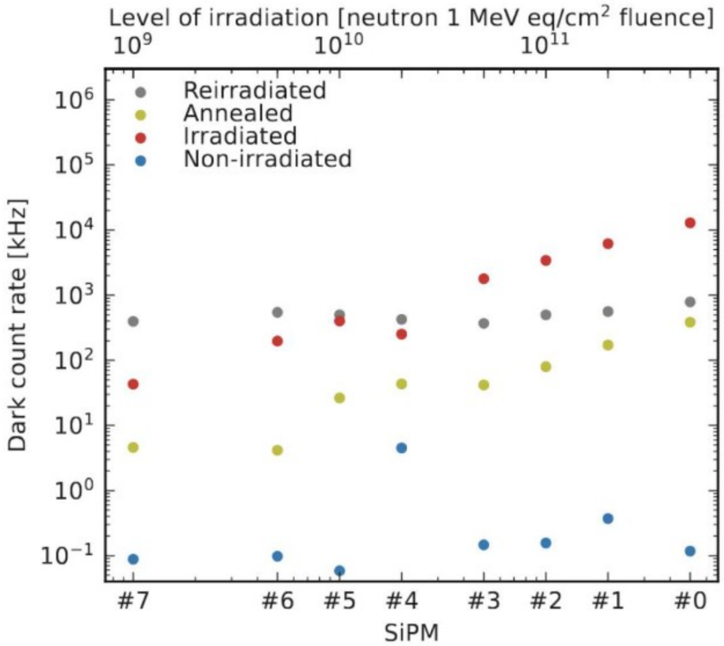
→ Ageing





→ Radiation damage





Problems

→ Environmental impact

CF_4 LHCb RICH2: **GWP = ¬ 7000 $_{\_CO2}$***

C_2F_6 ATLAS evap cooling (potential use): **GWP = ¬ 11000 $_{\_CO2}$***

C_3F_8 **F2 Chemicals** , Astor, ATLAS evap cooling: **GWP = ¬ 8500 $_{\_CO2}$***

C_4F_{10} LHCb RICH1, COMPASS RICH: **GWP = ¬ 8500 $_{\_CO2}$***

C_5F_{12} PP50: **GWP = ¬ 8500 $_{\_CO2}$***

C_6F_{14} PP1, **F2 Chemicals**: Liquid cooling, many expts: **GWP = ¬ 8000 $_{\_CO2}$***

For some perspective: SF6 GWP = ¬ 23000 $_{\_CO2}$*

- See references: 100 yr (average of 2nd, 4th & 5th assessment reports; AR2,4,5)
- https://www.ghgprotocol.org/sites/default/files/ghgp/Global-Warming-Potential-Values%20%28Feb%2016%202016%29_1.pdf

Take home

→ PID is used at most of the current experiment, and PID information can come from (almost) any device

→ PID requires complementarity

→ The richest field of development is on Cherenkov detectors (Belle II, ePIC, LHCb)

→ We all want the same things:

- → fast timing (10-100 ps)
- → high granularity
- → background resilience
- → rate tollerance

Thank you

Theoretical approaches

Somehow, the theoretical description of the Cherenkov radiation is never (to me) fully satisfactory. Different authors take different paths that highlight either one feature or another one.

Panofsky-Phillips : Assume from the very beginning that there is radiation
Frank-Tamm : Get the energy spectrum from the Maxwell equations in the medium
Jackson : Derive Frank-Tamm from the Fermi radiation theory.
Jelley : As Frank-Tamm

These lectures are the result of a tailoring of different parts (plus some personal additions), and you won't find them on any book (that I know about...)

The texts which are closer (but not identical) to the first part of this lecture are:
→ Hirose Akira, lecture notes (http://physics.usask.ca/~hirose/p812/notes.htm)
→ Jackson, classical Electrodynamics, 2^{nd} Ed.

Where to start from?

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The starting point of all this derivation are the covariant Maxwell equations

$$\nabla^2 \mathbf{A}(\mathbf{x}, t) - \frac{1}{c^2}\frac{\partial \mathbf{A}(\mathbf{x}, t)}{\partial t} = \mu \mathbf{J}(\mathbf{x}, t)$$

$$\nabla^2 \phi(\mathbf{x}, t) - \frac{1}{c^2}\frac{\partial \phi(\mathbf{x}, t)}{\partial t} = \frac{\rho(\mathbf{x}, t)}{\epsilon}$$

Lienard – Wiechert Potentials

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$$\nabla^2 \phi(\mathbf{x},t) - \frac{1}{c^2}\frac{\partial \phi(\mathbf{x},t)}{\partial t} = \frac{\rho(\mathbf{x},t)}{\epsilon}$$

$$\mathbf{J}(\mathbf{x}',t') = e\,\mathbf{v}(\mathbf{x}',t')\delta^3(\mathbf{x}' - \mathbf{r}_p(t'))$$

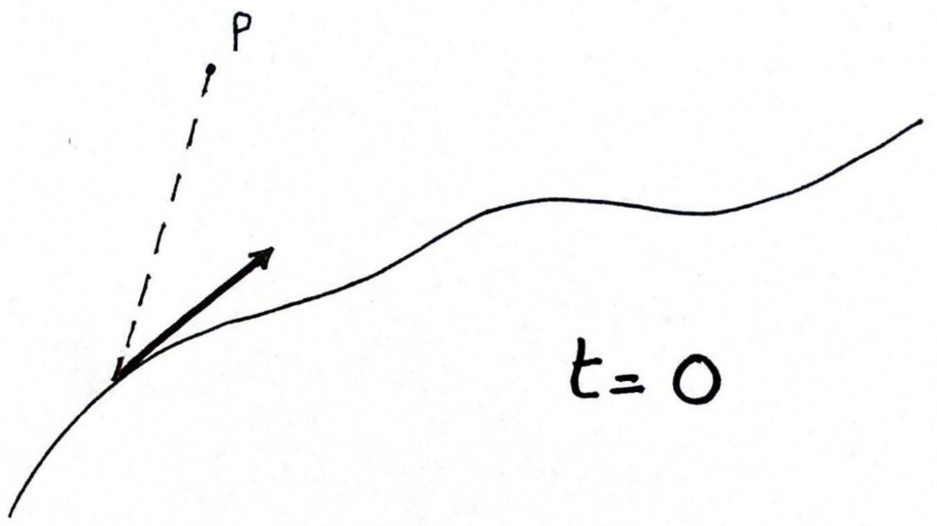
$$\rho(\mathbf{x}',t') = e\,\delta^3(\mathbf{x}' - \mathbf{r}_p(t'))$$

$$\mathbf{A}(\mathbf{x},t) = e\left[\frac{\boldsymbol{\beta}(t')}{[1-\boldsymbol{\beta}(t')\cdot\hat{\mathbf{n}}(t')]^3 R^2(t')}\right]_{ret}$$

$$\phi(\mathbf{x},t) = e\left[\frac{1}{[1-\boldsymbol{\beta}(t')\cdot\hat{\mathbf{n}}(t')]^3 R(t')}\right]_{ret}$$

The retardation effect

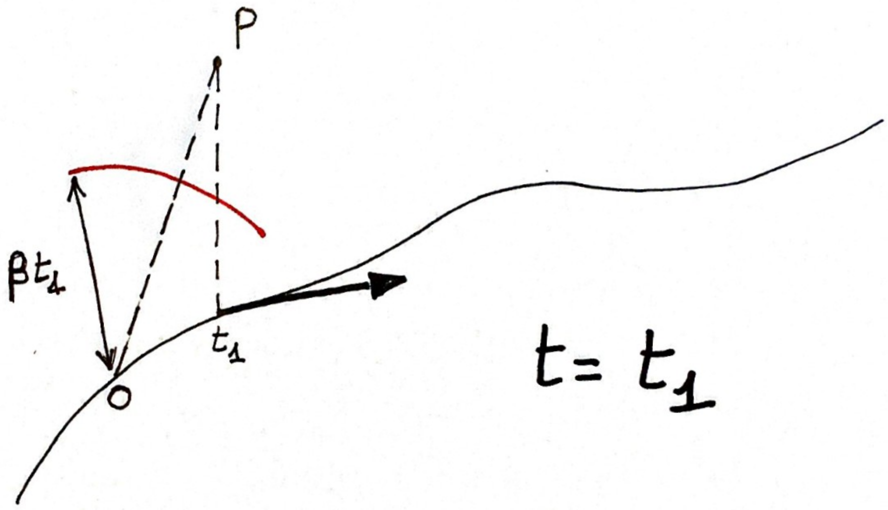
What does the “prime” means in the retarded potentials?



Let's imagine that a source starts moving at t=0. We want to know the field in the point P located at R from the source

Watch out: retardation is the very root of the Cherenkov effect !

The retardation effect



At the time t= t1 the source has moved, but the field perturbation has not reached P yet.

The retardation effect

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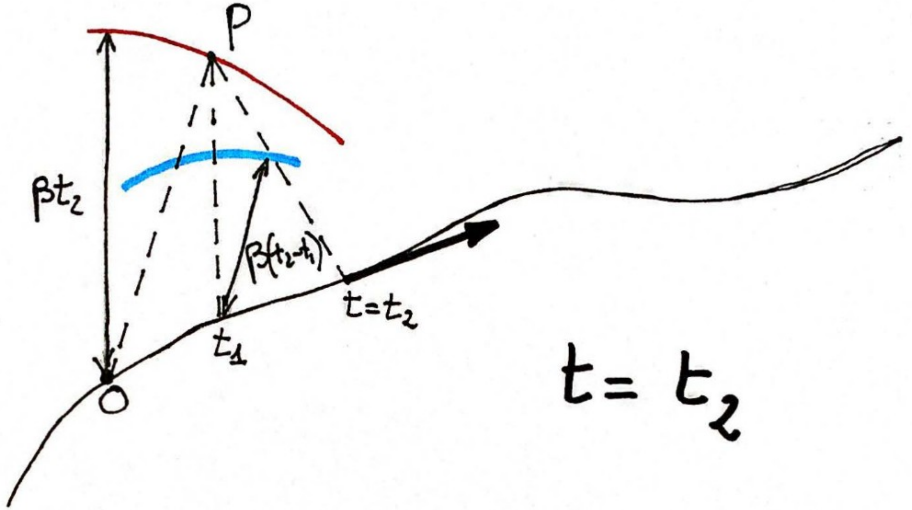


The field changes in P only at the time t2. At that point, the field in P is the one generated in O, but the source is now in a completely different position.

At the generic time t, the field in P is the one generated by the source at the t' time in the past, when it was at distance R(t') from P

$$\boxed{t = t' + \frac{R(t')}{c}}$$

The coordinate system

What is **n**? What is R? Where the hell is **x**?

$$\mathbf{A}(\mathbf{x},t) = e\left[\frac{\boldsymbol{\beta}(t')}{[1-\boldsymbol{\beta}(t')\cdot\hat{\mathbf{n}}(t')]^3 R^2(t')}\right]_{ret}$$

→ P is where the field is evaluated

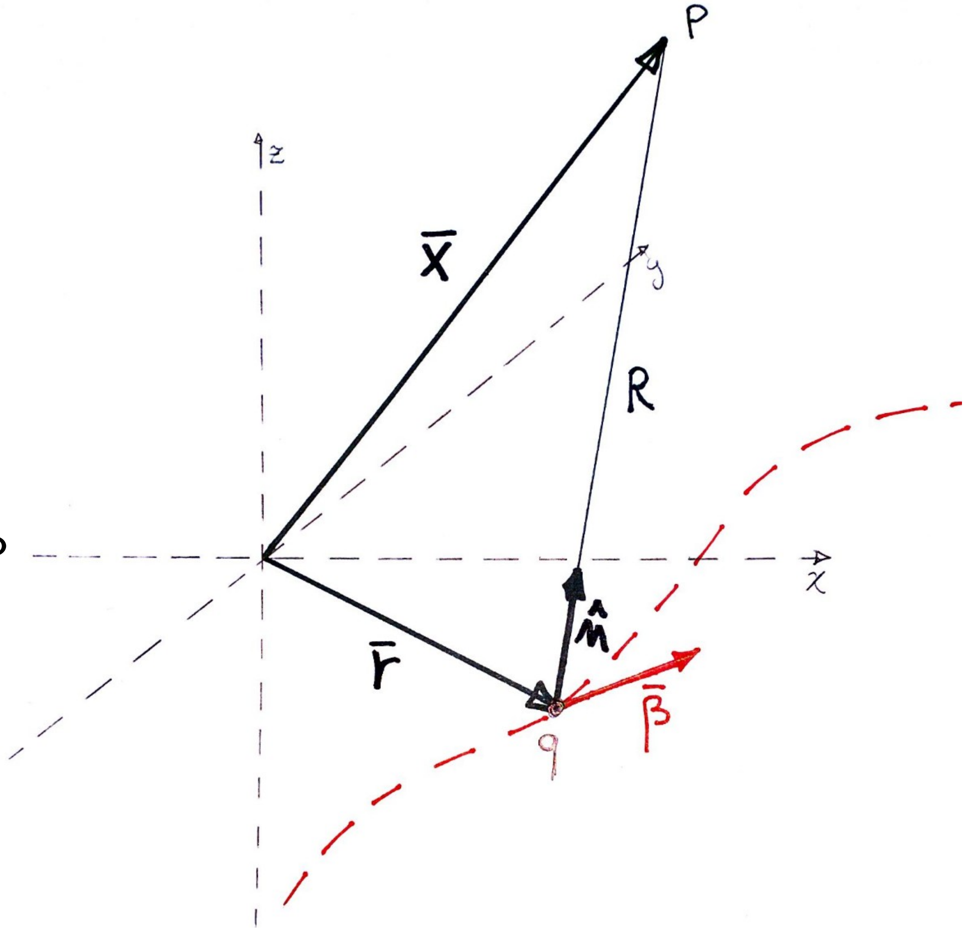
→ **x** is the coordinate of P.

→ R(t) is the distance between the source and P

→ **r**(t) is the position of the source

→ **n**(t) is the versor along R

Contains implicitly **x** : **n** = (**x** – **r**)/r



From the potential to the fields

From the 4-potential, we can easily (ahhahaahhahaha) derive the fields

$$\mathbf{A}(\mathbf{x},t) = e\left[\frac{\boldsymbol{\beta}(t')}{\left[1-\boldsymbol{\beta}(t')\cdot\hat{\mathbf{n}}(t')\right]^3 R^2(t')}\right]_{ret}$$

$$\phi(\mathbf{x},t) = e\left[\frac{1}{\left[1-\boldsymbol{\beta}(t')\cdot\hat{\mathbf{n}}(t')\right]^3 R(t')}\right]_{ret}$$

For the full calculations: Jackson 2^{nd} Ed, pp. 654-658

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$$\phi(\mathbf{x},t) = e\left[\frac{1}{[1-\boldsymbol{\beta}(t')\cdot\hat{\mathbf{n}}(t')]^3 R(t')}\right]_{ret}$$

$\Rightarrow$

$$\boxed{\begin{aligned}\mathbf{E} &= -\frac{\partial \mathbf{A}}{\partial t} - \vec{\nabla}\phi \\ \mathbf{B} &= \overline{\nabla} \times \mathbf{A}\end{aligned}}$$

Plug the potential in the field definition...

For the full calculations: Jackson 2nd Ed, pp. 654-658

From the potential to the fields

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$$\boxed{\begin{aligned}\mathbf{E} &= -\frac{\partial \mathbf{A}}{\partial t} - \vec{\nabla}\phi \\ \mathbf{B} &= \overline{\nabla} \times \mathbf{A}\end{aligned}}$$

… and get out the fields!

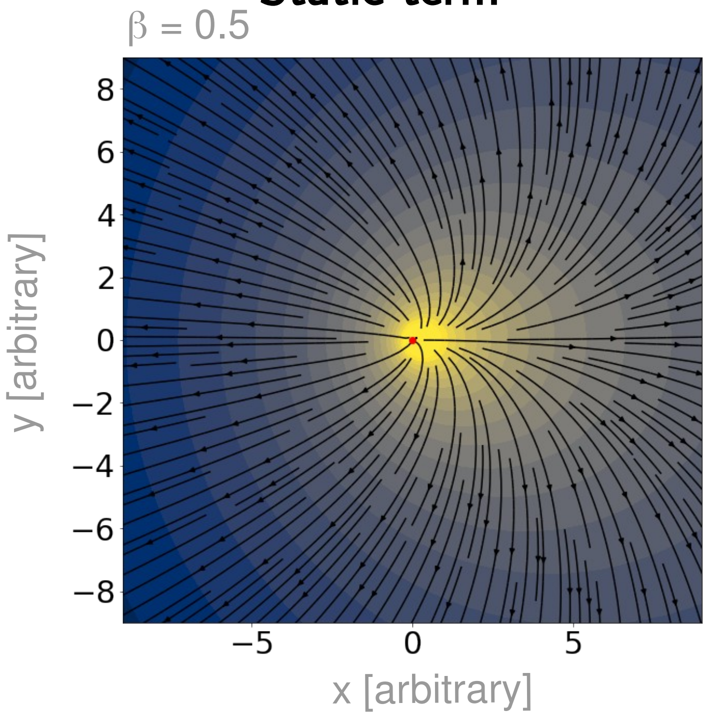
$$\boxed{\begin{aligned}\mathbf{E}(\mathbf{x},t) &= e\left[\frac{\hat{\mathbf{n}}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^3 R^2}\right]_{ret} + \frac{e}{c}\left[\frac{\hat{\mathbf{n}}\times((\hat{\mathbf{n}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}})}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^3 R}\right]_{ret} \\ \mathbf{B} &= [\mathbf{n}\times\mathbf{E}]_{ret}\end{aligned}}$$

For the full calculations: Jackson 2nd Ed, pp. 654-658

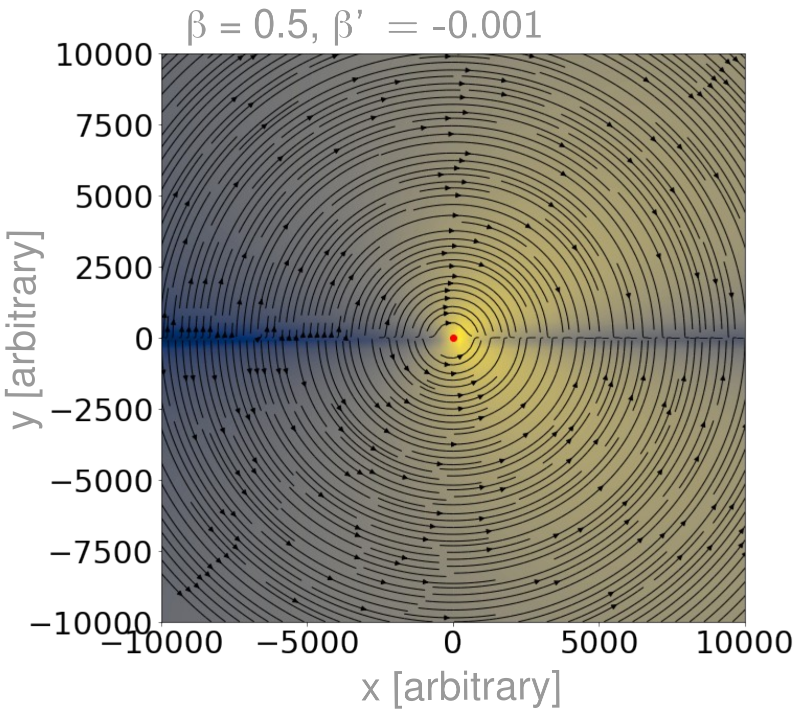
A closer look

$$\mathbf{E}(\mathbf{x},t) = e\left[\frac{\hat{\mathbf{n}}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^3R^2}\right]_{ret} + \frac{e}{c}\left[\frac{\hat{\mathbf{n}}\times((\hat{\mathbf{n}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}})}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^3R}\right]_{ret}$$

Static term

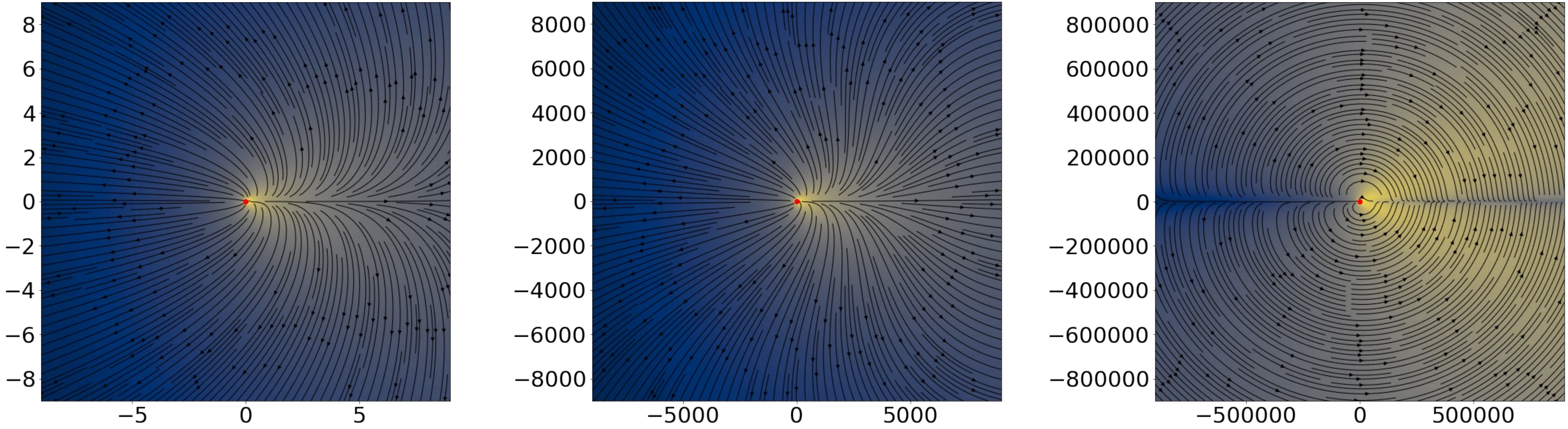


Dynamic term



A closer look

$$\mathbf{E}(\mathbf{x},t) = e\left[\frac{\hat{\mathbf{n}}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^3R^2}\right]_{ret} + \frac{e}{c}\left[\frac{\hat{\mathbf{n}}\times((\hat{\mathbf{n}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}})}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^3R}\right]_{ret}$$



Far field approximation: the Coulomb term is negligible at large distances

A closer look

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$$\mathbf{E}(\mathbf{x},t) = e\left[\frac{\hat{\mathbf{n}}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^3R^2}\right]_{ret} + \frac{e}{c}\left[\frac{\hat{\mathbf{n}}\times((\hat{\mathbf{n}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}})}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^3R}\right]_{ret}$$

There is also another, fundamental reason to neglect the Coulomb part in this context.

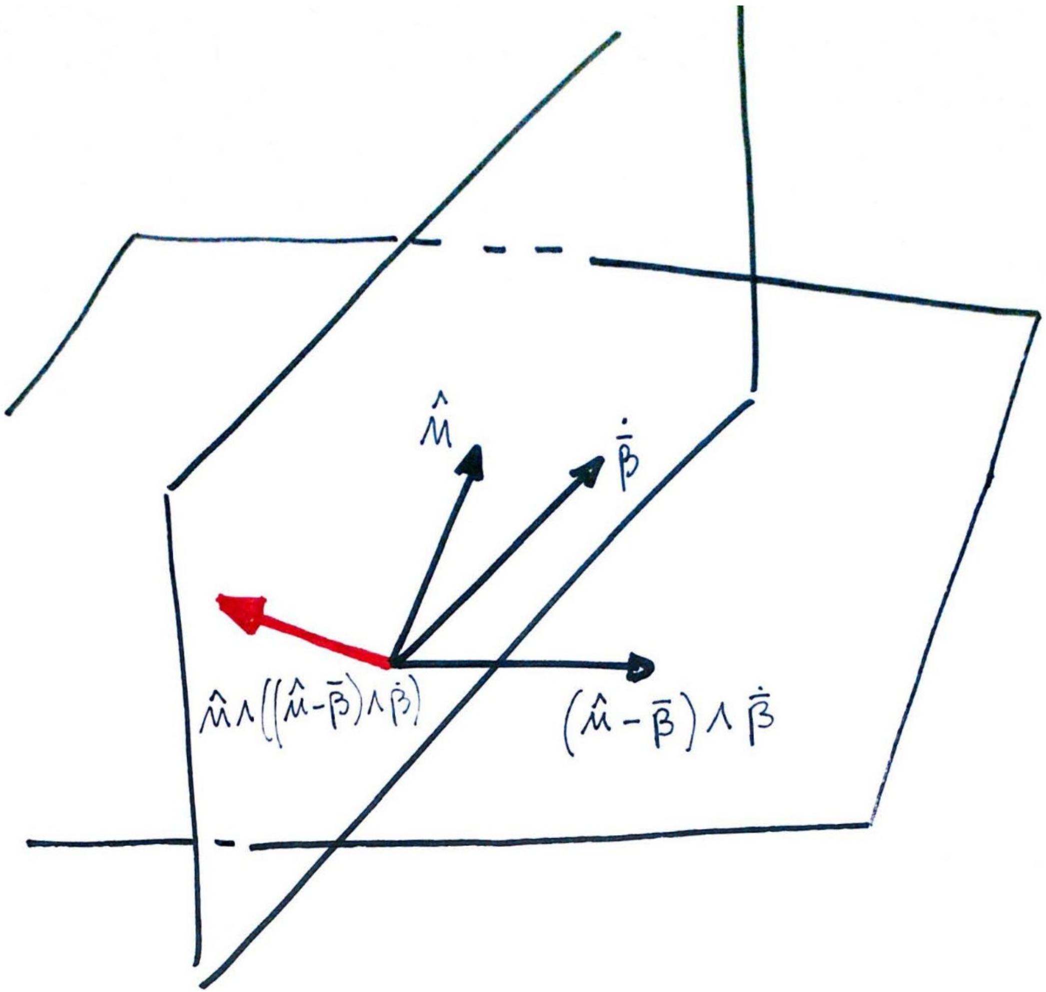
Can you figure it out?

Far field polarization

$$\mathbf{E}(\mathbf{x},t) = \frac{e}{c}\left[\frac{\hat{\mathbf{n}} \times ((\hat{\mathbf{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})}{(1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}})^3 R}\right]_{ret}$$

The far field is confined to the plane that comprises the origin, the source and the observation point, and is always orthogonal to the propagation direction

The radiation is linearly polarized!



How to get to the spectrum?

A this point we have a field generated **just because the source is moving**

$\rightarrow$ Derived in the vacuum, applies also in media re-defining b as b = v*n/c

$\rightarrow$ The field has an intriguing singularity at $\beta = 1$ ...

$$\mathbf{E}(\mathbf{x},t) = \frac{e}{c}\left[\frac{\hat{\mathbf{n}} \times ((\hat{\mathbf{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}})^3 R}\right]_{ret}$$

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To study its spectrum and angular distribution we will study the Intensity spectrum.

The intensity is defined in the standard way as the flux of the Poynting vector which can be also written in terms of the electric field only:

$$\frac{dI(\mathbf{x}, \omega)}{d\Omega} = \frac{c}{2\pi}|R \cdot \mathbf{E}(\mathbf{x}, \omega)|^2$$

For the full calculations of this part: Jackson 2nd Ed, pp. 668 and following

Facing the Fourier transform

Almost all the interesting part of the calculation boils down to computing this:

$$R \cdot \mathbf{E}(\mathbf{x}, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} R \cdot \mathbf{E}(\mathbf{x}, t) e^{i\omega t} dt$$

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Almost all the interesting part of the calculation boils down to computing this:

$$R \cdot \mathbf{E}(\mathbf{x}, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} R \cdot \mathbf{E}(\mathbf{x}, t) e^{i\omega t} dt$$

$$\mathbf{E}(\mathbf{x}, t) = \frac{e}{c} \left[\frac{\hat{\mathbf{n}} \times ((\hat{\mathbf{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}})^3 R} \right]_{ret}$$

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For the full calculations of this part: Jackson 2nd Ed, pp. 668 and following

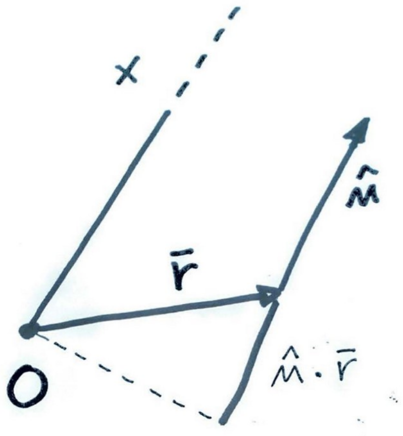
Facing the Fourier transform

How to deal with the retarded time?

$$t = t' + \frac{R(t')}{c}$$

$$t = t' + \frac{R(t')}{c} \approx t' + x/c - \frac{\hat{\mathbf{n}} \cdot \mathbf{r}(t')}{c}$$

In the far field, **x** and R are parallel



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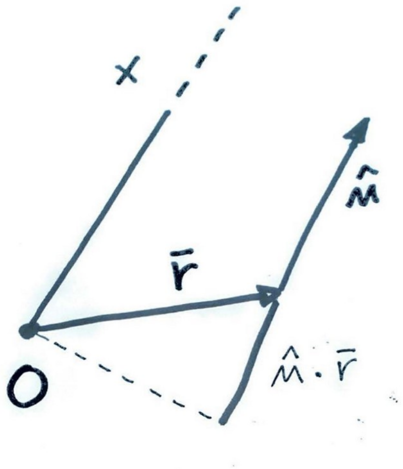

This becomes just and overall phase that can be neglected

Keep and eye on this...

$$\frac{dI(\mathbf{x},\omega)}{d\Omega} = \frac{e^2}{4\pi^2 c}\left| \int_{-\infty}^{+\infty} e^{i\omega\left(t' - \frac{\hat{\mathbf{n}}\cdot\mathbf{r}(t')}{c}\right)} \frac{\hat{\mathbf{n}} \times ((\hat{\mathbf{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})}{(1 - \boldsymbol{\beta}\cdot\hat{\mathbf{n}})^2} dt' \right|^2$$

$3 \to 2$ because of the change of variable in the differential

For the full calculations of this part: Jackson 2nd Ed, pp. 668 and following

Here the magic happens

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This is probably the most important and yet most controversial part of the calculation.

In the far field approx, the versor **n** is almost constant. Therefore, it can be proven that:

$$\frac{\hat{\mathbf{n}} \times ((\hat{\mathbf{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}})^2} = \frac{d}{dt}\left(\frac{\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \boldsymbol{\beta})}{1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}}}\right)$$

For the full calculations of this part: Jackson 2nd Ed, pp. 668 and following

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And the Fourier integral can be carried out by parts:

$$\frac{dI}{d\Omega} = \frac{e^2}{4\pi^2 c}\left|\left[e^{i\omega\left(t' - \frac{\hat{\mathbf{n}}\cdot\mathbf{r}}{c}\right)}\frac{\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \boldsymbol{\beta})}{1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}}}\right]_{-\infty}^{+\infty} + \int_{-\infty}^{+\infty} i\omega e^{i\omega\left(t' - \frac{\hat{\mathbf{n}}\cdot\mathbf{r}}{c}\right)}\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\boldsymbol{\beta})dt'\right|^2$$

For the full calculations of this part: Jackson 2nd Ed, pp. 668 and following (***not true, not even Jackson does this explicitly…***)

The "Cherenkov frame"

So far we treated the system in a rather general way, but for the case of Cherenkov radiation we can make few assumptions that greatly simplify the geometry:

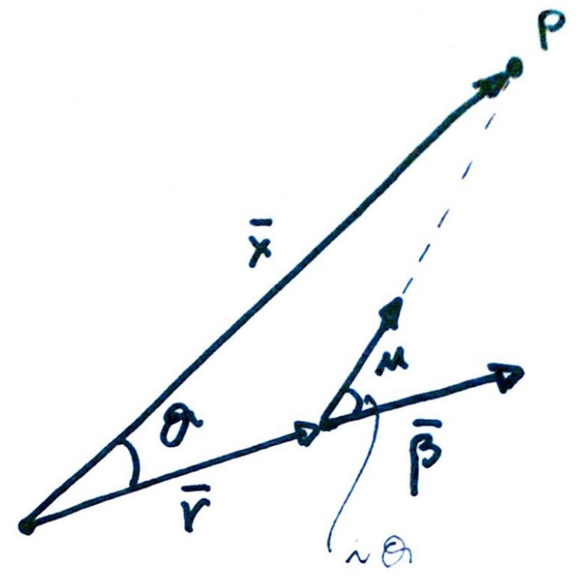
→ **The source moves on a line**

→ **We use the source direction as reference axis**

$$\hat{\mathbf{n}} \cdot \mathbf{r}/c = \hat{\mathbf{n}} \cdot \mathbf{v} \cdot t/c = \beta \cos\theta \cdot t$$

$$|\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \boldsymbol{\beta})| \approx \beta \sin\theta$$

$$1 - \boldsymbol{\beta} \cdot \hat{\mathbf{n}} \approx 1 - \beta \cos\theta$$



Solving the integral

$$\frac{d^2I(\theta,\omega)}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c}\left|\left[e^{i\omega t(1-\beta\cos\theta)}\frac{\beta\sin\theta}{1-\beta\cos\theta}\right]_{-\infty}^{+\infty} + \int_{-\infty}^{+\infty} i\omega e^{i\omega t(1-\beta\cos\theta)}\beta\sin\theta dt\right|^2$$

This part is zero since $\mathbf{n}$ and $\boldsymbol{\beta}$ are parallel at infinite.

$$\boxed{\frac{d^2I(\mathbf{x},\omega)}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c}\omega^2 v^2 \sin^2\theta\left|\int_{-\infty}^{+\infty} e^{i\omega t(1-\beta\cos\theta)}dt\right|^2}$$

We won!

This is an expression of the radiation intensity as function of frequency and angle

$$\frac{d^2 I(\mathbf{x},\omega)}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c}\omega^2 v^2 \sin^2\theta \left| \int_{-\infty}^{+\infty} e^{i\omega t(1-\beta\cos\theta)} dt \right|^2$$

$$\frac{d^2 I}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c}\omega^2 v^2 \sin^2\theta\, \delta^2(\omega(1-\beta\cos\theta))$$

We won! Did we?

This is an expression of the radiation intensity as function of frequency and angle

$$\frac{d^2 I(\mathbf{x},\omega)}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c}\omega^2 v^2 \sin^2\theta \left| \int_{-\infty}^{+\infty} e^{i\omega t(1-\beta\cos\theta)} dt \right|^2$$

Unfortunately this is a representation of a delta function

The square of the delta function is not integrable.
Everything diverges!

$$I = \frac{e^2}{4\pi^2 c} v^2 \int\int d\omega d\Omega \ \sin^2\theta \omega^2 \ 2\pi \ \delta^2[\omega(1-\beta\cos\theta)]$$

Rethinking our mistakes

So, what led us to create free, endless energy
(and incidentally blow up the whole universe in the process)?

$$\frac{d^2 I(\mathbf{x},\omega)}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c}\omega^2 v^2 \sin^2\theta \left| \int_{-\infty}^{+\infty} e^{i\omega t(1-\beta\cos\theta)} dt \right|^2$$

First, this object is intriguing, since it is well defined also for $\beta > 1$.
Perhaps this is the Cherenkov radiation, but we are missing some other contribution that culls the divergence...

Rethinking our mistakes

First hint:

$$\frac{d^2 I(\mathbf{x},\omega)}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c}\left|\int_{-\infty}^{+\infty} e^{i\omega\left(t' - \frac{\hat{\mathbf{n}}\cdot\mathbf{r}(t')}{c}\right)} \frac{\hat{\mathbf{n}} \times ((\hat{\mathbf{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})}{(1 - \boldsymbol{\beta}\cdot\hat{\mathbf{n}})^2} dt'\right|^2$$

This integral runs over an infinite range only formally.
The acceleration can be non-zero only over a finite time interval (otherwise we would violate the energy conservation principle), and so it acts as a damping function

$$\frac{d^2 I}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c}\left|\int_{-\infty}^{+\infty} i\omega e^{i\omega\left(t' - \frac{\hat{\mathbf{n}}\cdot\mathbf{r}}{c}\right)} \hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \boldsymbol{\beta}) dt'\right|^2$$

Here we removed the damping function and the integral is really over an infinite time!

Rethinking our mistakes

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Second hint:

$$\frac{d^2 I}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c}\left|\left[e^{i\omega(t'-\frac{\hat{\mathbf{n}}\cdot\mathbf{r}}{c})}\frac{\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\boldsymbol{\beta})}{1-\boldsymbol{\beta}\cdot\hat{\mathbf{n}}}\right]_{-\infty}^{+\infty} + \int_{-\infty}^{+\infty} i\omega e^{i\omega(t'-\frac{\hat{\mathbf{n}}\cdot\mathbf{r}}{c})}\hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\boldsymbol{\beta})dt'\right|^2$$

This part is zero since **n** and **β** are (anti-)parallel at infinite

Suddenly **n** is not treated as constant anymore…

Rethinking our approximations

In summary:

$\rightarrow$ By removing the explicit dependence on the acceleration, we turned an integral that was formally, but not actually, on an infinite range to a real integral over an infinite range.

$\rightarrow$ To solve it, we had to violate the far field approximation

How can we get out of this?

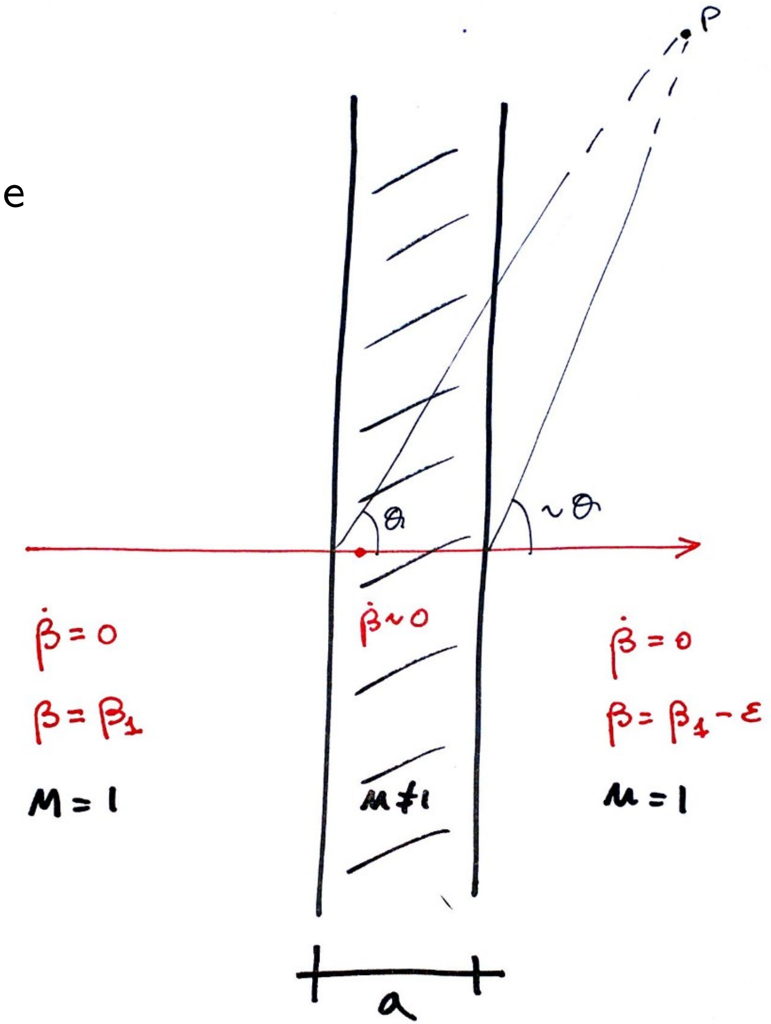
Rethinking our approximations

We have to introduce limits to out problem!

→ Take a slab of dielectric instead of an infinite one

→ Assume that β is almost constant while crossing the slab (but not strictly constant)

→ Assume that in the vacuum before and after there is no acceleration



More realistic dielectric

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Everything goes as before, but the integration range is now finite (the time to cross the slab)

$$\frac{d^2 I}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c}\left|\left[e^{i\omega t(1-\beta\cos\theta)}\frac{\beta\sin\theta}{(1-\beta\cos\theta)}\right]_{-a/2v}^{+a/2v} + \int_{-a/2v}^{+a/2v} i\omega e^{i\omega t(1-\beta\cos\theta)}\beta\sin\theta dt\right|^2$$

The calculation takes a bit but it's rather easy **if you assume to be able to treat β as a constant**

More realistic dielectric

$$\frac{d^2 I}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c} a^2 \, \omega^2 \, \sin^2\theta \left(\frac{\sin\alpha}{\alpha}\right)^2$$

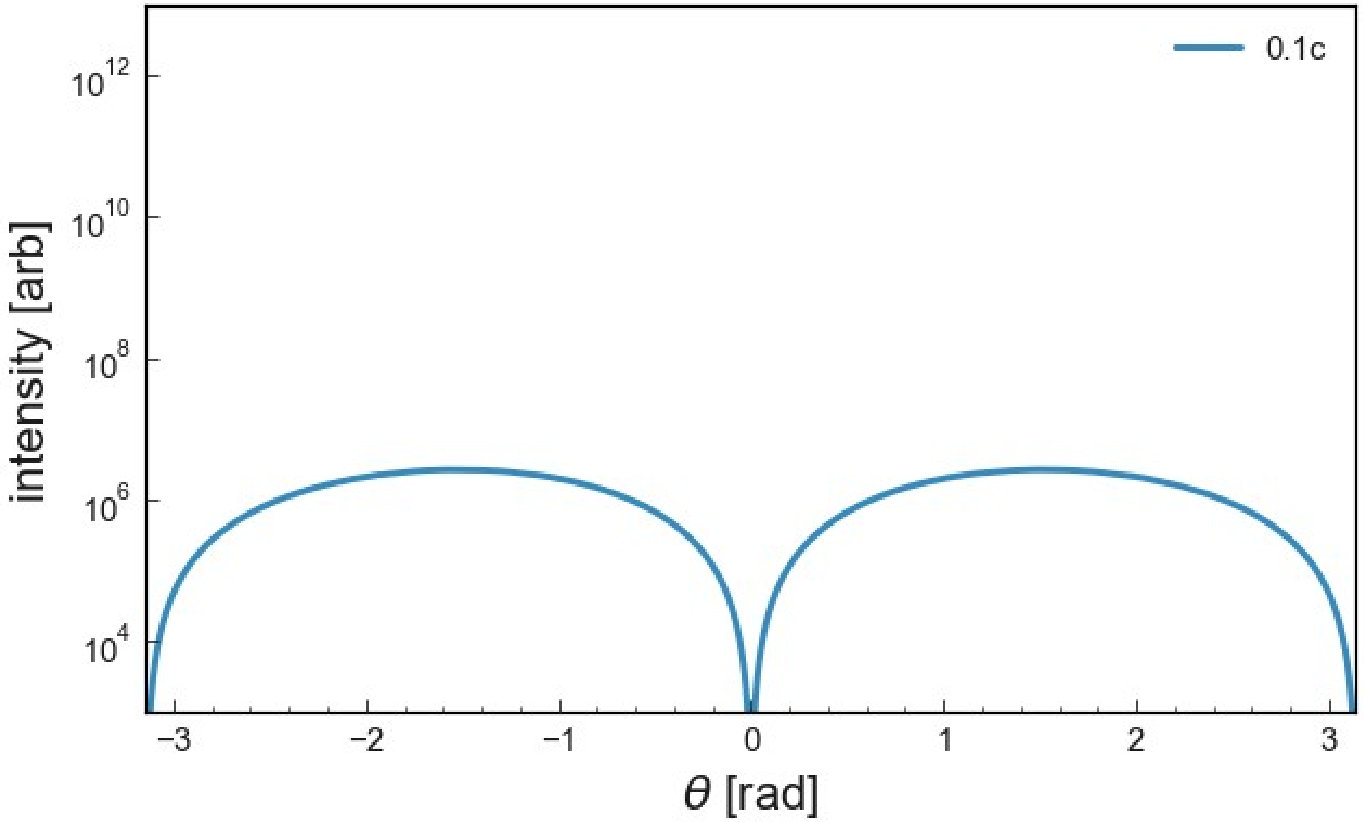
$$\alpha = \frac{1}{2}(1 - \beta\cos\theta)\omega\frac{a}{v}$$

Intensity of the radiation emitted by a particle of constant velocity when crossing a dielectric of thickness a.

β is IN the dielectric, so encodes the refractive index

Intensity distributions

OK, that was the last calculation! Now let's have a look at what we got.



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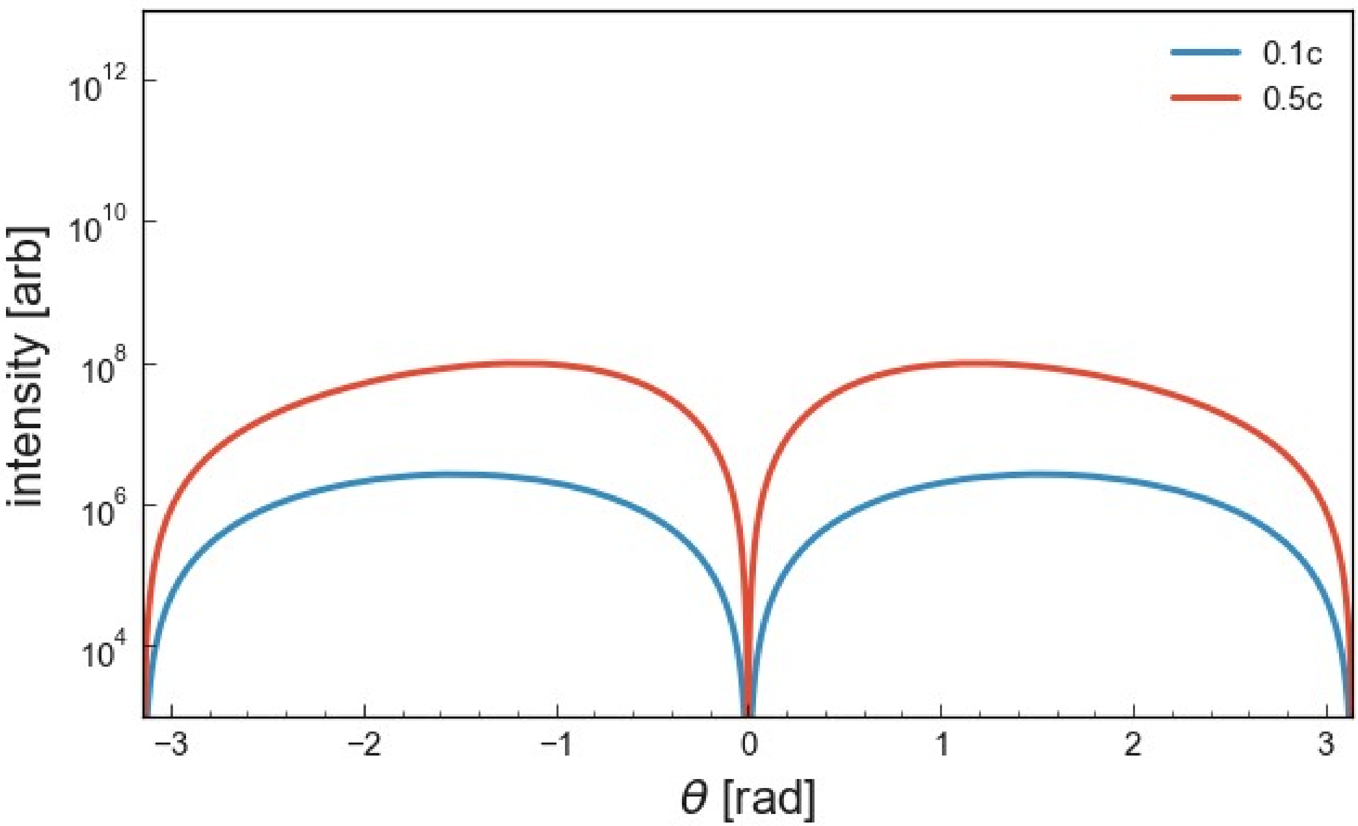

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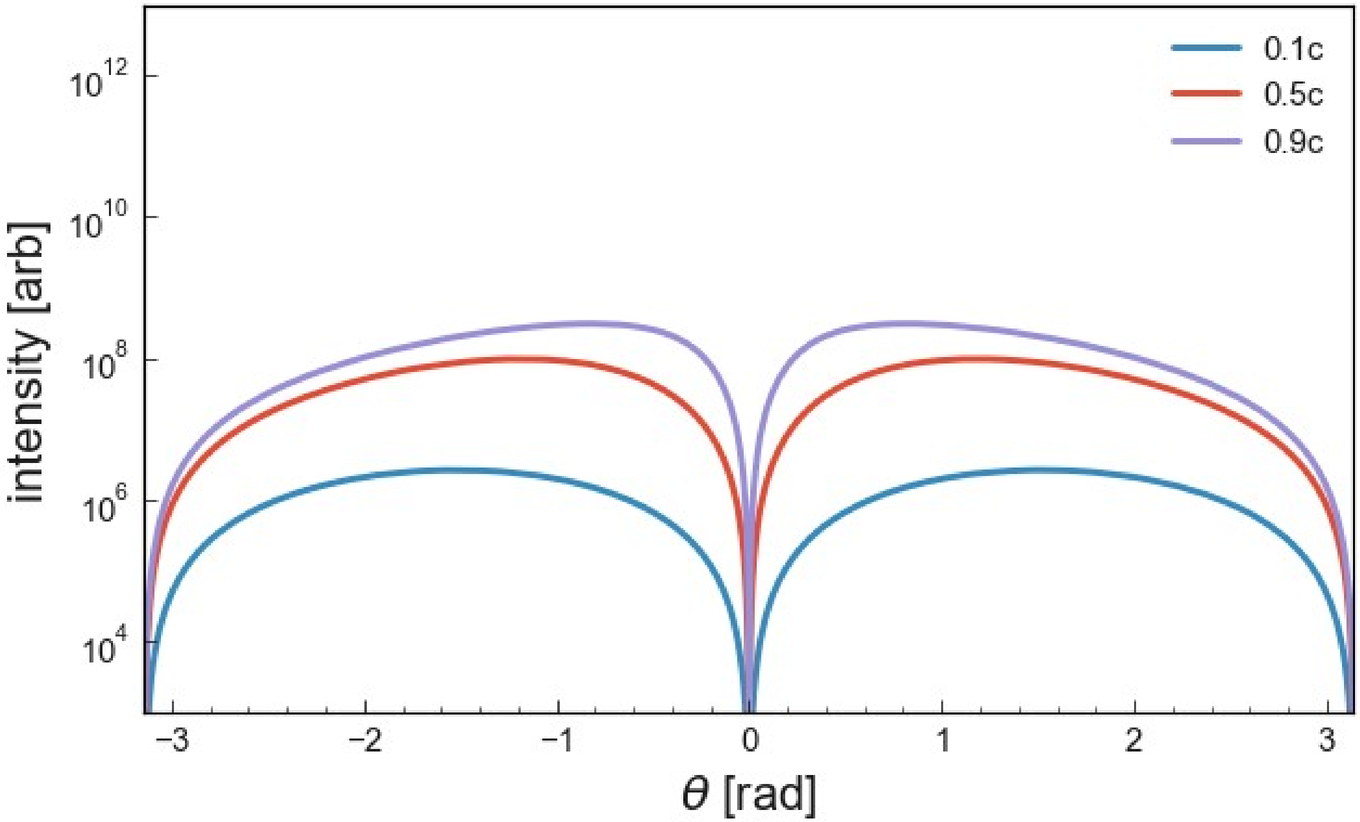

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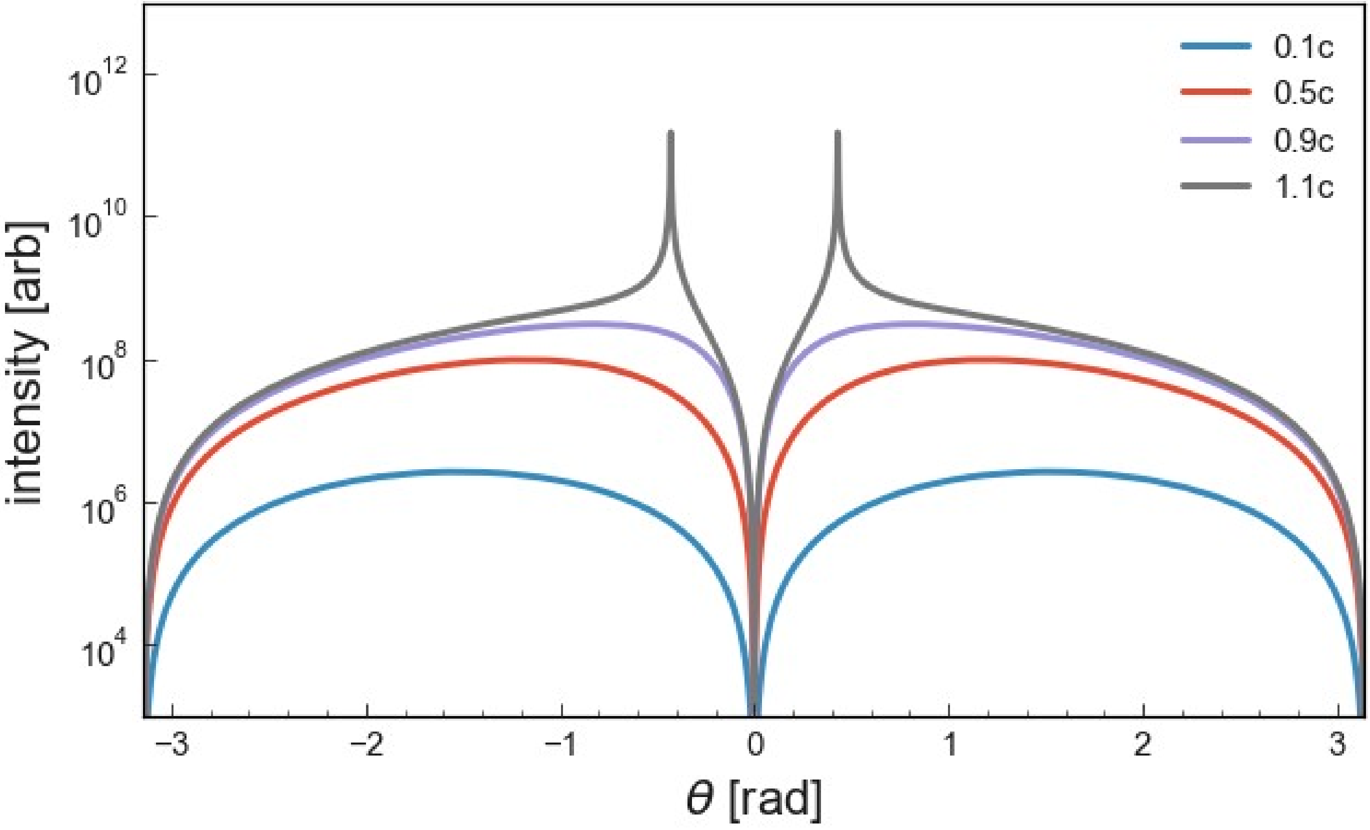
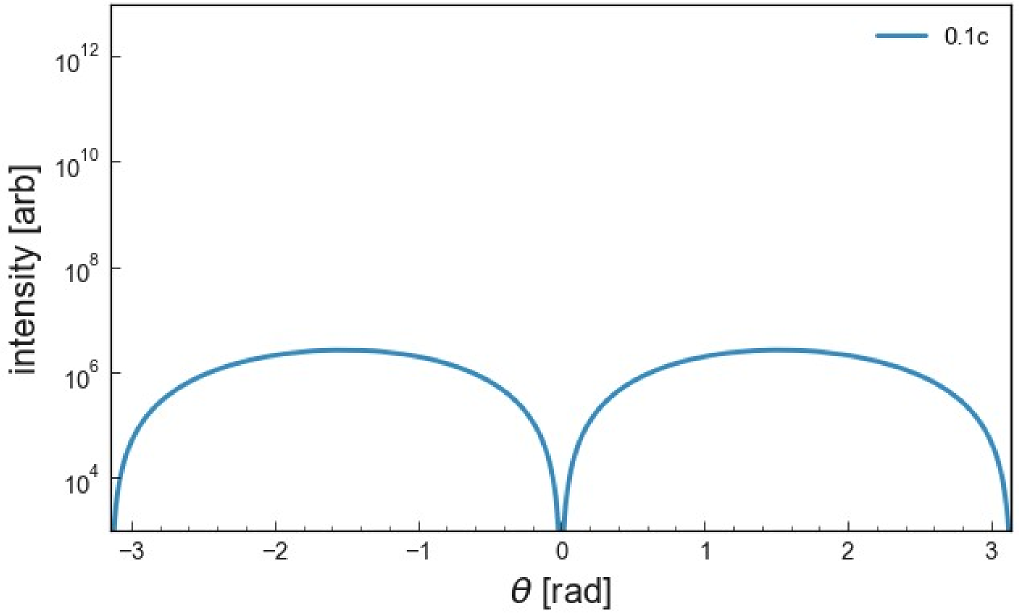


A closer look

Back to the formula, what is happening?



$$\alpha = \frac{1}{2}(1 - \beta\cos\theta)\omega\frac{a}{v}$$

~1

$$\frac{dI(\theta,\omega)}{d\Omega} = \frac{e^2}{4\pi^2 c}\, a^2\,\omega^2\,\sin^2\theta\left(\frac{\sin\alpha}{\alpha}\right)^2$$

~0, with oscillations

A closer look

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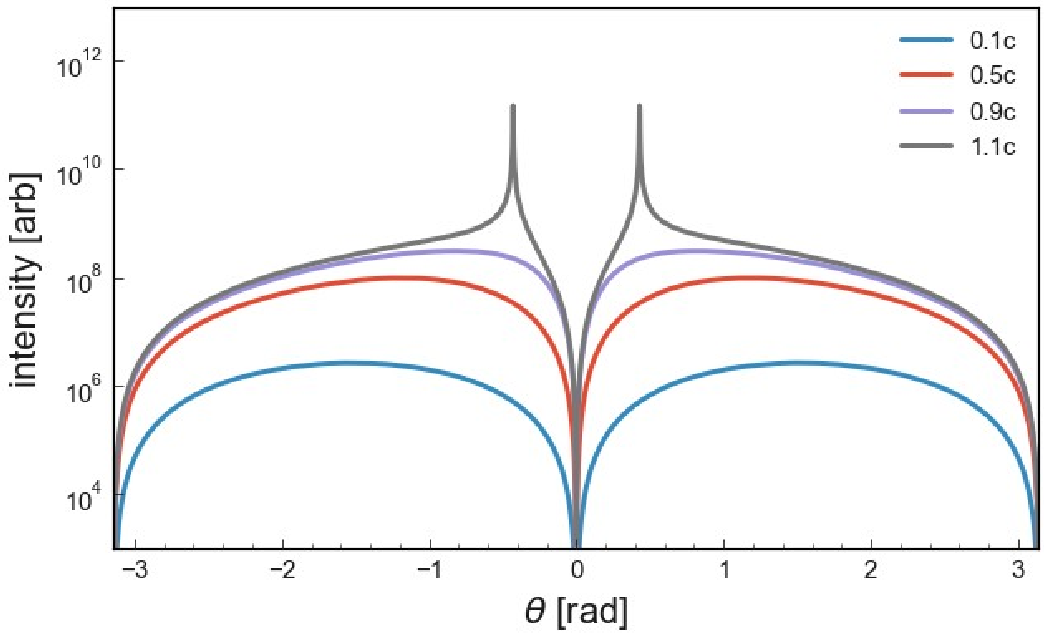


$$\alpha = \frac{1}{2}(1-\beta\cos\theta)\omega\frac{a}{v}$$

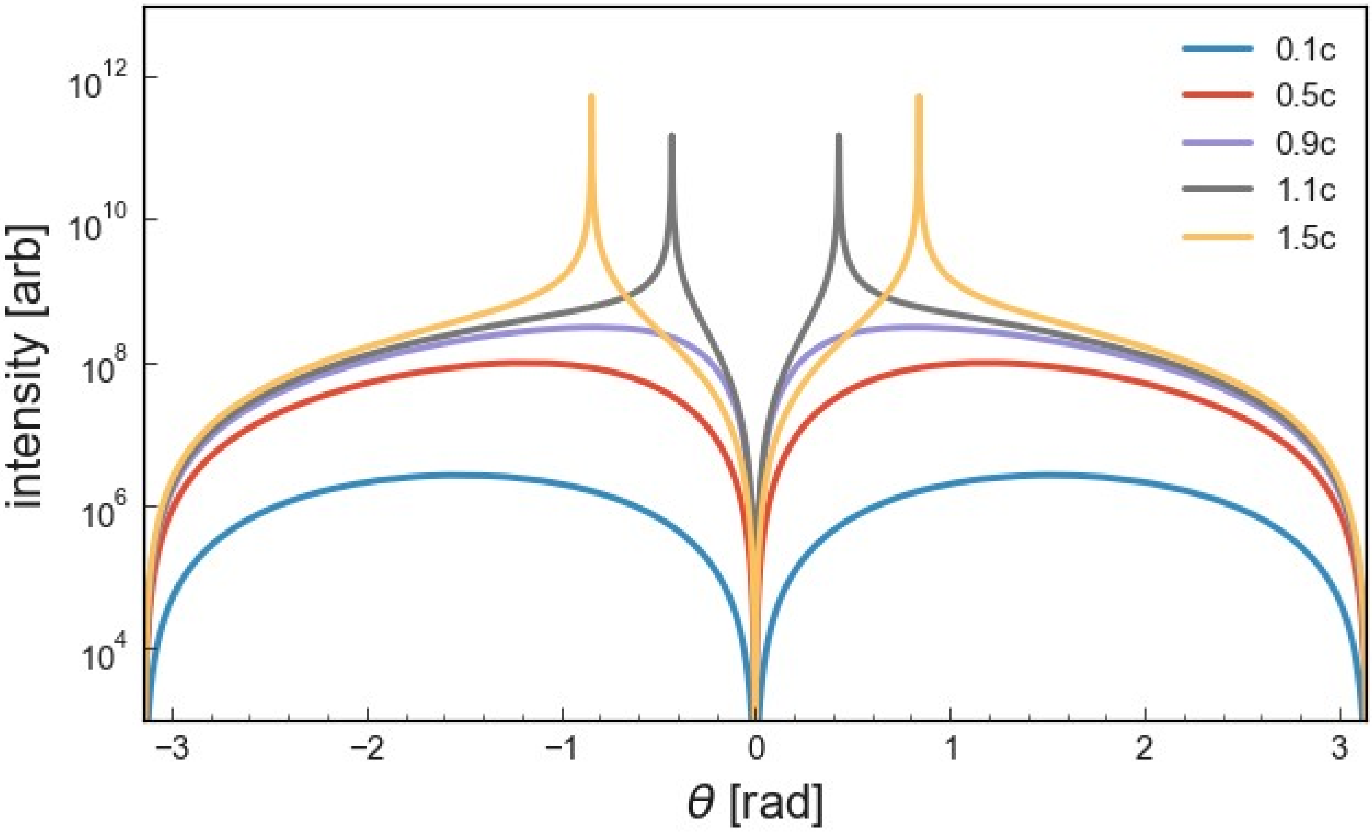
Goes to 0 at
cosθ = 1/β

$$\frac{d^2 I}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c}\, a^2\, \omega^2\, \sin^2\theta \left(\frac{\sin\alpha}{\alpha}\right)^2$$

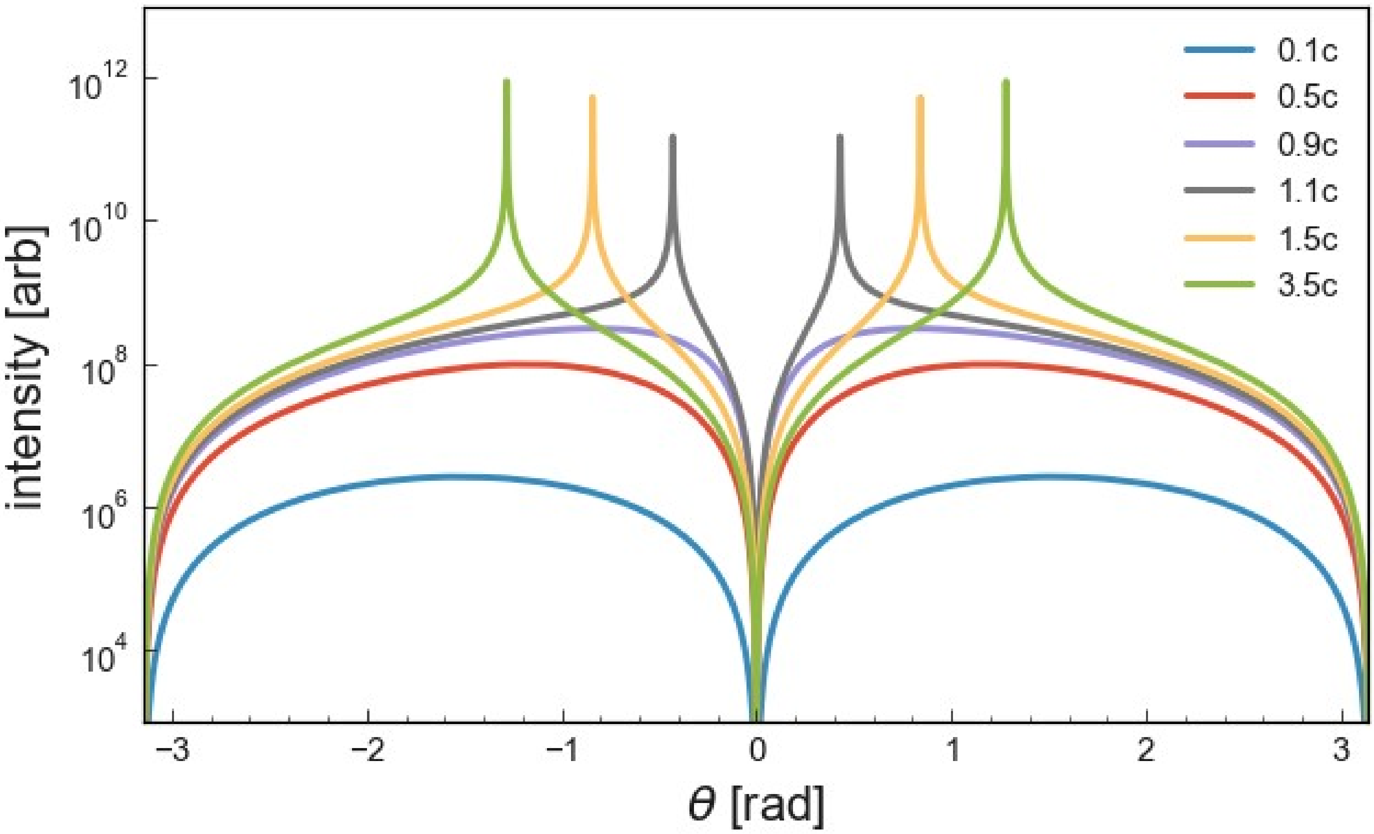
~ 1

Not a singularity, no infinite energy anymore

Increasing the speed



Increasing the speed



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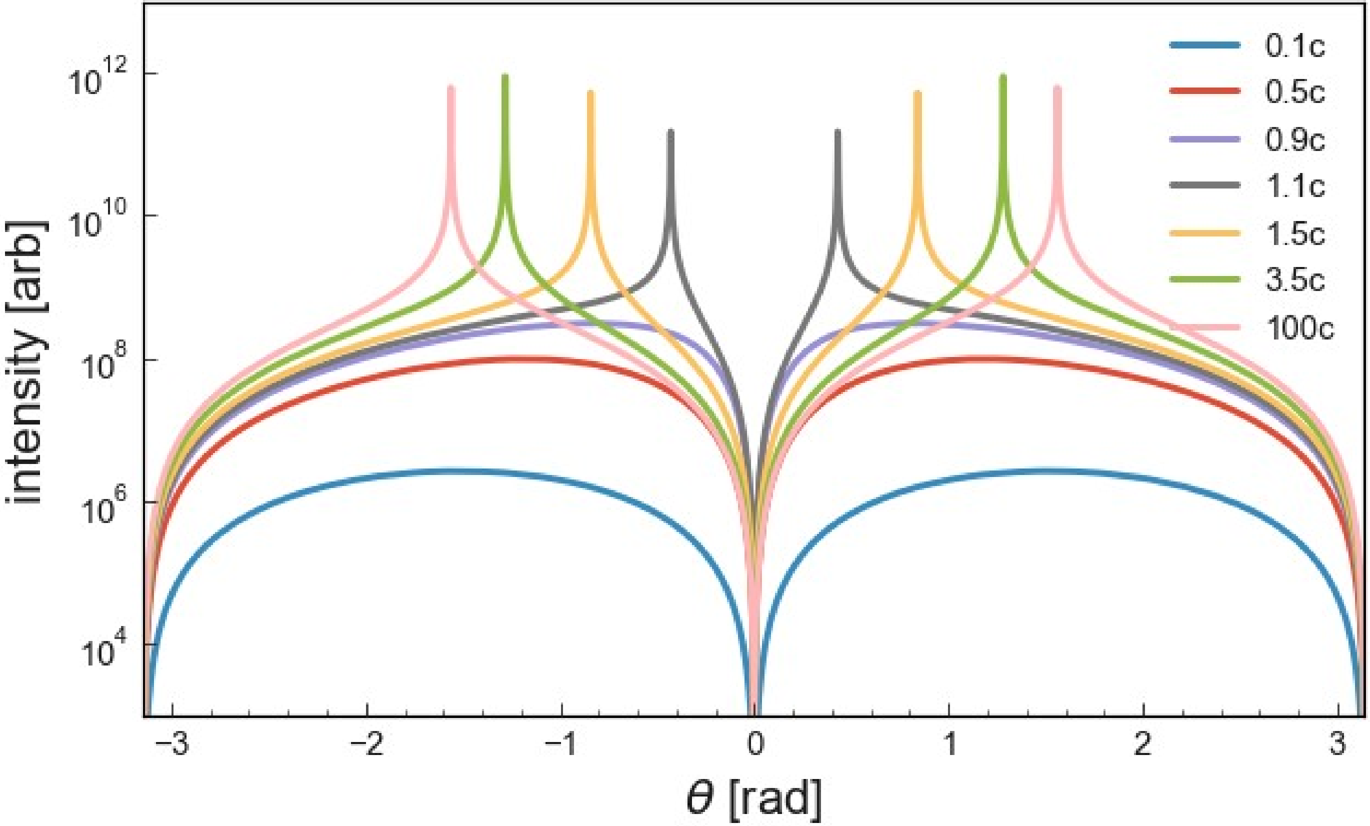

Towards the Frank-Tamm formula

How much radiation is emitted **per unit of dielectric?**

$$\frac{d^2 I}{d\Omega d\omega} = \frac{e^2}{4\pi^2 c}\, a^2\, \omega^2\, \sin^2\theta \left(\frac{\sin\alpha}{\alpha}\right)^2$$

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$$\alpha = \frac{1}{2}(1 - \beta\cos\theta)\omega\frac{a}{v}$$

Remember that a limit for sinc^2 is linked to the Dirac delta

$$\lim_{t\to\infty} \frac{1}{\pi t} \frac{sin^2[(x-a)t]}{(x-a)^2}$$