Present Status of Gravitational Wave Detection and Future Programs

Adalberto Giazotto

INFN-Pisa and

European Gravitational Observatory

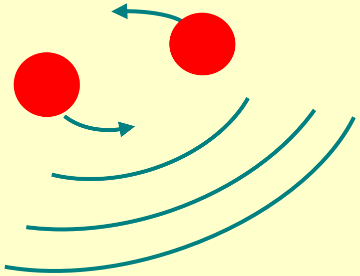
The Indirect Evidences of GW Existence

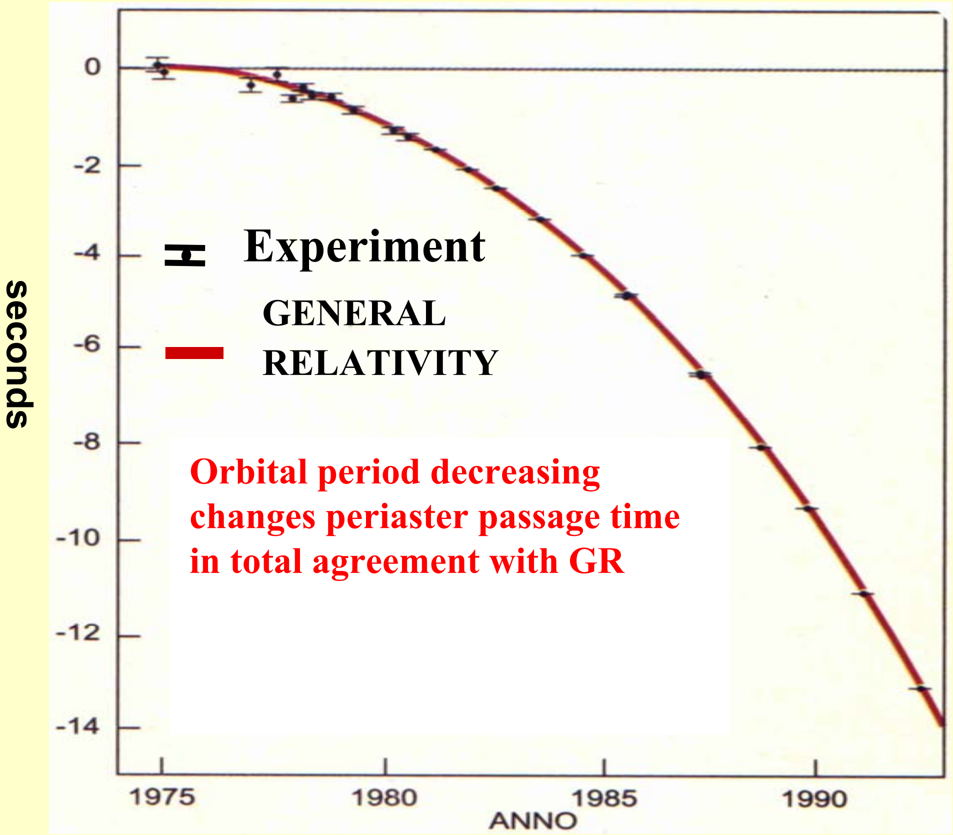
1974: First Discovery

Taylor and Hulse

Nobel Prize
1993

Coalescing Neutron Star System **PSR 1913+16**





Orbital period decreasing changes periaster passage time in total agreement with GR

Now there are about 6 similar systems, and the “double pulsar” PSR J0737-3039 is already overtaking 1913 in precision. All agree with GR

The GW Amplitude in TT system

For a GW propagating along X_3 we obtain the amplitude:

$$h^{TT}_{\mu\nu} = h^{TT}_{11}\begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&0\end{pmatrix} + h^{TT}_{12}\begin{pmatrix}0&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&0\end{pmatrix} = h^{+} e^{+}_{ik} + h^{x} e^{x}_{ik}$$

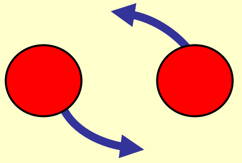
The polarizations e^{+}_{ik} and e^{x}_{ik} are exchanged with a π/4 rotation around x_3 axis i.e. GW are spin 2 massless fields.

In the limit of weak gravity, GW amplitude is proportional to the second time derivative of the source mass quadrupole moment:

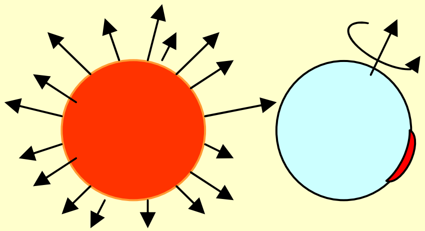
$$h_{\alpha\beta} = -\frac{2G}{c^{4} R_{o}}\left(\frac{\partial^{2}}{\partial t^{2}}\int \rho\left(x_{\alpha} x_{\beta}\right)^{TT} dV\right)_{t-R_{0}/c}$$

G Newton’s const.
R_0 Source distance
ρ Souce density distr.

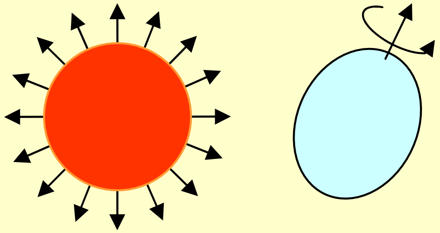
Large Asymmetry
“Large h”



Small Asymmetry
“Small h”

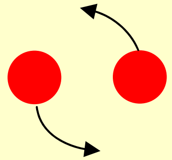


Axy-symmetry
“h=0”

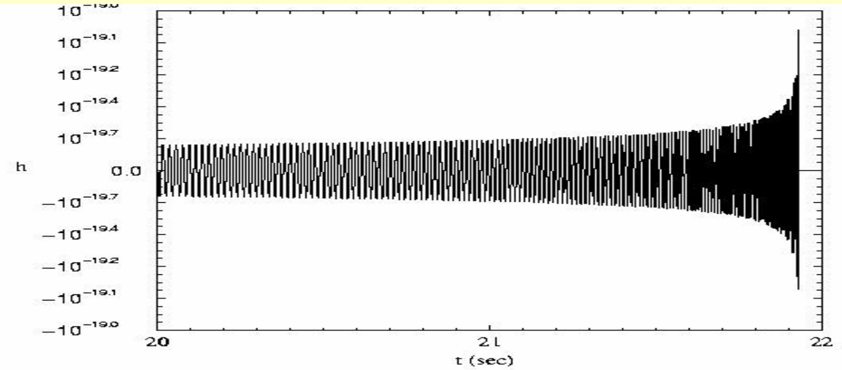


Some Gw Sources

1) Coalescing Binary Systems: NS and Black Holes



Rate~0,01/year in a 100 Mly sphere.

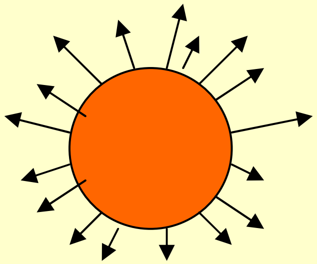


2)Supernovae Explosions

Explosions Rate:

Virgo Cluster (h~10^{-23}) ~30/year

Milky Way (h~10^{-20}) 1/30 years



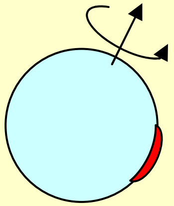
Small h

Asymmetry almost unpredictable

3) Periodic Sources :

For rotating Neutron Stars h very "Small" h< 10^{-25} .

Very long Integration time (1 year) increases S/N.

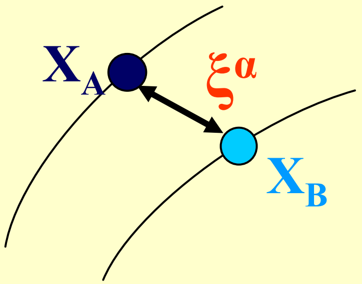


4) Big-Bang Cosmological BKG (CB): Since $\alpha_{GRAV}=10^{-39}$ Big-Bang matter is mainly transparent to GW. In the Virgo bandwidth we may observe GW emitted after 10^{-24}s from time zero.

The Detection of Gravitational Waves

F.A.E.Pirani in 1956 first proposed to measure Riemann Tensor by measuring relative acceleration of two freely falling masses. If A and B are freely falling particles, their separation $\xi^\alpha=(x_A-x_B)^\alpha$ satisfies the **Geodesic Deviation equation:**

$$\boxed{\frac{D^2\xi_\alpha}{d\tau^2} \Rightarrow \frac{1}{2}\ddot{h}^{TT}_{\alpha\beta}\,\xi^\beta} \Longrightarrow \boxed{\begin{array}{c}\text{Riemann Force}\\ F_\alpha = \frac{1}{2}M\ddot{h}^{TT}_{\alpha\beta}\,\xi^\beta\end{array}}$$

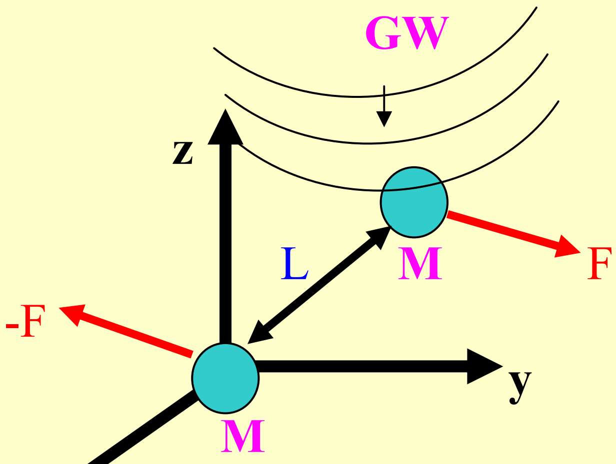


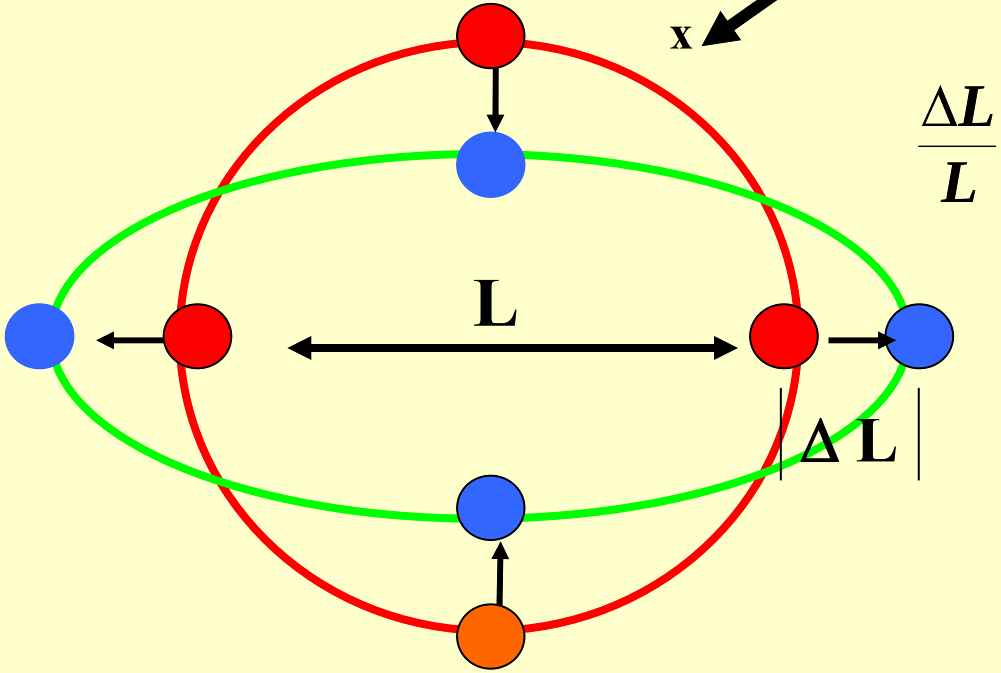
<u>The receiver is a device measuring space-time curvature</u> i.e. the relative acceleration of two freely falling masses or, equivalently, <u>their relative displacement</u>.

Gravitational Waves create tidal forces on the masses

Force increase with L until L<λ

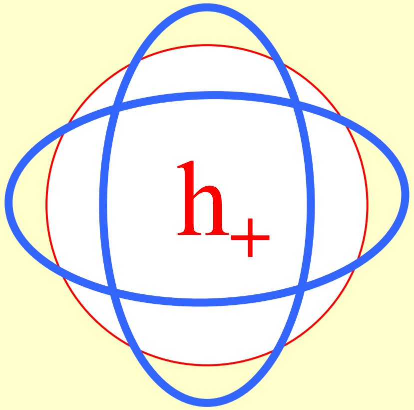
$$F_\beta = -\frac{1}{2} M L_\alpha \frac{d^2}{dt^2} h^\alpha_\beta$$

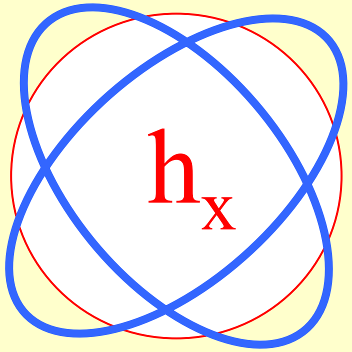




$$\frac{\Delta L}{L} \sim h < 10^{-22}$$

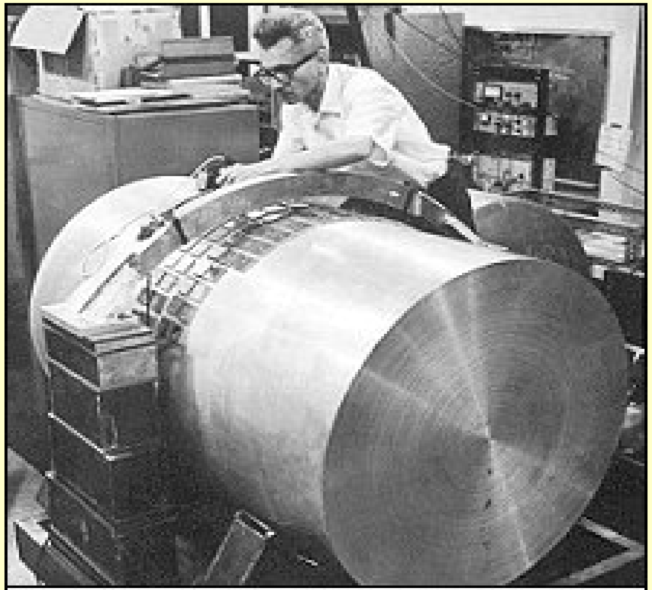
Effect of 2 Polarizations



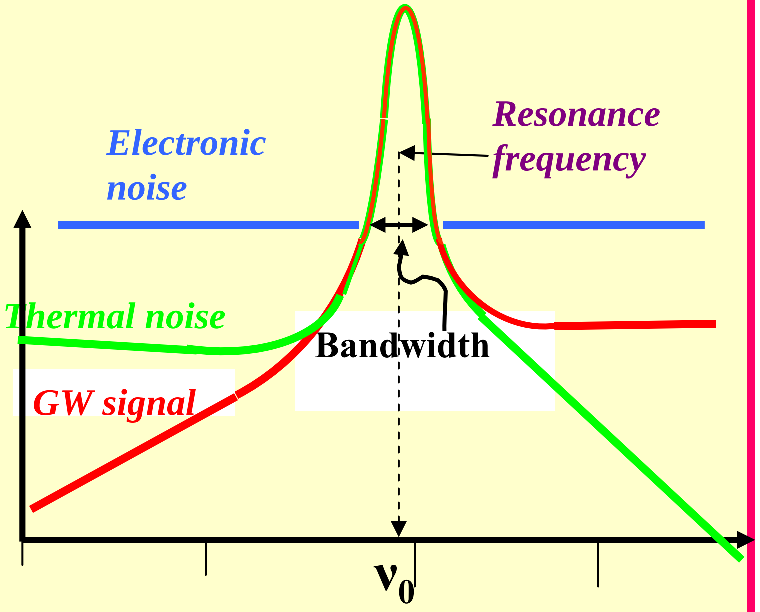


Early Detectors: Room Temperature Resonant Bars

In 1959 Joseph Weber was the first to build a GW detector working on the principles of Geodesic Deviation Equation.



JOSEPH WEBER WITH GRAVITATIONAL WAVE DETECTOR



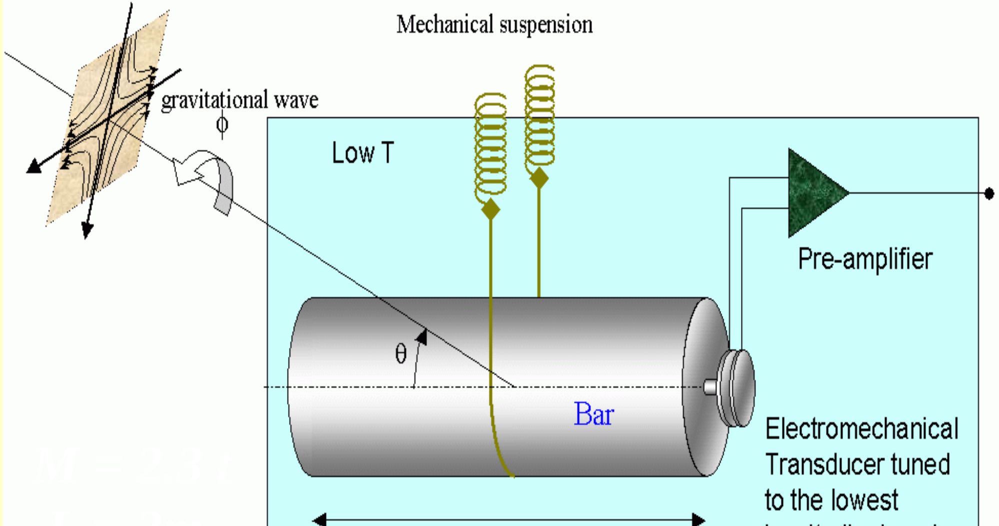
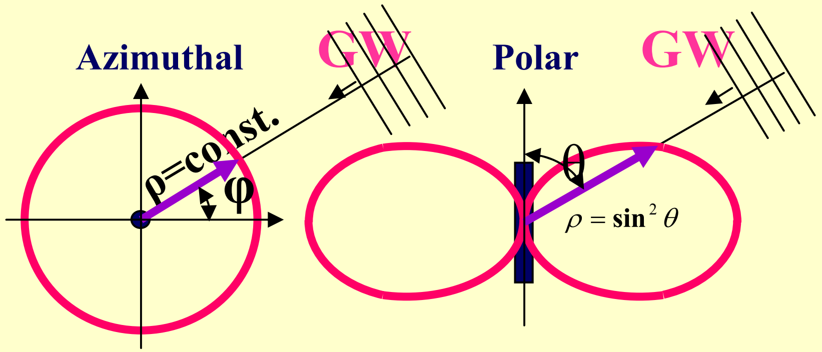


Figure courtesy of Massimo Cerdonio

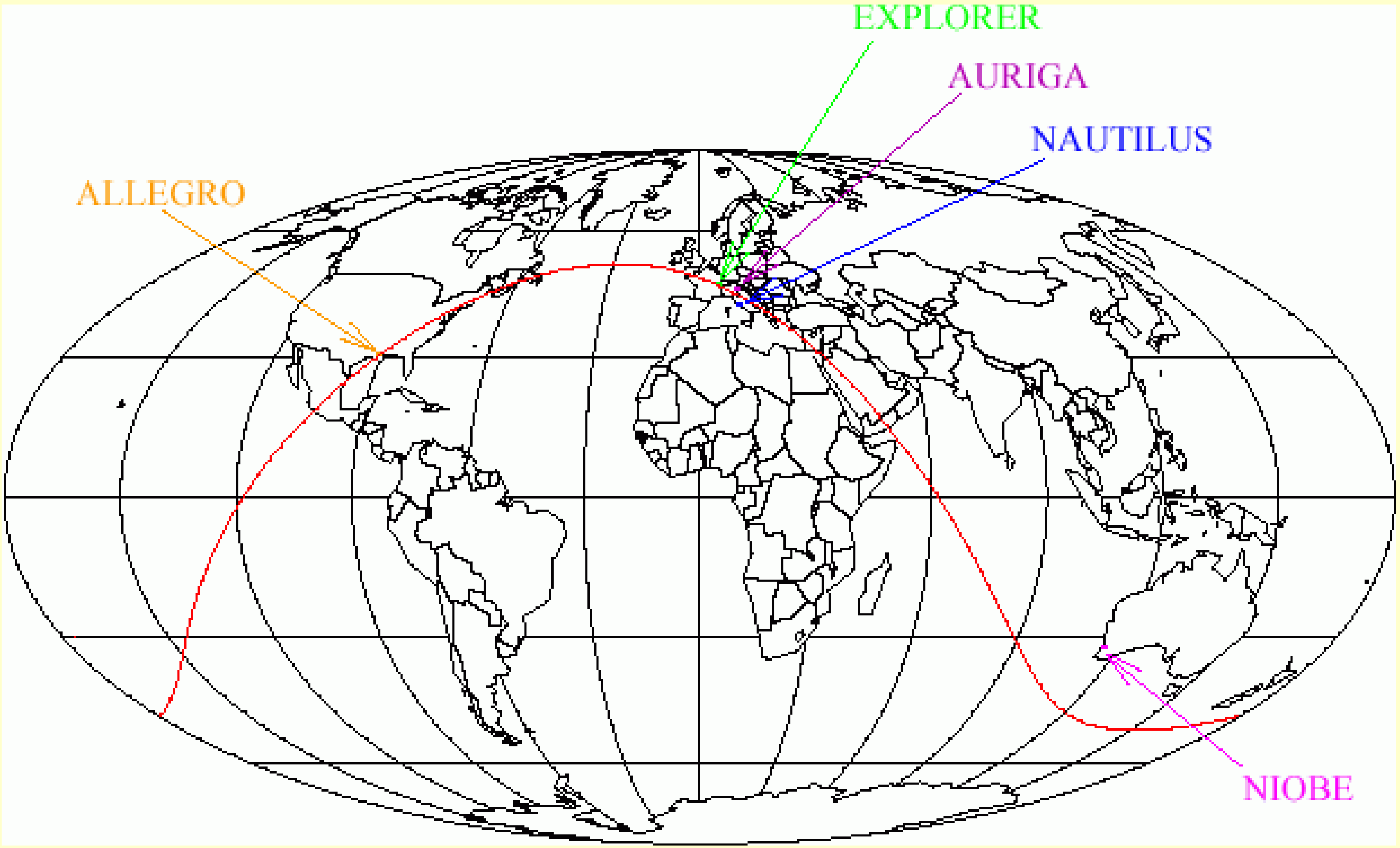
Antenna Pattern summed on polarizations



Cryogenic Bar Detectors

International Gravitational Event Collaboration

Cryogenic Bar Detectors network founded in 1997

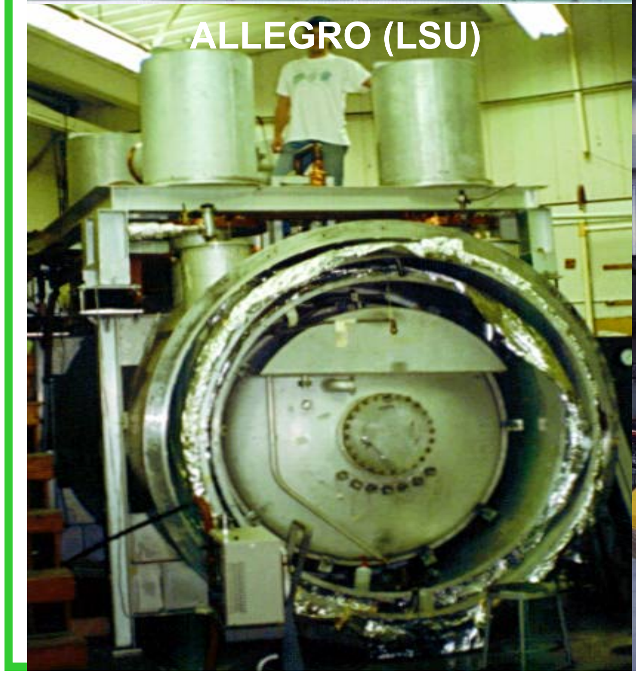


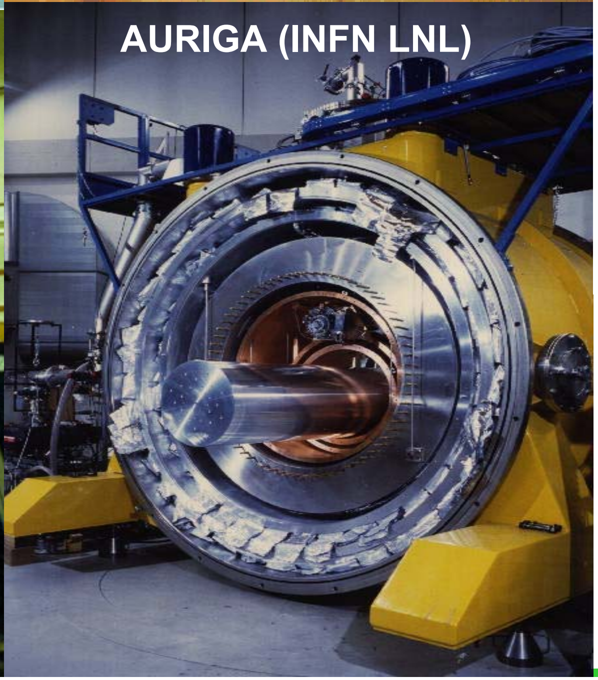
Courtesy E. Coccia

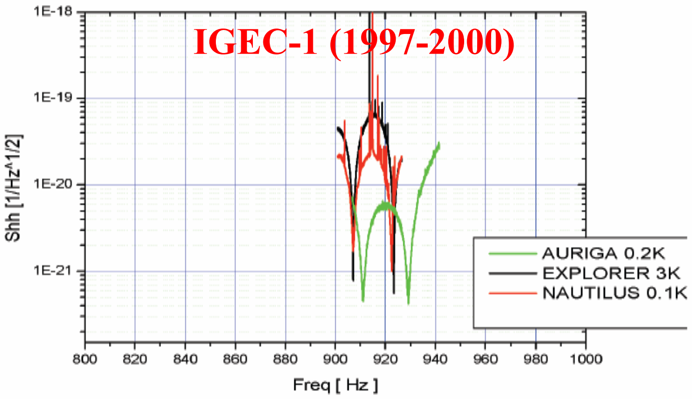
Cryogenic Bar Detectors Sensitivity&Stability

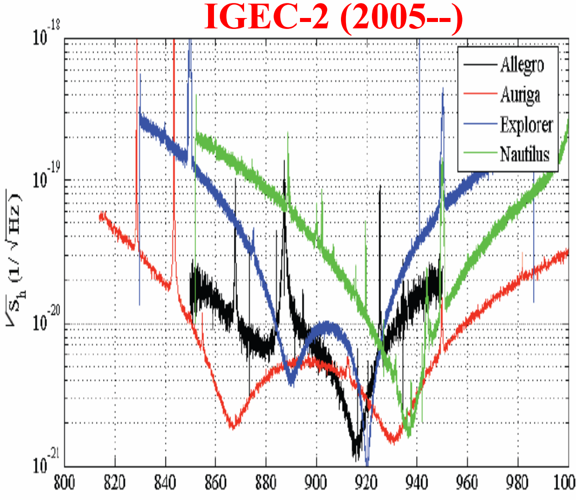

NAUTILUS (INFN LNF)

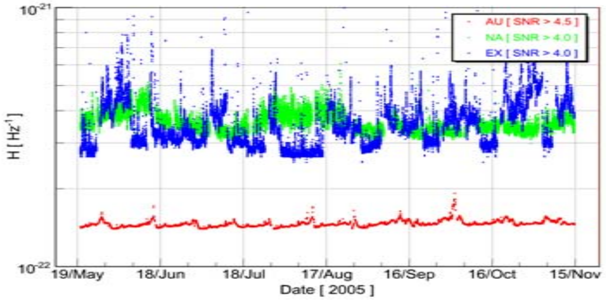

EXPLORER (INFN CERN)


ALLEGRO (LSU)


AURIGA (INFN LNL)


IGEC-1 (1997-2000)


IGEC-2 (2005--)

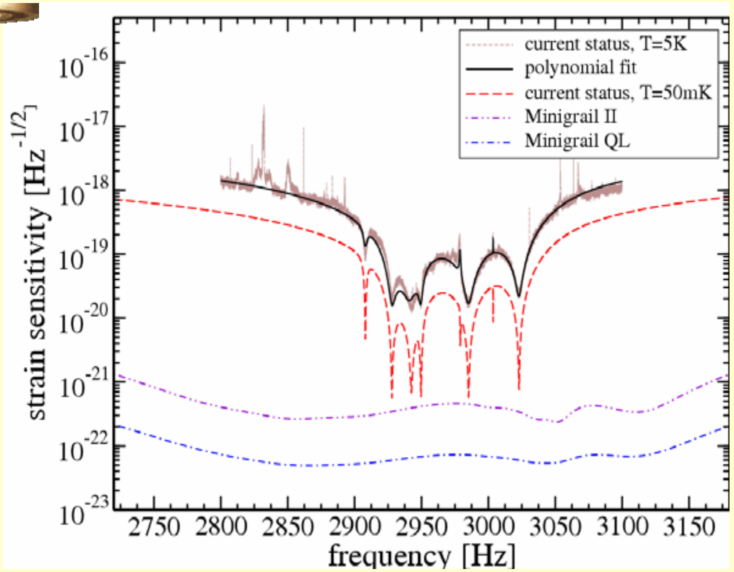


Bar Detectors situation at present

NIOBE (Perth) stopped operation and did not join IGEC-2
ALLEGRO (LSU) stopped operation in 2007

In 2006 INFN stopped R&D on Spherical Detectors and left running Auriga, Nautilus and Explorer on an annual evaluation.

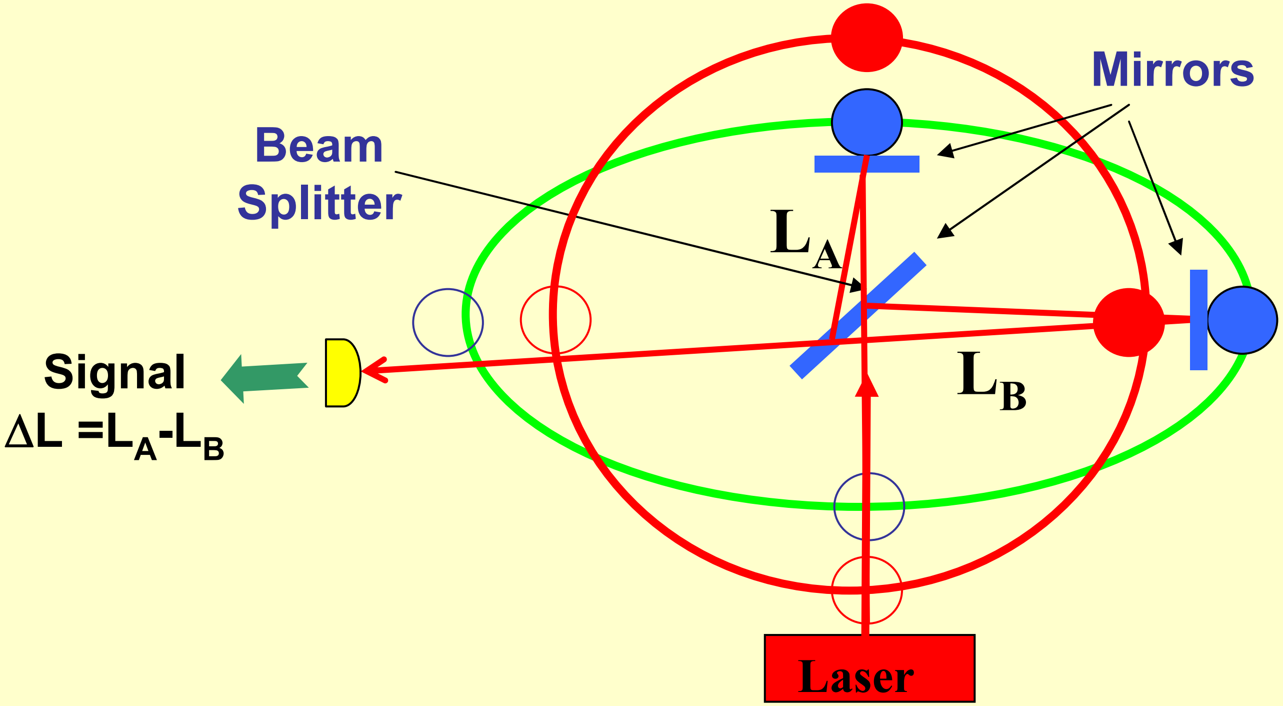
Spherical Detectors in commissionig phase are Minigrail in Leyden Univ. (Nd) and Mario Schenberg in S. Paulo Univ. (Br)



INTERFEROMETRIC DETECTORS

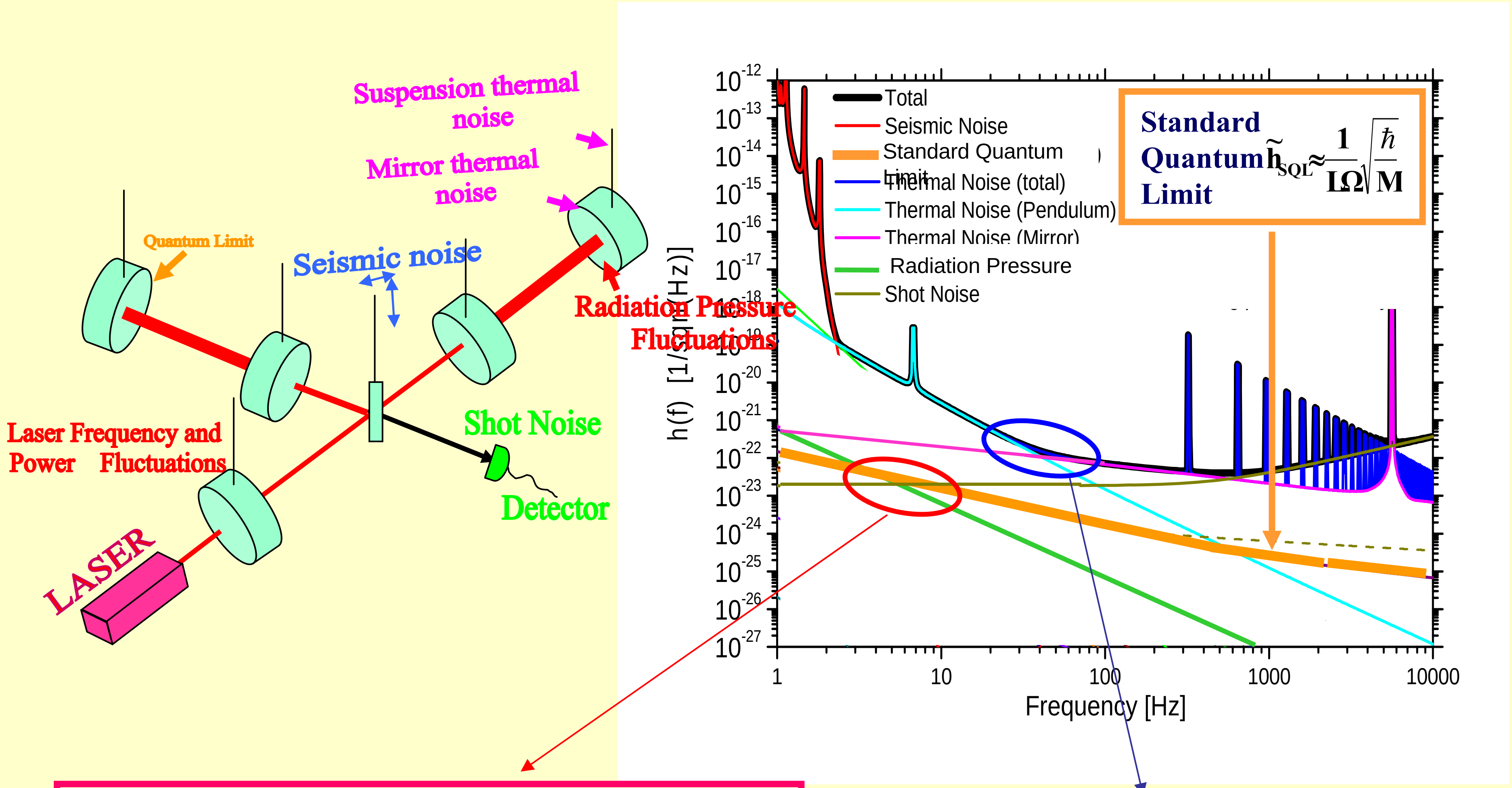
Large L → High sensitivity

Very Large Bandwidth 10-10000 Hz



Displacement sensitivity can reach ~10^{-19}-10^{-20} m, then, for measuring $\Delta L/L \sim 10^{-22}$ L_A and L_B should be km long.

Interferometer Noises



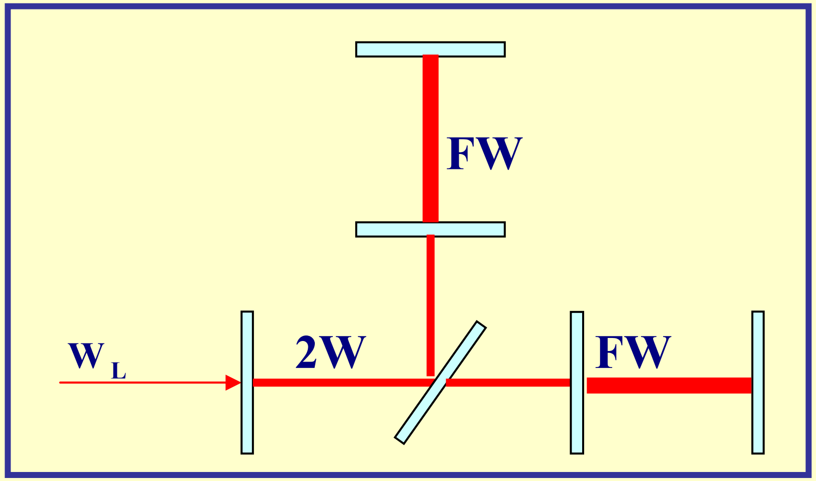
Standard Quantum Limit $\tilde{h}_{SQL} \approx \frac{1}{L\Omega}\sqrt{\frac{\hbar}{M}}$

Optical Noises can not be overcome in standard ITF but can with QND techniques. Radiation Pressure Fluctuations contribution to phase shift can be completely cancelled.

Thermal Noise, the more subtle, can perhaps be overcome by bringing Mirror temperature close to 4 K^0

Two Very Important Quantum Noises:

Shot noise and Radiation pressure Fluctuations



1) Shot Noise:

Uncertainty Prin. $\Delta\varphi\Delta N\geq 1$.
The phase of a coherent light beam fluctuates as:

$$\boxed{\varphi \geq \frac{1}{\sqrt{N}} = \sqrt{\frac{h\nu}{Wt}}}$$

2) Radiation Pressure Noise

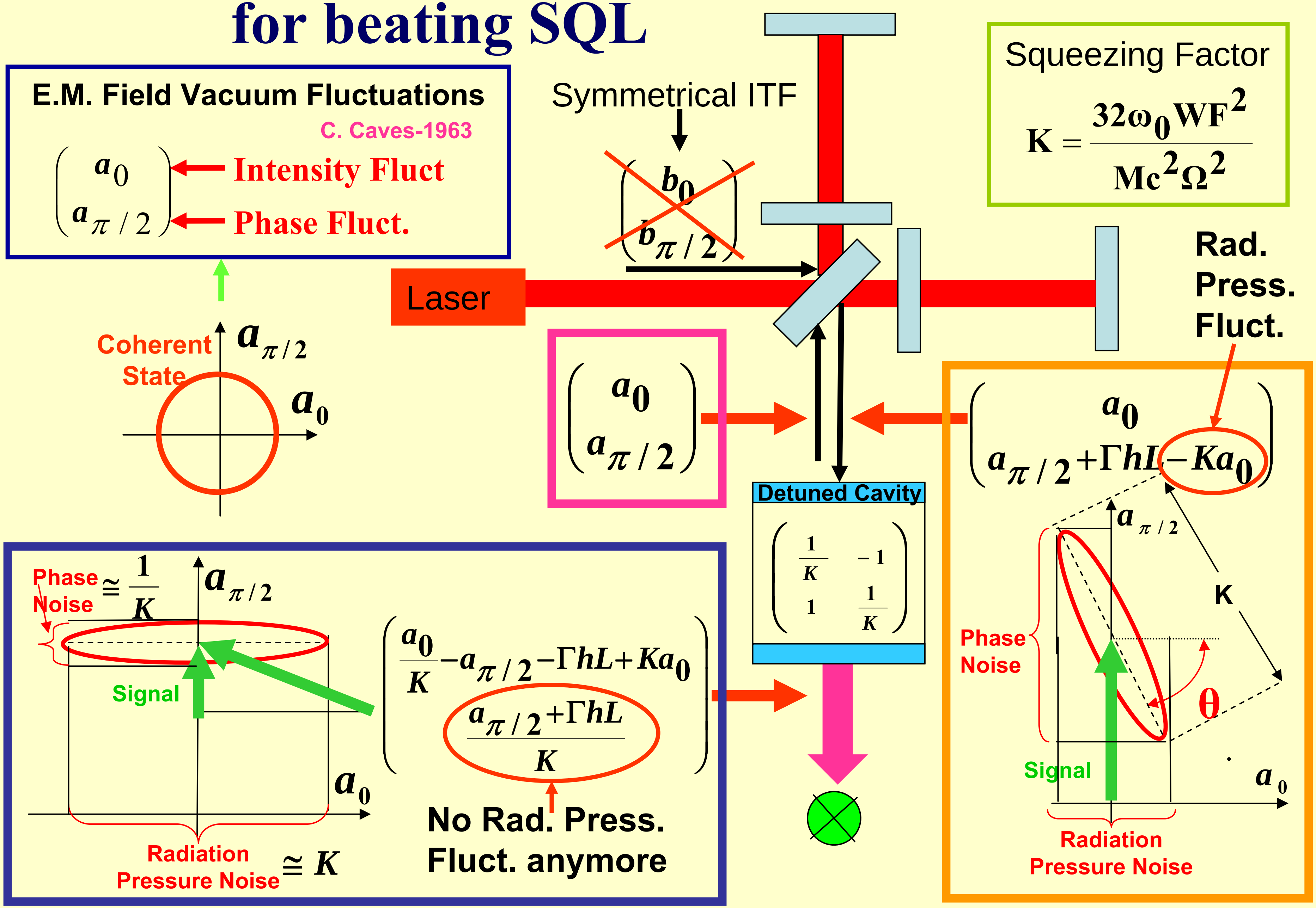
The photon number fluctuations create a fluctuating momentum on the mirrors of the FP cavities :

$$\boxed{\frac{\delta\tilde{P}}{\delta t} \approx F\frac{h\nu}{c}\sqrt{\frac{W}{h\nu}} = \frac{F}{c}\sqrt{h\nu W}}$$

The measurability cond. for Shot noise and Radiation Pressure noise is:

$$\boxed{\tilde{h}^2 > \overset{\text{RP}}{\left(\frac{2}{LM\Omega^2 c}\right)^2 h\nu W F^2} + \overset{\text{SN}}{\frac{\lambda^2}{16\pi^2}\frac{h\nu}{WF^2}}\,\frac{1+\left(\frac{\Omega F L}{c}\right)^2}{L^2}}$$

How to cancel Radiation Pressure Fluctuations for beating SQL



E.M. Field Vacuum Fluctuations

C. Caves-1963

$$\begin{pmatrix} a_0 \\ a_{\pi/2} \end{pmatrix}$$

Intensity Fluct

Phase Fluct.

Coherent State

$a_{\pi/2}$

a_0

Symmetrical ITF

$$\cancel{\begin{pmatrix} b_0 \\ b_{\pi/2} \end{pmatrix}}$$

Laser

Squeezing Factor

$$K = \frac{32\omega_0 W F^2}{M c^2 \Omega^2}$$

Rad. Press. Fluct.

$$\begin{pmatrix} a_0 \\ a_{\pi/2} \end{pmatrix}$$

$$\begin{pmatrix} a_0 \\ a_{\pi/2} + \Gamma h L \boxed{- K a_0} \end{pmatrix}$$

Detuned Cavity

$$\begin{pmatrix} \frac{1}{K} & -1 \\ 1 & \frac{1}{K} \end{pmatrix}$$

$a_{\pi/2}$

K

Phase Noise

θ

Signal

a_0

Radiation Pressure Noise

Phase Noise $\cong \frac{1}{K}$

$a_{\pi/2}$

Signal

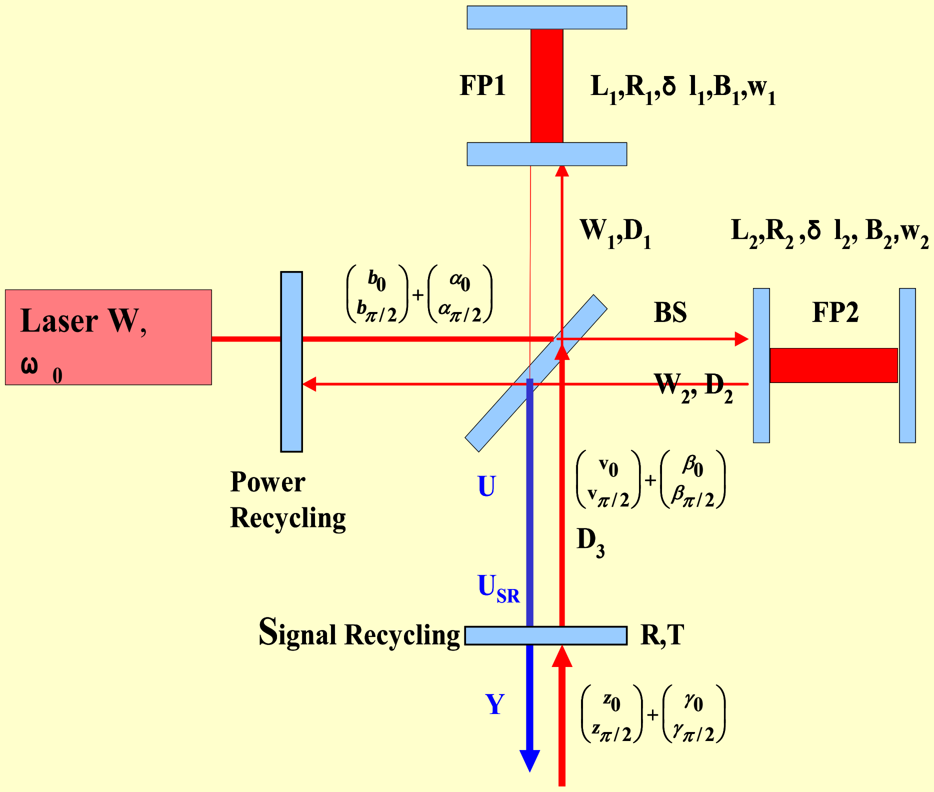
a_0

Radiation Pressure Noise $\cong K$

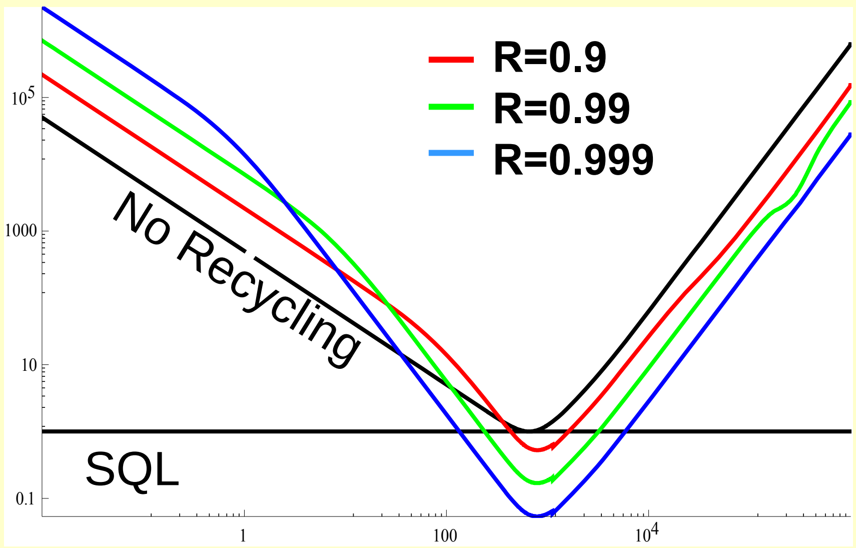
$$\begin{pmatrix} \frac{a_0}{K} - a_{\pi/2} - \Gamma h L + K a_0 \\ \boxed{\frac{a_{\pi/2} + \Gamma h L}{K}} \end{pmatrix}$$

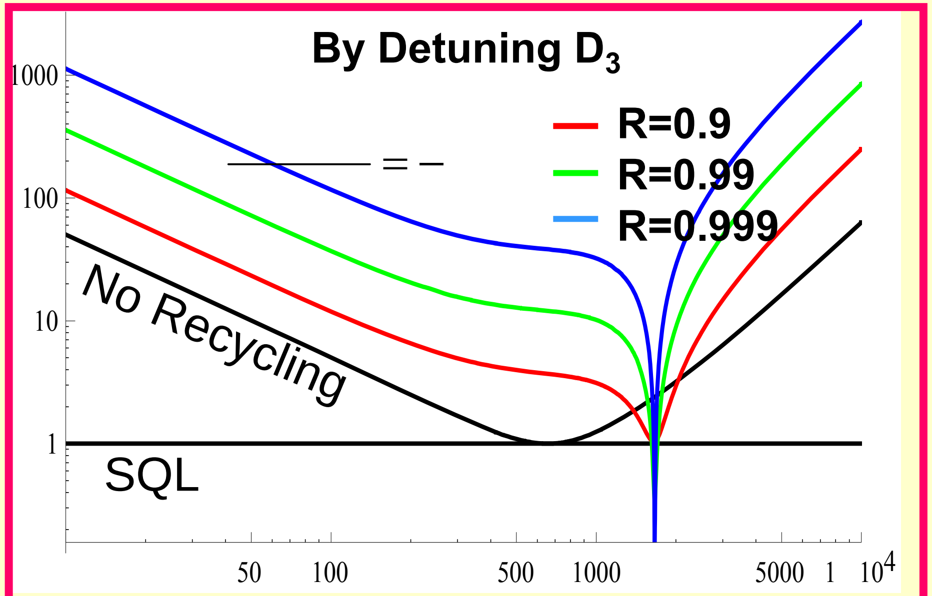
No Rad. Press. Fluct. anymore

Signal Recycling may produce rotations in the ΔN-Δφ $(a_0\ a_{\pi/2})$ space

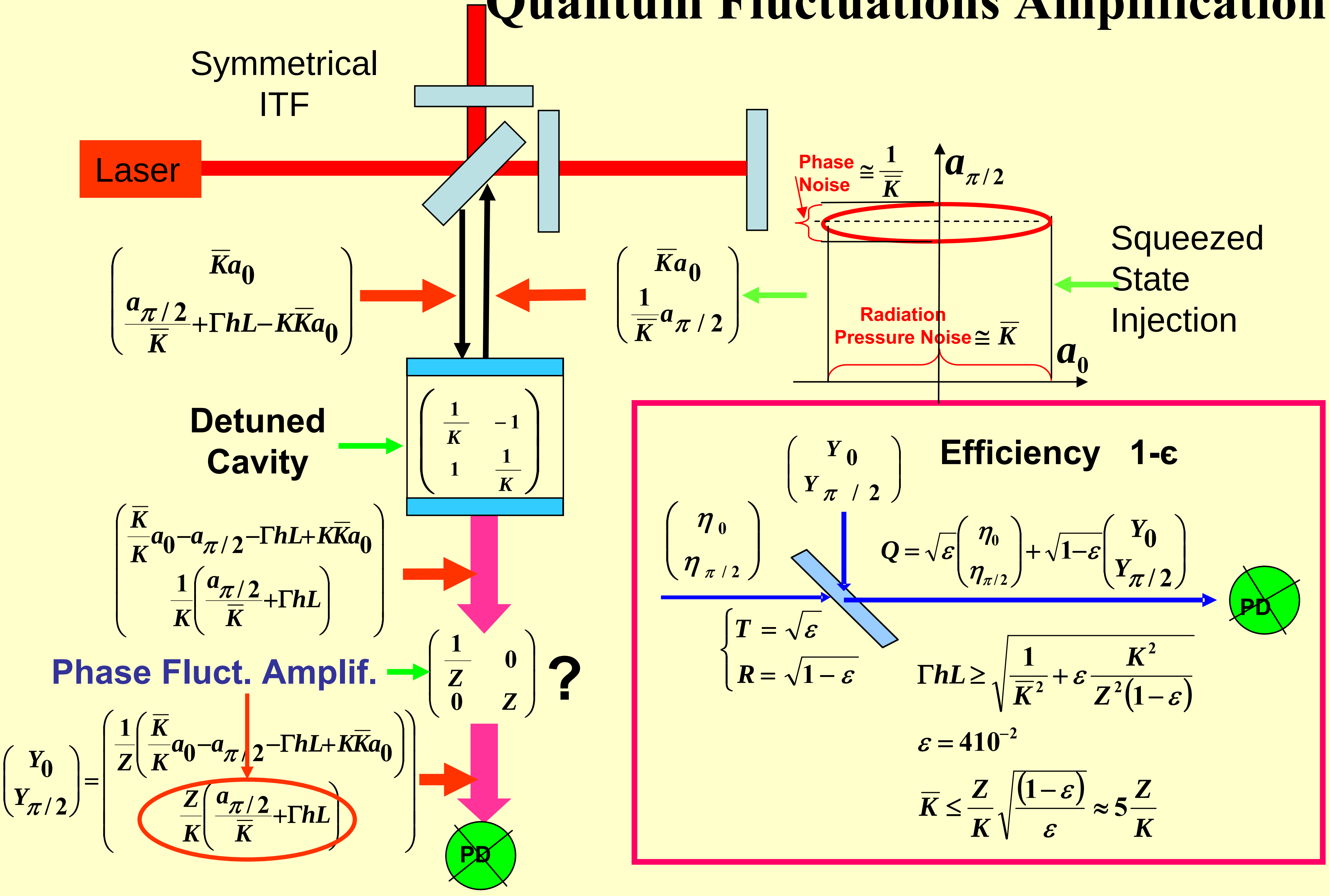


Varying D_3 at each frequecy for best S/N




By Detuning D_3

Squeezed Vacuum Injection and the Photodiode Problem: Quantum Fluctuations Amplification



$$\begin{pmatrix} \overline{K}a_0 \\ \frac{a_{\pi/2}}{\overline{K}}+\Gamma hL-K\overline{K}a_0 \end{pmatrix}$$

$$\begin{pmatrix} \overline{K}a_0 \\ \frac{1}{\overline{K}}a_{\pi/2} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{K} & -1 \\ 1 & \frac{1}{K} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\overline{K}}{K}a_0-a_{\pi/2}-\Gamma hL+K\overline{K}a_0 \\ \frac{1}{K}\left(\frac{a_{\pi/2}}{\overline{K}}+\Gamma hL\right) \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{Z} & 0 \\ 0 & Z \end{pmatrix}$$

$$\begin{pmatrix} Y_0 \\ Y_{\pi/2} \end{pmatrix}=\begin{pmatrix} \frac{1}{Z}\left(\frac{\overline{K}}{K}a_0-a_{\pi/2}-\Gamma hL+K\overline{K}a_0\right) \\ \frac{Z}{K}\left(\frac{a_{\pi/2}}{\overline{K}}+\Gamma hL\right) \end{pmatrix}$$

$$\begin{pmatrix} \eta_0 \\ \eta_{\pi/2} \end{pmatrix} \qquad \begin{pmatrix} Y_0 \\ Y_{\pi/2} \end{pmatrix}$$

$$\begin{cases} T=\sqrt{\varepsilon} \\ R=\sqrt{1-\varepsilon} \end{cases}$$

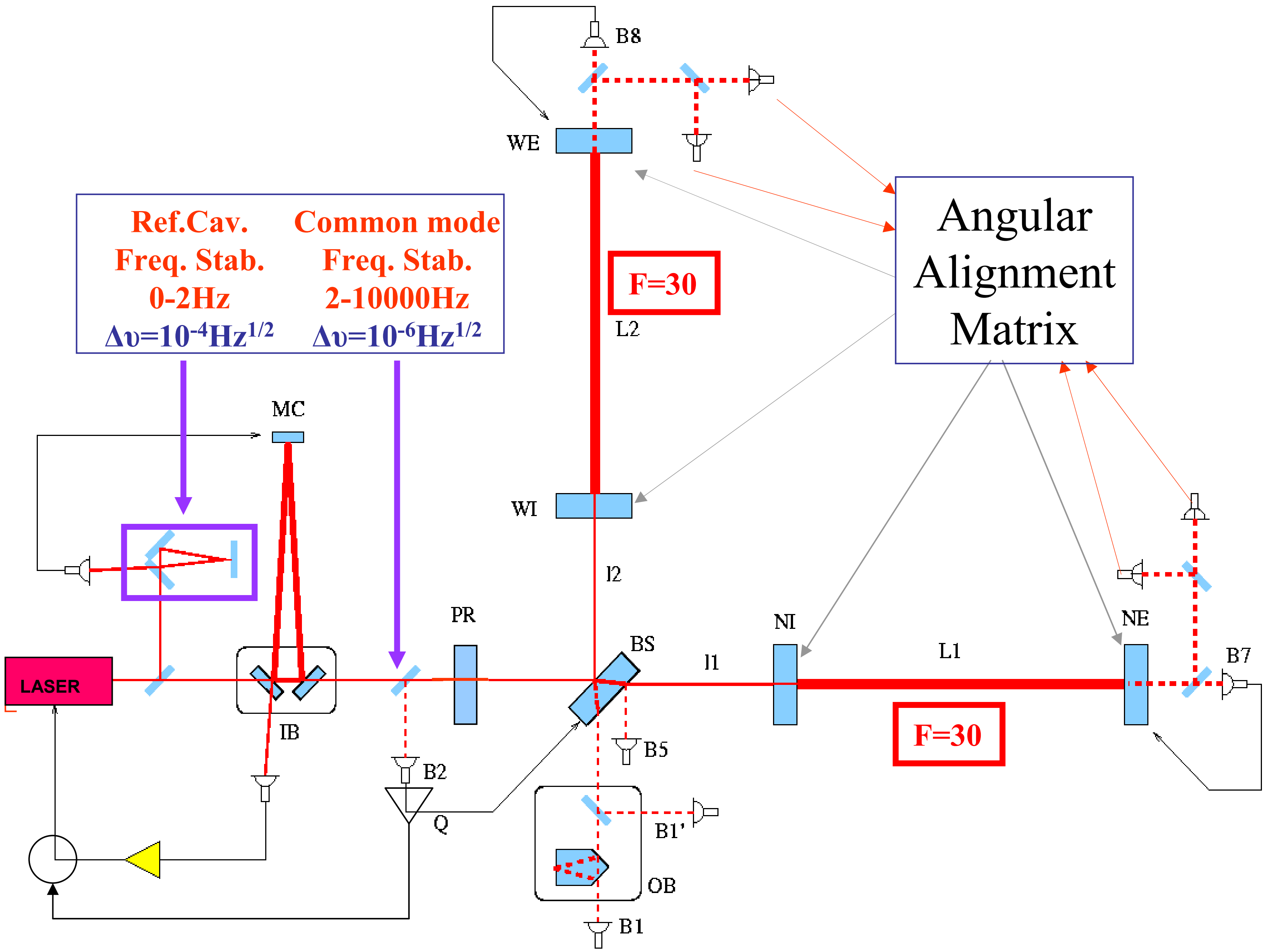
$$Q=\sqrt{\varepsilon}\begin{pmatrix} \eta_0 \\ \eta_{\pi/2} \end{pmatrix}+\sqrt{1-\varepsilon}\begin{pmatrix} Y_0 \\ Y_{\pi/2} \end{pmatrix}$$

$$\Gamma hL \geq \sqrt{\frac{1}{\overline{K}^2}+\varepsilon\frac{K^2}{Z^2(1-\varepsilon)}}$$

$$\varepsilon = 410^{-2}$$

$$\overline{K} \leq \frac{Z}{K}\sqrt{\frac{(1-\varepsilon)}{\varepsilon}} \approx 5\frac{Z}{K}$$

Virgo Diagram



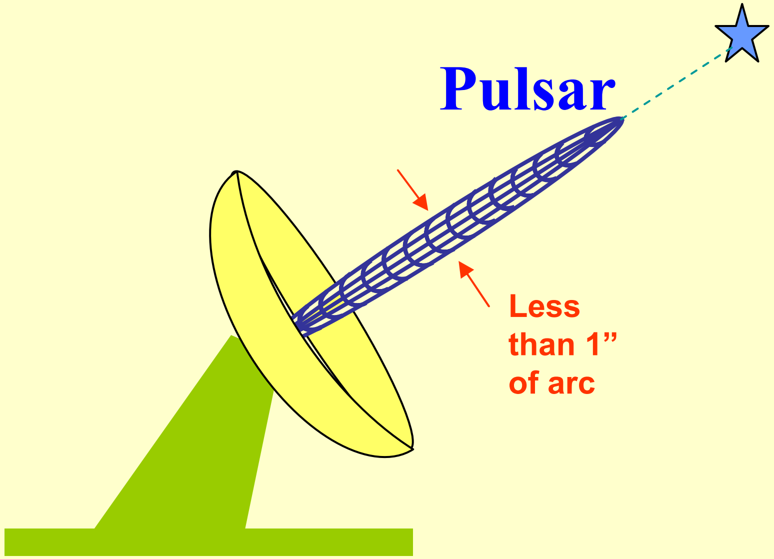
Ref.Cav.
Freq. Stab.
0-2Hz
$\Delta\upsilon=10^{-4}\mathrm{Hz}^{1/2}$

Common mode
Freq. Stab.
2-10000Hz
$\Delta\upsilon=10^{-6}\mathrm{Hz}^{1/2}$

Angular
Alignment
Matrix

GW Detectors have a very appealing Antenna pattern

Radiotelescope Antenna Pattern

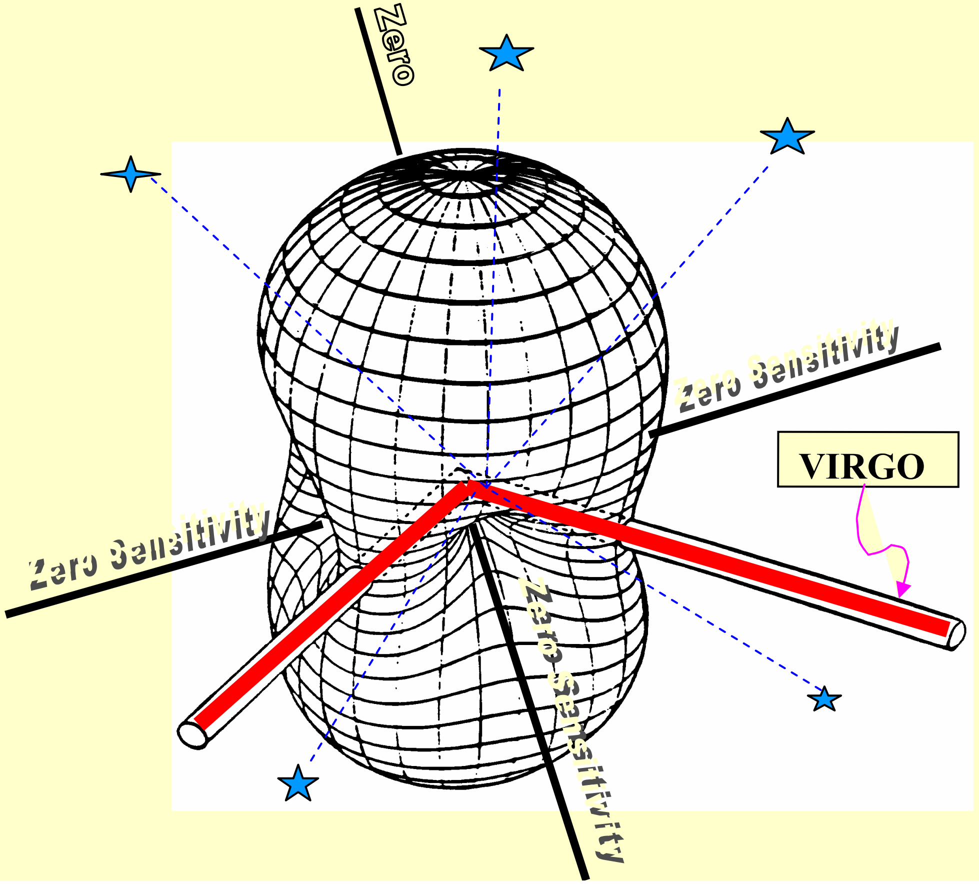


Sources are localyzed

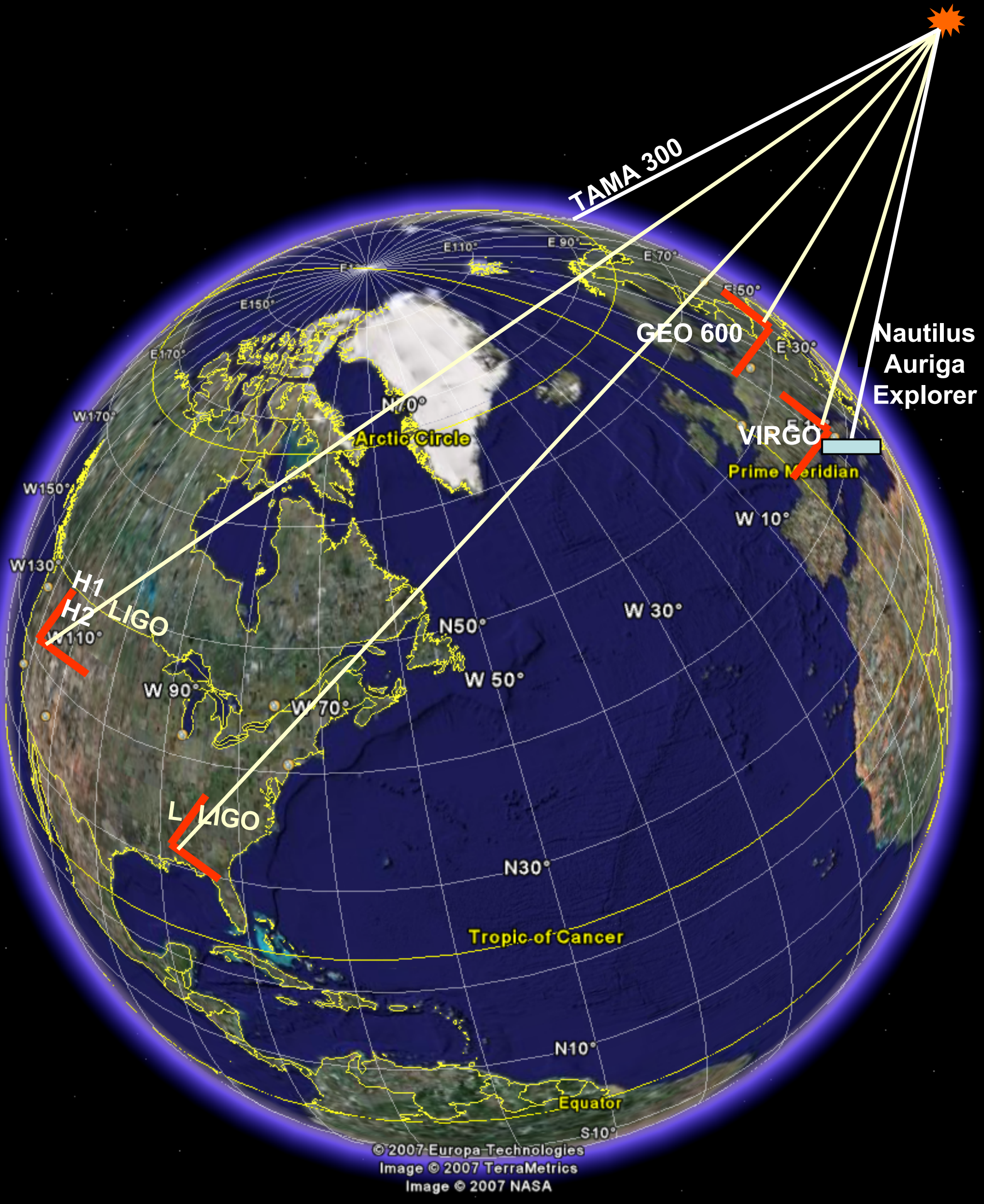
“Geometrically “

Interferometric GW Detector Antenna Pattern

ALL sky seen at once.



Global network of Detectors



© 2007 Europa Technologies
Image © 2007 TerraMetrics
Image © 2007 NASA

Coherent Analysis: why?

-Sensitivity increase

-Source direction determination from time of flight differences

-Polarizations measurement

-Test of GW Theory and GW Physical properties

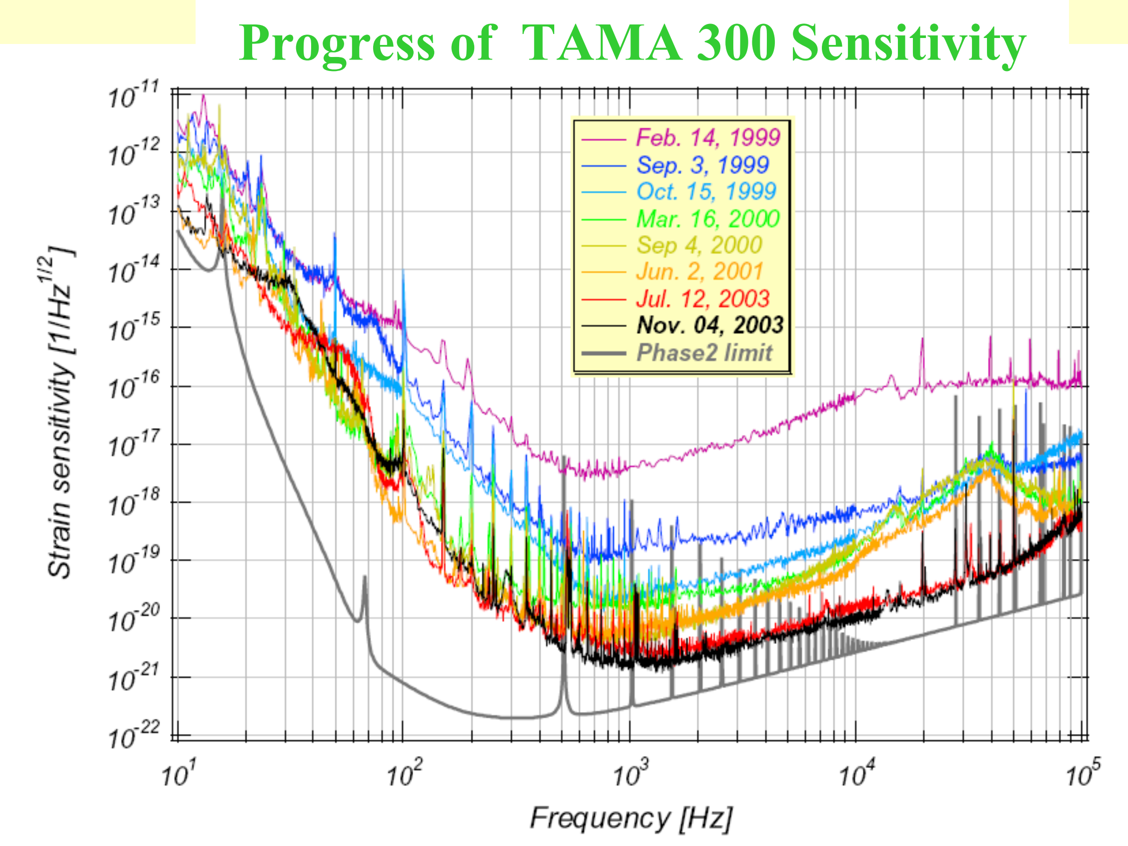
Astrophysical targets

- Far Universe expansion rate Measurement

-GW energy density in the Universe

-Knowledge of Universe at times close to Planck’s time

TAMA 300m-Tokyo

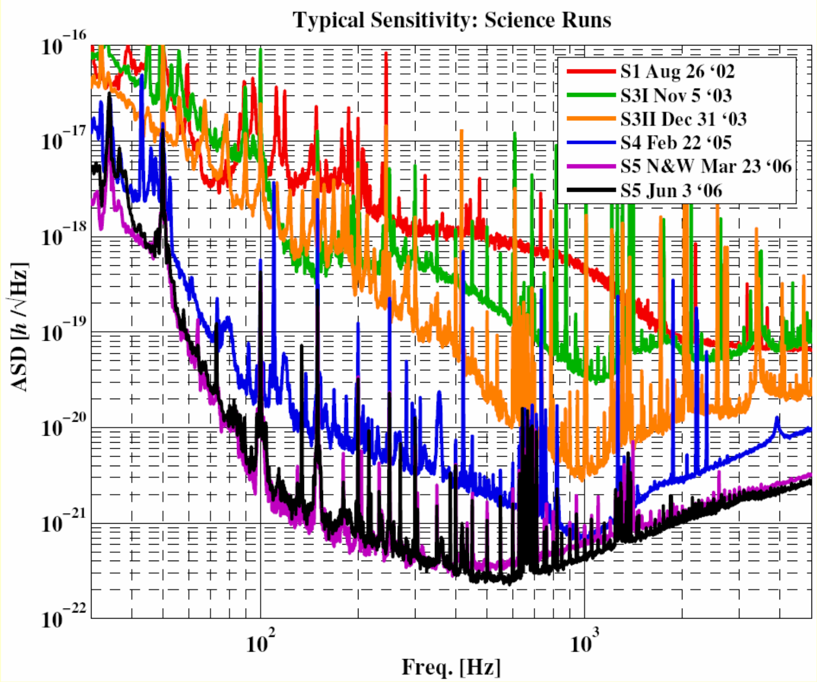


Progress of TAMA 300 Sensitivity

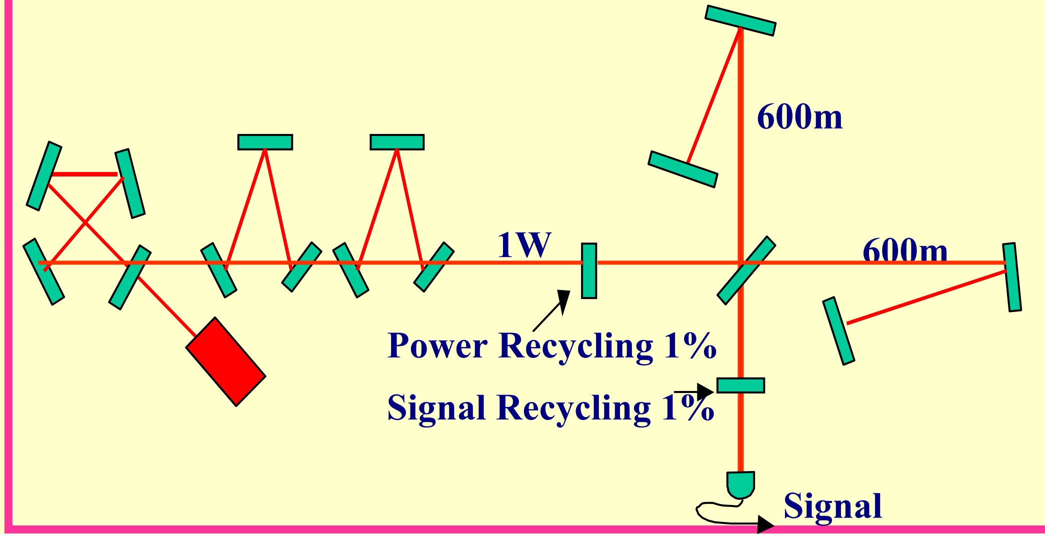
In 1999, TAMA is the first large ITF to start observations, in 2001 attained the world best sensitivity and made continuous observation more than 1000 hr with the highest sensitivity. Joint observations with LIGO/GEO during DT7-DT9

Best sensitivity : $h = 1.710^{-21} \, {}^{1}\!/\!\sqrt{Hz} \; @ \; 1\,KHz$ **Recycling gain of 4.5**

GEO 600 m- Hannover



GEO 600 is a Signal Recycling Interferometer



600m

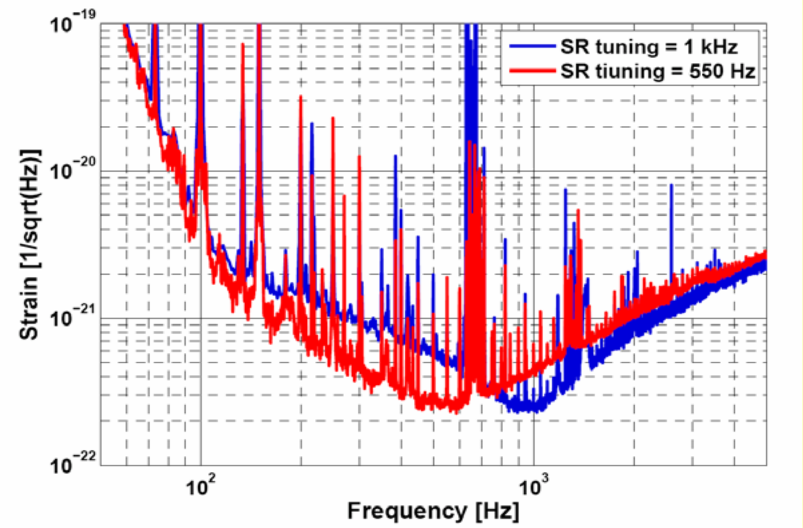
1W

600m

Power Recycling 1%

Signal Recycling 1%

Signal



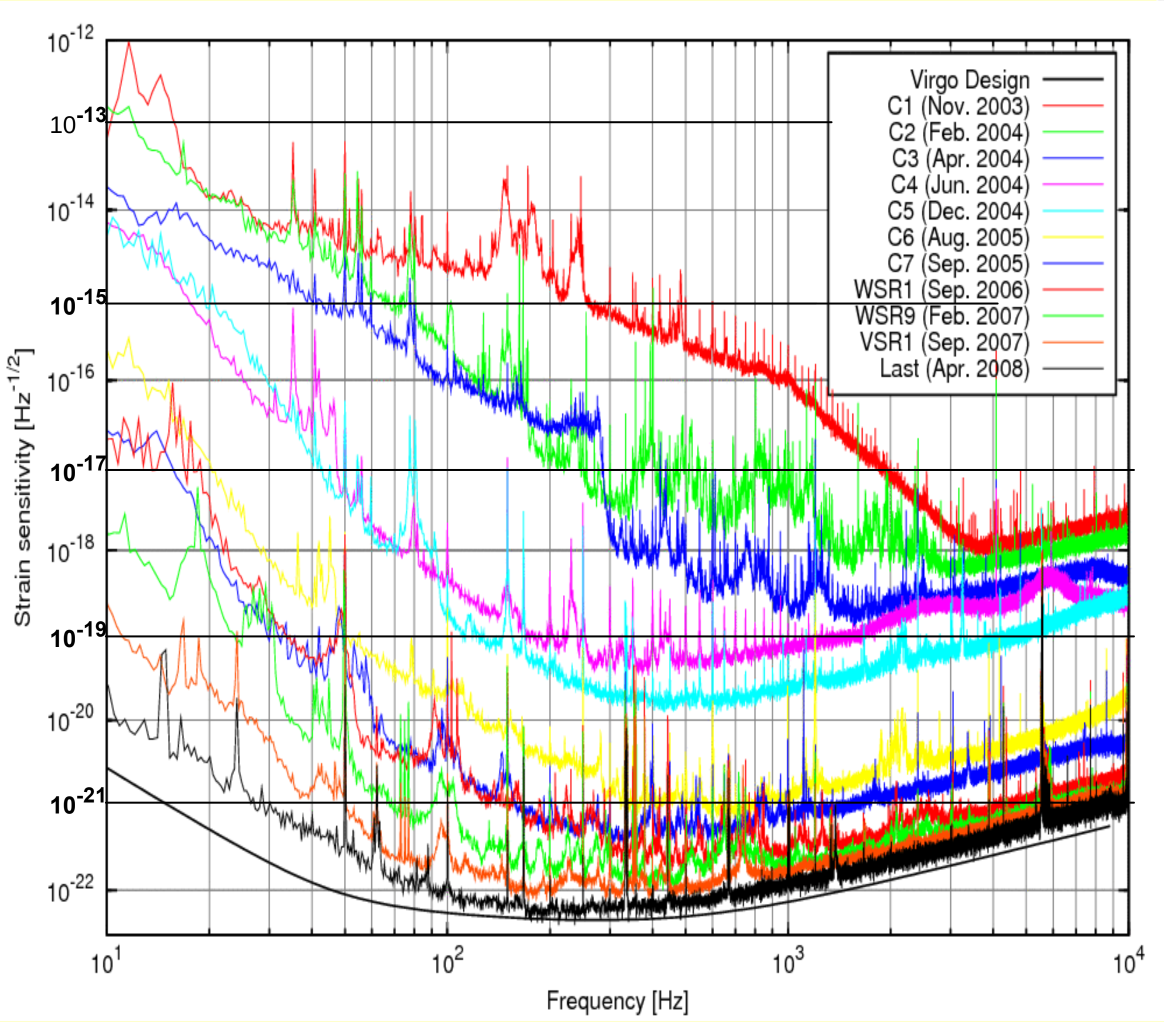
VIRGO

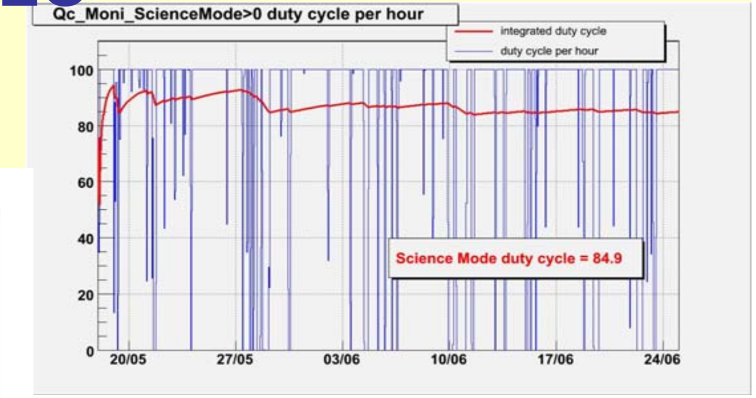
3 km-Cascina

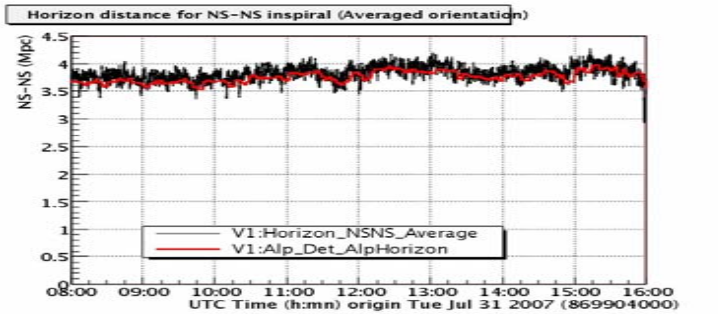
France (CNRS) 50%
Italy (INFN) 50%

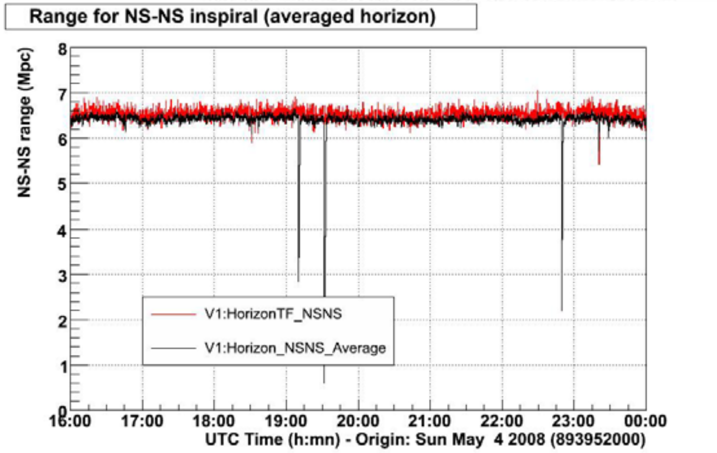
Virgo Sensitivity, Duty Cycle and Stability First 5 weeks (started 18/5/2007) of Coincidence with LIGO/GEO

Progress of Virgo Sensitivity





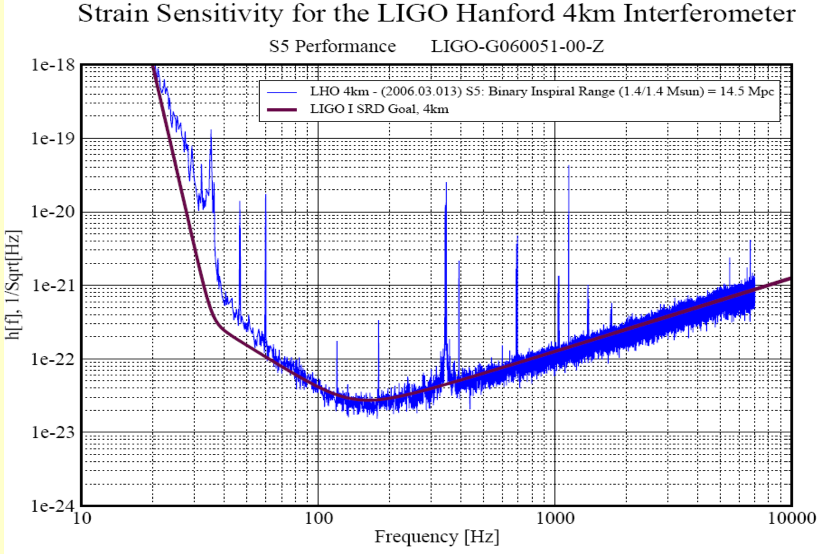




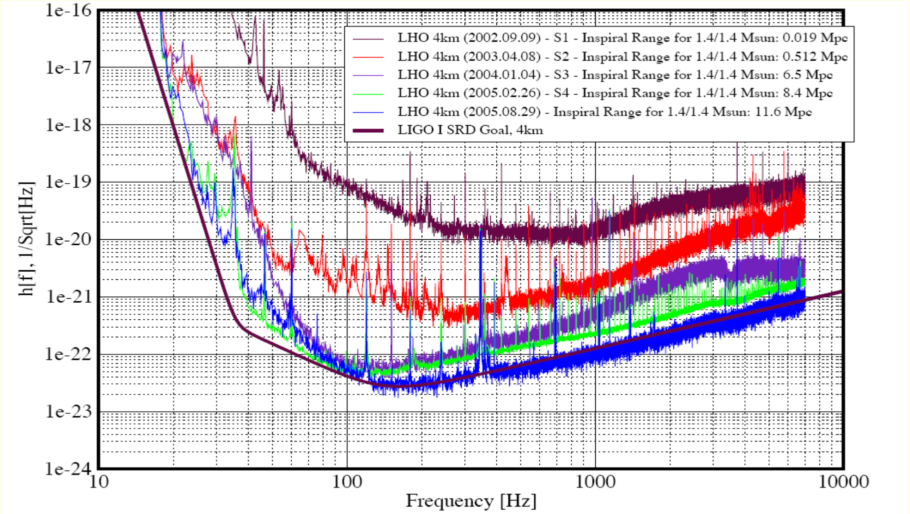
LIGO (Caltech&MIT)

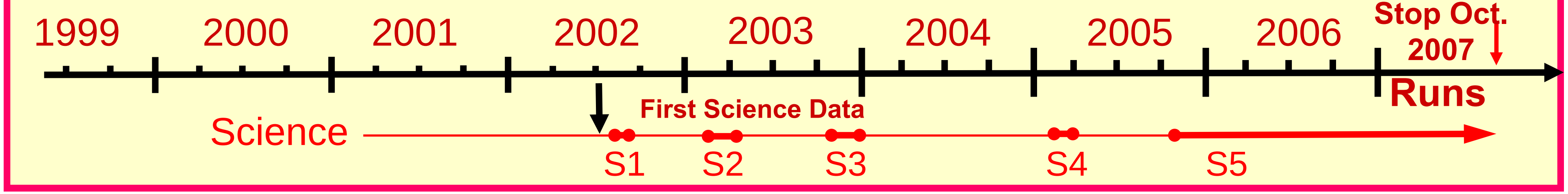
One Vacuum Tube with
2 ITF: 4 km and 2 km

4 km Arms


Strain Sensitivity for the LIGO Hanford 4km Interferometer

S5 Performance LIGO-G060051-00-Z





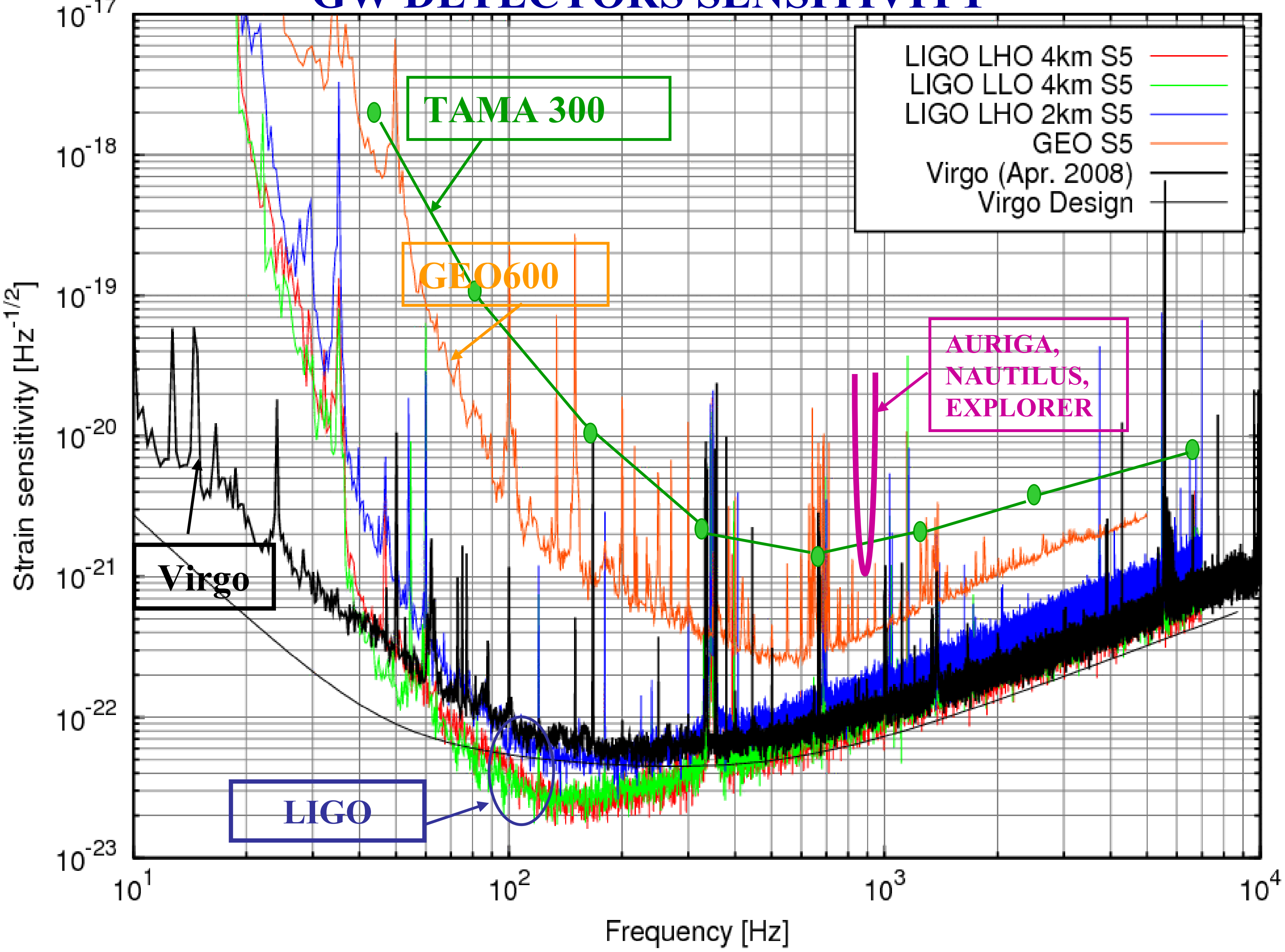
Stop Oct. 2007

Runs

Science

First Science Data

GW DETECTORS SENSITIVITY

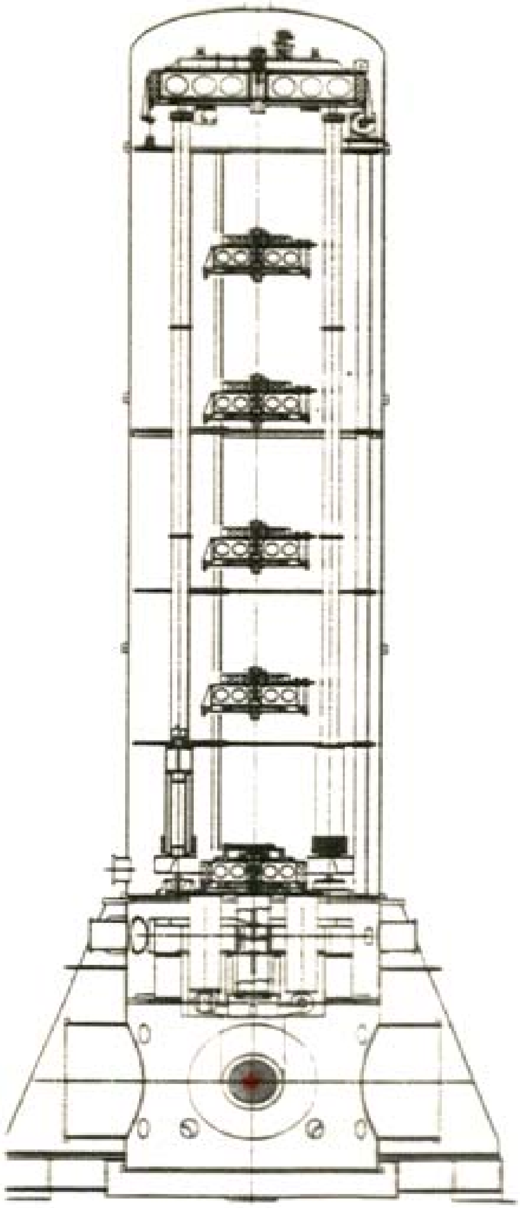


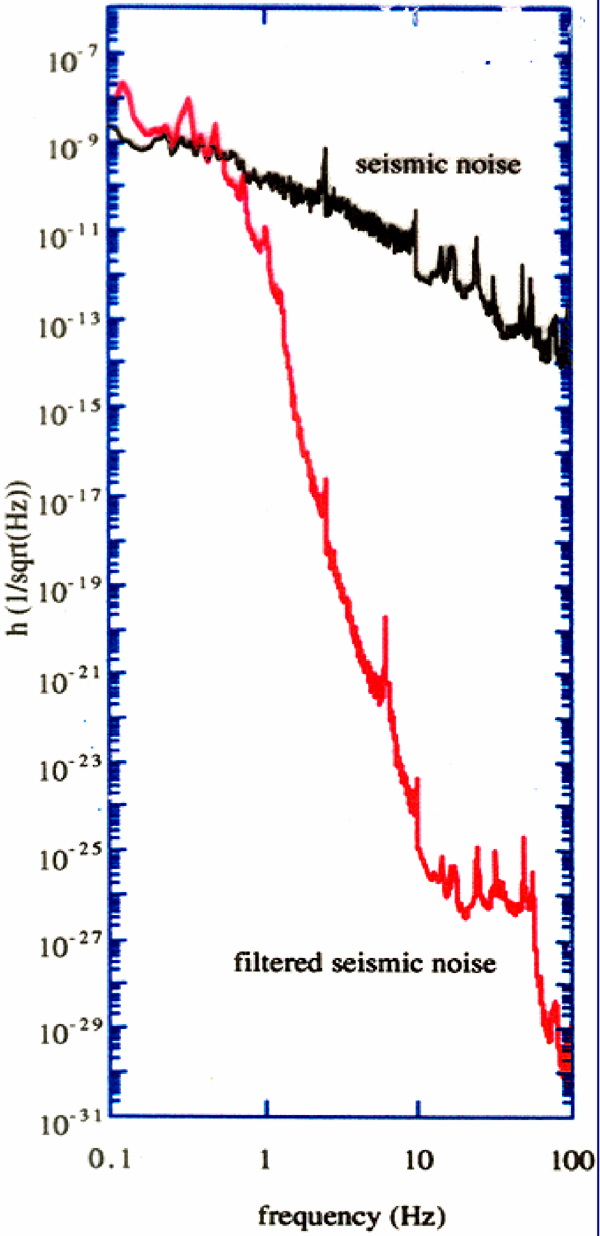
Some comments on Virgo low frequency performances

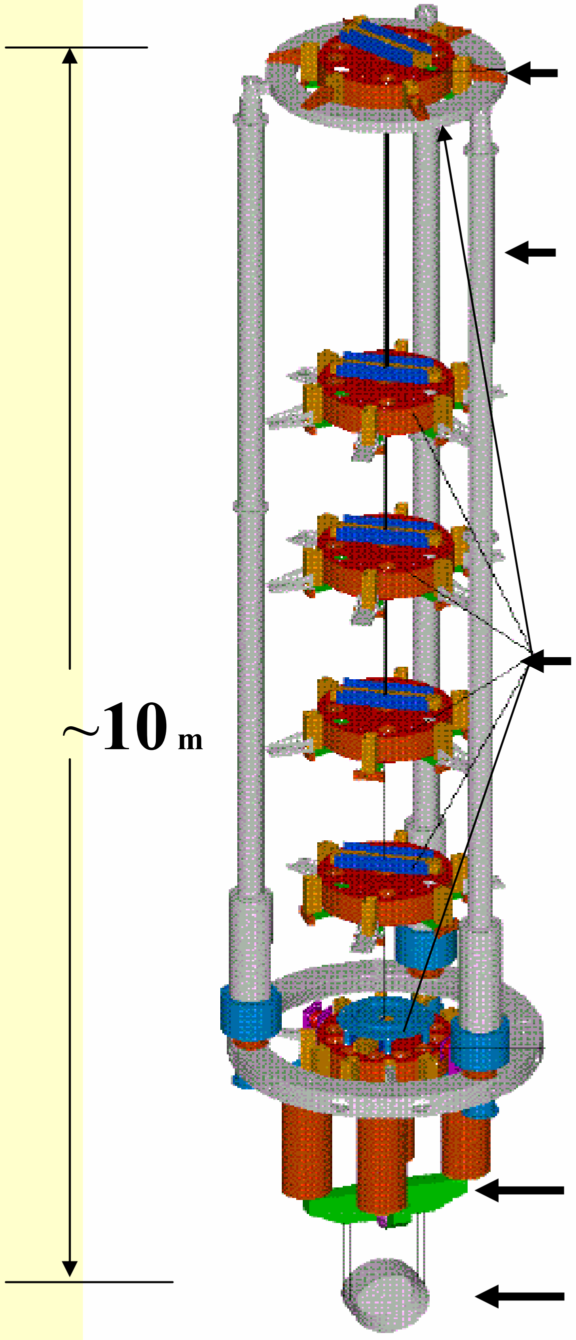
SUPERATTENUATORS

Isolate mirrors and optical benches from Seismic noise (10^{12} larger than signal @ 10 Hz)

6 SA for mirror suspension
3 SA for optical benches







Inertial Damping

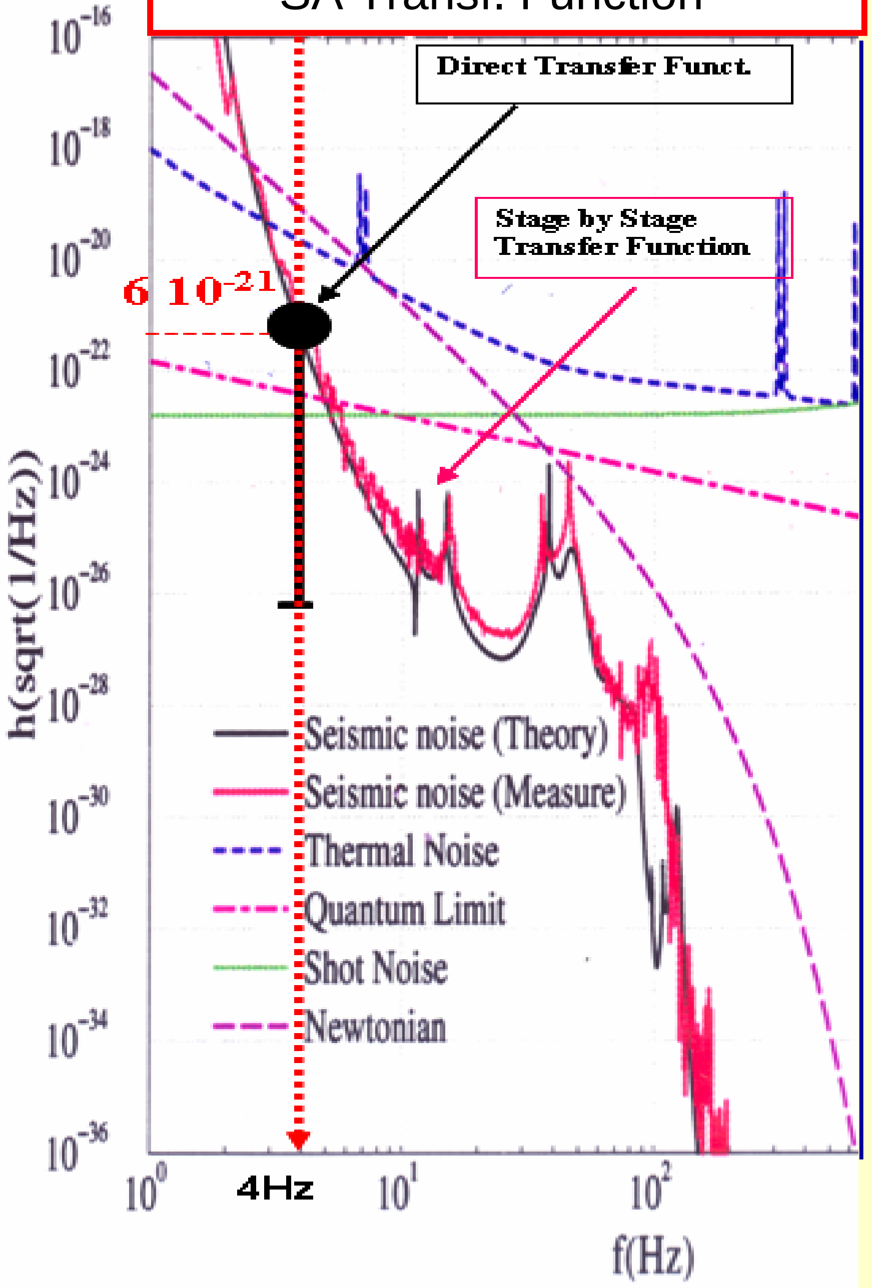
Inverted Pendulum
ν_{RES}=40 mHz

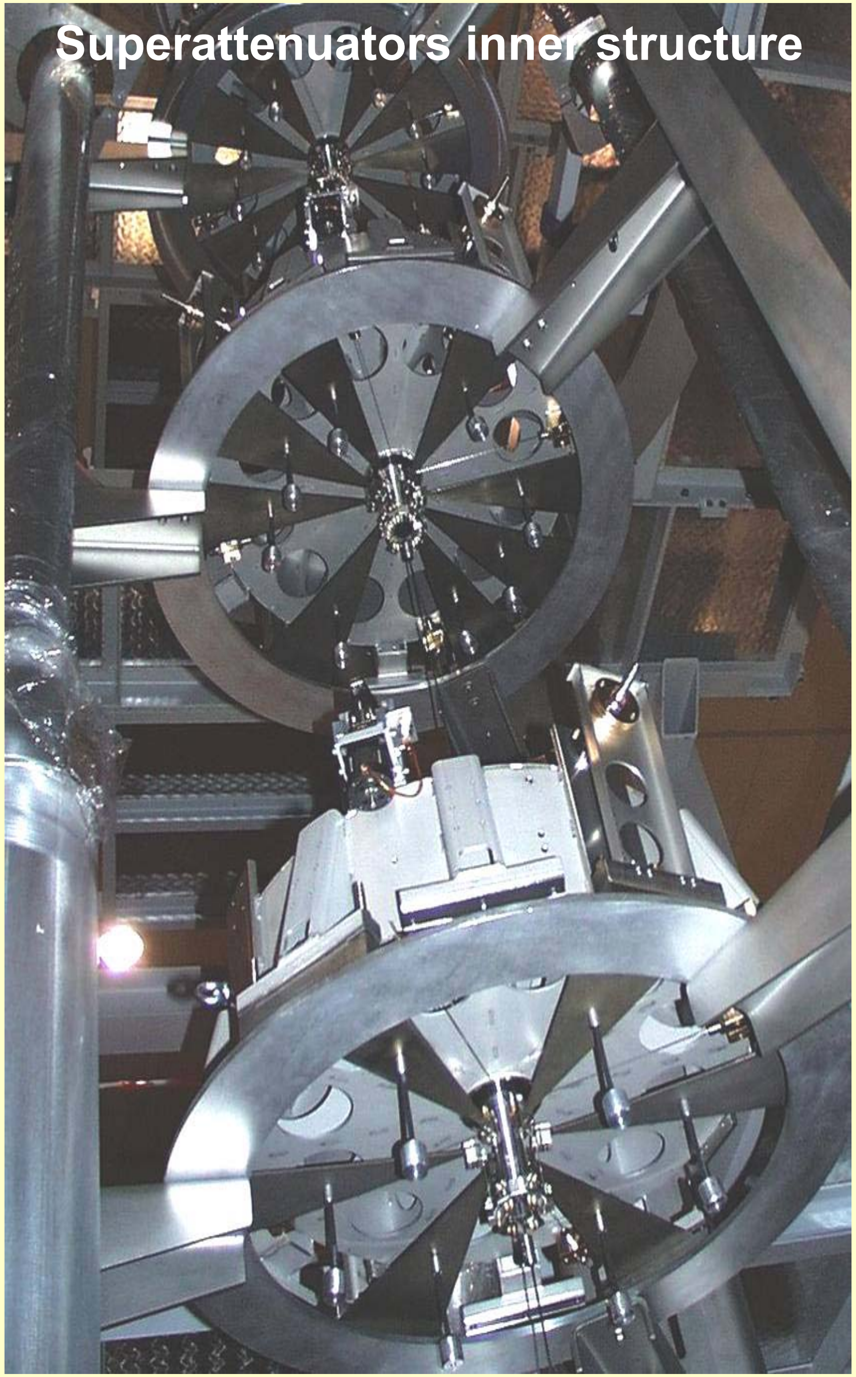
Mechanical Filters

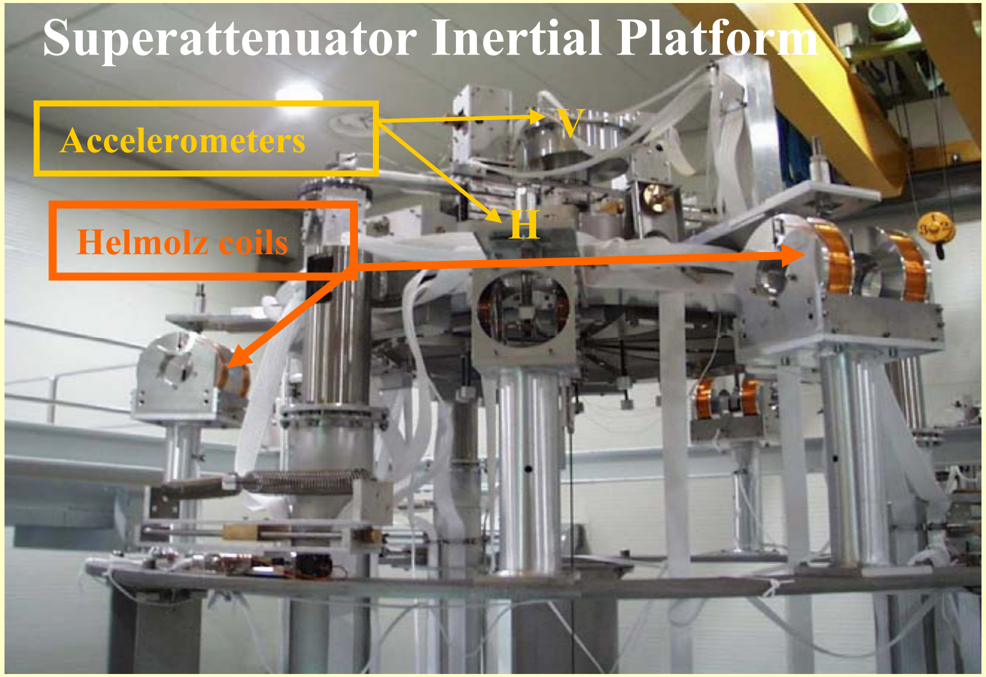
Marionet

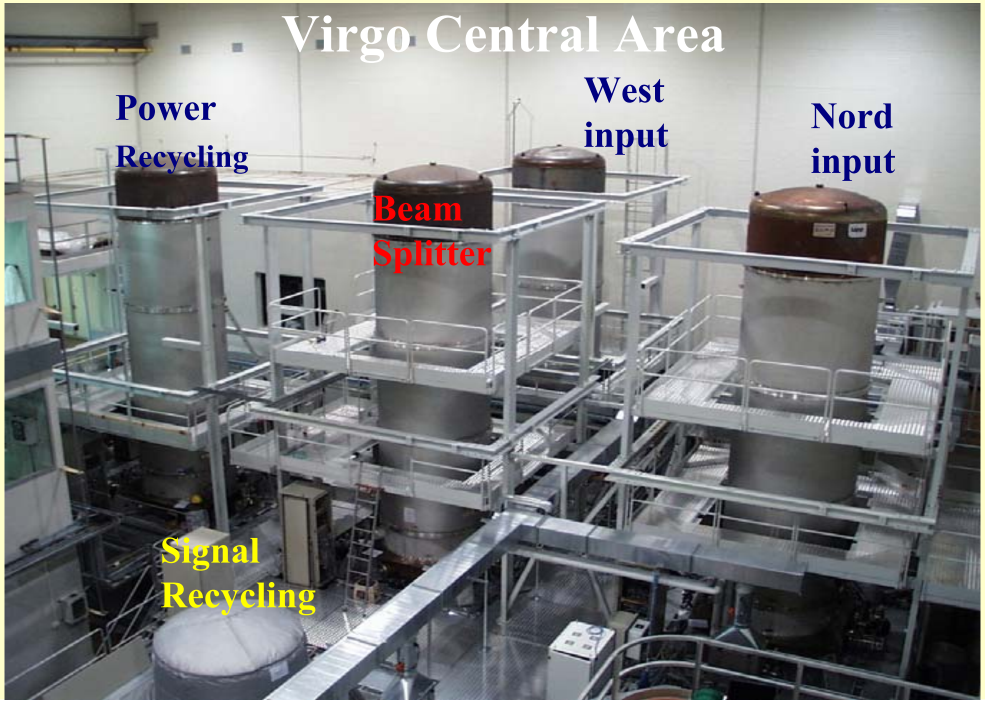
Mirror

SA Transf. Function

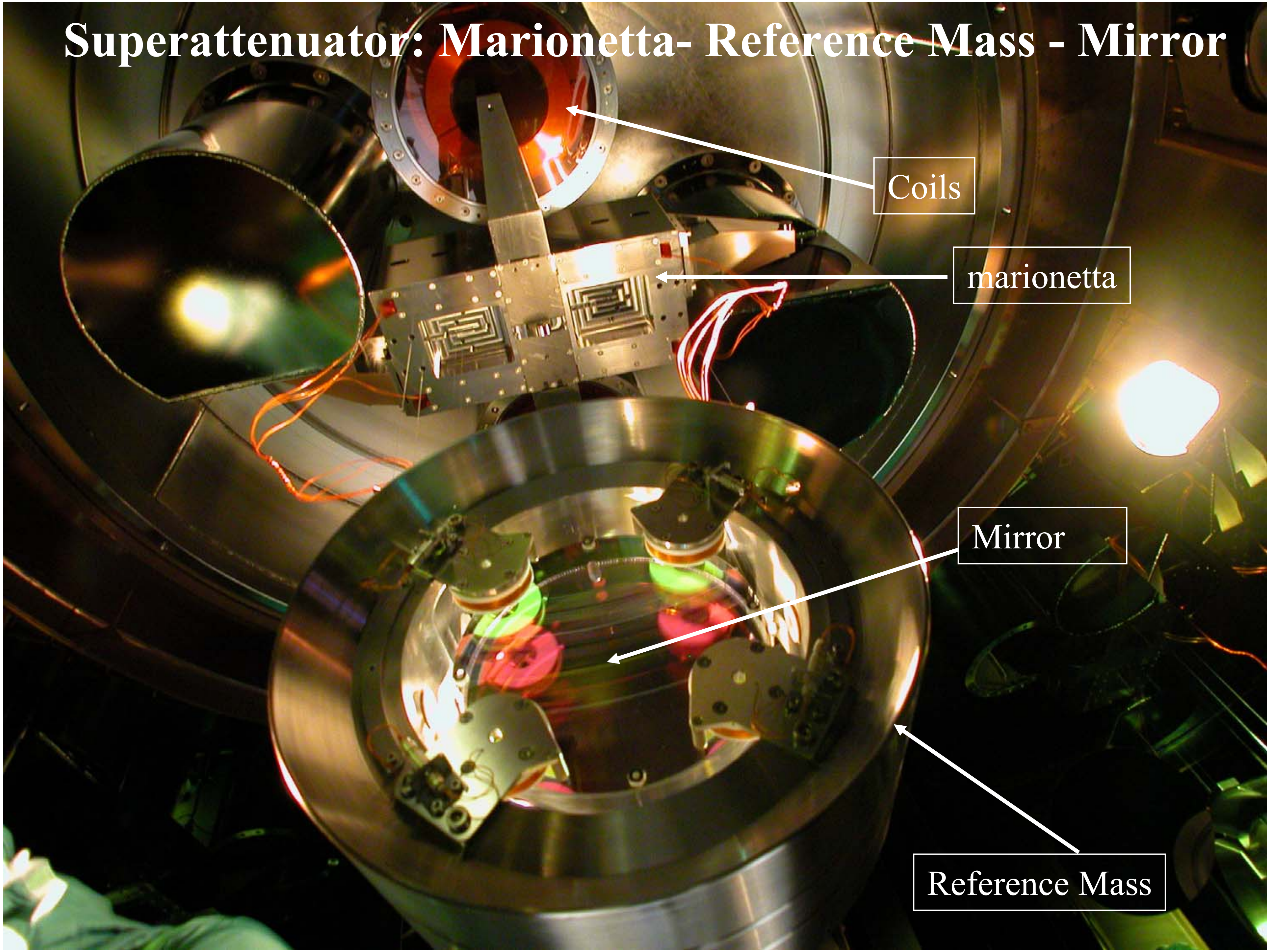



Superattenuators inner structure


Superattenuator Inertial Platform


Virgo Central Area

Superattenuator: Marionetta- Reference Mass - Mirror



GW DETECTION STATUS

IGEC: Network of Bar Detectors Started in 1997 (Auriga, Explorer, Nautilus, Allegro) for impulsive GW detection.

No evidence of a significant GW signal

LIGO-GEO600: GW from Pulsar (28 known)- ε < 10-5 – 10-6 (no mountains > 10 cm)- h upper limits: 2.10^{-24}@200Hz, 5.10^{-24}@400Hz, 10^{-23}@1KHz

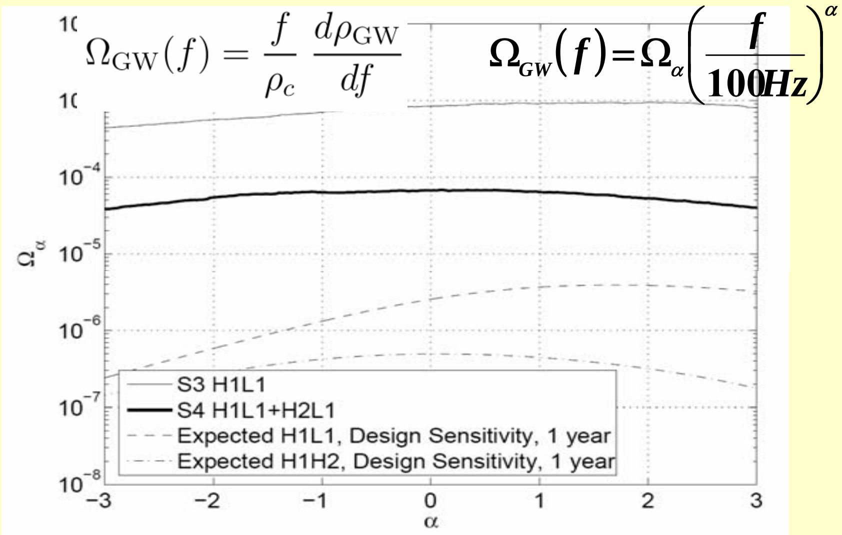
No evidence of a significant GW signal

LIGO,GEO600,TAMA: Up. lim.: Coalescing NS-NS <1 event/(gal.year) $2 < M_0 < 6$

Coalescing BH-BH <1 event/(gal.year) $10 < M_0 < 80$

No evidence of a significant GW signal

LIGO: Stockastic BKG

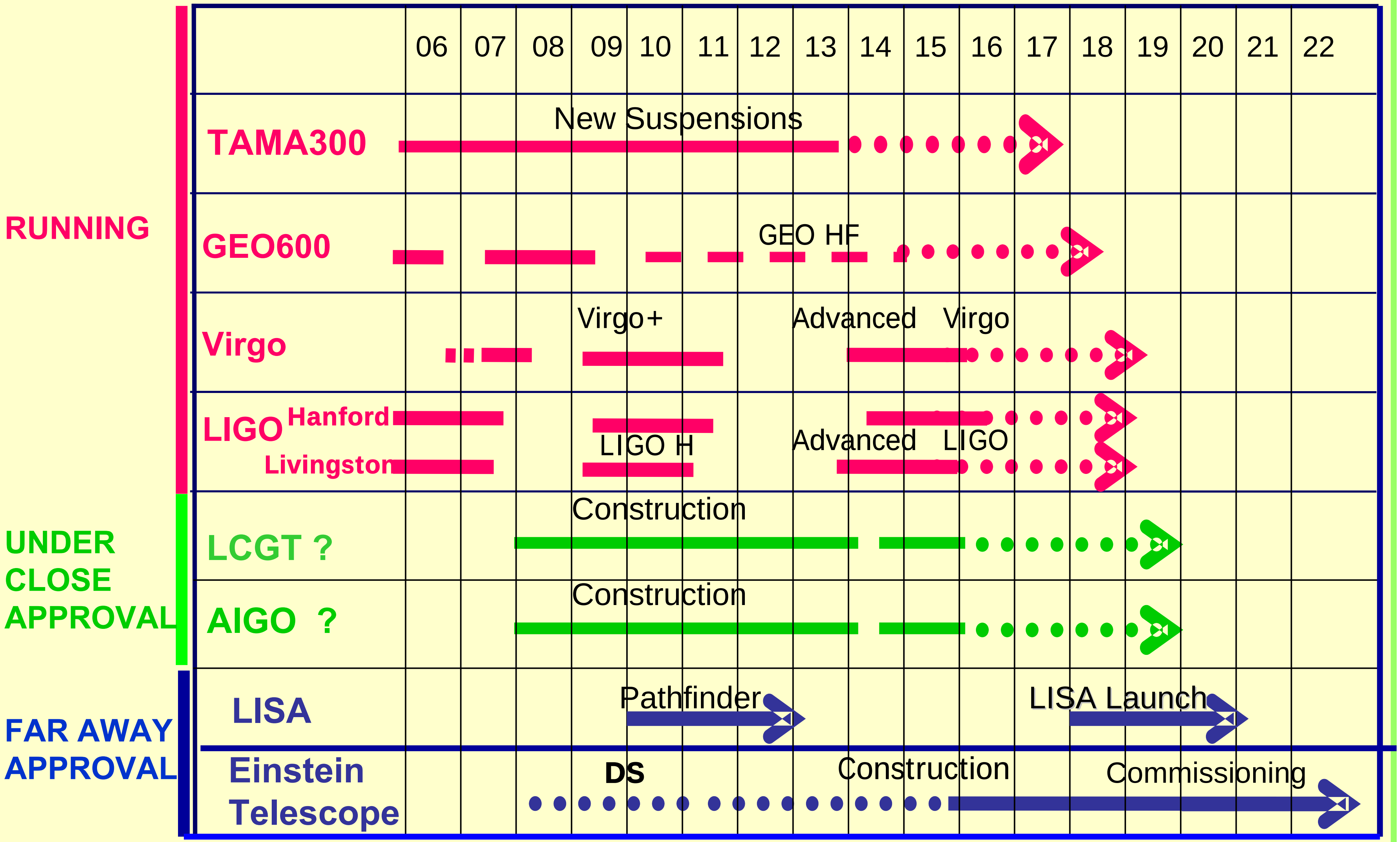


$$\Omega_{\rm GW}(f) = \frac{f}{\rho_c}\frac{d\rho_{\rm GW}}{df}$$

$$\Omega_{GW}(f)=\Omega_\alpha\left(\frac{f}{100Hz}\right)^\alpha$$

Virgo, LIGO, GEO 600:
May 18th-Oct. 2007 started common data taking and coherent analysis; main target impulsive events. Analysis running

The Future



06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22

RUNNING

TAMA300
New Suspensions

GEO600
GEO HF

Virgo
Virgo+
Advanced Virgo

LIGO Hanford
LIGO Livingston
LIGO H
Advanced LIGO

UNDER CLOSE APPROVAL

LCGT ?
Construction

AIGO ?
Construction

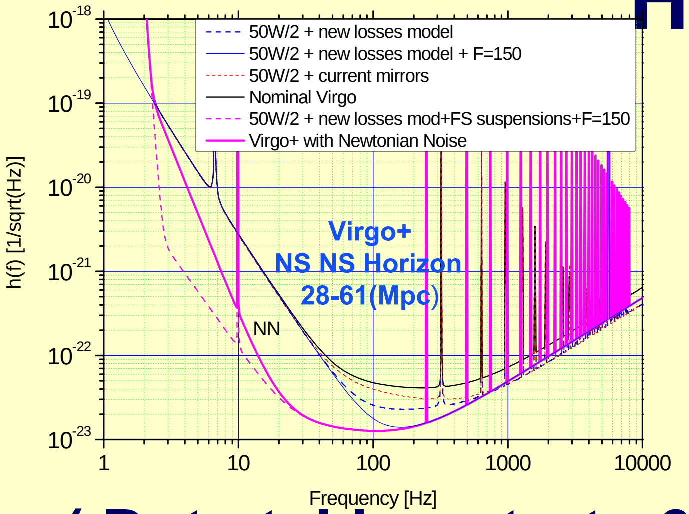
FAR AWAY APPROVAL

LISA
Pathfinder
LISA Launch

Einstein Telescope
DS
Construction
Commissioning

Virgo+

1) Cure low freq. Noise
2) Fused silica suspens
3) Increase arm finesse
4) Higher power laser



Henanced Ligo

1) DC readout
2) Higher laser power
3) Output modecleaner

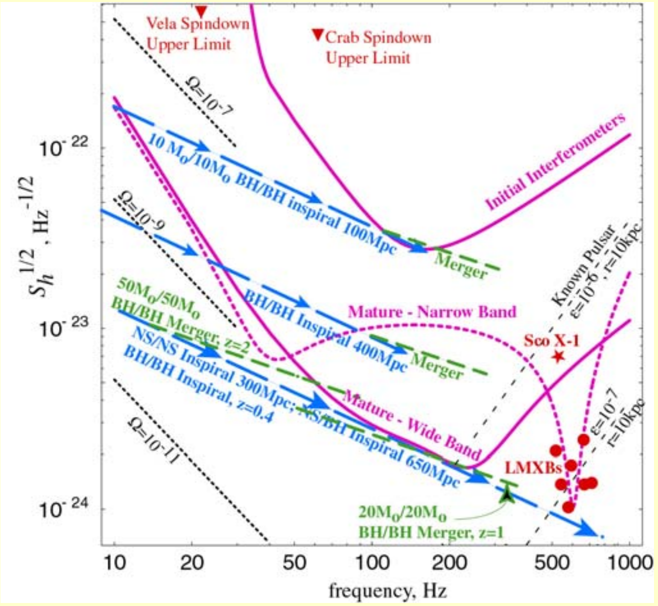
A factor of 2 improv. in sensitivity (8 in event rate)

(Data taking starts 6/2009)

Advanced Virgo

1)Larger mirror
2)Improved coatings
3)Higher laser power
4)DC readout

R&D underway
Design decisions late 2008

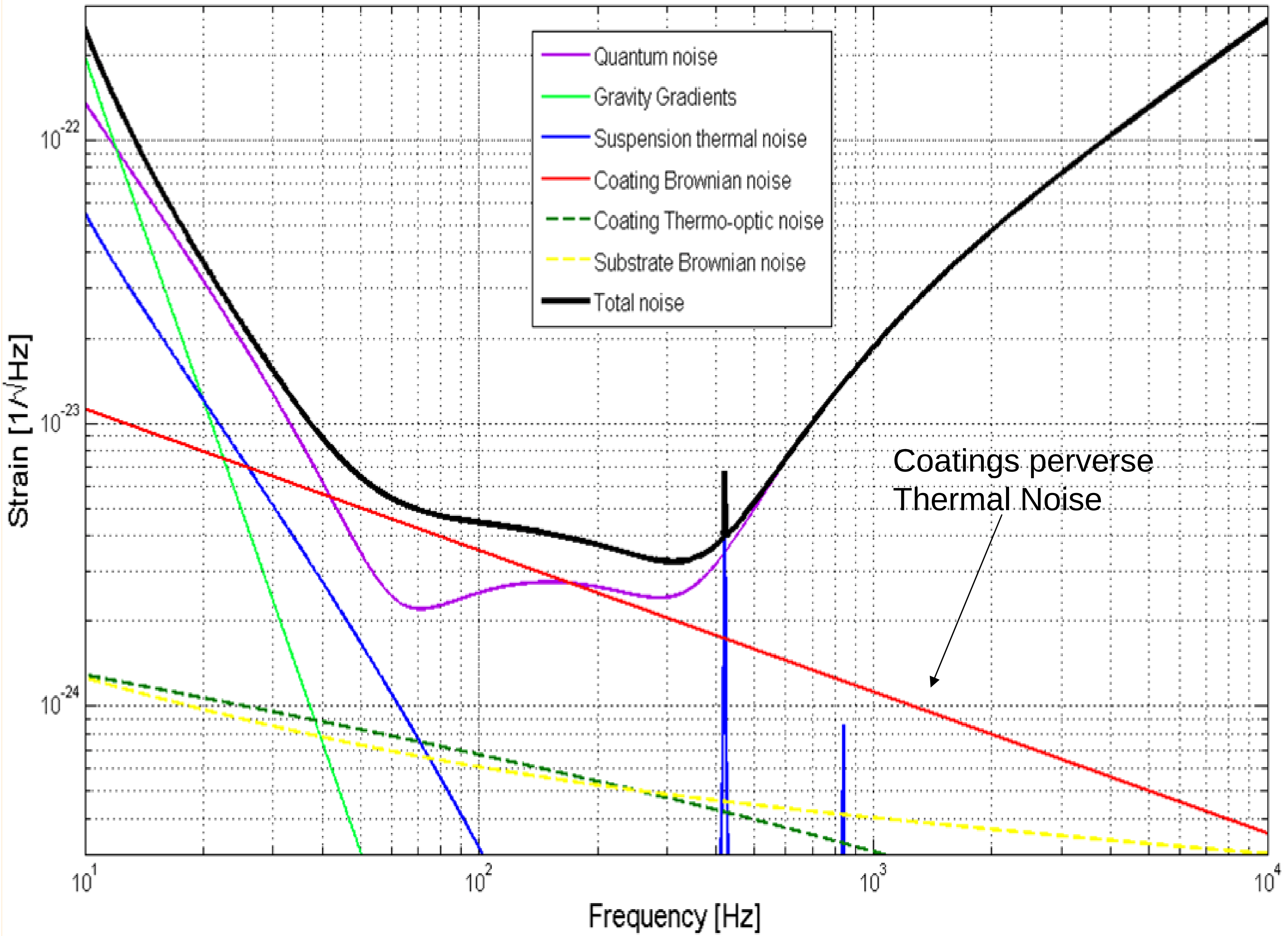


(Data taking starts 2014)

Advanced Ligo

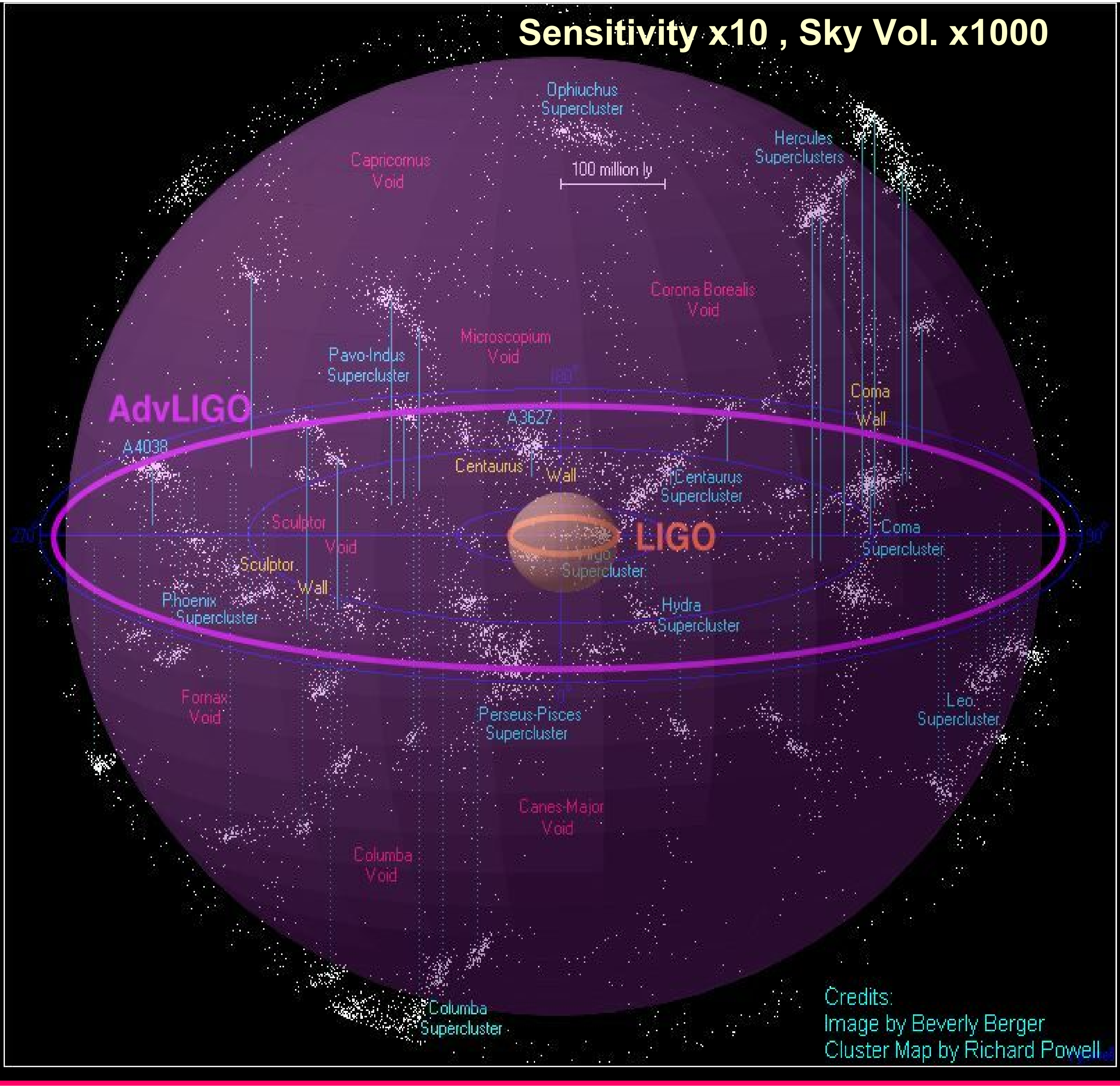
1)Active anti-seismic system operating to down to 10 Hz
2)Lower thermal noise suspensions and optics
3)Higher laser power
4)More sensitive and more flexible optical configuration

Parameter	*LIGO*	*Advanced LIGO*
Input Laser Power	10 W	180 W
Mirror Mass	10 kg	40 kg
Interferometer Topology	Power-recycled Fabry-Perot arm cavity Michelson	Dual-recycled Fabry-Perot arm cavity Michelson
GW Readout Method	RF heterodyne	DC homodyne
Optimal Strain Sensitivity	3 x 10^{-23} / rHz	Tunable, better than 5 x 10^{-24} / rHz
Seismic Isolation	f_{low} ~ 50 Hz	f_{low} ~ 10 Hz
Mirror Suspensions	Single Pendulum	Quadruple pendulum



Coatings perverse Thermal Noise

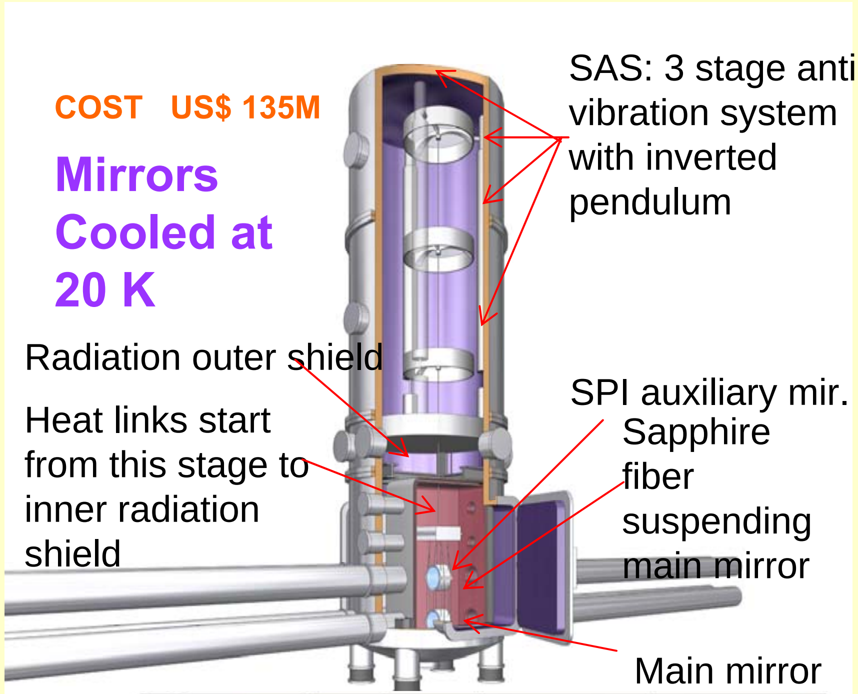
Sensitivity x10 , Sky Vol. x1000



Credits:
Image by Beverly Berger
Cluster Map by Richard Powell

Two Projects very important for the future

LCGT: A 3 km CRYOGENIC Interf. in Japan



COST US$ 135M

Mirrors Cooled at 20 K

SAS: 3 stage anti-vibration system with inverted pendulum

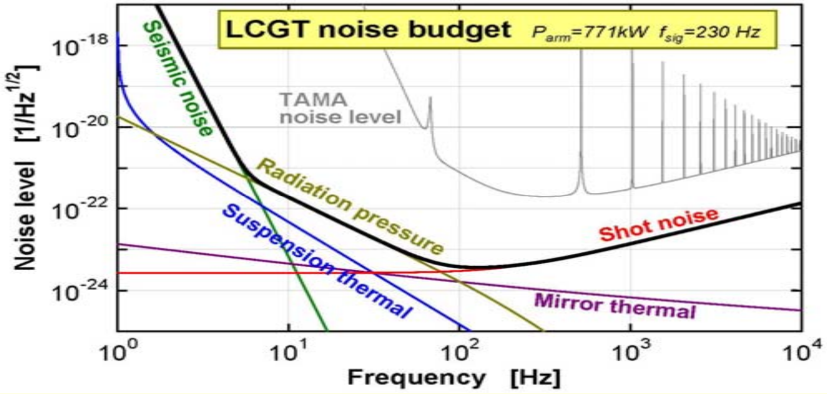
Radiation outer shield

Heat links start from this stage to inner radiation shield

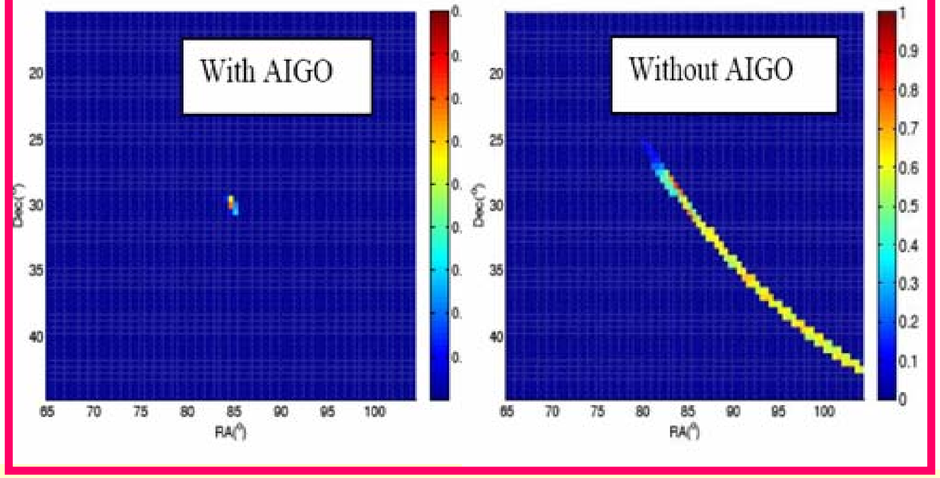
SPI auxiliary mir.

Sapphire fiber suspending main mirror

Main mirror

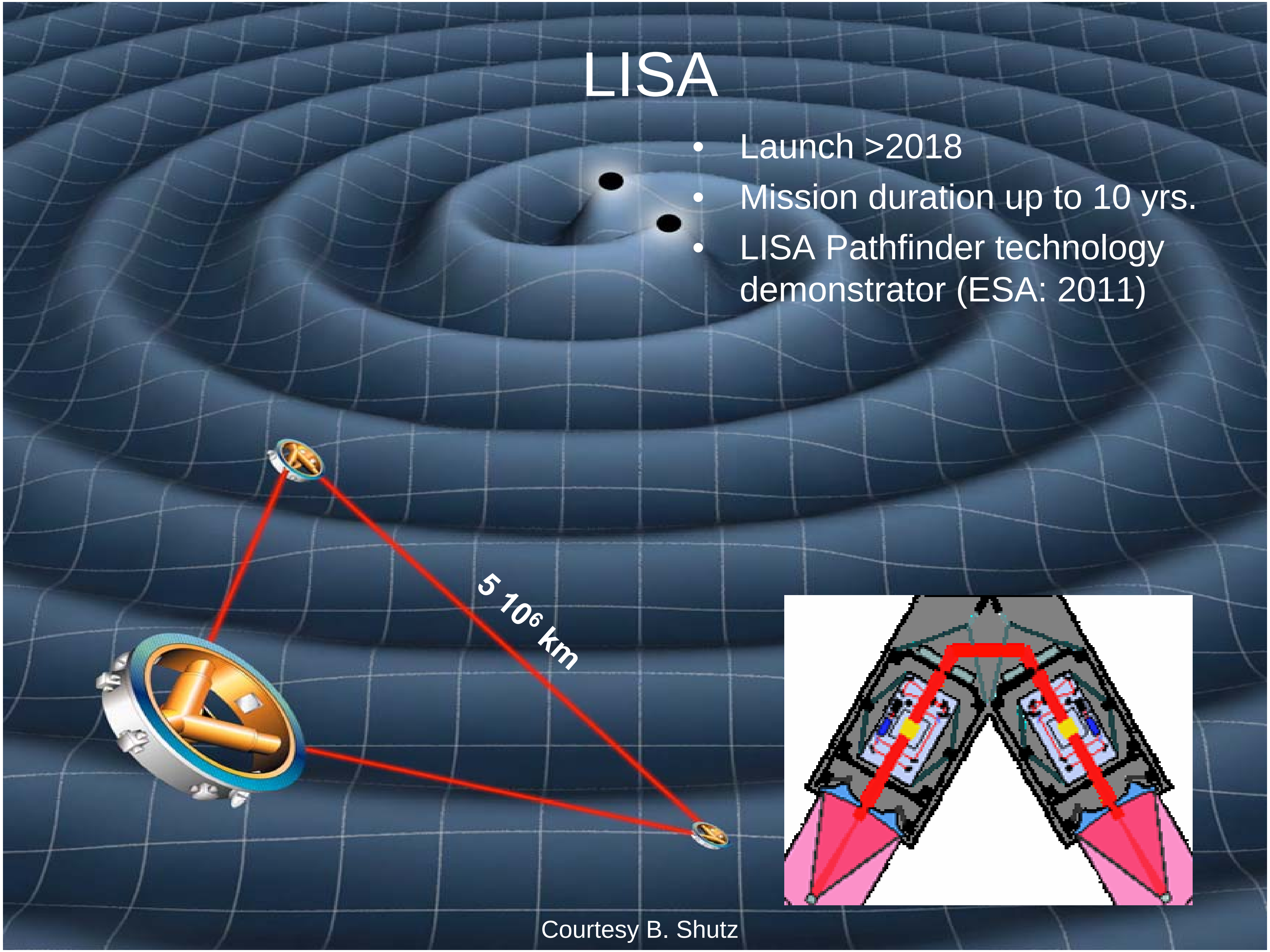


AIGO- A 5 km Interf. In Australia



LISA

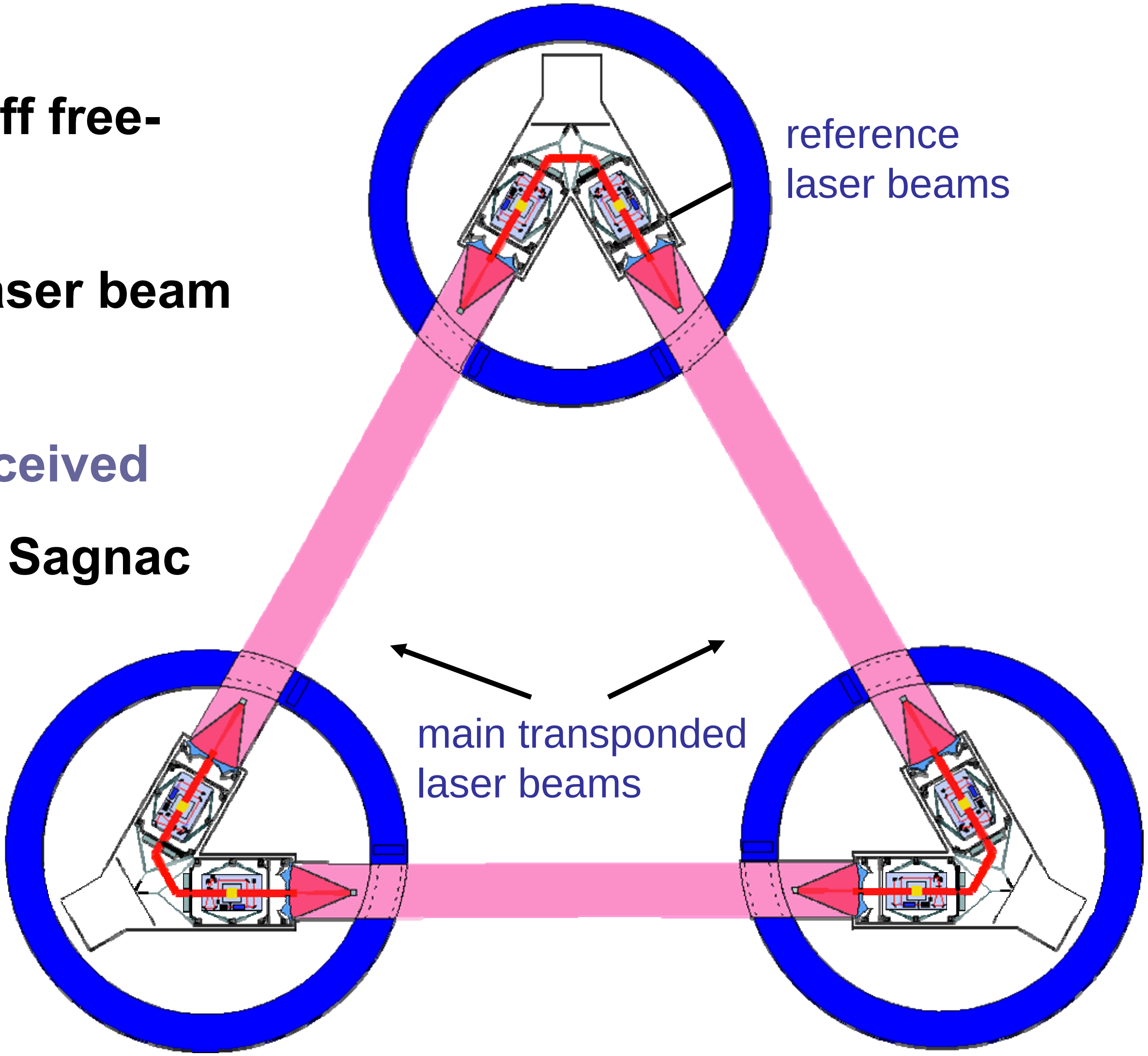
- Launch >2018
- Mission duration up to 10 yrs.
- LISA Pathfinder technology demonstrator (ESA: 2011)



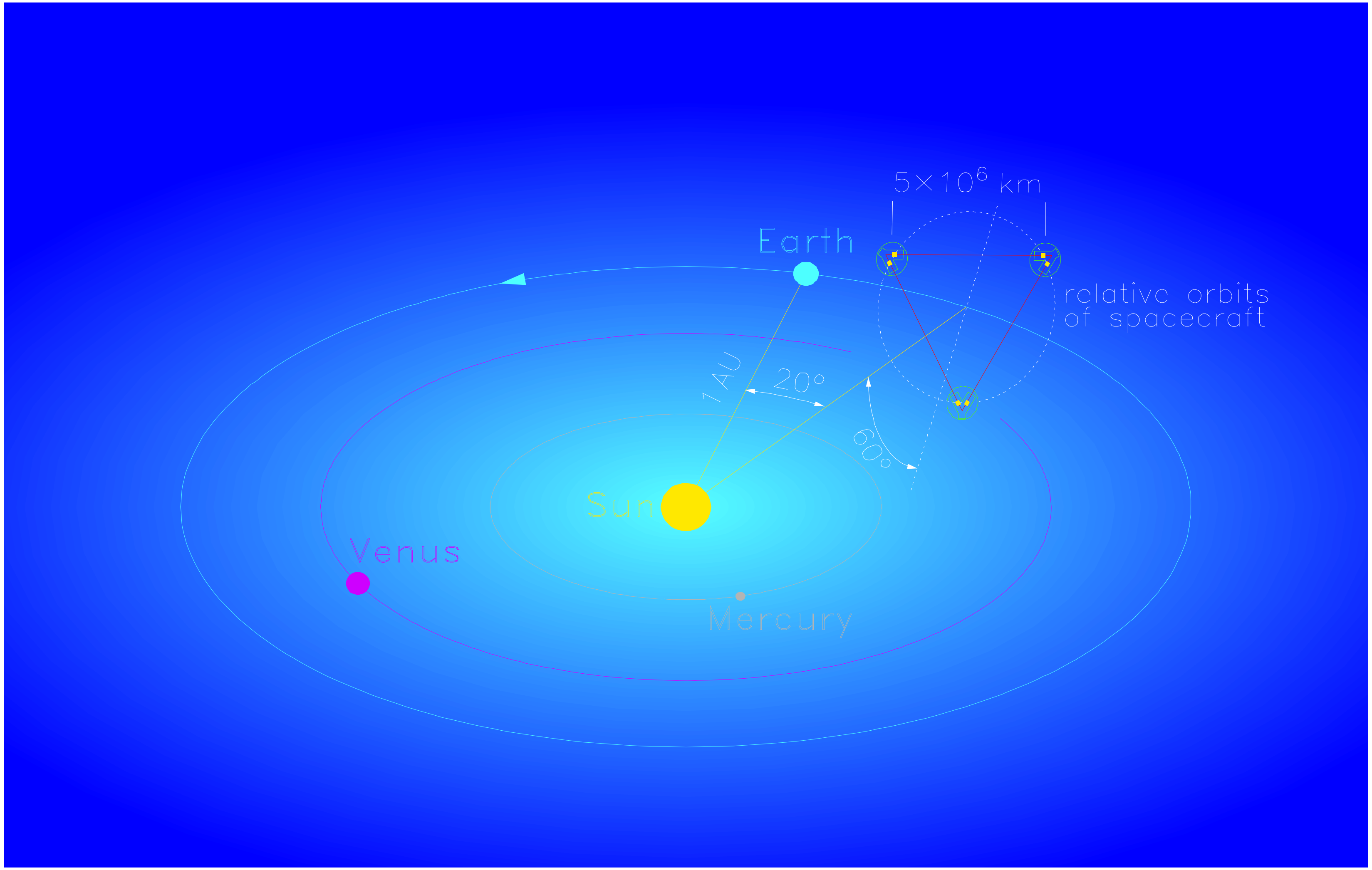
Courtesy B. Shutz

LISA Layout

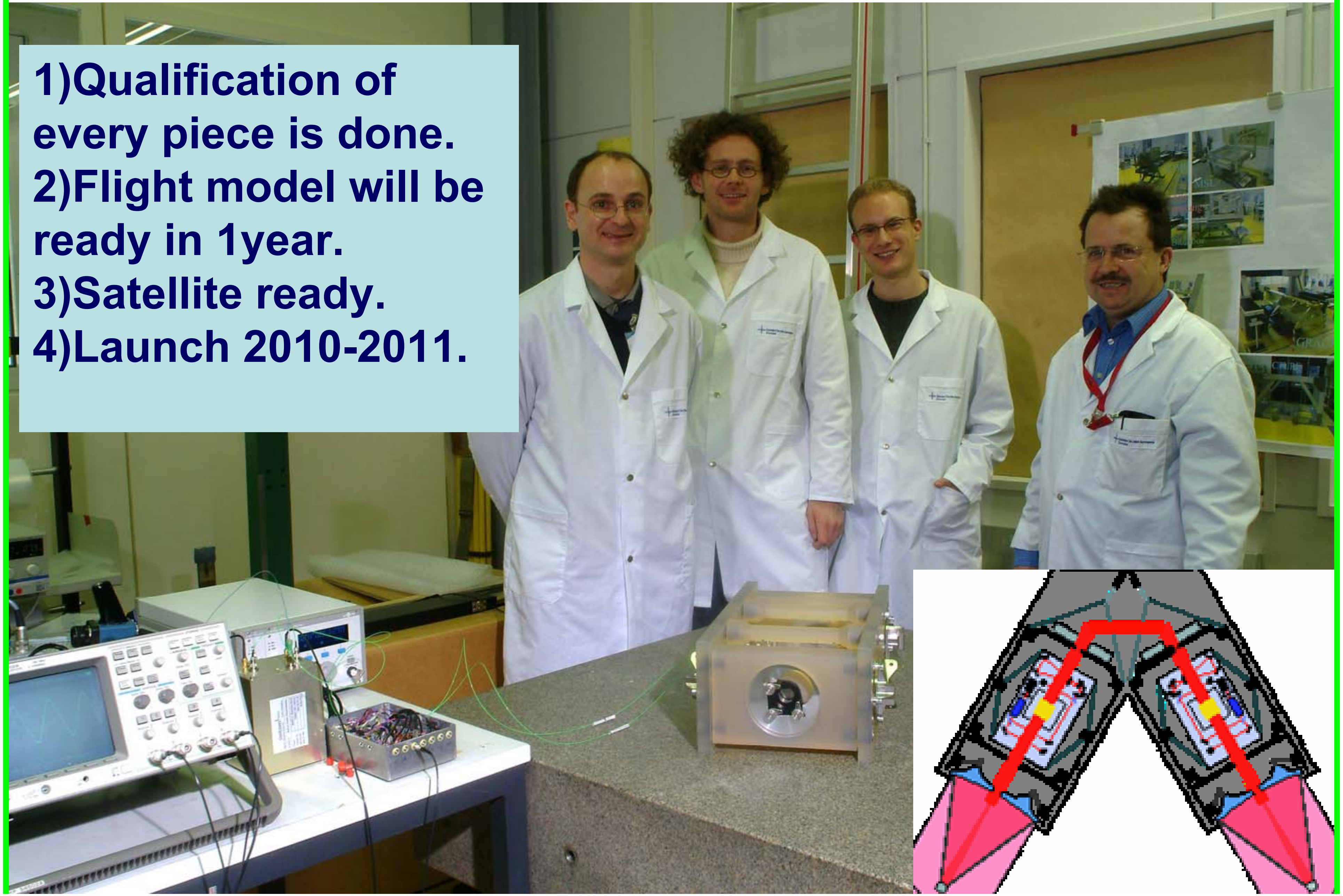
- **Laser beams reflected off free-flying test masses**
- **Diffraction widens the laser beam to many kilometers**
  - **0.7 W sent, 70 pW received**
- **Michelson with 3rd arm, Sagnac**
- **Can distinguish both polarizations of a GW**
- **Orbital motion provides direction information**



Cluster of 3 LISA spacecraft

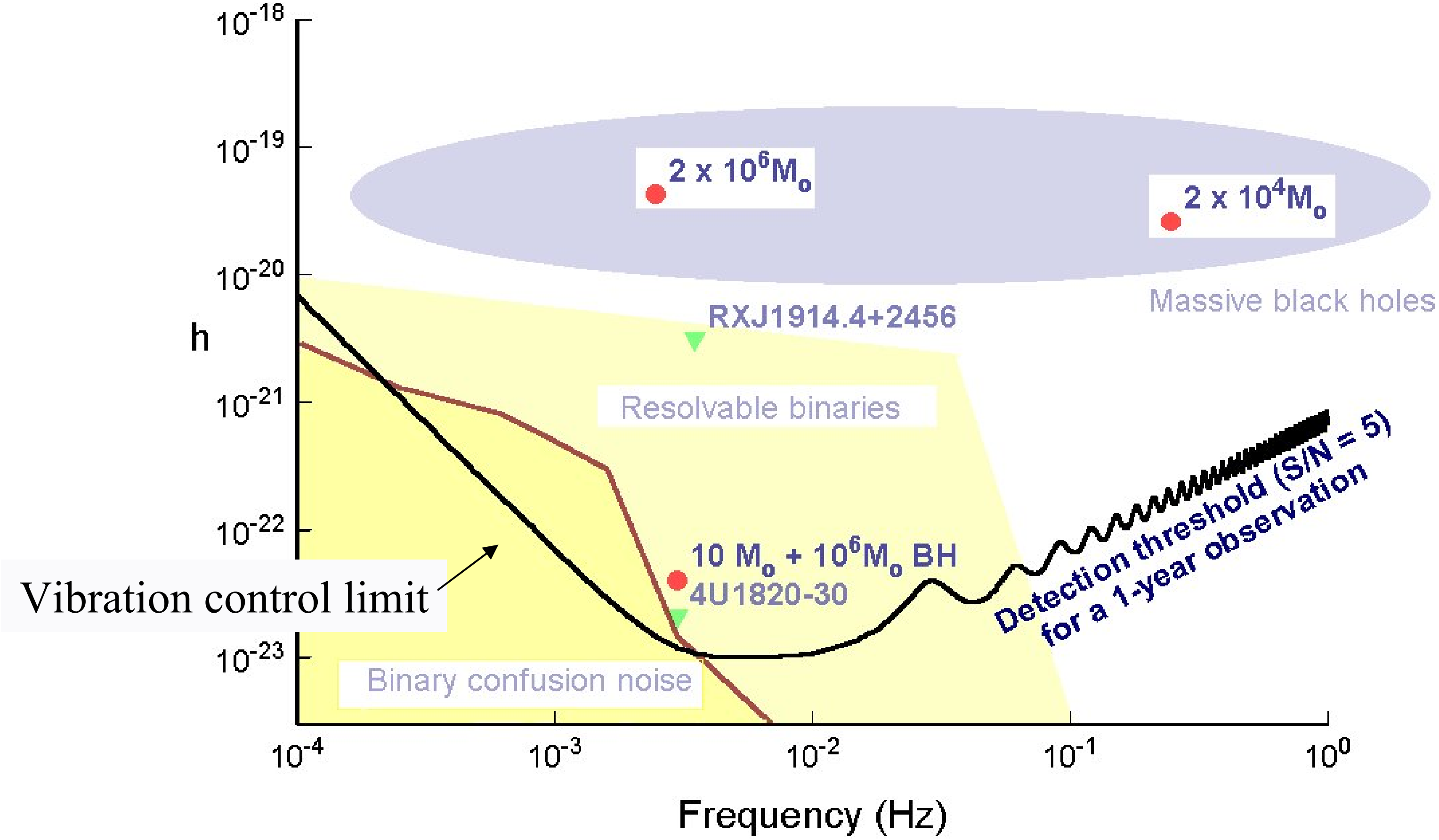


LISA Pathfinder

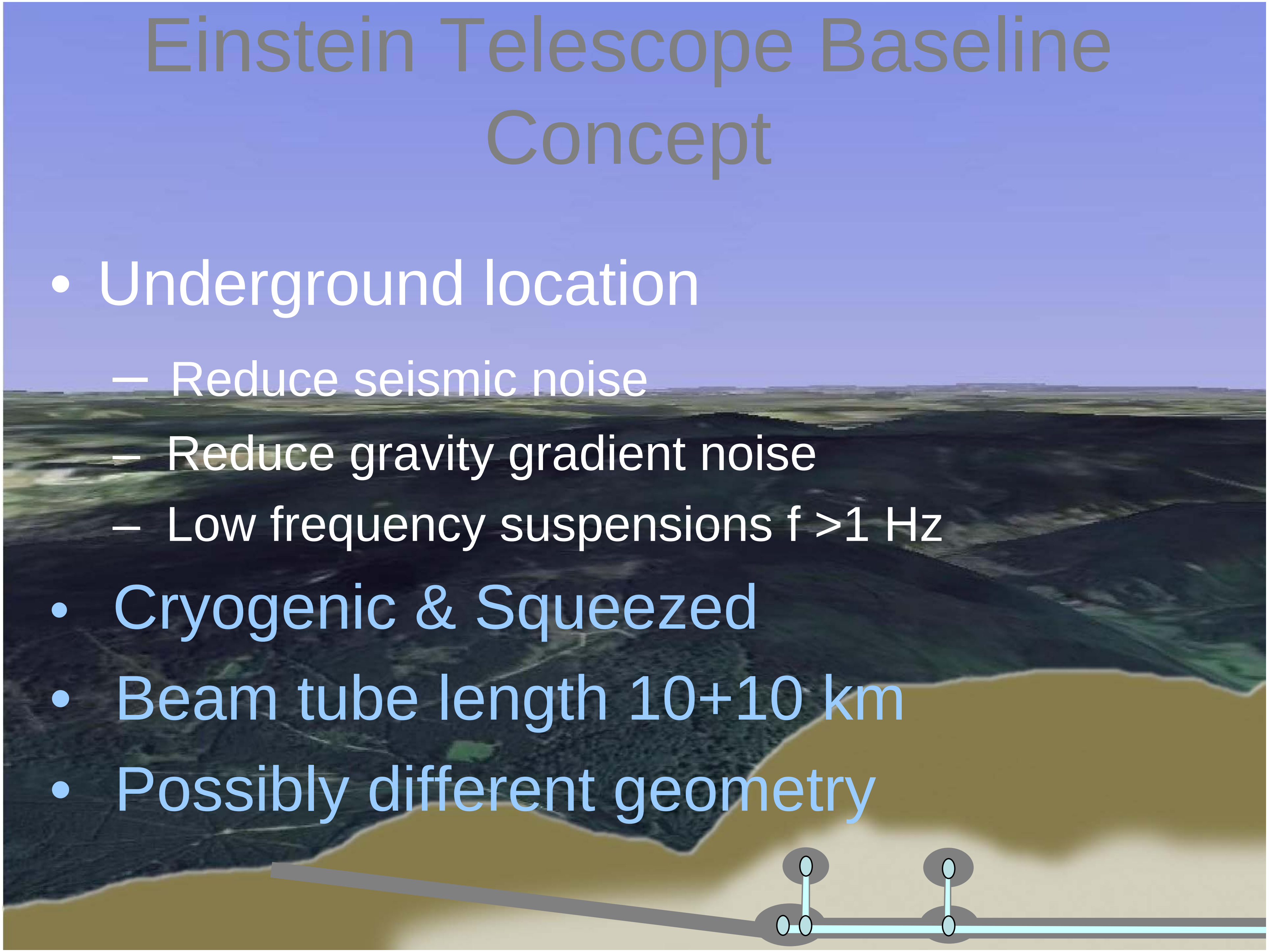


1)Qualification of every piece is done.
2)Flight model will be ready in 1year.
3)Satellite ready.
4)Launch 2010-2011.

LISA Sensitivity



Einstein Telescope Baseline Concept



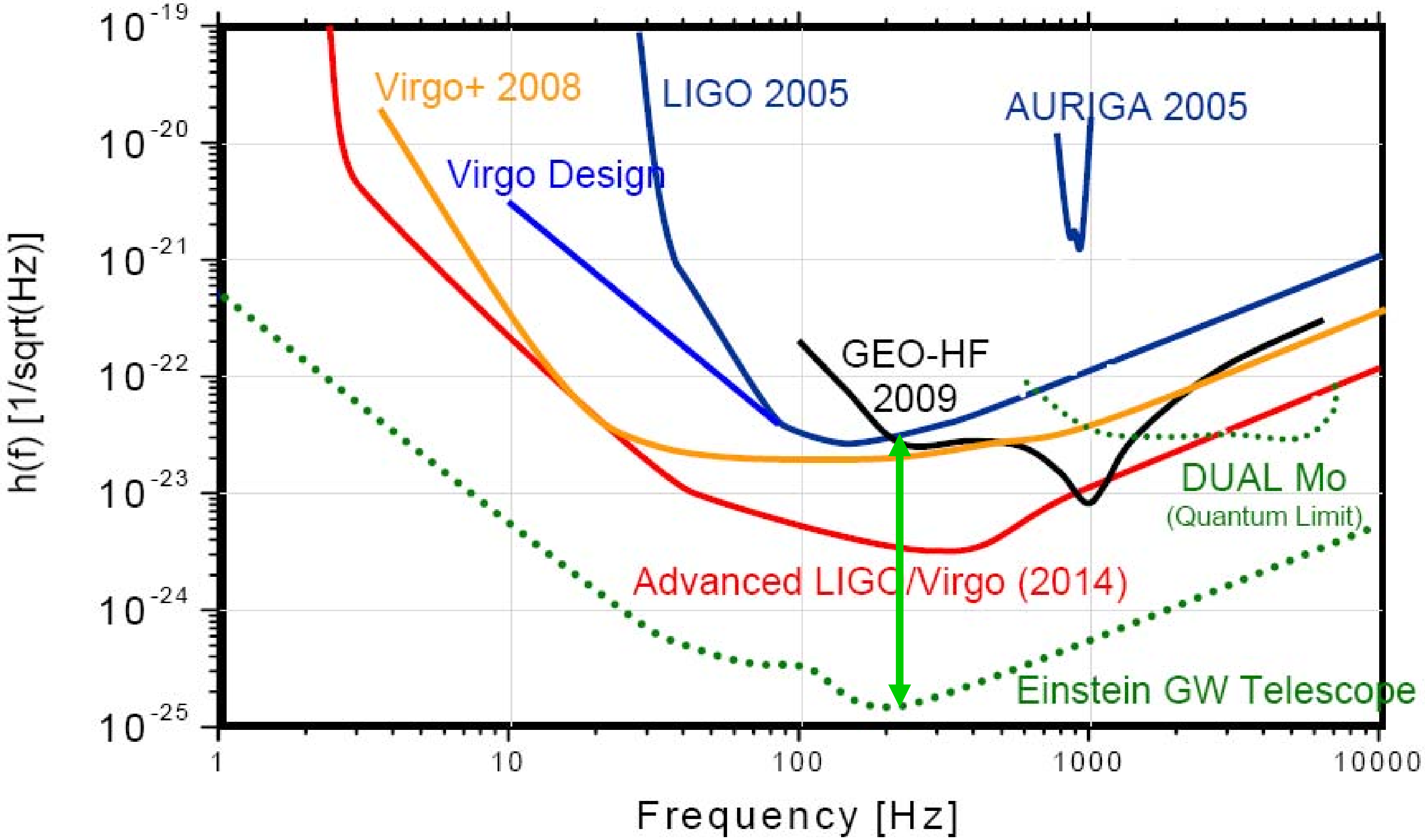
- Underground location
  - Reduce seismic noise
  - Reduce gravity gradient noise
  - Low frequency suspensions f >1 Hz
- Cryogenic & Squeezed
- Beam tube length 10+10 km
- Possibly different geometry

Einstein Telescope Configuration

1)ET will be the only surviving project. Virgo and LIGO will not have enough sensitivity for making a Network with ET

2)ET should be formed by at least 4 interferometers, well spaced in such a way to accurately measure source angle from time of flight differences. A wise decision could be in the same spirit as ESO whose telescopes are not in Europe. ET network should have at least one detector in southern hemisphere for better solving the “Inverse Problem”.

ET Sensitivity

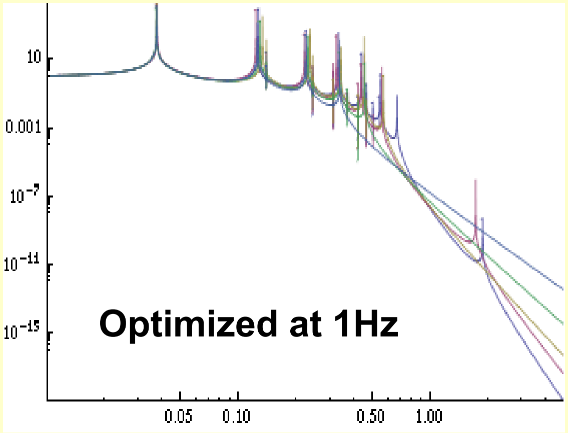


Harald Lück
for the European Gravitational-Wave Community

Some exercise: **Use of Superattenuators for ET**

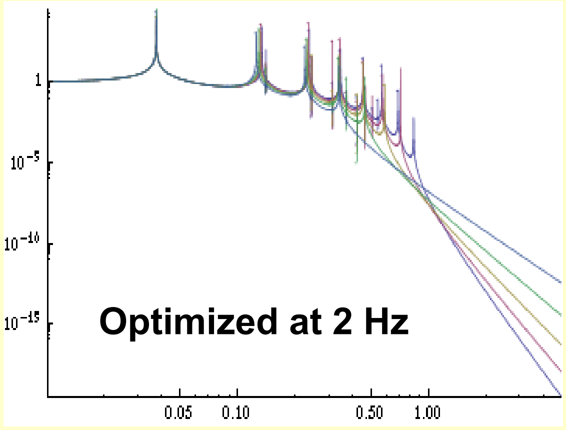
1) Inverted Pend. 40 mHz
2) 50 m tall mechanical filter chain

```
N     1 Hz       2 Hz
----------------------------
 {3, 1.6*10-7, 5.4*10-10}
 {4, 4.8*10-8, 3.3*10-11}
 {5, 2.6*10-8, 3.4*10-12}
 {6, 2.3*10-8, 6.7*10-12}
 {7, 2.1*10-8, 2.6*10-12}
```

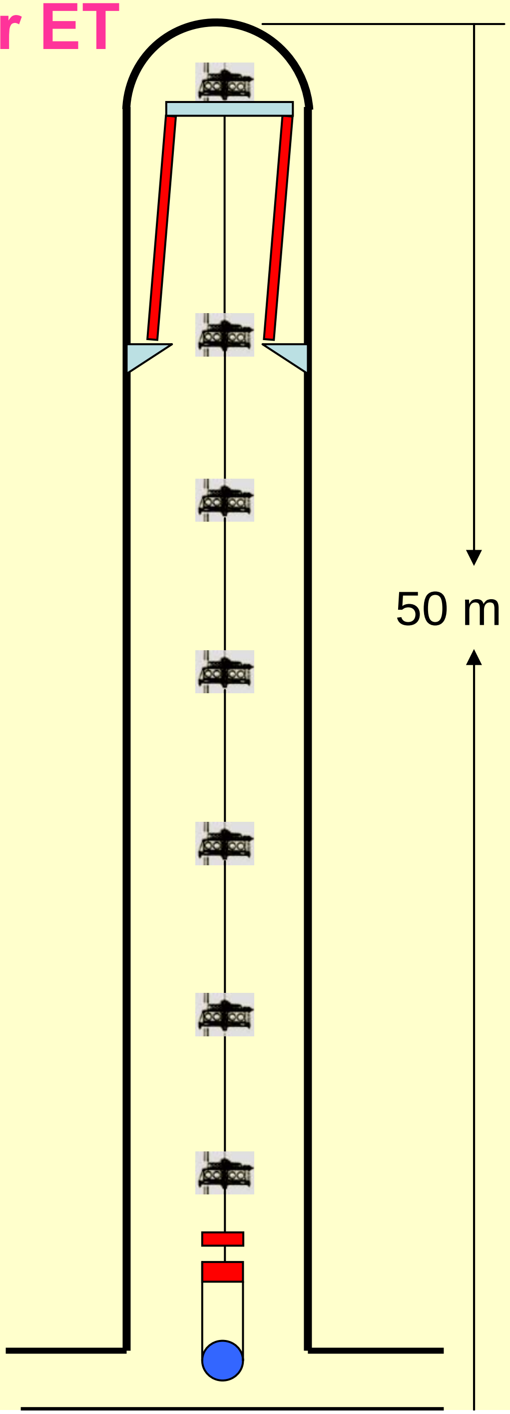


Optimized at 1Hz

```
N     1 Hz       2 Hz
----------------------------
 {3, 1.6*10-7, 5.4*10-10}
 {4, 4.8*10-8, 3.3*10-11}
 {5, 2.7*10-8, 3.3*10-12}
 {6, 2.8*10-8, 4.9*10-13}
 {7, 6.3*10-8, 1.0*10-13}
```



Optimized at 2 Hz



By assuming a seismic noise underground 10^{-8}m/sqrt(Hz) @1Hz, from the TF optimized at 2Hz we obtain h(2Hz)=10^{-25}/sqrt(Hz)<<10^{-22}/sqrt(Hz)

ET Prototyping

It is likely that the majority of ET noises can not be model, due to the extreme sensitivity needed:

a) Diffused light can not be model

b) e.m. fields can not be model

c) Ground Loops can not be model

Etc.........

To my opinion Interested nations should make a pool for building a full scale prototype .

Some Final Considerations

- **Bar detectors have grown up, by means of a fantastic technological effort, to enormous and unexpected sensitivity and operation stability. Their operation was so good as to create the first GW network.**

- **The big steps forward in the last decade has been in the Interferometers technology. They reached design sensitivity almost to 10 Hz and stability is so good (unespectedly) that we have created an efficient network. Virgo, now, is opening the very low frequency region and Advanced LIGO and Virgo will further reduce noise in this troublesome frequency region.**

- **Class Einstein, after what we have learned by the big machines, seems fesable with a very high probability of success. 1 Day of data of ET is equivalent to 10^6 days of data taking with Virgo or LIGO. This seems to be the right way to go for starting GW astronomy.**

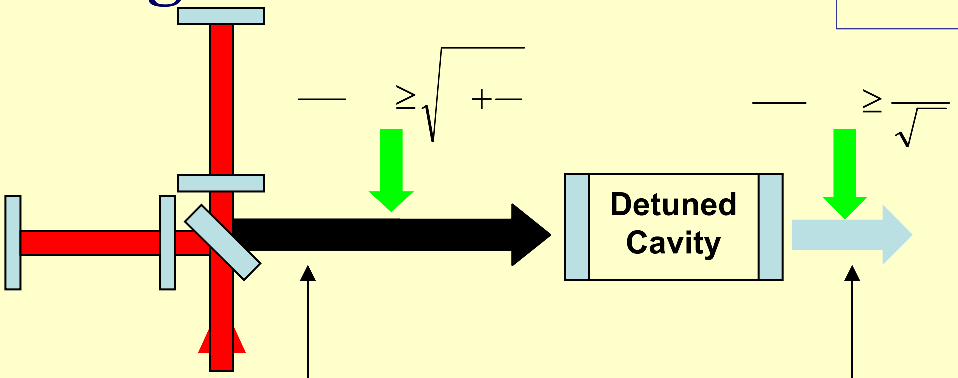
Modern Interferometers with QND Signal Readout

$$K = \frac{32\omega_0 W F^2}{M c^2 \Omega^2}$$

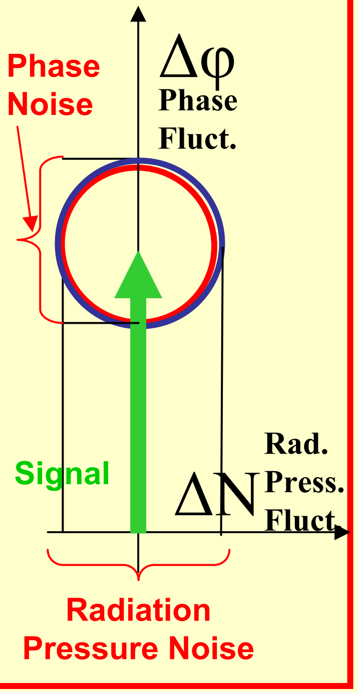
Uncertainty Principle:

$\Delta\varphi . \Delta N \sim 1$

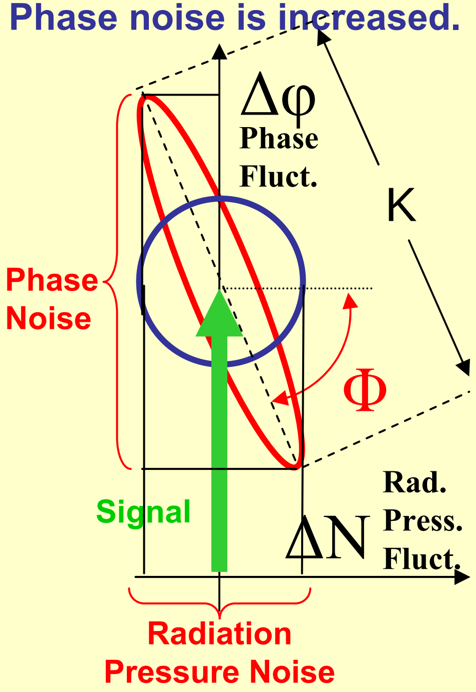
We only measure φ, the only one containing the signal, hence we can ignore ΔN.



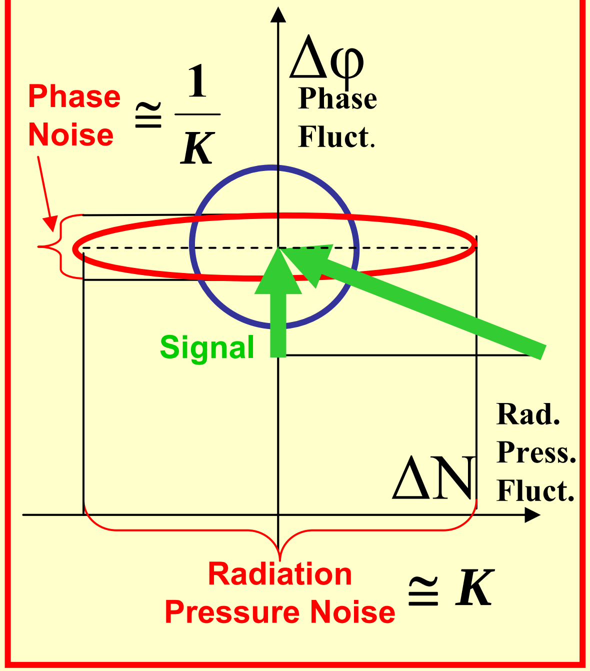
In a Fix Mirror ITF, Rad. Press. Fluct. can't move mirrors.



In a suspended Mirror ITF, Rad. Press. Fluct. move randomly mirrors, hence Phase noise is increased.

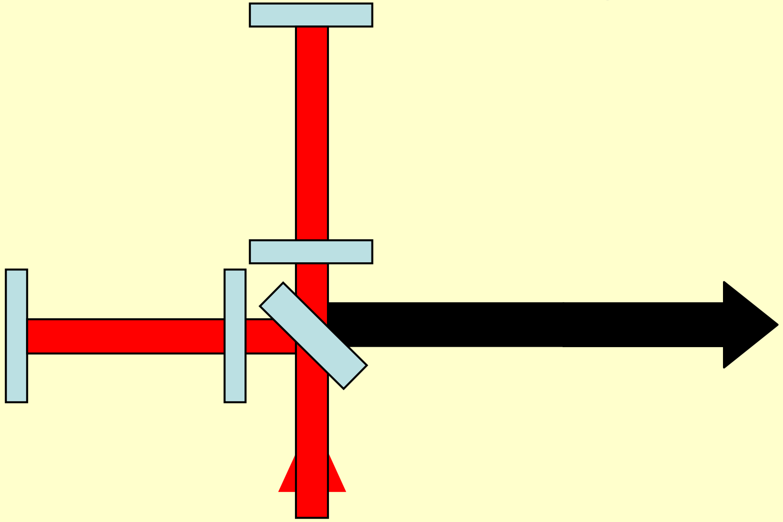


A Detuned Cavity can rotate in the Δφ, ΔN plane. Phase noise Δφ has been decreased at expenses of ΔN.

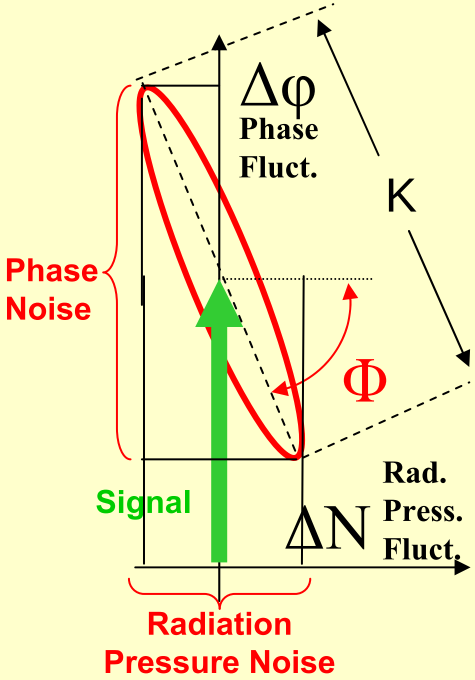


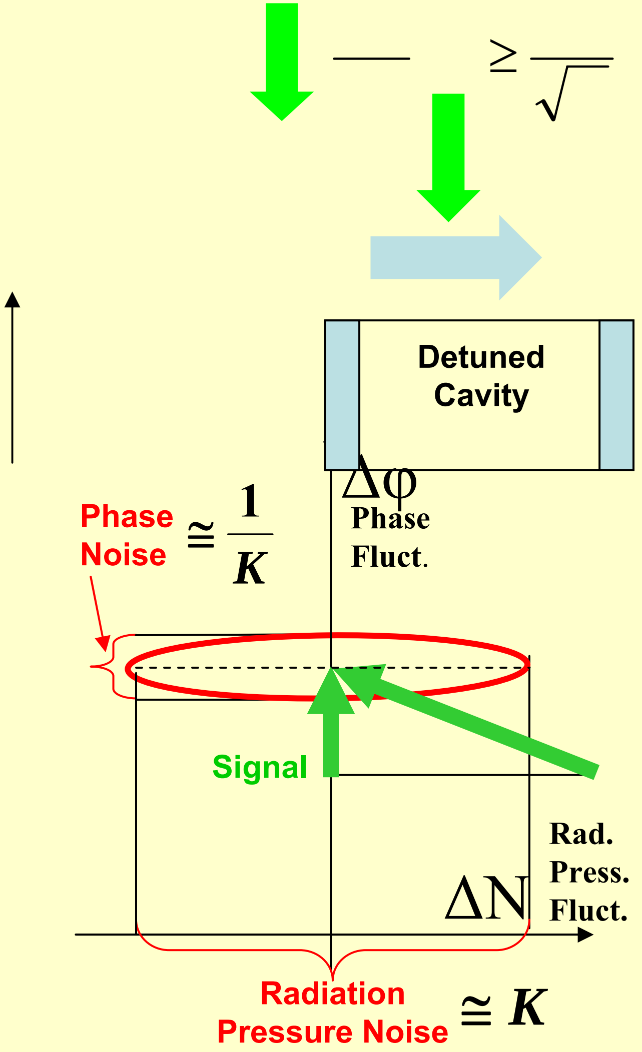
Modern Interferometers with QND Signal Readout

$$K = \frac{32\omega_0 W F^2}{M c^2 \Omega^2}$$



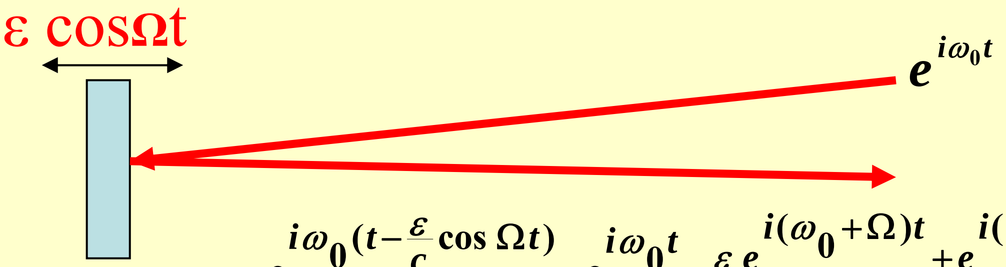
$$\begin{cases} \begin{pmatrix} a_0 \\ a_{\pi/2} + \Gamma h L - K a_0 \end{pmatrix} \\ K = \dfrac{32\omega_0 W F^2}{M c^2 \Omega^2} \end{cases}$$



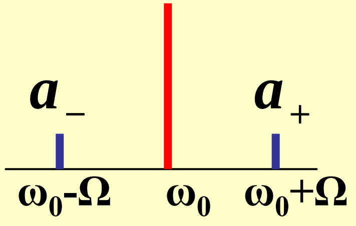


How to go below the SQL: Modern Interferometers with QND Signal Readout

In interferometric detectors, GW produce sidebands at frequency $\omega_0 \pm \Omega$ and emission of two correlated photons of frequency $\omega_0 \pm \Omega$:



$$e^{i\omega_0\left(t-\frac{\varepsilon}{c}\cos\Omega t\right)} \approx e^{i\omega_0 t} - \frac{\varepsilon}{c}\,\frac{e^{i(\omega_0+\Omega)t} + e^{i(\omega_0-\Omega)t}}{2}$$



$$\left[a_+, a_+'^{+}\right] = \left[a_-, a_-'^{+}\right] = 2\pi\delta(\Omega-\Omega')$$

The Total e.m. Field

$$\begin{cases} a_0(\Omega) = \dfrac{a_+ + a_-^+}{\sqrt{2}} \qquad a_{\pi/2}(\Omega) = \dfrac{a_+ - a_-^+}{i\sqrt{2}} \\ E_{Total} = C\left[\left(\alpha_0 + a_0(t)\right)\cos\omega_0 t + \left(\alpha_{\pi/2} + a_{\pi/2}(t)\right)\sin\omega_0 t\right] \end{cases}$$

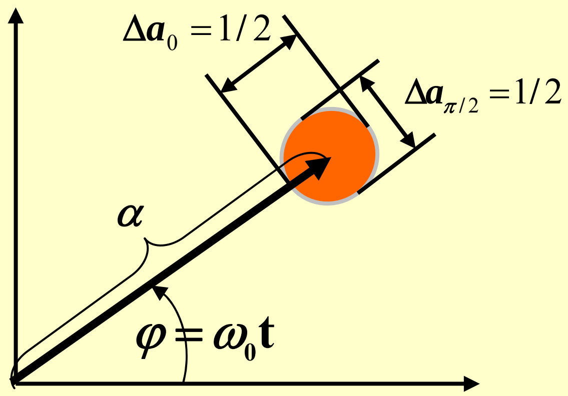
Classical Field Carrier

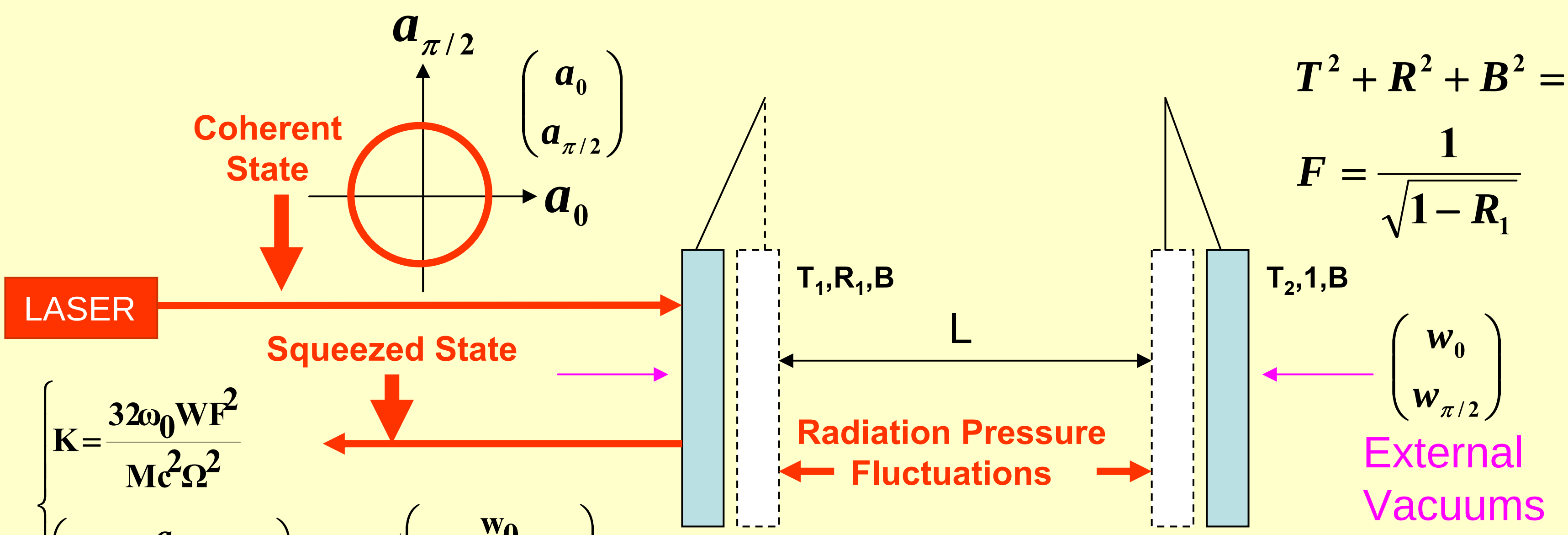
Quantum Field Quadratures

$$\begin{pmatrix} \alpha_0 \\ \alpha_{\pi/2} \end{pmatrix} + \begin{pmatrix} a_0(\Omega) \\ a_{\pi/2}(\Omega) \end{pmatrix}$$

←Intensity Fluct.

←Phase Fluct.

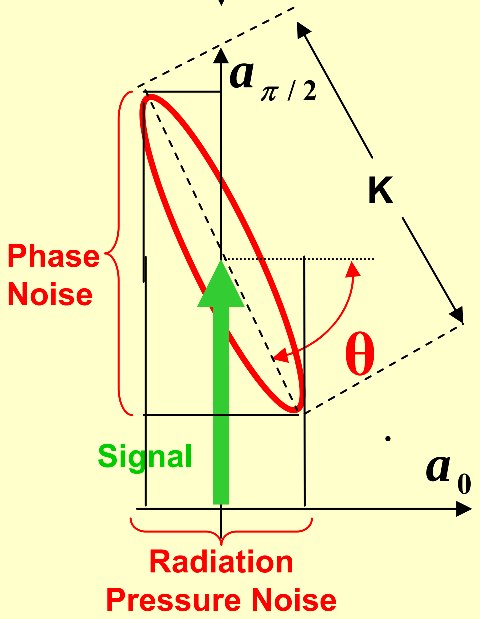




$$T^2 + R^2 + B^2 = 1$$

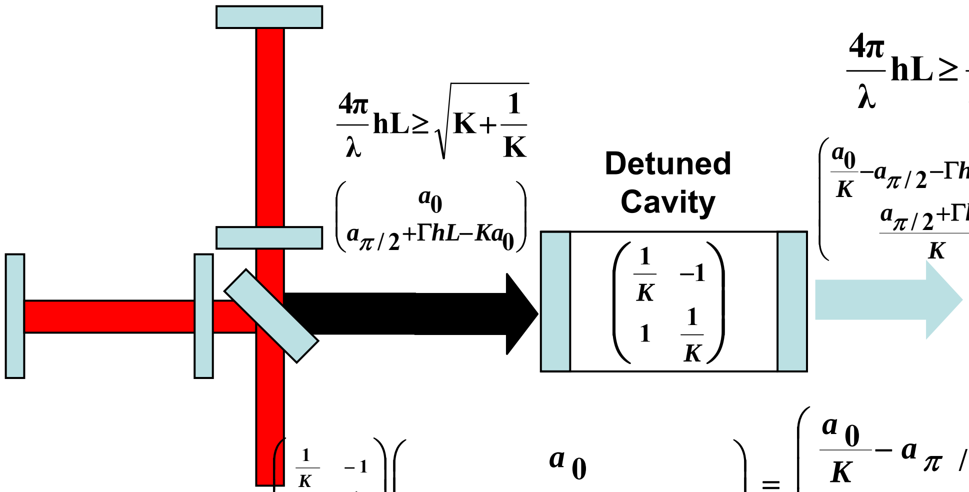
$$F = \frac{1}{\sqrt{1 - R_1}}$$

$$\begin{cases} K = \dfrac{32\omega_0 W F^2}{M c^2 \Omega^2} \\ \begin{pmatrix} a_0 \\ a_{\pi/2} + \Gamma h L - K a_0 \end{pmatrix} + 2 B F^{1/2} \begin{pmatrix} w_0 \\ w_{\pi/2} + \dfrac{K}{2} w_0 \end{pmatrix} \end{cases}$$



$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a_0 \\ a_{\pi/2} + \Gamma h L - K a_0 \end{pmatrix} = \begin{pmatrix} a_0 \cos\theta - a_0 \left(a_{\pi/2} + \Gamma h L - K\right) \\ \left(a_{\pi/2} + \Gamma h L\right) \sin\theta - a_0 \left(K \cos\theta - \sin\theta\right) \end{pmatrix}$$

If we find a way to rotate quadratures by an angle $\theta = arctgK$ we can get rid of Radiation Pressure Fluctuations in the phase shift channel



$$\frac{4\pi}{\lambda}hL \geq \sqrt{K+\frac{1}{K}}$$

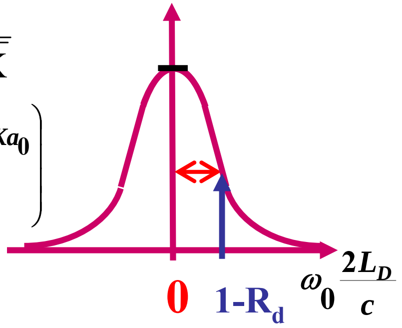
$$\begin{pmatrix} a_0 \\ a_{\pi/2}+\Gamma hL-Ka_0 \end{pmatrix}$$

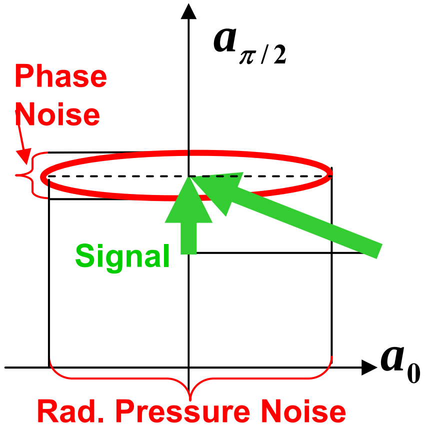
Detuned Cavity

$$\begin{pmatrix} \frac{1}{K} & -1 \\ 1 & \frac{1}{K} \end{pmatrix}$$

$$\frac{4\pi}{\lambda}hL \geq \frac{1}{\sqrt{K}}$$

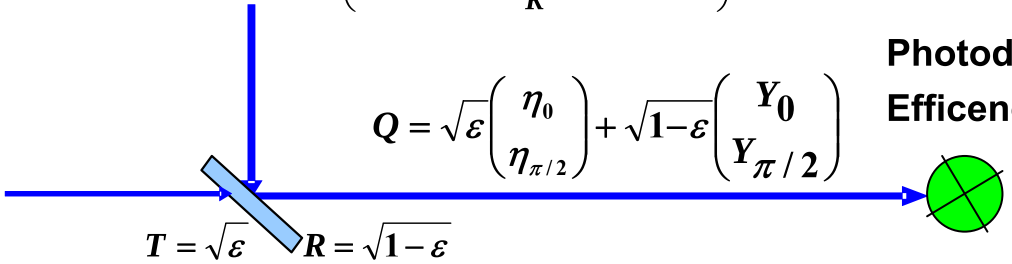
$$\begin{pmatrix} \frac{a_0}{K}-a_{\pi/2}-\Gamma hL+Ka_0 \\ \frac{a_{\pi/2}+\Gamma hL}{K} \end{pmatrix}$$





$$\begin{pmatrix} \frac{1}{K} & -1 \\ 1 & \frac{1}{K} \end{pmatrix}\begin{pmatrix} a_0 \\ a_{\pi/2}+\Gamma hL-Ka_0 \end{pmatrix} = \begin{pmatrix} \frac{a_0}{K}-a_{\pi/2}-\Gamma hL+Ka_0 \\ \frac{a_{\pi/2}+\Gamma hL}{K} \end{pmatrix}$$

$$\begin{pmatrix} Y_0 \\ Y_{\pi/2} \end{pmatrix} = \begin{pmatrix} \frac{a_0}{K}-a_{\pi/2}-\Gamma hL+Ka_0 \\ \frac{a_{\pi/2}+\Gamma hL}{K} \end{pmatrix}$$



$$Q=\sqrt{\varepsilon}\begin{pmatrix} \eta_0 \\ \eta_{\pi/2} \end{pmatrix}+\sqrt{1-\varepsilon}\begin{pmatrix} Y_0 \\ Y_{\pi/2} \end{pmatrix}$$

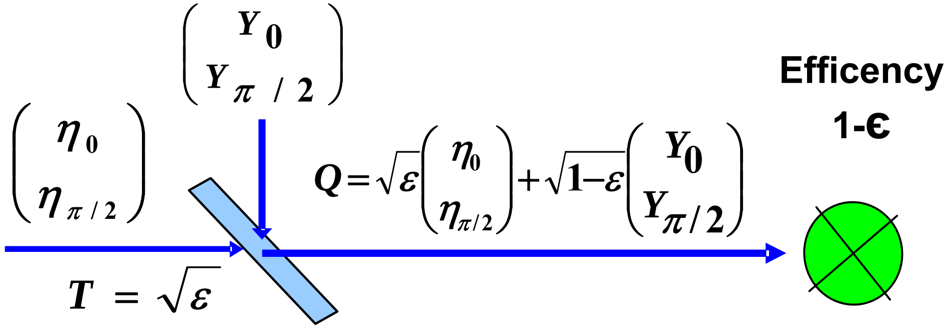
$T=\sqrt{\varepsilon}$ $R=\sqrt{1-\varepsilon}$

Photodiode Efficency 1-ε

$$\Gamma hL \geq \sqrt{1+K^2\frac{\varepsilon}{1-\varepsilon}}$$

$$\varepsilon = 410^{-2}$$

$$K \leq \sqrt{\frac{1-\varepsilon}{\varepsilon}} \approx 5$$



$\begin{pmatrix} Y_0 \\ Y_{\pi/2} \end{pmatrix}$

$\begin{pmatrix} \eta_0 \\ \eta_{\pi/2} \end{pmatrix}$

$Q = \sqrt{\varepsilon}\begin{pmatrix} \eta_0 \\ \eta_{\pi/2} \end{pmatrix} + \sqrt{1-\varepsilon}\begin{pmatrix} Y_0 \\ Y_{\pi/2} \end{pmatrix}$

Efficency

1-є

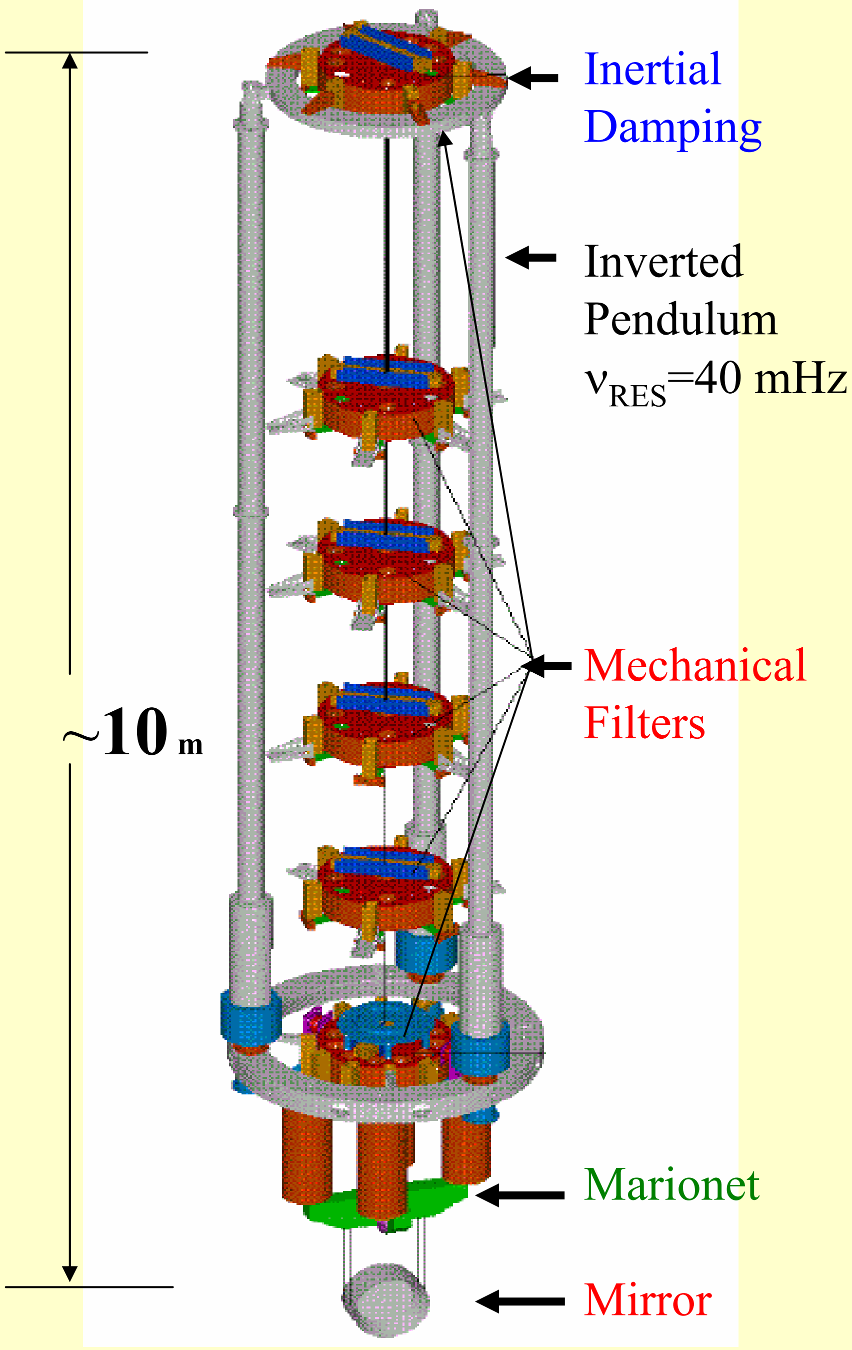
$T = \sqrt{\varepsilon}$

$R = \sqrt{1-\varepsilon}$

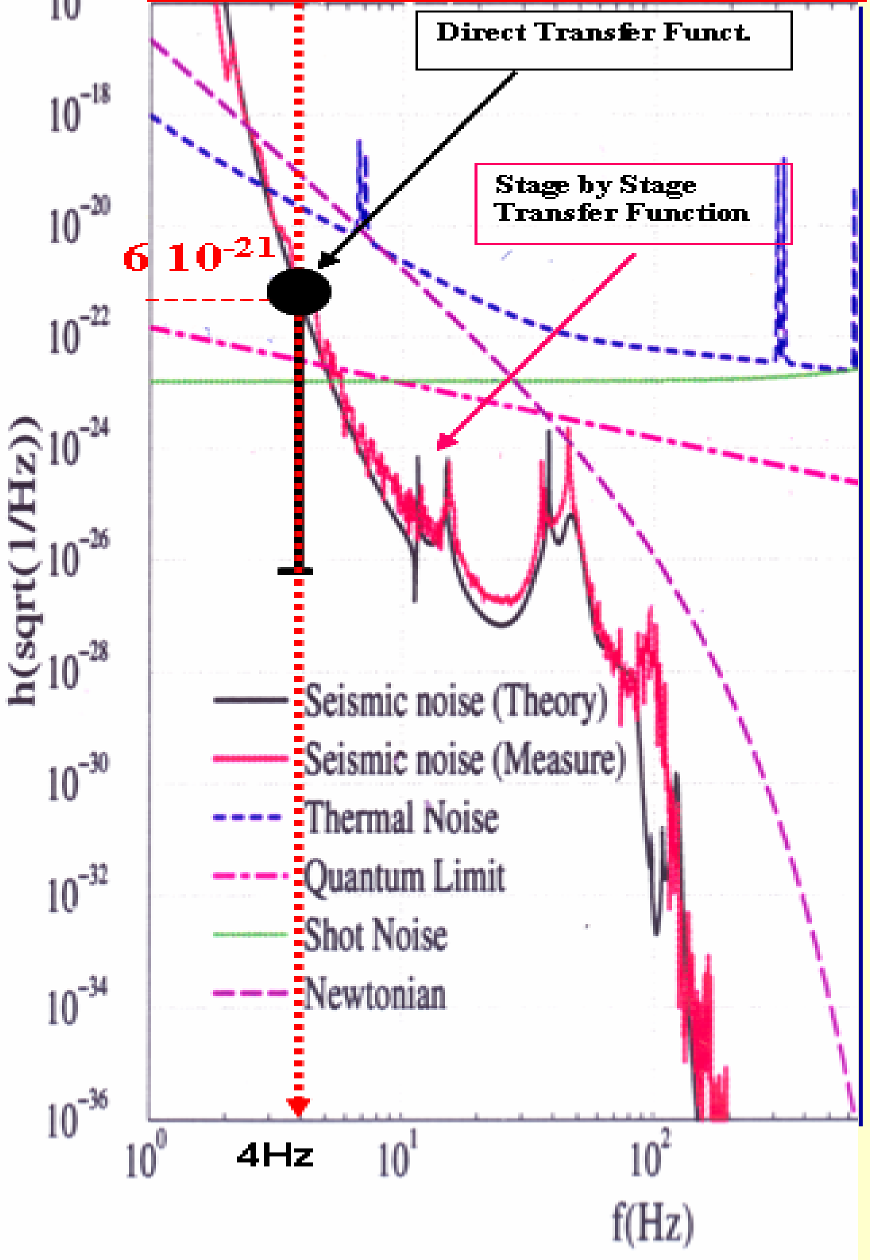
$$\Gamma h L \geq \sqrt{1 + K^2 \frac{\varepsilon}{1-\varepsilon}}$$

$$\varepsilon = 4 10^{-2}$$

$$K \leq \sqrt{\frac{1-\varepsilon}{\varepsilon}} \approx 5$$



SA Transf. Function



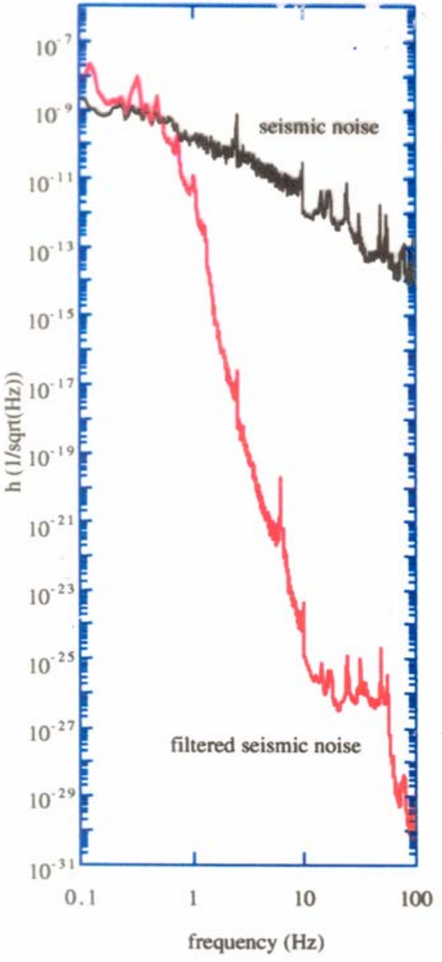
Some comments on Virgo low frequency performances

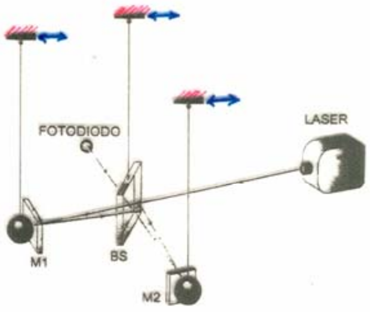
SUPERATTENUATORS

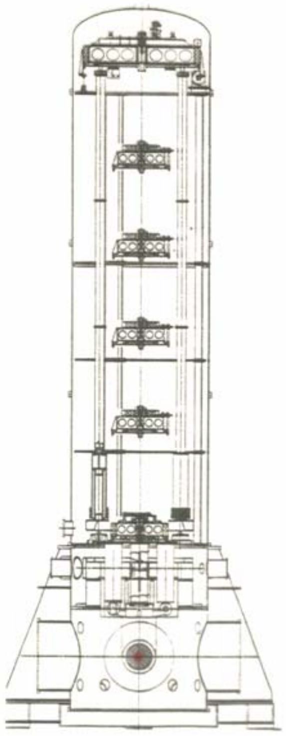
Isolate mirrors and optical benches from Seismic noise (10^{12} larger than signal @ 10 Hz)

6 SA for mirror suspension
3 SA for optical benches

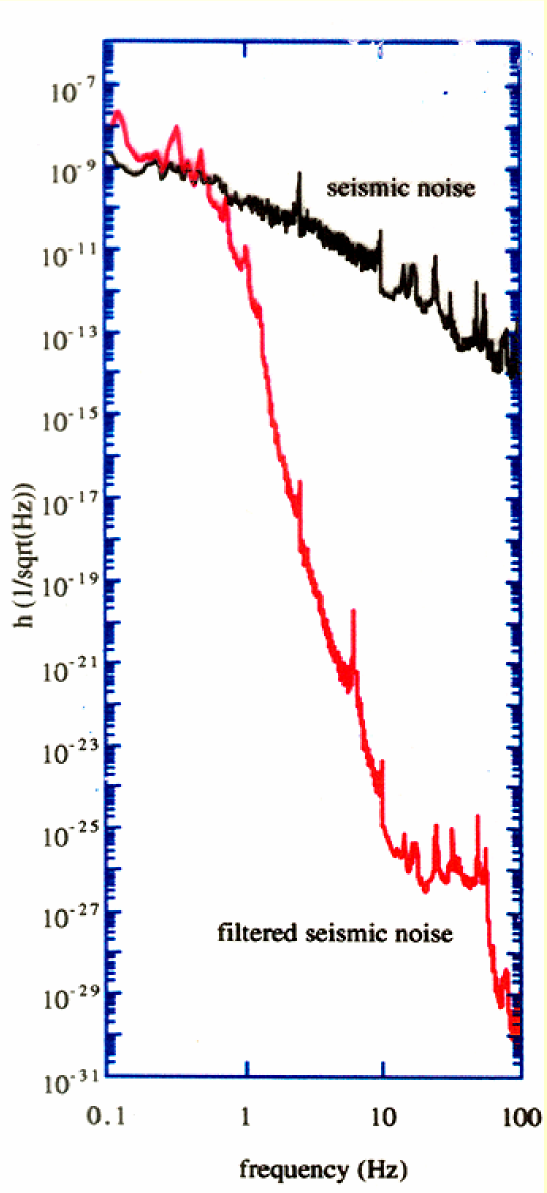
Seismic Noise determines mirrors suspensions design

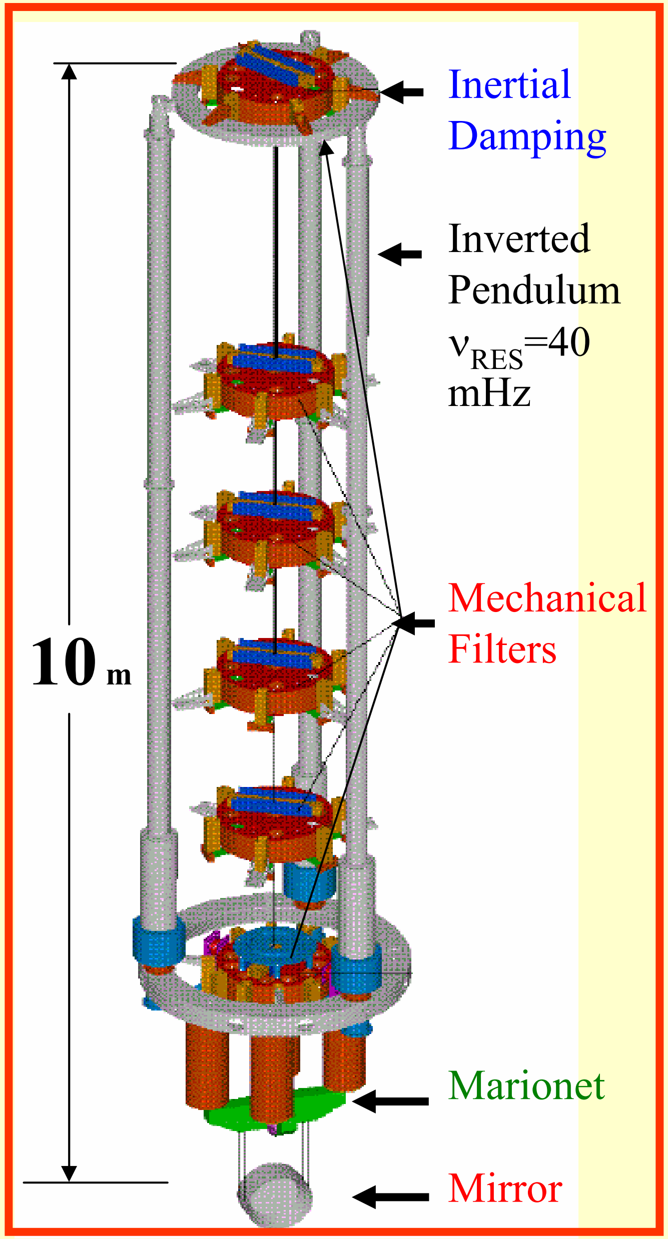






SUPERATTENUATORS: Isolate mirrors and optical benches from Seismic noise (10^{12} larger than signal @ 10 Hz)





Inertial Damping

Inverted Pendulum
ν_{RES}=40 mHz

Mechanical Filters

Marionet

Mirror

SA Transf. Function