



Study of the CALO BGO response

5/03/2025 - FOOT Physics Meeting

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The Calorimeter Calibration

- The calorimeter response linearity is affected by the Birk's law

$$\frac{dS}{dx} = \frac{A \frac{dE}{dx}}{1 + KB \frac{dE}{dx}}$$

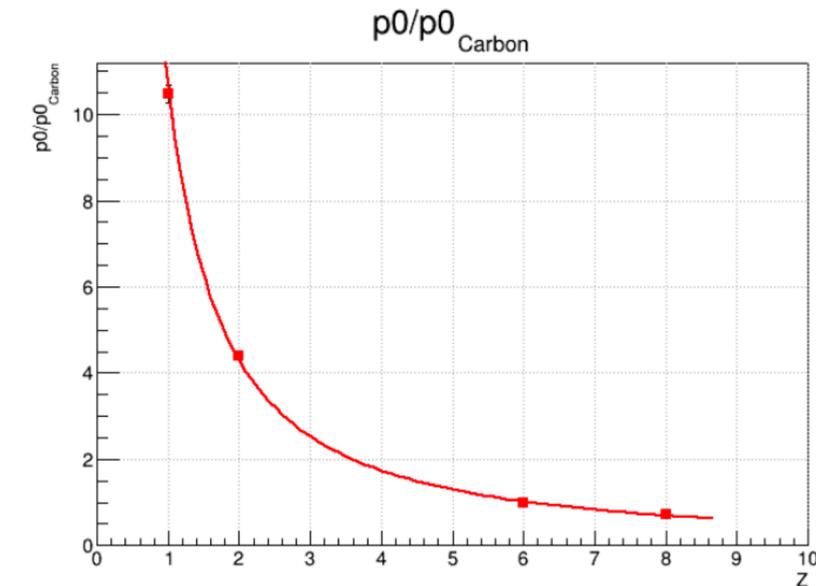
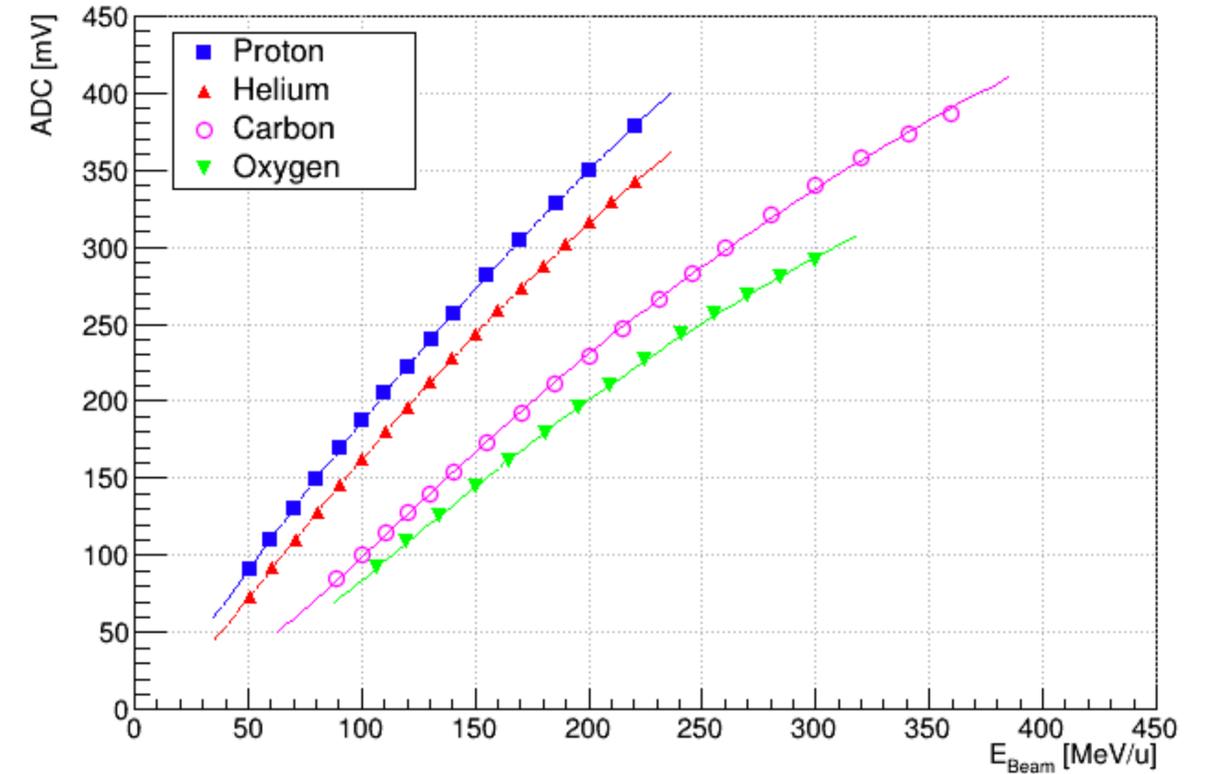
- The fit function is the Modified Birk's Function, that depends on three parameters:

$$ADC(E) = \frac{p_0 E^2}{1 + p_1 E + p_2 E^2}$$

- Parameters dependence on Z, modeled with the power law function

$$\frac{p_x}{p_{xCarbon}} = a_{0,x} Z^{a_{1,x}}$$

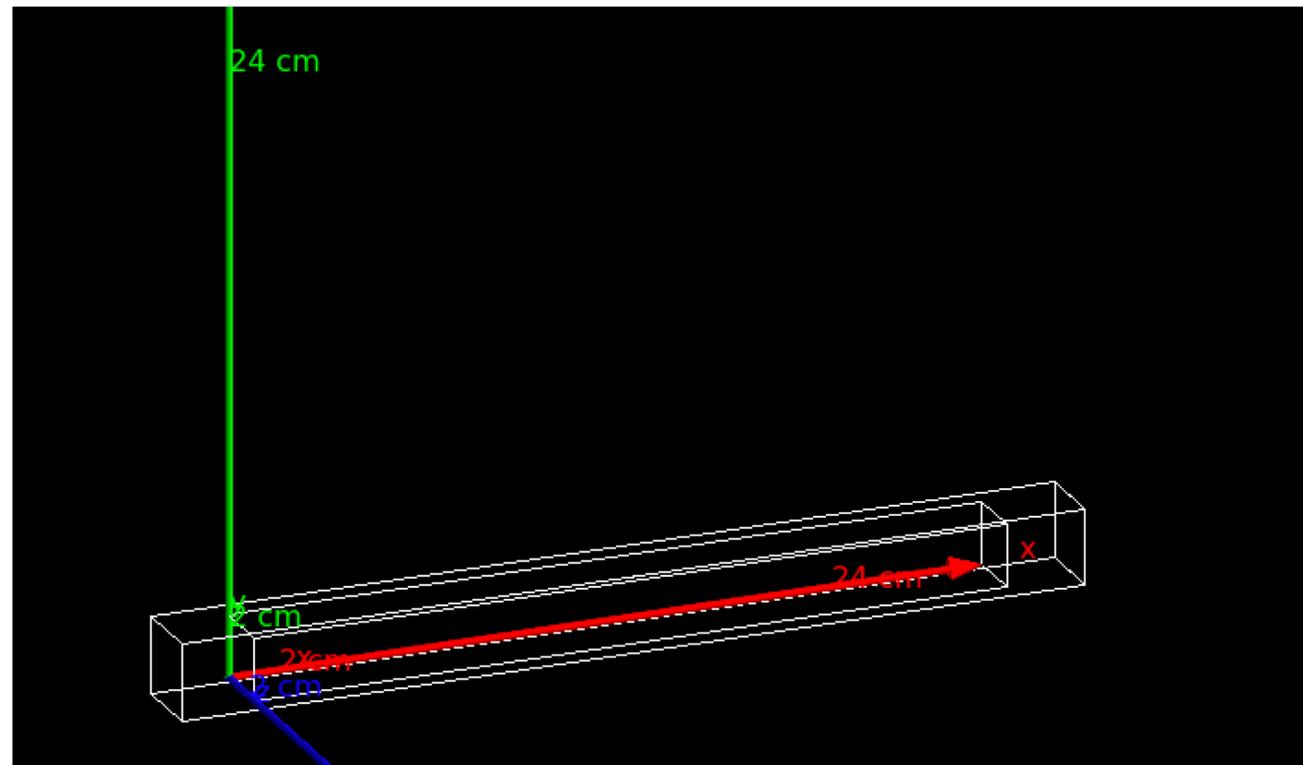
- There is an (unknown) dependence on Z



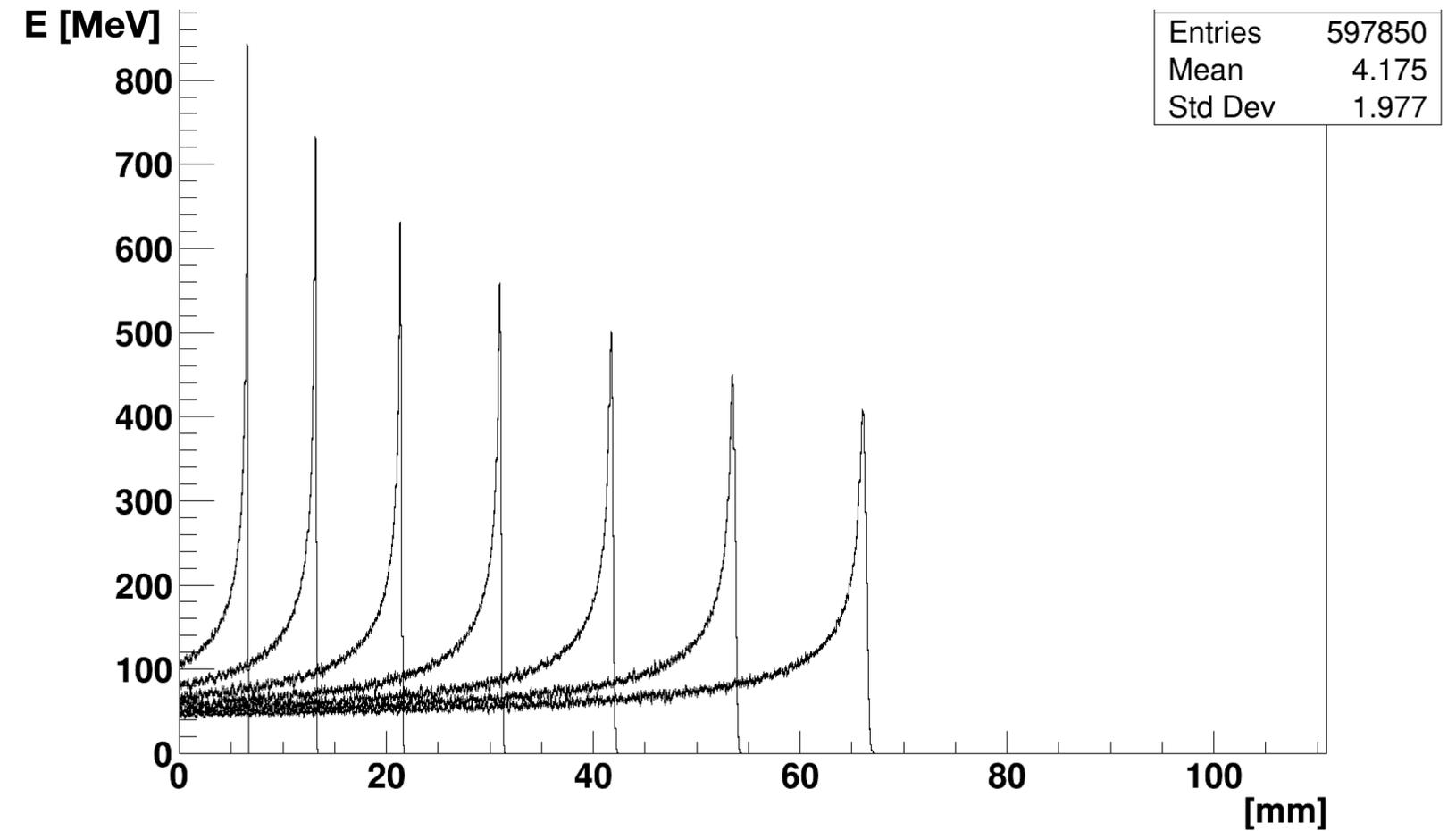
The goal of these studies is to understand these assumptions and the variation of the BGO response curve due to:

- Particle range variation
- Non-linearity related to optical pile up in the SiPM
- Different crossing ions species

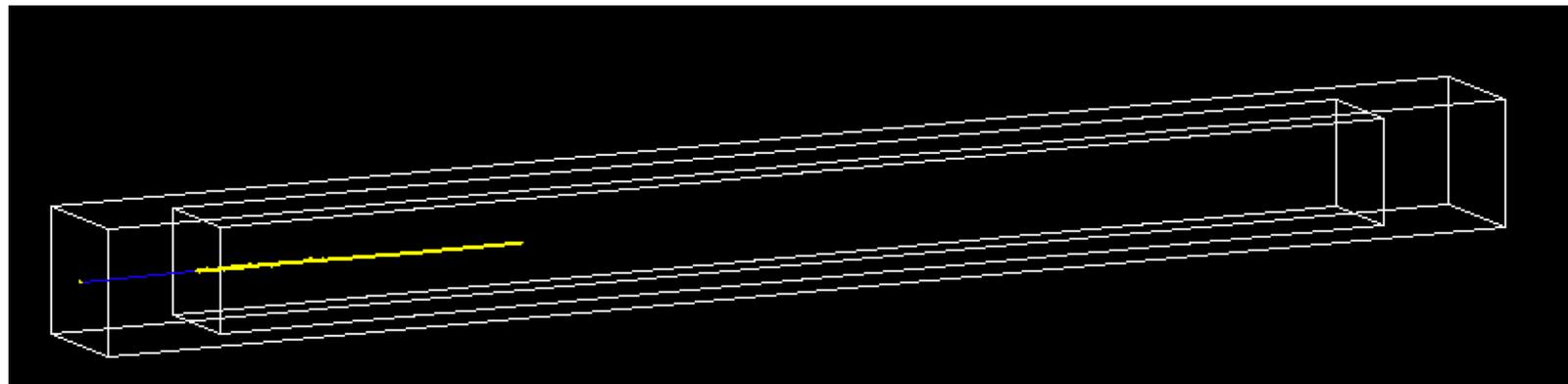
BGO crystal simulation with GEANT4



2x2x24cm BGO crystal within the world envelope



Bragg curve for Carbon E=100,150,200,250,300,350,400 MeV/u



E=400MeV/u Carbon sent against the crystal

Calculation of the integral of the Birk's law

Birk's law:

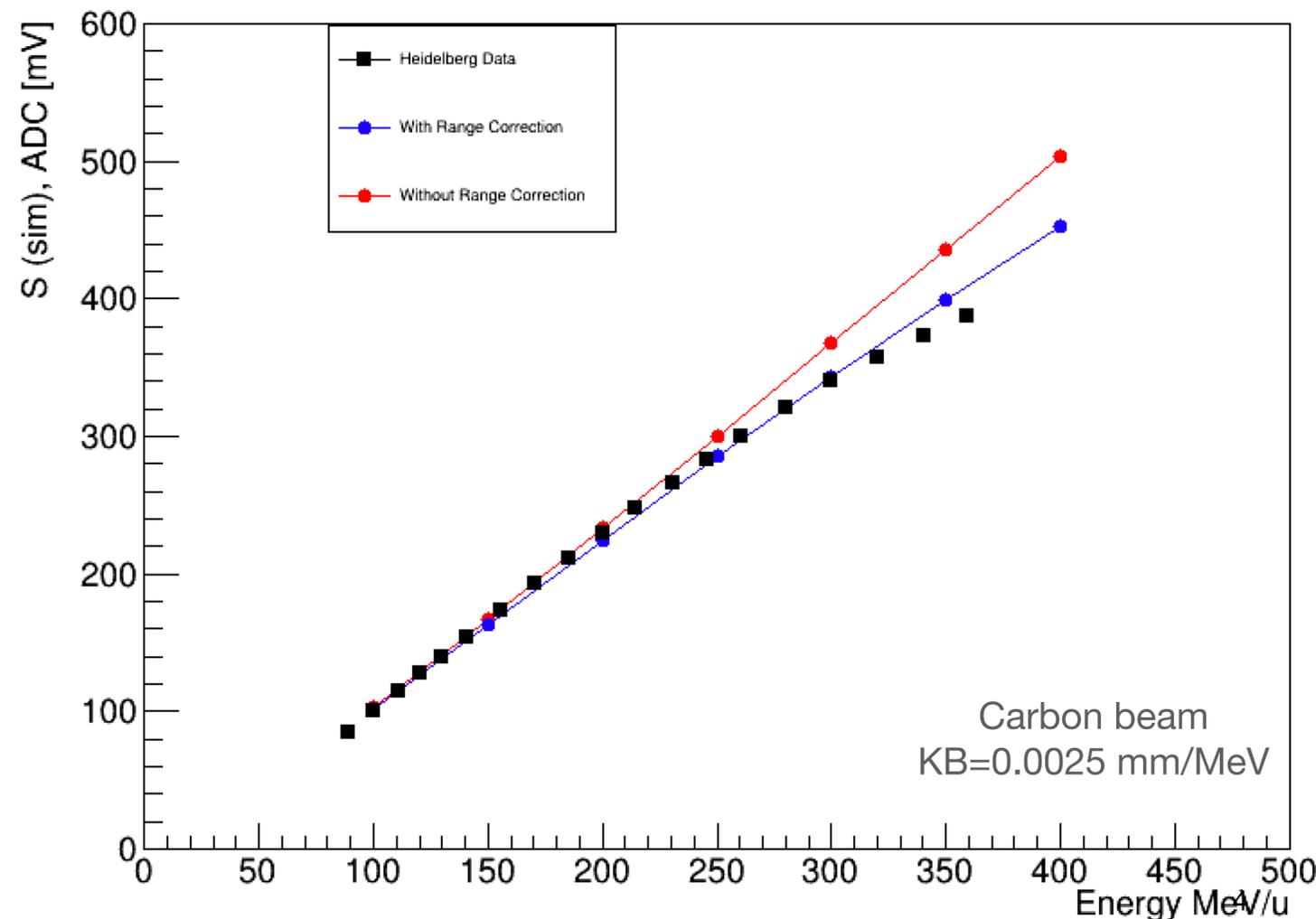
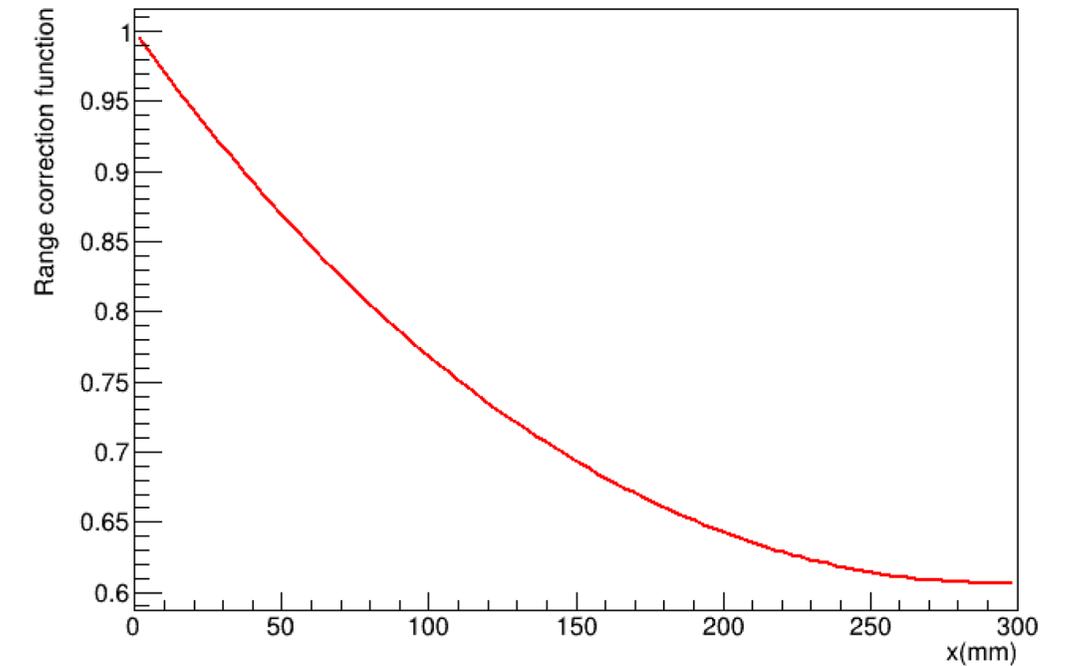
$$\frac{dS}{dx} = \frac{A \frac{dE}{dx}}{1 + KB \frac{dE}{dx}}$$

$$S = \sum_{i=1}^N \frac{dS}{dx} \cdot dx$$

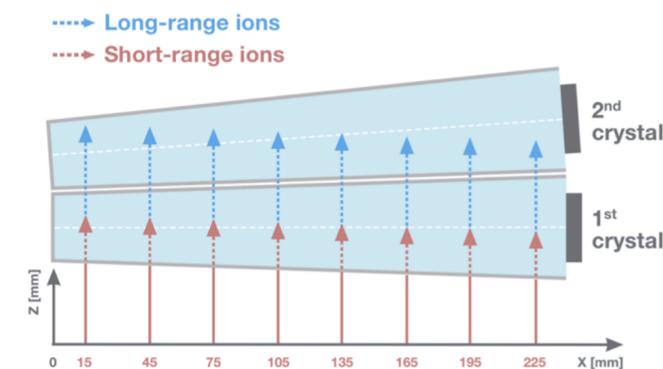
Light yield without range correction

$$S_r = \sum_{i=1}^N \frac{dS}{dx} \cdot f(x) \cdot dx$$

Light yield with range correction



$$f(x) = A \cdot [R \cdot e^{-\alpha(L-x)} + (1 - R)e^{-\alpha(L+x)}]$$



Beam	Maximum Amplitude	
	α (m ⁻¹)	R(%)
p70	4.0 ± 0.8	9.9 ± 1.4
p170	3.8 ± 2.6	10 ± 4
C115	3.9 ± 0.2	10.2 ± 0.4
C260	3.5 ± 0.2	10.3 ± 0.4

- The signal Maximum Amplitude decreases as a function of the distance from the front side of the BGO crystal (see L. Scavarda's thesis <https://repository.gsi.de/record/246963>)

Calculation of pile-up

Basic idea: each pixel in the SiPM has a dead time of **10 ns** -> Signal loss may be due to photon Pile-Up

Goal: calculation of the Pile-Up at the **maximum of the Wave Form**, within a 10 ns window

- In the analysis, the ADC value for a certain energy is taken at the maximum WF amplitude

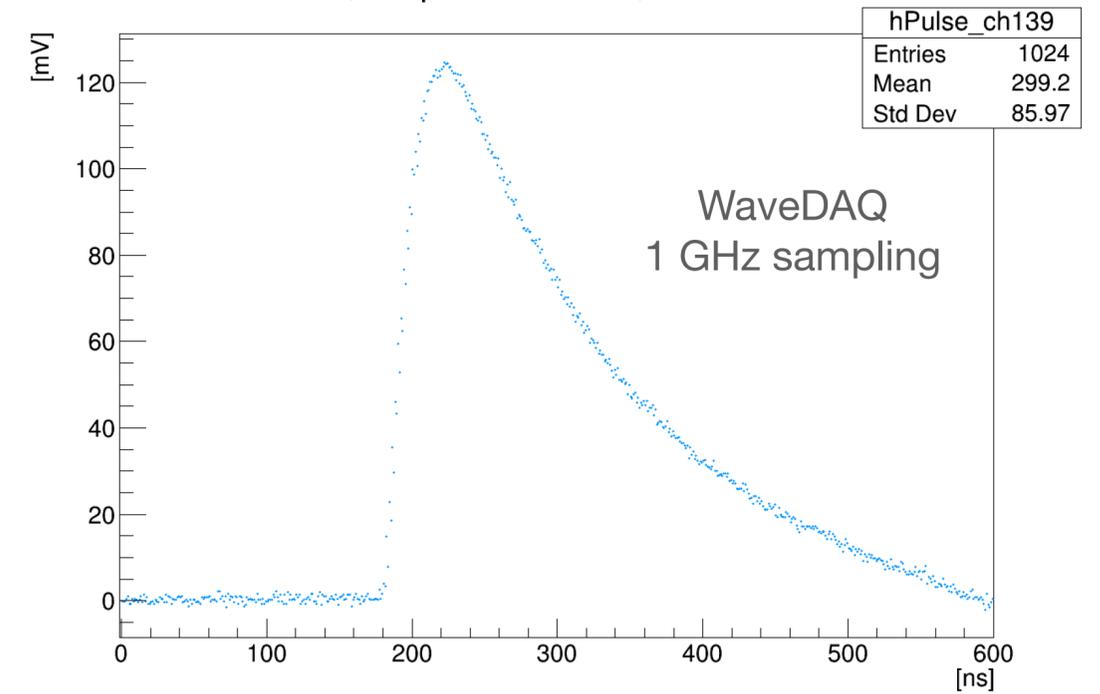
$$N_{ph} = LY \cdot E_{particle}(MeV)$$

Number of photons produced in the crystal

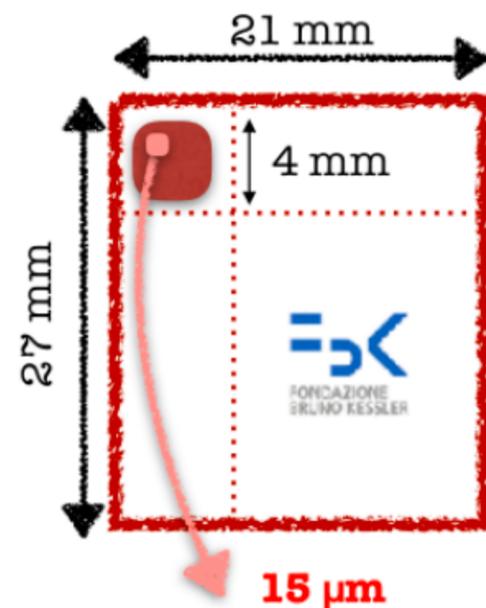
$$LY = 8000ph/MeV$$

BGO Light Yield

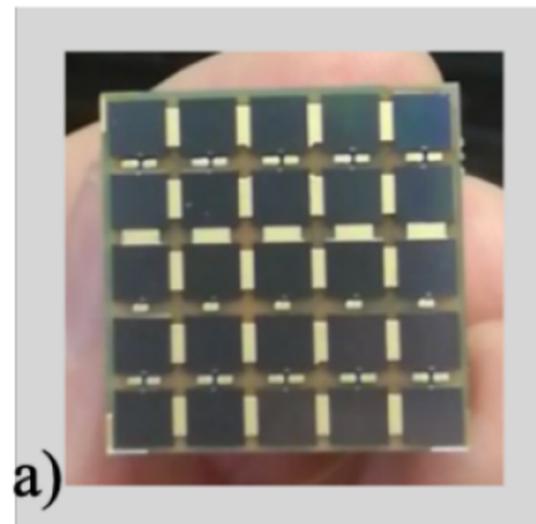
Ch: 139, Amp 124.542053, RMS 0.715679



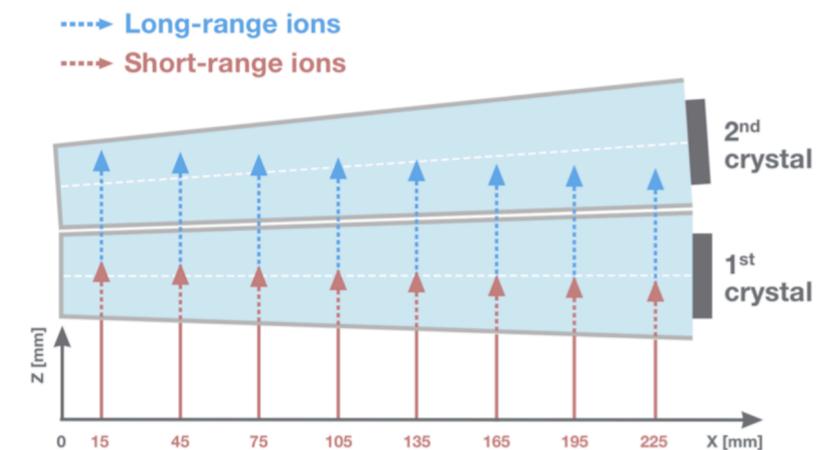
These photons reach the crystal back surface, where SiPM board is located



RGB-HD15



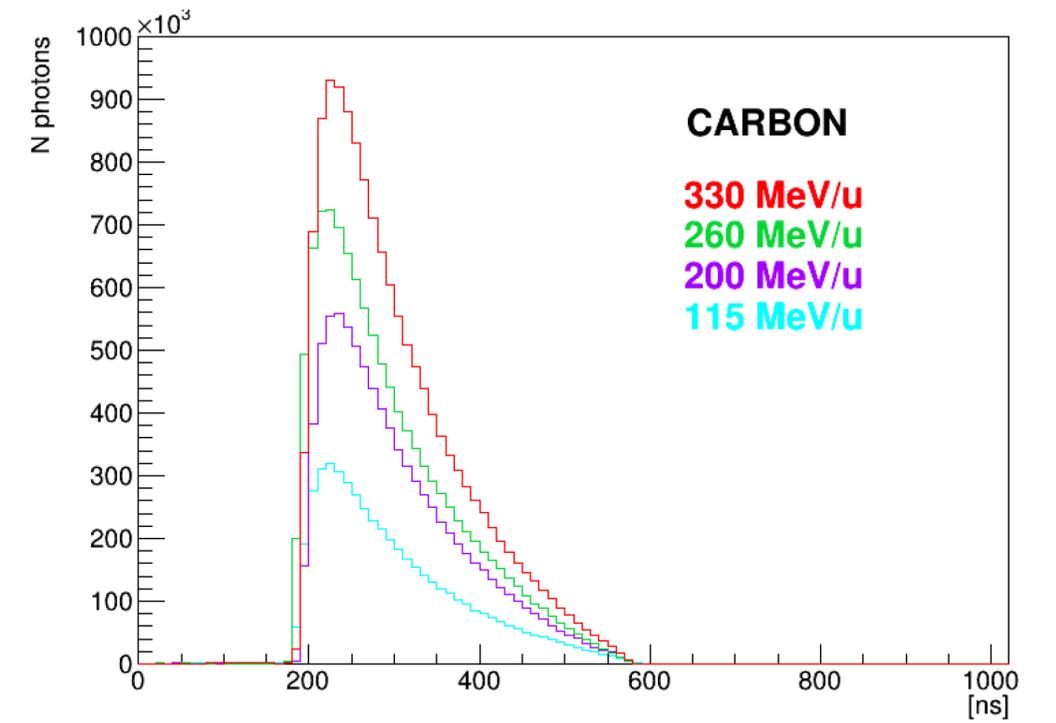
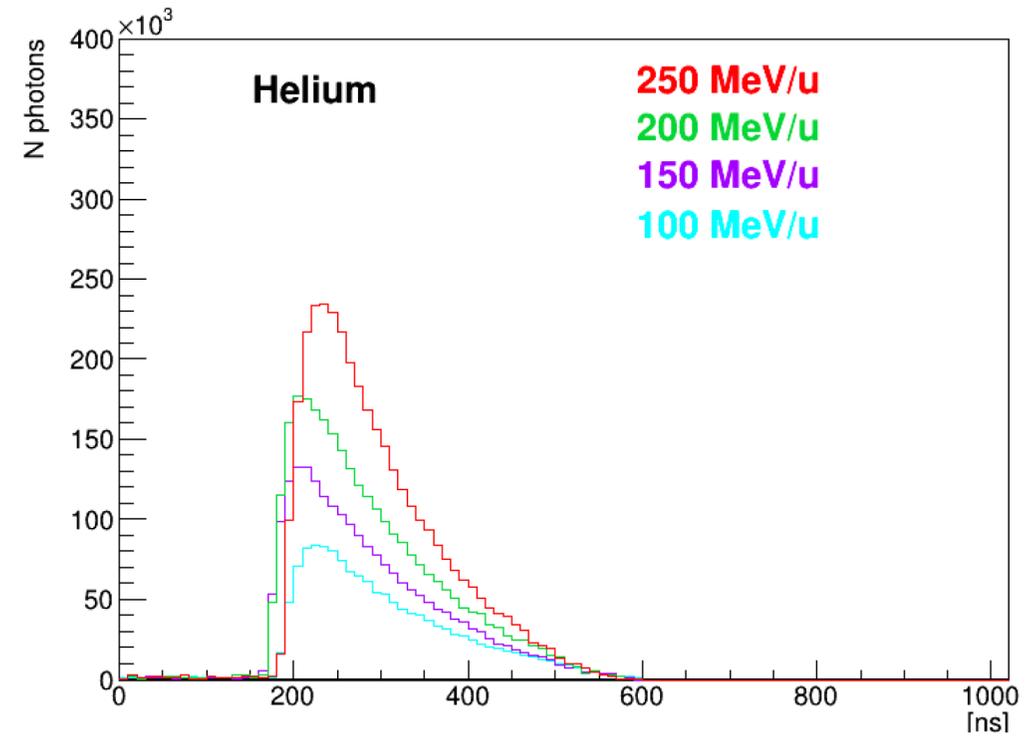
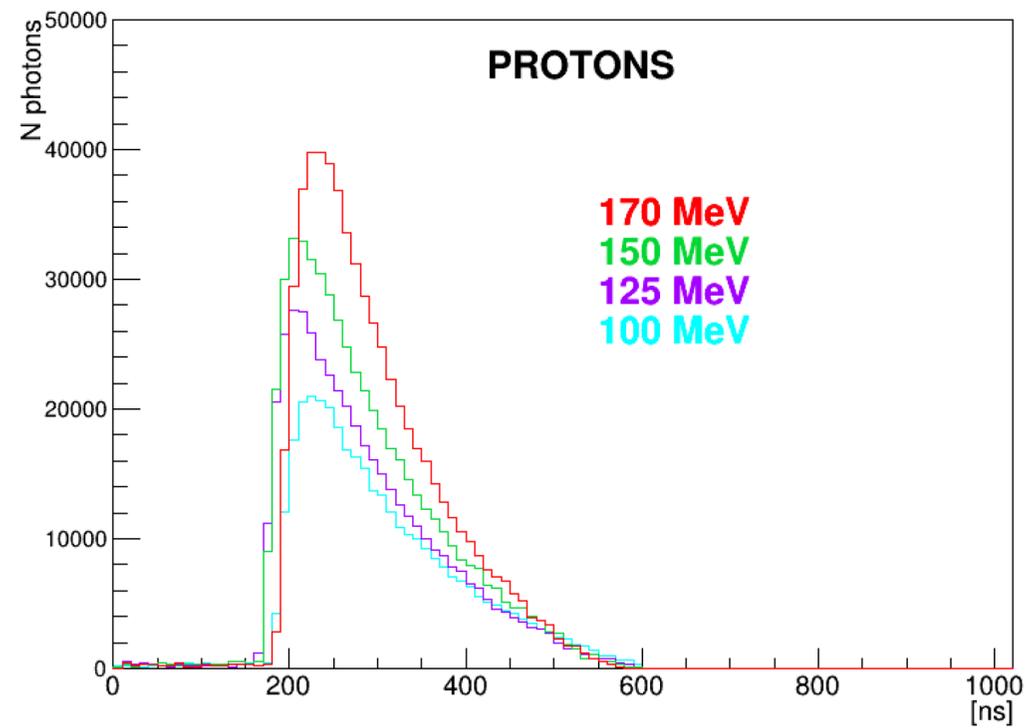
- SiPM surface does not cover the entire 3x3 mm² crystal face



71k microcells for each SiPM

Photons on SiPM

- Wave form functions normalized to the expected number of photons reaching the SiPM board
- This amount of photons, integrated over 10 ns, is distributed across 1.7M SiPM microcells



Pile-Up calculation

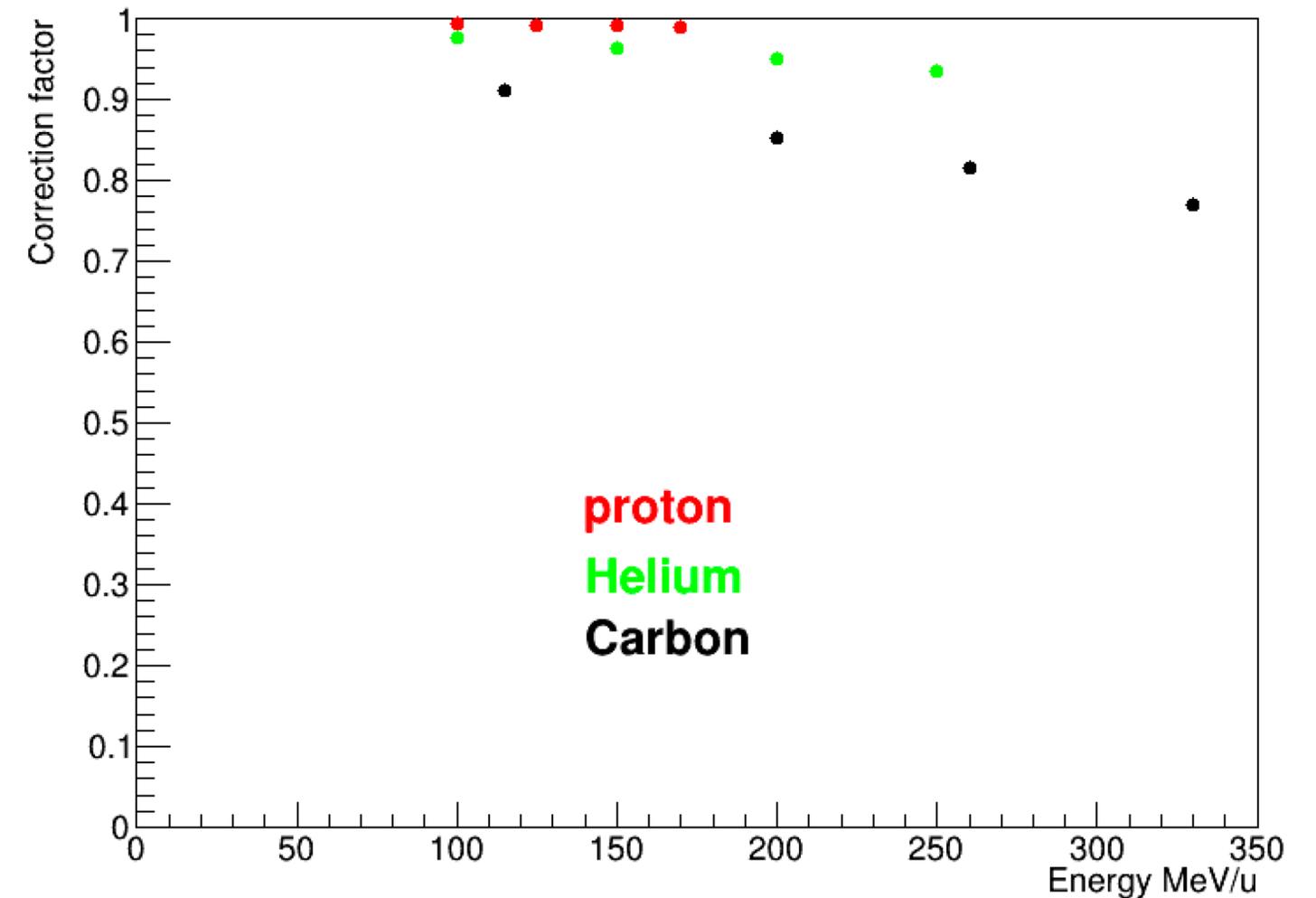
Expected number of SiPM microcells p containing k photons, after m photons are randomly distributed among N SiPM microcells
(<https://arxiv.org/pdf/1511.06528>)

$$p(N, m, k) = \frac{N}{k!} \left[\frac{m}{N} \right]^k e^{-m/N}$$

m : number of photons at maximum WF reaching the SiPM surface
 N : number of microcells

Signal loss calculated as:

$$y = 1 - \frac{N_{ph,tot} - N_{switchedon}}{N_{ph,tot}} = 1 - \frac{N_{ph,tot} - (N - p(N, m, k = 0))}{N_{ph,tot}}$$

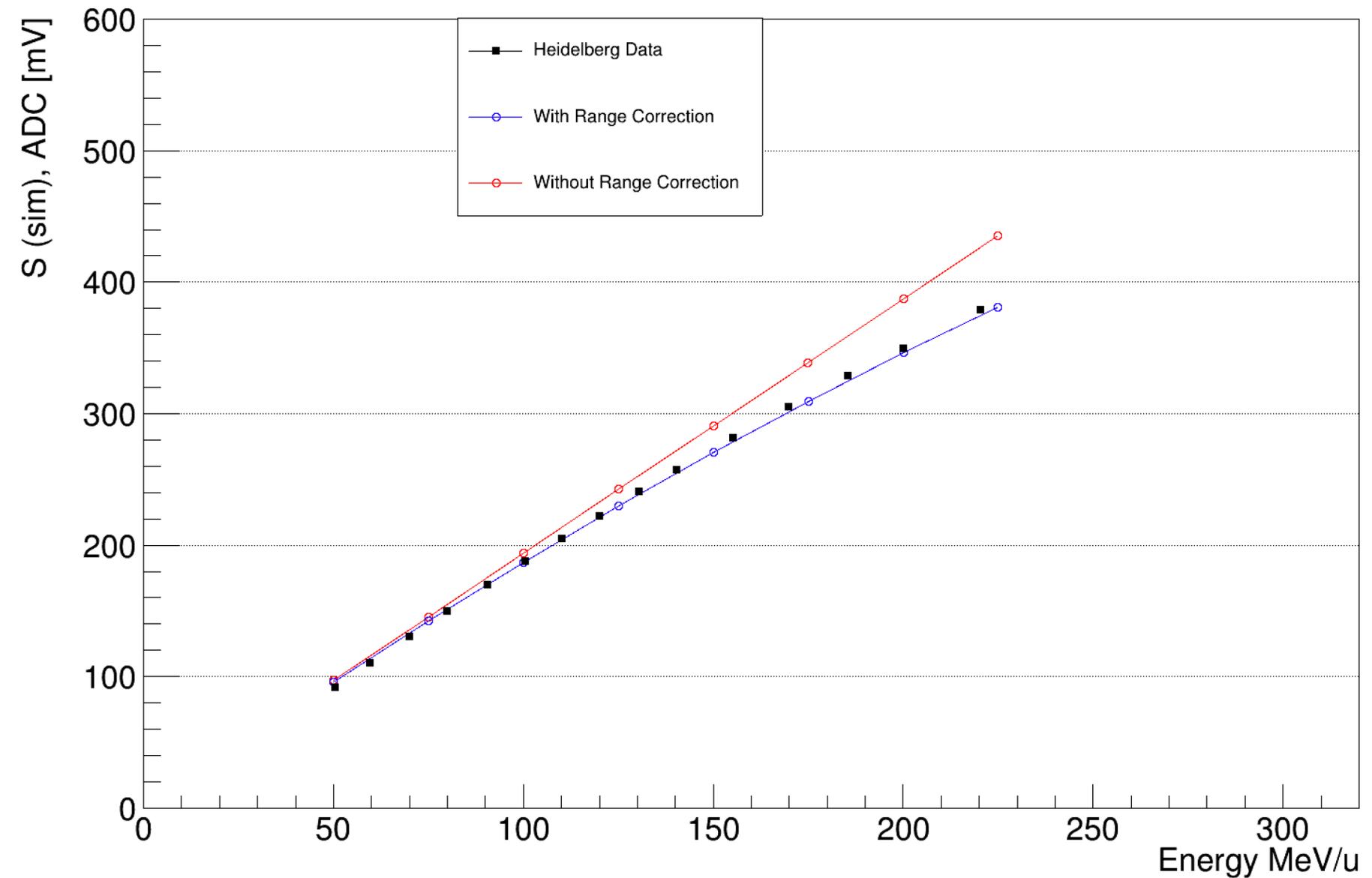


- Checked with Monte Carlo

Proton simulation

- The pile-up correction is very small for protons
- Check of what happens by assuming just the range correction for protons
- Comparison to Heidelberg data

$$S = \sum_{i=1}^N \frac{dS}{dx} \cdot dx$$



Protons, no pile up, KB=0

Summary and next steps

The goal of these studies is to understand the variation of the BGO response curve due to:

- Particle range variation
- Non-linearity related to optical pile up in the SiPM
- Different crossing ions species

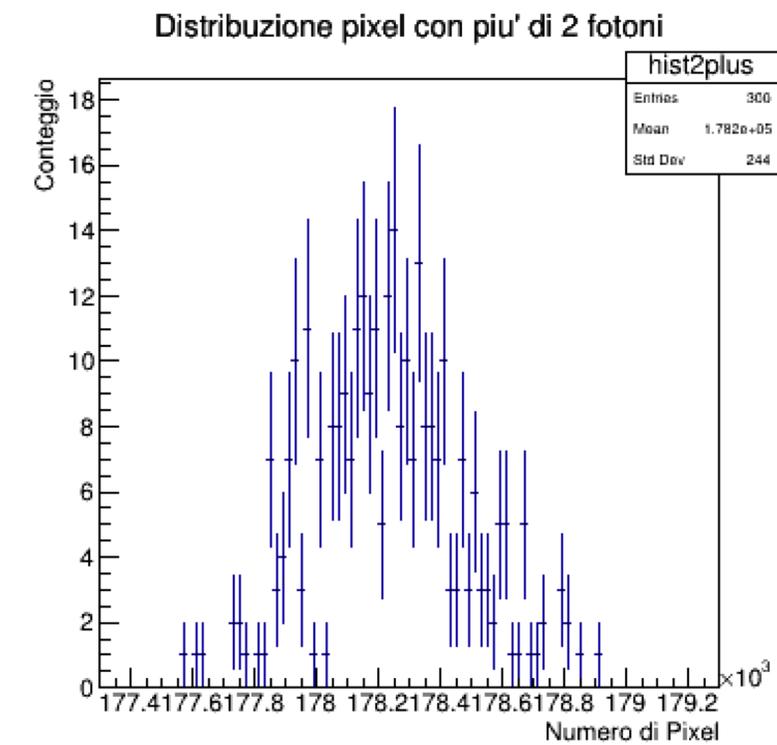
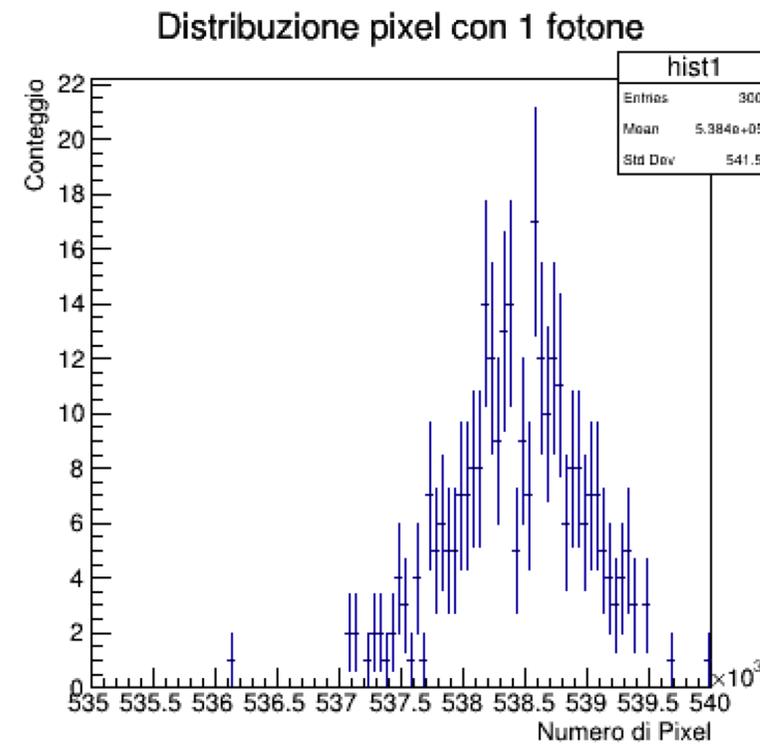
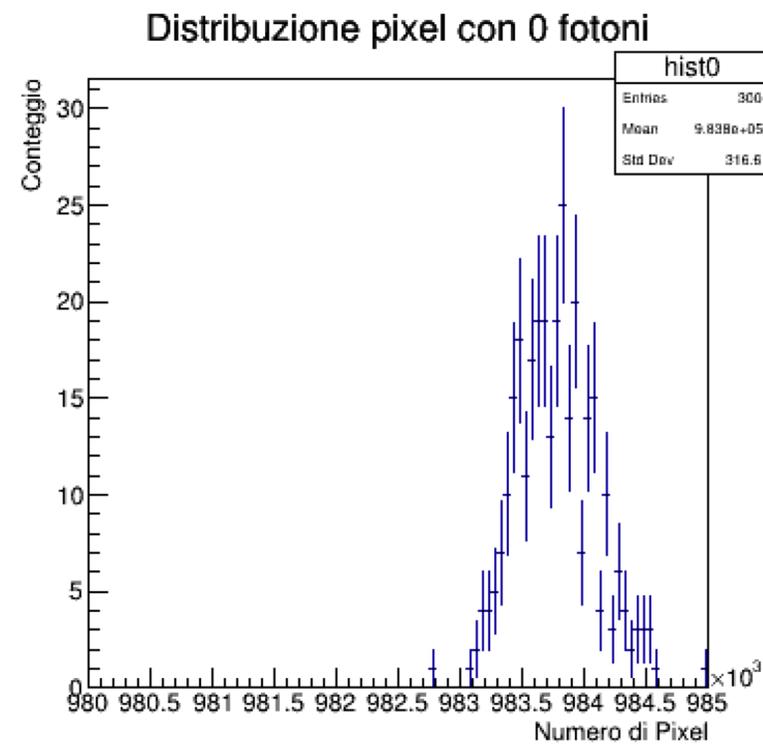
The pile-up on SiPM may lead to a non negligible signal reduction, in particular for $Z > 1$ particles

- the SiPM dead time may impact the amount of signal loss

We will now extend these studies to other particles

BACKUP

Monte Carlo Toy



For 330 MeV/u Carbon:
N photons at maximum=930461
N cells used= 1303x1303

From Poisson:
N pixel with 0 ph=983436
N pixel with 1 ph=538264
N pixels with at least 2 ph=178228

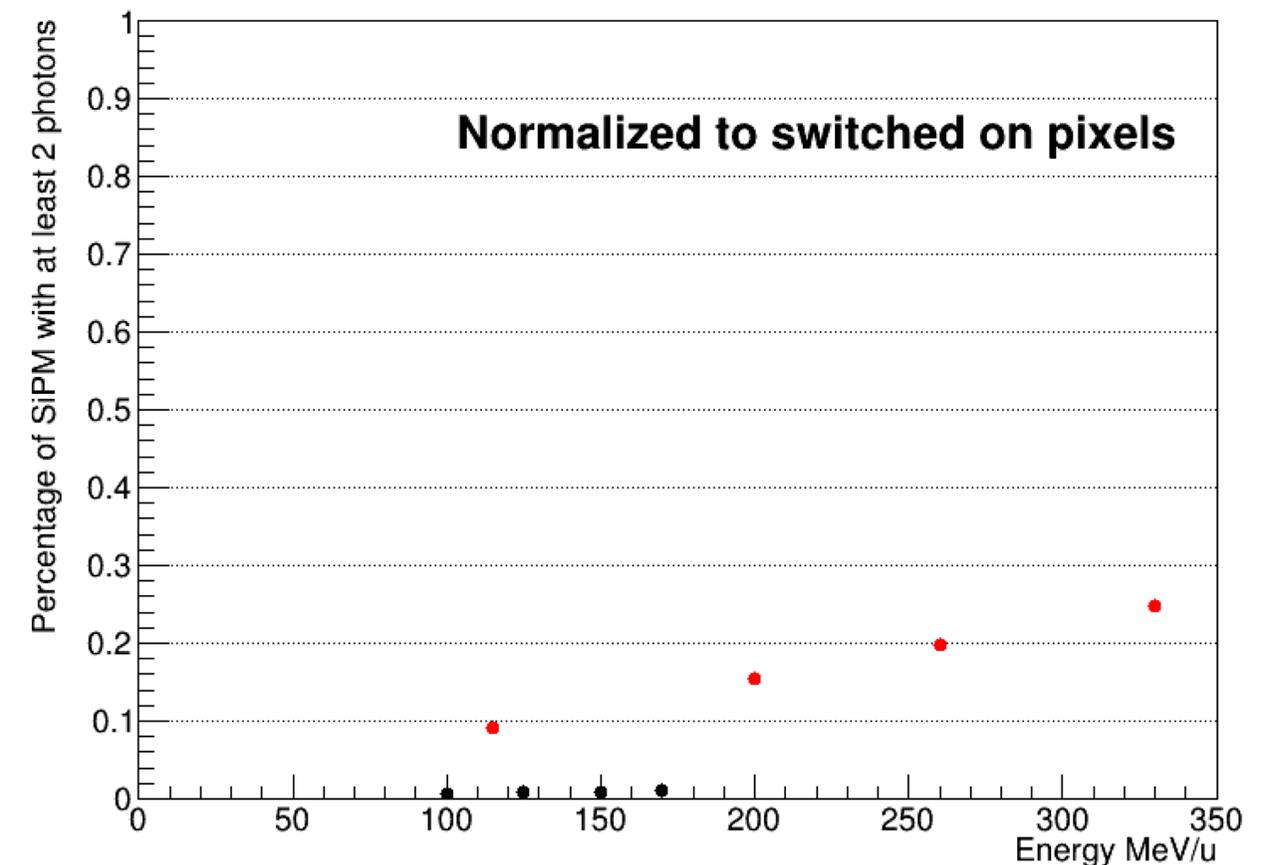
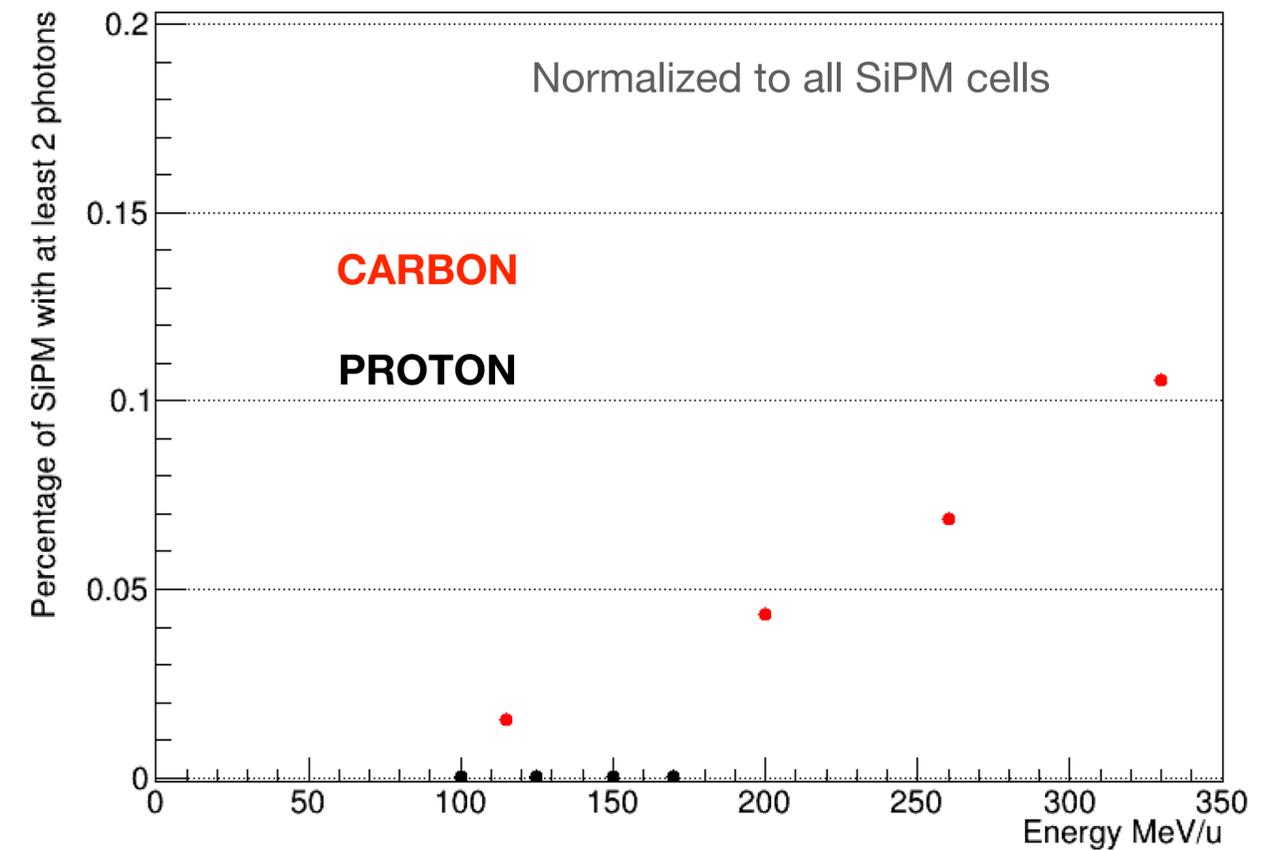
Pile-Up calculation $y = \frac{p(N, m, k > 1)}{N}$

Expected number of SiPM microcells p containing k photons, after m photons are randomly distributed among N SiPM microcells
 (<https://arxiv.org/pdf/1511.06528>)

$$p(N, m, k) = \frac{N}{k!} \left[\frac{m}{N} \right]^k e^{-m/N}$$

m : number of photons at maximum WF reaching the SiPM surface
 N : number of microcells

$$p(N, m, k > 1) = N - p(N, m, 0) - p(N, m, 1)$$



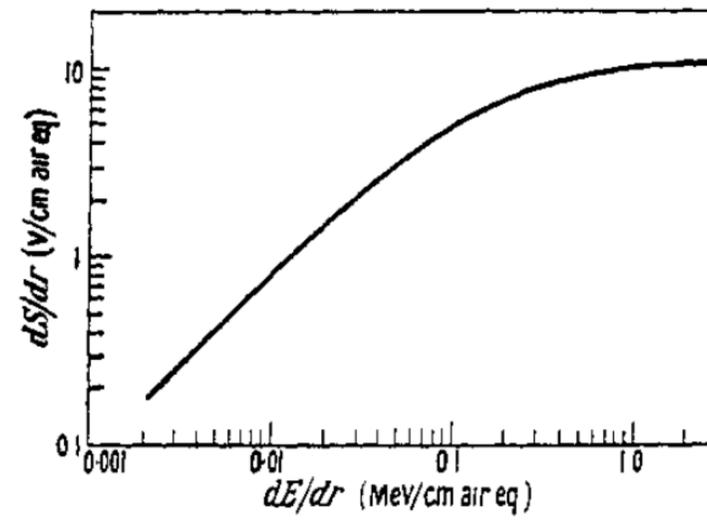
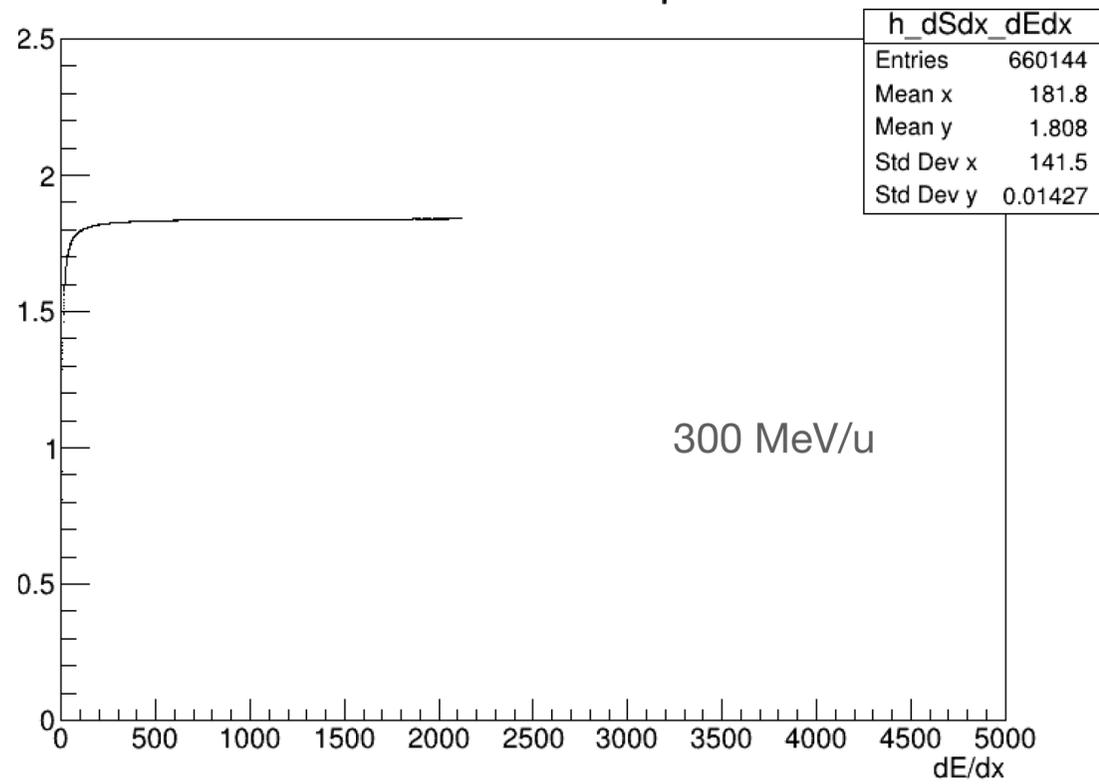


Figure 1. Specific fluorescence dS/dr plotted against specific energy loss dE/dr in anthracene.

$$\frac{dE}{dx} \approx \frac{C \cdot AZ^2}{E}, \quad (2)$$

which allows one to determine an analytical expression for the integral:

$$L(E, A, Z) = \int dL = \int_0^{x_{\max}} dx \frac{S \frac{dE}{dx}}{1 + KB \frac{dE}{dx}} = f_1 \cdot \left(E - f_2 \cdot AZ^2 \cdot \log \left(\frac{E + f_2 \cdot AZ^2}{f_2 \cdot AZ^2} \right) \right) \quad (3)$$

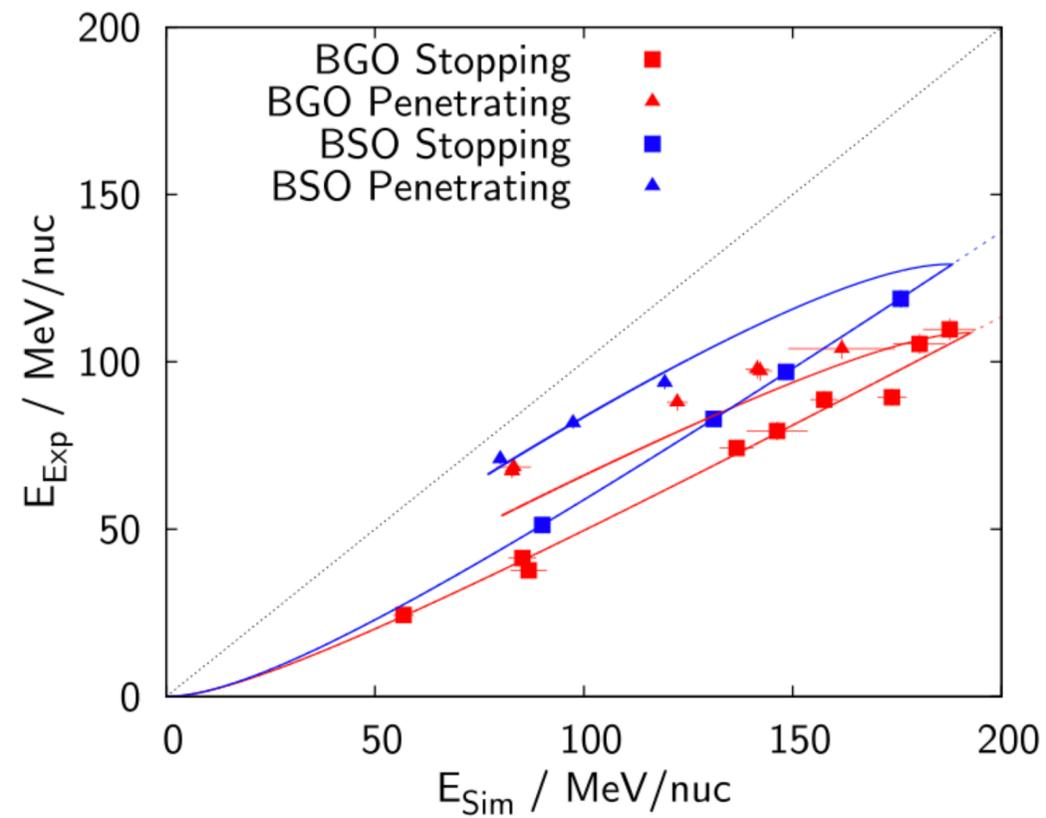


Fig. 5. Light yields for all six ions with fits of analytical light curve Eq. (3) to stopping particles (dashed lines). Numerical solution of Eq. (3) including stopping particles in 2 cm of crystal material (solid line). Particles without any quenching are expected to lie on the diagonal black dotted line with a slope of 1.