# **CR**/Propa

# the CRPropa framework for astroparticle propagation



#### Rafael Alves Batista

#### Sorbonne Université

- Institut d'Astrophysique de Paris (IAP)
- Lab. Physique Nucléaire et des Hautes Énergies (LPNHE)
  - rafael.alves\_batista@iap.fr
  - www.8rafael.com

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do we observe high-energy cosmic messengers, or do we

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# multimessenger propagation picture: neutrinos







# multimessenger propagation picture: neutrinos



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# multimessenger propagation picture: neutrinos



# neutrino oscillations

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pair production

 $\gamma + \gamma_{bg} \rightarrow e^+ + e^-$ 

-

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#### cosmological photon fields



# modelling the propagation

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injection spectrum initial composition source distribution source emissivity evolution



injection spectrum initial composition source distribution source emissivity evolution

#### propagation



injection spectrum initial composition source distribution source emissivity evolution



#### cookbook for astroparticle transport

particle interactions particle acceleration background photon fields background matter fields magnetic fields


injection spectrum initial composition source distribution source emissivity evolution



### cookbook for astroparticle transport

particle interactions particle acceleration background photon fields background matter fields magnetic fields

### outputs



injection spectrum initial composition source distribution source emissivity evolution



### cookbook for astroparticle transport

particle interactions particle acceleration background photon fields background matter fields magnetic fields

outputs

spectrum composition arrival directions arrival times



injection spectrum initial composition source distribution source emissivity evolution



### cookbook for astroparticle transport

particle interactions particle acceleration background photon fields background matter fields magnetic fields





injection spectrum initial composition source distribution source emissivity evolution



### cookbook for astroparticle transport

particle interactions particle acceleration background photon fields background matter fields magnetic fields













mixing all ingredients  $\rightarrow$  interpret (fit) observations based on models





mixing all ingredients  $\rightarrow$  interpret (fit) observations based on models





- mixing all ingredients  $\rightarrow$  interpret (fit) observations based on models
- this should be done *self-consistently for all messengers*





- mixing all ingredients  $\rightarrow$  interpret (fit) observations based on models
- this should be done *self-consistently for all messengers*
- need to *scan full parameter space* of uncertainties

























# the CRPropa framework







Alves Batista et al. JCAP 05 (2016) 038. arXiv:1603.07142 Alves Batista et al. JCAP 09 (2022) 035. arXiv:2208.00107

- publicly available Monte Carlo code
- propagation of high-energy cosmic rays, gamma rays, neutrinos, and electrons
- Galactic and extragalactic propagation
- modular structure
- parallelisation with OpenMP
- development on Github: https://github.com/CRPropa/CRPropa3

## the CRPropa framework for astroparticle propagation





## **CR**/Propa

fit UHECR measurements



Alves Batista, de Almeida, Lago, Kotera. JCAP 01 (2019) 002. arXiv:1806.10879

### gamma rays + IGMFs





Eichmann et al. JCAP 02 (2018) 036. arXiv:1701.06792



Hussain, Alves Batista, de Gouveia Dal Pino.

## applications



## **CR**/Propa

fit UHECR measurements



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### gamma rays + IGMFs





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Hussain, Alves Batista, de Gouveia Dal Pino.

### ... and much more!

## applications





Alves Batista et al. JCAP 05 (2016) 038. arXiv:1603.07142 Merten et al. JCAP 06 (2017) 046. arXiv:1704.07484 Alves Batista et al. JCAP 09 (2022) 035. arXiv:2208.00107

> Stochastic Differential Equations **3D** (galactic)

simple 1D

## **CRPropa. propagation modes**

**Boris-**Push 3D



Runge-Kutta 3D







## **CRPropa.** interaction processes







# multimessenger studies

## **CRPropa.** interaction processes



















































### Δx step

### resolution $(\Delta x)$

small: resolve interactions +

### large: speed up simulation $\bullet$




step

### resolution ( $\Delta x$ )

small: resolve interactions +

large: speed up simulation  $\bullet$ 

### magnetic field structure





step

### resolution ( $\Delta x$ )

small: resolve interactions +

large: speed up simulation +

### magnetic field structure

 resolve propagation regime (diffusive vs. ballistic)







step

### resolution $(\Delta x)$

small: resolve interactions

large: speed up simulation +

### magnetic field structure

 resolve propagation regime (diffusive vs. ballistic)

### geometry





step

### resolution ( $\Delta x$ )

small: resolve interactions +

large: speed up simulation  $\bullet$ 

### magnetic field structure

 resolve propagation regime (diffusive vs. ballistic)

### geometry

the "aiming" problem





# how to use CRPropa for gamma-ray studies?

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### example notebooks



### **CR**/Propa

#### Search docs

#### **PROJECT INFO**

- Installation
- Changelog
- FAQ
- How to cite CRPropa

#### USAGE

- **Basic Concepts**
- Modules
- Introduction to Python Steering

#### □ Extragalactic Propagation

- + 1D simulation
- 3D trajectories in a turbulent field
- 3D MHD models
- Photon Propagation
- Photon Propagation
- Example with secondary neutrinos
- Electromagnetic cascade example Plotting
- The targeting algorithm of CRPropa
- Propagation of Extragalactic CR in the Milky Way
- Galactic Cosmic Rays
- Diffusion of Cosmic Rays
- Acceleration
- Interfacing User Code
- Comparison of Propagation Modules (BP - CK)

Additional Resources

#### API

**Building Blocks** 

#### **Electromagnetic cascade example**

This is a simple 1D example of gamma-ray propagation over cosmological distances. Note that only pair production and inverse Compton scattering are relevant for the energy range of this example. Moreover, the radio background is negligible for the energy range below PeV.

```
[3]: from crpropa import *
    dsrc = redshift2ComovingDistance(0.14)
     electrons = True
     photons = True
    thinning = 0.90 # if 0, no thinning; speeds up the simulations considerably
    cmb = CMB()
     ebl = IRB_Gilmore12()
    crb = URB_Nitu21()
    sim = ModuleList()
    sim.add(SimplePropagation())
    sim.add(Redshift())
    sim.add(EMPairProduction(cmb, electrons, thinning))
    sim.add(EMPairProduction(ebl, electrons, thinning))
    # sim.add(EMPairProduction(crb, electrons, thinning))
    # sim.add(EMDoublePairProduction(cmb, electrons, thinning))
    # sim.add(EMDoublePairProduction(ebl, electrons, thinning))
    # sim.add(EMDoublePairProduction(crb, electrons, thinning))
    sim.add(EMInverseComptonScattering(cmb, photons, thinning))
    sim.add(EMInverseComptonScattering(ebl, photons, thinning))
    # sim.add(EMInverseComptonScattering(crb, photons, thinning))
    # sim.add(EMTripletPairProduction(cmb, electrons, thinning))
    # sim.add(EMTripletPairProduction(ebl, electrons, thinning))
    # sim.add(EMTripletPairProduction(crb, electrons, thinning))
    sim.add(MinimumEnergy(10 * GeV))
    obs = Observer()
    obs.add(Observer1D())
     obs.add(ObserverElectronVeto()) # we are only interested in photons
    output = TextOutput('cascade 1d.txt', Output.Event1D)
     output.setEnergyScale(eV)
     output.enable(output.WeightColumn) # this is required if thinning > 0
     output.disable(output.CandidateTagColumn) # not needed in this analysis
     obs.onDetection(output)
     source = Source()
    source.add(SourcePosition(Vector3d(dsrc, 0, 0)))
    source.add(SourceRedshift1D())
    source.add(SourceParticleType(22))
    source.add(SourcePowerLawSpectrum(10 * GeV, 10 * TeV, -1.5)) # intrinsic source spectrum
    # source.add(SourceEnergy(20 * TeV)) # a monochromatic intrinsic spectrum
    sim.add(obs)
    sim.setShowProgress(True)
    sim.run(source, 10000, True)
```

### example notebooks



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- Photon Propagation
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Plotting

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                                            obs.add(Observer1D())
                                            obs.add(ObserverElectronVeto()) # we are only interested in photons
                                            output = TextOutput('cascade_1d.txt', Output.Event1D)
                                            output.setEnergyScale(eV)
                                            output.enable(output.WeightColumn) # this is required if thinning > 0
                                            output.disable(output.CandidateTagColumn) # not needed in this analysis
                                             obs.onDetection(output)
                                             source = Source()
                                            source.add(SourcePosition(Vector3d(dsrc, 0, 0)))
                                            source.add(SourceRedshift1D())
                                             course add (Course Particle Ture (22))
https://crpropa.github.io/CRPropa3/
```

### example notebooks

☆ / Extragalactic Propagation / Electromagnetic cascade example





#### from crpropa import \*



### • define **Source** $\rightarrow$ initial conditions

#### from crpropa import \*



### • define **Source** $\rightarrow$ initial conditions

```
# source distribution: uniform with power-law spectrum
position = SourcePosition(Vector3d(distance, 0, 0))
direction = SourceDirection(Vector3d(-1, 0, 0)) # emit in the -x direction (1D simulation)
redshifts = SourceRedshift1D() # takes the positions and assign the corresponding redshifts
energySpectrum = SourcePowerLawSpectrum(energyMinimum, energyMaximum, -1)
particleType = SourceParticleType(22) # we are interested in gamma rays
source = Source()
source.add(position)
source.add(redshifts)
source.add(direction)
source.add(energySpectrum)
source.add(particleType)
```

#### from crpropa import \*

```
# general options
nEvents = 10000
energyMinimum = 1 * GeV
energyMaximum = 400 * TeV
redshift = 0.14
distance = redshift2ComovingDistance(redshift)
electrons = photons = True
thinning = 1.
cmb = CMB()
ebl = IRB_Gilmore12()
outputFile = 'sim/04-sim_gamma1D.txt'
```







### • define **Source** $\rightarrow$ initial conditions

## setting up a simulation. the observer

#### from crpropa import \*



- define Source  $\rightarrow$  initial conditions
- define **OBSERVER**  $\rightarrow$  region of interest, particle collector
  - saves outputs at a given time or + position,...

## setting up a simulation. the observer

#### from crpropa import \*



- define **Source**  $\rightarrow$  initial conditions
- define **OBSERVER**  $\rightarrow$  region of interest, particle collector
  - saves outputs at a given time or position,...

*# output # observer* 

observer.onDetection(output)

# setting up a simulation. the observer

#### from crpropa import \*

```
# general options
nEvents = 10000
energyMinimum = 1 * GeV
energyMaximum = 400 * TeV
redshift = 0.14
distance = redshift2ComovingDistance(redshift)
electrons = photons = True
thinning = 1.
cmb = CMB()
ebl = IRB_Gilmore12()
outputFile = 'sim/04-sim_gamma1D.txt'
```

```
outputType = Output.Event1D
output = TextOutput(outputFile, outputType)
output.disable(output.CandidateTagColumn)
output.enable(output.WeightColumn) # since we are using thinning
output.setEnergyScale(eV)
output.setLengthScale(Mpc)
observerType = Observer1D()
observer = Observer()
observer.add(observerType)
```





- define Source  $\rightarrow$  initial conditions
- define **OBSERVER**  $\rightarrow$  region of interest, particle collector
  - saves outputs at a given time or + position,...

# setting up a simulation. the interactions

#### from crpropa import \*



- define **Source**  $\rightarrow$  initial conditions
- define **OBSERVER**  $\rightarrow$  region of interest, particle collector
  - saves outputs at a given time or + position,...
- define interactions
  - e.g. pair production (with CMB, EBL, CRB), inverse Compton,...

# setting up a simulation. the interactions

#### from crpropa import \*



- define **Source**  $\rightarrow$  initial conditions
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  - saves outputs at a given time or position,...
- define **interactions** 
  - e.g. pair production (with CMB, EBL, CRB), inverse Compton, ...

# interactions z = Redshift()

# setting up a simulation. the interactions

#### from crpropa import \*

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# general options
nEvents = 10000
energyMinimum = 1 * GeV
energyMaximum = 400 * TeV
redshift = 0.14
distance = redshift2ComovingDistance(redshift)
electrons = photons = True
thinning = 1.
cmb = CMB()
ebl = IRB_Gilmore12()
outputFile = 'sim/04-sim_gamma1D.txt'
```

```
ppCMB = EMPairProduction(cmb, electrons, thinning)
ppEBL = EMPairProduction(ebl, electrons, thinning)
icsCMB = EMInverseComptonScattering(cmb, photons, thinning)
icsEBL = EMInverseComptonScattering(ebl, photons, thinning)
processes = [ppCMB, ppEBL, icsCMB, icsEBL, z]
```





- define Source  $\rightarrow$  initial conditions
- define **OBSERVER**  $\rightarrow$  region of interest, particle collector
  - saves outputs at a given time or + position,...

# setting up a simulation. the propagator

#### from crpropa import \*



- define **Source**  $\rightarrow$  initial conditions
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- define interactions
  - e.g. pair production (with CMB, EBL, CRB), inverse Compton, ...
- the propagation algorithm
  - 1D, 3D (Boris push, Cash-Karp, diffusion,...)

# setting up a simulation. the propagator

#### from crpropa import \*



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nEvents = 10000
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redshift = 0.14
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electrons = photons = True
thinning = 1.
cmb = CMB()
ebl = IRB_Gilmore12()
outputFile = 'sim/04-sim_gamma1D.txt'
```

# propagator: one-dimensional propagator = SimplePropagation(0.1 \* kpc, 100 \* kpc)





- define **Source**  $\rightarrow$  initial conditions
- define **OBSERVER**  $\rightarrow$  region of interest, particle collector
  - saves outputs at a given time or position,...
- define interactions
  - e.g. pair production (with CMB, EBL, CRB), inverse Compton, ...
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  - 1D, 3D (Boris push, Cash-Karp, diffusion,...)
- the **break conditions**  $\rightarrow$  when to stop simulating?

#### from crpropa import \*



- define **Source**  $\rightarrow$  initial conditions
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nEvents = 10000
energyMinimum = 1 * GeV
energyMaximum = 400 * TeV
redshift = 0.14
distance = redshift2ComovingDistance(redshift)
electrons = photons = True
thinning = 1.
cmb = CMB()
ebl = IRB_Gilmore12()
outputFile = 'sim/04-sim_gamma1D.txt'
```

# break conditions breakEnergy = MinimumEnergy(1 \* GeV)



- define **Source**  $\rightarrow$  initial conditions
- define **OBSERVER**  $\rightarrow$  region of interest, particle collector
  - saves outputs at a given time or position,...
- define **interactions** 
  - e.g. pair production (with CMB, EBL, CRB), inverse Compton, ...
- the propagation algorithm
  - 1D, 3D (Boris push, Cash-Karp, diffusion,...)
- the **break conditions**  $\rightarrow$  when to stop simulating?

## setting up a simulation. assembling the components

#### from crpropa import \*



- define **Source**  $\rightarrow$  initial conditions
- define **OBSERVER**  $\rightarrow$  region of interest, particle collector
  - saves outputs at a given time or position,...
- define **interactions** 
  - e.g. pair production (with CMB, EBL, CRB), inverse Compton, ...
- the propagation algorithm
  - 1D, 3D (Boris push, Cash-Karp, diffusion,...)
- the **break conditions**  $\rightarrow$  when to stop simulating?
- put everything in a **ModuleList**

## setting up a simulation. assembling the components

#### from crpropa import \*



- define **Source**  $\rightarrow$  initial conditions
- define **OBSERVER**  $\rightarrow$  region of interest, particle collector
  - saves outputs at a given time or position,...
- define interactions
  - e.g. pair production (with CMB, EBL, CRB), inverse Compton, ...
- the propagation algorithm
  - 1D, 3D (Boris push, Cash-Karp, diffusion, ...)
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#### from crpropa import \*

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# general options
nEvents = 10000
energyMinimum = 1 * GeV
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redshift = 0.14
distance = redshift2ComovingDistance(redshift)
electrons = photons = True
thinning = 1.
cmb = CMB()
ebl = IRB_Gilmore12()
outputFile = 'sim/04-sim_gamma1D.txt'
```

```
# assemble simulation components
sim = ModuleList()
sim.add(propagator)
for interaction in processes:
        sim.add(interaction)
sim.add(observer)
sim.add(breakEnergy)
sim.setShowProgress(True)
sim.run(source, nEvents, True)
```



	D	ID	E	ID0	EO	W
10	81.1649	22	4.833030e+12	22	4.921480e+12	1.0000
11	81.1649	22	1.899920e+09	22	1.934690e+09	1.0000
12	81.1649	22	1.409820e+11	22	1.435620e+11	1.0000
13	81.1649	22	1.259420e+10	22	1.282470e+10	1.0000
14	81.1649	22	3.121590e+11	22	3.178720e+11	1.0000
	•••	•••	•••	•••	•••	•••
95	81.1649	22	1.594810e+11	22	8.487520e+13	91.1634
96	81.1649	22	3.100800e+11	22	8.487520e+13	41.3962
97	81.1649	22	8.451480e+09	22	8.487520e+13	605.6770
98	81.1649	22	3.231870e+09	22	8.487520e+13	798.2140
99	81.1649	22	9.310190e+09	22	8.487520e+13	228.6540

90 rows × 8 columns



trajectory	7		םו ם		F		ΕÛ	w
length							EV	VV
		10	81.1649	22	4.833030e+12	22	4.921480e+12	1.0000
		11	81.1649	22	1.899920e+09	22	1.934690e+09	1.0000
		12	81.1649	22	1.409820e+11	22	1.435620e+11	1.0000
		13	81.1649	22	1.259420e+10	22	1.282470e+10	1.0000
		14	81.1649	22	3.121590e+11	22	3.178720e+11	1.0000
			•••	•••	•••		•••	•••
		95	81.1649	22	1.594810e+11	22	8.487520e+13	91.1634
		96	81.1649	22	3.100800e+11	22	8.487520e+13	41.3962
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		90 ro	ws × 8 col	umn	S			





E	ID0	EO	w
e+12	22	4.921480e+12	1.0000
+09	22	1.934690e+09	1.0000
e+11	22	1.435620e+11	1.0000
e+10	22	1.282470e+10	1.0000
e+11	22	3.178720e+11	1.0000
	•••	•••	•••
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detecte particle t	ed ype		at	energy detection				
trajectory		D	ID	E	ID0	EO	w	
length	10	81.1649	22	4.833030e+12	22	4.921480e+12	1.0000	
	11	81.1649	22	1.899920e+09	22	1.934690e+09	1.0000	
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	90 rc	ows × 8 co	lumns	S				



detected particle type			at	energy detection	emitted particle type		
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		•••		•••		•••	•••
	95	81.1649	22	1.594810e+11	22	8.487520e+13	91.1634
	96	81.1649	22	3.100800e+11	22	8.487520e+13	41.3962
	97	81.1649	22	8.451480e+09	22	8.487520e+13	605.6770
	98	81.1649	22	3.231870e+09	22	8.487520e+13	798.2140
	99	81.1649	22	9.310190e+09	22	8.487520e+13	228.6540
	90 ro	ws × 8 col					



	detecte	d			energy		emit	ted	
	particle type			at	detection		particle type		
		)							
-									
trajectory			D	ID	E	ID0	EO	w	
length 🚽		10	81.1649	22	4.833030e+12	22	4.921480e+12	1.0000	
		11	81.1649	22	1.899920e+09	22	1.934690e+09	1.0000	
		12	81.1649	22	1.409820e+11	22	1.435620e+11	1.0000	
		13	81.1649	22	1.259420e+10	22	1.282470e+10	1.0000	
		14	81.1649	22	3.121590e+11	22	3.178720e+11	1.0000	
				•••		•••	•••		
		95	81.1649	22	1.594810e+11	22	8.487520e+13	91.1634	
		96	81.1649	22	3.100800e+11	22	8.487520e+13	41.3962	
		97	81.1649	22	8.451480e+09	22	8.487520e+13	605.6770	
		98	81.1649	22	3.231870e+09	22	8.487520e+13	798.2140	
		99	81.1649	22	9.310190e+09	22	8.487520e+13	228.6540	
		00							
		90 rc	ws × 8 co	iumn	S				



	detected particle ty	d vpe		energy at detection		
trajectory			P			
lenath 👞		10	01 16 4 0	<b>U</b>	100000	
		10	81.1649	22	4.8330306	
		11	81.1649	22	1.899920e	
		12	81.1649	22	1.409820	
		13	81.1649	22	1.2594206	
		14	81.1649	22	3.121590	
			•••	•••		
		95	81.1649	22	1.594810	
		96	81.1649	22	3.100800	
		97	81.1649	22	8.451480e	
		98	81.1649	22	3.231870e	
		99	81.1649	22	9.310190e	
		90 rc	ows × 8 col	umn	s	





# modelling the propagation of gamma rays

Rafael Alves Batista | May 28, 2025 | The CRPropa framework






### intergalactic propagation of gamma rays





## interactions. pair production





## interactions. pair production







### interactions. pair production





### interactions. pair production

### **EBL dominates** at ~TeV energies





















### **CMB** dominates at ~TeV energies







# intergalactic magnetic fields (IGMFs)





### cosmic magnetic fields





### cosmic magnetic fields





### cosmic magnetic fields





### observational strategies

### **IGMF constraints with gamma rays. strategies**

strategy 1: point-like sources will appear extended

**strategy 2:** secondary gamma rays will arrive with time delays

**strategy 3:** combination of 1 and  $2 \rightarrow$  spectral changes



### probing IGMFs with gamma rays



### probing IGMFs with gamma rays







### probing IGMFs with gamma rays





### probing IGMFs with gamma rays







### probing IGMFs with gamma rays







### probing IGMFs with gamma rays







### CTA Consortium. JCAP 02 (2021) 048. arXiv:2010.01349







### CTA Consortium. JCAP 02 (2021) 048. arXiv:2010.01349







### CTA Consortium. JCAP 02 (2021) 048. arXiv:2010.01349



A-A

CHARD?





### CTA Consortium. JCAP 02 (2021) 048. arXiv:2010.01349



THE A

CHARD?





### CTA Consortium. JCAP 02 (2021) 048. arXiv:2010.01349



A-A

CHARD?





### constraining IGMFs with the ASTRI Mini-Array

### Vercellone et al. JHEAp 35 (2022) 1. arxiv:2208.03177







### constraining IGMFs with the ASTRI Mini-Array

































# plasma instabilities





### plasma instabilities: do they quench electromagnetic cascades?





### plasma instabilities: do they quench electromagnetic cascades?



pair production

 $\gamma + \gamma_{bg} \rightarrow e^+ + e^-$ 



### plasma instabilities: do they quench electromagnetic cascades?








 $e^{\pm} + \gamma_{bg} \rightarrow e^{\pm} + \gamma$ 





















#### canonical cascade photons



















#### plasma instabilities

energy losses: dE / dx

<image>





#### plasma instabilities

energy losses: dE / dx









#### canonical cascade photons

#### quenched cascade photons

38















#### the GRPLINST plugin for CRPropa





#### the GRPLINST plugin for CRPropa



# galactic propagation of gamma rays



#### galactic propagation of PeV gamma rays





Di Marco, Alves Batista, Sánchez-Conde. Phys. Rev. D 111 (2025) 083004. arXiv:2408.0818

# galactic propagation of PeV gamma rays



work by Gaetano Di Marco (Madrid)



Di Marco, Alves Batista, Sánchez-Conde. Phys. Rev. D 111 (2025) 083004. arXiv:2408.0818

# galactic propagation of PeV gamma rays



#### gamma-ray absorption in the interstellar radiation field

Di Marco, Alves Batista, Sánchez-Conde. Phys. Rev. D 111 (2025) 083004. arXiv:2408.0818









-0.9

# Lorentz invariance violation (LIV)







#### Lorentz invariance is *the* pillar of special relativity



- Lorentz invariance is *the* pillar of special relativity
- why LIV?
  - string theory +
  - loop quantum gravity  $\mathbf{+}$



- Lorentz invariance is *the* pillar of special relativity
- why LIV?
  - string theory +
  - loop quantum gravity  $\mathbf{+}$
- some observables
  - modified dispersion relations •
  - energy-dependent speed of light  $\bullet$



- Lorentz invariance is *the* pillar of special relativity
- why LIV?
  - string theory +
  - loop quantum gravity •
- some observables
  - modified dispersion relations •
  - energy-dependent speed of light +

#### modified dispersion relation

$$E_{\alpha}^{2} = p_{\alpha}^{2} \left[ 1 + \frac{m_{\alpha}^{2}}{p_{\alpha}^{2}} + \sum_{j=0}^{N} \chi_{j}^{\alpha} \left( \frac{p_{\alpha}}{E_{QG}^{\alpha}} \right) \right]$$





- Lorentz invariance is *the* pillar of special relativity
- why LIV?
  - string theory +
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#### modified dispersion relation

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leading order





- Lorentz invariance is *the* pillar of special relativity
- why LIV?
  - string theory +
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  - modified dispersion relations •
  - energy-dependent speed of light +

#### modified dispersion relation

$$E_{\alpha}^{2} = p_{\alpha}^{2} \left[ 1 + \frac{m_{\alpha}^{2}}{p_{\alpha}^{2}} + \sum_{j=0}^{N} \chi_{j}^{\alpha} \left( \frac{p_{\alpha}}{E_{\text{QG}}^{\alpha}} \right)^{j} \right]$$

$$\text{leading order}$$

$$E_{\alpha}^{2} \approx p_{\alpha}^{2} \left[ 1 + \frac{m_{\alpha}^{2}}{p_{\alpha}^{2}} + \chi_{n}^{\alpha} \left( \frac{p_{\alpha}}{E_{\text{QG}}^{\alpha}} \right)^{n} \right]$$











Saveliev & Alves Batista. Classical and Quantum Gravity 41 (2024) 115011. arXiv:2312.10803



#### LIV. modified mean free paths









#### simulation results









Saveliev & Alves Batista. Classical and Quantum Gravity 41 (2024) 115011. arXiv:2312.10803

#### simulation results

#### pair production + inverse Compton scattering + vacuum Cherenkov + photon decay







#### the LIVPROPA plugin for CRPropa





#### the LIVPROPA plugin for CRPropa



# galactic diffuse emission



#### diffuse galactic emission from CRs




### CR density [arbitrary units]

### diffuse galactic emission from CRs





















### diffuse galactic emission from CRs





0













### diffuse galactic emission from CRs







# dark matter



### improving predictions: WIMP annihillation







Arina et al. JCAP 03 (2024) 035. arXiv:2312.01153

### improving predictions: WIMP annihillation







### improving predictions: WIMP annihillation





work by

### Ignacio Martínez López (Madrid), and M.A. Sánchez-Conde (Madrid)



### improving predictions: WIMP annihillation





### axion-like particles. the ALPinist plugin for CRPropa





### axion-like particles. the ALPinist plugin for CRPropa









### **ALPinist. examples**





# summary & outlook







## extending CRPropa: write your own plugin





## extending CRPropa: write your own plugin





## extending CRPropa: write your own plugin



## **CR**/Propa

- public *open-source* framework for the propagation of high-energy particles
- many photon background and magnetic-field models
- enables a self-consistent interpretation of observations with **multiple messengers**
- **modular design** enables easy customisation for various applications in astroparticle physics
- easy to extend: write your own plugin!
- moving towards **CRPropa 3.3** (and planning **CRPropa 4**)
  - position-dependent photon fields +
  - hadronic interactions
  - improvement at lower energies
  - many photon backgrounds and magnetic-field models
- **Github** development
  - report possible issues there



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  - improvement at lower energies
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- **Github** development
  - report possible issues there



more information:

### crpropa.desy.de



### the developing team (currently)

S. Aerdker (Bochum) R. Alves Batista (Paris) J. Becker Tjus (Bochum) G. Di Marco (Madrid) J. Dörner (Bochum) K.-H. Kampert (Wuppertal) L. Merten (Bochum) L. Morejon (Wuppertal) G. Müller (Aachen) P. Reichherzer (Oxford) A. Saveliev (Kaliningrad) L. Schlegel (Bochum) G. Sigl (Hamburg) A. van Vliet (Abu Dhabi)

### and MANY contributors



### the CRPropa team

### CRPropa users and developers. **CRPropa Workshop 2022 (Madrid)**



back-up slides











(squared) centre of  $s = m^2 + E\varepsilon(1 - \beta\cos\theta)$ 

mass energy





(squared) centre of  $s = m^2 + E\varepsilon(1 - \beta\cos\theta)$ 

mass energy

### interaction length













(squared) centre of  $s = m^2 + E\varepsilon(1 - \beta\cos\theta)$ mass energy



particle













interaction

length









interaction

length







### the fate of the electrons



# flux attenuation $\Phi_o(E_o; z_s) = \Phi_s(E_o) \exp\left[-\tau(E_o, z_s)\right]$

### the fate of the electrons



flux  
attenuation
$$\Phi_o(E_o; z_s) = \Phi_s(E_o) \exp\left[-\tau(E_o, z_s)\right]$$
optical  
depth $\tau(E_o, z_s) = \int_0^{z_s} dz \ \lambda^{-1} \left(\frac{E_{o,s}}{1+z}, z\right) \frac{d\ell}{dz}$ 

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Addazi et al. Prog. Part. Nucl. Phys. 125 (2022) 103948. arXiv:2111.05659



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#### the fate of the electrons





# gamma rays. interactions during cosmological propagation









<b>^</b> -	┸	$^+$



Alves Batista, Saveliev, de Gouveia Dal Pino. MNRAS 489 (2019) 3836. arXiv:1904.13345

### plasma instabilities: quenching factors



model A;  $\alpha = 1.0$  $10^{3}$  $E_{\rm max} = 1 \times 10^{12} {\rm eV}$  $E_{\rm max} = 3 \times 10^{12} {\rm eV}$  $E_{\rm max} = 1 \times 10^{13} \ {\rm eV}$  $E_{\rm max} = 3 \times 10^{13} {\rm ~eV}$ z=0.14  $10^{2}$  –  $\cdot \frac{1}{2} \cdot \frac{1}{2}$  $10^{1}$  $10^{0}$ 10<sup>11</sup>  $10^{11}$  $10^{10}$  $10^{9}$ E [eV]model A;  $\alpha = 2$ . 2.0  $E_{\rm max} = 1 \times 10^{12} \ {\rm eV}$  $E_{\rm max} = 3 \times 10^{12} {\rm eV}$ 1.8- $E_{\rm max} = 1 \times 10^{13} \ {\rm eV}$  $E_{\rm max} = 3 \times 10^{13} \, {\rm eV}$ 1.6 -· 9 |· 4 -1.2 -1.0 0.8 -1013  $10^{12}$  $10^{9}$  $10^{10}$  $10^{11}$  $10^{14}$ E [eV]

Alves Batista, Saveliev, de Gouveia Dal Pino. MNRAS 489 (2019) 3836. arXiv:1904.13345

### plasma instabilities: quenching factors





Saveliev & Alves Batista. Classical and Quantum Gravity 41 (2024) 115011. arXiv:2312.10803



Saveliev & Alves Batista. Classical and Quantum Gravity 41 (2024) 115011. arXiv:2312.10803

- first complete simulations of gamma-ray propagation including LIV
  - modification of pair production
  - including inverse Compton scattering  $\bullet$
  - including vacuum Cherenkov +
  - including photon decay +



Saveliev & Alves Batista. Classical and Quantum Gravity 41 (2024) 115011. arXiv:2312.10803

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Saveliev & Alves Batista. Classical and Quantum Gravity 41 (2024) 115011. arXiv:2312.10803

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Saveliev & Alves Batista. Classical and Quantum Gravity 41 (2024) 115011. arXiv:2312.10803





#### vacuum Cherenkov and photon decay





















![](_page_239_Picture_4.jpeg)

![](_page_240_Figure_1.jpeg)

![](_page_240_Picture_4.jpeg)

![](_page_241_Figure_1.jpeg)

#### equation of motion

$$\left(E - i\frac{\partial}{\partial z} + \frac{1}{2E}\mathbb{M}\right)\overrightarrow{\mathscr{A}} = 0$$

![](_page_241_Picture_6.jpeg)

![](_page_241_Picture_7.jpeg)

![](_page_242_Figure_1.jpeg)

#### equation of motion

$$\left(E - i\frac{\partial}{\partial z} + \frac{1}{2E}\mathbb{M}\right)\overrightarrow{\mathscr{A}} = 0$$

#### mixing matrix

$$\begin{pmatrix} \Delta_{\parallel} \cos^{2} \varphi + \Delta_{\perp} \sin^{2} \varphi & \left( \Delta_{\parallel} - \Delta_{\perp} \right) \sin \varphi \cos \varphi & \Delta_{a\gamma} \cos \varphi \\ \left( \Delta_{\parallel} - \Delta_{\perp} \right) \sin \varphi \cos \varphi & \Delta_{\parallel} \sin^{2} \varphi + \Delta_{\perp} \cos^{2} \varphi & \Delta_{a\gamma} \sin \varphi \\ \Delta_{a\gamma} \cos \varphi & \Delta_{a\gamma} \sin \varphi & \Delta_{a} \end{pmatrix}$$

![](_page_242_Picture_8.jpeg)

![](_page_242_Picture_9.jpeg)

![](_page_243_Figure_1.jpeg)

#### equation of motion

$$\left(E - i\frac{\partial}{\partial z} + \frac{1}{2E}\mathbb{M}\right)\overrightarrow{\mathscr{A}} = 0$$

$$\begin{pmatrix} \Delta_{\parallel} \cos^{2} \varphi + \Delta_{\perp} \sin^{2} \varphi & \left( \Delta_{\parallel} - \Delta_{\perp} \right) \sin \varphi \cos \varphi & \Delta_{a\gamma} \cos \varphi \\ \left( \Delta_{\parallel} - \Delta_{\perp} \right) \sin \varphi \cos \varphi & \Delta_{\parallel} \sin^{2} \varphi + \Delta_{\perp} \cos^{2} \varphi & \Delta_{a\gamma} \sin \varphi \\ \Delta_{a\gamma} \cos \varphi & \Delta_{a\gamma} \sin \varphi & \Delta_{a} \end{pmatrix}$$

![](_page_243_Picture_9.jpeg)

![](_page_243_Picture_10.jpeg)