Axion production at finite density, systematically

WEIZMANN INSTITUTE OF SCIENCE

Konstantin Springmann

SN 1987A as seen from James Webb

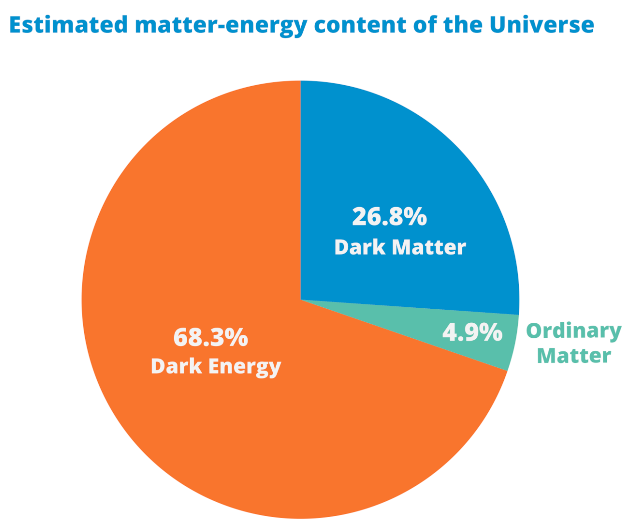
In collaboration with **Michael Stadlbauer (TUM,MPP)**, **Stefan Stelzl (EPFL)** and **Andreas Weiler (TUM)**

Based on

2410.10945, and 2410.19902

Also 2003.04903
with **Reuven Balkin (UCSC)** and **Javi Serra (IFT)**

Why Axions?



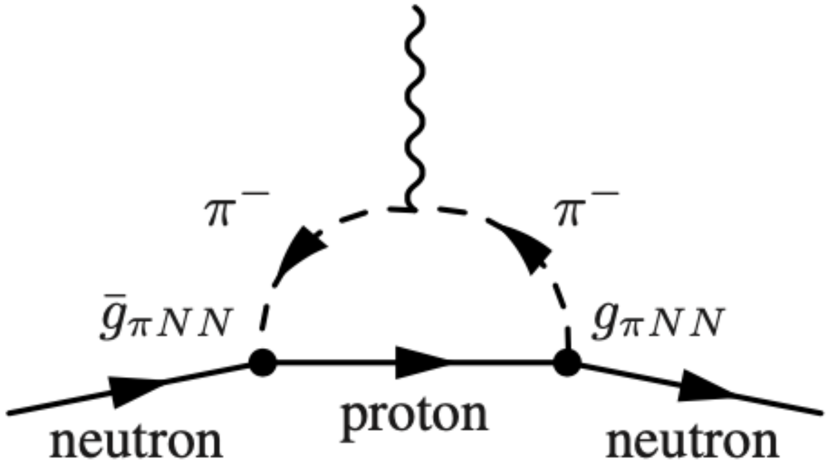
pseudo-Nambu-Goldstone boson which,

- **Can be DM**

- **Can solve Strong CP Puzzle** $|\bar{\theta}| \lesssim 10^{-10}$

$$\mathcal{L}_\chi \supset d_n \bar{n} \sigma^{\mu\nu} \gamma_5 n F_{\mu\nu}$$

nEDM $$d_n \approx \frac{e|\bar{\theta}|m_\pi^2}{m_n^3} \approx 10^{-16}|\bar{\theta}|e \text{ cm}$$



Crewther, Vecchia, Veneziano, Witten ('79)

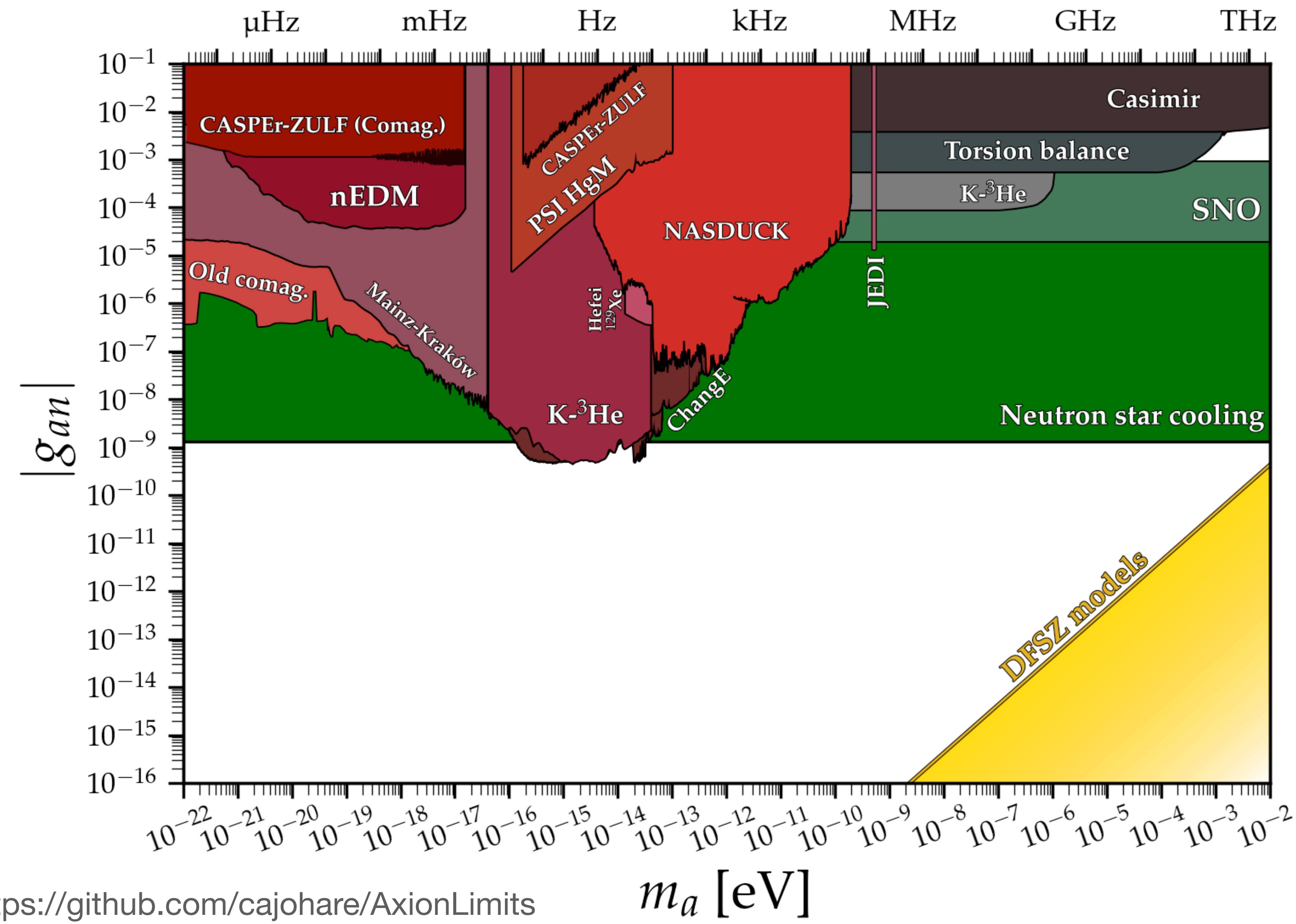
- **Sting theory, mediator to dark sector,…**

Axion-Neutron coupling

Some of the strongest bounds from SN and NS cooling

$$g_{an} = c_n \frac{m_n}{f_a}$$

For recent astro bounds see e.g. Mirizzi et al. '19 (SN), Buschmann et al. (NS) '21



https://github.com/cajohare/AxionLimits

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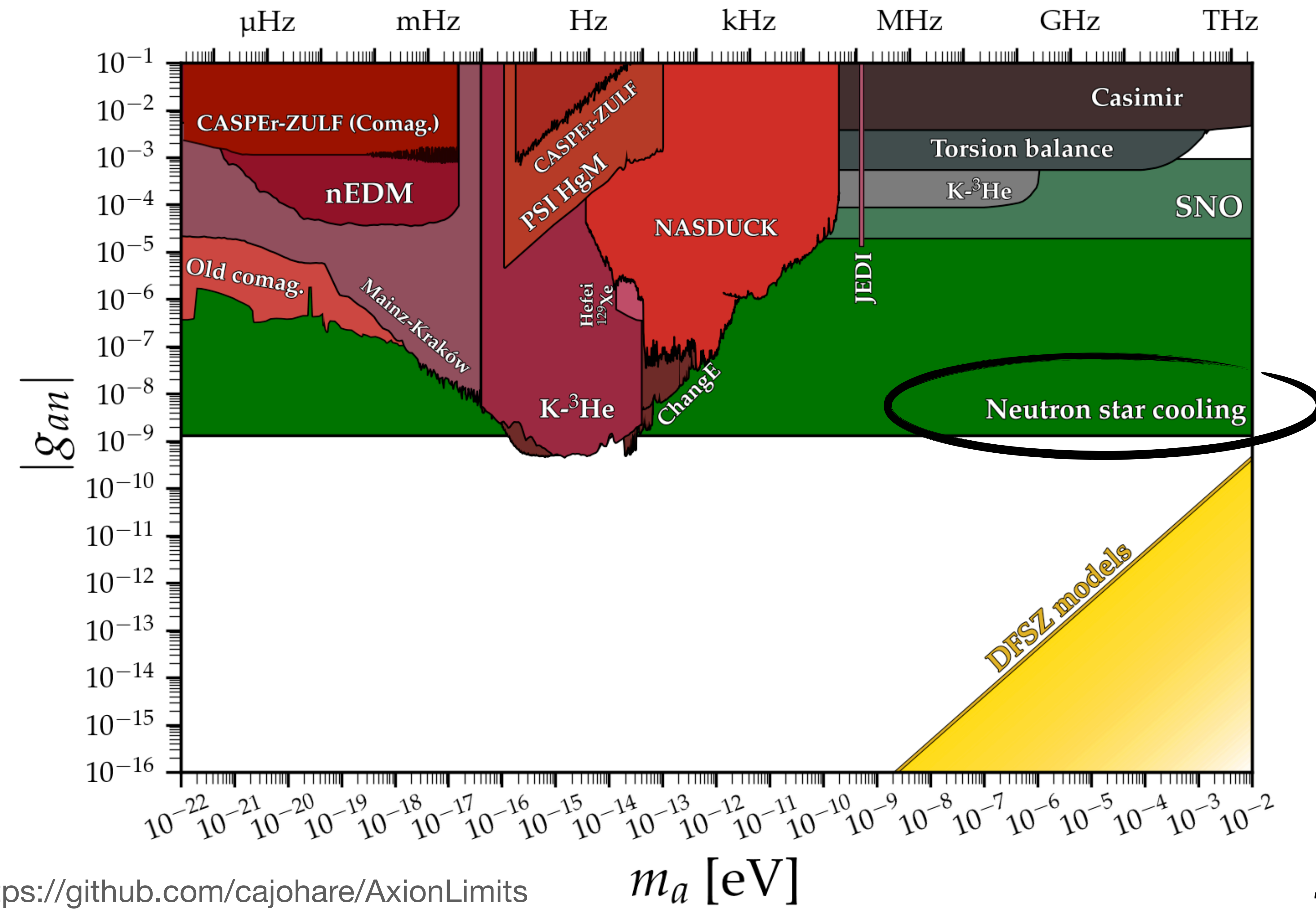

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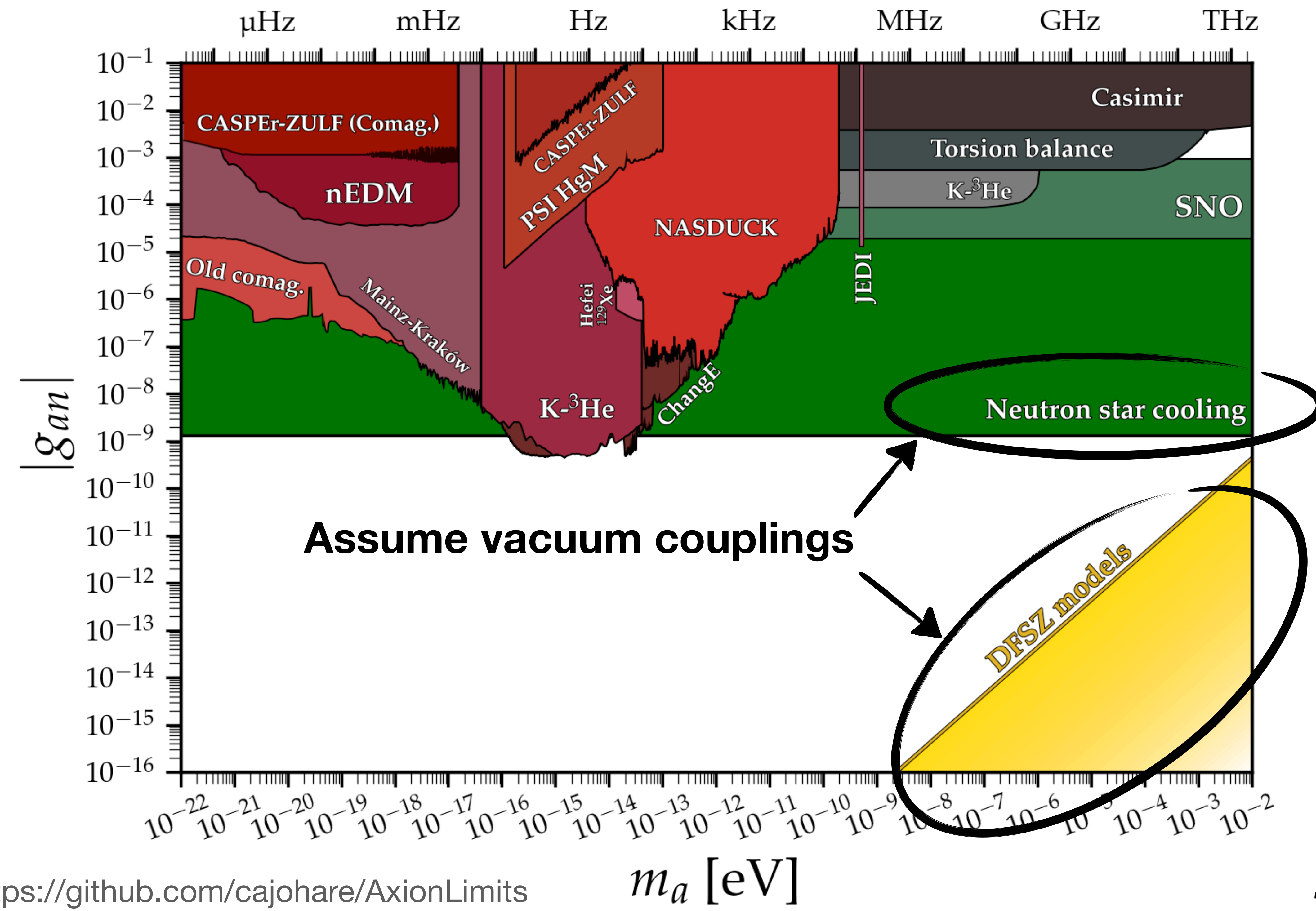

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Axion properties are highly susceptible to matter effects

Potential changes

Couplings to matter change

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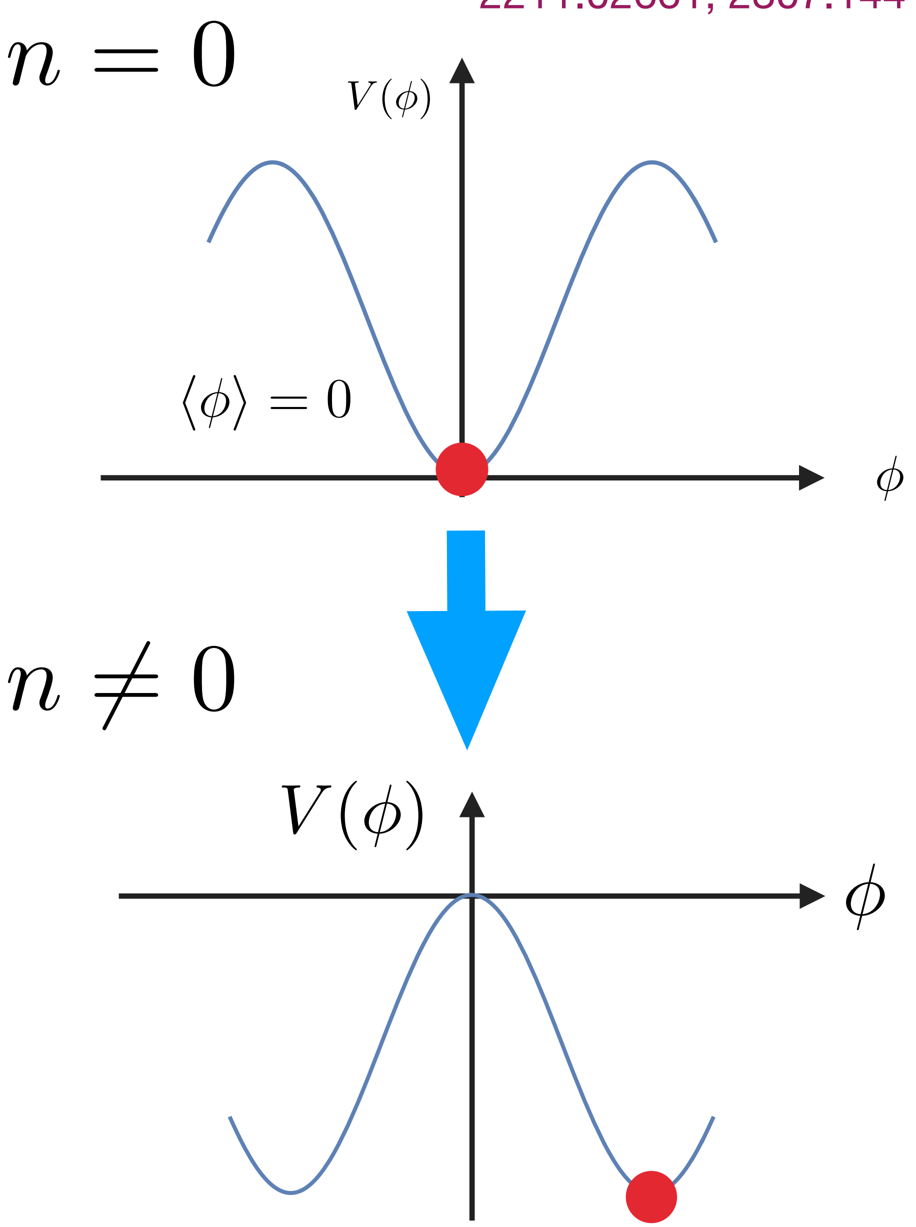
Potential changes

2211.02661, 2307.14418, 2408.07740



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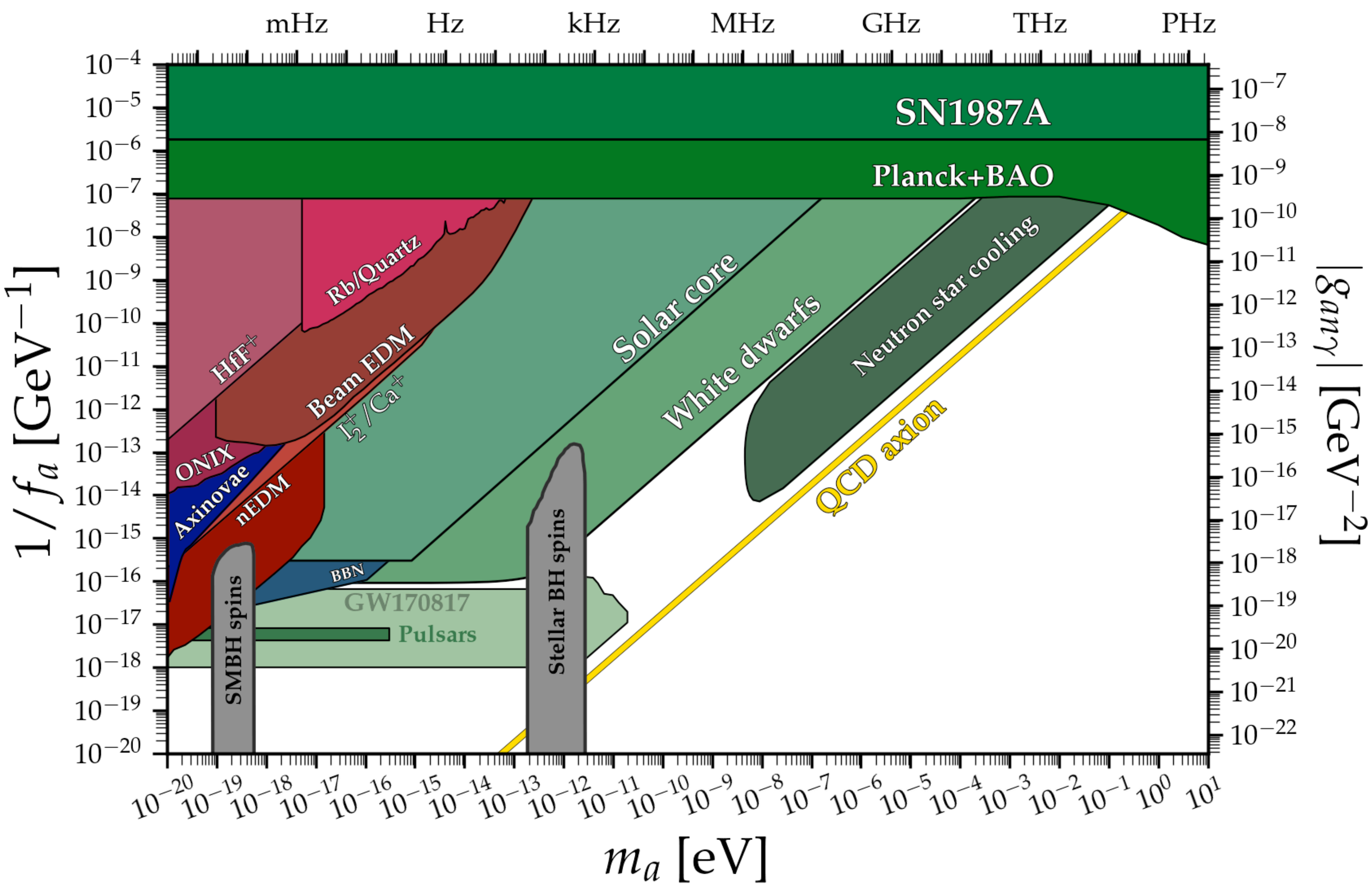

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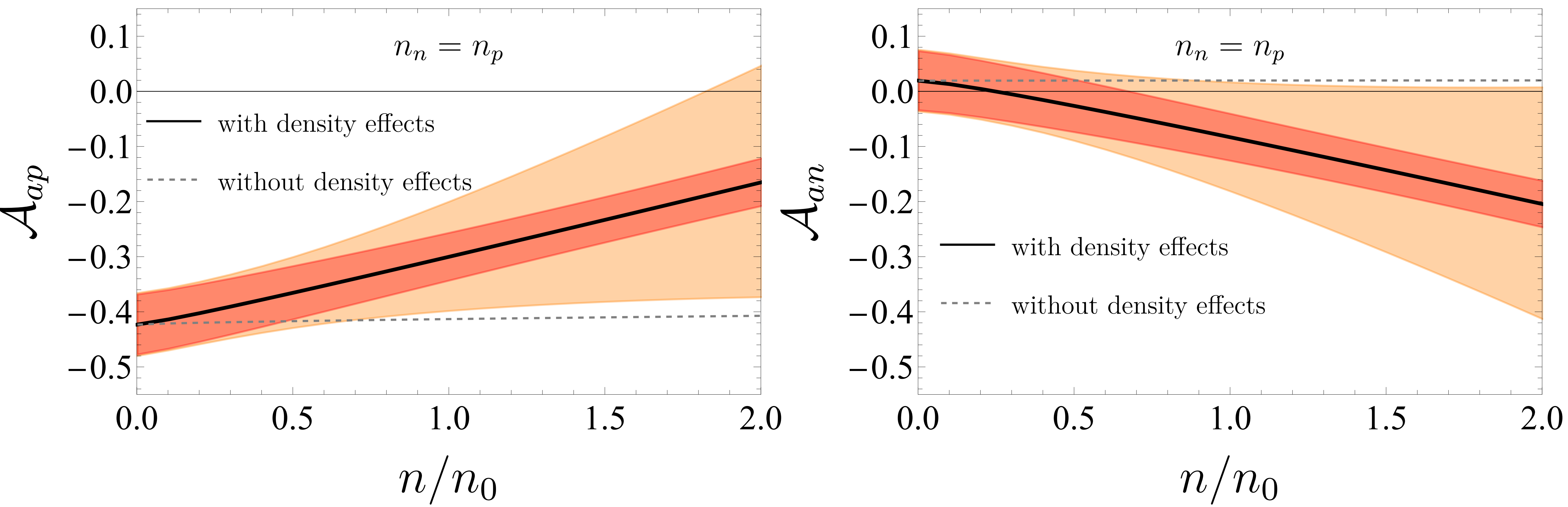
~~Potential changes~~

Couplings to matter change

2003.04903, 2410.10945

Proton **Neutron**



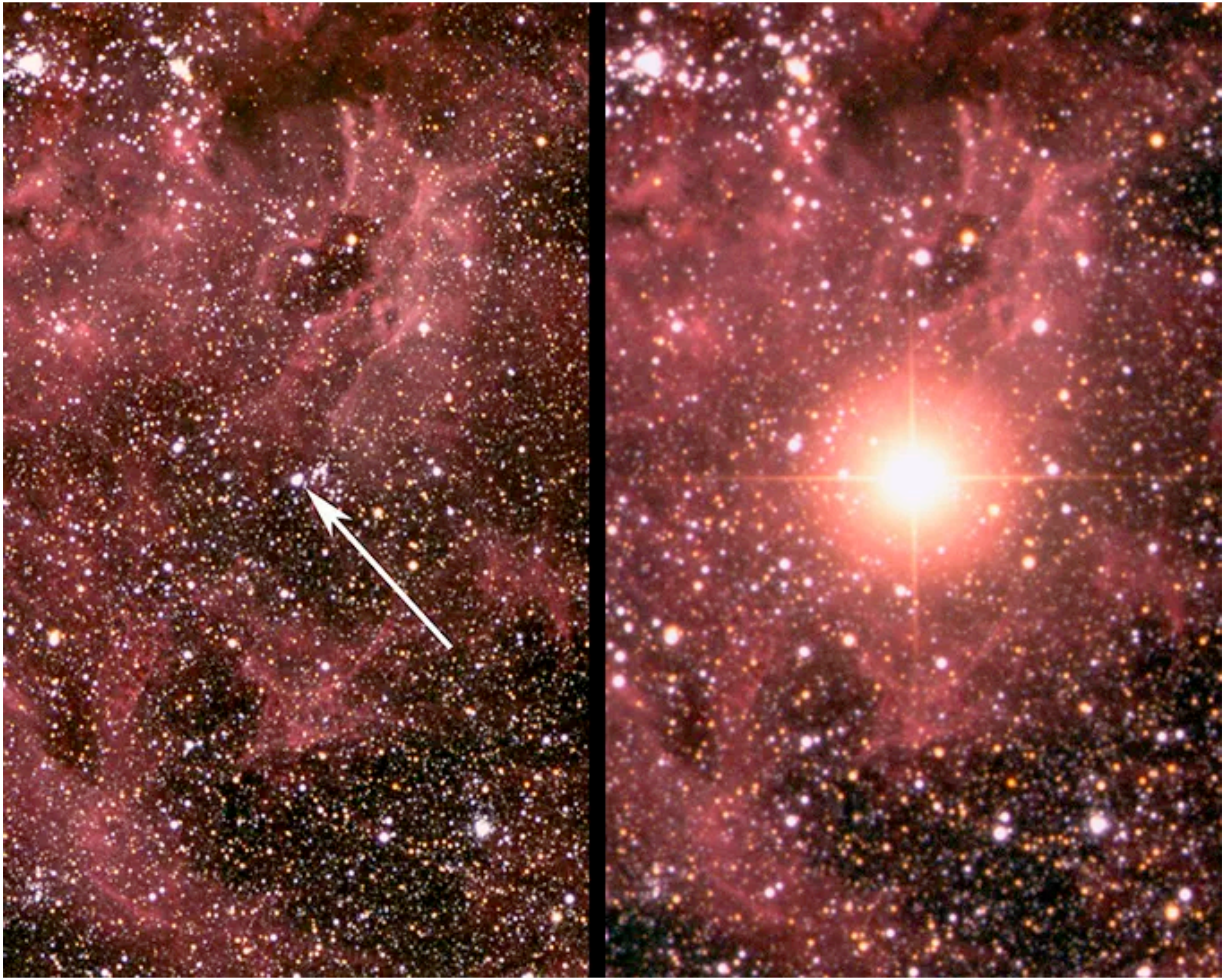
Results for KSVZ-type QCD axion

Outline

- Supernova bound on the QCD axion
- Axion EFTs
- Couplings in vacuum and finite density
- Supernova bound revisited
- Astrophobic axions

Bound from SN 1987A

Have observed a core-collapse (type II) SN in 1987 in the Large Magellanic Cloud



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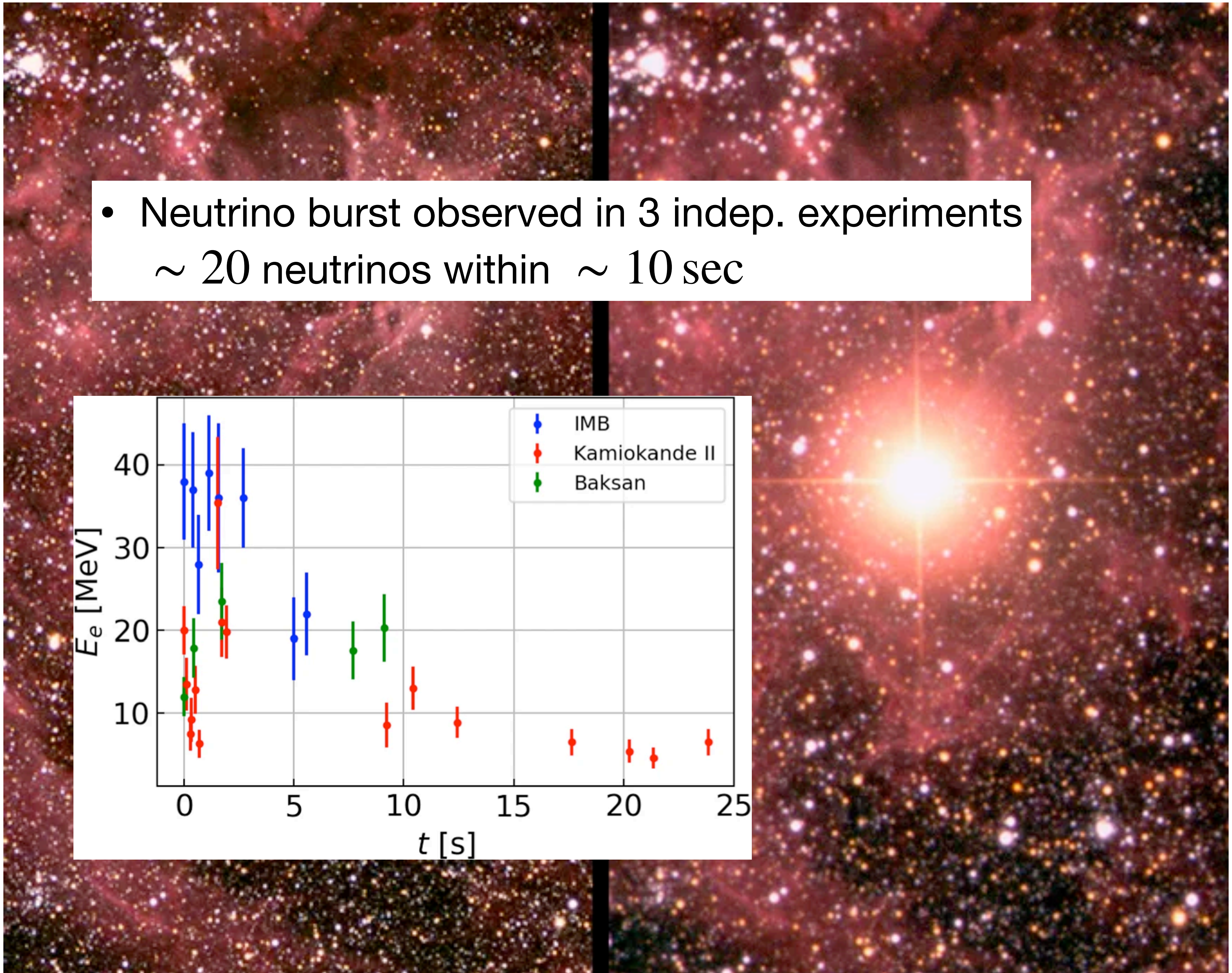

- Neutrino burst observed in 3 indep. experiments
  ~ 20 neutrinos within $\sim 10\,\mathrm{sec}$

Bound from SN 1987A

- If new lightly coupled particle gets produced, it could shorten the duration of the neutrino signal

Raffelt criterion: $$L_{\rm new} \lesssim L_\nu(t = 1{\rm s}) \simeq 3 \times 10^{52} {\rm erg\, s^{-1}}$$

Raffelt, Lect.Notes Phys. 741 (2008) 51-71

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- For QCD axion, this directly gives constraint on f_a

- Uncertainty in SN **dynamics** and **axion production**

Bar, Blum, D'Amico ('19)

Fransson et al. ('24)

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Focus on this

- Uncertainty in SN **dynamics** and **axion production**

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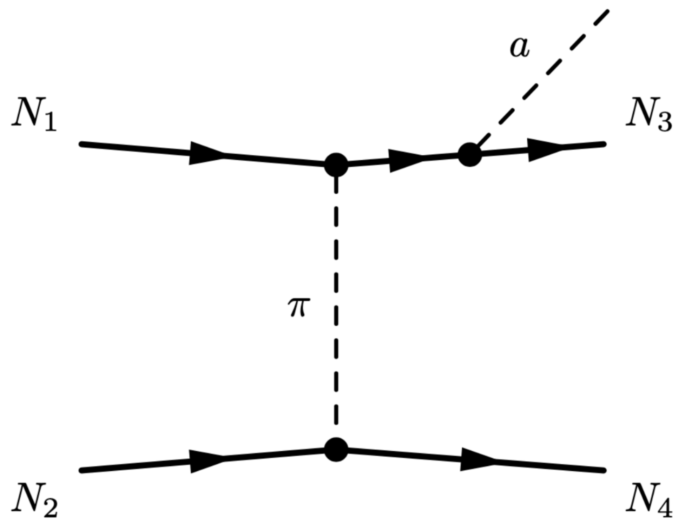
- Axions dominantly produced via Bremsstrahlung



Corrections to Bremsstrahlung

What has been done? Included corrections **phenomenologically**

- Multiply rate by fudge factors: $\Gamma_a = \Gamma_a^{\rm tree} \gamma_f \gamma_p \gamma_h$ Chang, Essig, McDermott ('18)

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Hannestad, Raffelt *Astrophys.J.* 507 (1998) 339-352
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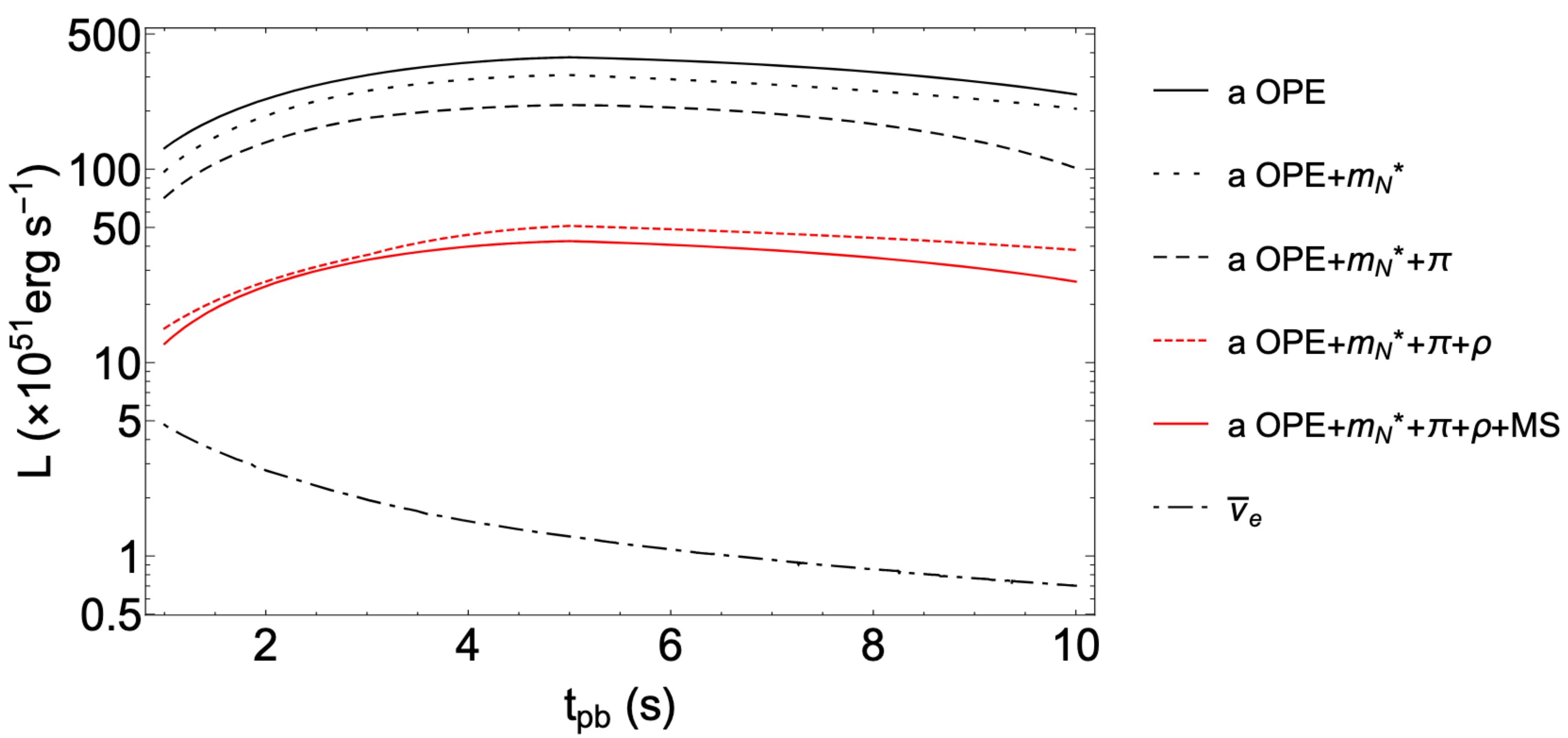

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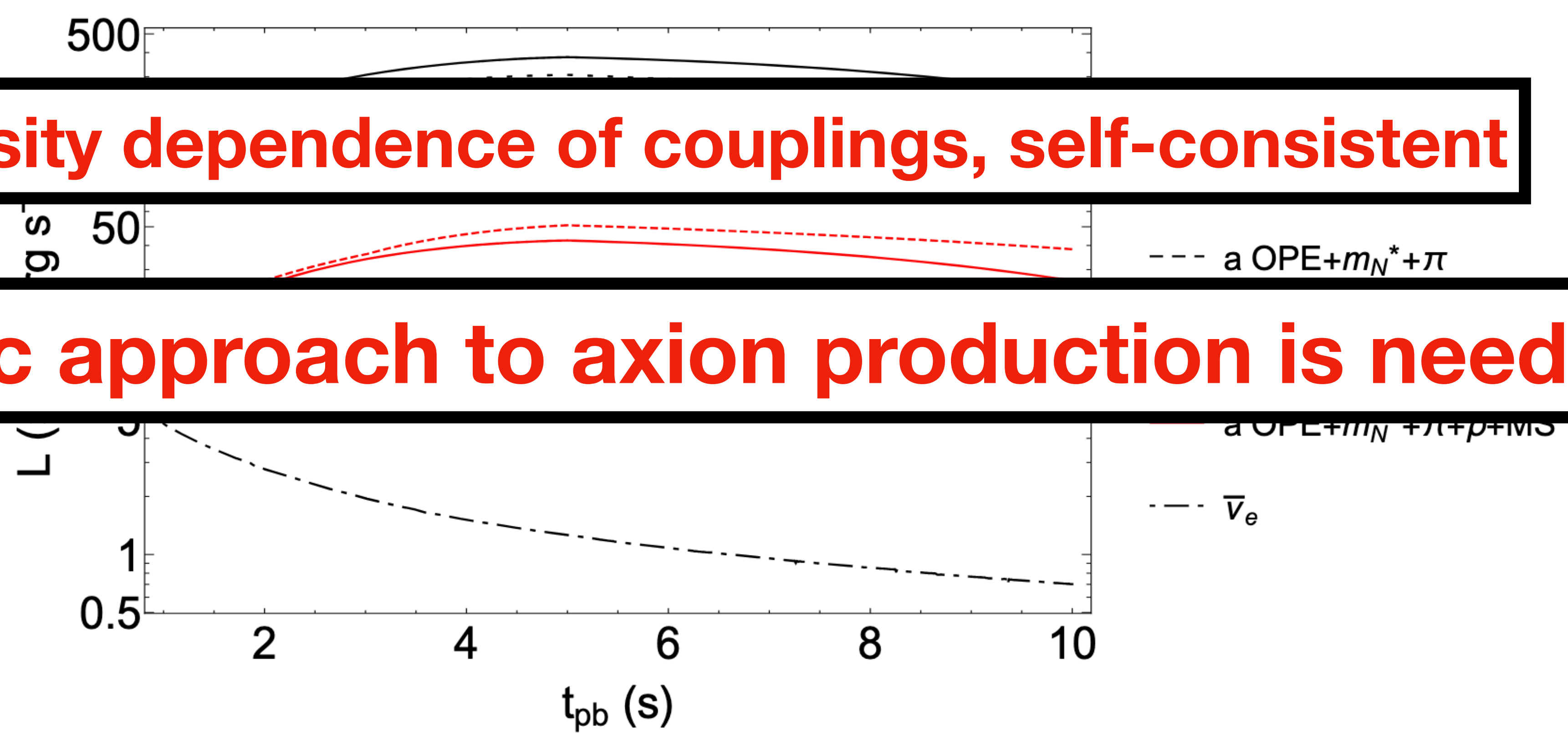

No density dependence of couplings, self-consistent

Systematic approach to axion production is needed

Axion-Nucleon coupling

- Axion-Nucleon coupling

$$\mathcal{L} \supset \frac{1}{f_a} \bar{N} c_N S \cdot \partial a N, \quad N = (p,n)^T \qquad N_f = 2$$

$$c_N = G_A c_{u-d} \tau^3 + G_0 c_{u+d} \mathbf{1}$$

Villadoro et.al. 15'

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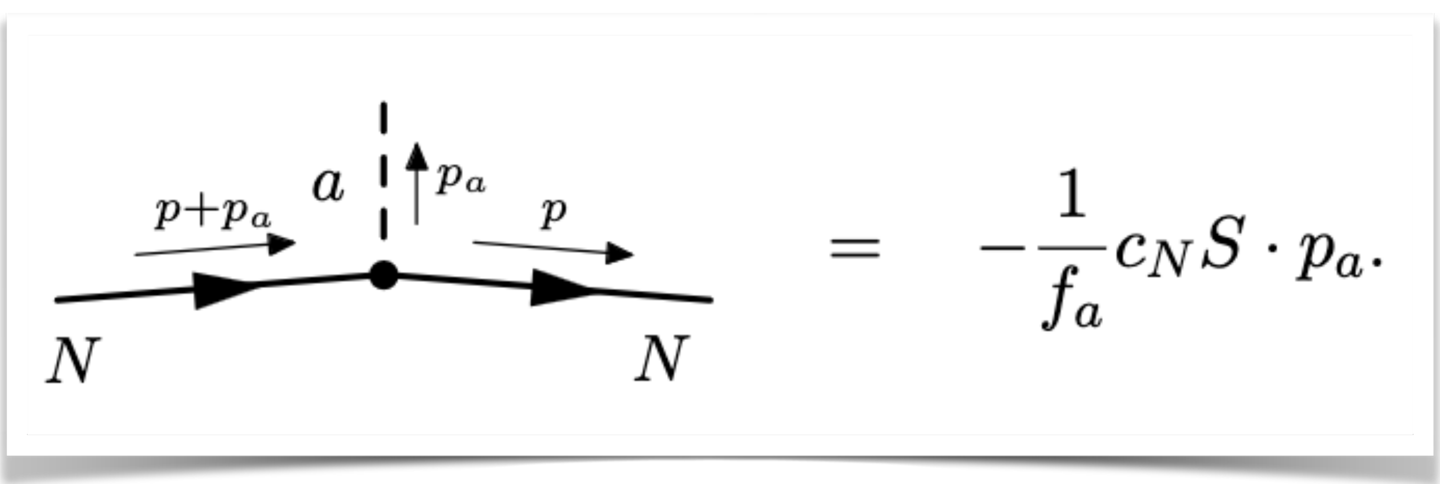

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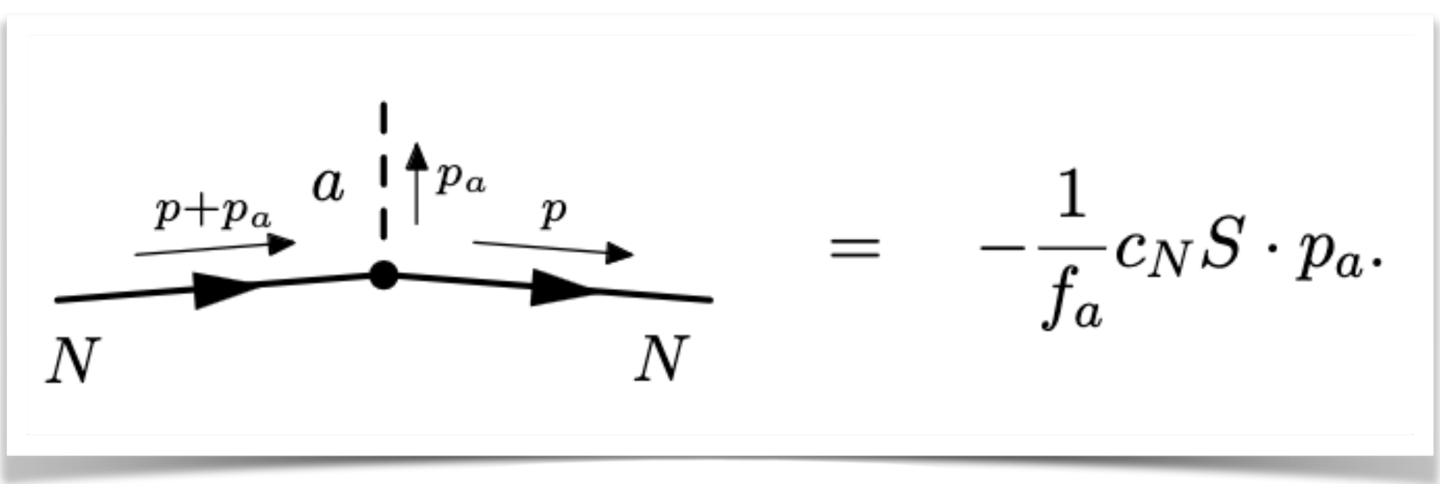

- KSVZ axion $c_p^{\rm KSVZ} = -0.47(3), \quad c_n^{\rm KSVZ} = +0.02(3)$

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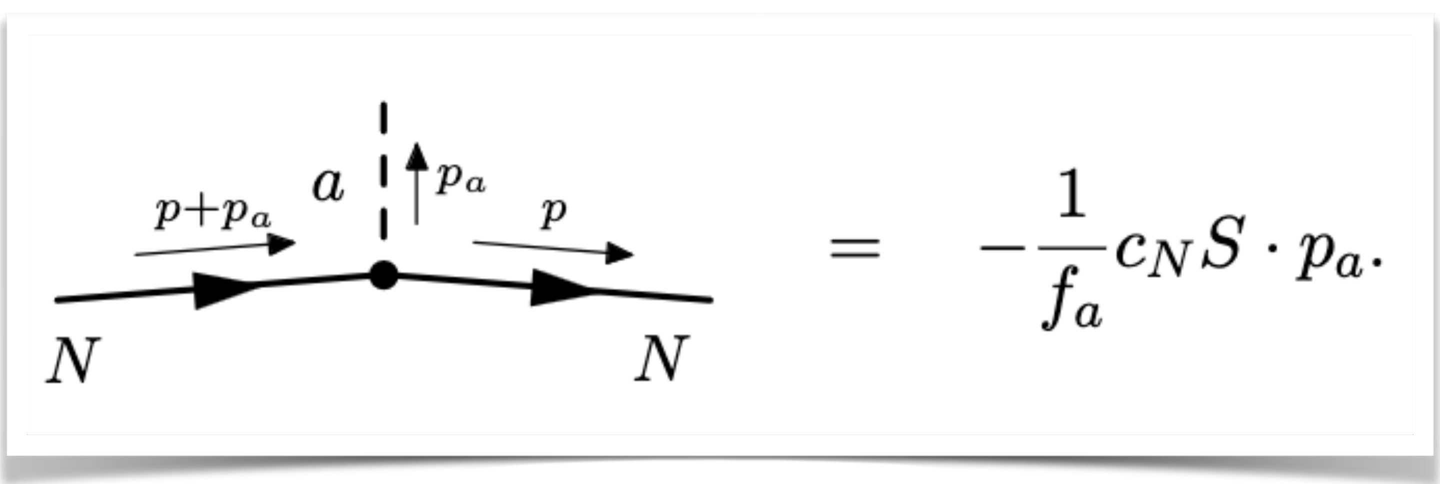

Compatible with zero due to accidental cancellation

- KSVZ axion $c_p^{\mathrm{KSVZ}} = -0.47(3), \quad c_n^{\mathrm{KSVZ}} = +0.02(3)$

Is this EFT valid in astrophysical environments?

This Hubble Space Telescope image shows Supernova 1987A within the Large Magellanic Cloud

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Not really…

- Typical momenta $k_F \simeq (3\pi^2 n_0)^{1/3} \simeq 260\,\mathrm{MeV}$ $n_0 \simeq 0.16\,\mathrm{fm}^{-3}$

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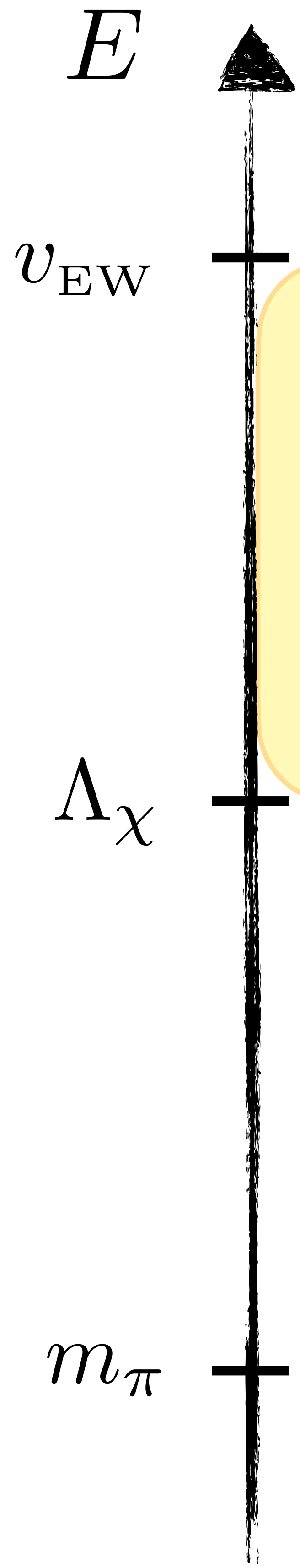
Need to construct EFT of pions and nuclons!

Axion EFTs



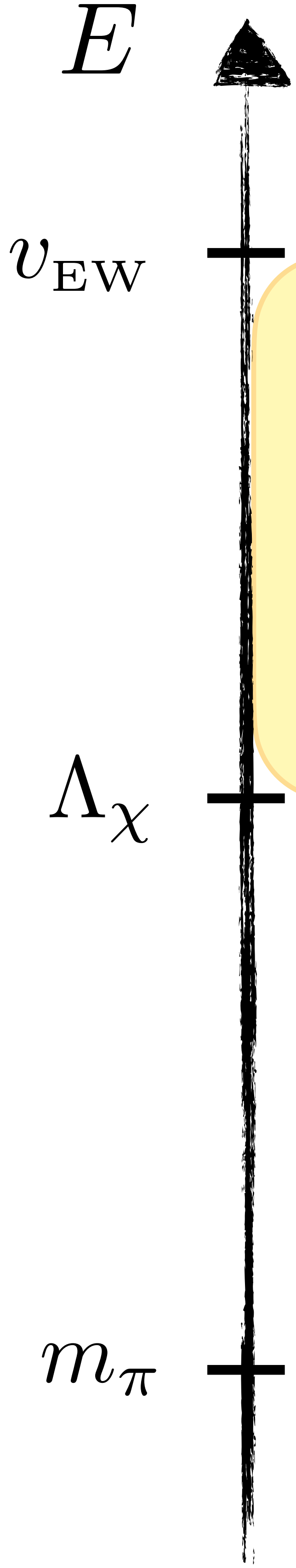
KSVZ axion $c_q^0 = 0$

After chiral quark rotation

$$\mathcal{L} \supset \bar{q} M_a q + \frac{\partial_\mu a}{f_a} \bar{q} \gamma^\mu \gamma_5 \left(c_q^0 - [Q_a]_q \right) q$$

$$M_a = e^{\frac{ia}{f_a} Q_a} M_q \qquad Q_a \equiv M_q^{-1} / \mathrm{Tr}(M_q^{-1})$$

Axion EFTs



Can be mapped to QCD Lagrangian with external sources

$$\mathcal{L} \supset -\bar{q}\left(s - i\gamma_5 p\right) q + \bar{q}\gamma^\mu \gamma_5 \left(a_\mu + a_\mu^s\right) q$$

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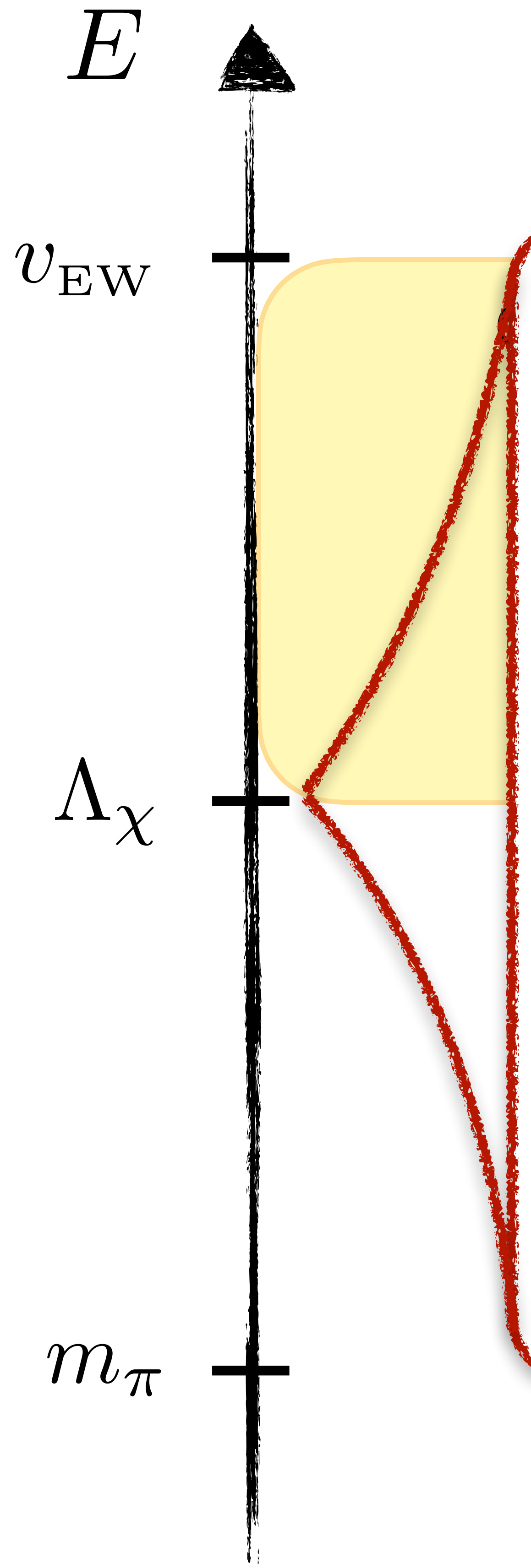


QCD confines: $|\langle \bar{q}_L q_R \rangle| \equiv B f_\pi^2$

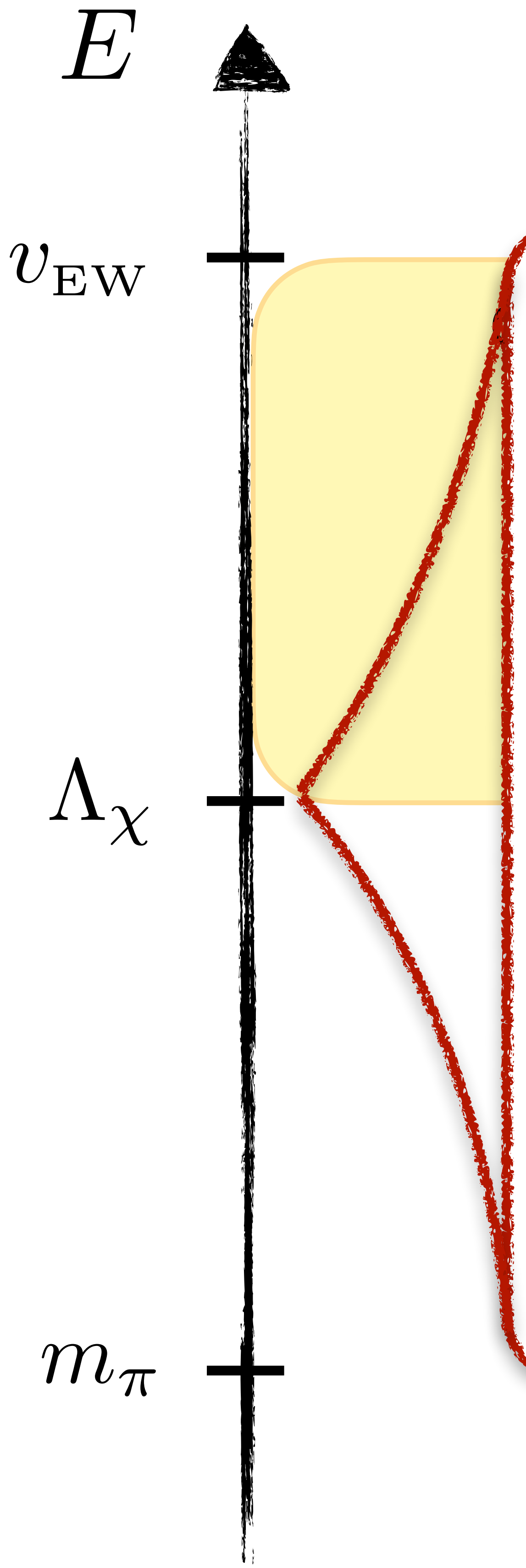
$$G_\chi = U(2)_L \times U(2)_R \to U(2)$$

- EFT w/ Mesons and Baryons:

  Chiral Perturbation Theory: spurion analysis

Weinberg ('79)
Gasser, Leutwyler ('85)

Axion EFTs



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$$p^\mu = m_N v^\mu + k^\mu \qquad \Psi(x) = e^{-i m_N v \cdot x} \left[N_v(x) + H_v(x) \right]$$

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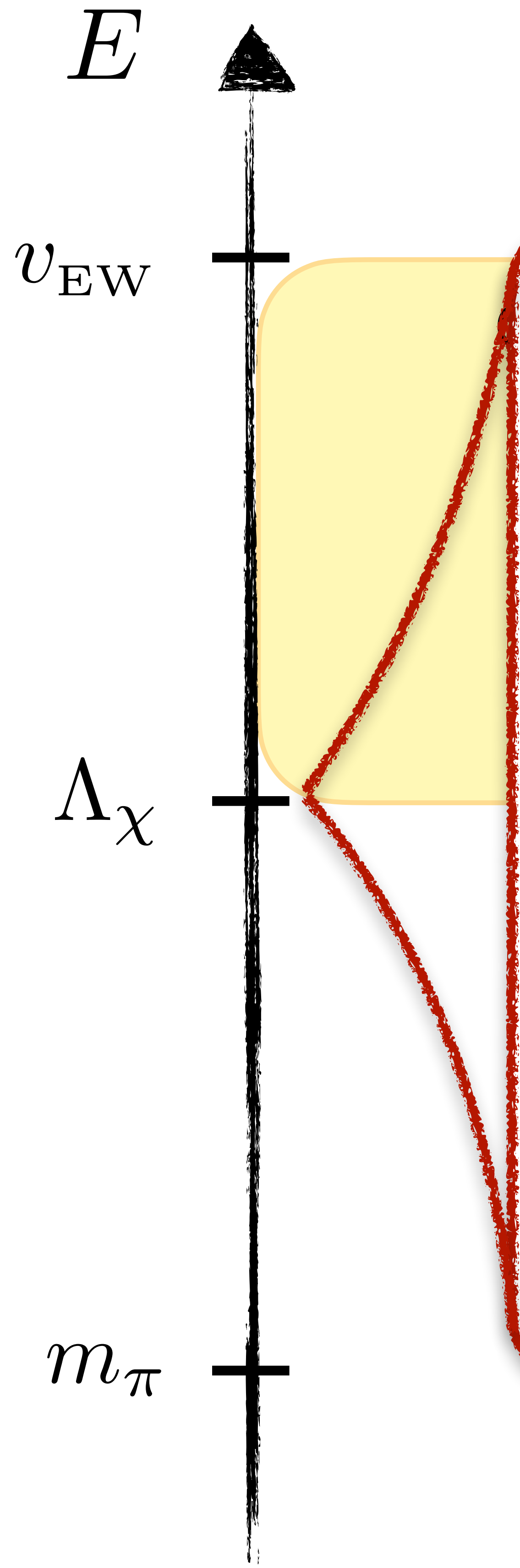

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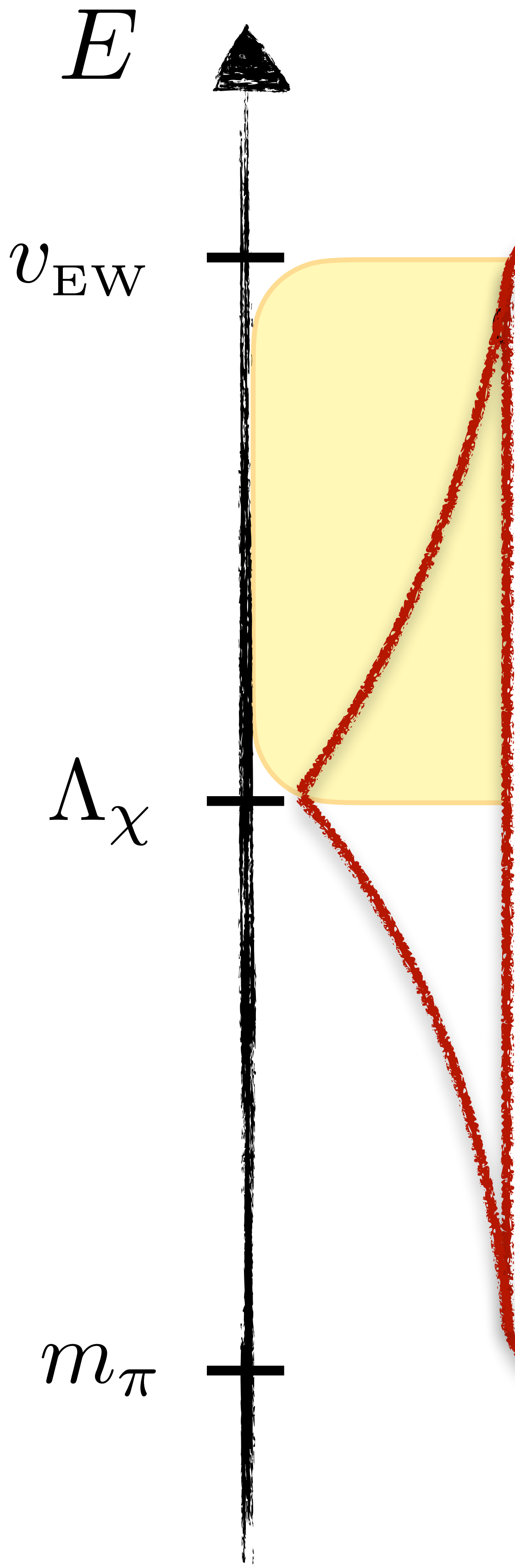
Axion EFTs



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$1\,\mathrm{GeV}$

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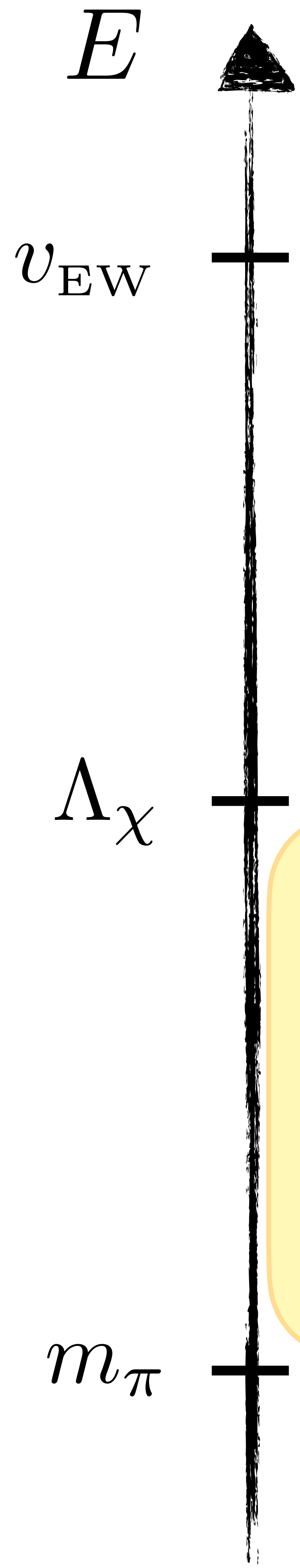
Resonances, e.g. $\Delta(1232)$

$\Lambda_\chi \sim (300 - 750)\,\mathrm{MeV}$

Axion EFTs



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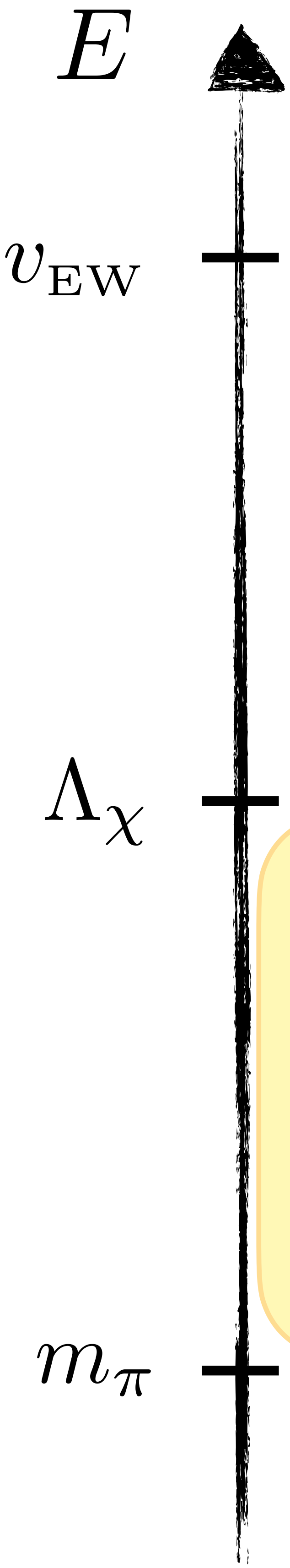
LO: $$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\nabla^\mu U (\nabla_\mu U)^\dagger + (\chi U^\dagger + \mathrm{h.c.})\right]$$

$$U = e^{i\pi^a\tau^a/f_\pi} \qquad \chi = 2BM_a$$

Axion EFTs



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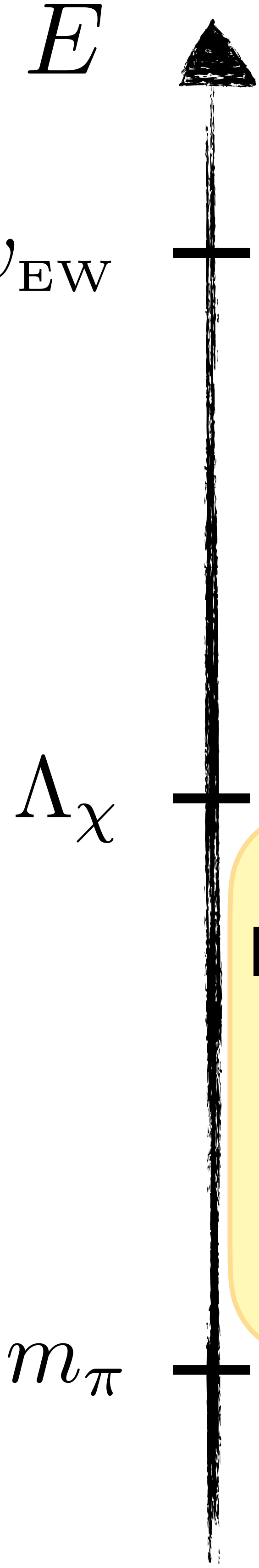
LO: $\hat{\mathcal{L}}_{\pi N}^{(1)} = \bar{N}\left(iv \cdot D + g_A S \cdot u + g_0 S \cdot \hat{u}\right) N$

$$\hat{u}_\mu = c_{u+d}\left(\frac{\partial_\mu a}{f_a}\right) + \ldots \qquad u_\mu = -\left(\frac{\partial_\mu \pi^a}{f_\pi}\right)\tau^a + c_{u-d}\left(\frac{\partial_\mu a}{f_a}\right)\tau_3$$

Axion EFTs



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NLO:

$$\begin{aligned}\hat{\mathcal{L}}_{\pi N}^{(2)} =& \bar{N}\Big[-\frac{1}{2m_N}\left(D^2 - (v\cdot D)^2 + ig_A\{S\cdot D, v\cdot u\} + ig_0\{S\cdot D, v\cdot \hat{u}\}\right) \\ &+ \hat{c}_1\langle\chi_+\rangle + \frac{\hat{c}_2}{2}(v\cdot u)^2 + \hat{c}_3(u\cdot u) + \frac{\hat{c}_4}{2}i\epsilon^{\mu\nu\rho\sigma}\left[u_\mu, u_\nu\right]v_\rho S_\sigma \\ &+ \hat{c}_5\tilde{\chi}_+ + \frac{\hat{c}_8}{4}(v\cdot u)(v\cdot\hat{u}) + \hat{c}_9(u\cdot\hat{u})\Big]N\end{aligned}$$

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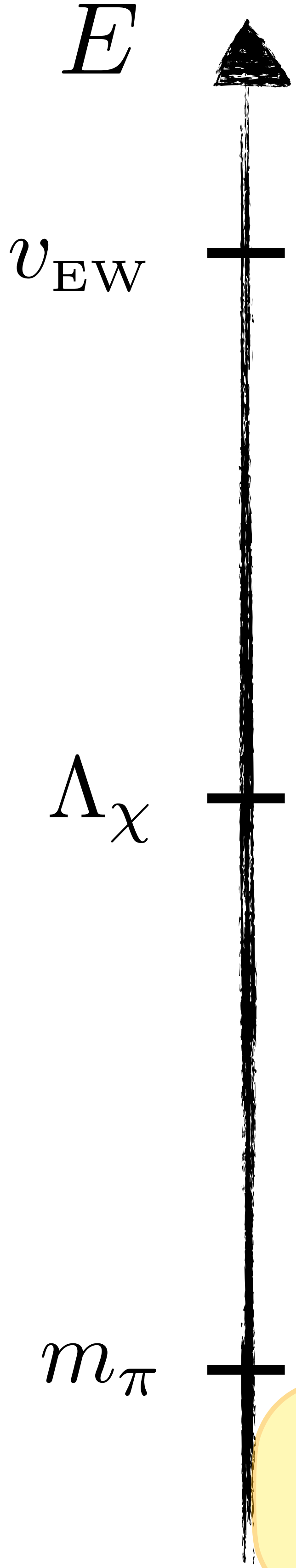

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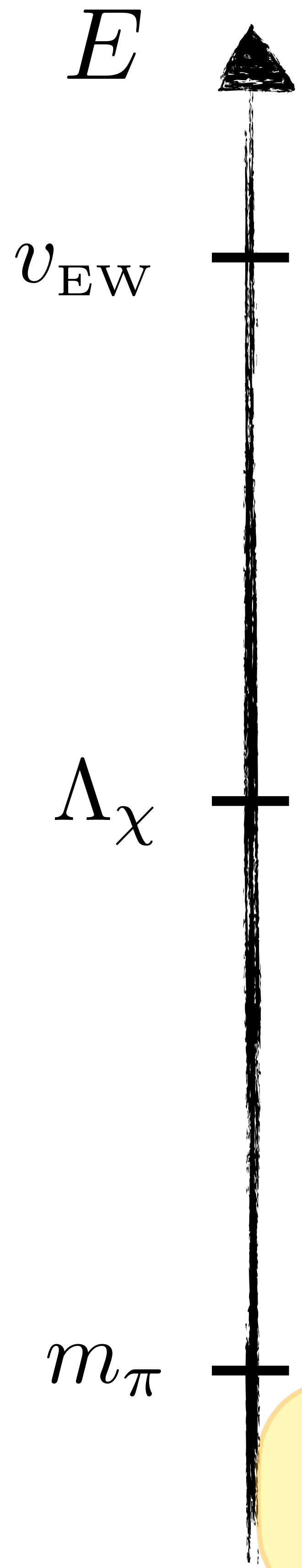
Integrate out pions: theory of baryons and axion

Axion EFTs



$$\mathcal{L} \supset -\bar{q}\left(s - i\gamma_5 p\right) q + \bar{q}\gamma^\mu\gamma_5\left(a_\mu + a_\mu^s\right) q$$

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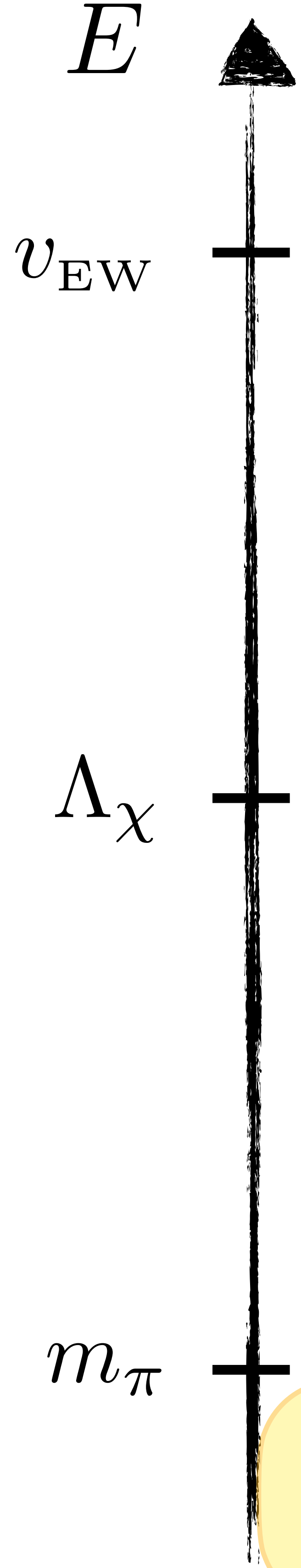
$$\begin{aligned}\hat{\mathcal{L}}^{(2)}_{\pi N} =& \bar{N}\Big[-\frac{1}{2m_N}\left(D^2 - (v\cdot D)^2 + i g_A\{S\cdot D, v\cdot u\} + i g_0\{S\cdot D, v\cdot\hat{u}\}\right)\\ &+ \hat{c}_1\langle\chi_+\rangle + \frac{\hat{c}_2}{2}(v\cdot u)^2 + \hat{c}_3(u\cdot u) + \frac{\hat{c}_4}{2} i\epsilon^{\mu\nu\rho\sigma}\left[u_\mu, u_\nu\right] v_\rho S_\sigma\\ &+ \hat{c}_5\tilde{\chi}_+ + \frac{\hat{c}_8}{4}(v\cdot u)(v\cdot\hat{u}) + \hat{c}_9(u\cdot\hat{u})\Big] N\end{aligned}$$

Match constants $G_A = g_A - \frac{g_A^3 m_\pi^2}{16\pi^2 f_\pi^2} + 4m_\pi^2\bar{d}_{16} + \frac{g_A m_\pi^3}{6\pi f_\pi^2}(2\hat{c}_4 - \hat{c}_3)$

Axion EFTs



$$\mathcal{L} \supset -\bar{q}\left(s - i\gamma_5 p\right)q + \bar{q}\gamma^\mu\gamma_5\left(a_\mu + a_\mu^s\right)q$$

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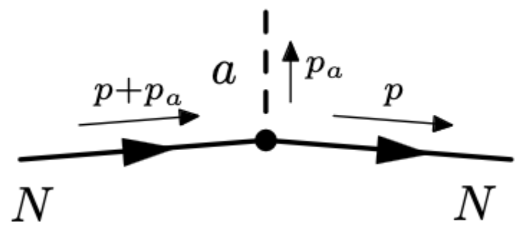
$$\begin{aligned}\hat{\mathcal{L}}^{(2)}_{\pi N} =& \bar{N}\Big[-\frac{1}{2m_N}\left(D^2 - (v\cdot D)^2 + ig_A\{S\cdot D, v\cdot u\} + ig_0\{S\cdot D, v\cdot \hat{u}\}\right)\\ &+ \hat{c}_1\left\langle \chi_+\right\rangle + \frac{\hat{c}_2}{2}(v\cdot u)^2 + \hat{c}_3(u\cdot u) + \frac{\hat{c}_4}{2}i\epsilon^{\mu\nu\rho\sigma}\left[u_\mu, u_\nu\right]v_\rho S_\sigma\\ &+\hat{c}_5\tilde{\chi}_+ + \frac{\hat{c}_8}{4}(v\cdot u)(v\cdot\hat{u}) + \hat{c}_9(u\cdot\hat{u})\Big]N\end{aligned}$$

$$\mathcal{L}_{aN} = \bar{N}\left[iv\cdot\partial + \frac{S\cdot\partial a}{f_a}\left(G_A c_{u-d}\tau^3 + G_0 c_{u+d}\mathbf{1}\right) + \sigma\left\langle \mathrm{Re}\left(M_a\right)\right\rangle + \ldots\right]N$$

Axion-Nuclon Coupling: Loop corrections

Corrections to the coupling can be calculated systematically in $\left(\frac{p}{4\pi f_\pi}\right)^\nu$

LO:



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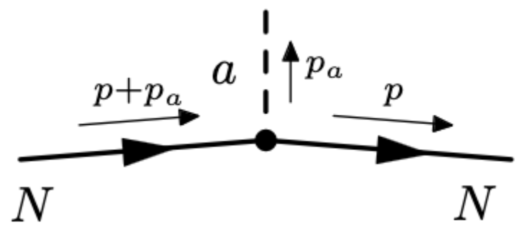
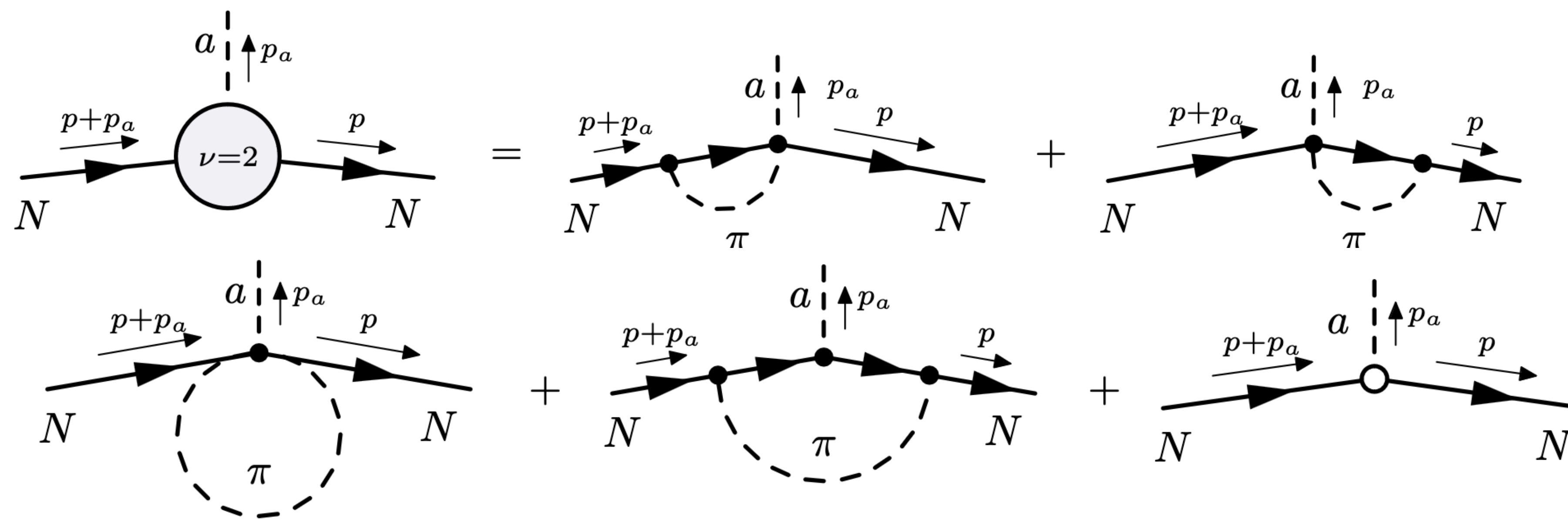


NLO:

$$\left(\frac{p}{4\pi f_\pi}\right)^2$$



Vonk, Guo, Meißner (2000)

Axion-Nuclon Coupling: Loop corrections

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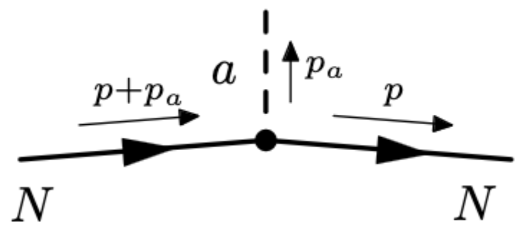
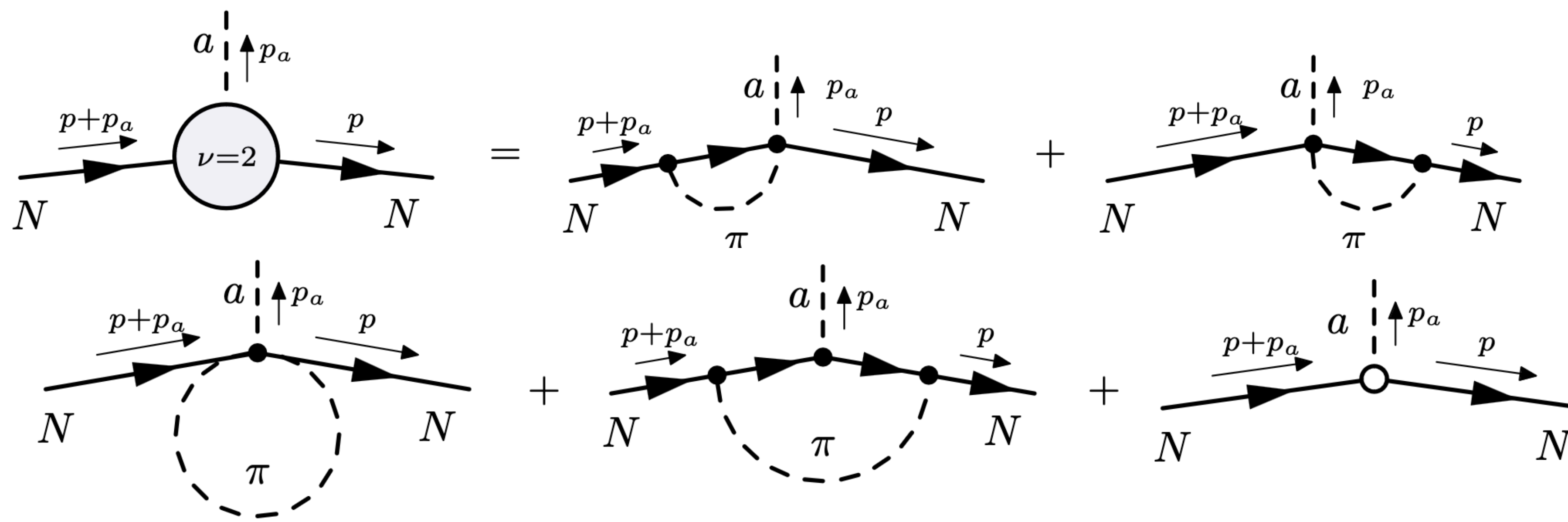
LO:



NLO:

$\left(\frac{p}{4\pi f_\pi}\right)^2$

Vonk, Guo, Meißner (2000)



NNLO:

Naively suppressed by $\left(\frac{p}{4\pi f_\pi}\right)^3$

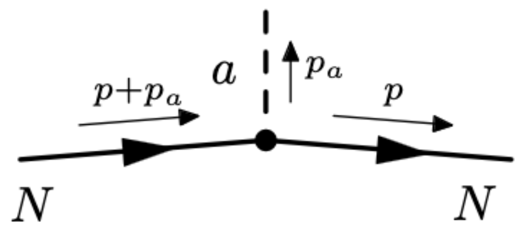
But low-lying $\Delta(1232)$ resonance enhances contribution $\left(\frac{p}{4\pi f_\pi}\right)^2 \left(\frac{p}{\Lambda_\chi}\right)$

$\Lambda_\chi \sim (300 - 750)\,\mathrm{MeV}$

Axion-Nuclon Coupling: Loop corrections

Corrections to the coupling can be calculated systematically in $\left(\frac{p}{4\pi f_\pi}\right)^\nu$

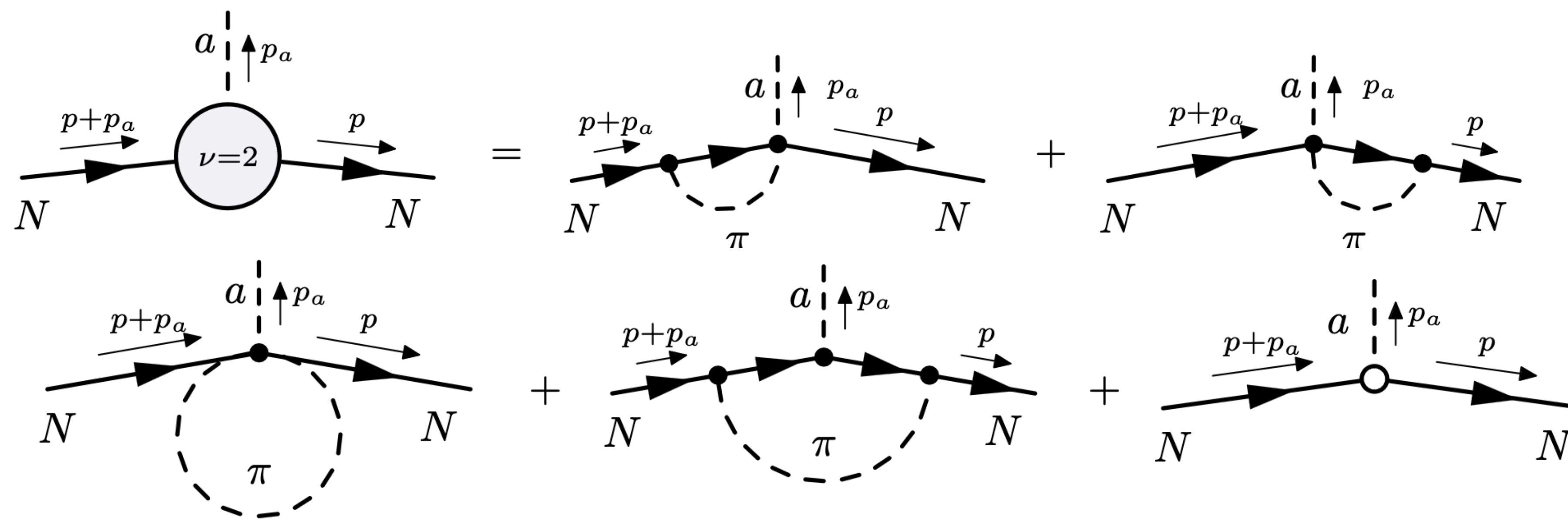
LO:



NLO:

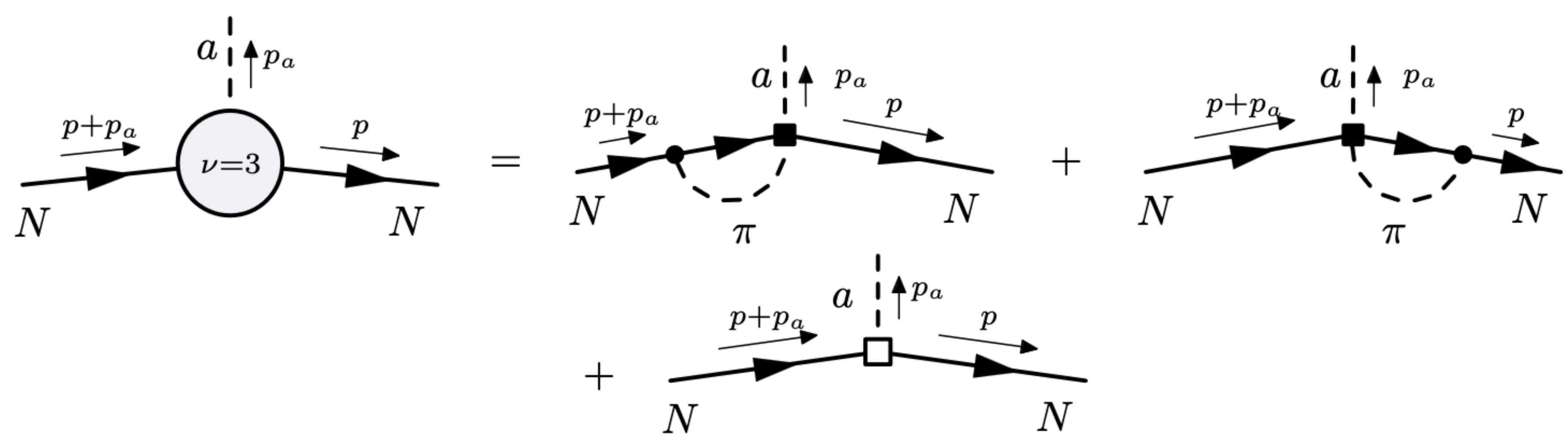
$$\left(\frac{p}{4\pi f_\pi}\right)^2$$

Vonk, Guo, Meißner (2000)



NNLO:

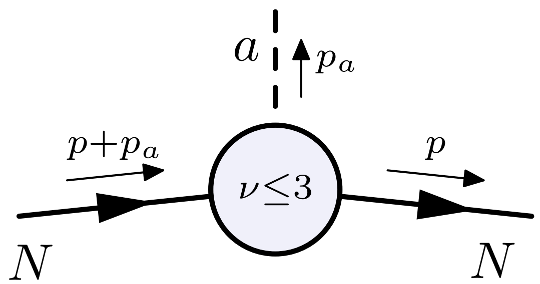
$$\left(\frac{p}{4\pi f_\pi}\right)^2 \left(\frac{p}{\Lambda_\chi}\right)$$



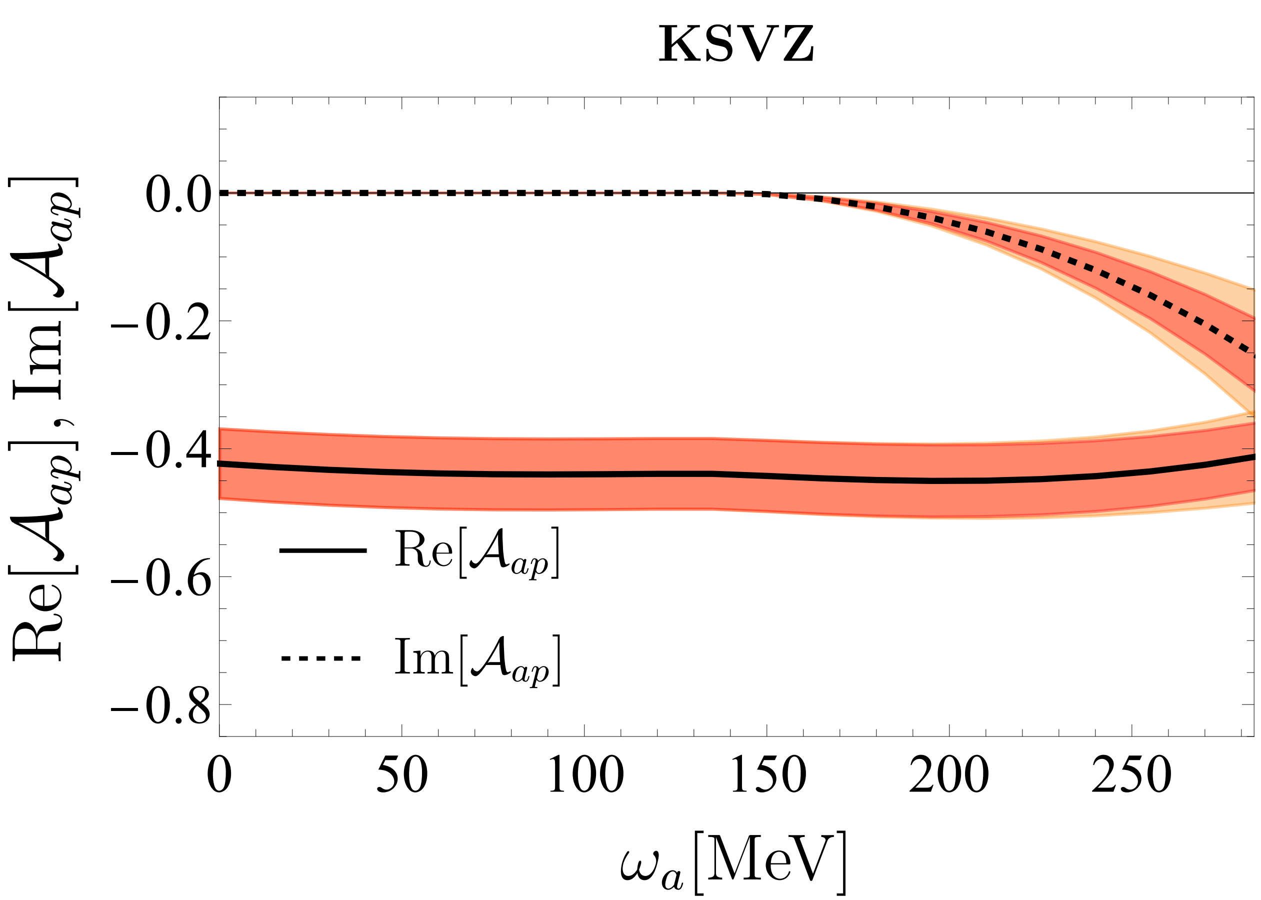
$$\Lambda_\chi \sim (300 - 750)\,\mathrm{MeV}$$

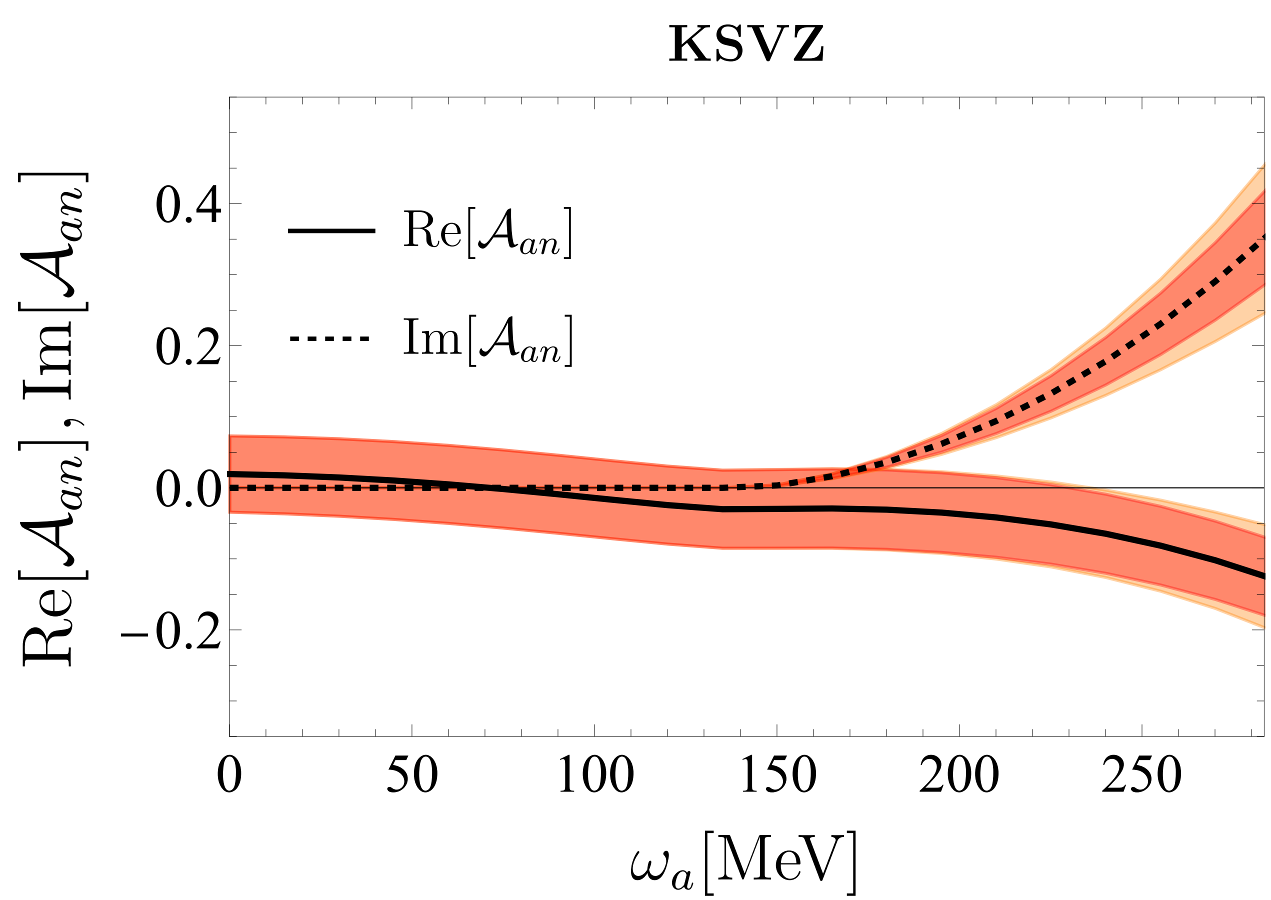
Axion-Nuclon Coupling: Loop corrections

Coupling depends on the axion energy! Can be written as



$$= -\frac{1}{f_a}\mathcal{A}^{\nu\leq 3}(\omega_a)\, S\cdot p_a - \frac{1}{f_a}\mathcal{B}^{\nu\leq 3}(\omega_a)\, S\cdot p$$





Axion-Nuclon Coupling: Loop corrections

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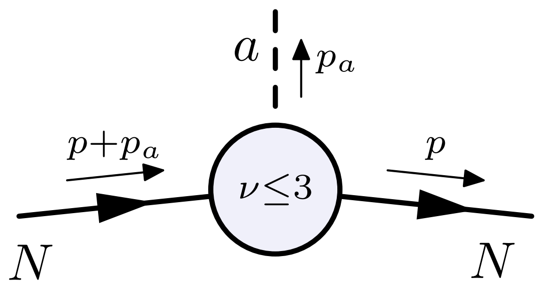

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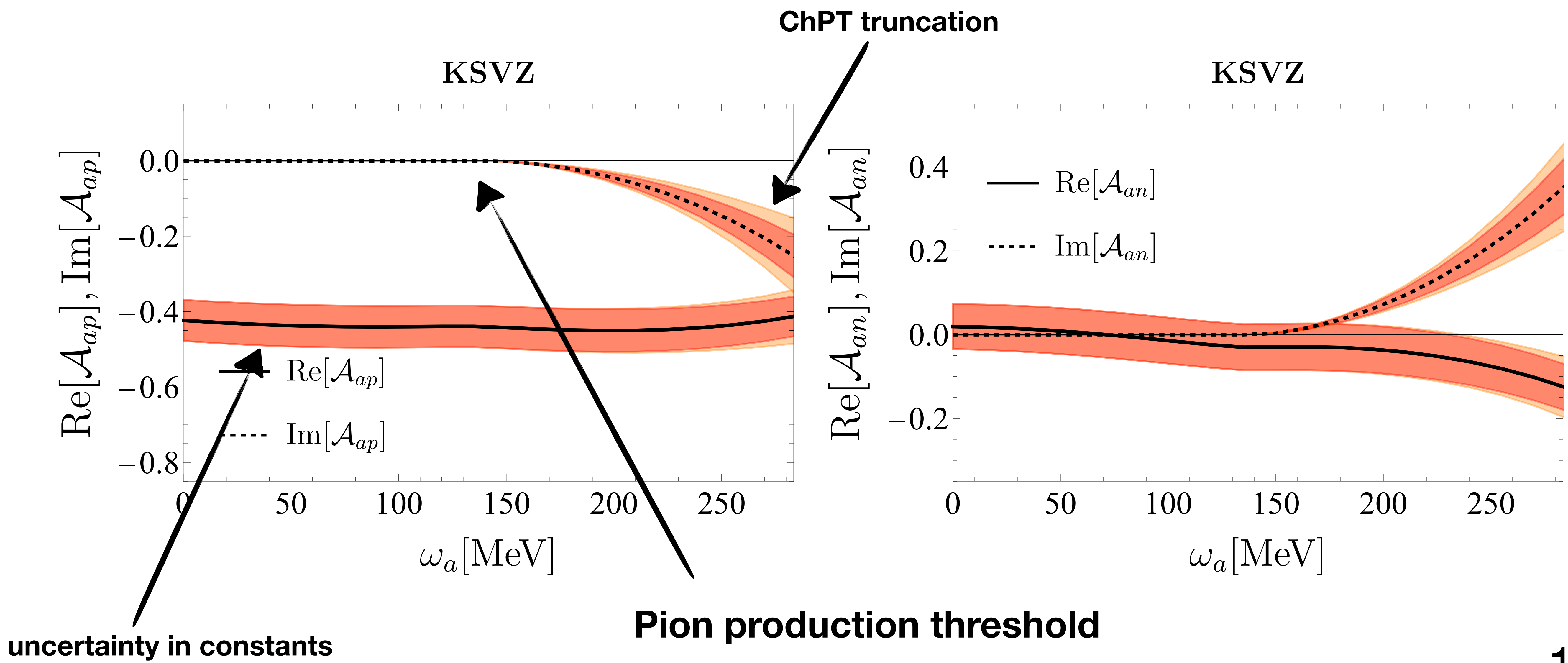
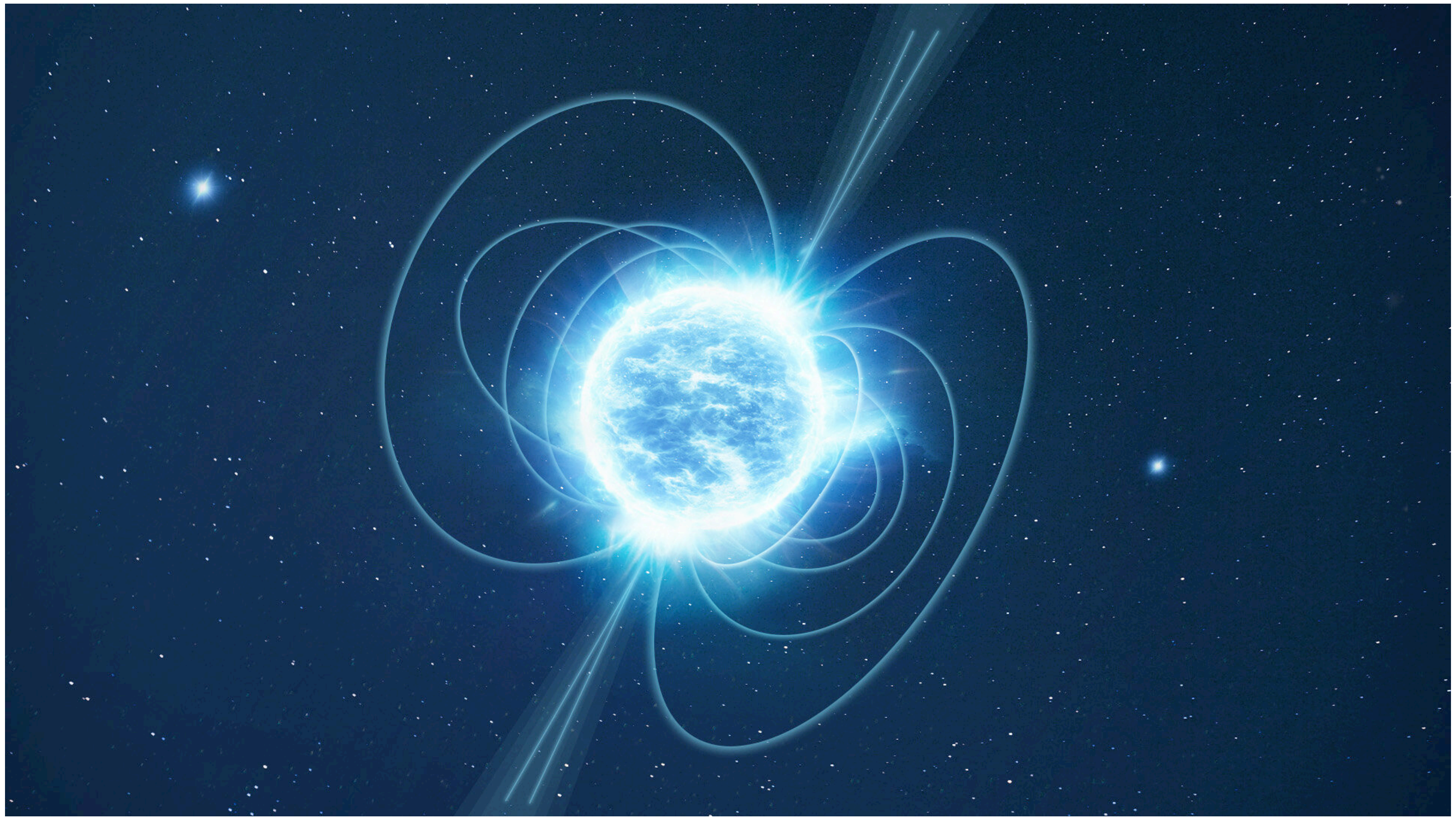


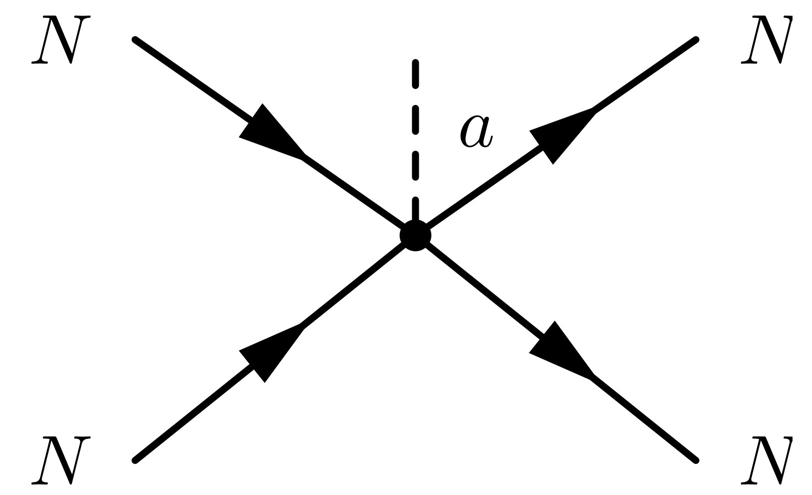
How does a density background change these couplings?



Axion-Nuclon Coupling: Finite density

- **Schematic example:**

$$\mathcal{L}^{(2)}_{\pi NN} = \frac{c_D}{2f_\pi^2\Lambda_\chi}(\bar{N}N)(\bar{N}S\cdot uN)$$



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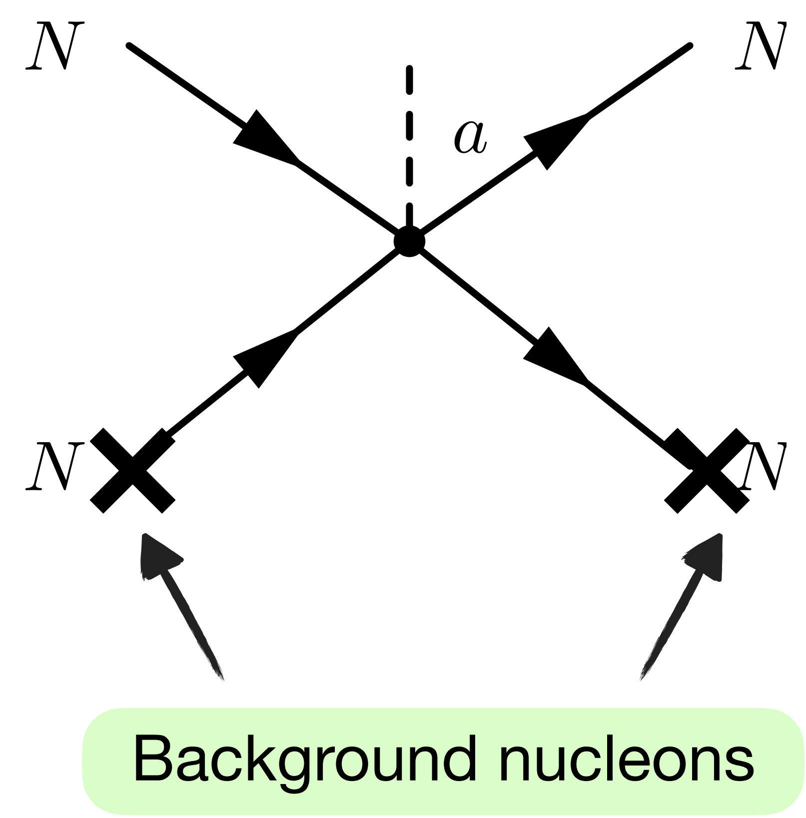


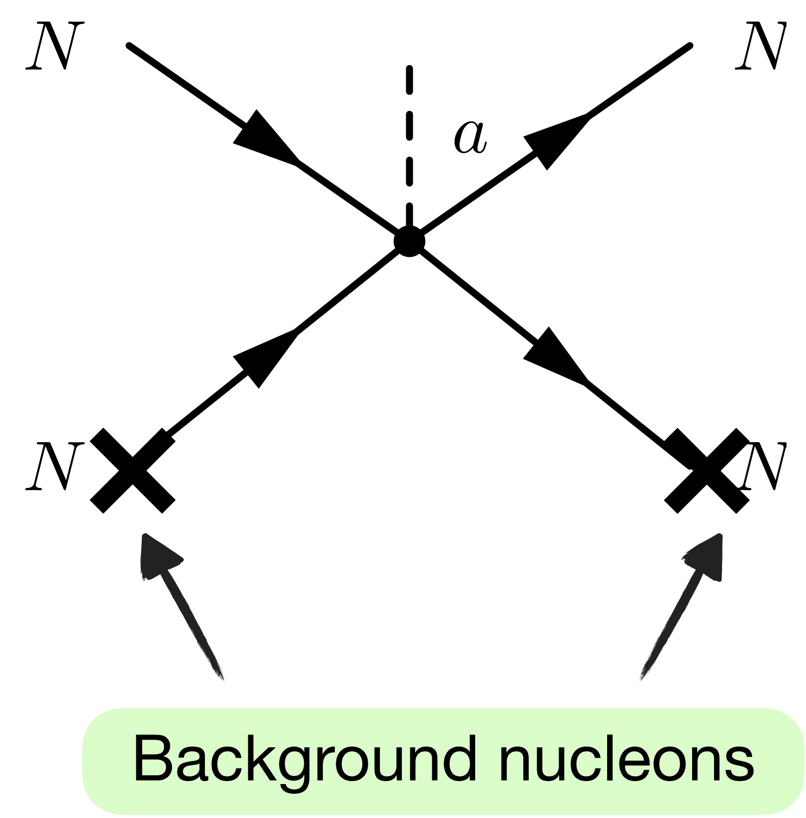
Axion-Nuclon Coupling: Finite density

- **Schematic example:**

$$\mathcal{L}^{(2)}_{\pi NN} = \frac{c_D}{2 f_\pi^2 \Lambda_\chi} (\bar{N}N)(\bar{N}S \cdot uN)$$

$$\langle \bar{N}N \rangle = n$$

Number density



- **Gives contribution to coupling:** $\sim \dfrac{k_f^3}{(4\pi f_\pi)^2 \Lambda_\chi}$

Axion-Nuclon Coupling: Finite density

- **Schematic example:**

$$\mathcal{L}^{(2)}_{\pi NN} = \frac{c_D}{2 f_\pi^2 \Lambda_\chi} (\bar{N}N)(\bar{N} S \cdot u N)$$

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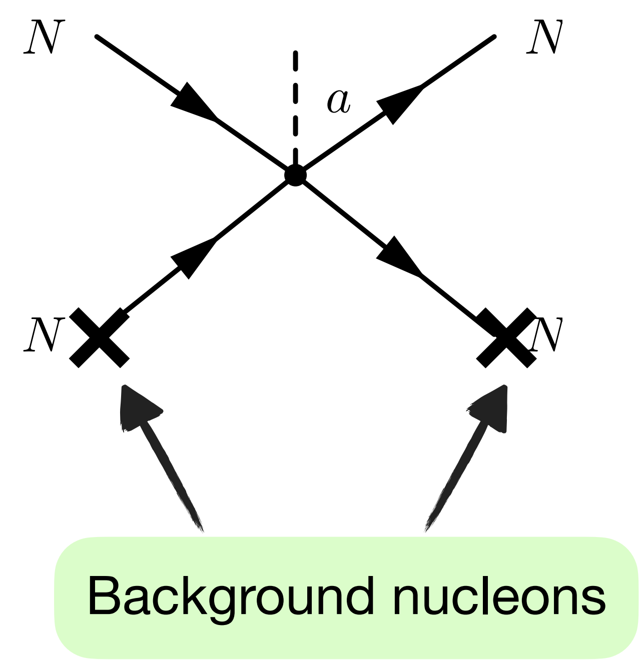
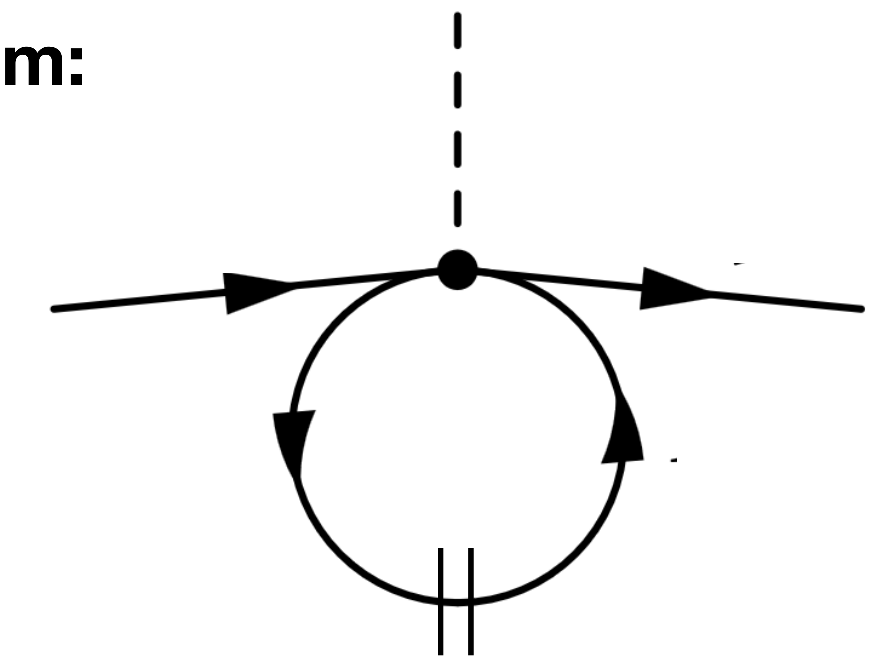
Number density



- **Systematically via QFT in Real-Time Formalism:**

Nucleon propagator at finite density

$$iG(k) = \frac{i}{k^0 + i\epsilon} - 2\pi\delta(k^0)\theta(k_f - |\vec{k}|)$$



Furnstahl, Serot ('91)
Ghosh, Grossman, Tangarife, Zu, Yu ('22)

Axion-Nuclon Coupling: Finite density

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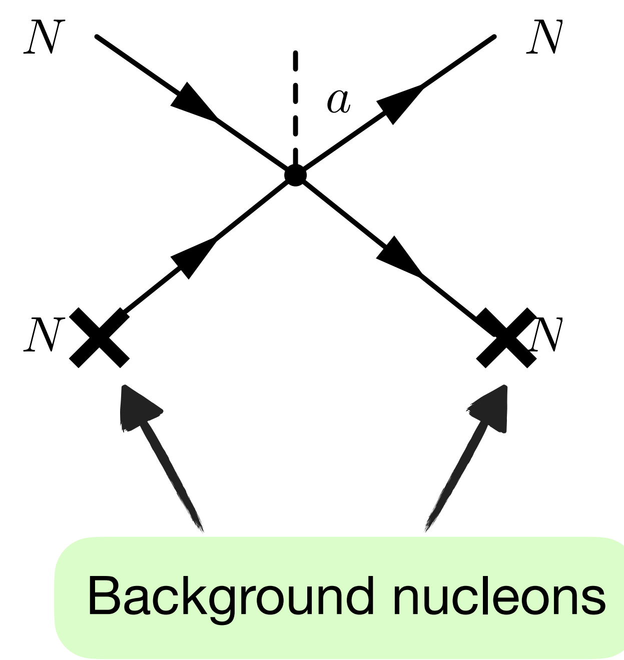
Number density



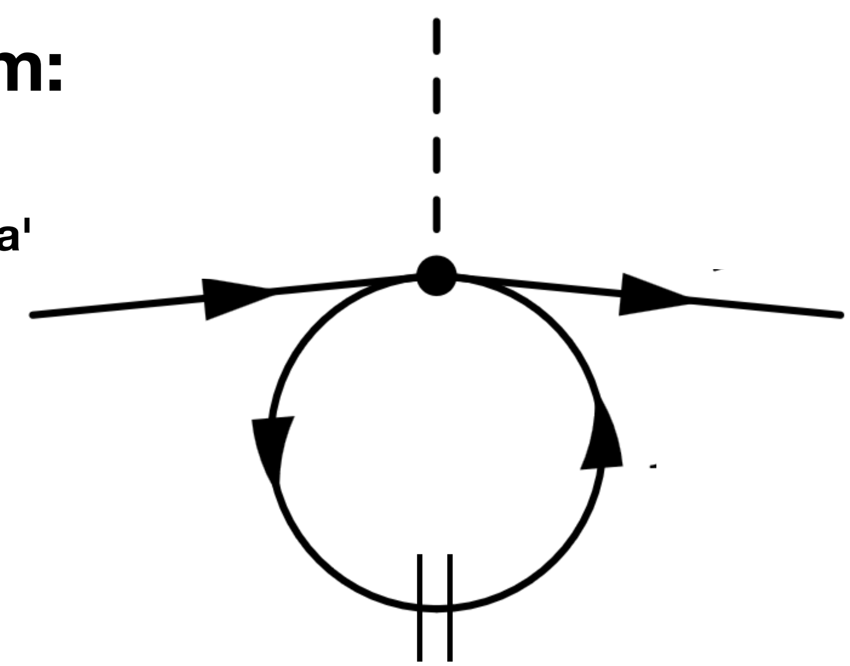
- **Systematically via QFT in Real-Time Formalism:**

Nucleon propagator at finite density

Filled 'Fermi sea'

$$iG(k) = \frac{i}{k^0 + i\epsilon} - 2\pi\delta(k^0)\theta(k_f - |\vec{k}|)$$

NR fermion propagator

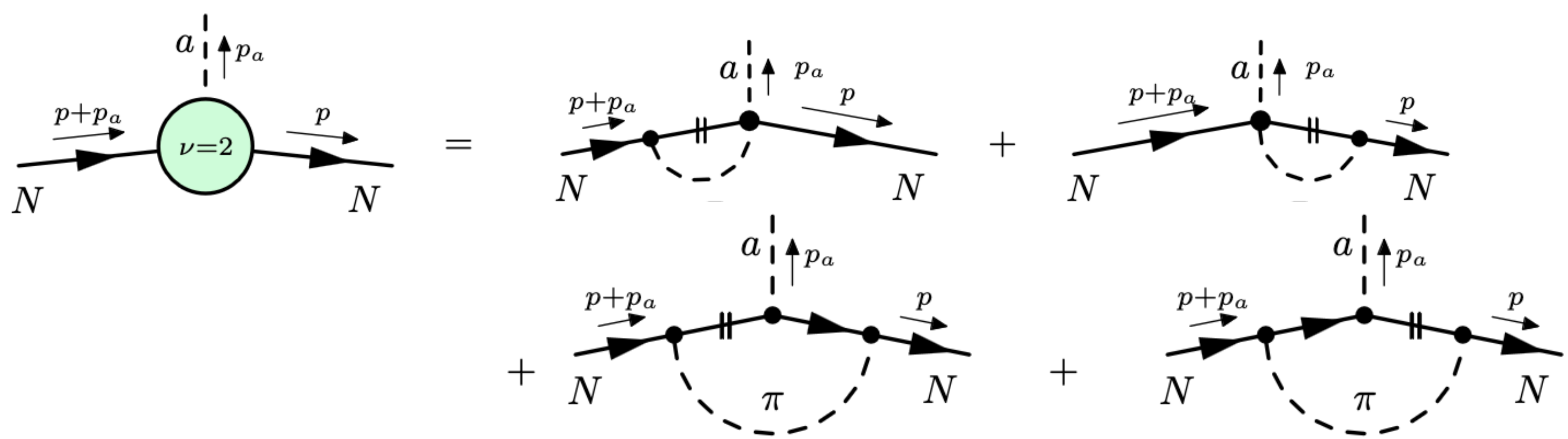


Furnstahl, Serot ('91)
Ghosh, Grossman, Tangarife, Zu, Yu ('22)

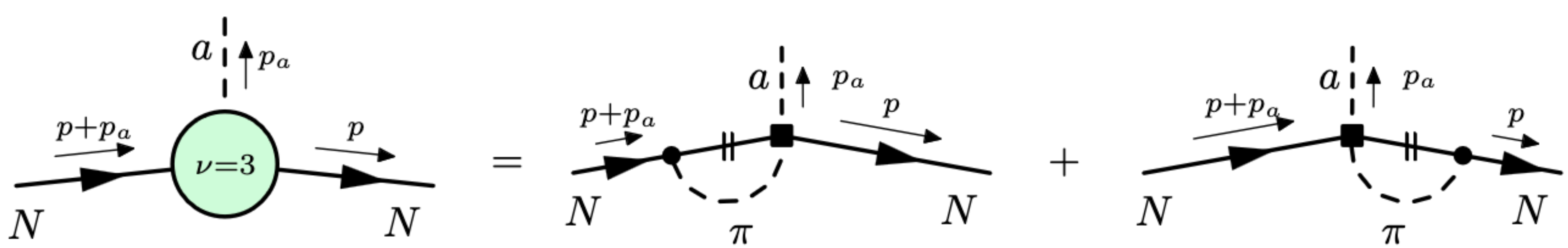
Axion-Nuclon Coupling: Finite density

Get corrections systematically $\left(\frac{p}{4\pi f_\pi}\right)^\nu \to \left(\frac{k_f}{4\pi f_\pi}\right)^\nu$

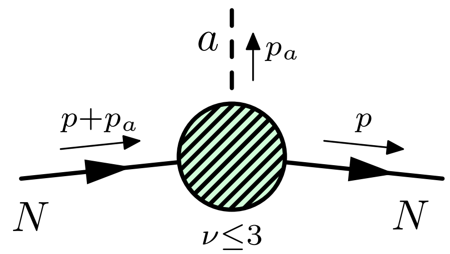
$$\left(\frac{k_f}{4\pi f_\pi}\right)^2$$



$$\left(\frac{k_f}{4\pi f_\pi}\right)^2 \left(\frac{k_f}{\Lambda_\chi}\right)$$

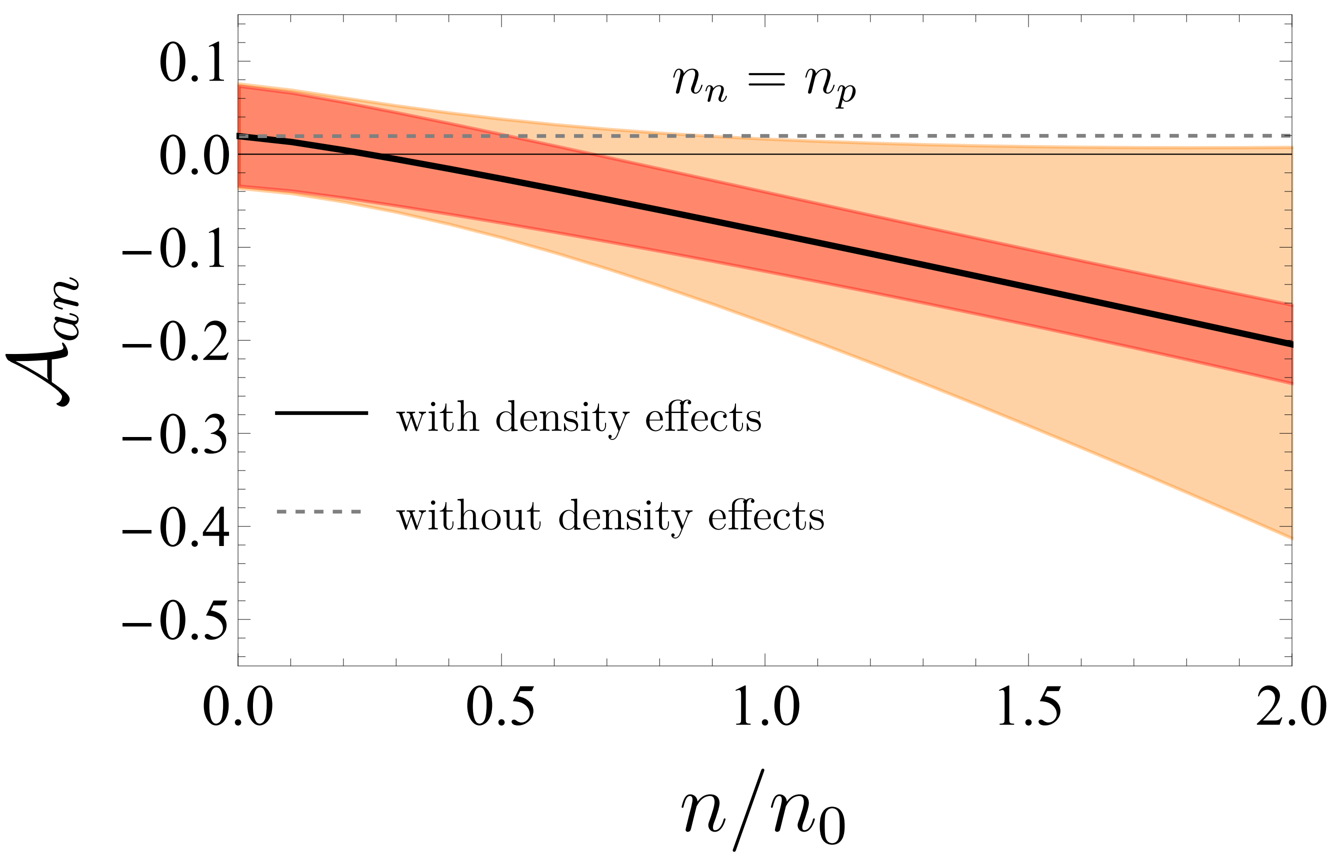


Axion-Nuclon Coupling: Finite density

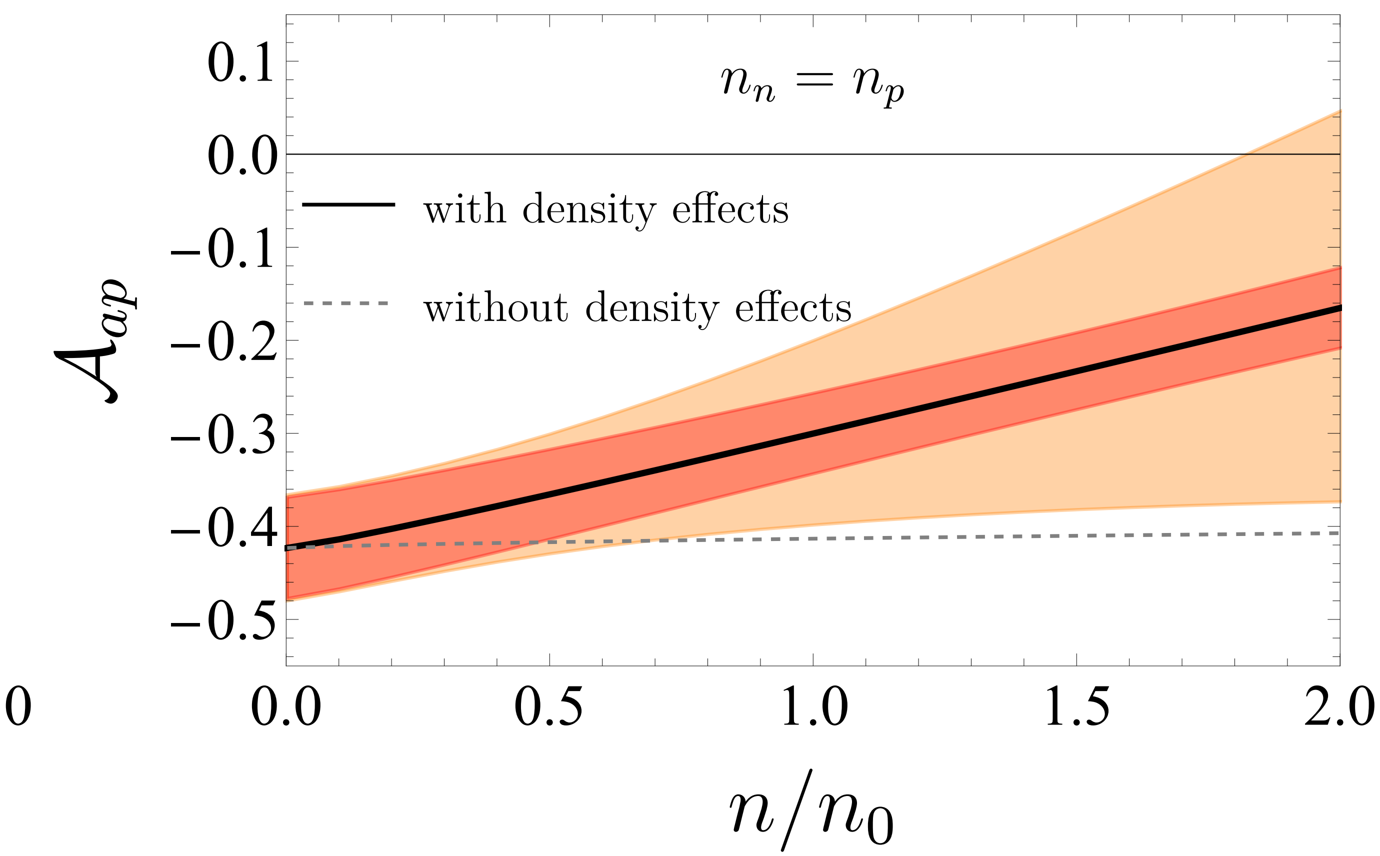


$$= -\frac{1}{f_a}\mathcal{A}^{\nu\leq 3}(k_f, p_a)\, S\cdot p_a - \frac{1}{f_a}\mathcal{B}^{\nu\leq 3}(k_f, p_a)\, S\cdot p$$

Neutron



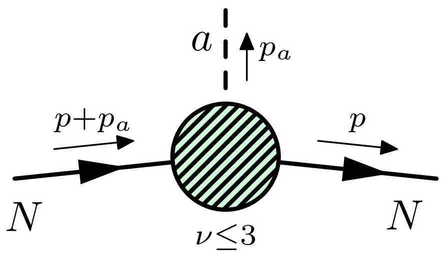
Proton



At finite density $\mathcal{A}_{an}^{\mathrm{KSVZ}}(n_0) = -0.1(4)(9)$

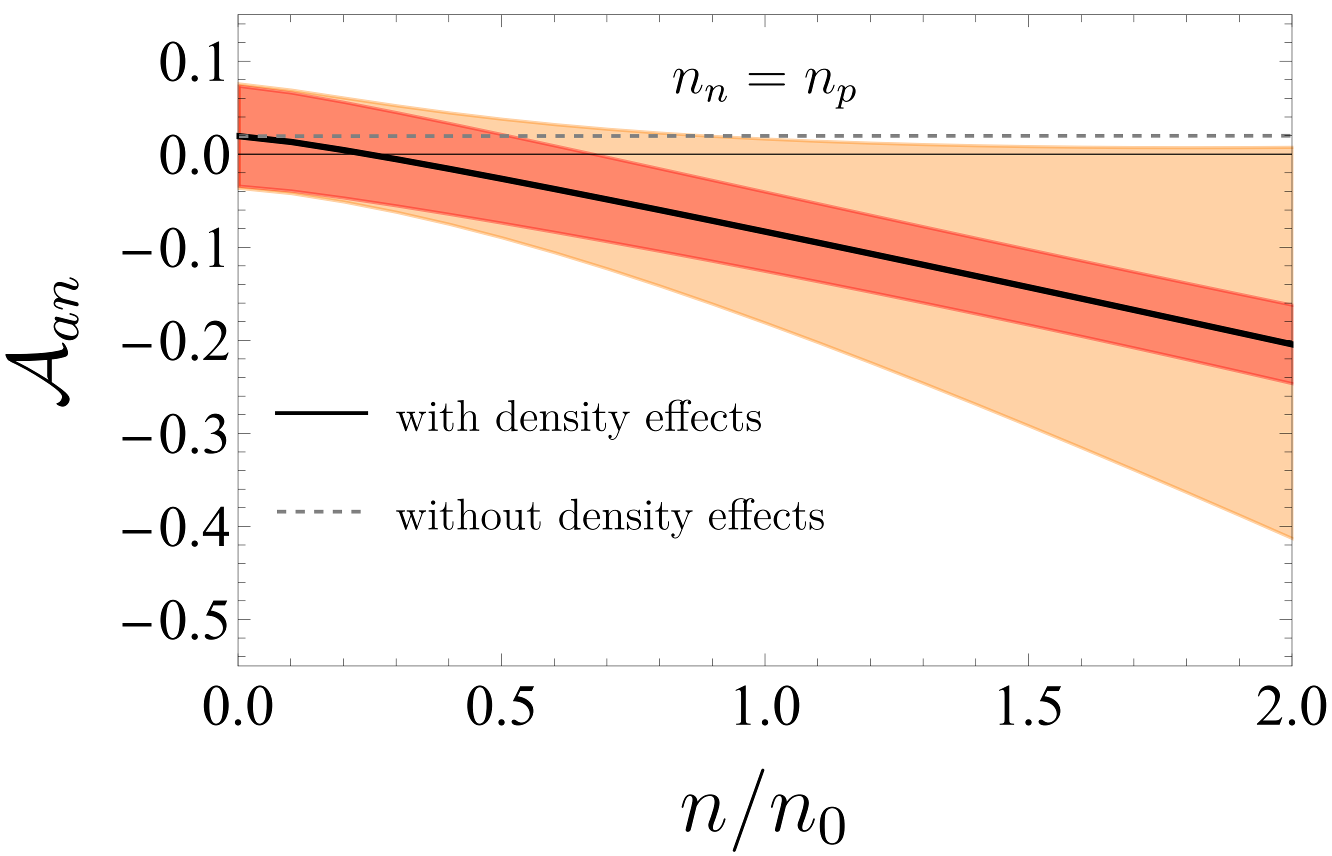
vs. vacuum $\mathcal{A}_{an}^{\mathrm{KSVZ}}(0) = 0.02(5)$

Axion-Nuclon Coupling: Finite density

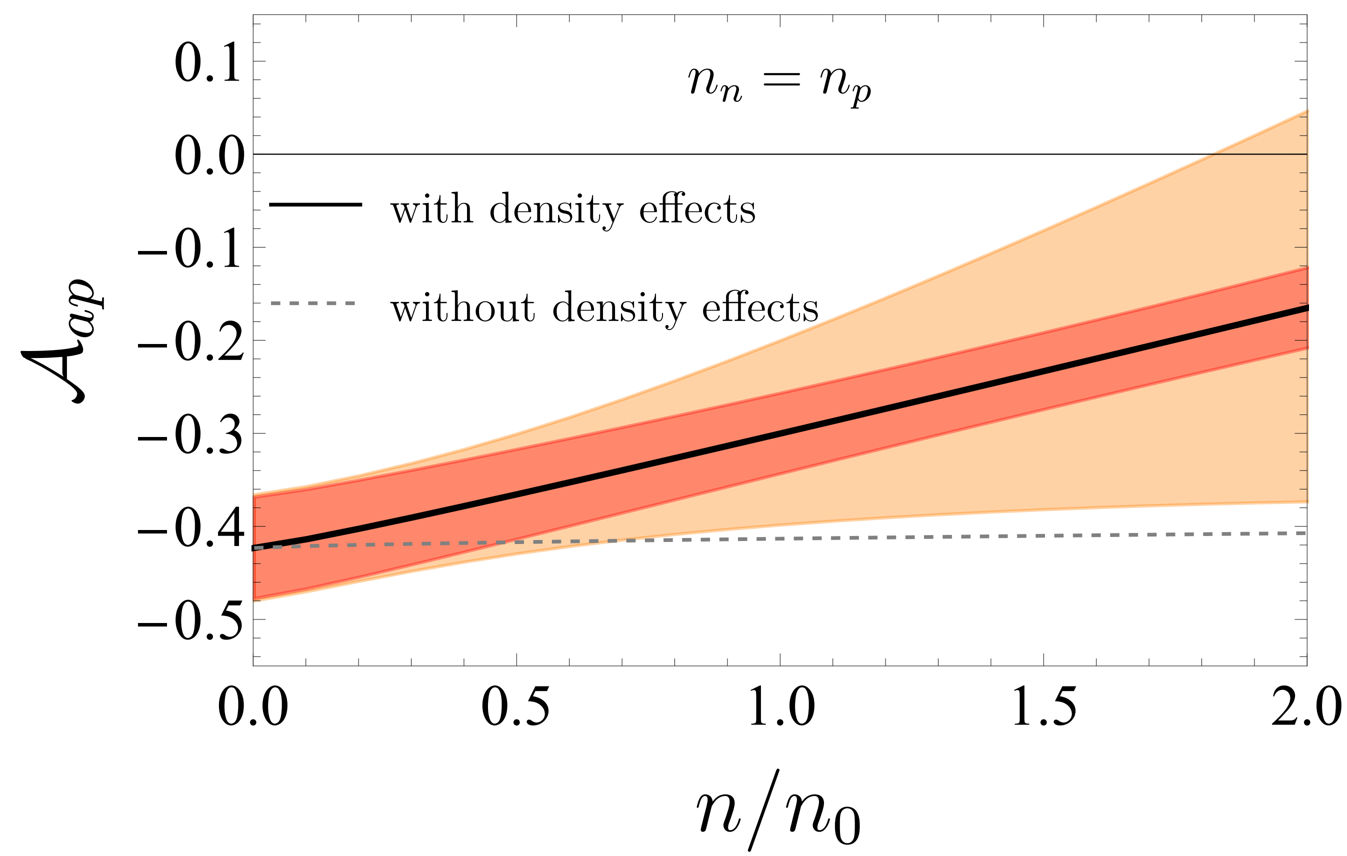


$$= -\frac{1}{f_a}\mathcal{A}^{\nu\leq 3}(k_f, p_a)\, S\cdot p_a - \frac{1}{f_a}\mathcal{B}^{\nu\leq 3}(k_f, p_a)\, S\cdot p$$

Neutron



Proton

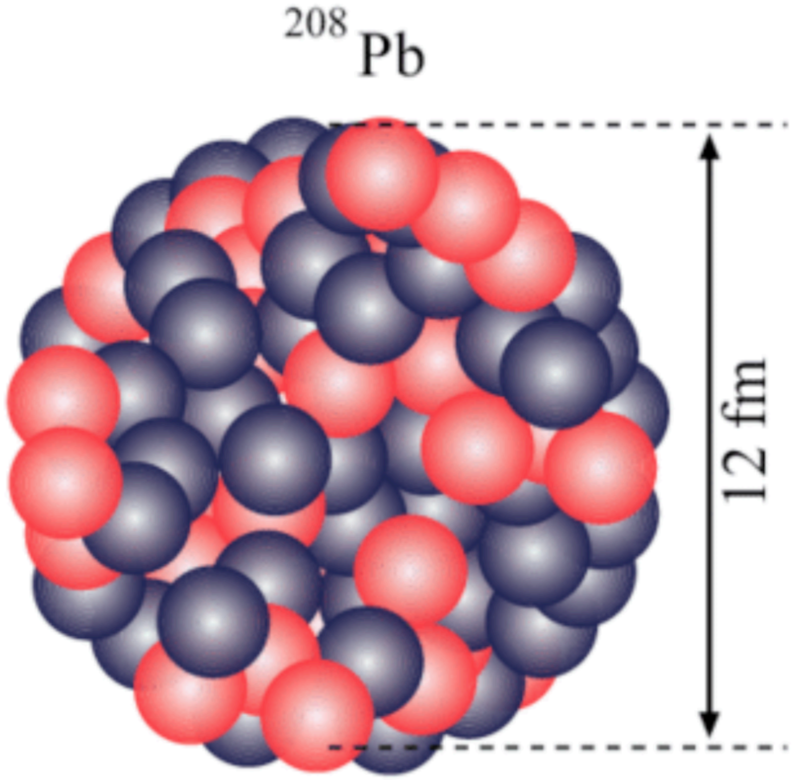


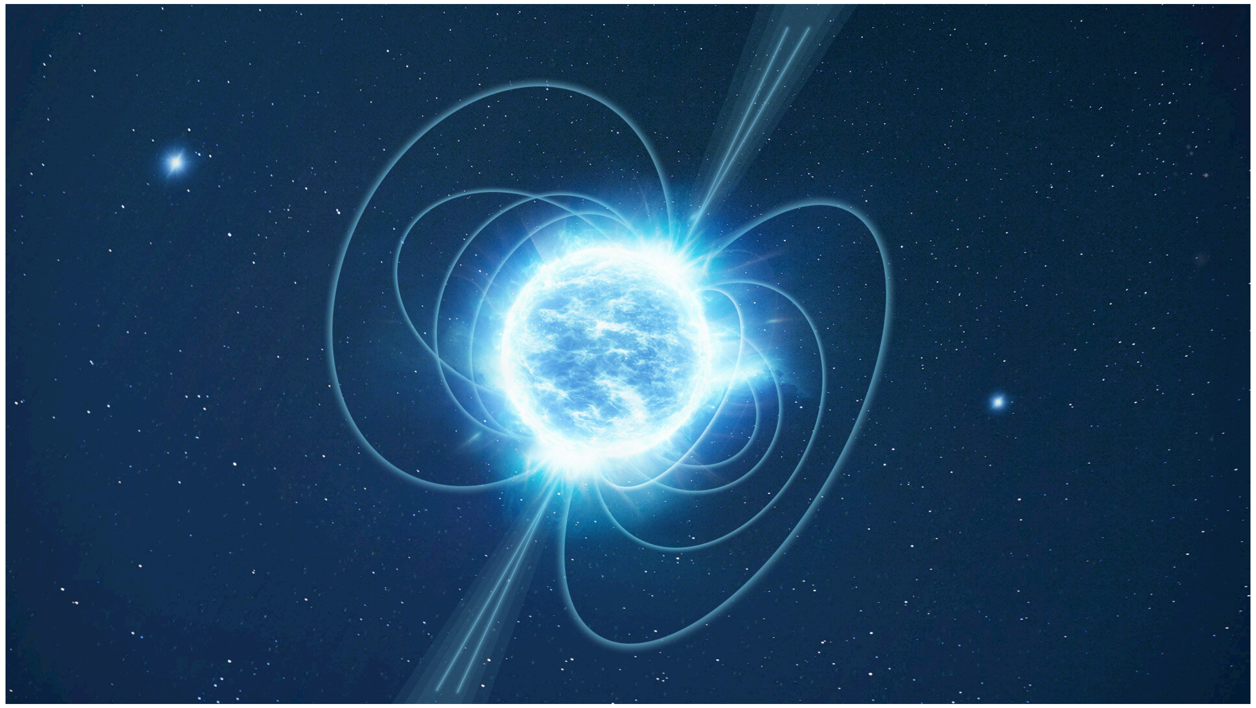
At finite density $\mathcal{A}_{an}^{\mathrm{KSVZ}}(n_0) = -0.1(4)(9)$

vs. vacuum $\mathcal{A}_{an}^{\mathrm{KSVZ}}(0) = 0.02(5)$

Accidental cancellation is lifted!

Implications for phenomenology





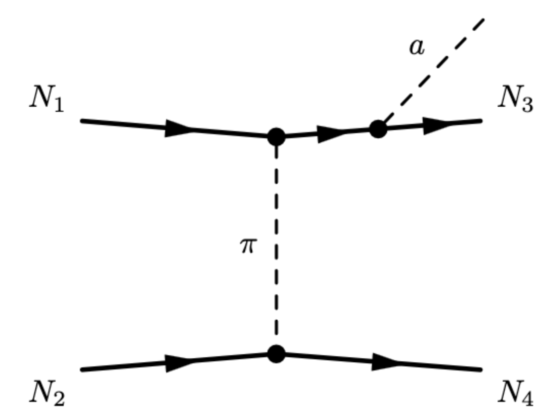
Supernova bound revisited

- Axion Luminosity

$$L_a = \int dr 4\pi r^2 \dot{\epsilon}_a(r)$$

With emissivity

$$\dot{\varepsilon}_a = \int \prod_{i=1}^{4} d\Pi_i d\Pi_a (2\pi)^4 S|\mathcal{M}|^2 \delta^{(4)} \left(\sum_i p_i - p_a\right) E_a f_1 f_2 (1 - f_3)(1 - f_4)$$



- Typically 1 pion exchange at tree level + pheno corrections

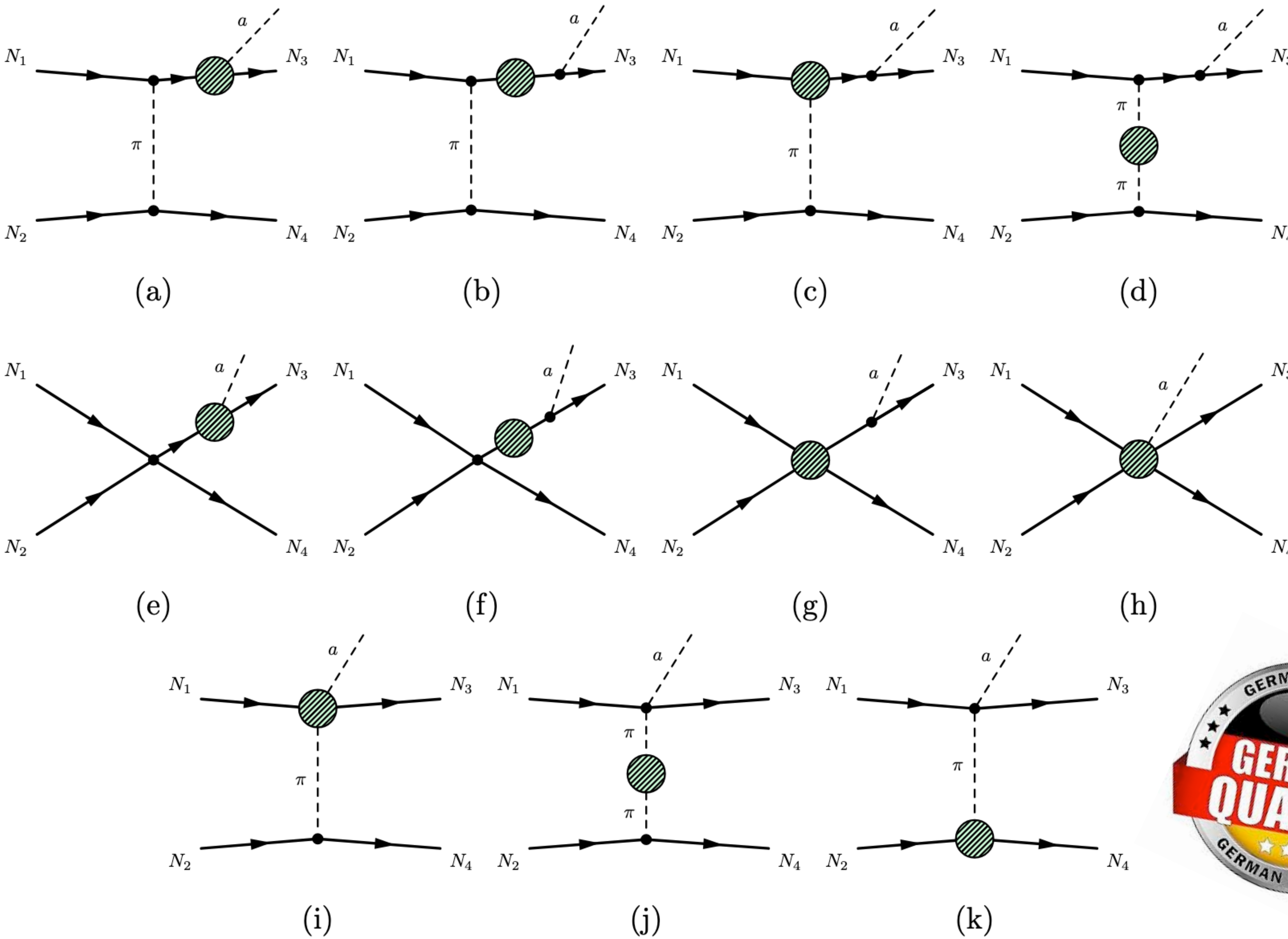
Chang, Essig, McDermott ('18) Carenza, Fischer, Giannotti, Guo, Martinez-Pinedo, Mirizzi ('19)

- **Outlined all relevant corrections diagramatically up to NNLO in chiral expansion**

Allows to systematically account for all effects from first principles!

Supernova bound revisited

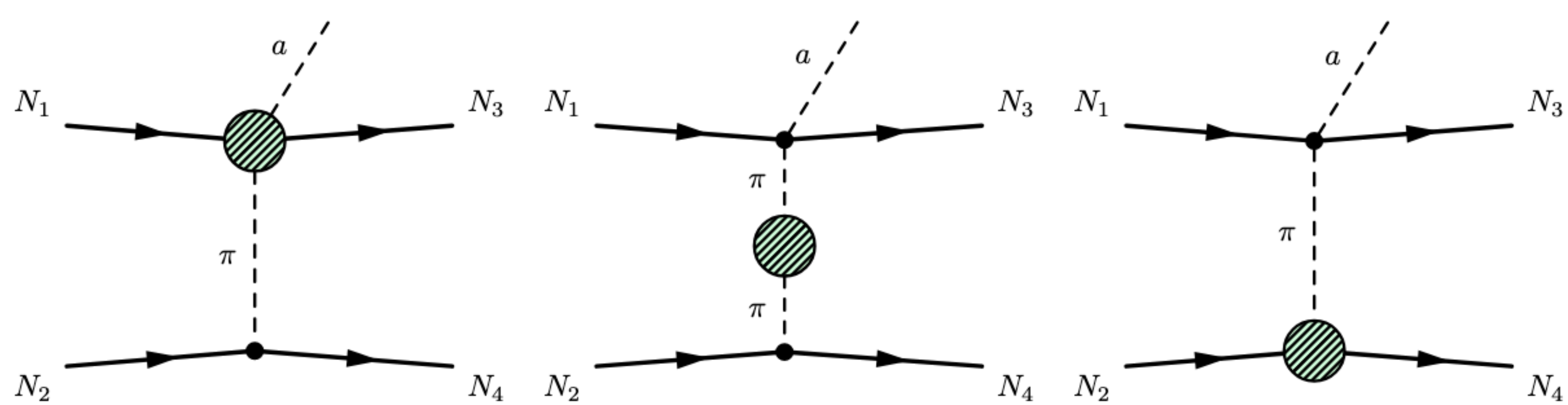
Relevant diagrams up to NLO



Supernova bound revisited

These are typically suppressed by $v \cdot k \simeq \dfrac{k^2}{2m_N}$

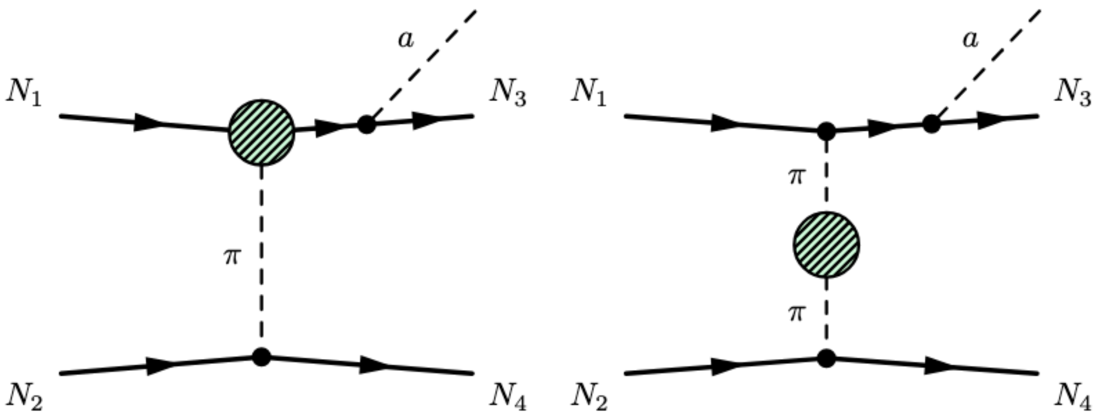
Choi, Kim, Seong, Shin ('21)



(i) (j) (k)

Supernova bound revisited

Neglected



(c) (d)

Modification of nuclear interaction:

- Fudge factor γ_p

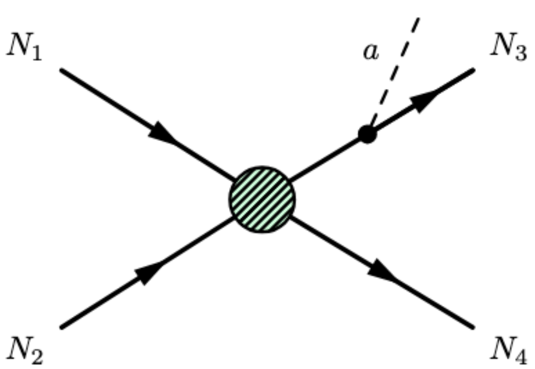
  Chang, Essig, McDermott ('18)

- Phenomenologically modelled

  Ericson, T., & Mathiot, J.-F. 1989, Phys. Lett. B, 219, 507

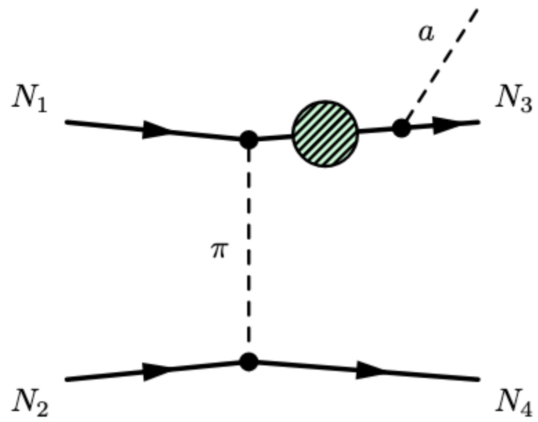
  Hannestad, Raffelt *Astrophys.J.* 507 (1998) 339-352

  Carenza, Fischer, Giannotti, Guo, Martinez-Pinedo, Mirizzi ('19)

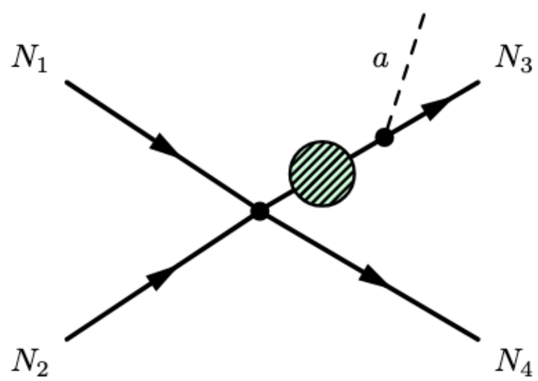


(g)

Supernova bound revisited



(b)



(f)

Modelled as nucleon re-scatterings

- Fudge factor γ_h

  Raffelt, Seckel ('88)

  Chang, Essig, McDermott ('18)

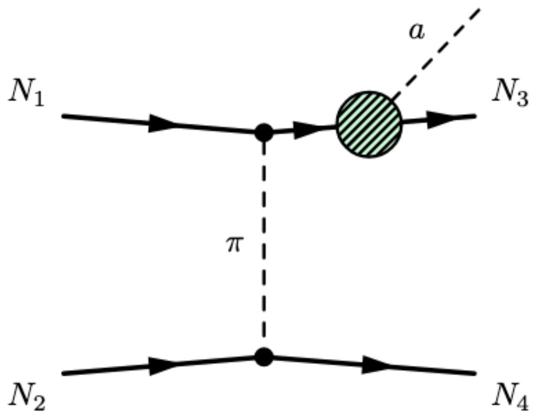
- Phenomenologically

  Raffelt, Seckel ('88)

  Carenza, Fischer, Giannotti, Guo, Martinez-Pinedo, Mirizzi ('19)

Neglected

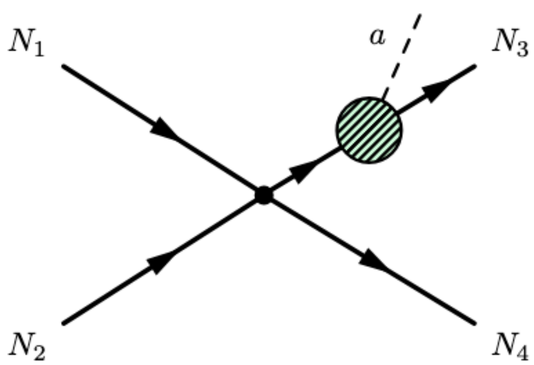
Supernova bound revisited



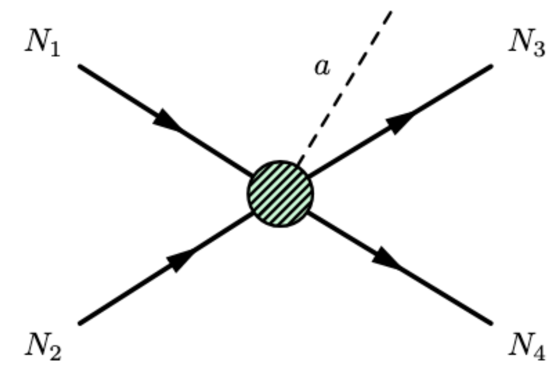
(a)

Outlined for the first time

KS, Stadlbauer, Stelzl, Weiler ('24)

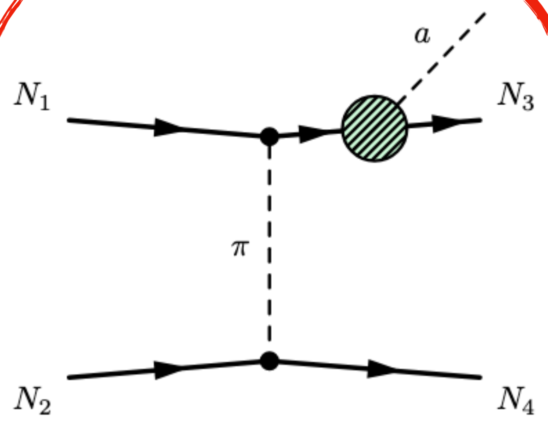


(e)

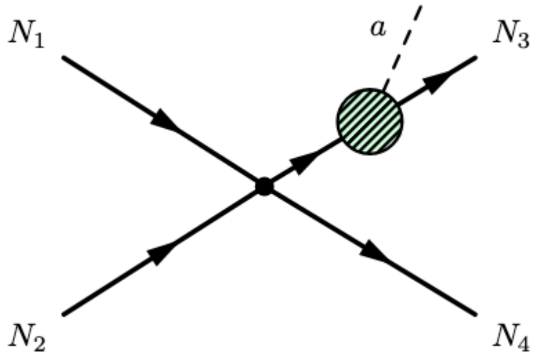


(h)

Supernova bound revisited



(a)

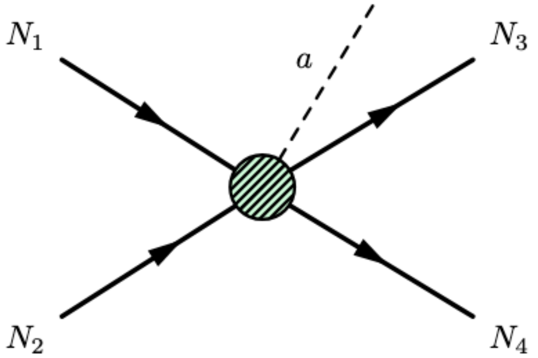


(e)

Outlined for the first time

KS, Stadlbauer, Stelzl, Weiler ('24)

Modified couplings



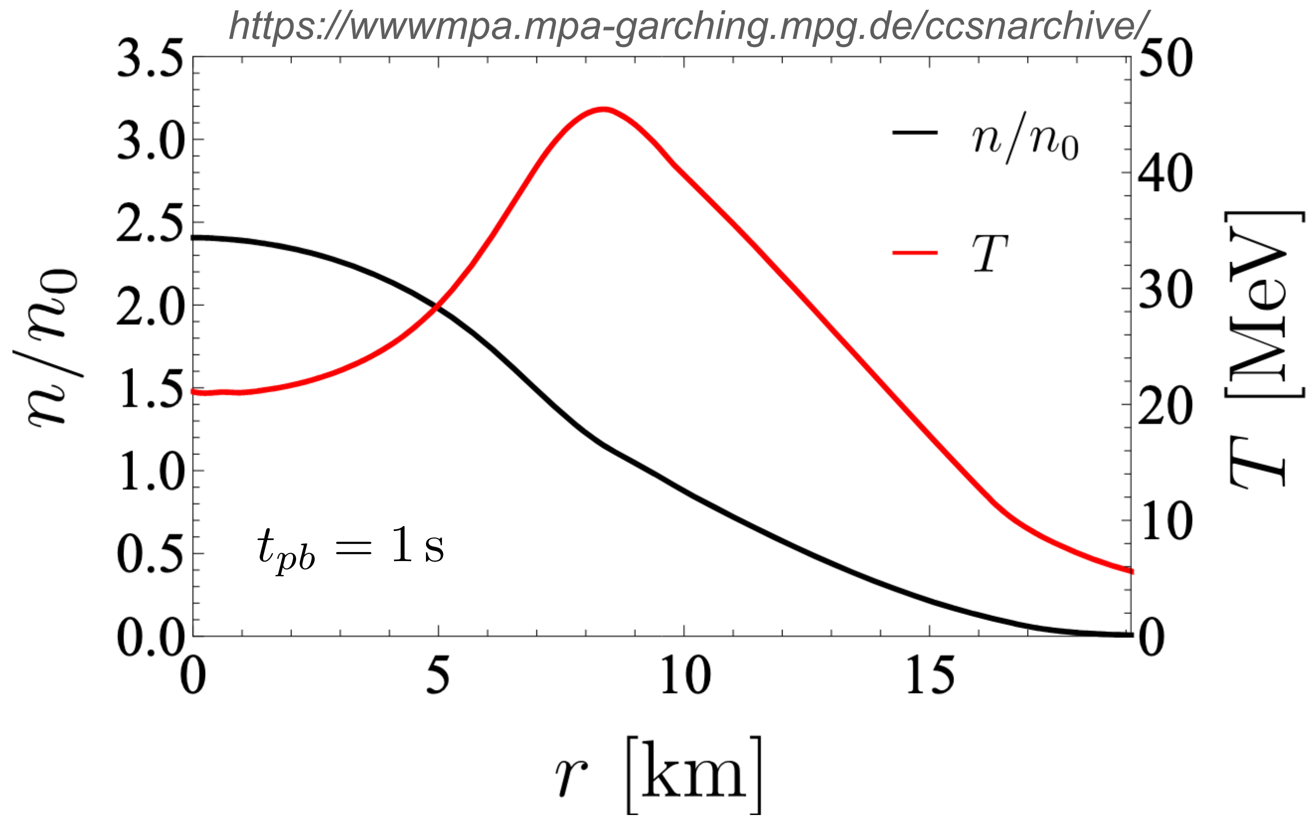
(h)

Focus on these for now.. but:

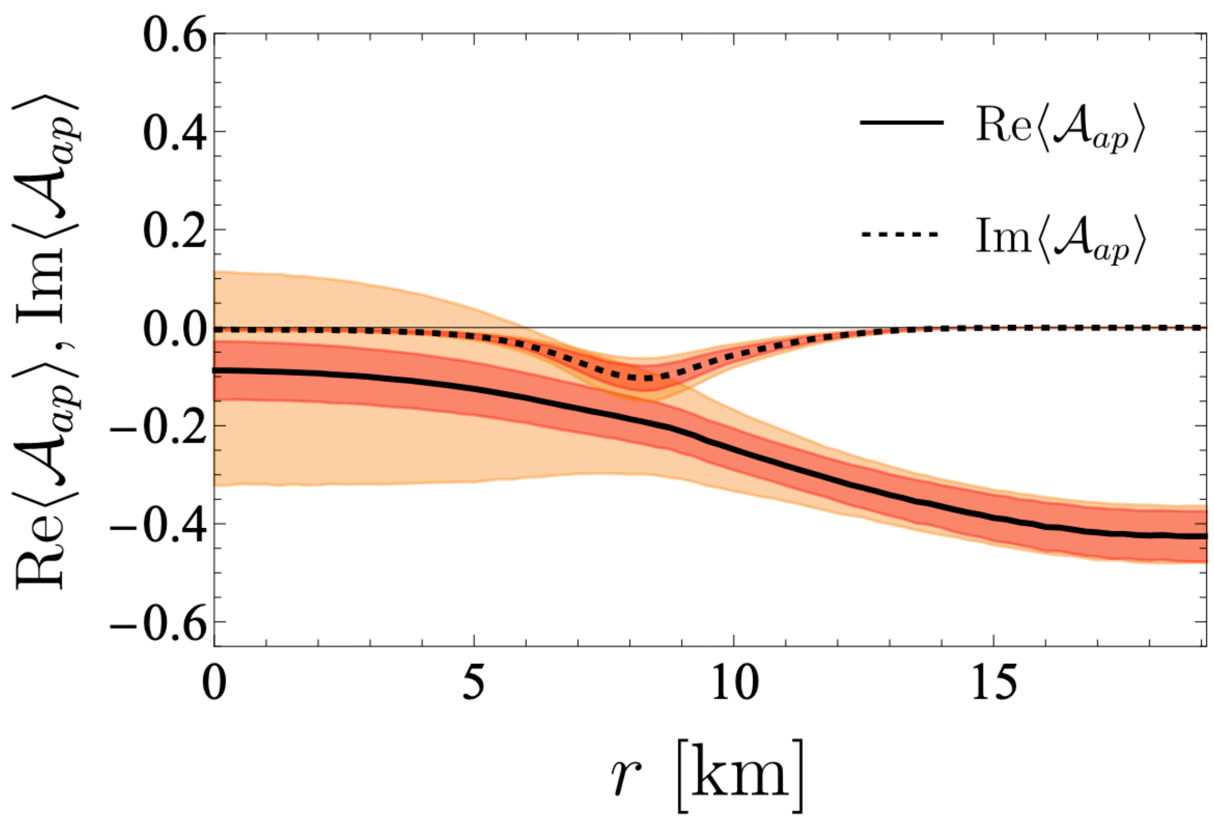
Fully systematic evaluation should take into account all diagrams up to given order

Supernova bound revisited

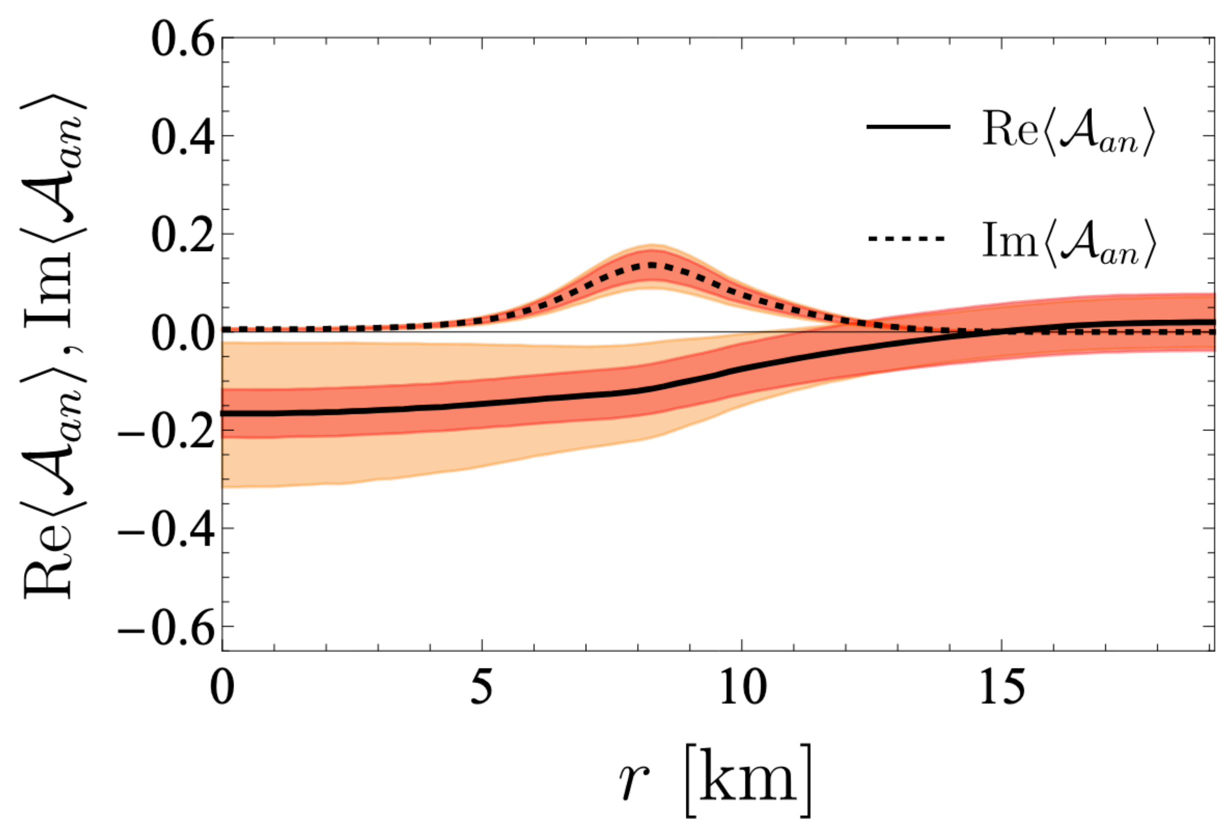
KSVZ axion couplings in a SN:



Proton



Neutron



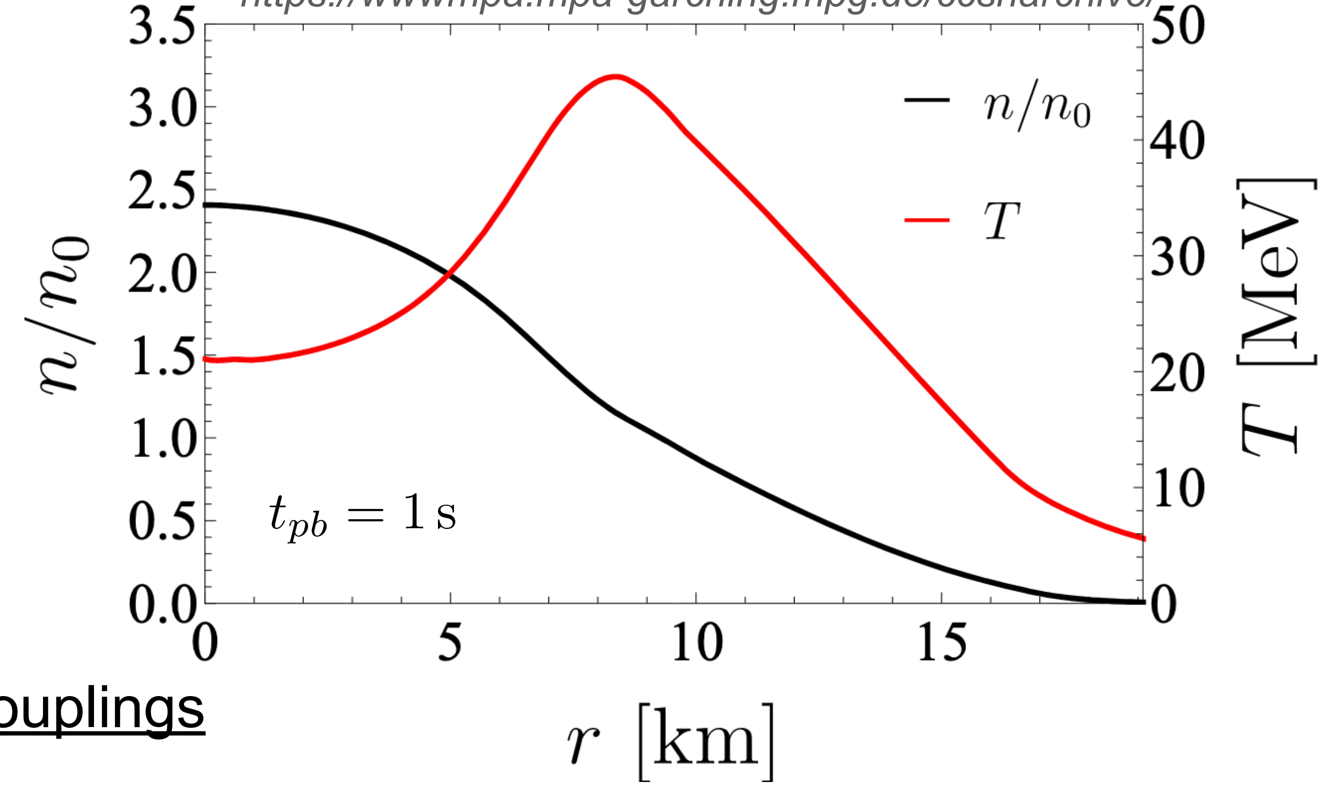
Supernova bound revisited

KSVZ axion couplings in a SN:

Available on GitHub

https://github.com/michael-stadlbauer/Axion-Couplings

https://wwwmpa.mpa-garching.mpg.de/ccsnarchive/



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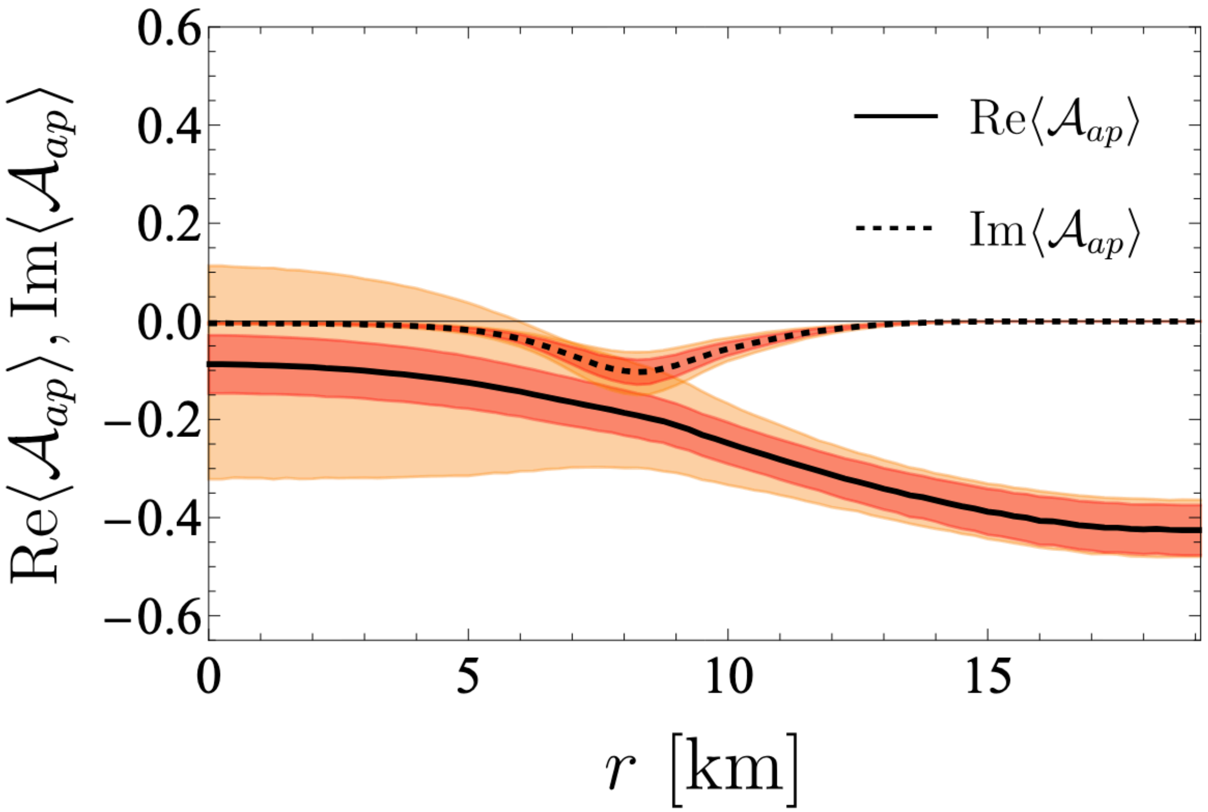

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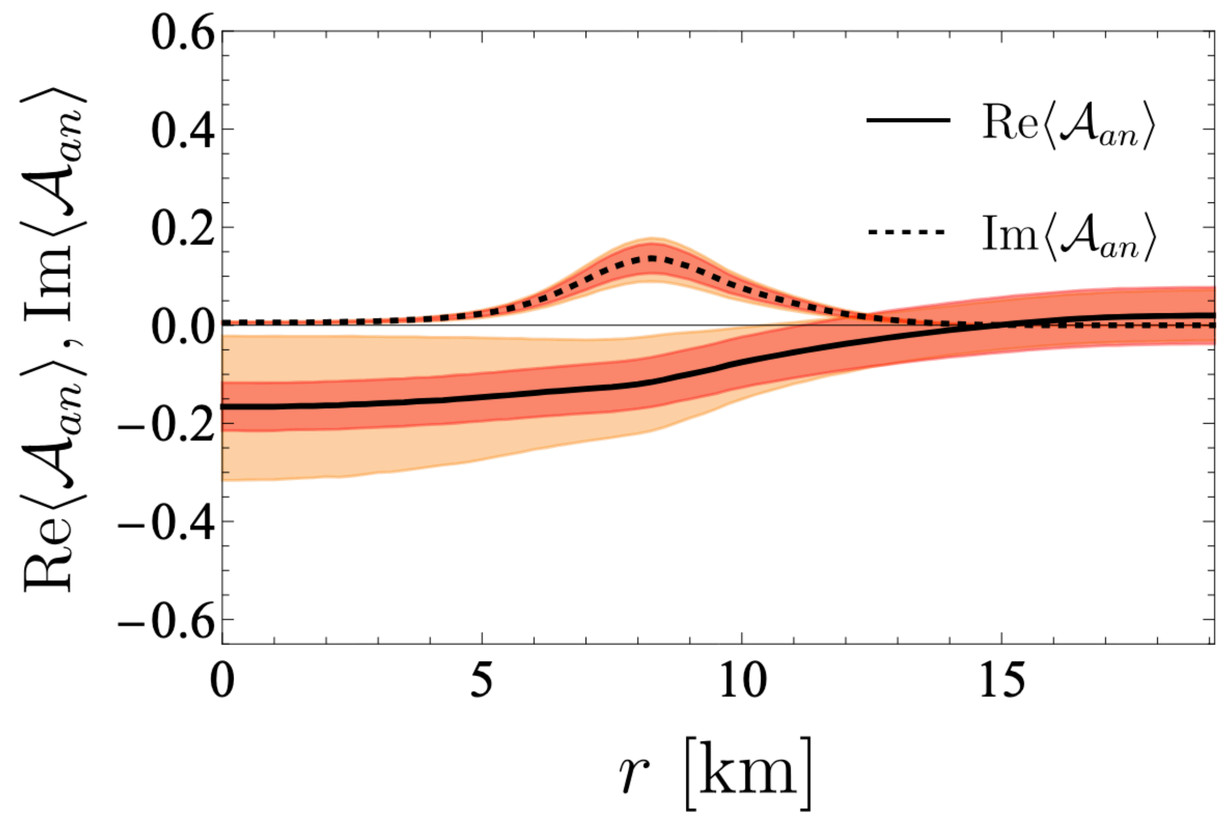
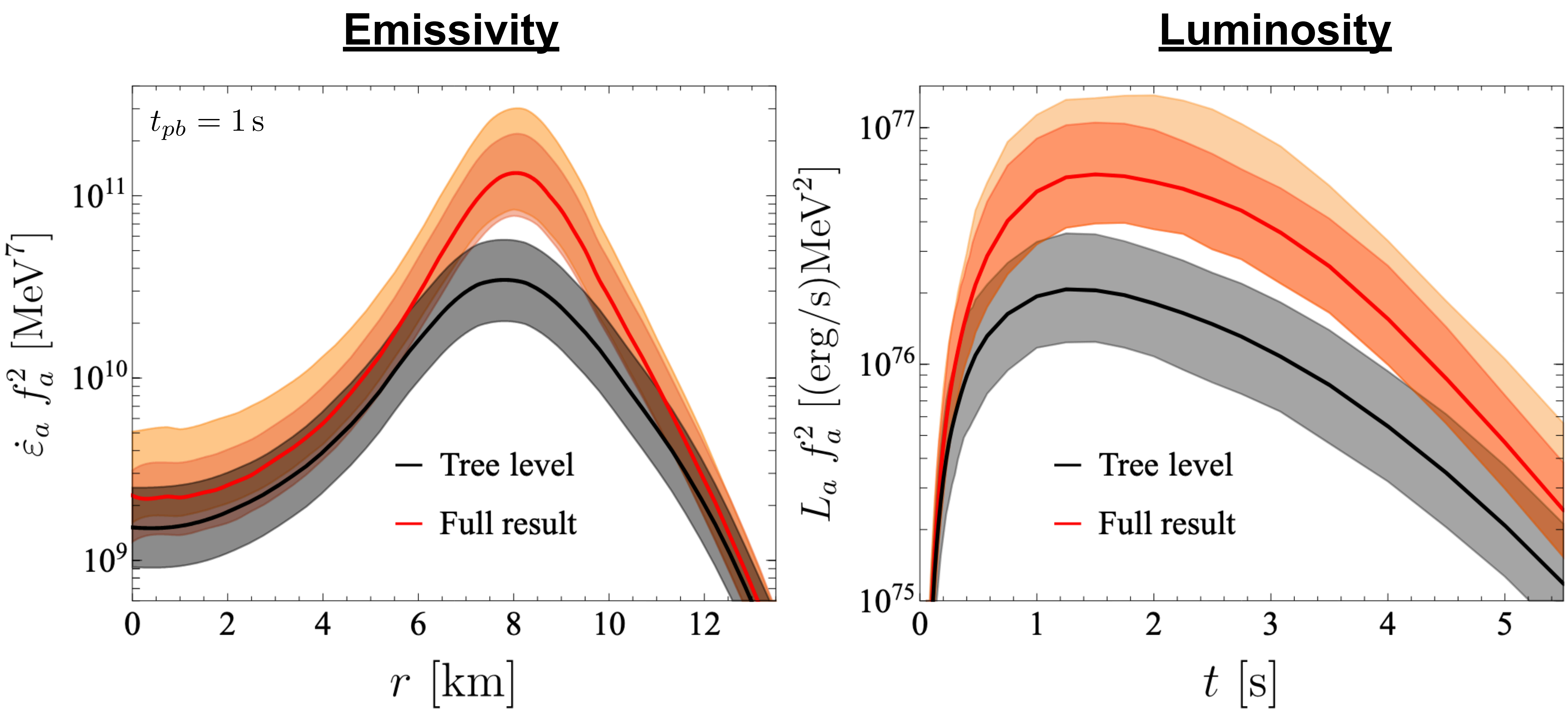


Supernova bound revisited

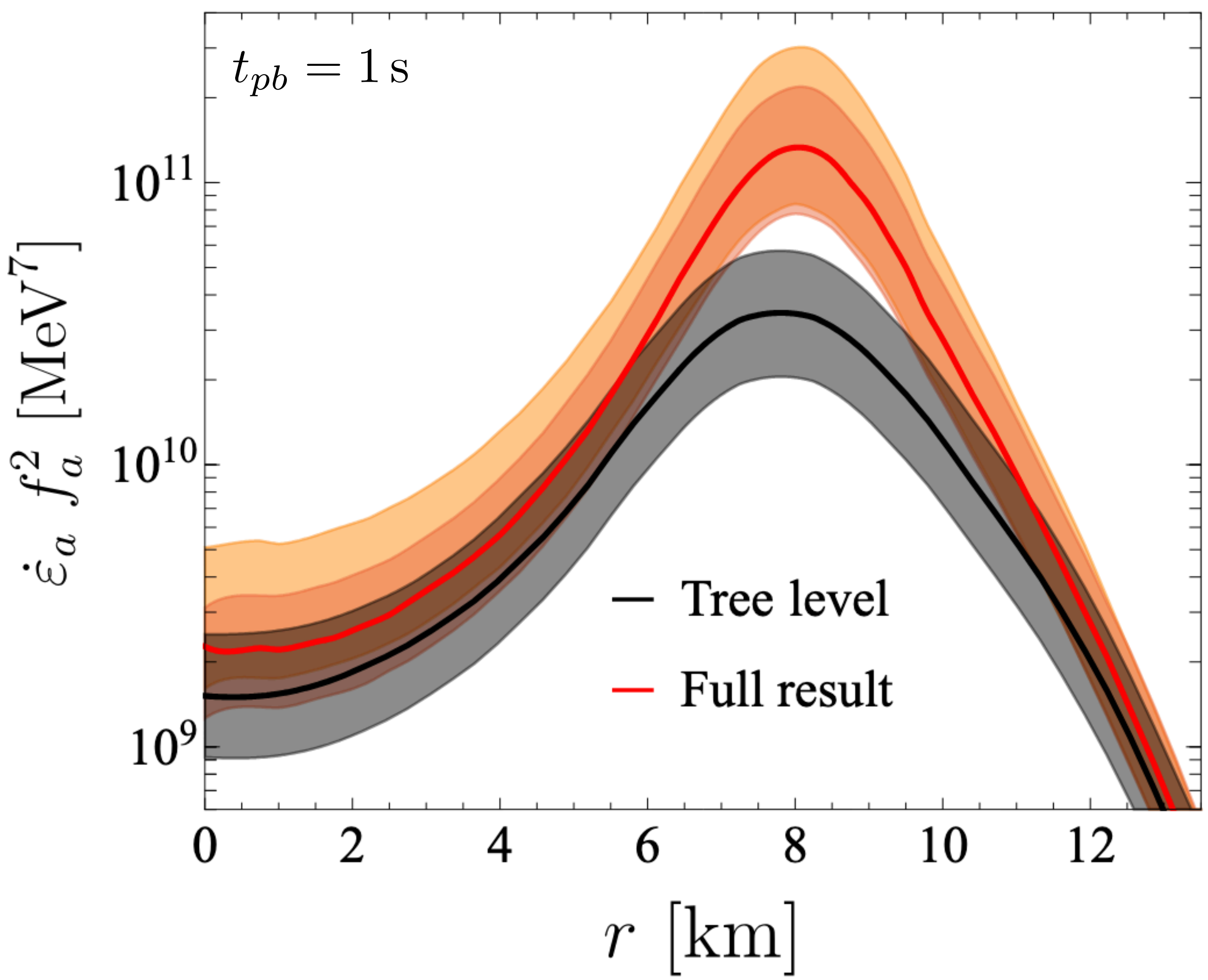
KSVZ axion



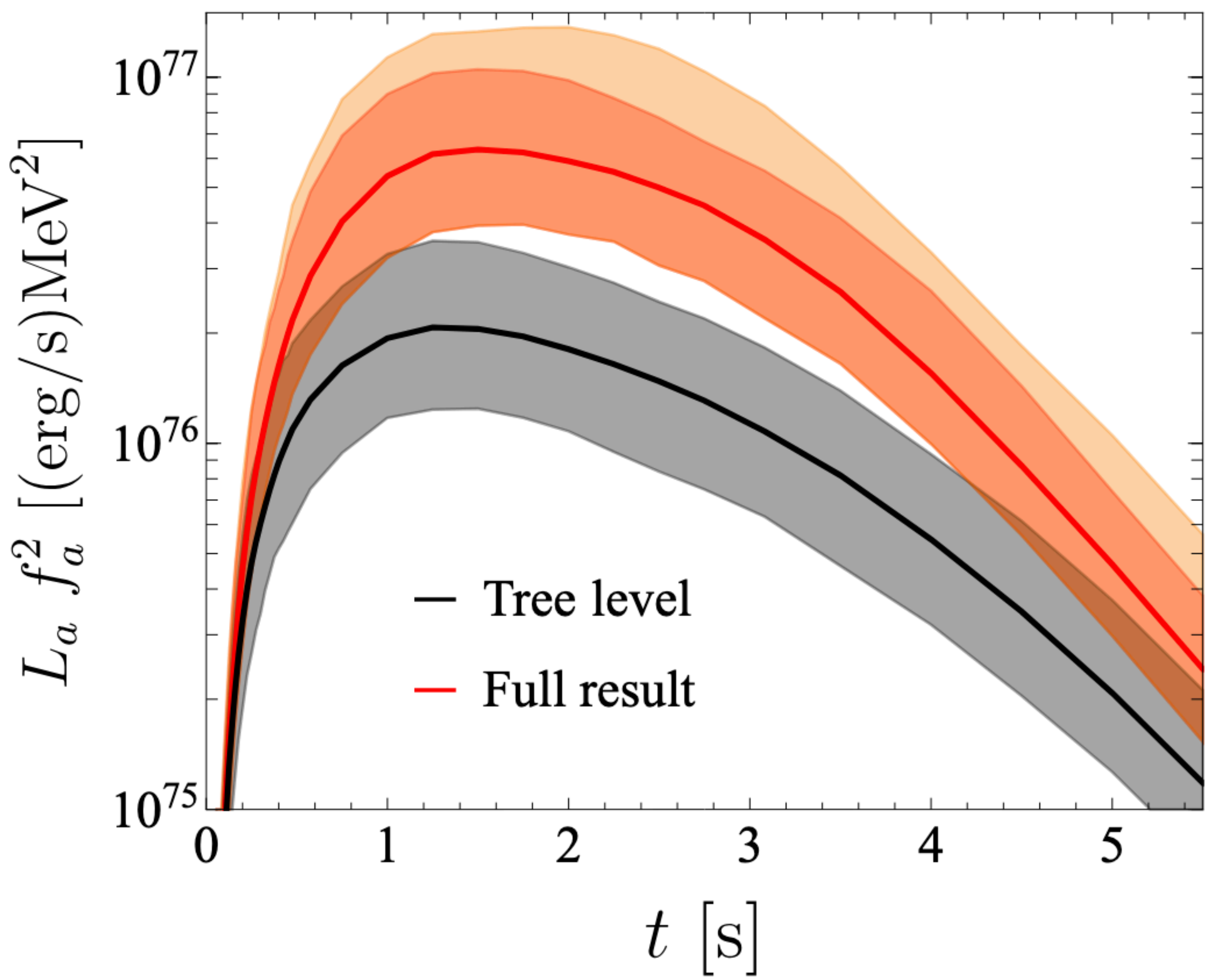
Supernova bound revisited

KSVZ axion

Emissivity



Luminosity



Tree level: $f_a \gtrsim 6.1^{+1.7}_{-1.4} \times 10^8\,\mathrm{GeV}, \quad m_a \lesssim 9.8^{+3.0}_{-2.2}\,\mathrm{meV}.$

Vertex corrections: $f_a \gtrsim 1.0^{+0.5}_{-0.2} \times 10^9\,\mathrm{GeV}, \quad m_a \lesssim 5.9^{+1.8}_{-2.0}\,\mathrm{meV}.$

Astrophobic axions

Derivative axion-nucleon couplings are **model-dependent**

Astrophobic axions

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$$\mathcal{L} \supset \frac{1}{f_a} \bar{N} c_N S \cdot \partial a N, \quad N = (p, n)^T$$

$$c_N = G_A c_{u-d} \tau^3 + G_0 c_{u+d} \mathbf{1}$$

$$c_{u-d} = c_u^0 - c_d^0 - \frac{1}{2}\frac{1-z}{1+z}$$

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Astrophobic models: manage to

$$c_{u+d} = c_{u-d} \simeq 0$$

DiLuzio, Mescia, Nardi, Panci, Ziegler ('17)

Badziak, Harigaya ('23)

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DiLuzio, Mescia, Nardi, Panci, Ziegler ('17)

Badziak, Harigaya ('23)

- These cancellations persist even at finite density and RGE evolution

Di Luzio, Giannotti, Mescia, Nardi, Okawa ('23)

Di Luzio, Fiorentino, Giannotti, Mescia, Nardi ('24)

see Vincenzo's talk in the afternoon

Astrophobic axions

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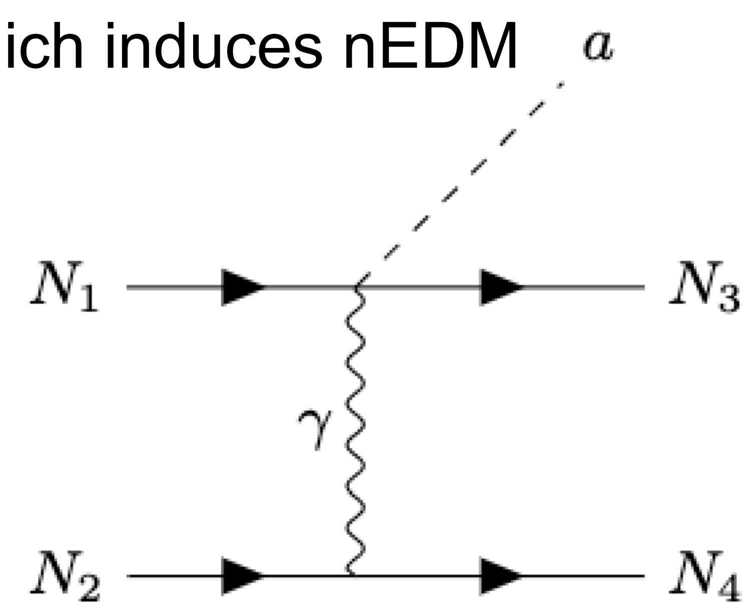
- Still want to solve CP problem so at least $aG\tilde{G}$ which induces nEDM

$$\mathcal{L}_a^{\mathrm{EDM}} = -\frac{i}{2}\frac{C_{aN\gamma}}{m_N}\frac{a}{f_a}\bar{N}\gamma_5\sigma_{\mu\nu}NF^{\mu\nu}$$



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Lucente, Mastrototaro, Carenza, DiLuzio, Giannotti, Mirizzi ('22)

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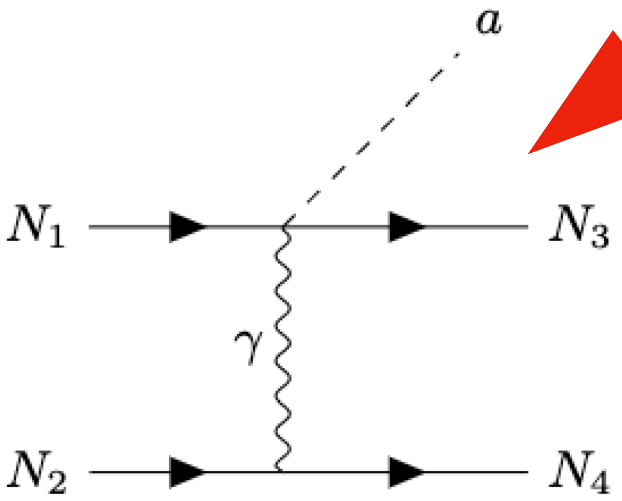
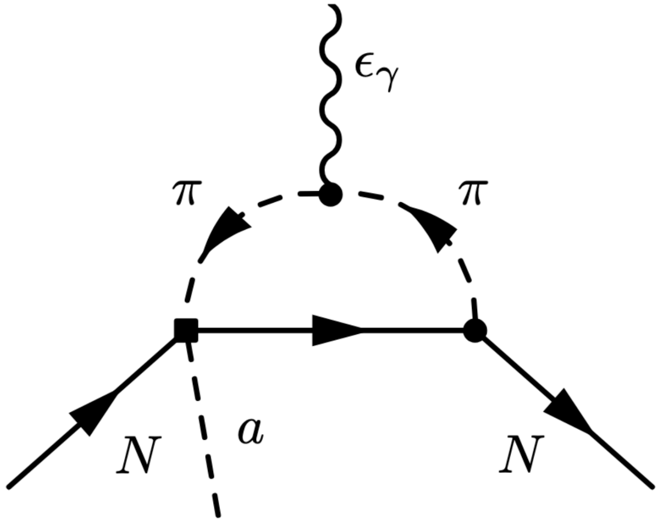
1 loop diagram



EDM induced at 1-loop:

Crewther, Vecchia, Veneziano, Witten ('79)
Schwartz, QFT



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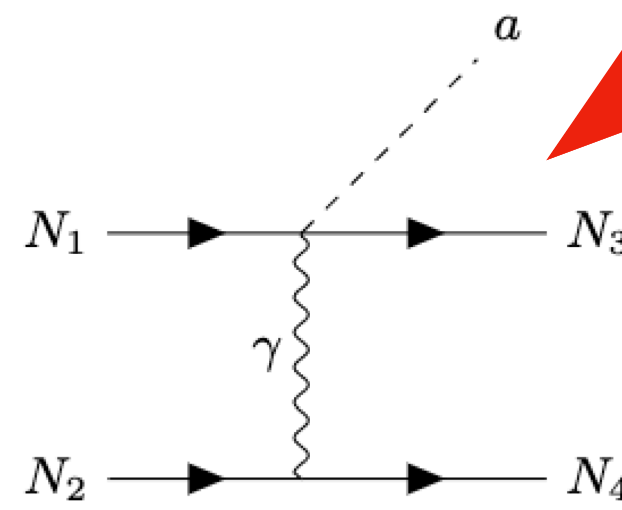

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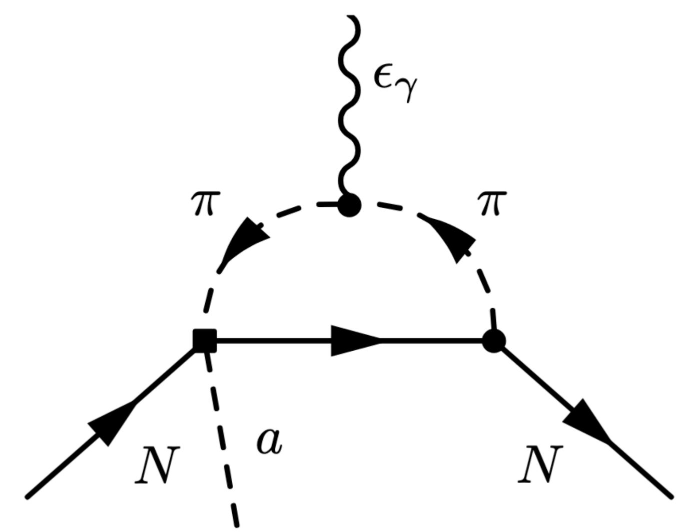

- Due to systematic approach, can identify (ir)relevant operator

$$\mathcal{L}_{\pi N}^{(2)} \supset -\hat{c}_5 m_\pi^2 \frac{4z}{(1+z)^2}\bar{N}\left(\frac{\pi^a a}{f_\pi f_a}\right)\tau^a N$$

Astrophobic axions

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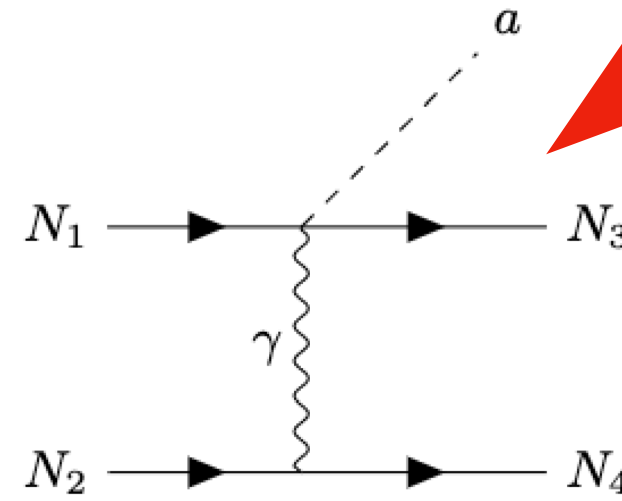

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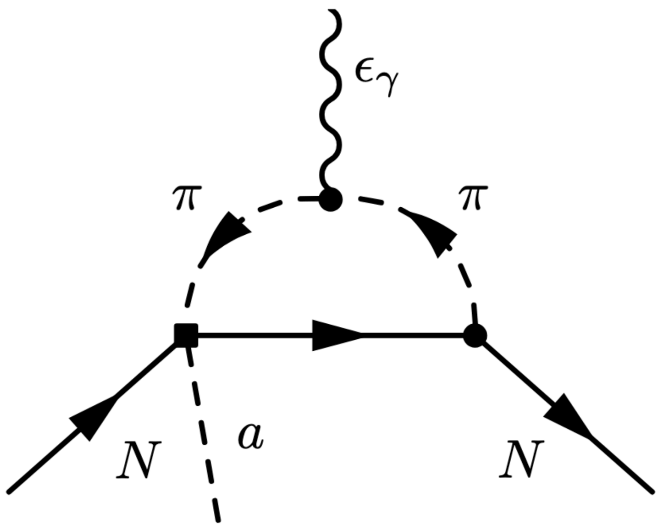

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- NLO, shift-symmetry breaking, isospin-breaking
- Size of EDM operator can be determined $\dfrac{C_{aN\gamma}}{m_N} \sim \dfrac{m_\pi^2}{(4\pi f_\pi)^2}\hat{c}_5$

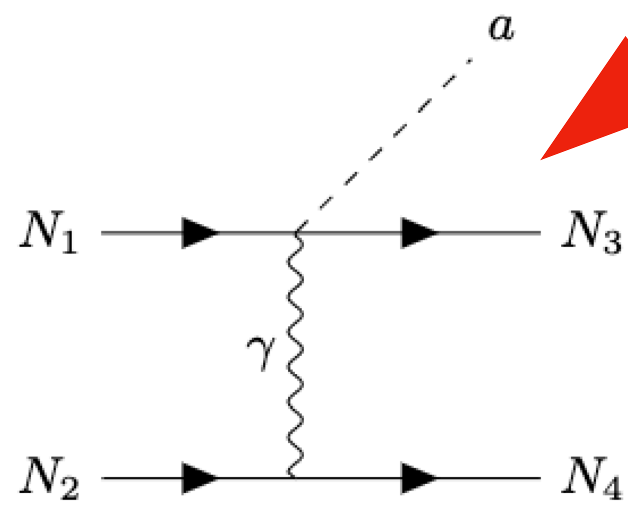
Crewther, Vecchia, Veneziano, Witten ('79)

Astrophobic axions

- Still want to solve CP problem so at least $aG\tilde{G}$ which induces nEDM

$$\mathcal{L}_a^{\text{EDM}} = -\frac{i}{2}\frac{C_{aN\gamma}}{m_N}\frac{a}{f_a}\bar{N}\gamma_5\sigma_{\mu\nu}NF^{\mu\nu}$$

1 loop diagram



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EDM induced at 1-loop:

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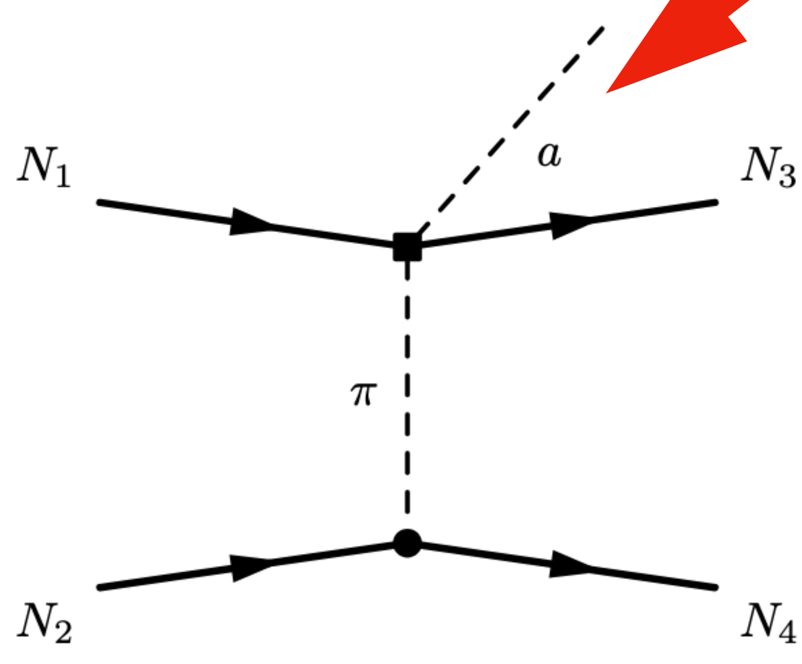
Schwartz, QFT

tree-level diagram

- Due to systematic approach, can identify (ir)relevant operator

$$\mathcal{L}_{\pi N}^{(2)} \supset -\hat{c}_5 m_\pi^2 \frac{4z}{(1+z)^2}\bar{N}\left(\frac{\pi^a a}{f_\pi f_a}\right)\tau^a N$$



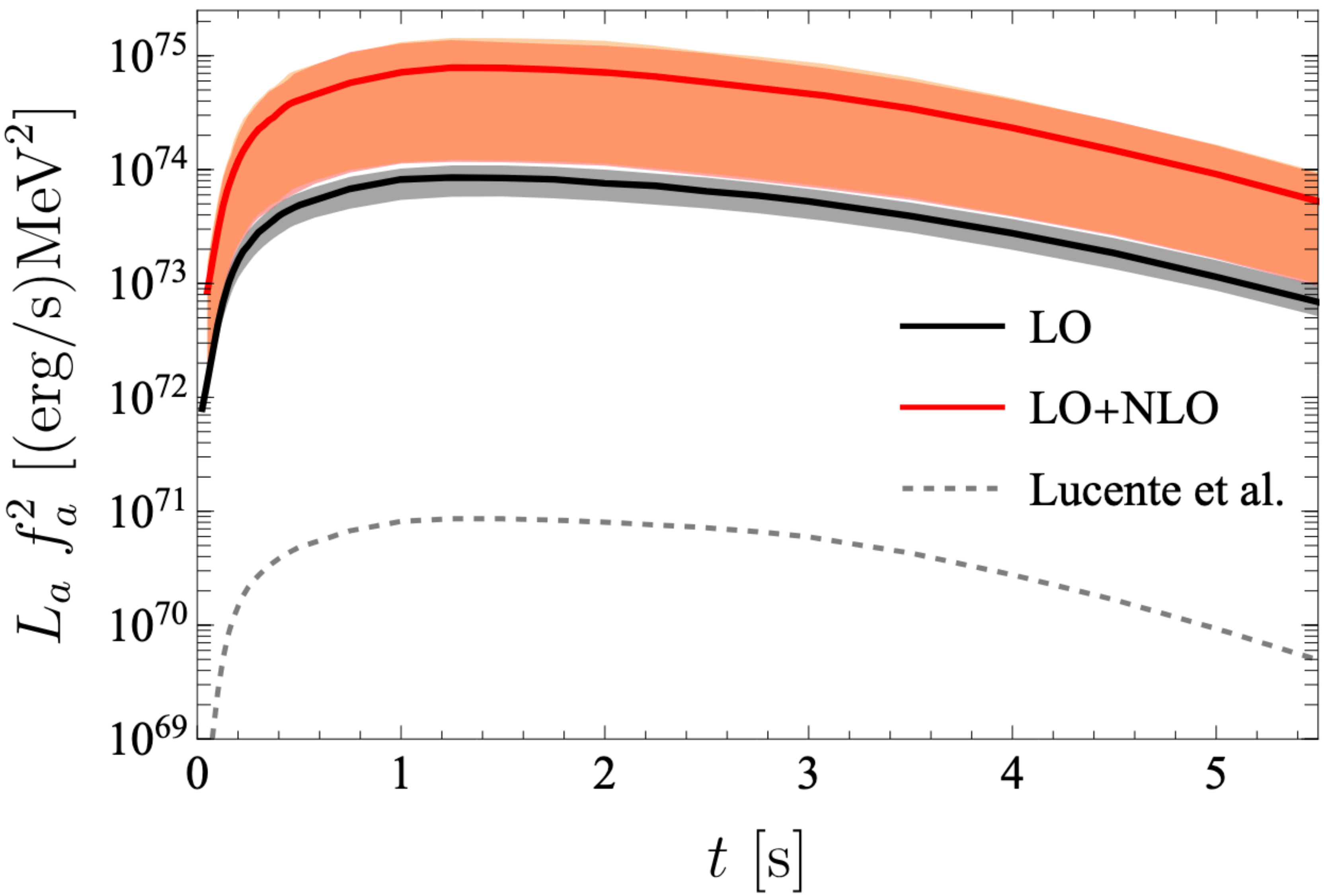
- Induces a **tree-level** diagram

KS, Stadlbauer, Stelzl, Weiler ('24)

Astrophobic axions

- Loose the loop-suppression compared to EDM operator

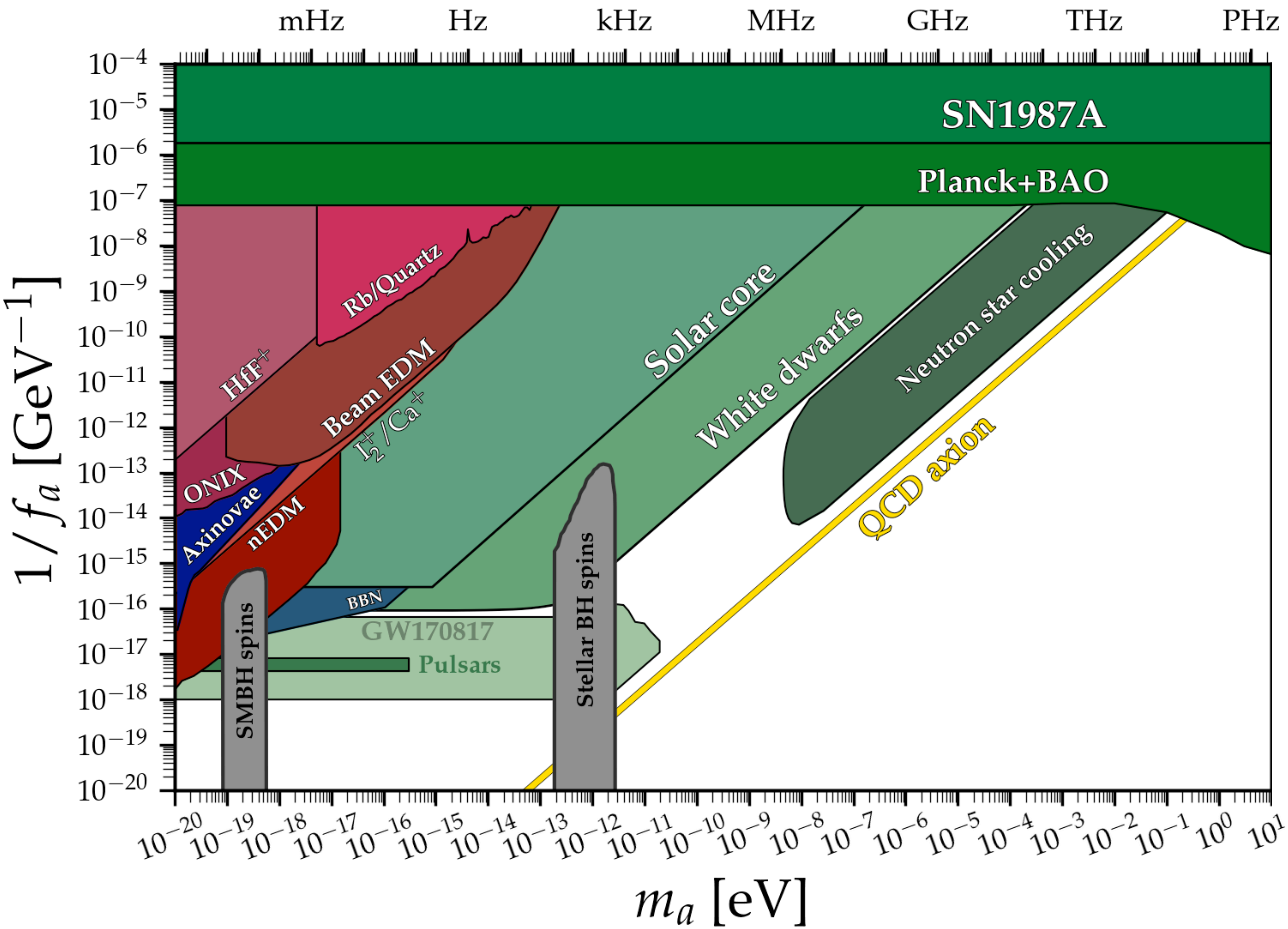
$$L_a^{\text{tree},\hat{c}_5} \simeq (4\pi)^4 L_a^{\text{EDM}} \simeq 10^4 L_a^{\text{EDM}}$$



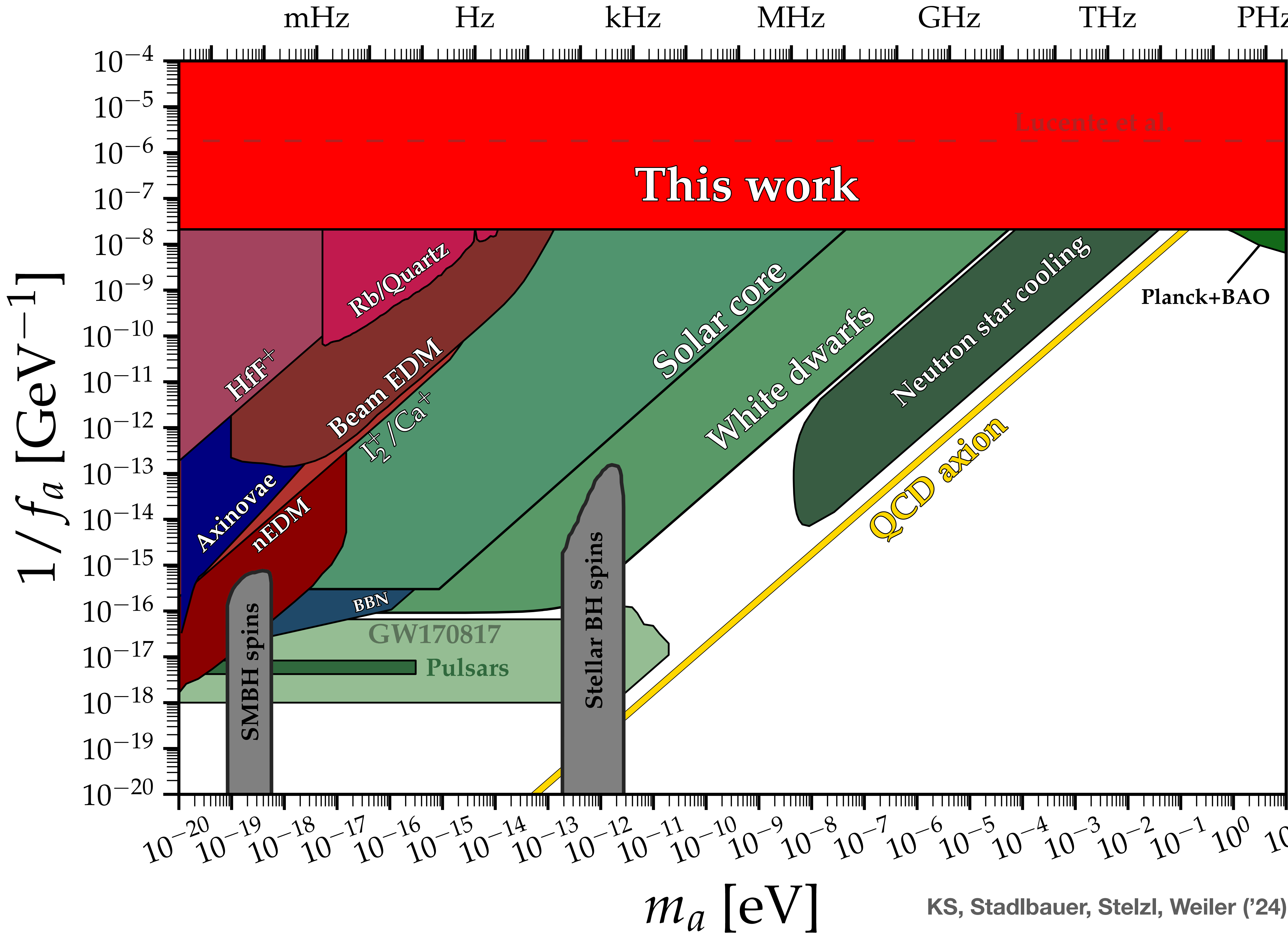
Strong universal bound on QCD axions: $f_a > 1.1^{+0.4}_{-0.6} \times 10^8 \,\text{GeV}, \quad (68\% \text{ C.L.})$

KS, Stadlbauer, Stelzl, Weiler ('24)

Astrophobic axions

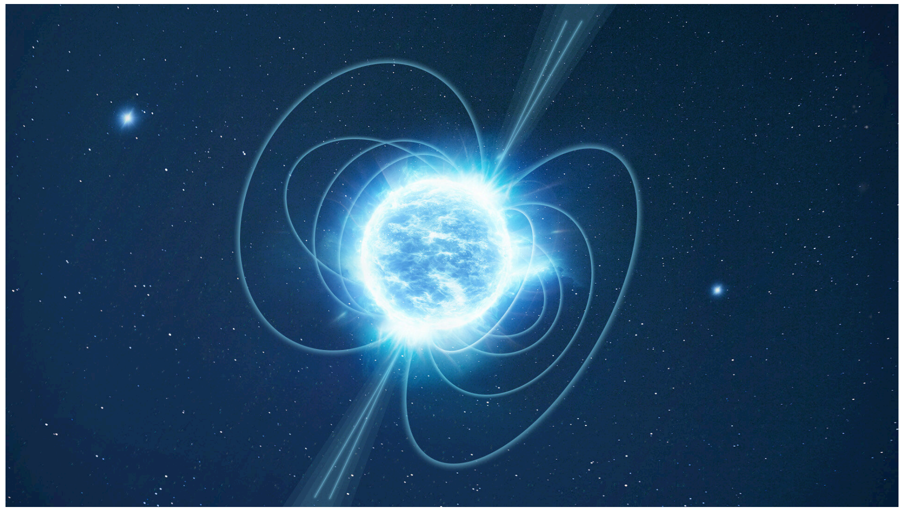


Astrophobic axions

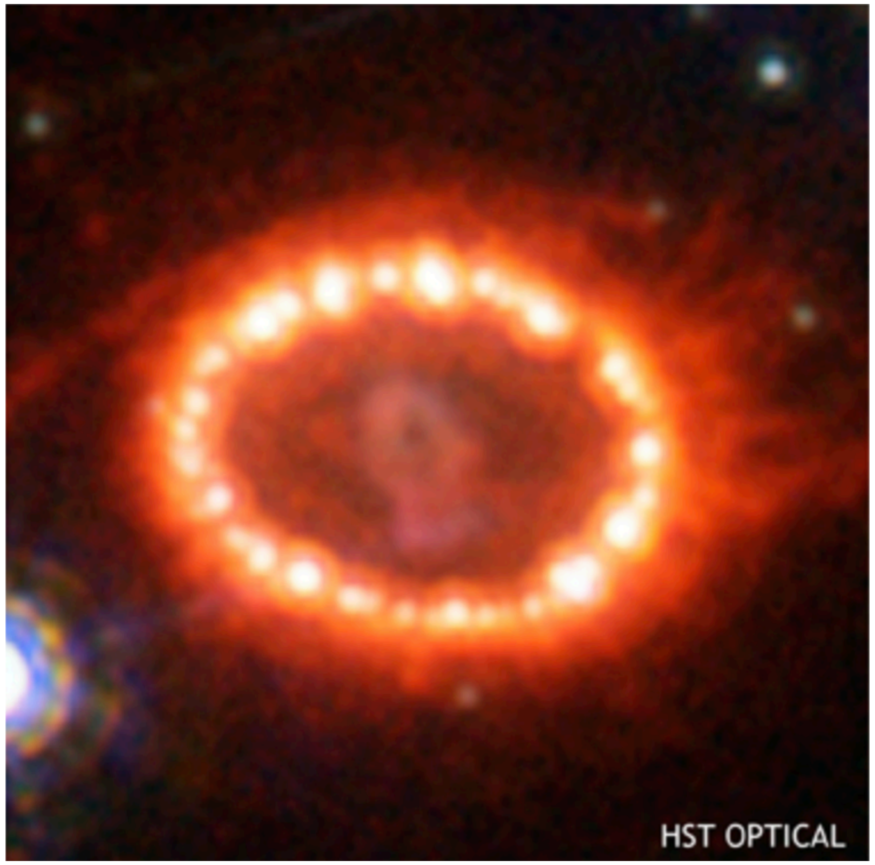


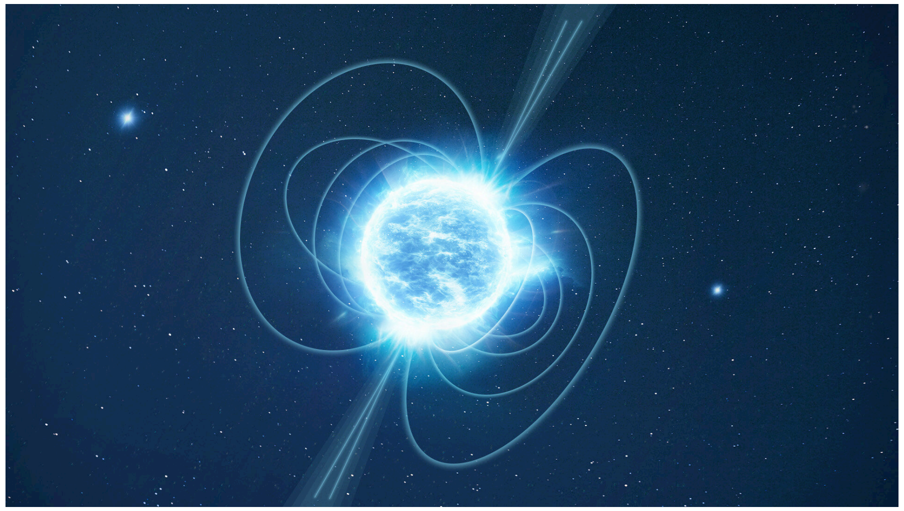
KS, Stadlbauer, Stelzl, Weiler ('24)

Conclusions

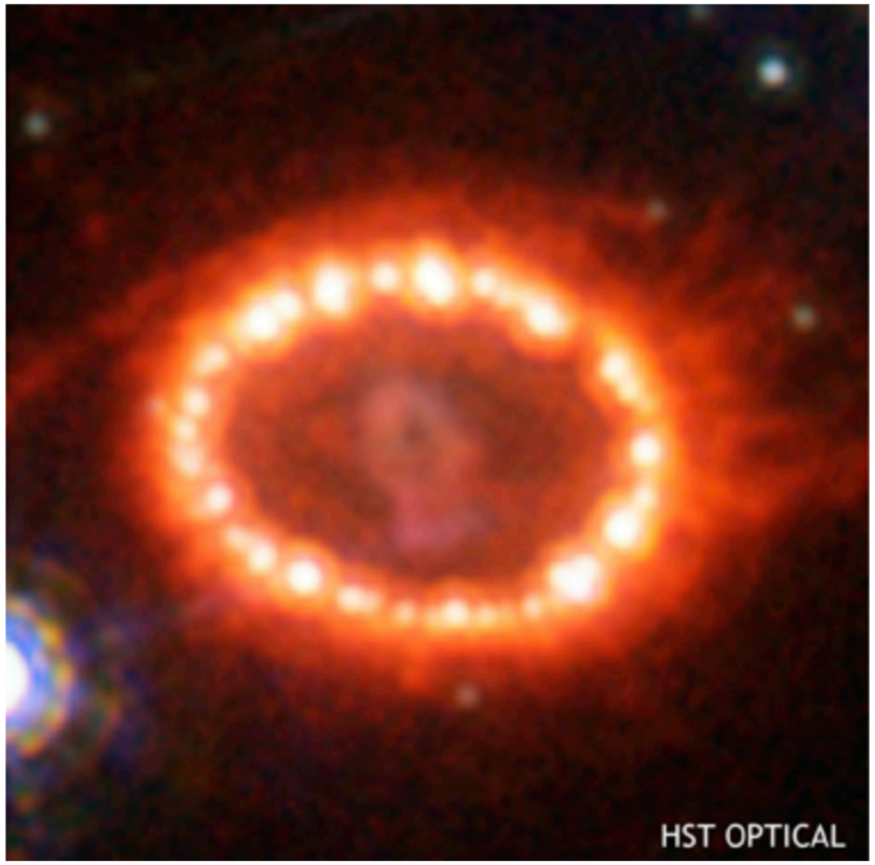


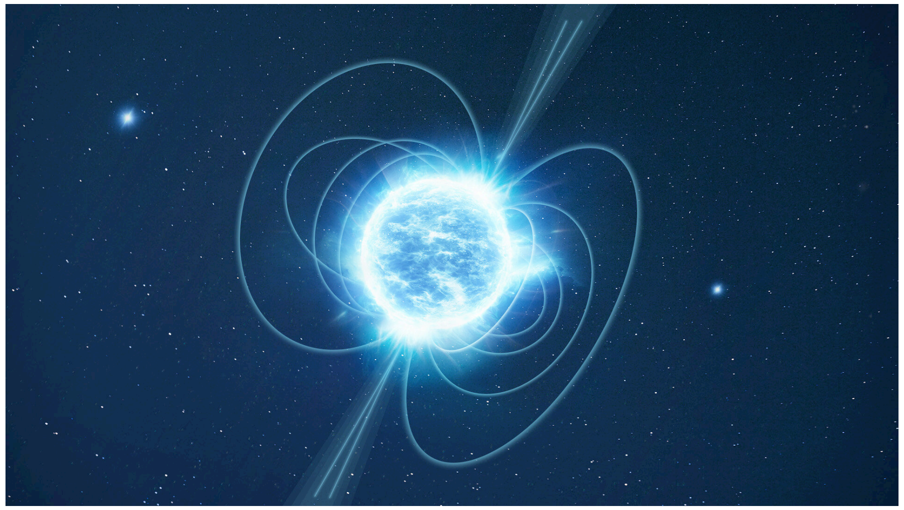
- QCD axion couplings are density dependent!
- Systematic calculation of axion couplings within ChPT
- Significant changes of supernova bound
- Large uncertainty at high densities
- Much to do: calculate ~150 diagrams, spin-correlations, …



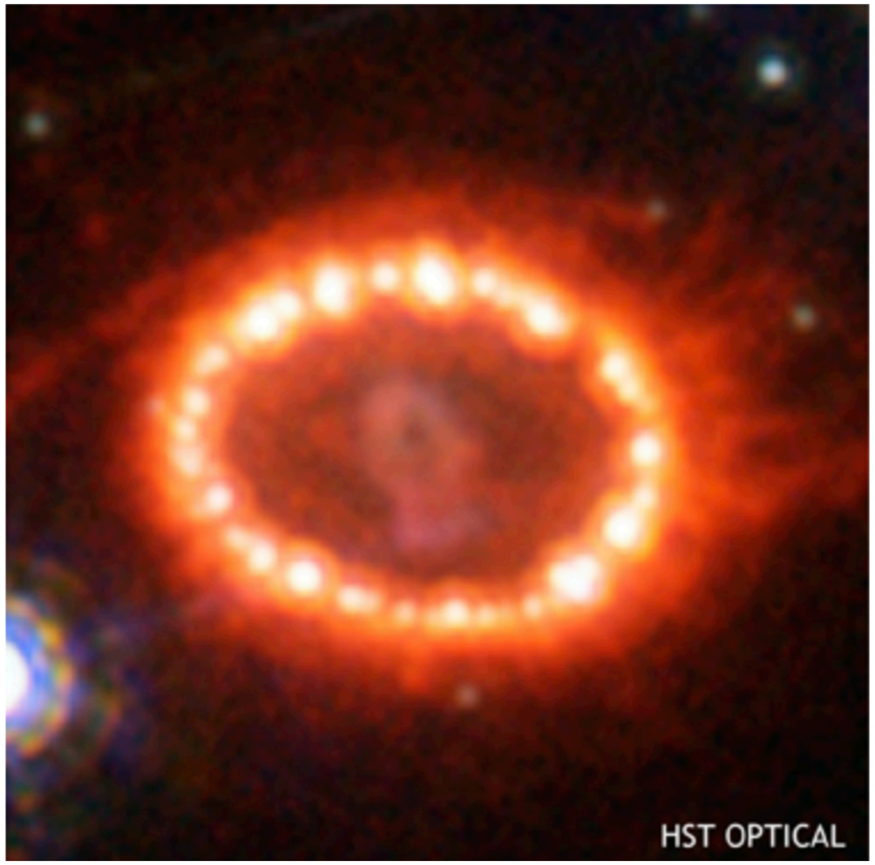


Thank you!





Backup



Couplings

1. Construction of effective axion—nucleon Lagrangian

Consider external sources coupled to QCD

$$\mathcal{L}_{\text{ext}} = \bar{q}\gamma_\mu \left(v^\mu + \frac{1}{3} v^\mu_{(s)} + \gamma_5 \left(a^\mu + a^\mu_{(s)} \right) \right) q - \bar{q} \left(s - i\gamma_5 p \right) q$$

Compare to axion — quark Lagrangian

$$\mathcal{L}_{a-q} = -\bar{q} \Big(\operatorname{Re} \mathcal{M}_a + i\gamma_5 \operatorname{Im} \mathcal{M}_a \Big) q + \bar{q}\gamma^\mu\gamma_5 \left(c_{u+d} \frac{\partial_\mu a}{2f_a} \mathbb{1} + c_{u-d} \frac{\partial_\mu a}{2f_a} \tau_3 \right) q,$$

Mapping:

$$\begin{aligned}
v^\mu &= v^\mu_{(s)} = 0, \\
a_\mu &= c_{u-d} \frac{\partial_\mu a}{2f_a} \tau_3, \\
a^{(s)}_\mu &= c_{u+d} \frac{\partial_\mu a}{2f_a} \mathbb{1}, \\
s &= \operatorname{Re} \mathcal{M}_a = \mathcal{M}_q \cos\left(\frac{a}{f_a} \mathcal{Q}_a \right), \\
p &= -\operatorname{Im} \mathcal{M}_a = -\mathcal{M}_q \sin\left(\frac{a}{f_a} \mathcal{Q}_a \right).
\end{aligned}$$

Couplings

1. Construction of effective axion—nucleon Lagrangian

Theory invariant under $U(2)_L \times U(2)_R$ for

$$
\begin{aligned}
r_\mu &\to V_R r_\mu V_R^\dagger + i V_R \partial_\mu V_R^\dagger, \\
\ell_\mu &\to V_L \ell_\mu V_L^\dagger + i V_L \partial_\mu V_L^\dagger, \\
\ell_\mu^{(s)} &\to \ell_\mu^{(s)} - \frac{1}{3}\partial_\mu \theta_L, \\
r_\mu^{(s)} &\to r_\mu^{(s)} - \frac{1}{3}\partial_\mu \theta_R, \\
(s - ip) &\to V_L\,(s - ip)\,V_R^\dagger, \\
(s + ip) &\to V_R\,(s + ip)\,V_L^\dagger.
\end{aligned}
$$

Symmetry breaking pattern

$$U(N_f)_L \times U(N_f)_R \xrightarrow{\langle \bar{q} q \rangle} U(N_f)_{L+R} = SU(N_f)_V \times U(1)_V$$

Mesons as pNGBs $\quad U(x) = \exp\left(i\frac{\boldsymbol{\pi}(x)}{f_\pi}\right), \quad U^\dagger U = \mathbb{1}, \quad \det U = 1,$

Transforms linearly $\quad U' = e^{-\frac{i}{3}(\theta_R - \theta_L)} V_R U V_L^\dagger, \quad V_{L/R} \in \mathrm{SU}(2)_{L/R}, \quad e^{-\frac{i}{3}(\theta_R - \theta_L)} \in U(1)_A.$

Couplings

1. Construction of effective axion—nucleon Lagrangian

Covariant Derivative
$$\hat{\nabla}_\mu U \equiv \left(\nabla_\mu - 2ia_\mu^{(s)}\right) U = \partial_\mu U - ir_\mu U + iU\ell_\mu - 2ia_\mu^{(s)} U$$
$$= \partial_\mu U - i\left[v_\mu, U\right] - i\left\{a_\mu, U\right\} - 2ia_\mu^{(s)} U$$

Mass spurion
$$\chi = 2B\mathcal{M}_a = 2B(s - ip),$$
$$\chi \overset{SU(2)_L \times SU(2)_R}{\longrightarrow} V_L \chi V_R^\dagger, \quad \chi \overset{\mathrm{P}}{\longrightarrow} \chi^\dagger, \quad \chi \overset{\mathrm{C}}{\longrightarrow} \chi^T.$$

Baryons: transform non-linearly
$$\Psi' = e^{-i\theta_V(x)} K(V_L(x), V_R(x), U(x))\Psi,$$

Compensator field
$$U(x) = u^2(x), \quad u(x) \to u'(x) = \sqrt{V_R U V_L^\dagger} \equiv V_R u K^{-1}(V_L, V_R, U)$$
$$K(V_L, V_R, U) = (u')^{-1} V_R u = \left(\sqrt{V_R U V_L^\dagger}\right)^{-1} V_R \sqrt{U}.$$

Couplings

1. Construction of effective axion—nucleon Lagrangian

Covariant Derivative

$$D_\mu \Psi = \left(\partial_\mu + \Gamma_\mu - i v_\mu^{(s)}\right) \Psi,$$

$$\Gamma_\mu = \frac{1}{2}\left[u^\dagger(\partial_\mu - i r_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger\right],$$

Vielbein (axial vector, isovector)

$$u_\mu = i\left[u^\dagger(\partial_\mu - i r_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\right] = i u^\dagger \nabla_\mu U u^\dagger.$$

$$u_\mu \overset{SU(2)_L \times SU(2)_R}{\longrightarrow} K u_\mu K^\dagger, \quad u_\mu \overset{\mathrm{P}}{\longrightarrow} -\mathcal{P}_\mu{}^\nu u_\nu, \quad u_\mu \overset{\mathrm{C}}{\longrightarrow} u_\mu^T.$$

Vielbein (axial vector, isoscalar)

$$\hat{u}_\mu = i u^\dagger(-2 i a_\mu^{(s)} U) u^\dagger = 2 a_\mu^{(s)},$$

Mass spurion

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u,$$

$$\chi_\pm \overset{SU(2)_L \times SU(2)_R}{\longrightarrow} K \chi_\pm K^\dagger, \quad \chi_\pm \overset{\mathrm{P}}{\longrightarrow} \pm\chi_\pm, \quad \chi_\pm \overset{\mathrm{C}}{\longrightarrow} \chi_\pm^T.$$

Couplings

1. Construction of effective axion—nucleon Lagrangian

Power Counting

$$\nu = 2L + \sum V_i \Delta_i, \quad \Delta_i = d_i + \frac{1}{2} n_i - 2,$$

$$U, u \sim O(1), \quad v_\mu, v_\mu^{(s)}, a_\mu, a_\mu^{(s)} \sim O(q), \quad s, p \sim O(q^2),$$

$$u_\mu, \hat{u}_\mu \sim O(q), \quad \chi_\pm \sim O(q^2).$$

With plane-wave solution

$$\psi_{\vec{p}}^{(+)(\alpha)}(\vec{x}, t) = u^{(\alpha)}(\vec{p}) e^{-ip\cdot x},$$

$$u^{(\alpha)}(\vec{p}) = \sqrt{E(\vec{p}) + m_N} \begin{pmatrix} \chi^{(\alpha)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E(\vec{p})+m_N} \chi^{(\alpha)} \end{pmatrix},$$

$$E(\vec{p}) = p^0 = \sqrt{\vec{p}^2 + m_N^2}.$$

Couplings

1. Construction of effective axion—nucleon Lagrangian

Power Counting

$$\Psi, \bar{\Psi} \sim O(1), \quad iD_\mu \Psi \sim O(1), \quad (iD_\mu - m_N)\Psi \sim O(q),$$
$$\mathbb{1}, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu} \sim O(1), \quad \gamma_5 \sim O(q).$$

LO ChPT Lagrangian

Mesons

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4} \mathrm{Tr}\Big[\hat{\nabla}_\mu U (\hat{\nabla}^\mu U)^\dagger + \big(\chi U^\dagger + \chi^\dagger U\big)\Big]$$

Baryons

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\not{D} - m_N + \frac{g_A}{2}\gamma_5 \not{u} + \frac{g_0}{2}\gamma_5 \hat{\not{u}}\right)\Psi.$$

NLO ChPT Lagrangian Baryons

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}\Bigg[c_1 \langle \chi_+ \rangle - \left(\frac{c_2}{8m_N^2}\langle u_\mu u_\nu \rangle D^{\mu\nu} + \text{h.c.}\right) + \frac{c_3}{2} u \cdot u + c_4 \frac{i}{4}[u_\mu, u_\nu]\sigma^{\mu\nu} + c_5 \tilde{\chi}_+$$
$$- \left(\frac{c_8}{8m_N^2}\langle \hat{u}_\mu u_\nu \rangle D^{\mu\nu} + \text{h.c.}\right) - \left(\frac{c_9}{8m_N^2}\langle \hat{u}_\mu \hat{u}_\nu \rangle D^{\mu\nu} + \text{h.c.}\right) + \frac{c_{10}}{2}\hat{u} \cdot u + \frac{c_{11}}{2}\hat{u} \cdot \hat{u}\Bigg]\Psi.$$

Couplings

1. Construction of effective axion—nucleon Lagrangian

Some expansions

$$\hat{u}_\mu = c_{u+d}\left(\frac{\partial_\mu a}{f_a}\right)\mathbb{1} + \ldots,$$

$$u_\mu = -\left(\frac{\partial_\mu \pi^a}{f_\pi}\right)\tau^a + c_{u-d}\left(\frac{\partial_\mu a}{f_a}\right)\tau_3 + c_{u-d}\left(\frac{\pi^a\pi^b\partial_\mu a}{2f_a f_\pi^2}\right)\left(\tau^b\delta^{3a} - \tau^3\delta^{ab}\right) + \ldots,$$

$$D_\mu = \partial_\mu + ic_{u-d}\left(\frac{\pi^a\partial_\mu a}{2f_\pi f_a}\right)\epsilon^{ab3}\tau^b + \ldots,$$

$$\widetilde{\chi}_+ = m_\pi^2\left(\frac{4m_u m_d}{(m_u+m_d)^2}\right)\left(\frac{\pi^a a}{f_\pi f_a}\right)\tau^a + \ldots\ .$$

Couplings

1. Construction of effective axion—nucleon Lagrangian

Heavy Baryon Limit

Nucleon momentum $p^\mu = m_N v^\mu + q^\mu, \quad \text{with} \quad v \cdot q \ll 1.$

Nucleon field $\Psi(x) = e^{-im_N v\cdot x}\left[\mathcal{N}_v(x) + \mathcal{H}_v(x)\right]$

$$\mathcal{N}_v \equiv e^{im_N v\cdot x} P_{v+}\Psi, \quad \mathcal{H}_v \equiv e^{im_N v\cdot x} P_{v-}\Psi, \qquad P_{v\pm} = \frac{1 \pm \not{v}}{2}$$

Use LO EOM and integrate out the heavy component

LO HBChPT Lagrangian
$$\boxed{\hat{\mathcal{L}}^{(1)}_{\pi N} = \bar{N}_v \left(iv\cdot D + g_A S_v \cdot u + g_0 S \cdot \hat{u}\right) N_v,}$$

Couplings

1. Construction of effective axion—nucleon Lagrangian

Heavy Baryon Limit

NLO HBChPT Lagrangian

$$\begin{aligned}\hat{\mathcal{L}}_{\pi N}^{(2)} = \bar{N}\Big[&-\frac{1}{2m_N}\left(D^2 - (v\cdot D)^2 + ig_A\{S\cdot D, v\cdot u\} + ig_0\{S\cdot D, v\cdot \hat{u}\}\right)\\ &+ \hat{c}_1\langle\chi_+\rangle + \frac{\hat{c}_2}{2}(v\cdot u)^2 + \hat{c}_3(u\cdot u) + \frac{\hat{c}_4}{2} i\epsilon^{\mu\nu\rho\sigma}[u_\mu, u_\nu]v_\rho S_\sigma\\ &+ \hat{c}_5\widetilde{\chi}_+ + \frac{\hat{c}_8}{4}(v\cdot u)(v\cdot\hat{u}) + \hat{c}_9(u\cdot\hat{u})\Big]N,\end{aligned}$$

Couplings

2. Finite Density Propagator

Real-time formalism

Generating functional along some contour

$$Z[j,\bar{j}] = \mathcal{N}\int \mathcal{D}[\psi,\bar{\psi}]\exp\left(i\int_C d^4x\,(\mathcal{L}+\mu\bar{\psi}\gamma_0\psi+\bar{j}\psi+j\bar{\psi})\right)$$

$$= Z_0\exp\left(i\int d^4y\,\mathcal{L}_{\mathrm{I}}\left[\frac{\delta}{i\delta\bar{j}},\frac{\delta}{-i\delta j}\right]\right)$$

$$\times\exp\left(-i\int d^4x\int d^4z\,\bar{j}(x)G_0^{(c)}(x-z)j(z)\right)$$

Free propagator $\left[i\partial\!\!\!/+\mu\gamma_0-m\right]G_0^{(c)}(x)=\delta^{(4)}(x).$

$$iG_0^{(c)}(x)=\theta_c(t)\left\langle\psi(x)\bar{\psi}(0)\right\rangle_0-\theta_c(-t)\left\langle\bar{\psi}(0)\psi(x)\right\rangle_0,$$

Couplings

2. Finite Density Propagator

Real-time formalism

Explicitly calculate $iG_0^{(c)}(x) = \theta_c(t)\,\langle \psi(x)\bar{\psi}(0)\rangle_0 - \theta_c(-t)\,\langle \bar{\psi}(0)\psi(x)\rangle_0\,,$

Use

$$\psi(x) = \sum_s \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}\left(a_k^s u_k^s e^{-ikx} + b_k^{s\dagger} v_k^s e^{+ikx}\right) e^{i\mu t},$$

$$\bar{\psi}(x) = \sum_s \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}\left(a_k^{s\dagger} \bar{u}_k^s e^{+ikx} + b_k^{s} \bar{v}_k^s e^{-ikx}\right) e^{-i\mu t},$$

with KMS boundary conditions

$$\langle a_k^s a_p^{s'\dagger}\rangle = (2\pi)^3\delta^{ss'}\delta^{(3)}(\vec{k}-\vec{p})\left[1-n_F(\omega_k,\mu)\right],$$

$$\langle b_k^{s\dagger} b_p^{s'}\rangle = (2\pi)^3\delta^{ss'}\delta^{(3)}(\vec{k}-\vec{p})n_F(\omega_k,-\mu).$$

$$iG_0^{(c)}(x) = (i\partial\!\!\!/ + m)\int \frac{d^4k}{(2\pi)^4} e^{-ikx}\rho_0(k)\left[n_F(-\omega_k,-\mu)\theta_c(t) - n_F(\omega_k,\mu)\theta_c(-t)\right]$$

Couplings

2. Finite Density Propagator

Real-time formalism

Propagator

$$iG_0^{(c)}(x) = (i\partial\!\!\!/ + m) \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \rho_0(k) \left[n_F(-\omega_k, -\mu)\theta_c(t) - n_F(\omega_k, \mu)\theta_c(-t) \right]$$

Spectral density

$$\rho_0(k) = 2\pi \left[\theta(k_0) - \theta(-k_0)\right] \delta(k^2 - m^2)$$
$$= i \left[\theta(k_0) - \theta(-k_0)\right] \left(\Delta_{0F}(k) - \Delta_{0F}^{\dagger}(k) \right)$$

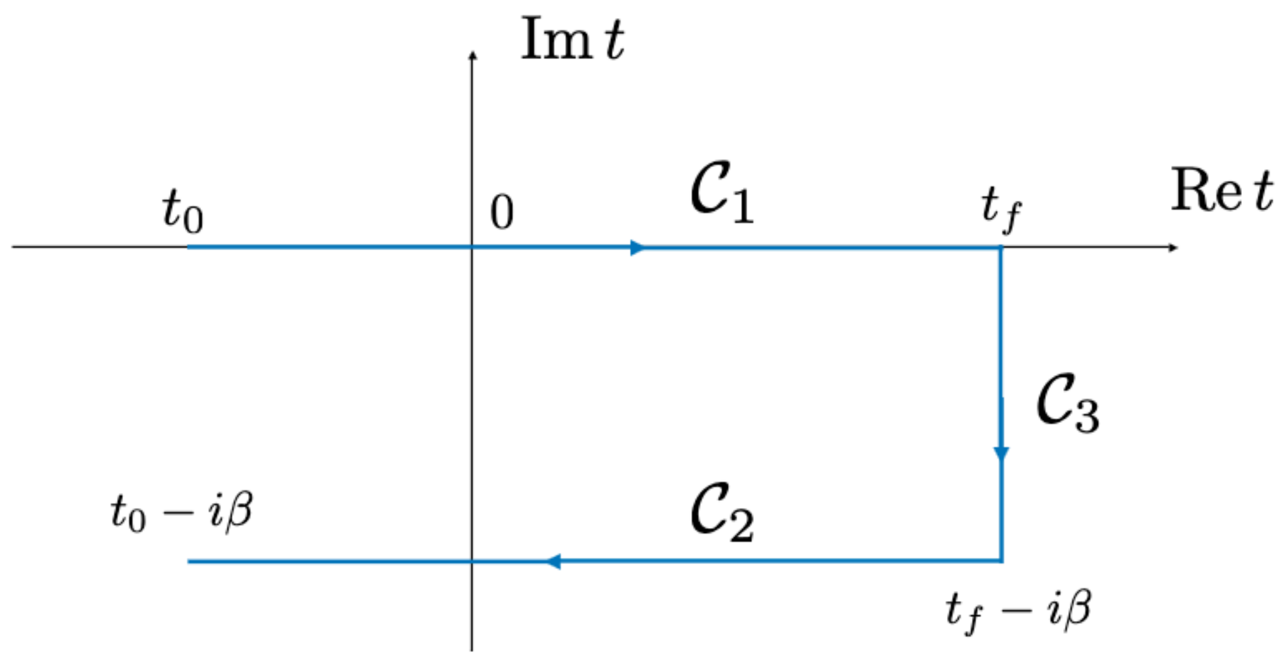
$$\Delta_{0F}(k) = \frac{1}{k^2 - m^2 + i\epsilon}.$$

Couplings

2. Finite Density Propagator

Real-time formalism

Contour



Generating functional factorizes $\lim_{t_f \to \infty} \lim_{t_0 \to -\infty} Z[\bar{j}, j] = Z_{12}[\bar{j}, j] Z_3[\bar{j}, j]$

with

$$Z_{12}\left[j, \bar{j}\right] = Z_0 \exp\left(i \int d^4y \left\{\mathcal{L}_{\mathrm{I}}\left[\frac{\delta}{i\delta\bar{j}_1}, \frac{\delta}{-i\delta j_1}\right] - \mathcal{L}_{\mathrm{I}}\left[\frac{\delta}{i\delta\bar{j}_2}, \frac{\delta}{-i\delta j_2}\right]\right\}\right)$$
$$\times \exp\left(-i \int d^4x \int d^4z \bar{j}_{(k)}(x) G_0^{(kl)}(x-z) j_l(z)\right),$$

Couplings

2. Finite Density Propagator

Real-time formalism

Propagator becomes 2x2 matrix

$$\sin^2 \Theta(k) = n_F(\omega_k, \mu)$$

$$\begin{aligned} i\boldsymbol{G}(k) &= i\boldsymbol{G}_{0F}(k) + i\boldsymbol{G}_{0T}(k) \\ &= \begin{pmatrix} iG_{0F}(k) & \\ & -iG^{\dagger}_{0F}(k) \end{pmatrix} - 2\pi(\not{k} + m)\delta(k^2 - m^2)\sin\Theta(k) \begin{pmatrix} \sin\Theta(k) & \cos\Theta(k) \\ \cos\Theta(k) & \sin\Theta(k) \end{pmatrix} \end{aligned}$$

In zero temperature limit, propagator diagonalizes

$$\begin{aligned} i\boldsymbol{G}^{(11)}(k) &= (\not{k} + m)\left[\frac{i}{k^2 - m^2 + i\epsilon} - 2\pi\delta(k^2 - m^2)\theta(k^0)\theta(k_F - |\vec{k}|)\right] \\ &\simeq \frac{i}{l^0 + i\epsilon} - 2\pi\delta(l^0)\theta(p^0)\theta(k_F - |\vec{l}|) \end{aligned}$$

Couplings

3. Particle production rates

As in vacuum

$$\Gamma(p_a) = \frac{1}{E_a}\mathrm{Im}\,\Pi(p_a)$$

At finite density, self-energy is given by the pole of

$$i\boldsymbol{\Delta}^{(11)}(p_a) = i\Delta(p_a) - n_B(\omega_a)\left[i\Delta(p_a) - i\Delta^*(p_a)\right]$$

$$i\Delta(p_a) = \frac{i}{p_a^2 - m_a^2 - \Pi(p_a) + i\epsilon}$$

In zero temperature limit, given by 11-component

$$\Gamma_i(p_a) = \frac{1}{E_a}\mathrm{Im}\Pi_{11}(p_a)$$

Therefore

$$d\Gamma = d\Gamma_i(p_a)\frac{V}{(2\pi)^3}d^3p_a,$$ and $$d\dot{\epsilon}_a = \frac{E_a}{V}d\Gamma$$

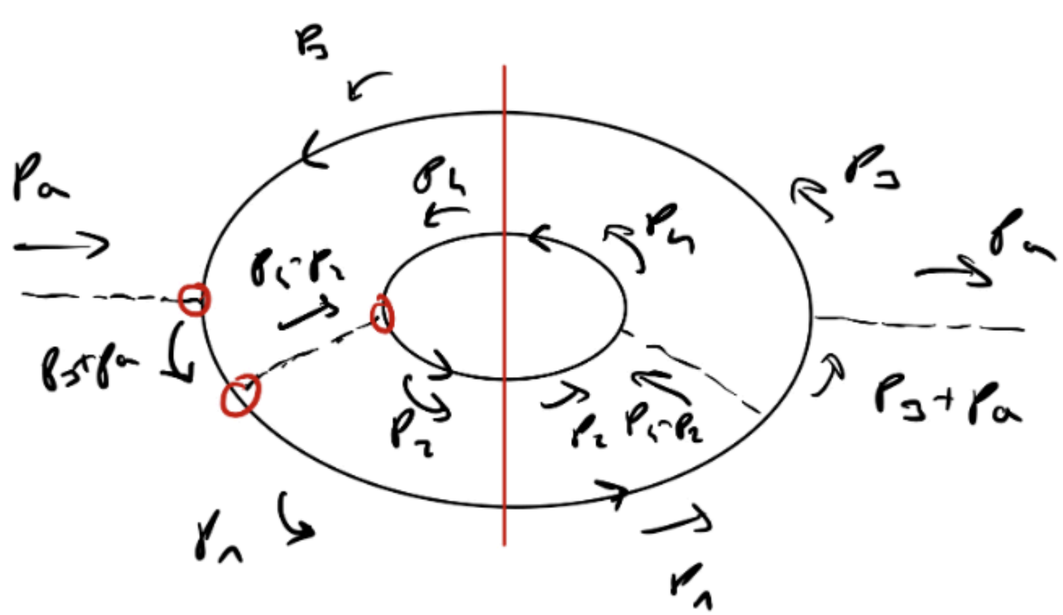
Couplings

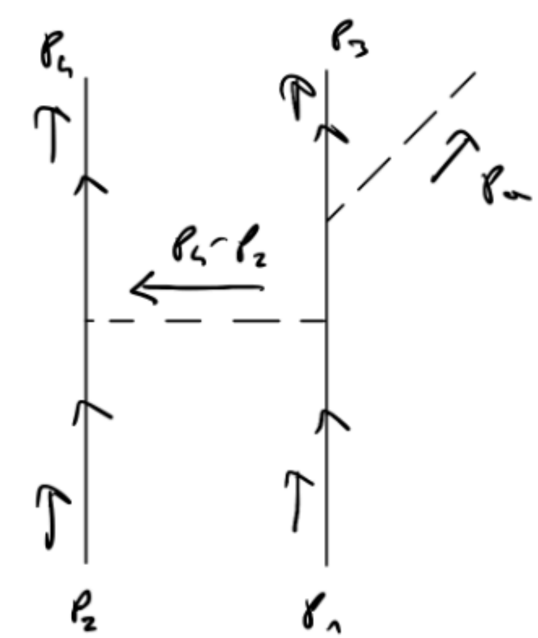
3. Particle production rates

Emissivity $$\dot{\epsilon}_a = \int \Gamma_i(p_a) E_a \frac{d^3p_a}{(2\pi)^3} = \int 2\mathrm{Im}\Pi_{11}(p_a) E_a d\Pi_a$$

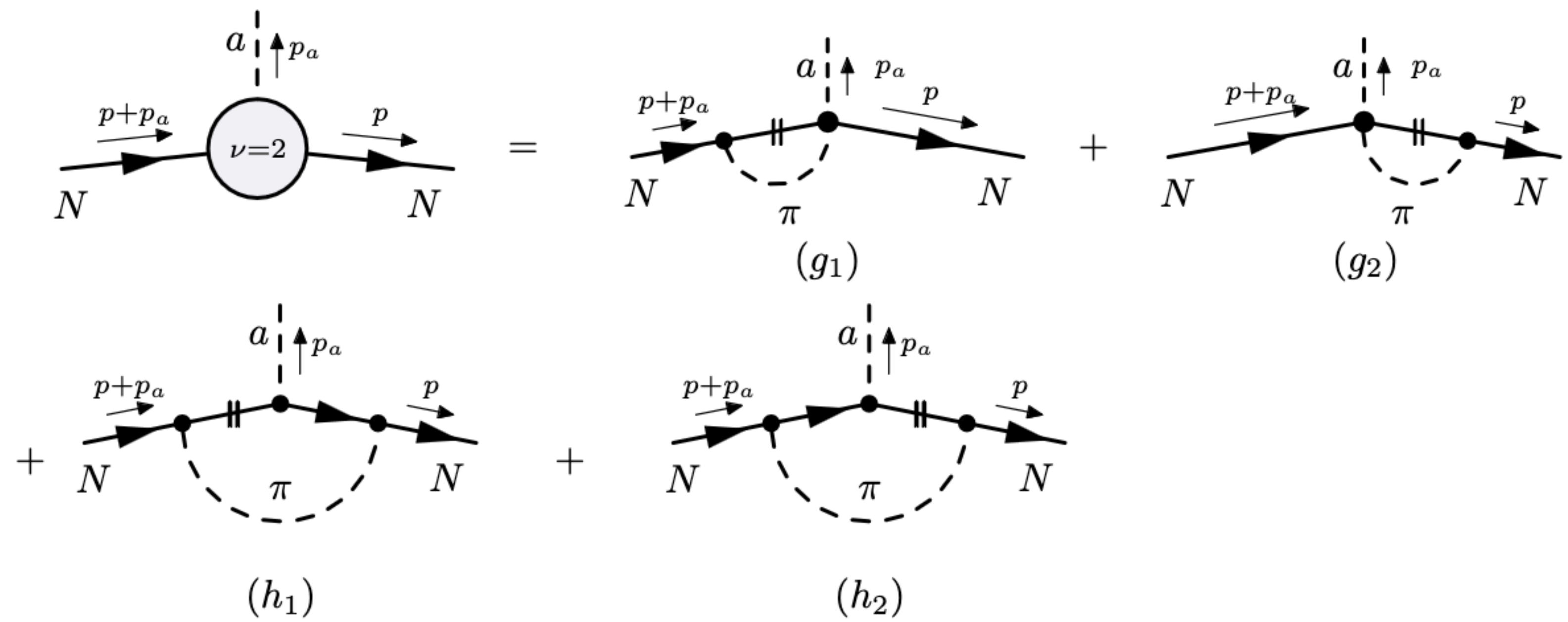
$$d\Pi_a = d^3p_a/(2\pi)^3 2E_a$$

For Bremsstrahlung: Apply cutting rules





Example calculation of finite density loops



$$(h_1)+(h_2)=\int\frac{d^4k}{(2\pi)^4}\left[-\frac{g_A}{2f_\pi}\vec{\sigma}\cdot(\vec{k}-\vec{p})\tau^a\right]\left[\frac{i}{k^0}-2\pi\delta(k^0)\theta(k_f-|\vec{k}|)\right]\left[\frac{c_N}{2f_a}\vec{\sigma}\cdot\vec{p}_a\right]$$
$$\times\left[\frac{i}{k^0+p_a^0}-2\pi\delta(k^0+p_a^0)\theta(k_f-|\vec{k}+\vec{p}_a|)\right]\left[\frac{g_A}{2f_\pi}\vec{\sigma}\cdot(\vec{k}-\vec{p})\tau^b\right]\left[\frac{-i\delta^{ab}}{m_\pi^2-(k-p)^2}\right].$$