Università degli Studi di Padova
Dipartimento di Fisica e Astronomia Galileo Galilei
INFN Padova
Istituto Nazionale di Fisica Nucleare
Sezione di Padova

Unveiling Imprints from Dark Symmetries: Signatures of Axion Portal to Scalar DM

Tommaso Sassi

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Breakthrough beyond the state of the art (in a nutshell)

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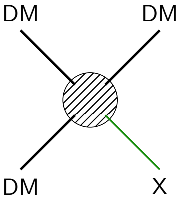
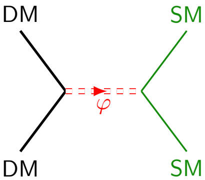
In literature





Fermionic DM

[Y. Cai and A. P. Spray: 1509.08481]

[F. D'Eramo and J. Thaler: 1003.5912]

Vector DM

[F. D'Eramo *et al.*: 1210.7817]

[T. Hambye, M.H.G. Tytgat: 0907.1007]

[C.Arina *et al.*: 0912.4496v2]

Scalar DM

[G. Belanger *et al.*: 1202.2962]

Fermionic DM

[S. Gola *et al.*: 2106.00547 (N_R)]

[L. Fitzpatrick *et al.*: 2306.03128 (gluon)]

[G. Armando *et al.*: 2310.05827 (leptophilic)]

Vector DM

[K. Kaneta *et al.*: 1611.01466 (A'_μ)]

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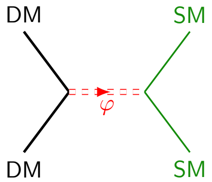
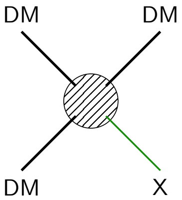
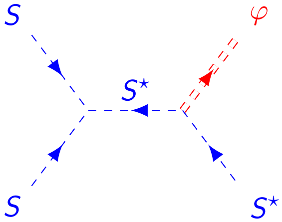




NEW!: SCALAR DM

[$F.D'Eramo, TS$: $arXiv$:2502.19491]

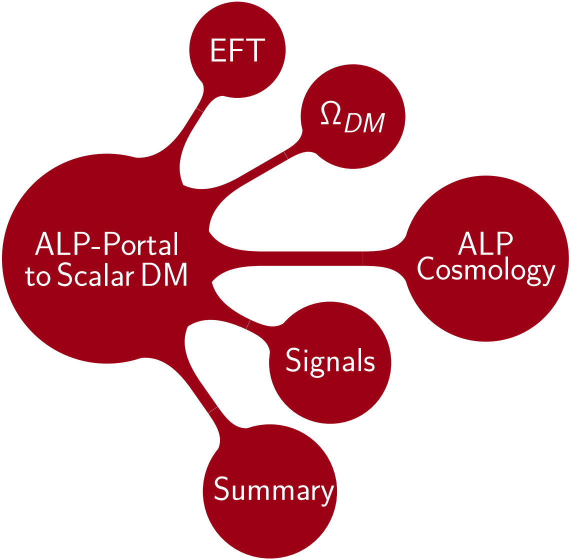


$\mathbb{Z}_3$ symmetry

Physical $\mathcal{C}_S \dfrac{\partial_\mu \varphi}{2 f_\varphi} S^\dagger \, i \overleftrightarrow{\partial^\mu} \, S$?

ALP-SM couplings $(g,\, \gamma,\, \psi)$

Outline


EFT
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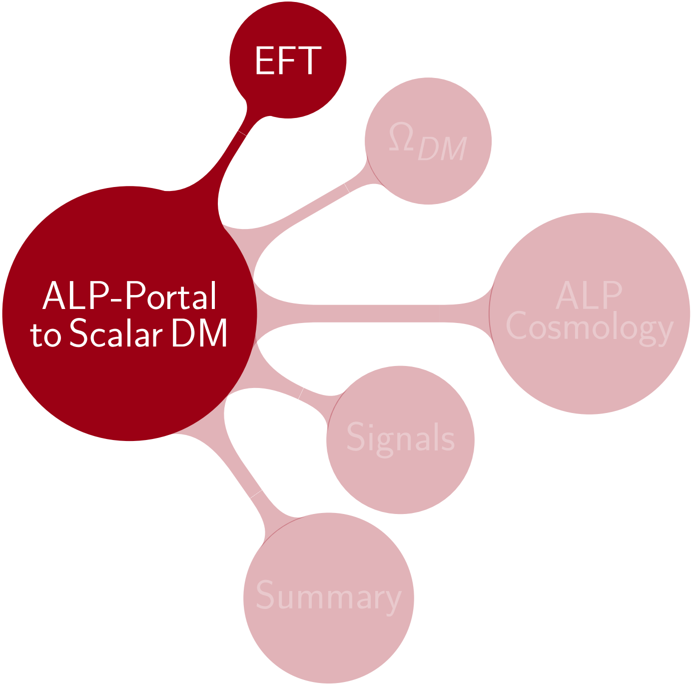
ALP-Portal to Scalar DM
ALP Cosmology
Signals
Summary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Derivative basis interaction

We start from the dimension 5 EFT (SM $+$ S, φ) in the derivative basis ($\varphi \to \varphi +$ const)

$V_{\text{mix}}(S, H)$ negligible [G.Arcadi *et al.*: 2403.15860] . . .

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$$\mathcal{L}^{(\partial)}_{S\varphi} = \mathcal{C}_S \frac{\partial_\mu \varphi}{2 f_\varphi} S^\dagger \, i \overleftrightarrow{\partial^\mu} S + \mathcal{O}\left(\frac{1}{f_\varphi^2}\right) ,$$

$$\mathcal{L}^{(\partial)}_{\varphi\, SM} = \frac{\varphi}{8\pi f_\varphi} \mathcal{C}_V \, \alpha_V \, V_{\mu\nu} \widetilde{V}^{\mu\nu} + \frac{\partial_\mu \varphi}{2 f_\varphi} \left(\mathcal{C}_H \, H^\dagger \, i \overleftrightarrow{D^\mu} \, H \right) + \\ + \frac{\partial_\mu \varphi}{2 f_\varphi} \left(\mathcal{C}_{\psi_L} \overline{\psi_L} \gamma^\mu \psi_L + \overline{\psi_R} \gamma^\mu \psi_R \right) + \mathcal{O}\left(\frac{1}{f_\varphi^2}\right) .$$

Changing basis

φ-dependent rotations remove derivative couplings
[Georgi, Kaplan, Randall: Phys. Lett. B, 169:73–78, 1986]

$$S \to \exp\left[i\,\mathcal{C}_{\mathcal{S}}\frac{\varphi}{2f_\varphi}\right] S\,, \quad H \to \exp\left[i\,\mathcal{C}_{H}\frac{\varphi}{2f_\varphi}\right] H\,, \quad \psi \to \exp\left[i\,\mathcal{C}_{\psi}\frac{\varphi}{2f_\varphi}\right] \psi$$

and φ-SM interactions are redefined [M. Bauer *et al.*: 2012.12272]

$$\mathcal{L}^{(\mathrm{N}\partial)}_{\varphi\,SM} = \frac{\varphi}{8\pi f_\varphi}\mathcal{C}'_V\,\alpha_V\,V_{\mu\nu}\tilde{V}^{\mu\nu} + \frac{\varphi}{2f_\varphi}\mathcal{C}'_\psi\,\overline{\psi_L}\,H\,\psi_R + \mathcal{O}\left(\frac{1}{f_\varphi^2}\right)\,,$$

while $\mathscr{L}_{\mathcal{S}\varphi}$... it's gone $\cancel{\mathscr{L}_{\mathcal{S}\varphi}}$!
But wait, where did the ALP-DM interaction go!?

Dark Matter Potential

The stabilizing symmetry acting on the DM is *crucial*! $\mathbb{Z}_3$

$$V_S^{(\partial)}(S) = m_S^2 S^\dagger S + \frac{1}{3!}(A\,S^3 + A^* S^{\dagger\,3}) + \frac{\lambda_S}{4}(S^\dagger S)^2$$

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$A \leq 3\sqrt{\lambda_S} m_S$ [G. Belanger *et al.*: 1211.1014]

$S \to \exp\left[i\,\mathcal{C}_S \frac{\varphi}{2f_\varphi}\right] S$

$$V_S^{(\mathrm{N}\partial)}(S) = V_S^{(\partial)}\left(e^{i\,\mathcal{C}_S \frac{\varphi}{2f_\varphi}} S\right)$$

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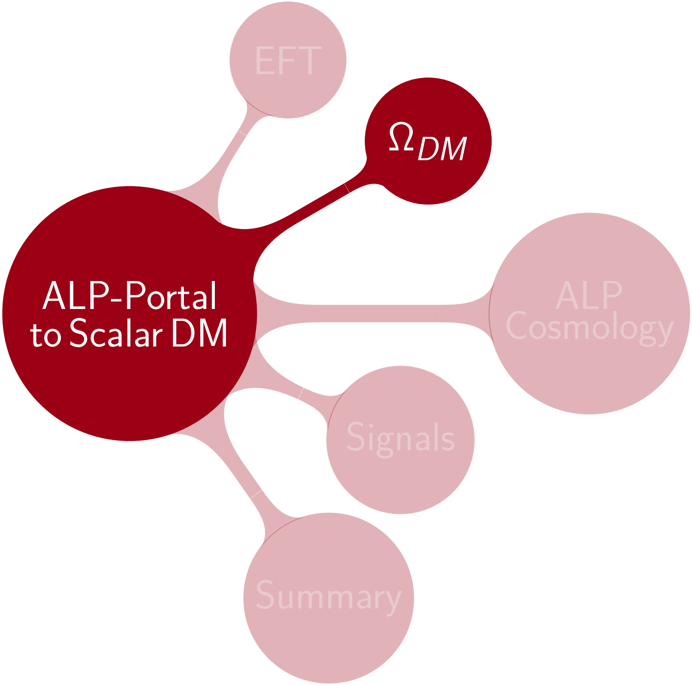
$$V_S^{(\mathrm{N}\partial)}(S) = V_S^{(\partial)}\left(e^{i\,\mathcal{C}_S \frac{\varphi}{2f_\varphi}} S\right)$$

$$\lambda_{S\varphi} \equiv i\frac{3}{2}\frac{\mathcal{C}_S A}{f_\varphi}$$

$$V_S^{(N\partial)}(S) \supset \frac{1}{3!}\left(\lambda_{S\varphi} S^3 + \lambda_{S\varphi}^* S^{\dagger\,3}\right)\varphi$$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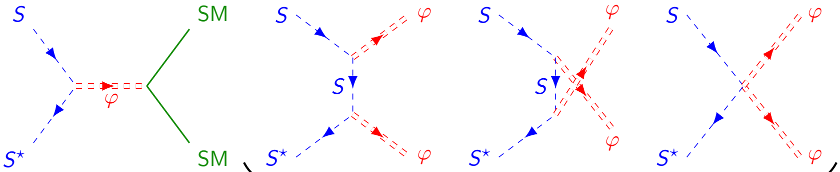
Dark Matter Freeze-Out

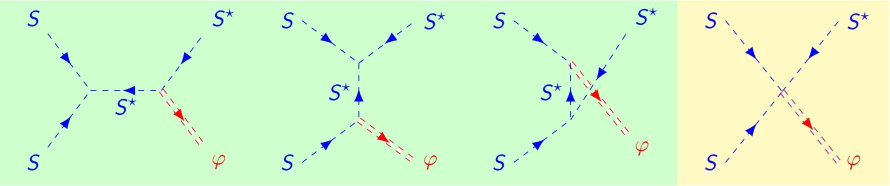
Standard DM annihilations are absent . . .



vanishing on-shell

[$O(1/f_\varphi^4)$, see UV completion]

Only *semi-annihilations* are active at tree-level at $O(1/f_\varphi^2)$!

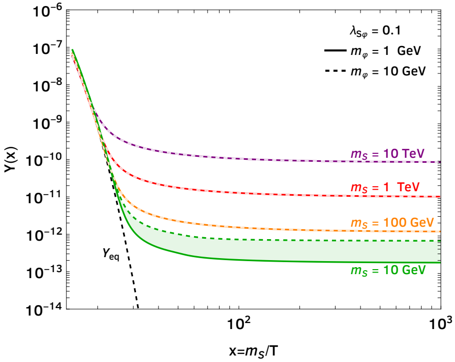


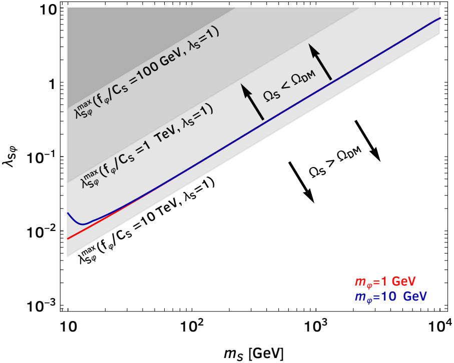
∂ basis

$N\partial$ basis

Relic density I: $N\partial$ Basis

$m_S \in [10,\ 10^4]$ GeV $\qquad m_\varphi \in [1,\ 10]$ GeV

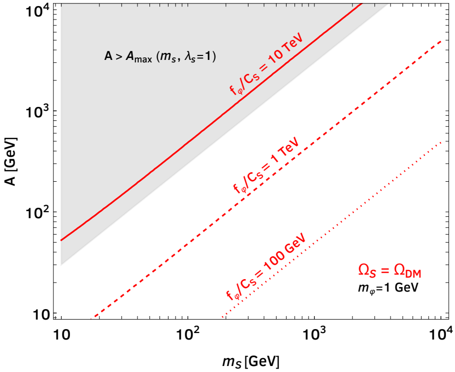


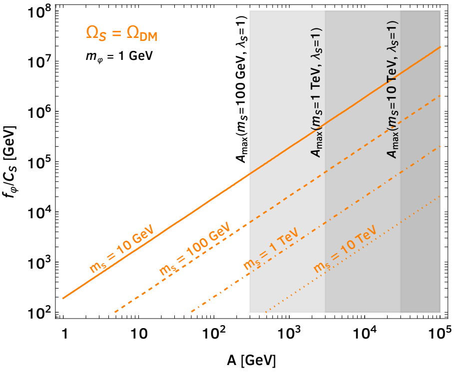


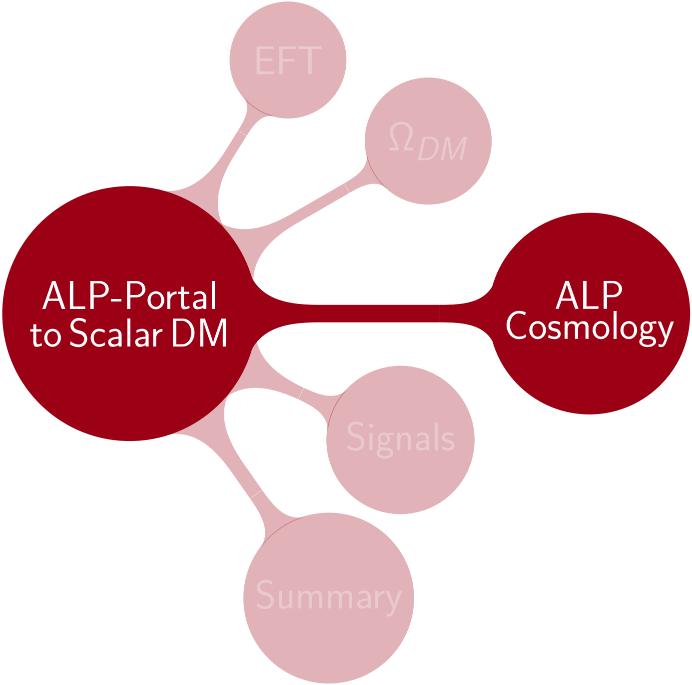
Relic density II: ∂ Basis

Unravel the three-dimensional parameters space $(m_S,\ A,\ f_\varphi)$

$$\lambda_{S\varphi} \equiv i\frac{3}{2}\frac{\mathcal{C}_S A}{f_\varphi}\ , \qquad A \leq 3\sqrt{\lambda_S}\, m_S$$



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General requirements

1. Short lifetime $\tau_\varphi \ll 1\,\mathrm{sec}$

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   - Escape BBN constraints [M. Kawasaki *et al.*: 1709.01211]
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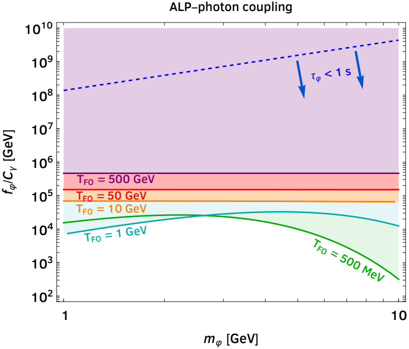
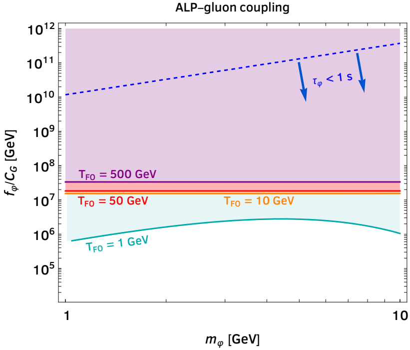
2. Thermal equilibrium at DM Freeze-Out
   - $T_{DS} = T_\gamma$
   - Sizeable ALP-SM couplings
   - $\mathcal{C}_i$ hierarchy?
   - $\left.\dfrac{\gamma_\alpha(T)}{H(T)s(T)}\right|_{T=T_{FO}} \gtrsim 1$

ALP coupling to SM gauge bosons





No $\frac{\mathcal{C}_\gamma}{\mathcal{C}_S}$ hierarchy needed

[D. Cadamuro, J. Redondo: 1110.2895]

No $\frac{\mathcal{C}_G}{\mathcal{C}_S}$ hierarchy needed

[F. D'Eramo, F. Hajkarim, S. Yun: 2108.05371]

[K. Bouzoud, J. Ghiglieri: 2404.06113]

ALP coupling to fermions

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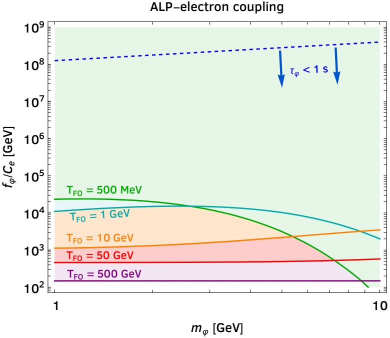
[A. Czarnecki *et al.*: 1110.2171]

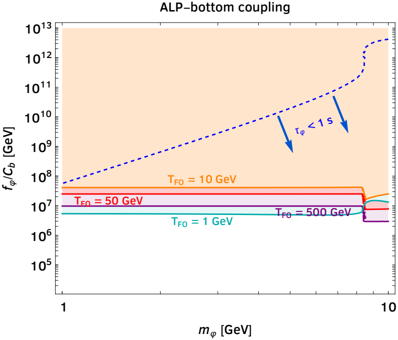
[M. Bauer *et al.*: 1708.00443]

[J. L. Feng *et al.*: 9709411]

[A. Crivellin *et al.*: 1402.1173]

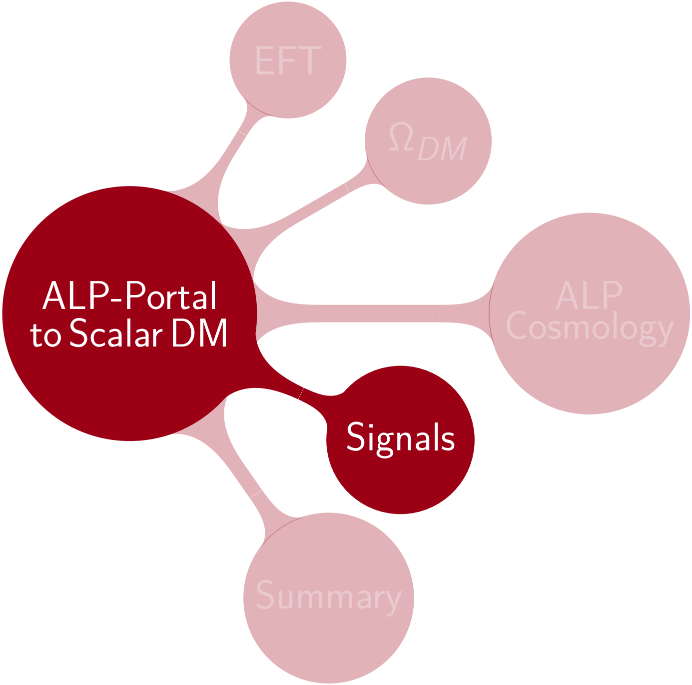
Warning: QCD and threshold effects in action





Need for small $\frac{\mathcal{C}_e}{\mathcal{C}_S}$ hierarchy

No need for $\frac{\mathcal{C}_b}{\mathcal{C}_S}$ hierarchy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What to Expect

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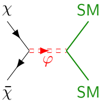
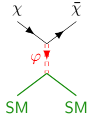
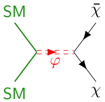
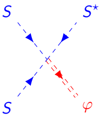
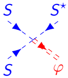
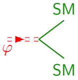
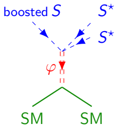
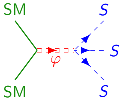
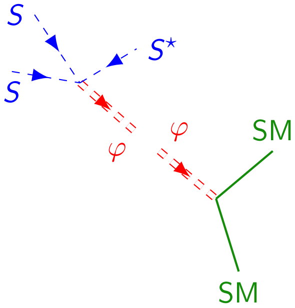
	Freeze-Out	Indirect	Direct	Collider
Fermion ($\mathbb{Z}_2$)	 [M. Cirelli *et al.*: 2406.01705, T.R. Slatyer: 1710.05137]		 [E. Del Nobile *et al.*: 1307.5955]	
Scalar ($\mathbb{Z}_3$)		 followed by  [Queiroz, Siqueira: 1901.10494]	 [Berger *et al.*: 1912.05558] [Kamenetskaia *et al.*: 2501.12117]	 [Toma: 2109.05911]

One-Step Cascade Semi-Annihilation



In progress ... [F.D'Eramo, S.Manconi, TS]

$$\frac{dN_X}{dx_S} = \frac{4}{3\zeta}\int_{x_\varphi^{min}}^{x_\varphi^{max}} \frac{dN_X}{dx_\varphi}\frac{dx_\varphi}{\sqrt{x_\varphi^2 - \epsilon_X^2}},$$

$$\zeta = \frac{4}{3}\frac{p_\varphi^{(GF)}}{m_S} \qquad x_\varphi = \frac{E_X}{m_S}$$

Key features

- $\varphi \to \mathrm{SM}\,\overline{\mathrm{SM}}$ shapes the spectrum
- Softer and broadened spectra for $\epsilon_\varphi \ll 1$
- Channel closes for $\epsilon_X \to 1$

Indirect Detection: γ-Ray Spectra from Semi-Annihilation

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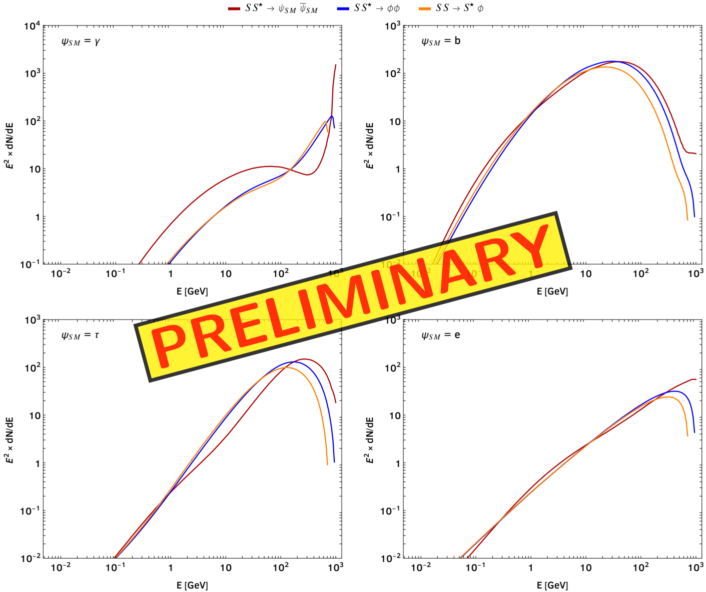

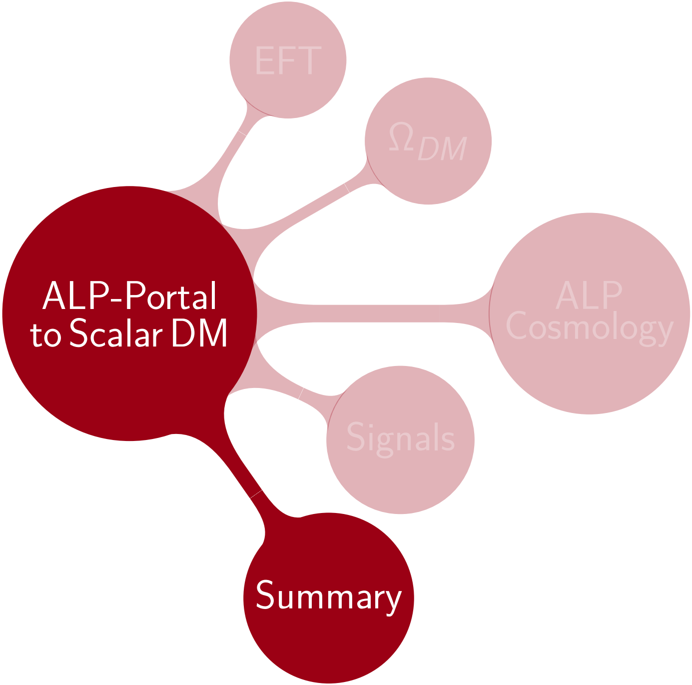
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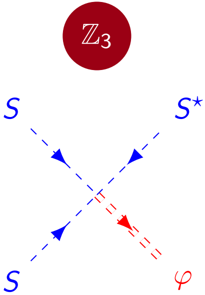
NEW! ALP-portal to *scalar* DM
[F.D'Eramo, TS: arXiv:2502.19491]

Governed by *semi-annihilations*



Imprints of symmetries on phenomenology

- DM relic independent of $\mathcal{C}_{\varphi-SM}$
- Accessible parameters space
- Peculiar phenomenology (suppressed DD, interesting ID)

Thank you!

SPARES

EFT: Derivative basis

$$\mathcal{L}^{(\partial)}_{\varphi\, SM} = \frac{\varphi}{8\pi f_\varphi} \sum_V \mathcal{C}_V \alpha_V V_{\mu\nu} \widetilde{V}^{\mu\nu} + \frac{\partial_\mu \varphi}{2 f_\varphi} \left(\mathcal{H}^\mu_\varphi + \mathcal{F}^\mu_\varphi \right) + \mathcal{O}\left(\frac{1}{f_\varphi^2} \right) ,$$

$$V = G,\ W,\ B\ ,$$

$$\mathcal{H}^\mu_\varphi = H^\dagger \overleftrightarrow{D^\mu} H\ ,$$

$$\mathcal{F}^\mu_\varphi = \sum_{j=1}^{3} \Big[\mathcal{C}^j_Q\, \bar{Q}^j_L \gamma^\mu Q^j_L + \mathcal{C}^j_u\, \bar{u}^j_R \gamma^\mu u^j_R + \mathcal{C}^j_d\, \bar{d}^j_R \gamma^\mu d^j_R +$$

$$+\ \mathcal{C}^j_E\, \bar{E}^j_L \gamma^\mu E^j_L + \mathcal{C}^j_e\, \bar{e}^j_R \gamma^\mu e^j_R \Big]$$

EFT: Non-Derivative basis

$$\mathcal{L}^{(\mathrm{N}\partial)}_{\varphi\, SM} = \frac{\varphi}{2f_\varphi}\mathcal{P}_\varphi + \frac{\varphi}{8\pi f_\varphi}\sum_V \mathcal{C}'_V \alpha_V V_{\mu\nu}\tilde{V}^{\mu\nu} + \mathcal{O}\left(\frac{1}{f_\varphi^2}\right) \;,$$

$$\begin{aligned}\mathcal{P}_\varphi = -i\sum_{j=1}^{3}\Big[&(\mathcal{C}_u^j - \mathcal{C}_Q^j - \mathcal{C}_H) y_u^j \,\bar{Q}_L^j \tilde{H} u_R^j + \\ &(\mathcal{C}_d^j - \mathcal{C}_Q^j + \mathcal{C}_H) y_d^j \,\bar{Q}_L^j H d_R^j + \\ &+ (\mathcal{C}_e^j - \mathcal{C}_E^j + \mathcal{C}_H) y_e^j \,\bar{E}_L^j H e_R^j\Big] + \mathrm{h.c.}\;.\end{aligned}$$

$N\partial$- Wilson Coefficients

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$$
\begin{aligned}
\mathcal{C}'_G &= \mathcal{C}_G - \sum_{j=1}^{3}\left(\mathcal{C}_Q^j - \frac{1}{2}\mathcal{C}_u^j - \frac{1}{2}\mathcal{C}_d^j\right) \ , \\
\mathcal{C}'_W &= \mathcal{C}_W - \sum_{j=1}^{3}\left(\frac{3}{2}\mathcal{C}_Q^j + \frac{1}{2}\mathcal{C}_E^j\right) \ , \\
\mathcal{C}'_B &= \mathcal{C}_B - \sum_{j=1}^{3}\left(\frac{1}{6}\mathcal{C}_Q^j + \frac{1}{2}\mathcal{C}_E^j - \frac{4}{3}\mathcal{C}_u^j - \frac{1}{3}\mathcal{C}_d^j - \mathcal{C}_e^j\right) \ .
\end{aligned}
$$

Counting Wilson Coefficients

We count the number of independent $\mathcal{C}$ to be:

$$
\begin{array}{lr}
\mathcal{C}_V \times 3 \qquad V = G, W, B & + \\
\mathcal{C}_S & + \\
\{\mathcal{C}_u, \mathcal{C}_c, \mathcal{C}_t\} = \left\{ \frac{-\mathcal{C}_Q^1 + \mathcal{C}_u^1}{2}, \frac{-\mathcal{C}_Q^2 + \mathcal{C}_u^2}{2}, \frac{-\mathcal{C}_Q^3 + \mathcal{C}_u^3}{2} \right\} & + \\
\{\mathcal{C}_d, \mathcal{C}_s, \mathcal{C}_b\} = \left\{ \frac{-\mathcal{C}_Q^1 + \mathcal{C}_d^1}{2}, \frac{-\mathcal{C}_Q^2 + \mathcal{C}_d^2}{2}, \frac{-\mathcal{C}_Q^3 + \mathcal{C}_d^3}{2} \right\} & + \\
\{\mathcal{C}_e, \mathcal{C}_\mu, \mathcal{C}_\tau\} = \left\{ \frac{-\mathcal{C}_E^1 + \mathcal{C}_e^1}{2}, \frac{-\mathcal{C}_E^2 + \mathcal{C}_e^2}{2}, \frac{-\mathcal{C}_E^3 + \mathcal{C}_e^3}{2} \right\} & = \\
\hline
 & 13
\end{array}
$$

Boltzmann Equation for DM

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$$\frac{dY_S}{d\ln x} = -\left(1 - \frac{1}{3}\frac{d\ln g_{*s}}{d\ln x}\right)\frac{s(x)\,\langle\sigma_{SS\to S^\star\varphi}v_{\text{Møl}}\rangle}{H(x)}\left[Y_S^2 - Y_S\,Y_S^{\text{eq}}\right]$$

$$\langle\sigma_{SS\to S^\star\varphi}v_{\text{Møl}}\rangle = \frac{\int_{4m_S^2}^{\infty} ds\,\sqrt{s}\,\left(s-4m_S^2\right)\,\sigma_{SS\to S^\star\varphi}(s)\,K_1\left[\sqrt{s}/T\right]}{8m_S^4\,TK_2[m_S/T]^2}$$

$$\simeq \frac{\lambda_{S\varphi}^2}{128\pi\,m_S^2}\sqrt{9-10\left(\frac{m_\varphi}{m_S}\right)^2+\left(\frac{m_\varphi}{m_S}\right)^4}+\mathcal{O}\left(\frac{T}{m_S}\right)$$

Boltzmann Equation for ALPs

$$\frac{dY_\varphi}{d\ln x} = -\left(1 - \frac{1}{3}\frac{d\ln g_{*s}}{d\ln x}\right)\frac{\sum_\alpha \gamma_\alpha(x)}{H(x)s(x)}\left(1 - \frac{Y_\varphi}{Y_\varphi^{\mathrm{eq}}}\right)$$

$$\gamma_{\mathcal{B}_1\mathcal{B}_2\to\varphi} = n_\varphi^{\mathrm{eq}}\frac{K_1[m_\varphi/T]}{K_2[m_\varphi/T]}\Gamma_{\varphi\to\mathcal{B}_1\mathcal{B}_2}$$

Inverse Decays

$$\begin{aligned}\gamma_{\mathcal{B}_1\mathcal{B}_2\to\mathcal{B}_3\varphi} &= n_{\mathcal{B}_1}^{\mathrm{eq}} n_{\mathcal{B}_2}^{\mathrm{eq}}\langle\sigma_{\mathcal{B}_1\mathcal{B}_2\to\mathcal{B}_3\varphi}v_{\text{Møl}}\rangle \\ &= n_{\mathcal{B}_3}^{\mathrm{eq}} n_\varphi^{\mathrm{eq}}\langle\sigma_{\mathcal{B}_3\varphi\to\mathcal{B}_1\mathcal{B}_2}v_{\text{Møl}}\rangle\end{aligned}$$

Binary scatterings

ALP coupled to SM gauge bosons: scattering rates

$$\gamma^{(\gamma)}_{\rm scattering} = \frac{\alpha_{\rm em}^3}{144\pi^2}\left(\frac{\mathcal{C}_\gamma}{f_\varphi}\right)^2 T^3\, n_Q(T)\left[\log\left(\frac{T^2}{m_\gamma^2(T)}\right) + 0.8194\right]$$

$$\gamma^{(G)}_{\rm scattering} = \frac{2\zeta(3)\mathcal{D}_G}{\pi^3}\left(\frac{\mathcal{C}_G\,\alpha_s}{8\pi f_\varphi}\right)^2 T^6\, F_G(T)$$

ALP coupled to SM gauge bosons: lifetime

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$$\Gamma_{\varphi\to\gamma\gamma} = \frac{\mathcal{C}_\gamma^2 \alpha_{em}^2 m_\varphi^3}{256\pi^3 f_\varphi^2}$$

$$\downarrow$$

$$\tau_{\varphi\to\gamma\gamma} = \Gamma^{-1}_{\varphi\to\gamma\gamma} \approx 0.5\,\mathrm{s}\left(\frac{1}{\mathcal{C}_\gamma}\right)^2 \left(\frac{0.01}{\alpha_{\rm em}}\right)^2 \left(\frac{1\,\mathrm{GeV}}{m_\varphi}\right)^3 \left(\frac{f_\varphi}{10^8\,\mathrm{GeV}}\right)^2$$

$$\Gamma_{\varphi\to gg} = 8\times\frac{\mathcal{C}_G^2 \alpha_s^2 m_\varphi^3}{256\pi^3 f_\varphi^2}$$

$$\downarrow$$

$$\tau_{\varphi\to gg} = \Gamma^{-1}_{\varphi\to gg} \approx 0.7\,\mathrm{s}\left(\frac{1}{\mathcal{C}_G}\right)^2 \left(\frac{0.3}{\alpha_s}\right)^2 \left(\frac{1\,\mathrm{GeV}}{m_\varphi}\right)^3 \left(\frac{f_\varphi}{10^{10}\,\mathrm{GeV}}\right)^2$$

ALP coupled to SM fermions

Thermalization receives three main contributions:

$$\gamma^{(\psi)}_{\mathrm{Inv-D}} = n^{\mathrm{eq}}_{\varphi} \frac{K_1[m_\varphi/T]}{K_2[m_\varphi/T]} \Gamma_{\varphi\to\bar{\psi}\psi}$$ Inverse Decays

$$\gamma^{(\psi)}_{\mathrm{Pair}} = (n^{\mathrm{eq}}_{\psi})^2 \langle \sigma_{\psi\bar{\psi}\to V\varphi} v_{\text{Møl}} \rangle$$ Fermion Pair Annihilation

$$\gamma^{(\psi)}_{\mathrm{Compton}} = 2 \times n^{\mathrm{eq}}_{\psi} n^{\mathrm{eq}}_{V} \langle \sigma_{\psi V\to\psi\varphi} v_{\text{Møl}} \rangle$$ Inverse Compton

Scattering cross sections

Accounting for m_φ effects:

$$\sigma_{\psi\bar{\psi}\to V\varphi}(s) = \frac{\mathcal{C}_\psi^2 g_V^2}{N_{\rm avg}\,\pi f_\varphi^2}\frac{m_\psi^2}{s(s-4m_\psi^2)(s-m_\varphi^2)}\times$$

$$\times\left[(s^2-4m_\varphi^2 m_\psi^2+m_\varphi^4)\tanh^{-1}\left(\sqrt{1-\frac{4m_\psi^2}{s}}\right)-s\,m_\varphi^2\sqrt{1-\frac{4m_\psi^2}{s}}\,\right]$$

$$\sigma_{\psi V\to\psi\varphi}(s) = \frac{\mathcal{C}_\psi^2 g_V^2}{N_{\rm avg}\,\pi f_\varphi^2}\frac{\sqrt{\lambda(s,m_\psi,m_\varphi)}\,m_\psi^2}{s^2(s-m_\psi^2)^3}\times$$

$$\times\left[4s^2\frac{\lambda(s,m_\psi,m_\varphi)+m_\varphi^4}{\sqrt{\lambda(s,m_\psi,m_\varphi)}}\coth^{-1}\left(\frac{s+m_\psi^2-m_\varphi^2}{\sqrt{\lambda(s,m_\psi,m_\varphi)}}\right)+\right.$$

$$\left.+\,m_\varphi^2\left(7s^2+2\,s\,m_\psi^2-m_\psi^4\right)-\left(s-m_\psi^2\right)^2\left(3s-m_\psi^2\right)\right]$$

ALP lifetime I: Electron

$$\Gamma_{\varphi \to e^+ e^-} = \frac{\mathcal{C}_e^2 m_\varphi m_e^2}{8\pi f_\varphi^2} \sqrt{1 - \frac{4 m_e^2}{m_\varphi^2}}$$

$$\tau_{\varphi \to gg} = \Gamma^{-1}_{\varphi \to gg} \approx 0.7\,\mathrm{s} \left(\frac{1}{\mathcal{C}_G}\right)^2 \left(\frac{0.3}{\alpha_G}\right)^2 \left(\frac{1\,\mathrm{GeV}}{m_\varphi}\right)^3 \left(\frac{f_\varphi}{10^{10}\,\mathrm{GeV}}\right)^2$$

ALP lifetime II: Bottom

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$$\Gamma_\varphi = \begin{cases} \Gamma_{\varphi\to b\bar{b}} = 3 \times \dfrac{\mathcal{C}_b^2 m_\varphi m_b^2}{8\pi f_\varphi^2}\sqrt{1-\dfrac{4m_b^2}{m_\varphi^2}} & , \quad m_\varphi > 2m_b \\ \begin{cases} \Gamma_{\varphi\to\psi'\bar{\psi}'} = N_c^{\psi'} \times \dfrac{\mathcal{C}_{\psi'}^2 m_\varphi m_{\psi'}^2}{8\pi f_\varphi^2}\sqrt{1-\dfrac{4m_{\psi'}^2}{m_\varphi^2}} \\ \Gamma_{\varphi\to VV} = \mathcal{D}_V \dfrac{\mathcal{C}_V^2 \alpha_V^2 m_\varphi^3}{256\pi^3 f_\varphi^2} \end{cases} & , \quad m_\varphi \leq 2m_b \end{cases}$$

$$\mathcal{C}_{\psi'} \simeq -\mathcal{C}_b\, N_c^b \frac{Y_b^2}{8\pi^2} \ln\left(\frac{f_\varphi}{m_\varphi}\right), \quad \mathcal{C}_\gamma = 2\,\mathcal{C}_\psi\, N_c^\psi Q_\psi^2 \left(1 - \frac{\arcsin^2(x_\psi)}{x_\psi^2}\right), \quad \mathcal{C}_G = \mathcal{C}_b \left(1 - \frac{\arcsin^2(x_\psi)}{x_\psi^2}\right)$$

$$\tau_\varphi = \begin{cases} \Gamma^{-1}_{\varphi\to b\bar{b}} \approx 1.4\,\mathrm{s} \left(\dfrac{1}{\mathcal{C}_b}\right)^2 \left(\dfrac{10\ \mathrm{GeV}}{m_\varphi}\right) \left(\dfrac{f_\varphi}{5\times 10^{12}\ \mathrm{GeV}}\right)^2, & m_\varphi > 2m_b \\ \Gamma^{-1}_{\varphi\to gg} \approx 0.6\,\mathrm{s} \left(\dfrac{1}{\mathcal{C}_b}\right)^2 \left(\dfrac{2\ \mathrm{GeV}}{m_\varphi}\right)^7 \left(\dfrac{f_\varphi}{5\times 10^{8}\ \mathrm{GeV}}\right)^2, & m_\varphi \leq 2m_b \end{cases}$$

A Possible UV Completion I

The UV theory contains extra fields and a global U_φ symmetry

Field	Φ	S	Ψ_L	Ψ_R
Type	Complex scalar	Complex scalar	Weyl	Weyl
U_φ charge	-3	1	ξ_Ψ	$\xi_\Psi - 3$

$$\mathcal{L}_{\rm UV} \supset -\left(y_\Psi \Phi^\dagger \bar{\Psi}_L \Psi_R + \text{h.c.}\right) + \overset{\ll 1}{\cancel{\lambda_{\Phi H}}}\, \Phi^\dagger \Phi\, H^\dagger H + \overset{\ll 1}{\cancel{\lambda_{\Phi S}}}\, \Phi^\dagger \Phi\, S^\dagger S + \left(\kappa_\star \Phi S^3 + \text{h.c.}\right)$$

A Possible UV Completion II

After Φ taking a VEV $\Phi \to \left(\frac{v_\Phi + \rho}{\sqrt{2}}\right) \exp\left[i \frac{\varphi}{v_\Phi}\right] \ldots$

$$\mathcal{L}_\varphi = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{v_\Phi}{\sqrt{2}}\left[\kappa_\star \exp\left[i\frac{\varphi}{v_\Phi}\right] S^3 + y_\Psi \exp\left[-i\frac{\varphi}{v_\Phi}\right]\bar{\Psi}_L \Psi_R + \mathrm{h.c.}\right]$$

and integrating out heavy modes $\ldots$

$$\mathcal{L}_\varphi^{N\partial} \supset \frac{\varphi}{8\pi v_\Phi}\left[\alpha_G G_{\mu\nu}\widetilde{G}^{\mu\nu} + 2Y_\Psi^2 \alpha_B B_{\mu\nu}\widetilde{B}^{\mu\nu}\right] + \boxed{\frac{v_\Phi}{\sqrt{2}}\left[\kappa_\star \exp\left[i\frac{\varphi}{v_\Phi}\right] S^3 + \mathrm{h.c.}\right]}$$

$\mathbb{Z}_3$ *arises!*

Matching UV completion onto our EFT

The UV theory matches onto our EFT as ($\mathcal{C}_G = 1$):

$$f_\varphi = v_\Phi \,, \qquad \mathcal{C}_B = 2Y_\Psi^2 \,, \qquad A = 3\sqrt{2}\,\kappa_\star v_\Phi$$

Higher order potential terms are naturally suppressed:

$$V_S(S) \supset \frac{\lambda_5}{4!}\frac{1}{v_\Phi}S^4 S^\dagger \,, \qquad \lambda_5 = \frac{6\sqrt{2}k_\star \lambda_{\Phi S}}{\lambda_\Phi} \ll 1$$

φ-dependent rotations $S \to e^{[i\varphi/(3f_\varphi)]}S$ recover ∂-basis Lagrangian up to dim-6 interaction terms

$$\delta\mathcal{L}^{(6)}_{\rm INT} = \frac{\mathcal{C}_S'^2}{4f_\varphi^2}|\partial_\mu\varphi|^2 S^\dagger S \qquad \mathcal{C}_S = \mathcal{C}_S' = \frac{2}{3}$$

Indirect detection spectra I

ALP particles are produced in $SS \to S^\star \varphi$ processes with

$$E_\varphi^{(\mathrm{GF})} = \frac{3}{4} m_S \left(1 + \frac{\epsilon_\varphi^2}{3}\right) \qquad p_\varphi^{(\mathrm{GF})} = m_S \sqrt{\lambda\left(1, \frac{1}{2}, \frac{\epsilon_\varphi}{2}\right)} \qquad \epsilon_\varphi = \frac{m_\varphi}{m_S}$$

Subsequent visible $\varphi \to XX$ decay products are isotropic and monochromatic

$$\frac{dN_X}{dE'_X \, d\cos\theta'} = \delta\left(E'_X - \frac{m_\varphi}{2}\right) \qquad \int dE_X \, d\cos\theta \frac{dN_X}{dE_X \, d\cos\theta} = 2$$

Indirect detection spectra II

Kinematic variables in the GF are related to those in the ALP rest frame

$$E_X^{(\mathrm{GF})} = \gamma_L (E'_X + \beta_L \, p'_X \cos\theta') \ .$$

Differential energy injection spectra in the GF are computed as

$$\frac{dN_X}{dE_X} = \int d\cos\theta' dE'_X \ \frac{dN_X}{dE'_X \, d\cos\theta'} \ \delta\left(E_X - E_X^{(\mathrm{GF})}\right) \ .$$

Indirect detection spectra III

Introduce dimensionless variables

$$\epsilon_X \equiv \frac{2m_X}{m_\varphi}, \qquad x_\varphi \equiv \frac{2E'_X}{m_\varphi}, \qquad x_S \equiv \frac{E_X}{m_S}$$

and apply this change of variable to the differential spectrum

$$\frac{dN_X}{dx_S} = 2\int d\cos\theta' dx_\varphi \frac{dN_X}{dx_\varphi\, d\cos\theta'} \times$$

Master Formula

$$\times\, \delta\left(2x_S - \frac{3}{4}\left(x_\varphi\left(1+\frac{\epsilon_\varphi^2}{3}\right) + \zeta\sqrt{x_\varphi^2 - \epsilon_X^2}\cos\theta'\right)\right)$$

Indirect detection spectra IV

Upon isotropy assumption and performing the $\int_{-1}^{1} d\cos\theta$ integral

$$\frac{dN_X}{dx_S} = \frac{4}{3\zeta}\int_{x_\varphi^{\min}}^{x_\varphi^{\max}} \frac{dx_\varphi}{\sqrt{x_\varphi^2 - \epsilon_X^2}}\frac{dN_X}{dx_\varphi}\,,$$

$$x_\varphi^{\min} = \frac{8x_S\left(3+\epsilon_\varphi^2\right) - 3\zeta\sqrt{64x_S^2 - \epsilon_X^2\left[\left(3+\epsilon_\varphi^2\right) - 9\zeta^2\right]}}{\left(3+\epsilon_\varphi^2\right) - 9\zeta^2}\,,$$

$$x_\varphi^{\max} = \mathrm{Min}\left\{1, \frac{8x_S\left(3+\epsilon_\varphi^2\right) + 3\zeta\sqrt{64x_S^2 - \epsilon_X^2\left[\left(3+\epsilon_\varphi^2\right) - 9\zeta^2\right]}}{\left(3+\epsilon_\varphi^2\right) - 9\zeta^2}\right\}\,.$$