

Growth of Cosmic Strings with Field-Dependent Tension

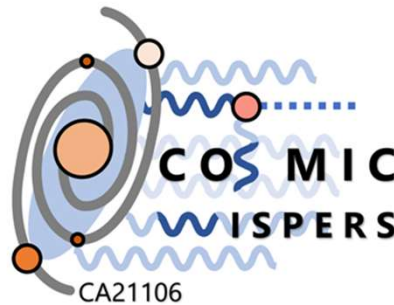
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3rd General Meeting of Cost Action Cosmic WISPers,
Sofia 09/09/2025



ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA



Based on: LB, M. Cicoli, F. Pedro: 2503.11293

Why Cosmic Strings?

Cosmic Strings: one-dimensional objects emerging in BSM theories [**Vilenkin, Shellard: 2000**].

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In String Theory, **Cosmic String Networks** can form:

- From **brane-antibrane annihilation** at the end of inflation (see [**Cicoli et al:2024**] for recent progress)
- From **closed strings** which **grow** and percolate into a network [**Conlon et al: 2024**]

Once formed, **networks** reach a **scaling regime** [**Martins, Shellard:1996, 2000; Revello, Villa:2024**]:

$$\rho_s \sim \rho_{\text{bkg}}$$

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The network **emits GWs** through production of loops [Allen, Caldwell: 1992; Lewicki et al: 2018; Ghoshal, Revello, Villa: 2025].

Recently proposed to fit the **PTA signal** [Ellis et al.:2023; Avgoustidis et al.:2025]

In scaling regime: can give information about the **cosmological history of the Early Universe!**

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Goal: study the mechanism underlying **formation** of a string network through **comoving growth** of elementary strings in Type IIB String Theory.

Setup: population of **closed circular loops** nucleated **after inflation** can **grow to cosmological** size and form a network if their **tension decreases** sufficiently fast [**Conlon et al.:2024; Revello, Villa:2024**]

Well-established case: **fundamental strings** with tension controlled by **kinating volume** of the internal dimensions.

We analyse the problem using Dynamical System Techniques, and find this mechanism works in **more general situations!**

Overview

- **Part I: Strings with Time-Dependent Tension**
- **Part II: Strings with Field-Dependent Tension and Dynamical System Approach**
- **Part III: Cosmic Strings in Type IIB String Theory**

Part I:

Strings with Time-Dependent Tension

Equations of Motion

Nambu-Goto action for a string with spacetime-dependent tension:

$$S_{\text{NG}} = - \int d^2\sigma \mu(x) \sqrt{-\gamma}$$

with $x^\mu = x^\mu(\sigma)$ and $\gamma_{ab} = \frac{\partial x^\mu}{\partial \sigma^a} \frac{\partial x^\nu}{\partial \sigma^b} g_{\mu\nu}$ in FLRW spacetime $ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$

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Varying the action [Emond et al. :2021]:

$$\frac{1}{\mu\sqrt{-\gamma}} \partial_a (\mu\sqrt{-\gamma} \gamma^{ab} x_{,b}^\mu) + \Gamma_{\nu\rho}^\mu \gamma^{ab} x_{,a}^\nu x_{,b}^\rho - \frac{\partial^\mu \mu}{\mu} = 0$$

Equations of Motion of the Circular Loop

Gauge-fixing:

$$\sigma^0 = x^0 = t$$

and

$$\frac{\partial \mathbf{x}}{\partial \sigma^0} \cdot \frac{\partial \mathbf{x}}{\partial \sigma^1} = 0$$

Circular-loop ansatz:

$$\mathbf{x}(t) = R(t) (\cos \theta, \sin \theta, 0)$$

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Equations of motion ($\mu(x) \equiv \mu(t)$):

$$\frac{\dot{\epsilon}}{\epsilon} = H - a^2 \dot{R}^2 \left(2H + \frac{\dot{\mu}}{\mu} \right)$$

$$\ddot{R} + H \dot{R} + \frac{R}{\epsilon^2} + \left(2H + \frac{\dot{\mu}}{\mu} \right) (1 - a^2 \dot{R}^2) \dot{R} = 0$$

Where:

$$\epsilon^2 \equiv \frac{a^2 R^2}{1 - a^2 \dot{R}^2} \quad \longrightarrow \quad \boxed{\epsilon(t) = a(t) R_{\max}(t)}$$

Physical Radius of the String

Growth Condition

Growth in comoving coordinates if:

$$\frac{\dot{\epsilon}}{\epsilon} > H$$

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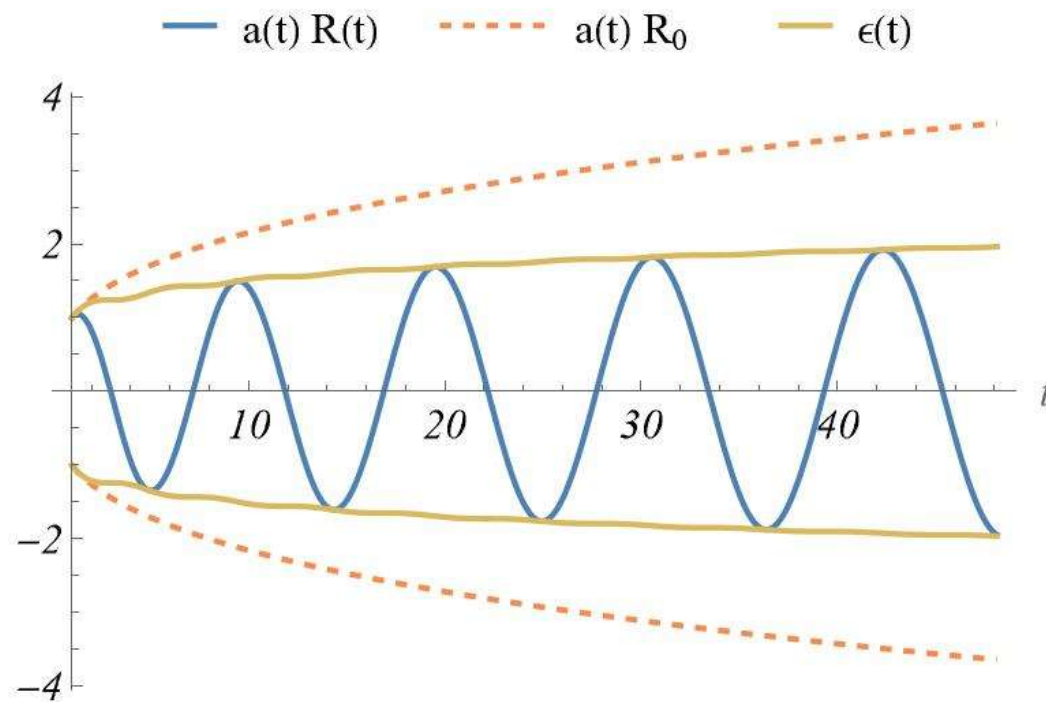
In expanding Universe, growth assured by the growth condition [Conlon et al.: 2024] :

$$2H + \frac{\dot{\mu}}{\mu} < 0$$

If string tension decreases sufficiently fast with the scale factor, the physical radius of the string grows faster than Hubble radius and loops can percolate!

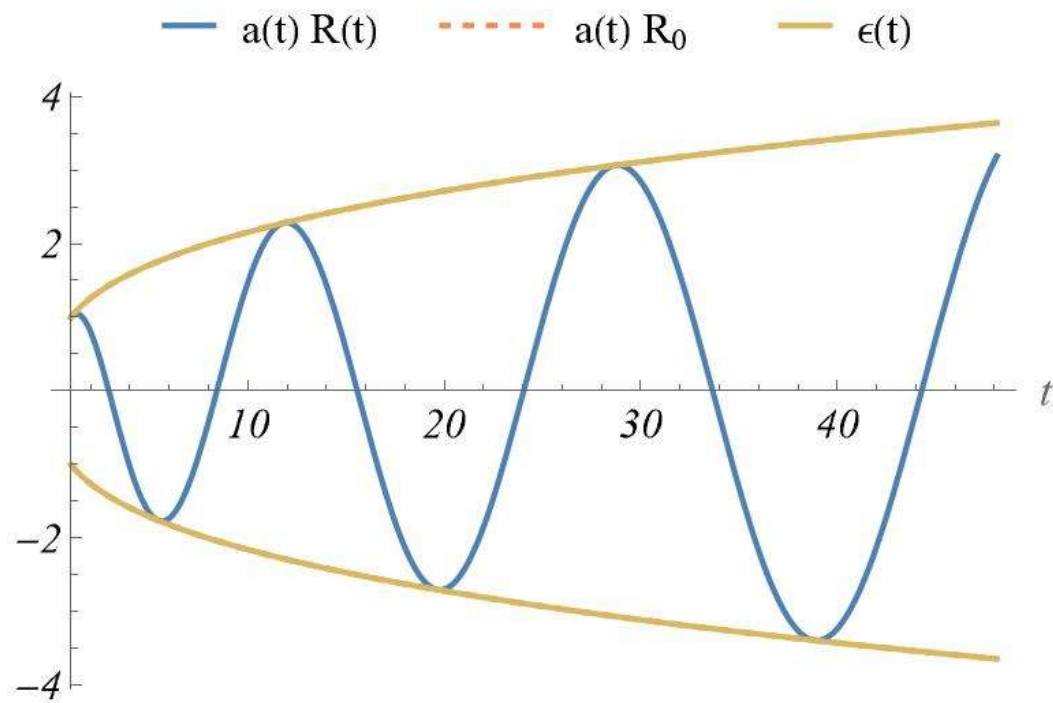
$$\mu \sim a^{-q}, \quad q > 2 \implies \text{Net Growth!}$$

$$\mu \sim a^{-q}$$



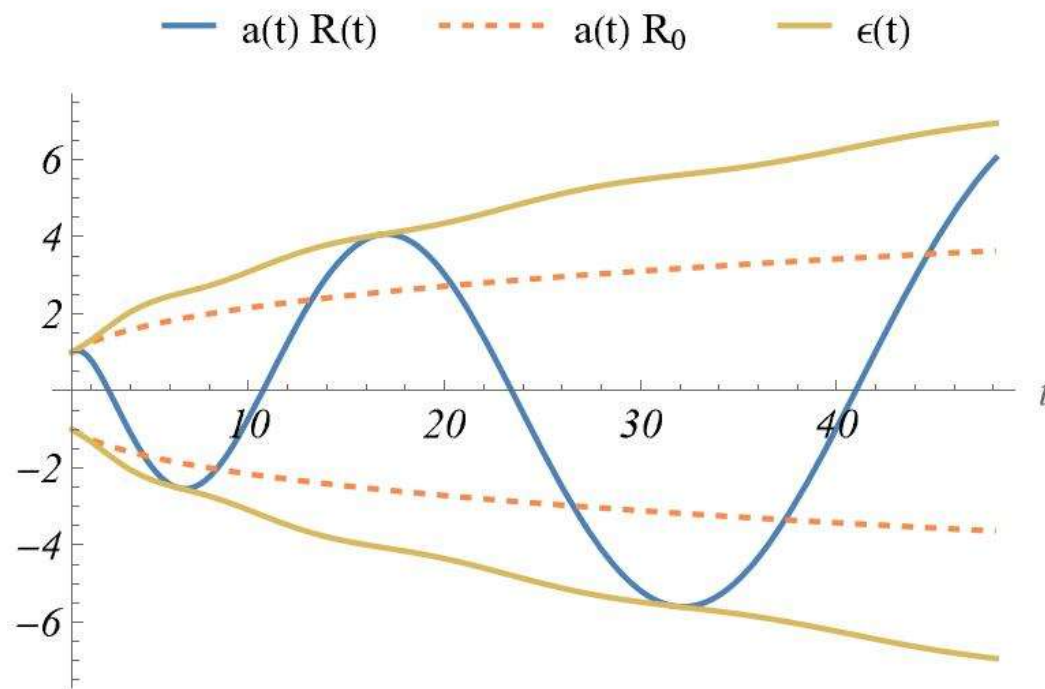
$$q < 2$$

$$\mu \sim a^{-q}$$



$$q = 2$$

$$\mu \sim a^{-q}$$



$$q > 2$$

Part II:
Strings with Field-Dependent Tension and
Dynamical System Approach

Field-Dependent Tension

Suppose time-variation of tension set by time-dependence of a scalar field ϕ

Consider a population of string loops with exponential tension:

$$\mu = \mu_0 e^{-\xi \frac{\phi}{M_p}}$$

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In number of efoldings $N = \ln a$ the growth condition becomes ($M_p = 1$) :

$$2 - \xi \phi' < 0$$

Depends on ξ and dynamics of ϕ !

We can study it using Dynamical System (DS) Techniques!

The Dynamical System Setup

Flat FLRW Universe filled with background perfect fluid with eos:

$$p_f = \omega \rho_f$$

Initial population of string loops with negligible energy density (however, see [Conlon et al.:2025]):

$$\rho_s \ll \rho_\phi, \rho_f$$

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Define phase space variables:

$$X = \frac{\phi'}{\sqrt{6}} \qquad Y = \frac{1}{H} \sqrt{\frac{V(\phi)}{3}}$$

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Friedmann equation:

$$X^2 + Y^2 + \frac{\rho_f}{3H^2} = 1 \Rightarrow X^2 + Y^2 \leq 1$$

The Dynamical System Setup

Consider an exponential potential:

$$V = V_0 e^{-\lambda\phi}$$

Klein-Gordon and Continuity eq. in phase space [Copeland, Liddle, Wands:1998]:

$$X' = -3X + \lambda\sqrt{\frac{3}{2}}Y^2 + \frac{3}{2}X [2X^2 + (\omega + 1)(1 - X^2 - Y^2)]$$

$$Y' = -\lambda\sqrt{\frac{3}{2}}XY + \frac{3}{2}Y [2X^2 + (\omega + 1)(1 - X^2 - Y^2)] .$$

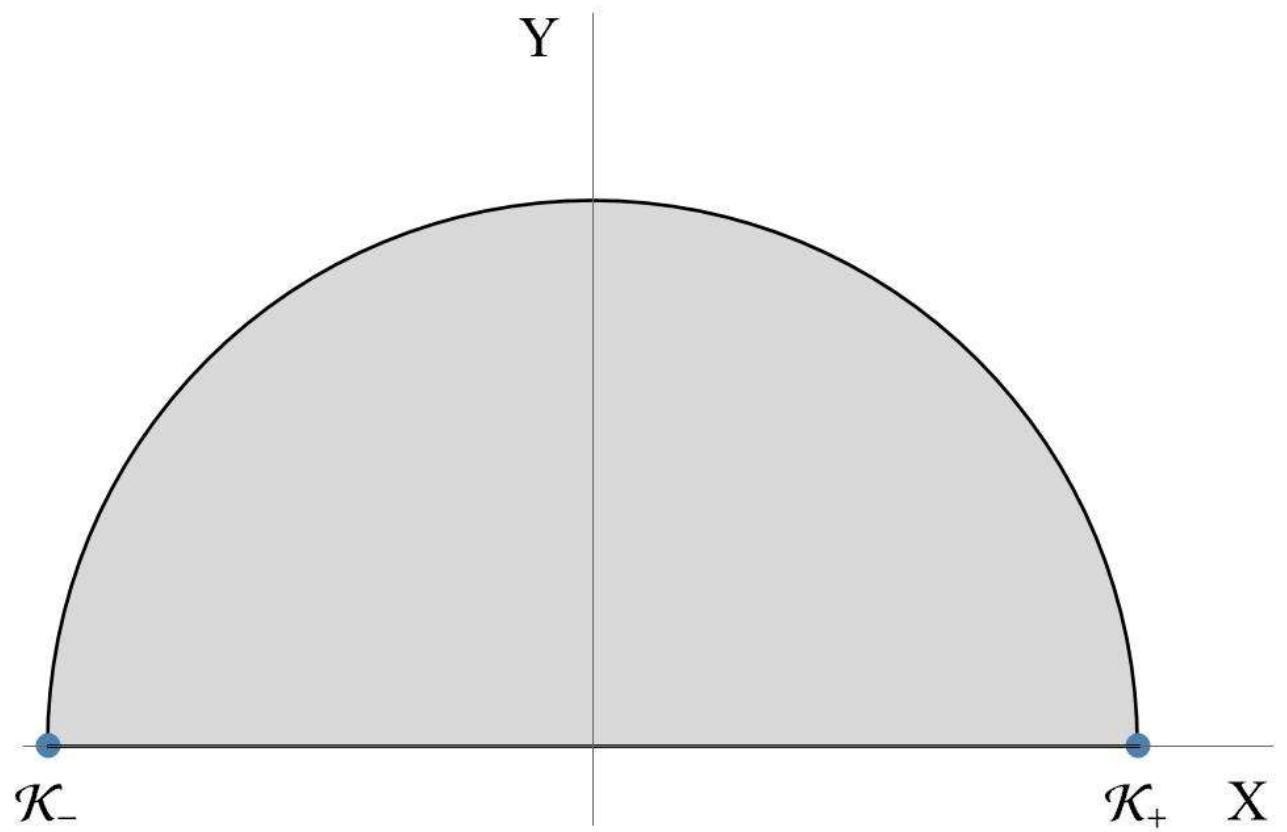
Fixed points fully classified!

Phase Space and Fixed Points

$$X = \frac{\phi'}{\sqrt{6}}$$

$$Y = \frac{1}{H} \sqrt{\frac{V(\phi)}{3}}$$

FP	X	Y
$\mathcal{K}_+$	1	0
$\mathcal{K}_-$	-1	0
$\mathcal{F}$	0	0
$\mathcal{M}$	$\frac{\lambda}{\sqrt{6}}$	$\sqrt{1 - \frac{\lambda^2}{6}}$
$\mathcal{S}$	$\sqrt{\frac{3}{2}} \frac{\omega+1}{\lambda}$	$\sqrt{\frac{3(1-\omega^2)}{2\lambda^2}}$



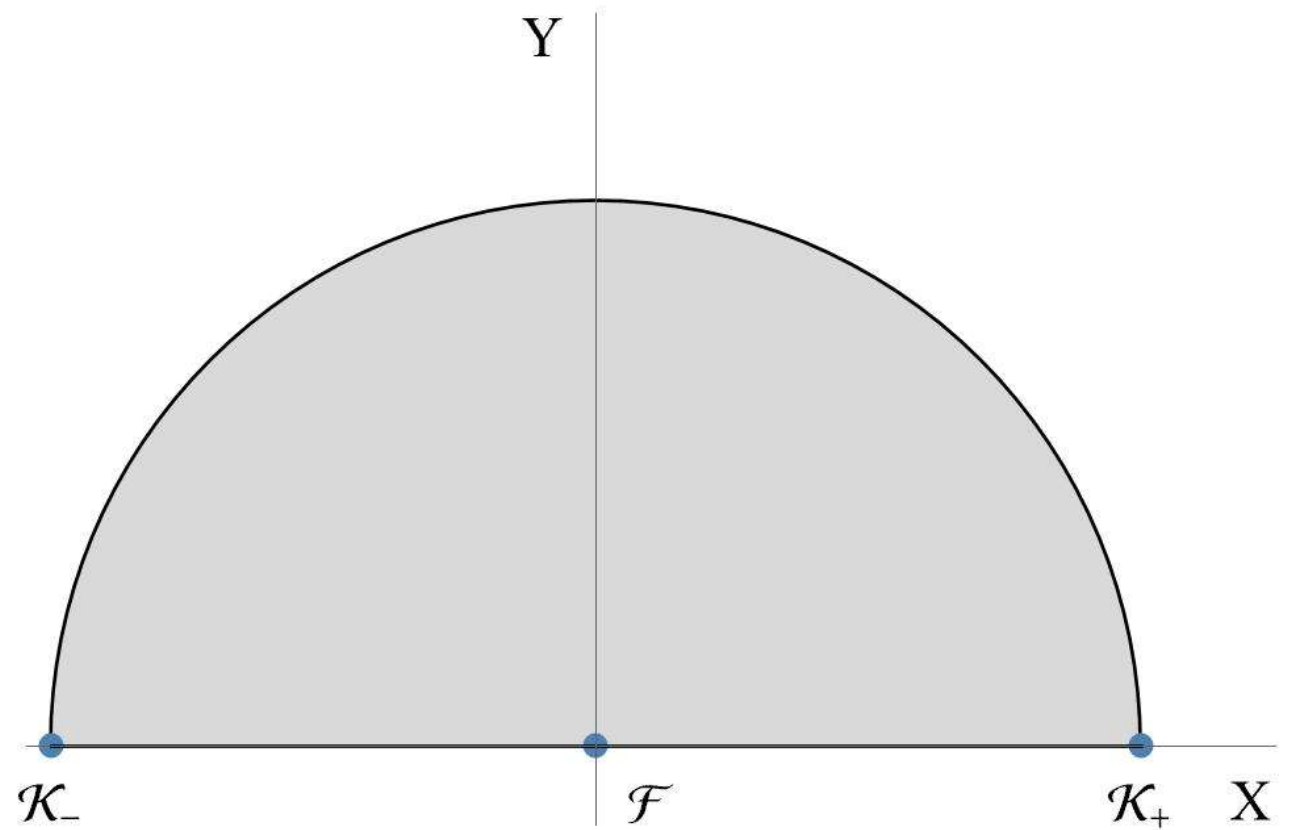
Kinating fixed points

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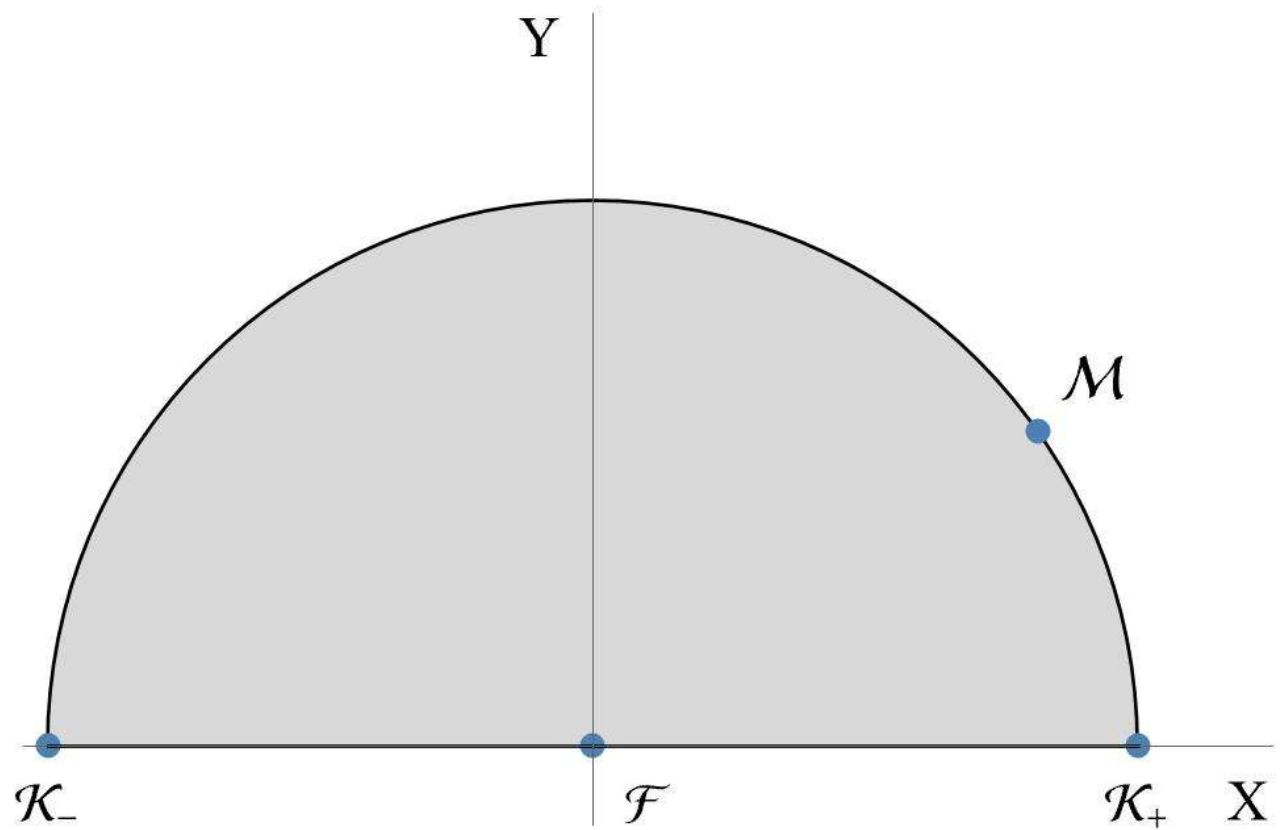
Fluid domination fixed point

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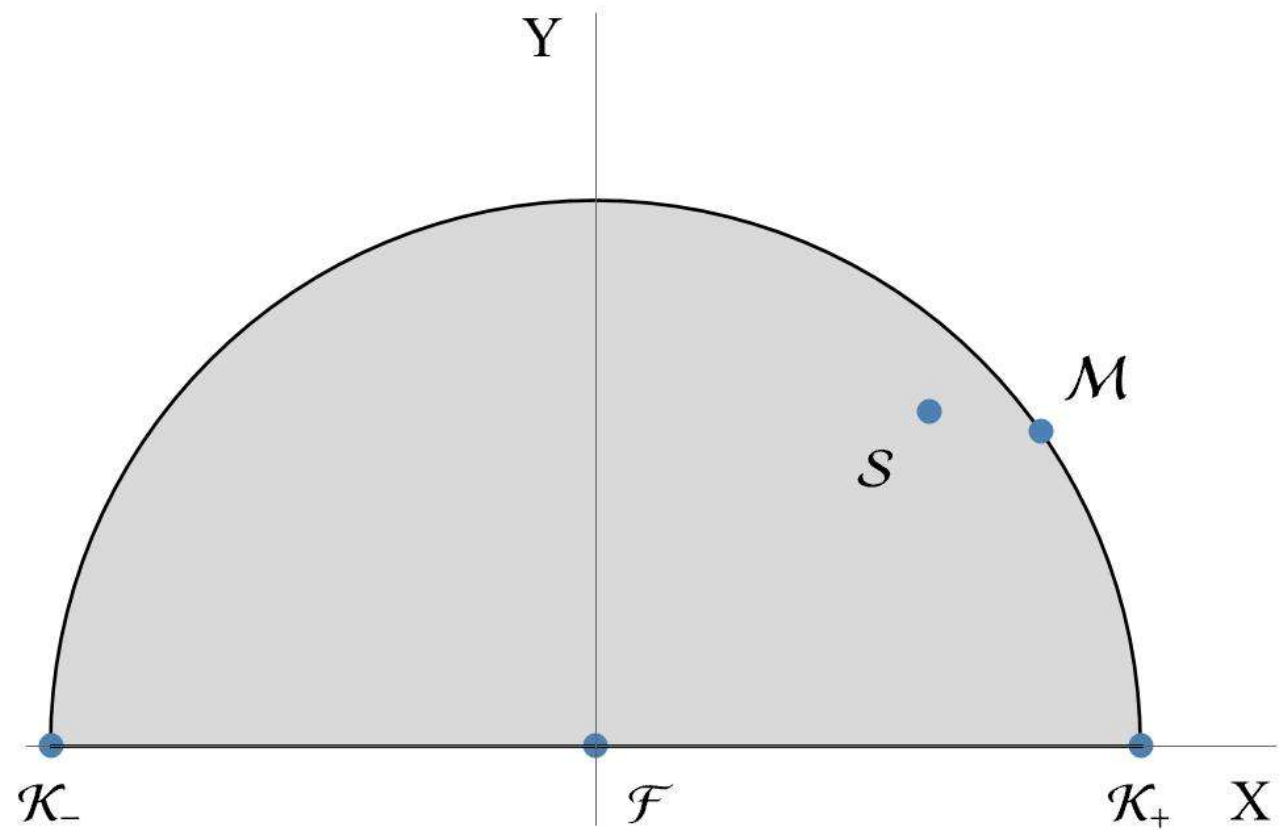
Field domination fixed point

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Scaling fixed point

Growth Condition and Phase Space Constraints

In **phase space**, the growth condition identifies a **Growth Region (GR)**:

$$X > \sqrt{\frac{2}{3}} \frac{1}{\xi}$$

If $\xi > 0$

$$X < \sqrt{\frac{2}{3}} \frac{1}{\xi}$$

If $\xi < 0$

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Can we push the fixed points of the dynamical system within the GR?

Fixed Points of Growth

FP	X	Y	Existence	Existence and Growth when $\xi > \sqrt{2/3}$
$\mathcal{K}_+$	1	0	$\forall \lambda$ and $\forall \omega$	$\forall \lambda$ and $\forall \omega$
$\mathcal{K}_-$	-1	0	$\forall \lambda$ and $\forall \omega$	$\forall \lambda$ and $\forall \omega$
$\mathcal{F}$	0	0	$\forall \lambda$ and $\forall \omega$	never
$\mathcal{M}$	$\frac{\lambda}{\sqrt{6}}$	$\sqrt{1 - \frac{\lambda^2}{6}}$	$ \lambda < \sqrt{6}$	$\frac{2}{ \xi } < \lambda < \sqrt{6}$
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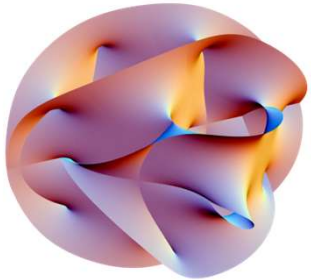
Depending on the values of λ , ω and ξ , $\mathcal{K}_\pm$, $\mathcal{M}$ and $\mathcal{S}$ can be in the GR!

Part III:

Cosmic Strings in Type IIB String Theory

Type IIB Compactifications and Moduli

**D = 10 TYPE IIB
STRING THEORY**



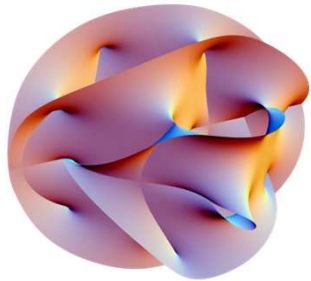
Calabi-Yau



**D=4 $\mathcal{N}=1$
SUGRA EFT**

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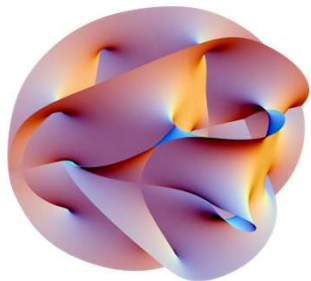
Moduli
Scalar Fields
in the EFT



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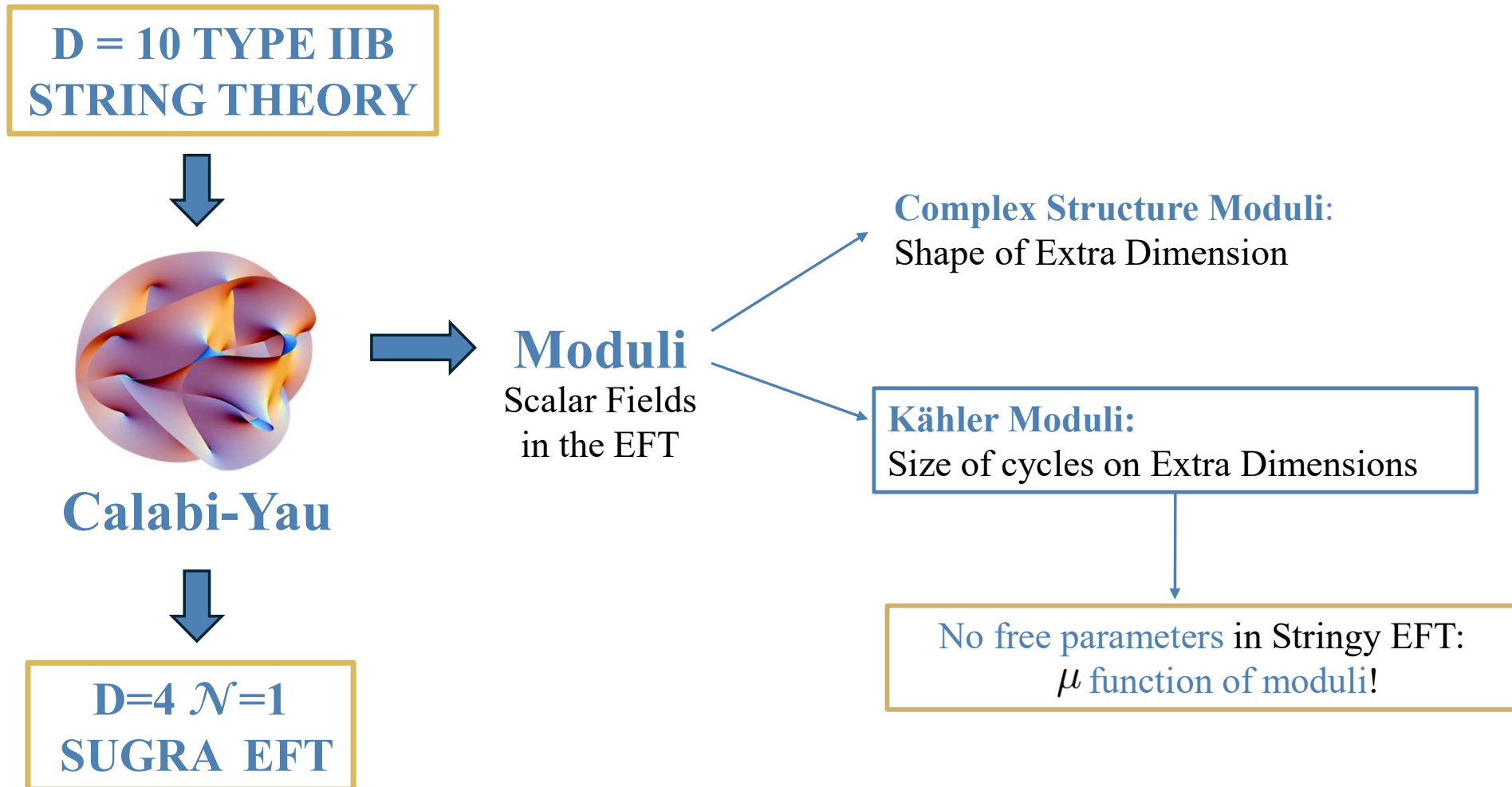


Moduli
Scalar Fields
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Complex Structure Moduli:
Shape of Extra Dimension

Kähler Moduli:
Size of cycles on Extra Dimensions

Type IIB Compactifications and Moduli



Fundamental Strings

Fundamental strings (F-strings) in 4d are produced by string-scale energy effects.

Tension (in Einstein frame):

$$\mu \sim M_s^2 = \frac{\sqrt{g_s} M_p^2}{4\pi\mathcal{V}}$$

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Depends on two moduli: $\mathcal{V}$ and $g_s = e^\varphi$

In terms of the dilaton:

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No GR in the phase space of φ

Volume Mode: Growth Region

Tension dependence on $\mathcal{V}$

$$\mu \sim \frac{M_p^2}{\mathcal{V}}$$

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Tension dependence on $\mathcal{V}$

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In the **simplest** case:

$$\mathcal{V} = \tau^{3/2}$$

Canonically normalised field:

$$\frac{\chi}{M_p} = \sqrt{\frac{3}{2}} \ln \tau = \sqrt{\frac{2}{3}} \ln \mathcal{V}$$

Tension dependence on χ

$$\mu \sim M_p^2 e^{-\sqrt{\frac{3}{2}} \frac{\chi}{M_p}} \implies \boxed{\xi = \sqrt{\frac{3}{2}} > \sqrt{\frac{2}{3}}}$$

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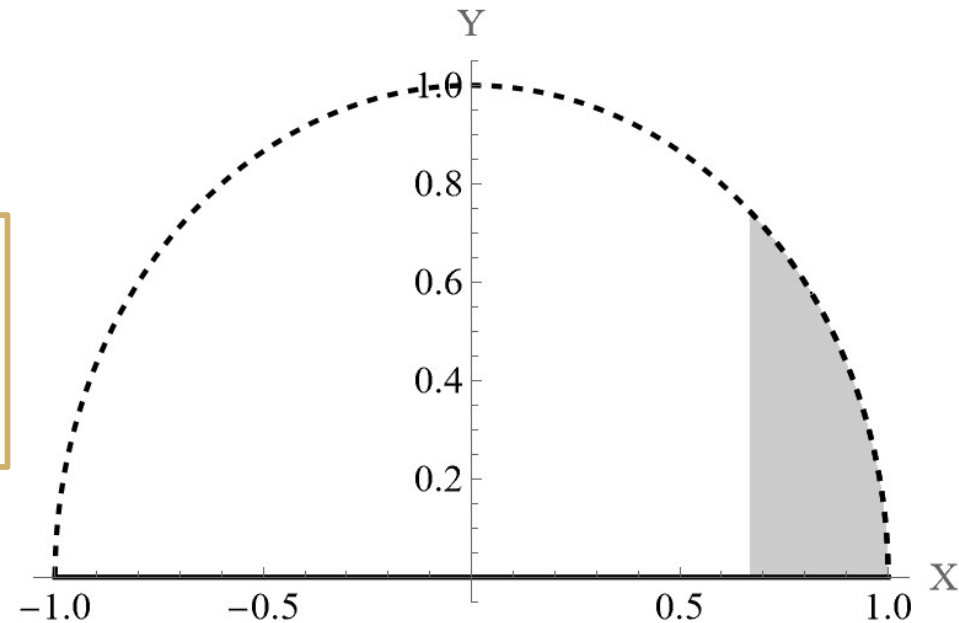
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Canonically normalised field:

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Growth region exists!

$$X > \frac{2}{3}$$



Tension dependence on χ

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Volume Mode: Kination

$$\mu \sim M_p^2 e^{-\sqrt{\frac{3}{2}} \frac{\chi}{M_p}}$$

Typical potential for volume ($M_p = 1$):

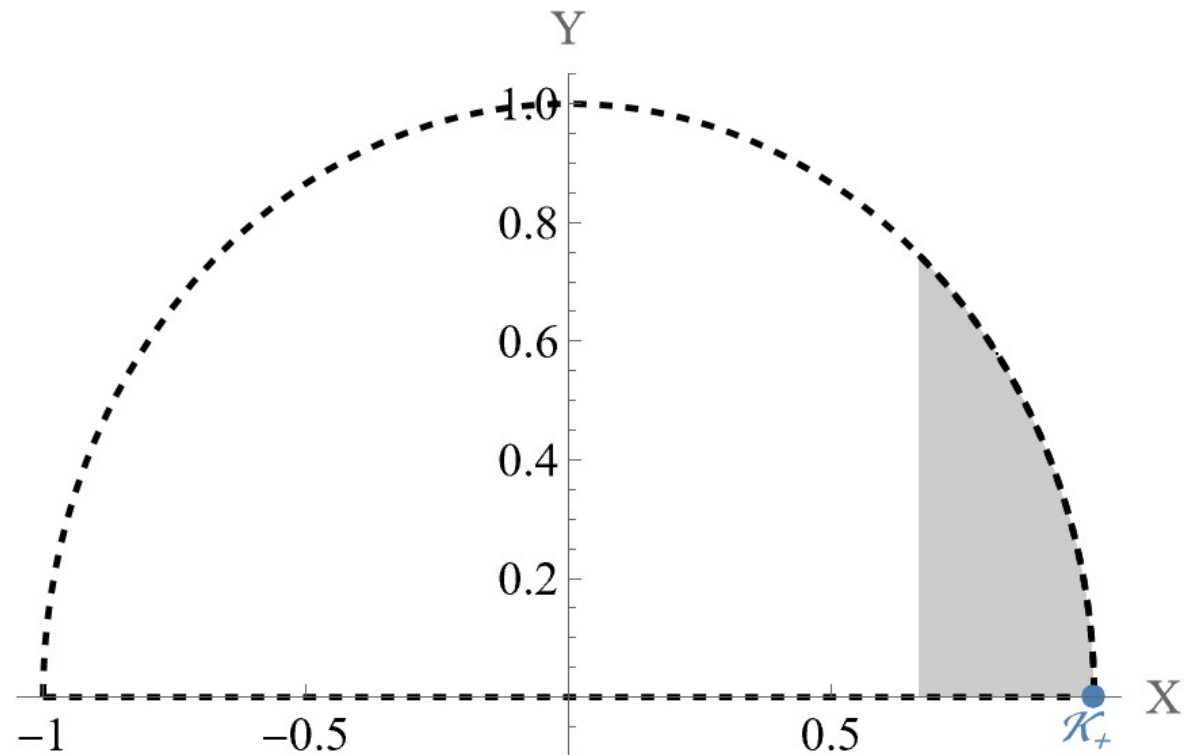
$$V \sim \frac{\alpha}{\mathcal{V}^p} \Rightarrow V = V_0 e^{-\lambda \chi}$$

We can use our DS techniques!

$\mathcal{K}_+ = (1, 0)$ fixed point in the growth region [Conlon et al.: 2024]

But it is unstable!

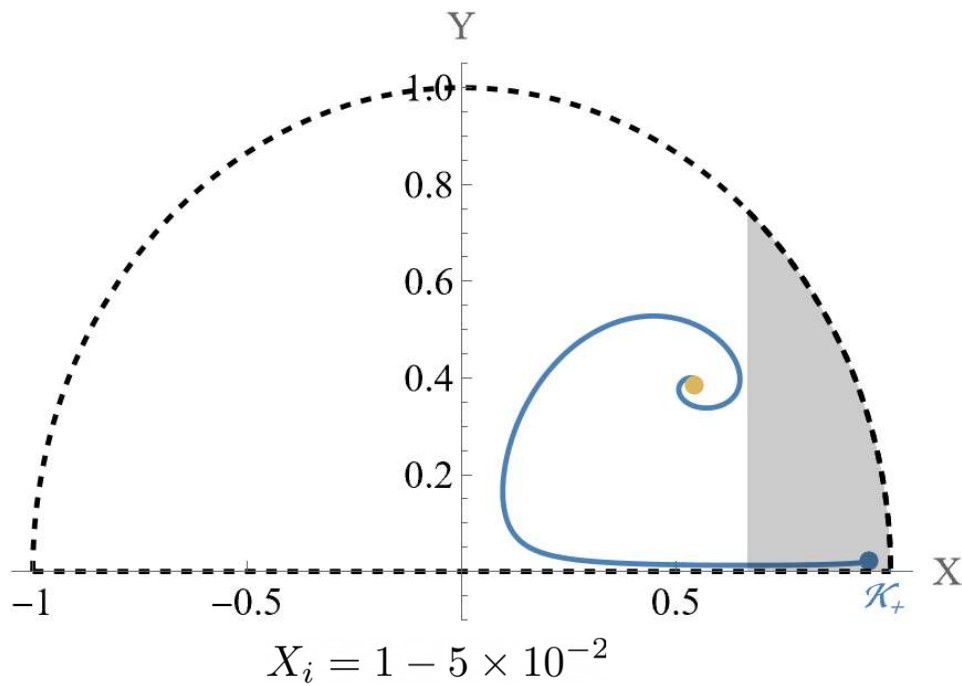
$$\rho_{\text{kin}} \sim a^{-6}$$



Volume Mode: Almost Kination

$$V = V_0 e^{-\lambda \chi}$$

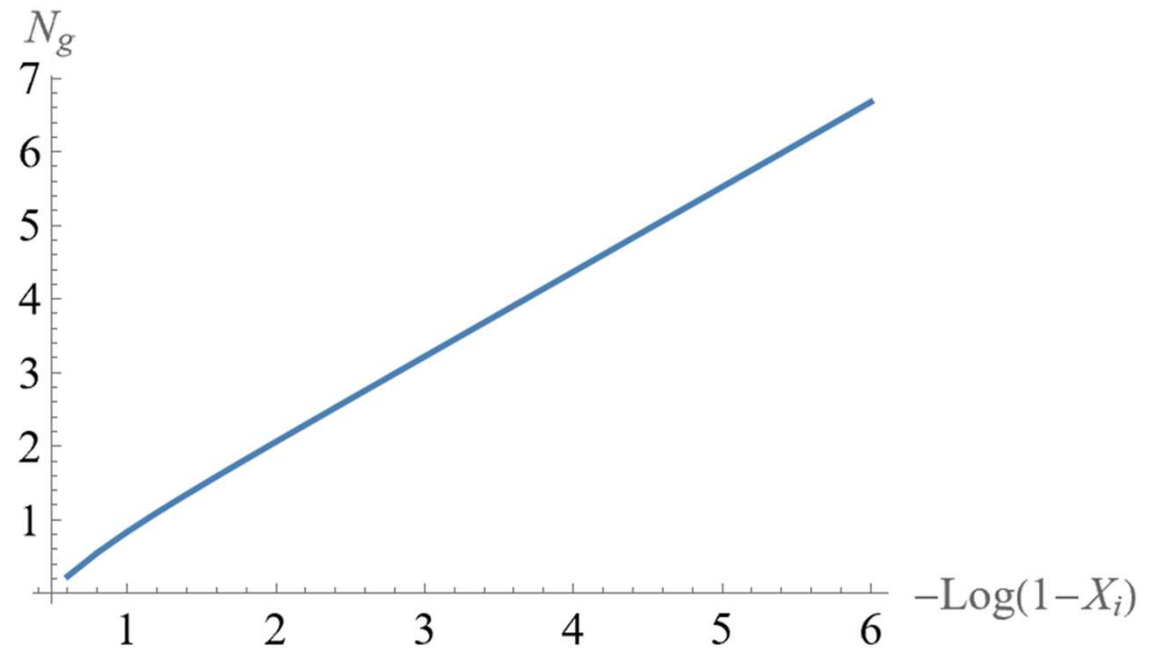
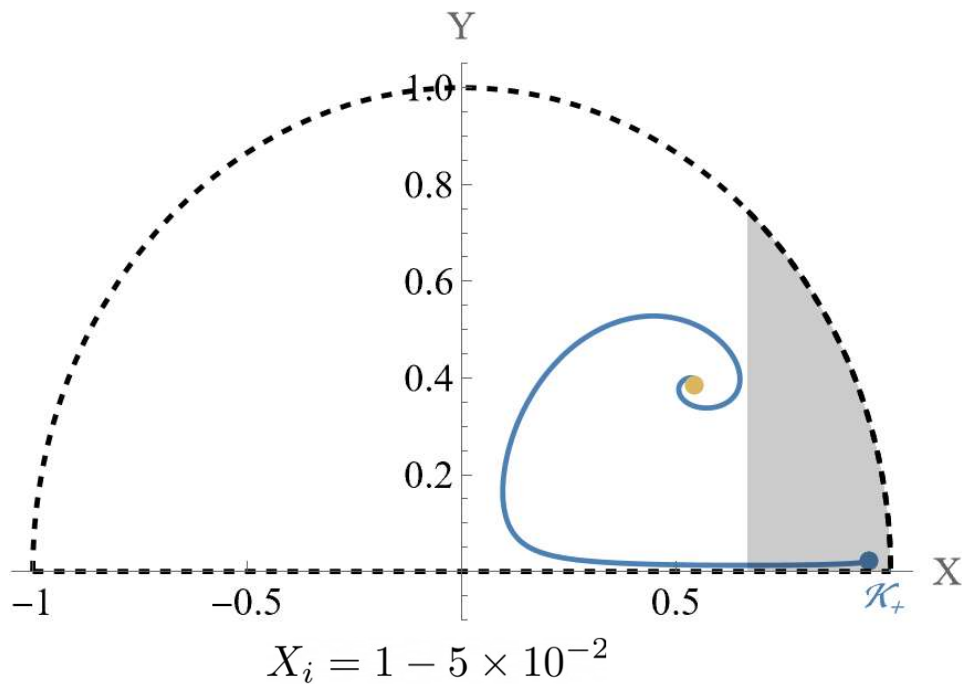
Starting close to $\mathcal{K}_+$, the system spends **some time** in the **GR**, but then exits.



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Volume Mode: Field Domination Fixed Point

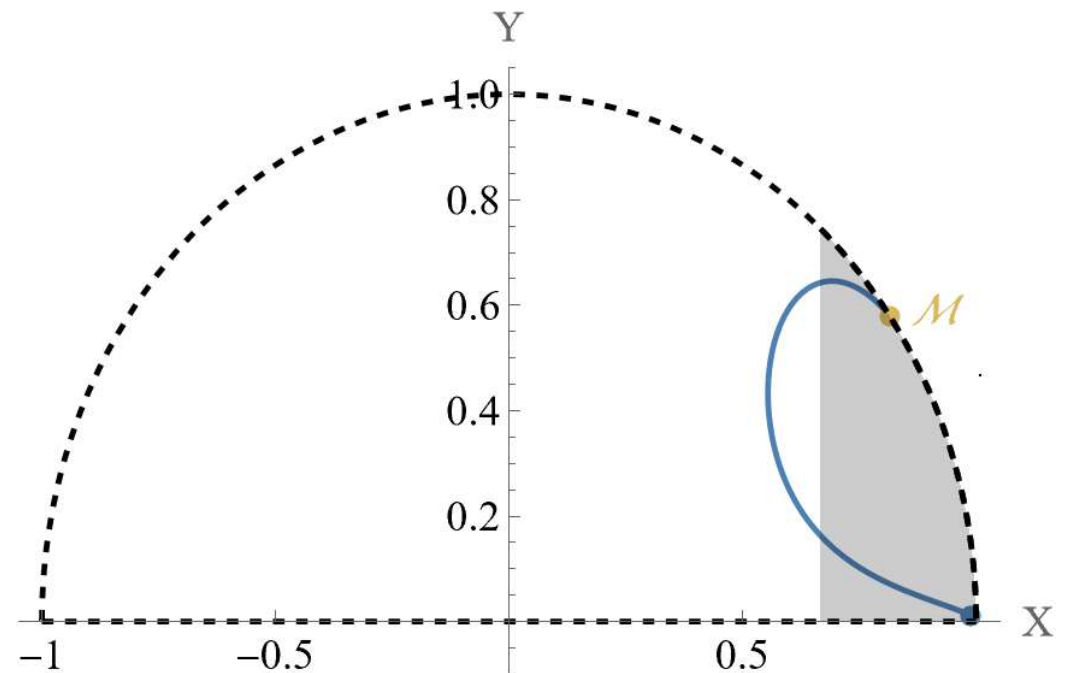
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$$\xi = \sqrt{\frac{3}{2}}$$

$\mathcal{M}$ in GR for:

$$2\sqrt{\frac{2}{3}} < \lambda < \sqrt{6} \implies \frac{4}{3} < p < 2$$

For such values, $\mathcal{M}$ is **unstable**



$$\lambda = 2$$

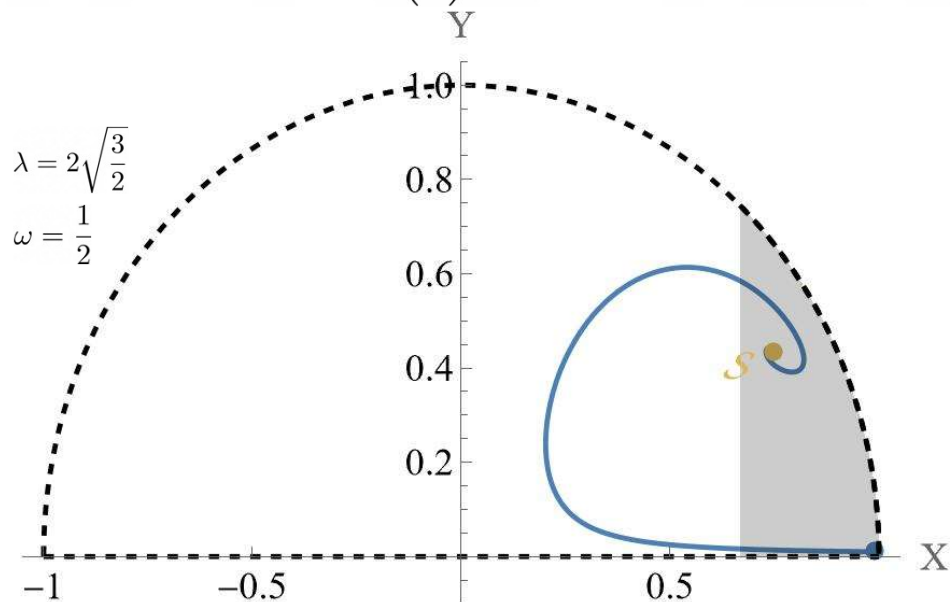
Volume Mode: Scaling Fixed Point

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$\mathcal{S}$ is in GR for:

$$\sqrt{3(\omega+1)} < \lambda < (\omega+1) \left(\frac{3}{2}\right)^{3/2} \implies \sqrt{2(\omega+1)} < p < \frac{3}{2}(\omega+1)$$



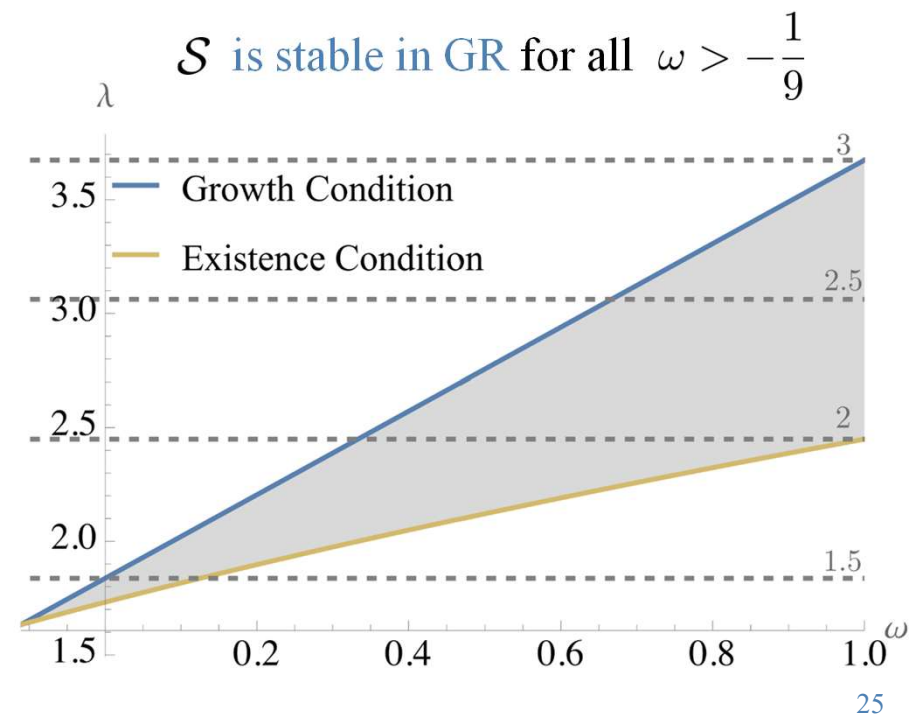
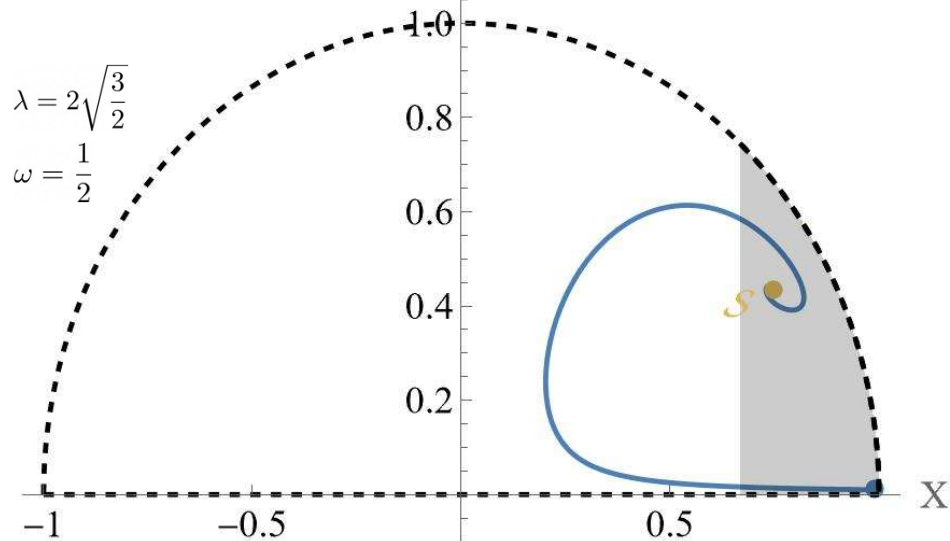
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Effective Strings in Type IIB

Other source of strings in the EFT: p -brane wrapping a $(p-1)$ -cycle Σ_{p-1}

Effective Strings in Type IIB

Other source of strings in the EFT: **p-brane** wrapping a **(p-1)-cycle** Σ_{p-1}

If $\text{Vol}(\Sigma_{p-1}) = t_{p-1} \ell_s^{p-1}$:

$$\boxed{\mu \sim M_s^2 t_{p-1}} \longrightarrow \text{Additional moduli-dependence!}$$

Effective Strings in Type IIB

Other source of strings in the EFT: **p-brane** wrapping a **(p-1)-cycle** Σ_{p-1}

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In **Type IIB** these strings emerge if:

- A **D3-brane** wraps a **2-cycle** of volume: $t \ell_s^{-2} \implies \mu \sim M_s^2 t$
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The simplest case: $\mathcal{V} \propto t^3 \propto \tau^{3/2}$ allows for **no GR** [Conlon et al:2024]:

- For **D3's**:

$$\boxed{\xi = \sqrt{\frac{2}{3}}}$$

- For **5-branes**:

$$\boxed{\xi < \sqrt{\frac{2}{3}}}$$

Fibred Calabi-Yau's

Next-to-simplest case: CY with a **fibred structure**!

$\mathbb{P}^1$ base fibred by a $\mathbb{T}^4$ or a $K3$ surface.

$$\mathcal{V} = \frac{1}{2} k_{122} t_1 t_2^2 = \kappa \tau_b \sqrt{\tau_f}$$

Relation between **2- and 4-cycles**:

$$\tau_i = \frac{\partial \mathcal{V}}{\partial t_i} \implies t_2 = \sqrt{\frac{2\tau_f}{k_{122}}} \quad t_1 = \frac{1}{\sqrt{2}k_{122}} \frac{\tau_b}{\sqrt{\tau_f}}$$

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Idea: keep **volume fixed** and **vary the orthogonal direction**

$$u = \frac{1}{\kappa} \frac{\tau_f}{\tau_b} = e^{\sqrt{3}\phi/M_p}$$

Canonically Normalised Cycles and Tension

Tension **linear in cycle volume**: just check **canonical normalisation** at fixed volume! ($M_p = 1$)

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$$|\xi| > \sqrt{\frac{2}{3}}$$

Same $|\xi|$ **mirror GRs!**

Growth Region and Kination

$$\mu \sim M_s^2 e^{\pm \frac{2}{\sqrt{3}} \phi/M_p}$$

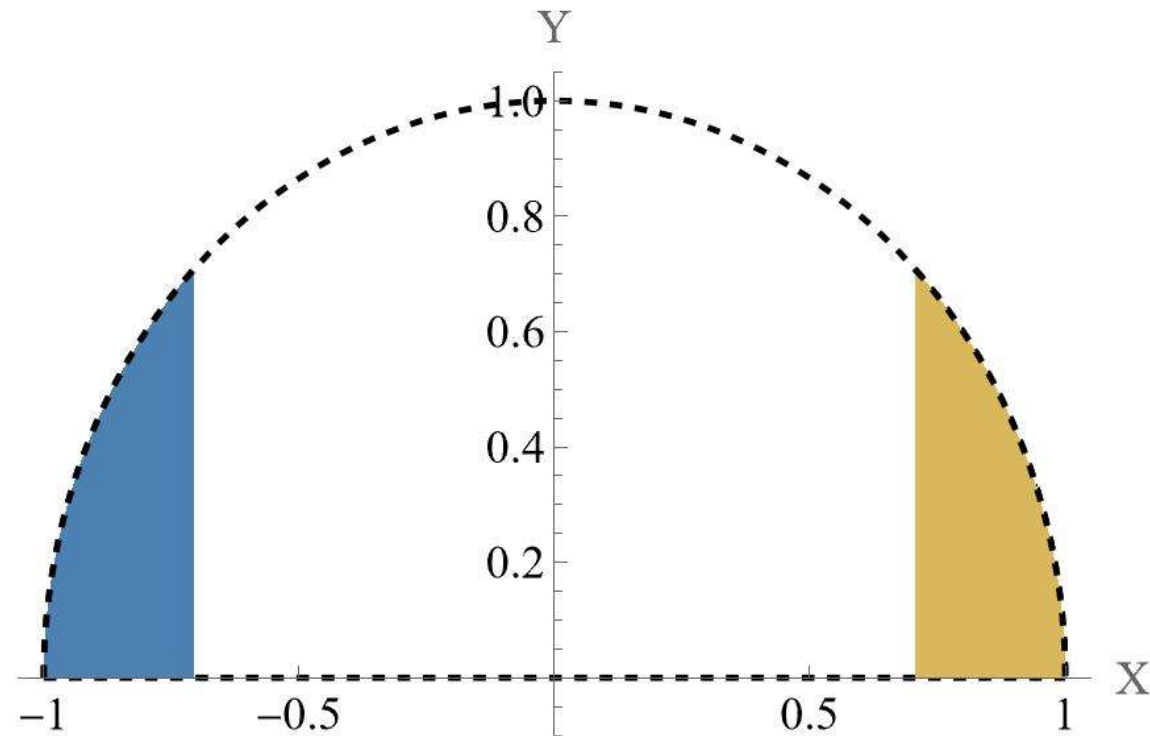
Growth Regions:

$$X < -\frac{1}{\sqrt{2}}$$

fibre 4-cycle

$$X > \frac{1}{\sqrt{2}}$$

base 2-cycle



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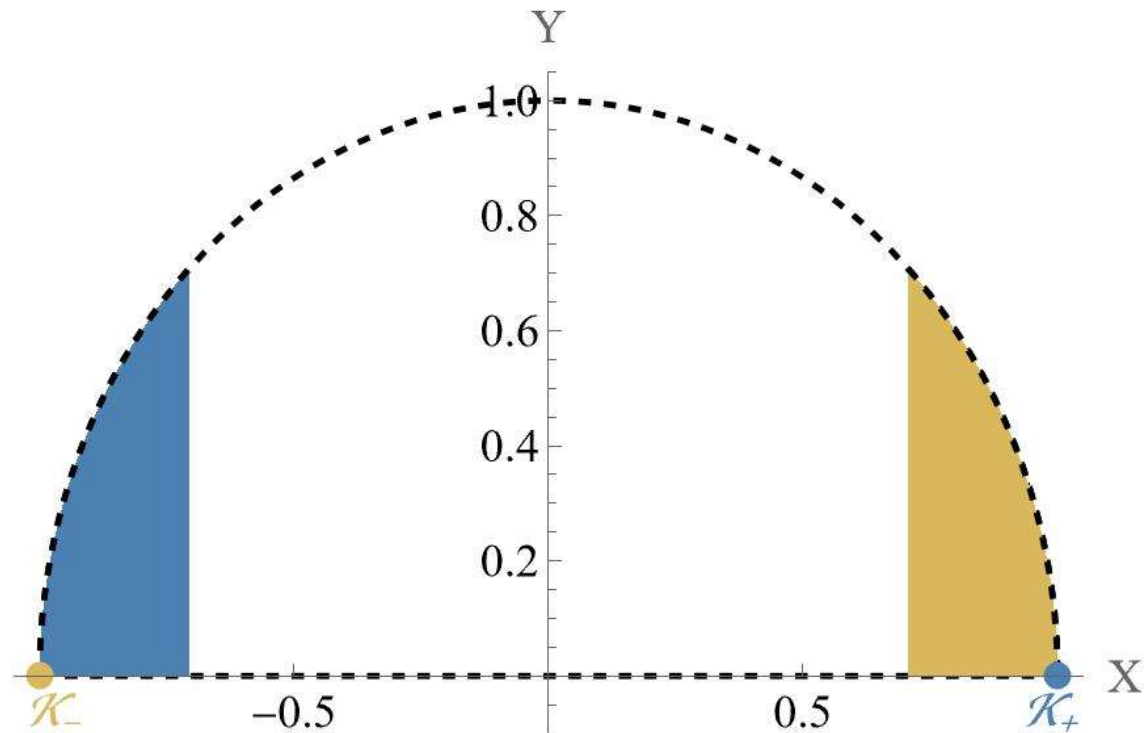
$$X > \frac{1}{\sqrt{2}}$$

base 2-cycle

Perturbative potential:

$$V \sim \frac{\beta}{\tau_f^q} \sim V_0 e^{-\lambda \phi}$$

Kinating fixed points $\mathcal{K}_{\pm}$ within one GR.



Field Domination Fixed Points

FP	X	Y	Existence	Existence and Growth when $ \xi > \sqrt{2/3}$
$\mathcal{M}$	$\frac{\lambda}{\sqrt{6}}$	$\sqrt{1 - \frac{\lambda^2}{6}}$	$ \lambda < \sqrt{6}$	$\frac{2}{ \xi } < \lambda < \sqrt{6}$

$$|\xi| = \frac{2}{\sqrt{3}}$$

$\mathcal{M}$ in GR for:

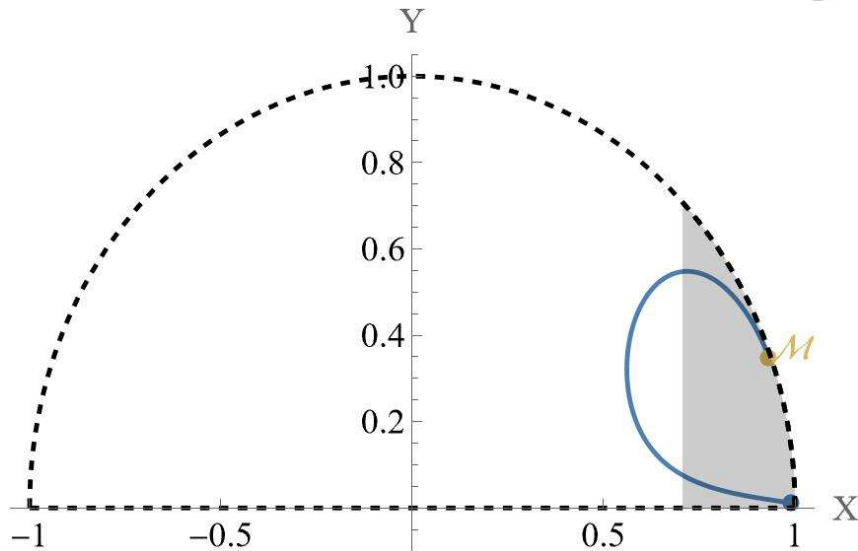
$$\sqrt{3} < \lambda < \sqrt{6} \implies \frac{3}{2} < q < \frac{3}{\sqrt{2}}$$

base 2-cycle

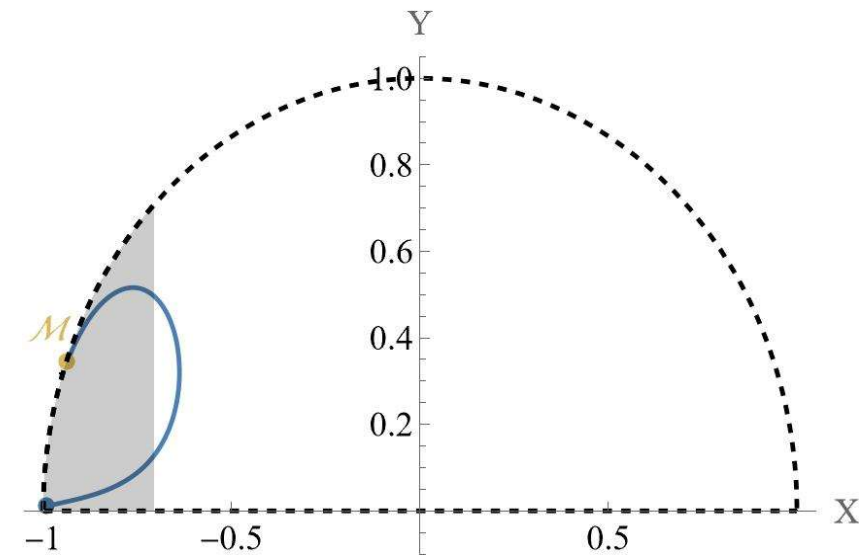
$$-\sqrt{6} < \lambda < -\sqrt{3} \implies -\frac{3}{\sqrt{2}} < q < -\frac{3}{2}$$

fibre 4-cycle

Unstable! If $\mathcal{S}$ exists, the system will evolve toward it



$$|\lambda| = \frac{4}{\sqrt{3}}$$



Scaling Fixed Points

FP	X	Y	Existence	Existence and Growth when $ \xi > \sqrt{2/3}$
$\mathcal{S}$	$\sqrt{\frac{3}{2}} \frac{\omega+1}{\lambda}$	$\sqrt{\frac{3(1-\omega^2)}{2\lambda^2}}$	$\lambda^2 > 3(\omega+1)$	$\sqrt{3(\omega+1)} < \lambda < \frac{3}{2}(\omega+1) \xi $ with $\sqrt{\omega+1} > \frac{2}{\sqrt{3} \xi }$

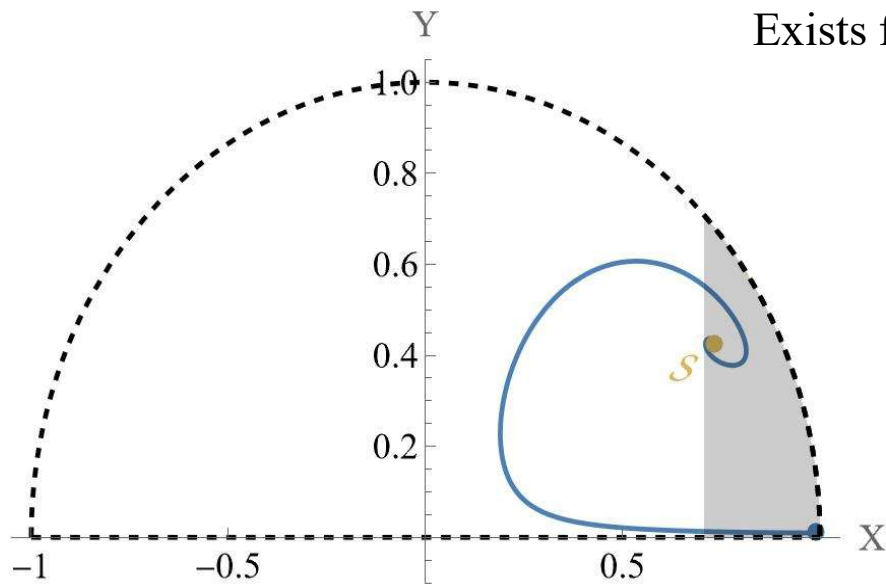
$$|\xi| = \frac{2}{\sqrt{3}}$$

$\mathcal{S}$ is in GR for:

$$\sqrt{3(\omega+1)} < \lambda < \sqrt{3}(\omega+1) \implies \frac{1}{2}\sqrt{\omega+1} < q < \frac{1}{2}(\omega+1)$$

$$-\sqrt{3}(\omega+1) < \lambda < -\sqrt{3(\omega+1)} \implies -\frac{1}{2}(\omega+1) < q < -\frac{1}{2}\sqrt{\omega+1}$$

base 2-cycle

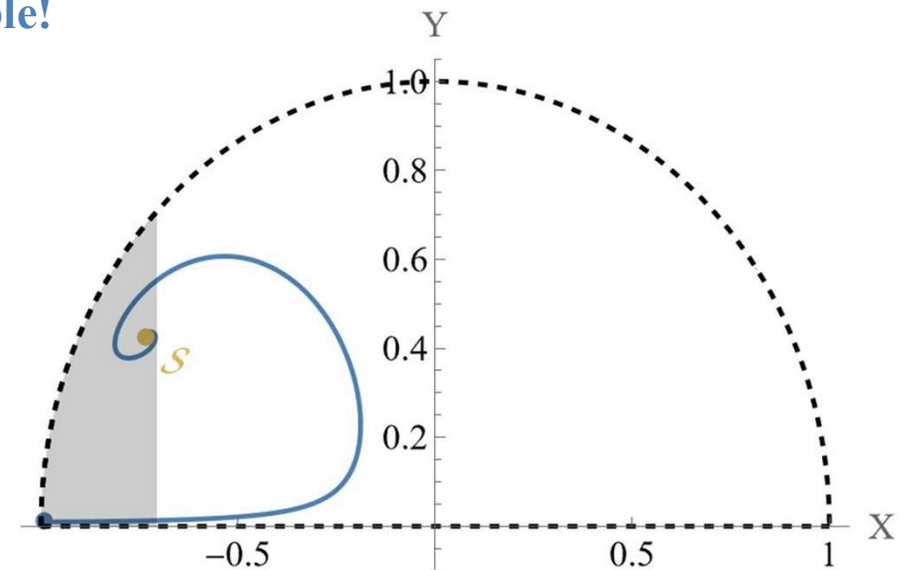


Exists for all $\omega > 0$ **Stable!**

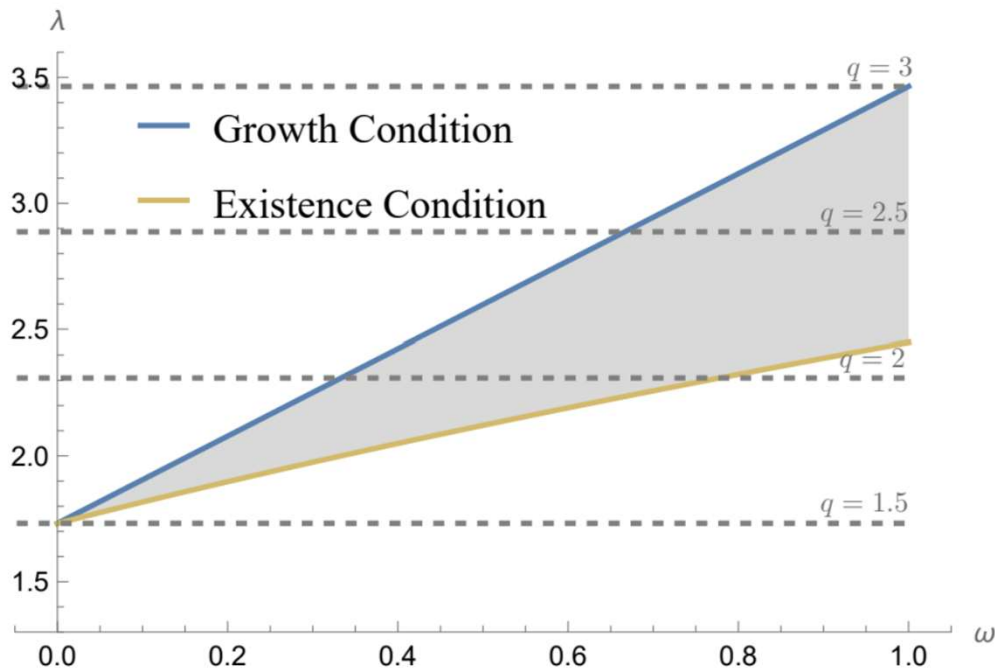
$$|\lambda| = 2$$

$$\omega = 1/3$$

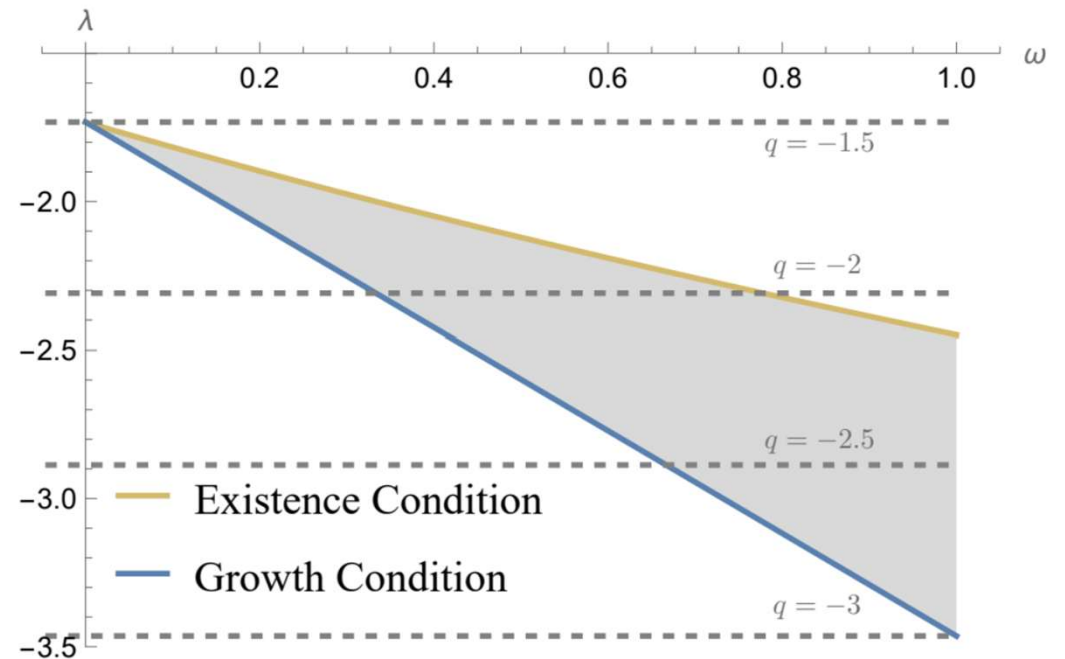
fibre 4-cycle



Scaling Fixed Points in GR



base 2-cycle



fibre 4-cycle

Summary

- Cosmic Strings from Superstring theory are phenomenologically interesting!
- Their tension depends on a modulus and can be studied as a Dynamical System
- Fundamental Strings can Grow even if the volume is not kinating, but scaling
- Effective strings from Fibred CYs can also grow in comoving size when the cycle evolves

Outlook

- What happens when the string fluid is no longer negligible? [Conlon et al: 2025]
- What happens when the modulus reaches its minimum? How does the string fluid energy density behave? [LB, Cicoli, Pedro: WIP]
- Can Effective String give rise to observable signals in GW or particle emission? [LB, Revello, Villa: WIP]
- How do we produce the initial population of strings? How do we estimate their number density?

Thank You For Your Attention!

Backup

Cosmic Strings

In **QFT**, solitonic field configurations in the presence of **U(1) SSB** [Kibble: 1976]:

- **Global Strings:** from **global** U(1) SSB, also called **axion strings** [Vilenkin, Everett: 1982; Lazarides, Shaqfi: 1982].
- **Local Strings:** from **gauge** U(1) SSB [Nielsen, Olsen: 1973]

In **String Theory**, **1-dimensional** objects in the EFT:

- **Fundamental (F-) Strings:** **fundamental strings** of string theory in 4D
- **D1-Strings:** **D-branes** with 1 extended spacelike dimension
- **EFT (or Axion) Strings:** **Dp-branes wrapped around (p-1)-cycles** in the internal dimensions

+ **Local Strings** realized from SSB in gauge sectors

Constraints and Observational Features

Tension of a Cosmic String Network strongly constrained by CMB and PTA.

Constraint from CMB [Charnok et al.:2016]:

$$G\mu \lesssim 10^{-7} - 10^{-8}$$

Constraint from PTA [Ellis et al.:2023; Marfatia et al. :2023; Avgoustidis et al.:2025]:

$$G\mu \lesssim 10^{-11} - 10^{-12}$$

Best fit possible for cosmic superstrings in Large Volume Scenario! [Ghoshal, Revello, Villa:2025]

(Super)String Theory Crash Course

Superstring Theory is one the most promising candidates for Quantum Gravity.

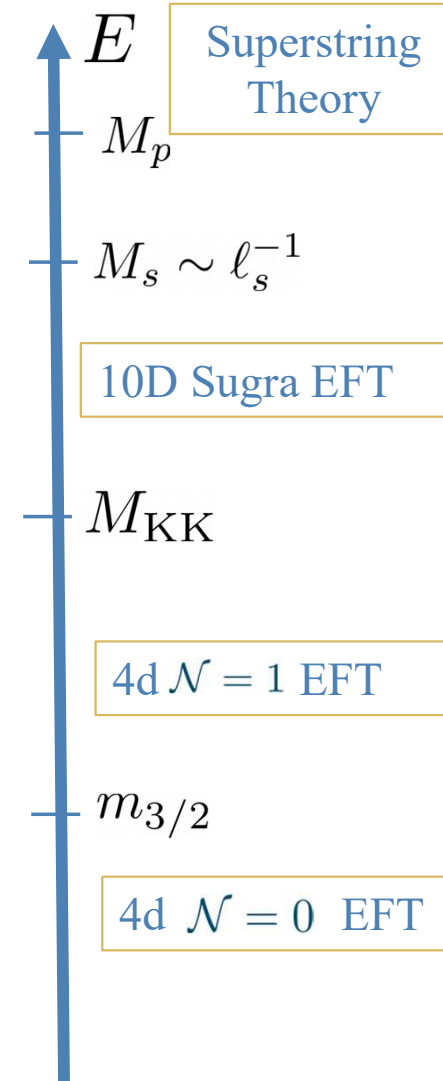
The fundamental objects are one-dimensional strings with length: $\ell_s = 2\pi\sqrt{\alpha'}$

Quantization of the Superstring automatically gives a massless spin-2 field: graviton!

Multiple Types of Superstring Theory, related by dualities

Requires 10 (or 11) dimensions for consistency (Weyl anomaly cancellation)

Needs compactification to make contact with phenomenology!



Tension of Effective Strings

Tension From **dim. reduction** can get a **NG action** out of **DBI-like part** of actions:

$$S_p \simeq -T_p \int_{\mathcal{W}_{p+1}} \sqrt{-g_{(p+1)}} (1 + \dots) d^{p+1}\zeta \quad \xrightarrow{\mathcal{W}_{p+1} = \Sigma_{p-1} \times \mathcal{W}_2} \quad S_p \supset -T_p \int_{\mathcal{W}_2} \sqrt{-g_{(2)}} \text{Vol}(\Sigma_{p-1}) d^2\sigma$$

$$\text{If } \text{Vol}(\Sigma_{p-1}) = t_{p-1} \ell_s^{p-1}$$

$$\mu \sim T_p M_s^{-p+1} t_{p-1} \quad \xrightarrow{T_p \sim M_s^{p+1}} \quad \boxed{\mu \sim M_s^2 t_{p-1}}$$