Axion Quality: Challenges and Pathways to Improvement

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Axion Quality: Challenges and Pathways to Improvement

- The strong CP problem:
  - *the axion solution*
- The dirty side of the Axion Solution:
  - *origin and quality problem of* $U(1)_{PQ}$
- Overview of solutions for the axion problem
- Recent developments in connection with Flavour

Strong CP Problem

QCD violates CP symmetry

$$\delta\mathcal{L}_{\rm QCD} = \theta\,\frac{\alpha_s}{8\pi} G\tilde{G}$$

Natural expectation

$$\theta \sim \mathcal{O}(1)$$

Gauge-invariant + renormalizable
non-perturbative QCD (instantons)

Strong CP Problem & The Axion

Strong CP problem

$$\delta\mathcal{L}_{\rm QCD} = \theta\,\frac{\alpha_s}{8\pi}G\tilde{G}$$

$$|\theta| \lesssim 10^{-10}$$

Electric Dipole Moment for Neutron

Why so small??

promote θ to a dynamical field,
which relaxes to zero via QCD dynamics

$$\theta \to \frac{a}{f_a} \qquad \to \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

with $\langle a \rangle = 0$

Axion: a new spin-0 boson → *pNGB of a spontaneously broken global* U(1)$_{\rm PQ}$ *with QCD anomaly*

$$\Phi = f_a e^{ia/f_a}$$

U(1)$_{\rm PQ}$

$$a \to a + \alpha f_a$$

Strong CP Problem & The Axion

Strong CP problem

$$\delta\mathcal{L}_{\rm QCD} = \theta\,\frac{\alpha_s}{8\pi} G\tilde{G}$$

$$|\theta| \lesssim 10^{-10}$$

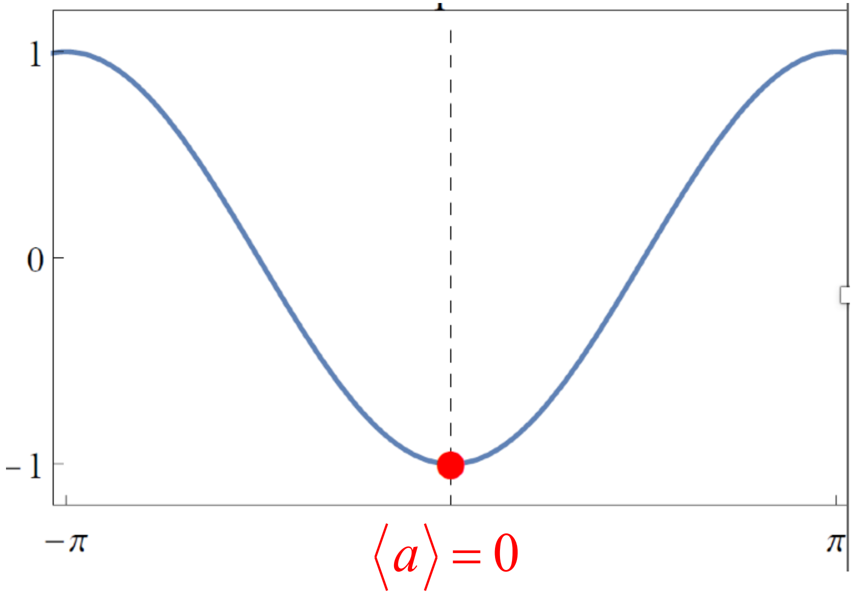
Electric Dipole Moment for Neutron

$$\rightarrow \frac{a}{f_a}\frac{\alpha_s}{8\pi} G\tilde{G}$$

Axion potential

$$V_a \approx -\Lambda^4_{\rm QCD}\cos\left(\frac{a}{f_a}\right)$$

by QCD PQ-breaking effects at low-energy



Vafa-Witten theorem

$$V(a) \geq V(0)$$

→ minimum at a=0
CP is preserved!

The Axion Solution: the good

- **Axion:** ***a new spin-0*** → ***pNGB of a QCD anomalous global*** **U(1)$_{\text{PQ}}$**

1. axion mass $m_a \approx 1\ \text{eV}\,\dfrac{10^7\,\text{GeV}}{f_a}$

2. axion couplings to photons, nucleons, electrons $\propto 1/f_a$

$$\rightarrow \frac{a}{f_a}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{a\,\mu\nu} + \frac{a}{f_a}\frac{\alpha}{8\pi}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial_\mu a}{f_a}\bar{f}\gamma^\mu\gamma_5 f$$

$(f = p, n, e)$

The lighter the axion, the weaker are its interactions!

(being not yet excluded & detected)

Invisible **long-lived light particle → <u>DM candidate</u> ☺**

The Axion Solution: the bad

- ☐ **Origin of U(1)$_{PQ}$: *Do global symmetries exist in nature*?**

  - ✓ **Global symmetries** act uniformly across spacetime (e.g., rotating all fields by the same phase)

    → *Broadly, this is at odds with the local nature of spacetime deformations in Quantum Gravity.*

    → Opposite to gauge symmetries, which are inherently local

***Global symmetries* are not believed to be fundamental,**
presumably an accident at low-energy!?!

The Axion Solution: the bad

- **Origin of U(1)$_{\mathrm{PQ}}$:** ***a global symmetry might not exist in nature***

***Quantum Gravity Conjecture*:→ no-global symmetry**

Susskind ’95 — Black Hole information paradox

Hawking radiation

Banks, Seiberg ‘11 — No global symmetries in String theory

Harlow, Ooguri ‘18 — No global symmetries in ADS/CFT

Quantum Gravity must break all global symmetries at any **$[d]$**

$$\Delta\mathcal{L}_{\rm UV}^{\cancel{\rm PQ}} \sim \frac{1}{\Lambda_{\rm UV}^{d-4}}\mathcal{O}^{[d]} \qquad \Lambda_{\rm UV} = M_{\rm Pl} \simeq 1.2\times 10^{19}\,{\rm GeV} \gg \Lambda_{\rm QCD}$$

The Axion Solution: the bad

❑ **Origin of U(1)$_{PQ}$: *a global symmetry might not exist in nature***

- ✓ Global symmetries explicitly broken by Quantum Gravity

$$\Delta\mathcal{L}_{\mathrm{UV}}^{\cancel{\mathrm{PQ}}} \sim \frac{1}{\Lambda_{\mathrm{UV}}^{d-4}}\mathcal{O}^{[d]} \qquad [d]>0$$

- ✓ **Moreover, U(1)$_{PQ}$** is anomalous: absent in nature as real symmetry

→ ***PQ broken already at the renormalizable level:*** $d \leq 4$

$$\mathcal{O}^{[d\leq 4]} \rightarrow \Phi^d + \mathrm{h.c}, \qquad \Phi = f_a e^{ia/f_a}$$

Therefore, U(1)$_{PQ}$ is not fundamental: *its origin presumably/desirable an accident* (e. g., emerging by imposing Lorentz and Gauge invariance ex. baryon/lepton number) *or approximate by unknown mechanism* (χ sym)

The Axion Solution: the bad

Benchmark axion models → DFSZ and KSVZ

SM quarks
2 Higgs (H_u, H_d) + **Singlet**

Zhitnitsky ’80, Dine, Fischler, Srednicki ’81

DFSZ

BSM quark Q_L
1 Higgs + **Singlet**

KSVZ

Kim ’79, Shifman, Vainshtein, Zakharov ’80

PQ already broken at renormalizable level

$$\boxed{\mu_H^2 H_u H_d}$$

$$\boxed{\mu_Q \bar{Q}_L Q_R}$$

The U(1)$_{PQ}$ symmetry is imposed by hand, no explanation for its origin

The Axion Solution: the bad

☐ **Quality problem of U(1)$_{PQ}$:** *axion solution fragile at* d>4?

☺ **Ideally,** PQ symmetry emerges at low-energy accidentally → protected at d ≲ 4 ⇒ new gauge symmetries!

☹ **BUT** UV physics (ex. gravity?) still violates the PQ symmetry at h.o:

→ gauge-invariant PQ-breaking operators of d>4 are still generated at low energies

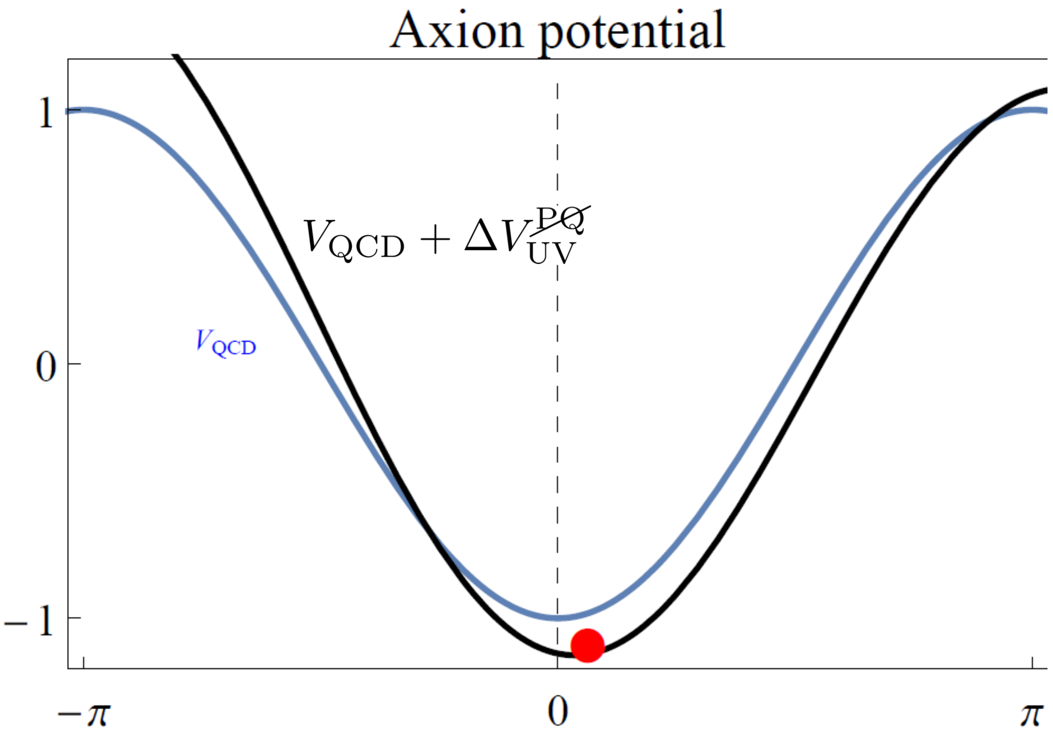
$$\Delta\mathcal{L}_{\rm UV}^{\cancel{\rm PQ}} \sim \frac{\Phi^d}{\Lambda_{\rm UV}^{d-4}} + {\rm h.c} \qquad\qquad \Phi = f_a e^{i a/f_a}$$

☹ **New contribution to the axion potential** → axion solution lost!

$$\hookrightarrow \quad \Delta V_{\rm UV}^{\cancel{\rm PQ}}(\theta_{\rm eff}) \sim \left(\frac{f_a}{\Lambda_{\rm UV}}\right)^{d-4} f_a^4$$

Quality problem of U(1)$_{\text{PQ}}$

- **The minimum of the potential is shifted**



$\langle a \rangle \neq 0$

$\langle \theta_{\text{eff}} \rangle \neq 0$

$$\delta \mathcal{L}_{\text{QCD}} = \theta \frac{\alpha_s}{8\pi} G\tilde{G}$$

Then, the minimum is CP-violating!

Strong CP problem strike back

$$\Delta V_{\text{UV}}^{\cancel{\text{PQ}}}(\theta_{\text{eff}}) \sim \left(\frac{f_a}{\Lambda_{\text{UV}}}\right)^{d-4} f_a^4$$

Quality problem of $\mathrm{U(1)_{PQ}}$

We solve the Strong CP problem only if

$$\langle\theta_{\mathrm{eff}}\rangle < 10^{-10} \qquad \text{(neutron EDM)}$$

Minimizing the full potential, $V_{\mathrm{QCD}} + \Delta V_{\mathrm{UV}}^{\cancel{\mathrm{PQ}}}$, it translates to

$$\boxed{\left(\frac{f_a}{\Lambda_{\mathrm{UV}}}\right)^{d-4} f_a^4} \lesssim \boxed{10^{-10}}\,\boxed{\Lambda_{\mathrm{QCD}}^4} \quad \text{QCD}$$

UV PQ-breaking physics

Experimental bound

$$\underline{\Delta V_{\mathrm{UV}}^{\cancel{\mathrm{PQ}}}(\theta_{\mathrm{eff}})} \sim \left(\frac{f_a}{\Lambda_{\mathrm{UV}}}\right)^{d-4} f_a^4 \qquad \underline{\frac{\alpha_s}{32\pi^2} G\tilde{G}}$$

Quality problem of U(1)$_{\mathrm{PQ}}$

We solve the Strong CP problem only if

$$\langle \theta_{\mathrm{eff}} \rangle < 10^{-10} \qquad \text{(neutron EDM)}$$

Minimizing the full potential, $V_{\mathrm{QCD}} + \Delta V_{\mathrm{UV}}^{\cancel{\mathrm{PQ}}}$, it translates to

$$\color{red}{\lambda} \left(\frac{f_a}{\Lambda_{\mathrm{UV}}} \right)^{d-4} f_a^4 \lesssim 10^{-10} \Lambda_{\mathrm{QCD}}^4$$

- **For physically well-motivated scales**

Dark Matter $f_a \sim 10^{9-11}\ \mathrm{GeV}$

Gravity $\Lambda_{\mathrm{UV}} \sim M_{\mathrm{Pl}}$

⇒

PQ must be preserved up to **d ≥ 10** for **λ~1**

High-Quality PQ Symmetry

Quality problem of U(1)$_{\rm PQ}$

We solve the Strong CP problem only if

$$\langle \theta_{\rm eff} \rangle < 10^{-10} \qquad \text{(neutron EDM)}$$

Minimizing the full potential, $V_{\rm QCD} + \Delta V_{\rm UV}^{\cancel{\rm PQ}}$, it translates to

$$\lambda \left(\frac{f_a}{\Lambda_{\rm UV}} \right)^{d-4} f_a^4 \lesssim 10^{-10} \Lambda_{\rm QCD}^4$$

- ❖ **For physically well-motivated scales**

Dark Matter $f_a \sim 10^{9-11}\ {\rm GeV}$

Gravity $\Lambda_{\rm UV} \sim M_{\rm Pl}$

⇒

PQ breaking **d < 10** for $\lambda \sim 1$
must be forbidden

High-Quality PQ Symmetry

Quality problem of $\mathrm{U(1)_{PQ}}$

We solve the Strong CP problem only if

$$\langle \theta_{\rm eff} \rangle < 10^{-10} \qquad \text{(neutron EDM)}$$

Minimizing the full potential, $V_{\rm QCD} + \Delta V_{\rm UV}^{\cancel{\rm PQ}}$, it translates to

More generally
$$\boxed{\frac{f_a^n v_I^{d-n}}{\Lambda_{\rm UV}^{d-4}} \lesssim 10^{-10} \Lambda_{\rm QCD}^4}$$

$$\mathcal{O}^{[d]} \to \Phi_I^d \to v_I e^{i a_I / v_I}$$

We can reduce d in presence of more vevs, such $v_{\rm I} < f_a$

- **For physically well-motivated scales**

Dark Matter $f_a \sim 10^{9-11}\ \mathrm{GeV}$

Gravity $\Lambda_{\rm UV} \sim M_{\rm Pl}$

PQ must be preserved up to **d ≥ 10** for λ~1

High-Quality PQ Symmetry

Solutions to the PQ Quality Problem

$$\lambda \left(\frac{f_a}{\Lambda_{\rm UV}} \right)^{d-4} f_a^4 \lesssim 10^{-10} \Lambda_{\rm QCD}^4$$

- **Low-scale** f_a

- **Gravitational suppression of coupling** $\lambda \ll 1$

- **Gauge protection of PQ-breaking operators at** $d>10$

Solutions to the PQ Quality Problem

$$\lambda \left(\frac{f_a}{\Lambda_{\rm UV}}\right)^{d-4} f_a^4 \lesssim 10^{-10} \Lambda_{\rm QCD}^4$$

- **Low-scale** f_a @d>4

U(1)$_{\rm PQ}$ origin not addressed
accidental/approximate for d≤4

- **Gravitational suppression of coupling** $\lambda \ll 1$

U(1)$_{\rm PQ}$ *only* ***approximate***
due to tiny breaking terms

- **Gauge protection of PQ-breaking operators at** d>10

U(1)$_{\rm PQ}$ ***accidental****/exact* up
to d>10 terms

Solutions to the PQ Quality Problem

$$\lambda\left(\frac{f_a}{\Lambda_{\mathrm{UV}}}\right)^{d-4} f_a^4 \lesssim 10^{-10}\Lambda_{\mathrm{QCD}}^4$$

❖ **Low-scale** f_a @d>4 ➡ **Challenge:** evade astrophysical bounds, $f_a \gtrsim 10^8$ GeV

for example, *by modifying the* $m_a - f_a$ *relation:*

$$m_a \propto \frac{\Lambda_{\mathrm{QCD}}^2}{f_a}$$

➡

may arise in mirror SM models (or QCD or GUT models)

extra contribution to m_a from hidden gauge sector with PQ anomaly:

$$\Lambda_D \gg \Lambda_{\mathrm{QCD}}$$

Rubakov ‘97, Berezhiani, Gianfagna, Giannotti ‘01,
Gianfagna Giannotti Nesti ’04, Gaillard, Gavela et al. ‘18, …

Solutions to the PQ Quality Problem

$$\lambda\left(\frac{f_a}{\Lambda_{\mathrm{UV}}}\right)^{d-4} f_a^4 \lesssim 10^{-10}\Lambda_{\mathrm{QCD}}^4$$

❖ Low-scale f_a @d>4

❖ **Gravitational suppression of coupling $\lambda \ll 1$**

⇨ Non-perturbative quantum gravitational effects may generate PQ-breaking operators

⇒ studied in the context of Euclidian action of Wormholes configurations

$$\lambda \sim e^{-S_A}$$

☺ Lee '88, Giddings, Strominger '88: $S \sim M_{\mathrm{Pl}}/f_a$ No quality problem

☹ Abbott, Wise '88, Alvey, Escudero '20 $S \sim \log M_{\mathrm{Pl}}/f_a$ Quality problem

Solutions to the PQ Quality Problem

$$\lambda\left(\frac{f_a}{\Lambda_{\mathrm{UV}}}\right)^{d-4} f_a^4 \lesssim 10^{-10}\Lambda_{\mathrm{QCD}}^4$$

- **Low-scale** f_a @d>4

U(1)$_{\mathrm{PQ}}$ origin not addressed
accidental/approximate for d<4

- **Gravitational suppression of coupling** $\lambda \ll 1$

U(1)$_{\mathrm{PQ}}$ *only* ***approximate***
due to tiny breaking terms

- **Gauge protection of PQ-breaking operators at** d>10

U(1)$_{\mathrm{PQ}}$ ***accidental****/exact* up
to d>10 terms

❤

Gauge Protection to PQ symmetries

Benchmark axion models → DFSZ and KSVZ

- **discrete gauge symmetries** can make U(1)$_{PQ}$ accidental and protect to higher order

$$\boxed{\cancel{\mu_H^2 H_u H_d}} \qquad \boxed{\cancel{\mu_Q \bar{Q}_L Q_R}}$$

e.g. not allowed by discrete gauge

Ringwald, Saikawa ‘15

- Discrete gauge symmetries:

$$\mathbb{Z}_n : \quad \Phi \to e^{i\frac{2\pi}{n}}\Phi \quad \Rightarrow \quad \Delta\mathcal{L}_{\mathrm{UV}}^{\cancel{\mathrm{PQ}}} = \overset{\text{allowed}}{\frac{\Phi^n}{\Lambda_{\mathrm{UV}}^{n-4}}} \quad \Rightarrow \quad n \geq 10$$

High-Quality PQ

$\Downarrow$

It can emerge from a U(1) gauge symmetry:

$$\Phi \to e^{i\alpha}\Phi$$

$$S \to e^{in\alpha}S$$

$$\langle S \rangle : \quad U(1) \to Z_n$$

$$\alpha = \frac{2\pi}{n}$$

Gauge Protection to PQ symmetries

- Discrete gauge symmetries:

$$\mathbb{Z}_n : \quad \Phi \to e^{i\frac{2\pi}{n}}\Phi \quad \Longrightarrow \quad \Delta\mathcal{L}_{\mathrm{UV}}^{\cancel{\mathrm{PQ}}} = \frac{\Phi^n}{\Lambda_{\mathrm{UV}}^{n-4}} \quad \Longrightarrow \quad n \geq 10$$

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$$\Phi \to e^{i\alpha}\Phi \qquad \langle S\rangle : \; U(1) \to Z_n$$

$$S \to e^{in\alpha}S \qquad \alpha = \frac{2\pi}{n}$$

Krauss and Wilczek ´89, Dias et al. ´03 Carpenter et al.´09, Harigaya et al. ´13

- Abelian gauge U(1):

Barr and Seckel ´92

$$\Phi_1 \to e^{iq_1\alpha}$$

$$\Phi_2 \to e^{iq_2\alpha}$$

$$\Longrightarrow \quad \Delta\mathcal{L}_{\mathrm{UV}}^{\cancel{\mathrm{PQ}}} = \frac{(\Phi_1^\dagger)^{q_2}(\Phi_2)^{q_1}}{\Lambda_{\mathrm{UV}}^{q1+q2-4}}$$

HQ PQ

$$\Longrightarrow \quad q_1 + q_2 \geq 10$$

- Non-Abelian gauge group:

Di Luzio et al. ’17
Darme’, Nardi ‘21
+Smarra ‘22

$$SU(N) \otimes SU(N) \quad \Phi \sim (N, \bar{N}) \quad \Longrightarrow \quad \Delta\mathcal{L}_{\mathrm{UV}}^{\cancel{\mathrm{PQ}}} = \frac{\det \Phi}{\Lambda_{\mathrm{UV}}^{N-4}}$$

$$\mathrm{U}(1)_{\mathrm{PQ}}\text{:} \; \Phi \to e^{i\alpha}\Phi$$

HQ PQ

$$\Longrightarrow \quad N \geq 10$$

Gauge Protection to PQ symmetries

- Discrete gauge symmetries:

High-Quality PQ

$$\mathbb{Z}_n : \quad \Phi \to e^{i\frac{2\pi}{n}}\Phi \quad \Longrightarrow \quad \Delta\mathcal{L}_{\mathrm{UV}}^{\cancel{\mathrm{PQ}}} = \frac{\Phi^n}{\Lambda_{\mathrm{UV}}^{n-4}} \quad \Longrightarrow \quad n \geq 10$$

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It can emerge from a U(1) gauge symmetry:

$$\Phi \to e^{i\alpha}\Phi \qquad \langle S \rangle : \; U(1) \to Z_n$$

$$S \to e^{in\alpha}S \qquad \alpha = \frac{2\pi}{n}$$

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- Abelian gauge U(1):

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$$\Phi_1 \to e^{iq_1\alpha} \qquad \Phi_2 \to e^{iq_2\alpha} \quad \Longrightarrow \quad \Delta\mathcal{L}_{\mathrm{UV}}^{\cancel{\mathrm{PQ}}} = \frac{(\Phi_1^\dagger)^{q_2}(\Phi_2)^{q_1}}{\Lambda_{\mathrm{UV}}^{q1+q2-4}}$$

HQ PQ

$$\Longrightarrow \quad q_1 + q_2 \geq 10$$

- Non-Abelian gauge group:

Di Luzio et al. ’17
Darme’, Nardi ‘21
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$$SU(N) \otimes SU(N)$$

$$SU(9)_{\mathrm{GUT}}$$

Georgi et al. ’81

$$SU(N) \to SO(N)$$

Ardu et al. ´20

Gauge Protection to PQ symmetries

- Discrete gauge symmetries (vertical): : $\mathbb{Z}_n$

- Abelian gauge U(1) (vertical): U_1

- Non-Abelian gauge group (vertical): $SU(N) \otimes SU(N)$ $SU(9)_{\mathrm{GUT}}$ $SU(N) \to SO(N)$

These examples solve the PQ quality problem to dimension *N*

BUT they have some limitations:

i) *ad-hoc gauge symmetries → solution to PQ-quality problem hidden in the UV*

ii) *lack of testability of the UV mechanism addressing the quality problem*

- **Are possible *models with low-energy signatures? explaining additional SM puzzles (like SM flavor structure, Dark Matter)?***

Gauge Protection to PQ symmetries

$$\lambda\left(\frac{f_a}{\Lambda_{\rm UV}}\right)^{d-4} f_a^4 \lesssim 10^{-10}\Lambda_{\rm QCD}^4$$

- **Two possibilities to enlarging the SM gauge group through** *a new gauge interactions*

- or unrelated to SM families
  ⇒ *vertical gauge protection*

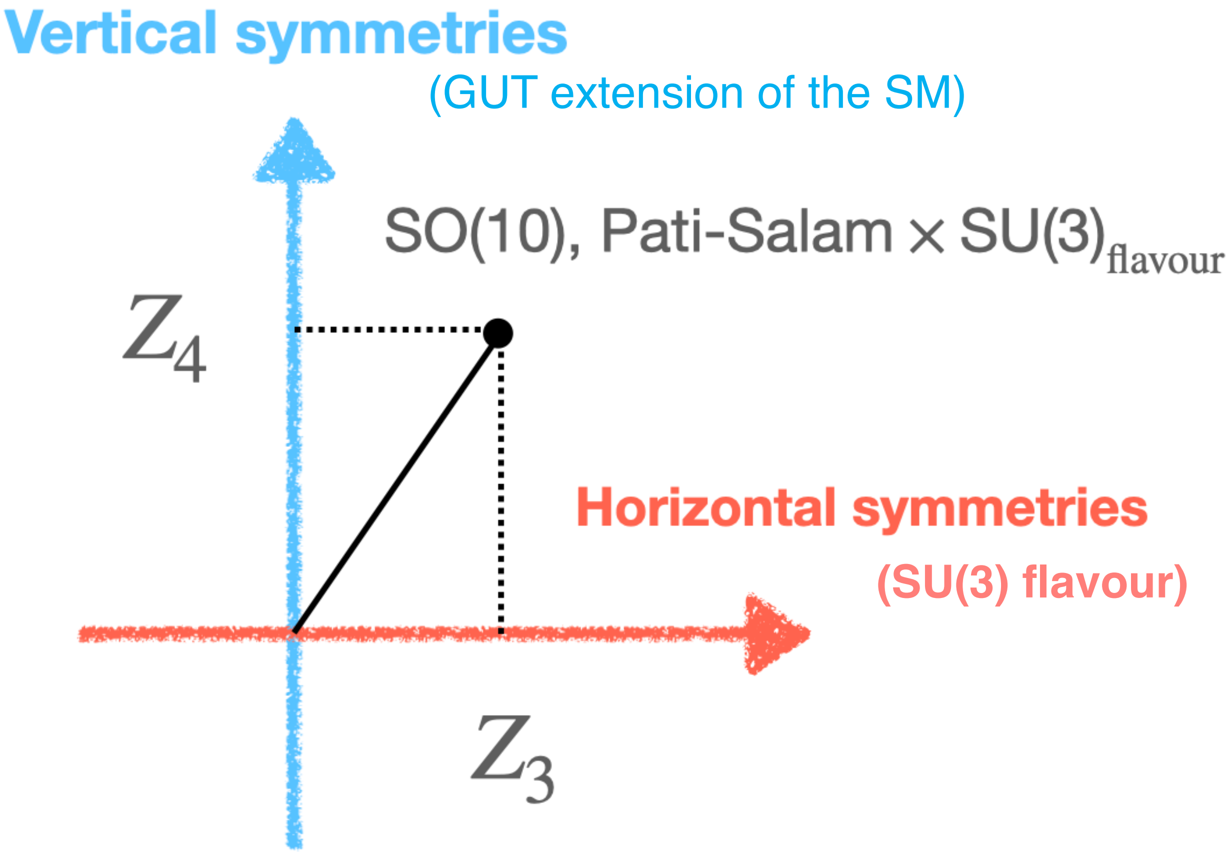
  $\mathbb{Z}_n$ U_1 $SU(9)_{\rm GUT}$

- or related to SM families
  ⇒ *horizontal gauge protection*

  $SU(N)\otimes SU(N)$ Darme', Nardi, Smarra '22

SO(10)xSU(3)$_F$: Di Luzio ´20
SU(4)$_{PS}$ xSU(3)$_F$: Di Luzio, Ladini, FM & Susic ´25

Axion quality from Flavour and GUT



Vertical symmetries

(GUT extension of the SM)

SO(10), Pati-Salam × $SU(3)_{\text{flavour}}$

Z_4

Horizontal symmetries

(SU(3) flavour)

Z_3

SO(10): Di Luzio ´20

$SU(4)_{PS}$: Di Luzio, Ladini, FM & Susic ´25:

→ $SU(4)_{PS}$ group easier than SO(10) to work out details

Axion quality from the interplay of vertical and horizontal gauge symmetries

Axion quality from Flavour and GUT

Vertical structure = GUT extension of the SM

+

Horizontal structure = Gauged flavour symmetries

SO(10): Di Luzio ´20

$SU(4)_{PS}$: Di Luzio, Ladini,
FM & Susic ´25

nice features

- No ad-hoc symmetries for PQ protection
- PQ breaking tied to to PS/flavor scales
- SM flavour structure
- Axion – flavour connection
- High-Quality PQ predicts new sub-eV fermions
- Cosmological signatures from new fermions

Axion quality from the interplay of vertical and horizontal gauge symmetries

Pati-Salam – axion flavour model

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2, 0)$	$(4, 2, 1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0, 1/2)$	$(4, 1, 2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0, 1/2)$	$(1, 1, 1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0, 0)$	$(1, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0, 0)$	$(15, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0, 0)$	$(10, 1, 3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0, 0)$	$(4, 1, 2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0, 0)$	$(15, 1, 3)$	$+1$	$\mathbf{1}$	$+1$	1	0

- **Gauge group:** $SU(4)_{\mathrm{PS}} \otimes SU(2)_L \otimes SU(2)_R \otimes SU(3)_{f_R}$
- **Too many fields but each plays an important role!**

❖ **Fermion sector**

Q_L and Q_R contain SM fermions(quarks, leptons) plus ν_R (**bonus in PS**)

→ only Q_R charged under $SU(3)_{fR}$: to avoid EW-scale flavour gauge bosons

Ψ_R → anomalons: BSM fermions needed to cancel anomalies of $SU(3)_{fR}$

Pati-Salam – axion flavour model

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❖ Scalar Sector

Φ and Σ contain SM Higgs → $\overline{Q_L}\, Q_R \Phi + \overline{Q_L}\, Q_R \Sigma$ for realistic Yukawa sector

Δ → neutrino Majorana mass term → $Q_R Q_R \Delta^*$ (seesaw type I)

χ → avoid spurious global symmetries together with Δ

ξ → needed for accidental PQ to arise: $\Delta\chi^2\xi$

Accidental global PQ symmetry

$$V = \Phi\Sigma^*\xi + \Phi\Sigma^*(|\Sigma|^2 + |\Delta|^2 + |\chi|^2 + \xi^2) + \Sigma^{*2}(\Phi^2 + \Delta^2) + \Delta\chi^2\xi + \text{h.c.}$$

SSB and vev hierarchy

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
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SSB chain: $\underline{SU(4)_{\mathrm{PS}} \otimes SU(2)_L \otimes SU(2)_R} \otimes \underline{SU(3)_{f_R}} \otimes \underline{U(1)_{\mathrm{PQ}}}$

$$\xrightarrow{\langle \Delta, \chi, \xi \rangle} \underline{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y} \xrightarrow{\langle \Phi, \Sigma \rangle} SU(3)_c \otimes U(1)_{\mathrm{EM}}$$

Large vevs: $\langle \Delta, \chi, \xi \rangle \sim V \sim 10^9 - 10^{14}\ \mathrm{GeV}$

PS, Flavour, PQ SSB

Small vevs: $\langle \Phi, \Sigma \rangle \sim v \sim 10^2\ \mathrm{GeV}$

EW SSB

Hierarchy

$$v \ll V \ll M_{\mathrm{Pl}}$$

Axion embedding

Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$\mathrm{SU}(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$\mathrm{U}(1)_{\mathrm{PQ}}$
Q_L	$(1/2,0)$	$(4,2,1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
Q_R	$(0,1/2)$	$(4,1,2)$	$+i$	$\mathbf{3}$	$e^{i2\pi/3}$	1	$+1$
Ψ_R	$(0,1/2)$	$(1,1,1)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	8	$+2$
Φ	$(0,0)$	$(1,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
Σ	$(0,0)$	$(15,2,2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Sigma \geq 2$	$+2$
Δ	$(0,0)$	$(10,1,3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0,0)$	$(4,1,2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0,0)$	$(15,1,3)$	$+1$	$\mathbf{1}$	$+1$	1	0

- Axion a: (mostly) a combination of polar modes in Δ and χ

Peccei-Quinn
scale

$$f_a = \frac{V_\chi V_\Delta}{3\sqrt{V_\chi^2 + 4V_\Delta^2}}$$

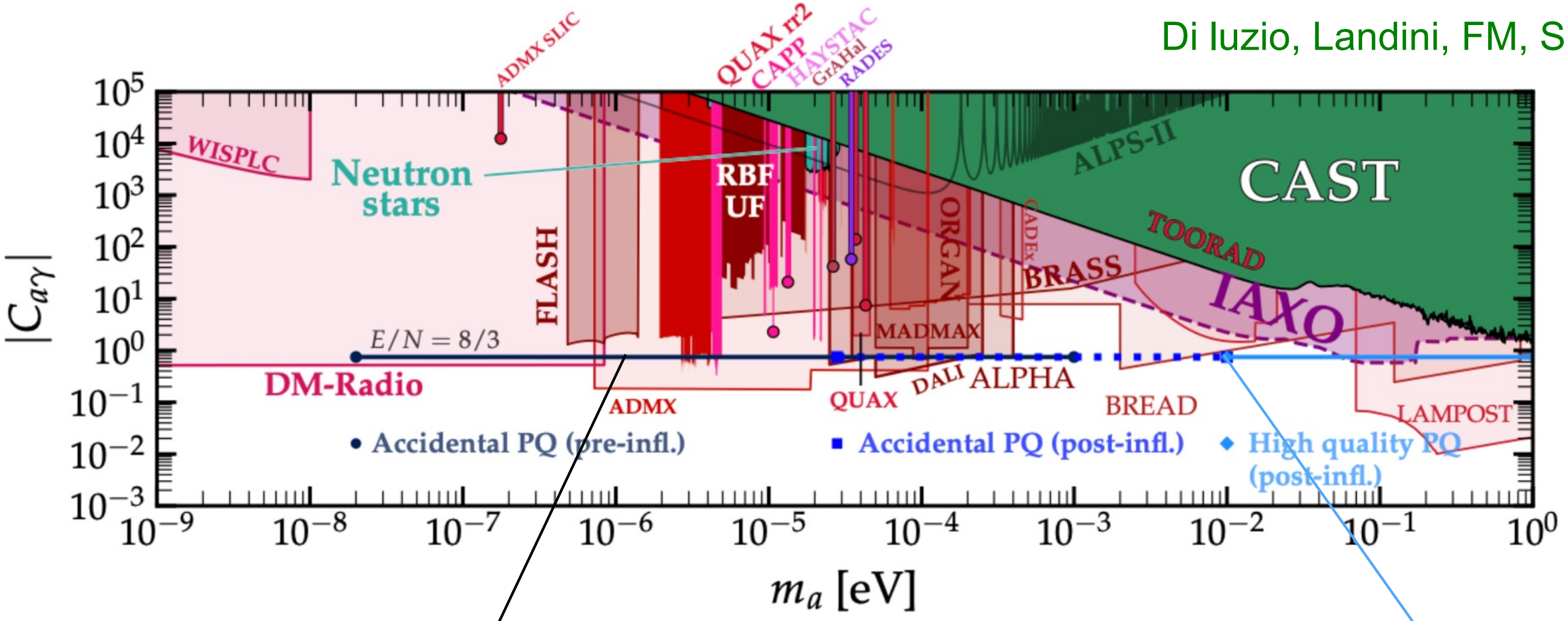
$V_\Delta,\ V_\chi \sim 10^9 - 10^{14}$ GeV

➡ related to PS and flavor breaking scales

Pati-Salam – axion flavour model

Axion mass range: **high-quality** PQ vs **accidental** PQ scenario

Di Iuzio, Landini, FM, Susic ’25



E/N=8/3 DFSZ-like

high-quality PQ

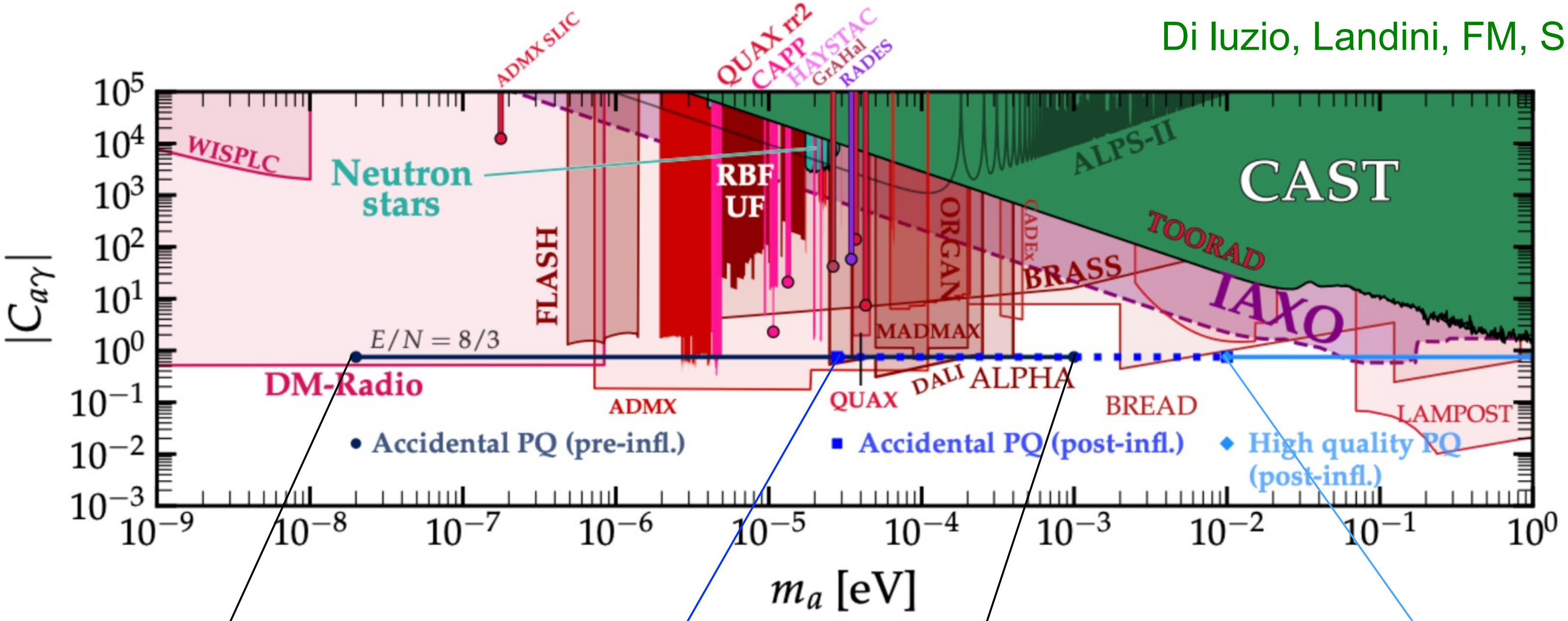
$$\boxed{m_a \gtrsim 0.01\ \text{eV}}$$

$$\mathcal{L}_{a\gamma} = \frac{\alpha_{\rm em} C_{a\gamma}}{8\pi f_a}\, a\, F^{\mu\nu}\tilde{F}_{\mu\nu} \qquad C_{a\gamma} = E/N - 1.92(4)\,,$$

Pati-Salam – axion flavour model

Axion mass range: **high-quality** PQ vs **accidental** PQ scenario

Di Luzio, Landini, FM, Susic '25



$f_a^{\cancel{B-L}} \lesssim 10^{14}$ GeV

$f_a \lesssim 10^{12}$ GeV

$f_a \gtrsim 10^{10}$ GeV

seesaw
scale

DM axion
overclosure

iso-curvature
fluctuations

high-quality PQ

$m_a \gtrsim 0.01$ eV

a PQ theory tells us where to search in an otherwise huge param. space!

Axion quality in PS – flavor model

- Axion a: (mostly) a combination of polar modes in Δ and χ

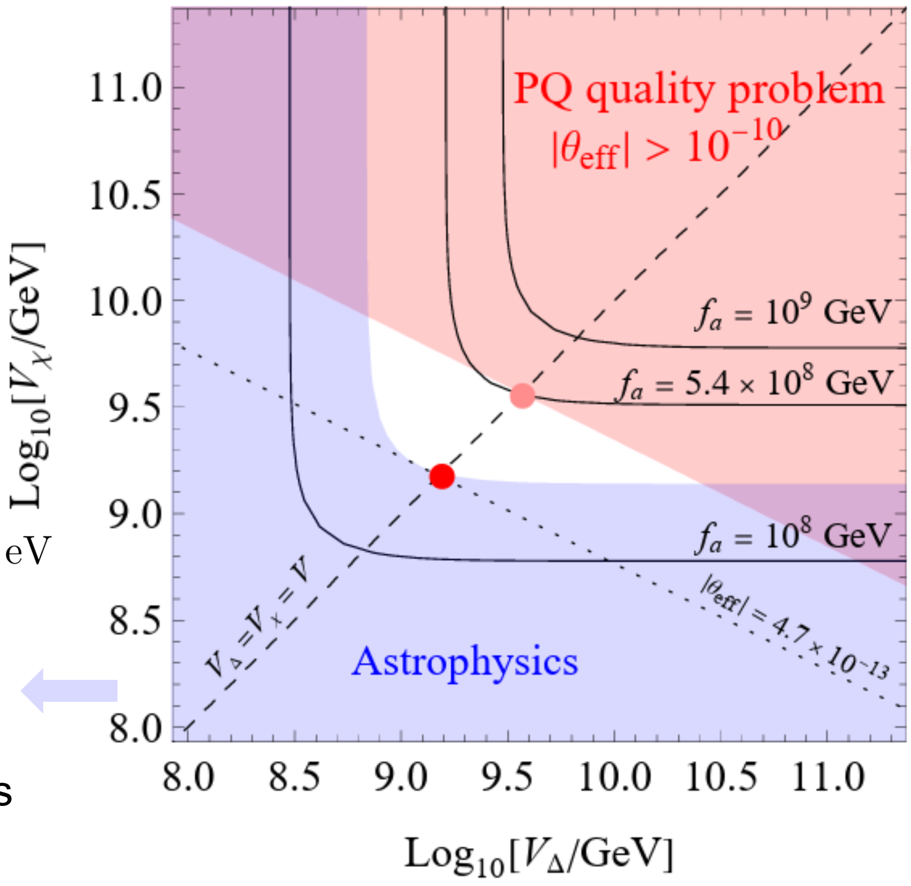
- Axion decay constant f_a:

$$f_a = \frac{V_\chi V_\Delta}{3\sqrt{V_\chi^2+4V_\Delta^2}}$$

● $f_a \gtrsim 2.3 \times 10^8\ \text{GeV}$

$m_a \lesssim 0.02\ \text{eV}$

axion-electron coupling
from
red giants+white dwarfs



$$\frac{v^2 V_\Delta^2 V_\chi^4}{M_{\rm Pl}^4} \gtrsim 10^{-10}\chi_{\rm QCD}^4$$

allowed

● $f_a \lesssim 5.6 \times 10^8\ \text{GeV}$

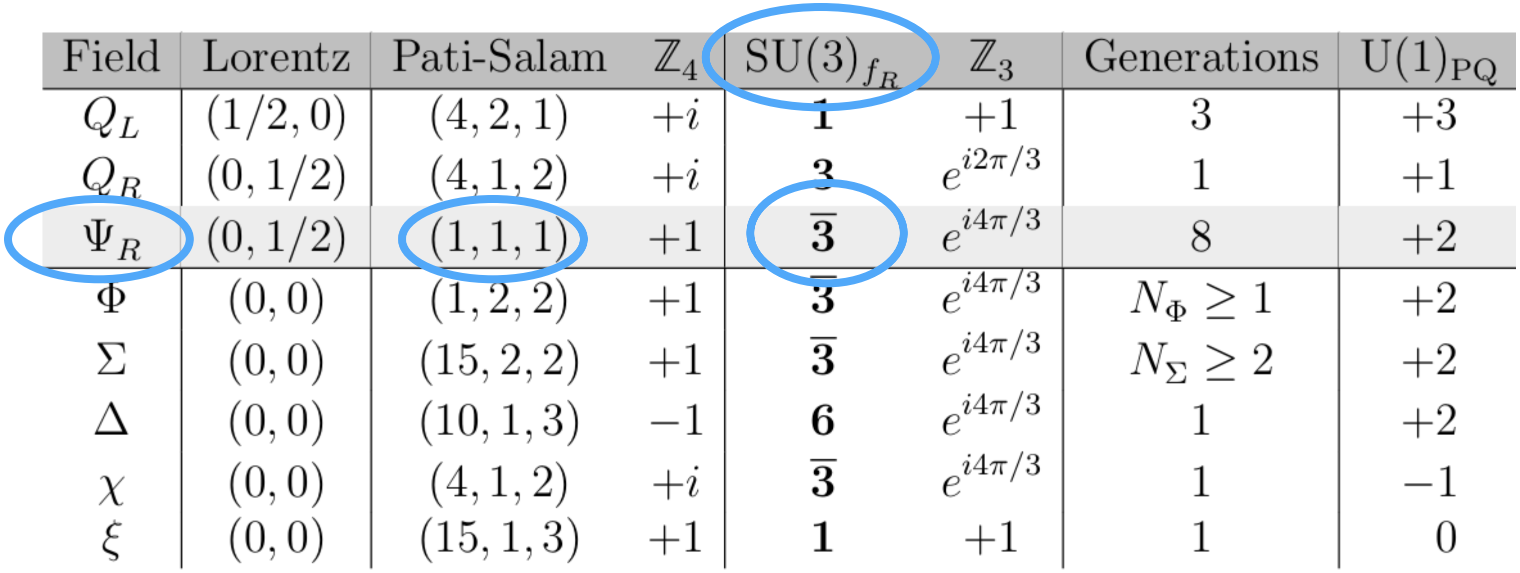
$m_a \gtrsim 0.01\ \text{eV}$

Minimal value allowed

$|\theta_{\rm eff}^{\rm min}| \simeq 4.7 \times 10^{-13}$

Testable by future
proton/neutron EDM
experiments

Anomalon phenomenology



Field	Lorentz	Pati-Salam	$\mathbb{Z}_4$	$SU(3)_{f_R}$	$\mathbb{Z}_3$	Generations	$U(1)_{PQ}$
Q_L	$(1/2, 0)$	$(4, 2, 1)$	$+i$	$\mathbf{1}$	$+1$	3	$+3$
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Φ	$(0, 0)$	$(1, 2, 2)$	$+1$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	$N_\Phi \geq 1$	$+2$
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Δ	$(0, 0)$	$(10, 1, 3)$	-1	$\mathbf{6}$	$e^{i4\pi/3}$	1	$+2$
χ	$(0, 0)$	$(4, 1, 2)$	$+i$	$\overline{\mathbf{3}}$	$e^{i4\pi/3}$	1	-1
ξ	$(0, 0)$	$(15, 1, 3)$	$+1$	$\mathbf{1}$	$+1$	1	0

Flavour gauge anomaly cancellation requires new exotic fermions: *anomalons*

GUT (SM) singlet – only charged under the flavor group

Anomalons→ new testable dynamics directly related to the solution of PQ quality problem and the flavor puzzle

Anomalon phenomenology

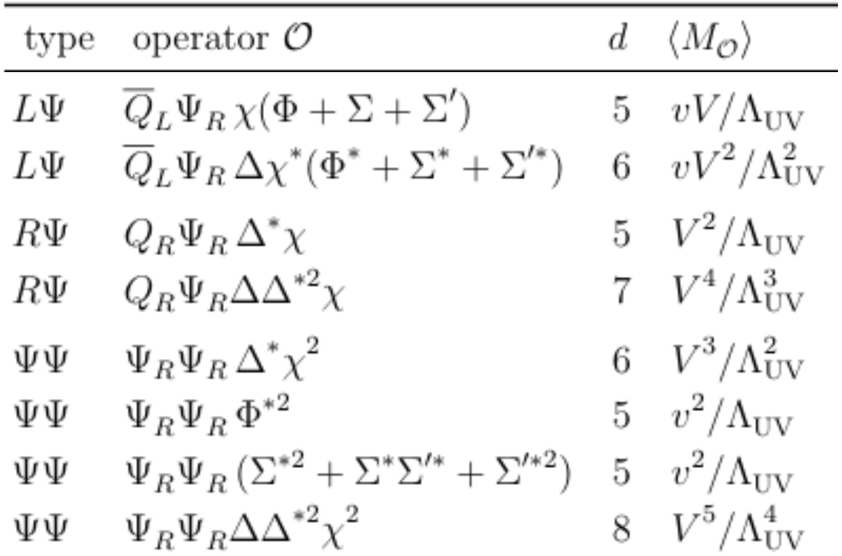
Anomalons are massless at the renormalizable level

$$\Psi_R\Psi_R \quad Q_R\Psi_R \quad \bar{Q}_L\Psi_R$$

$$\Psi_R\Psi_R\phi \quad Q_R\Psi_R\phi \quad \bar{Q}_L\Psi_R\phi$$

forbidden by gauge invariance → no ψ_L

Higher-dimensional operators can generate their mass



type	operator $\mathcal{O}$	d	$\langle M_{\mathcal{O}}\rangle$
$L\Psi$	$\overline{Q}_L\Psi_R\,\chi(\Phi+\Sigma+\Sigma')$	5	$vV/\Lambda_{\rm UV}$
$L\Psi$	$\overline{Q}_L\Psi_R\,\Delta\chi^*(\Phi^*+\Sigma^*+\Sigma'^*)$	6	$vV^2/\Lambda_{\rm UV}^2$
$R\Psi$	$Q_R\Psi_R\,\Delta^*\chi$	5	$V^2/\Lambda_{\rm UV}$
$R\Psi$	$Q_R\Psi_R\Delta\Delta^{*2}\chi$	7	$V^4/\Lambda_{\rm UV}^3$
$\Psi\Psi$	$\Psi_R\Psi_R\,\Delta^*\chi^2$	6	$V^3/\Lambda_{\rm UV}^2$
$\Psi\Psi$	$\Psi_R\Psi_R\,\Phi^{*2}$	5	$v^2/\Lambda_{\rm UV}$
$\Psi\Psi$	$\Psi_R\Psi_R\,(\Sigma^{*2}+\Sigma^*\Sigma'^*+\Sigma'^{*2})$	5	$v^2/\Lambda_{\rm UV}$
$\Psi\Psi$	$\Psi_R\Psi_R\Delta\Delta^{*2}\chi^2$	8	$V^5/\Lambda_{\rm UV}^4$

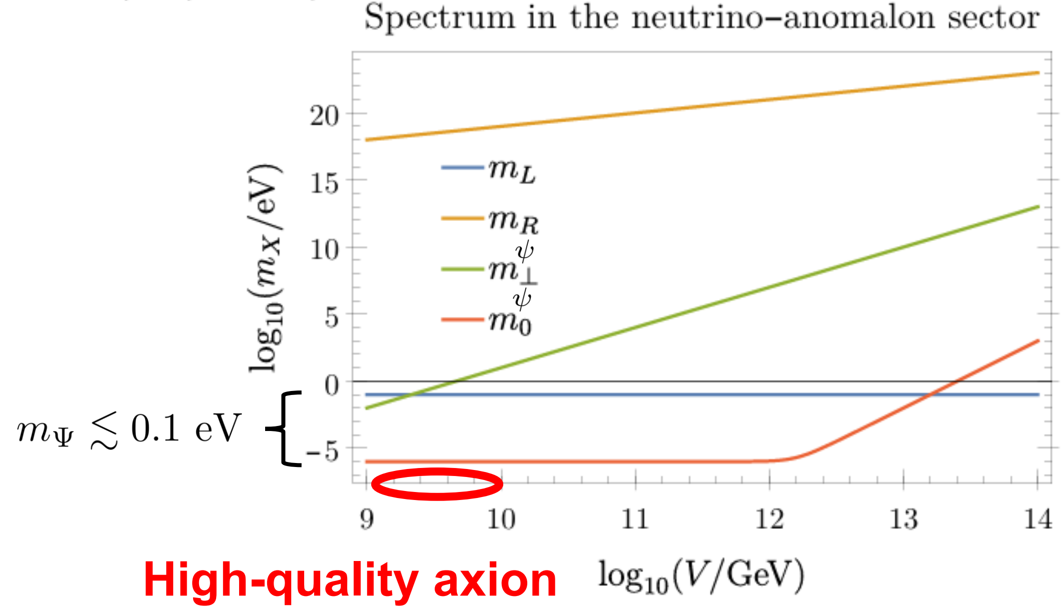
Anomalons → potential test of the quality mechanism!

Anomalon phenomenology

Being SM-singlets, they mix with neutrinos !

$$\frac{1}{2}\begin{pmatrix}(\overline{\nu}_L)^I\\(\nu_R)^A\\(\Psi_R)^K{}_A\end{pmatrix}^T\begin{pmatrix}(M_{LL})_{IJ} & (M_{LR})_{IB} & (M_{L\Psi})_{IN}{}^B\\(M_{RL})_{AJ} & (M_{RR})_{AB} & (M_{R\Psi})_{AN}{}^B\\(M_{\Psi L})_K{}^A{}_J & (M_{\Psi R})_K{}^A{}_B & (M_{\Psi\Psi})_K{}^A{}_N{}^B\end{pmatrix}\begin{pmatrix}(\overline{\nu}_L)^J\\(\nu_R)^B\\(\Psi_R)^N{}_B\end{pmatrix}$$

see-saw type I



$m_\Psi \lesssim 0.1$ eV

High-quality axion

High-quality PQ predicts sub-eV anomalons

Anomalon phenomenology

Being SM-singlets, they mix with neutrinos !

and they interact with the SM via

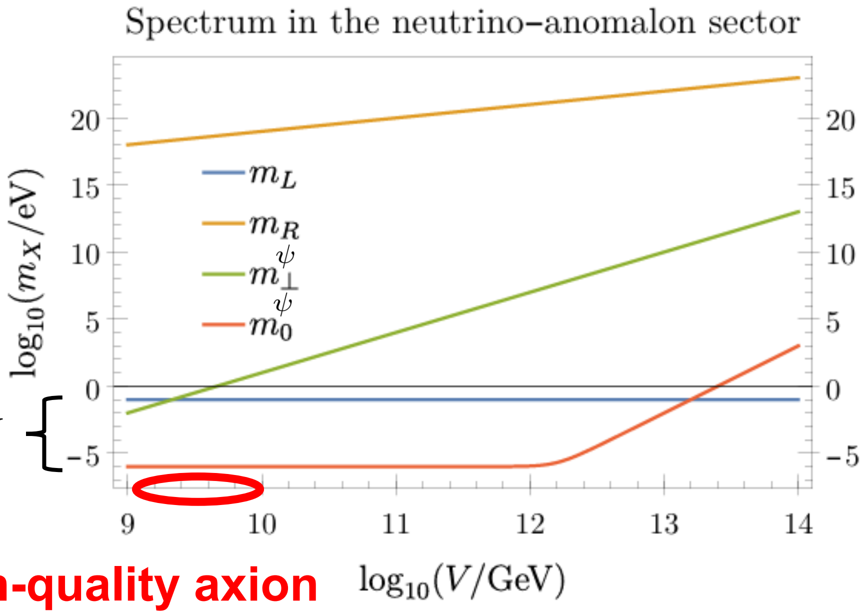
i) flavor gauge interactions

ii) neutrino mixing

Dark Radiation

$$\Delta N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\rho_\Psi}{\rho_\gamma}$$

$m_\Psi \lesssim 0.1$ eV

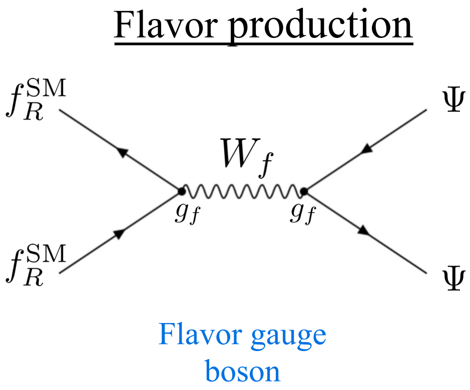


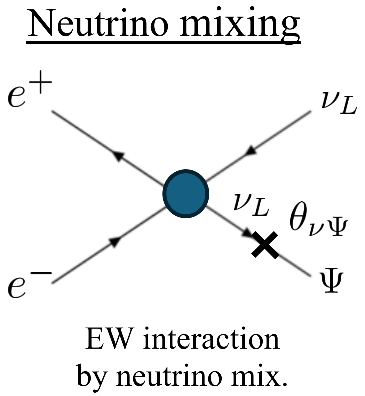
High-quality axion

High-quality PQ predicts sub-eV anomalons

Anomalon phenomenology

How are they produced in the early Universe?





⇩

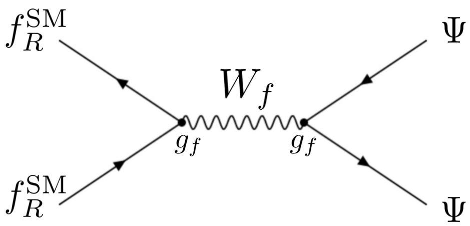
- $SU(3)_{fR}$ gauge bosons at the PS scale of 10^9 GeV

Anomalons → potential test of the quality mechanism!

Anomalon phenomenology

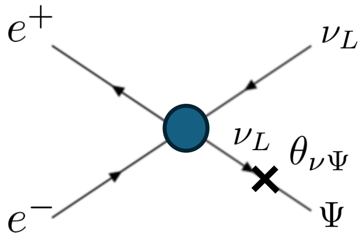
How are they produced in the early Universe?

Flavor production



Flavor gauge
boson

Neutrino mixing



EW interaction
by neutrino mix.

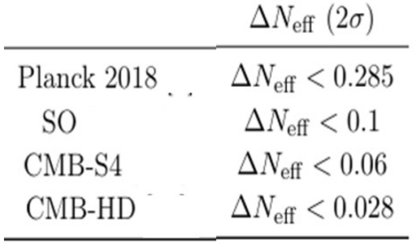
$$Y_\Psi = n_\Psi / s$$

Large couplings $g_f, \theta_{\nu\Psi} \sim \mathcal{O}(1)$

Thermalization with SM bath

$$\Delta N_{\rm eff}^{\rm TH} \simeq 1.13 \frac{N_\Psi}{24}\left(\frac{106.75}{g_s(T_{\rm dec})}\right)^{4/3} > 0.285$$

already excluded by PLANCK 18'!



	$\Delta N_{\rm eff}$ (2σ)
Planck 2018	$\Delta N_{\rm eff} < 0.285$
SO	$\Delta N_{\rm eff} < 0.1$
CMB-S4	$\Delta N_{\rm eff} < 0.06$
CMB-HD	$\Delta N_{\rm eff} < 0.028$

Very small
couplings $g_f, \theta_{\nu\Psi} \ll \mathcal{O}(1)$

No thermalization / freeze-in production

$$\Delta N_{\rm eff}^{\rm FI} \simeq 56.96\, Y_\Psi / g_s(T_{\rm FI})^{1/3} \ll 0.014$$

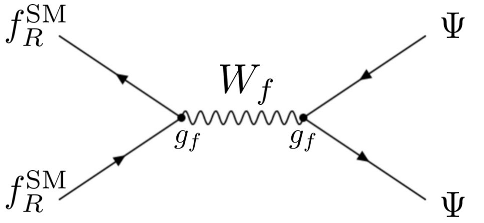
not observable!

Anomalons → potential test of the quality mechanism!

Anomalon phenomenology

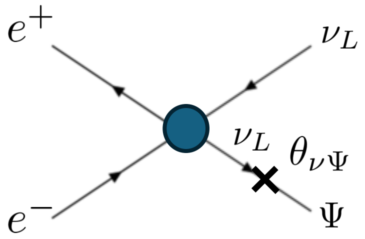
How are they produced in the early Universe?

Flavor production



Flavor gauge boson

Neutrino mixing



EW interaction by neutrino mix.

$$Y_\Psi = n_\Psi / s$$

Large couplings $g_f, \theta_{\nu\Psi} \sim \mathcal{O}(1)$

Thermalization with SM bath

$$\Delta N_{\mathrm{eff}}^{\mathrm{TH}} \simeq 1.13 \frac{N_\Psi}{24} \left(\frac{106.75}{g_s(T_{\mathrm{dec}})}\right)^{4/3} > 0.285$$

already excluded by PLANCK 18'!

Intermediate regime

$$0.014 \lesssim \Delta N_{\mathrm{eff}}^{\mathrm{FI}} \lesssim 1.13$$

More precise computation is needed

Very small couplings $g_f, \theta_{\nu\Psi} \ll \mathcal{O}(1)$

No thermalization / freeze-in production

$$\Delta N_{\mathrm{eff}}^{\mathrm{FI}} \simeq 56.96\, Y_\Psi / g_s(T_{\mathrm{FI}})^{1/3} \ll 0.014$$

not observable!

Anomalons → potential test of the quality mechanism!

Conclusions

❖ The axion hypothesis provides a well motivated BSM scenario

- solves the strong CP problem
- provides a DM candidate
- **is unambiguously testable by detecting the axion**

❖ The Peccei Quinn symmetry must be protected by UV sources
(*quality problem*)

⬇

GUT + flavor gauge symmetries provide the desired protection

● No *ad-hoc* symmetries introduced

⬇

Connection between PQ and flavor structure

● Phenomenological signatures

⬇

Cosmology of anomalons

☹ more precise computations
+ anomalons as extended neutrino sectors
+ flavor observable connection

Много благодаря

THANK YOU!

SM fermions embedding

$$(\overline{Q}_L)^I{}_{ai} = \begin{pmatrix} \overline{u}^I_{L1} & \overline{d}^I_{L1} \\ \overline{u}^I_{L2} & \overline{d}^I_{L2} \\ \overline{u}^I_{L3} & \overline{d}^I_{L3} \\ \overline{\nu}^I_{L} & \overline{e}^I_{L} \end{pmatrix} , \qquad (Q_R)^{ai'A} = \begin{pmatrix} u^A_{R1} & d^A_{R1} \\ u^A_{R2} & d^A_{R2} \\ u^A_{R3} & d^A_{R3} \\ \nu^A_{R} & e^A_{R} \end{pmatrix}$$

Axion quality in PS – flavor model

$$\Delta\mathcal{L}_{\rm UV}^{\cancel{PQ}} \sim \frac{\mathcal{O}^{[d]}}{M_{\rm Pl}^{d-4}}$$

$$V_\chi \sim V_\Delta \sim V$$

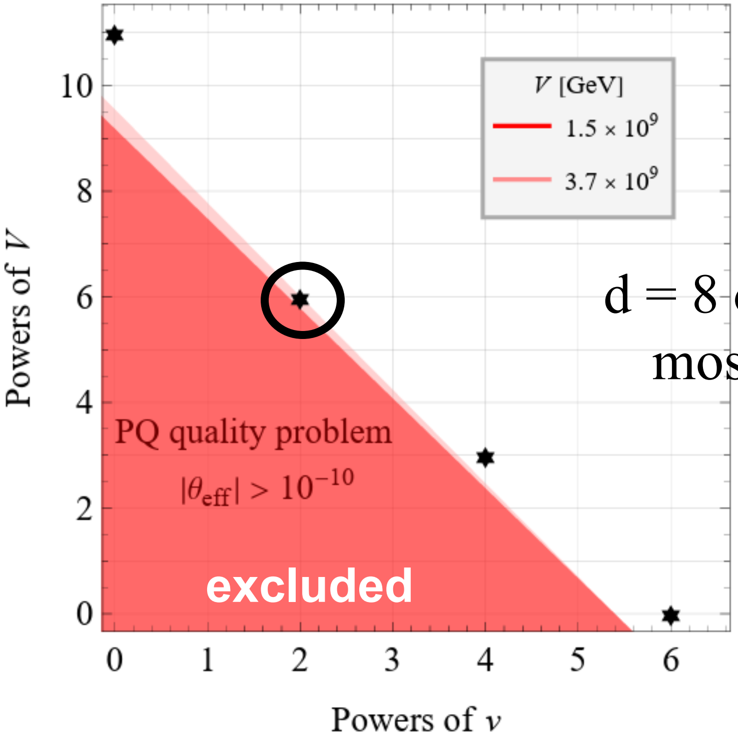
$$V_\Phi \sim V_\Sigma \sim v$$

$$(\Delta\Delta^*)\Delta^3\chi^{*6} \to V^{11}/M_{\rm Pl}^7$$

$$\underline{\Phi^{2-k}\Sigma^k\Delta^2\chi^{*4} \to v^2V^6/M_{\rm Pl}^4}$$

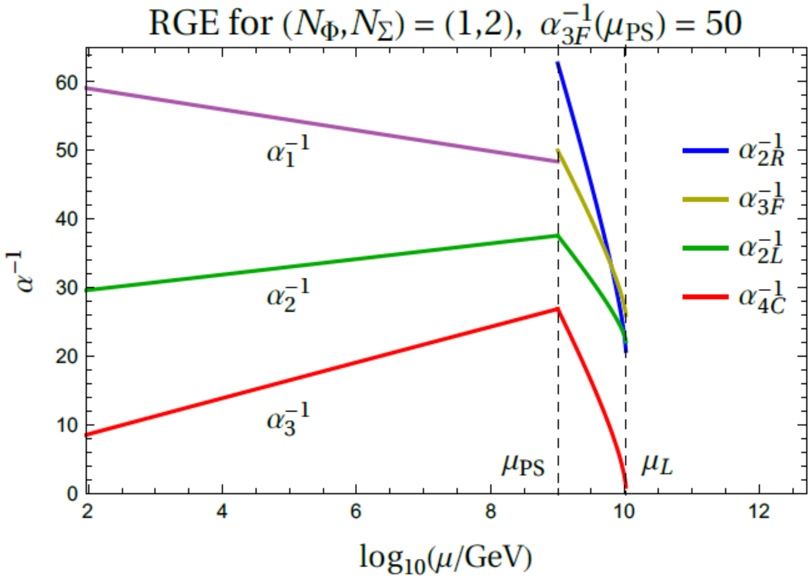
$$\Phi^{4-k}\Sigma^k\Delta\chi^{*2} \to v^4V^3/M_{\rm Pl}^3$$

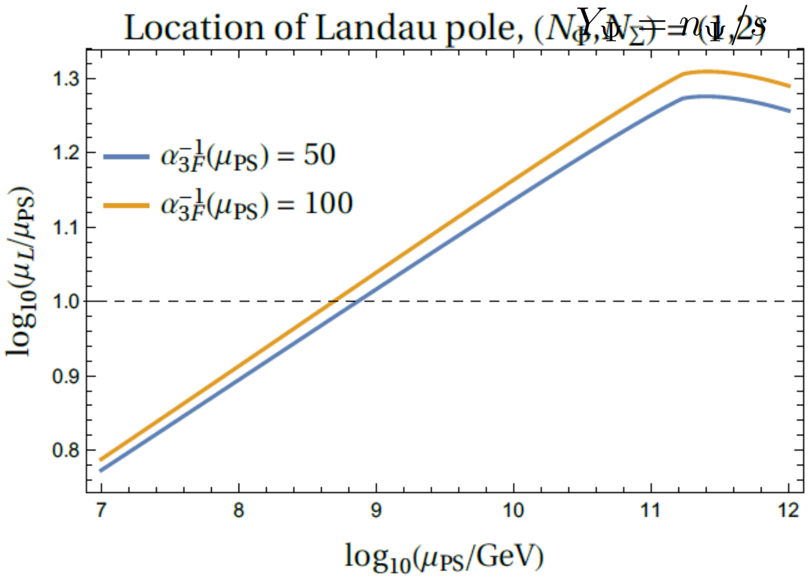
$$\Phi^{4-k}\Sigma^k\Sigma^2 \to v^6/M_{\rm Pl}^2$$



d = 8 operator is the most dangerous

Perturbativity





Landau pole of gauge couplings at scale O(10) x PS scale

New physics around the LP scale → We must require that this new physics is PQ-conserving in order not to worsten the quality problem!

Domain Wall problem

Bias term $\mathcal{V}_{\rm bias} = -2\Xi V^4 \cos\left(\frac{a}{V}+\delta\right)$ $\quad V = N_{\rm DW} f_a$

➡ $t_{\rm decay} \approx\ 5\times 10^{-5}\ {\rm s}\ \left(\frac{10^{-50}}{\Xi}\right)\left(\frac{12}{N_{\rm DW}}\right)^4\left(\frac{m_a}{0.02\ {\rm eV}}\right)^3$

Matching the bias term to $\Phi^{2-k}\Sigma^k\Delta^2\chi^{*4} \to v^2 V^6/M_{\rm Pl}^4$

➡ $\Xi \sim \left(\frac{v}{M_{\rm Pl}}\right)^2\left(\frac{V}{M_{\rm Pl}}\right)^2 \sim 10^{-52}(N_{\rm DW}/12)^2(0.02\ {\rm eV}/m_a)^2$

➡ $t_{\rm decay} \sim 10^{-3}\ {\rm sec}\left(\frac{12}{N_{\rm DW}}\right)^6\left(\frac{m_a}{0.02\ {\rm eV}}\right)^5$

DW decay before BBN ➡ $f_a \gtrsim 4\times 10^8\ {\rm GeV}$

Anomalon phenomenology

Being SM-singlets, they mix with neutrinos !

↓

New (light) fermions, **interact with the SM** via

i) **flavor** gauge interactions

ii) **neutrino** mixing

$$M_{\nu\Psi} = \begin{pmatrix} \frac{v^2 V}{\Lambda^2} & yv & l\,\frac{vV}{\Lambda} & \tilde{l}\,\frac{vV^2}{\Lambda^2} \\ .. & V & r\,\frac{V^2}{\Lambda} & \tilde{r}\,\frac{V^4}{\Lambda^3} \\ .. & .. & \frac{V^3}{\Lambda^2} & \frac{v^2}{\Lambda} + \frac{V^5}{\Lambda^4} \\ .. & .. & .. & \frac{v^2}{\Lambda} + \frac{V^5}{\Lambda^4} \end{pmatrix}$$

→ In upper-left 2×2: **see-saw** type I

Axion quality in PS – flavor model

Let’s introduce the higher-dimensional operators

$$\Delta\mathcal{L}_{\mathrm{UV}} \sim \frac{\mathcal{O}^{[d]}}{M_{\mathrm{Pl}}^{d-4}} \quad \Longrightarrow \quad \frac{\langle\mathcal{O}\rangle^{[d]}}{M_{\mathrm{Pl}}^{d-4}} \sim \frac{v^{n} V^{d-n}}{M_{\mathrm{Pl}}^{d-4}}$$

$\Downarrow$ which ones are dangerous for PQ quality?

i) Gauge-invariant

ii) PQ- breaking

} *Easy checks*

iii) check non-vanishing explicit index contraction

iv) non-vanishing vacuum contribution $\langle\mathcal{O}\rangle \neq 0$

} *Not trivial*

Anomalon spectrum

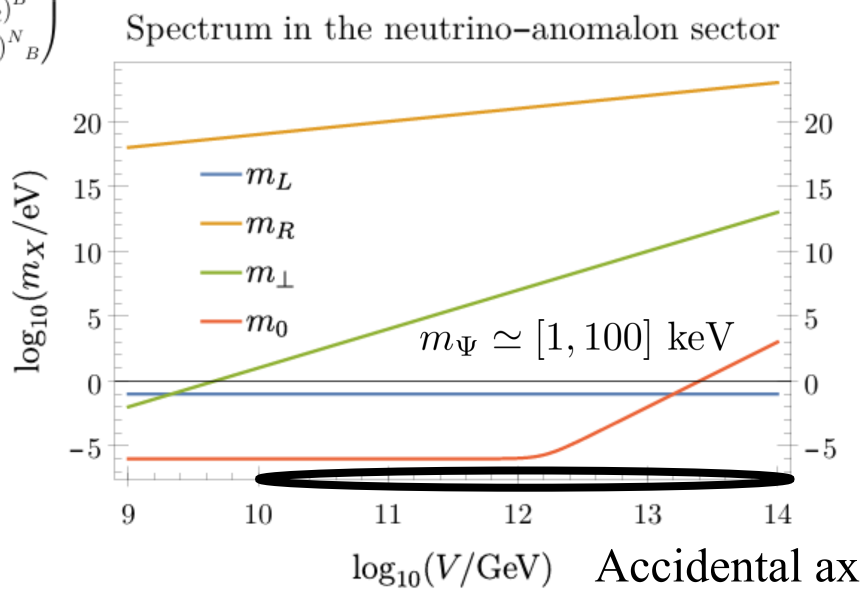
$$\frac{1}{2}\begin{pmatrix} (\overline{\nu}_L)^I \\ (\nu_R)^A \\ (\Psi_R)^K{}_A \end{pmatrix}^T \begin{pmatrix} (M_{LL})_{IJ} & (M_{LR})_{IB} & (M_{L\Psi})_{IN}{}^B \\ (M_{RL})_{AJ} & (M_{RR})_{AB} & (M_{R\Psi})_{AN}{}^B \\ (M_{\Psi L})_K{}^A{}_J & (M_{\Psi R})_K{}^A{}_B & (M_{\Psi\Psi})_K{}^A{}_N{}^B \end{pmatrix} \begin{pmatrix} (\overline{\nu}_L)^J \\ (\nu_R)^B \\ (\Psi_R)^N{}_B \end{pmatrix}$$

Dark radiation

+

<u>Dark Matter</u>

Analogous to sterile neutrino DM



Accidental axion

Axion astrophysical limits

Axion couplings with matter $g_{aX} = c_X m_X / f_a$

$$c_p = -0.45 \quad + 0.29\cos^2\beta - 0.15\sin^2\beta\,,$$

$$c_n = +0.013 - 0.14\cos^2\beta + 0.27\sin^2\beta\,.$$

$$c_e = 0.33\sin^2\beta$$

SN 1987A → $g_{an}^2 + 0.61 g_{ap}^2 + 0.53 g_{an} g_{ap} \lesssim 8.3 \times 10^{-19}$

Red giants+white dwarfs → $g_{ae} \lesssim 2 \times 10^{-13}$

Saturating the perturbative bound $\tan\beta = 0.25$ → $f_a \gtrsim 2 \times 10^8\ \mathrm{GeV}$

dominated by SN 1987A

$$\tan\beta \equiv \frac{v_u}{v_d} = \frac{\sqrt{\sum_A |v_A^{u\Phi}|^2 + \sum_A |v_A^{u\Sigma}|^2 + \sum_A |v_A^{u\Sigma'}|^2}}{\sqrt{\sum_A |v_A^{d\Phi}|^2 + \sum_A |v_A^{d\Sigma}|^2 + \sum_A |v_A^{d\Sigma'}|^2}}\,.$$

Including radiative corrections to axion-electron coupling a stronger bound is obtained → $f_a \gtrsim 2.3 \times 10^8\ \mathrm{GeV}$

Quality problem of $U(1)_{PQ}$

We solve the Strong CP problem only if

$$\langle \theta_{\rm eff} \rangle < 10^{-10} \qquad \text{(neutron EDM)}$$

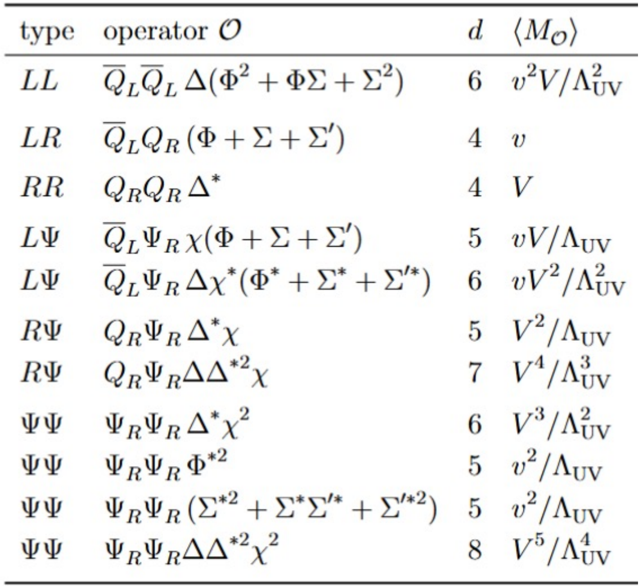
Minimizing the full potential, $V_{\rm QCD} + \Delta V_{\rm UV}^{\cancel{\rm PQ}}$, it translates to

$$\boxed{\left(\frac{f_a}{\Lambda_{\rm UV}}\right)^{d-4} f_a^4} \lesssim 10^{-10} \Lambda_{\rm QCD}^4$$

$$f_a \ll \Lambda_{\rm UV}$$

The lowest-dimensional PQ-breaking operators are the most dangerous!

Anomalon – neutrino mixings



type	operator $\mathcal{O}$	d	$\langle M_{\mathcal{O}} \rangle$
LL	$\overline{Q}_L \overline{Q}_L \, \Delta(\Phi^2 + \Phi\Sigma + \Sigma^2)$	6	$v^2 V/\Lambda_{\rm UV}^2$
LR	$\overline{Q}_L Q_R \, (\Phi + \Sigma + \Sigma')$	4	v
RR	$Q_R Q_R \, \Delta^*$	4	V
$L\Psi$	$\overline{Q}_L \Psi_R \, \chi(\Phi + \Sigma + \Sigma')$	5	$vV/\Lambda_{\rm UV}$
$L\Psi$	$\overline{Q}_L \Psi_R \, \Delta\chi^*(\Phi^* + \Sigma^* + \Sigma'^*)$	6	$vV^2/\Lambda_{\rm UV}^2$
$R\Psi$	$Q_R \Psi_R \, \Delta^* \chi$	5	$V^2/\Lambda_{\rm UV}$
$R\Psi$	$Q_R \Psi_R \Delta \Delta^{*2} \chi$	7	$V^4/\Lambda_{\rm UV}^3$
$\Psi\Psi$	$\Psi_R \Psi_R \, \Delta^* \chi^2$	6	$V^3/\Lambda_{\rm UV}^2$
$\Psi\Psi$	$\Psi_R \Psi_R \, \Phi^{*2}$	5	$v^2/\Lambda_{\rm UV}$
$\Psi\Psi$	$\Psi_R \Psi_R \, (\Sigma^{*2} + \Sigma^* \Sigma'^* + \Sigma'^{*2})$	5	$v^2/\Lambda_{\rm UV}$
$\Psi\Psi$	$\Psi_R \Psi_R \Delta \Delta^{*2} \chi^2$	8	$V^5/\Lambda_{\rm UV}^4$

$$\begin{pmatrix} M_{LL} & M_{LR} & M_{L\Psi_\perp} & M_{L\Psi_0} \\ M_{RL} & M_{RR} & M_{R\Psi_\perp} & M_{R\Psi_0} \\ M_{\Psi_\perp L} & M_{\Psi_\perp R} & M_{\Psi_\perp \Psi_\perp} & M_{\Psi_\perp \Psi_0} \\ M_{\Psi_0 L} & M_{\Psi_0 R} & M_{\Psi_0 \Psi_\perp} & M_{\Psi_0 \Psi_0} \end{pmatrix} \sim \begin{pmatrix} \frac{v^2 V}{\Lambda_{\rm UV}^2} & yv & l\,\frac{vV}{\Lambda_{\rm UV}} & \tilde{l}\,\frac{vV^2}{\Lambda_{\rm UV}^2} \\ .. & V & r\,\frac{V^2}{\Lambda_{\rm UV}} & \tilde{r}\,\frac{V^4}{\Lambda_{\rm UV}^3} \\ .. & .. & \frac{V^3}{\Lambda_{\rm UV}^2} & \frac{v^2}{\Lambda_{\rm UV}} + \frac{V^5}{\Lambda_{\rm UV}^4} \\ .. & .. & .. & \frac{v^2}{\Lambda_{\rm UV}} + \frac{V^5}{\Lambda_{\rm UV}^4} \end{pmatrix}$$