Report on track reconstruction with Gaussian Sum Filter method

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Track reconstruction in a Cherenkov neutrino telescope

- electromagnetic showers
- Cherenkov photon scattering
- ^{40}K beta-decay background

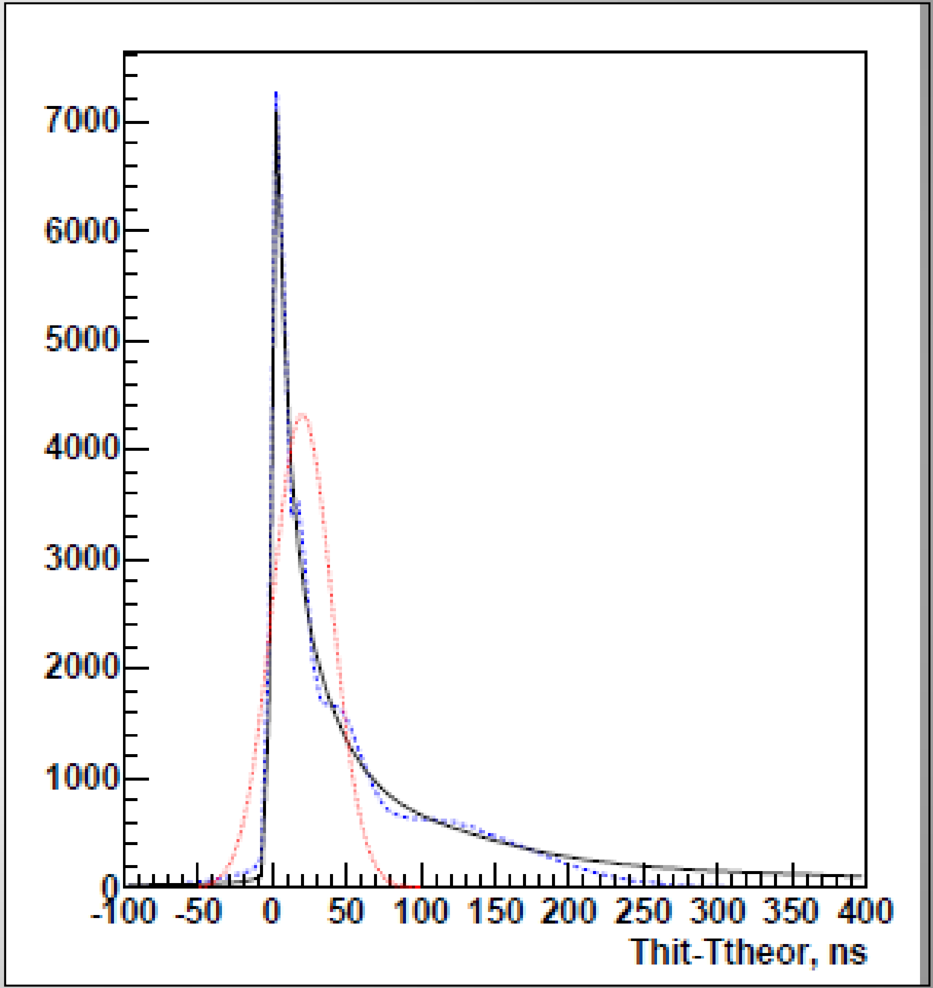
The arrival time depends on the track parameters (time, direction and position) as well as on the PMTs position (i.e. the detector geometry) and track reconstruction deals in a non-linear problem with VERY non-gaussian measument errors

We have non-linear problem with non-gaussian distributions

A method to take into account non-Gaussian distributions of measurement errors is the Gaussian-Sum Filter, based on the Kalman Filter approach

Gaussian Sum Filter (GSF)

The distribution of the measurement errors may be approximated by sum of several Gaussian distributions, so that a Gaussian mixture model becomes the natural description: the core corresponds to the principal component of the Gaussian mixture, and the tails can be modeled by one or several additional Gaussians.



solid — Monte-Carlo,
red dot — «best» approximation with one gaussian
blue dash — approximation with sum of five gaussians,

Gaussian Sum Filter (GSF)

so, approximating non-gaussian measurement errors distribution as sum of several gaussians:

$$p(\epsilon_k)=\sum_1^m p_i \varphi(\epsilon_k, \mu_i, \sigma_i) \quad , \quad \sum_1^m p_i=1$$

State vector becomes multicomponent, because one measurement produce one estimation of state vector for each gaussian components of the measurement error

$$x_{k|k}^{ij}=x_{k|k-1}^{j}+C_{k|k}^{ij} H^T \sigma_i^{-2}\left(y_k - H x_{k|k-1}^{j} - \mu_i\right)$$

with covariation matrix

$$C_{k|k}^{ij}=\left(\left(C_{k|k-1}^{j}\right)^{-1}+H^T \sigma_i^{-2} H\right)^{-1}$$

Gaussian Sum Filter

State vector now is a combination of state vector components

$$x_{k|k} = \sum_{l=1}^{n_k} q_k^l x_{k|k}^l$$

With corresponding covariation matrix

$$C_{k|k} = \sum_{l=1}^{n_k} q_k^l C_{k|k}^l + \sum_{l=1}^{n_k} \sum_{m>l}^{n_k} q_k^l q_k^m \left(x_{k|k}^l - x_{k|k}^m\right)\left(x_{k|k}^l - x_{k|k}^m\right)^T$$

and Weights

$$q_k^l = q_k^{ij} = p_i q_{k-1} \varphi\left(y_k ; H x_{k|k-1}^j, \sigma_i^2 + H C_{k|k-1}^j H^T\right)$$

Index «l» (and «m») join indexes «i» and «j» in one sequence.

full result is weighted mean of accumulated components

Gaussian Sum Filter

In our study, we assume that the distribution of the measurement errors ε_k can be modelled by a Gaussian mixture with five components:

$$p(\varepsilon_k) = \sum_{i=1}^{5} p_i \cdot \varphi(\varepsilon_k; \mu_i, \sigma_i)$$

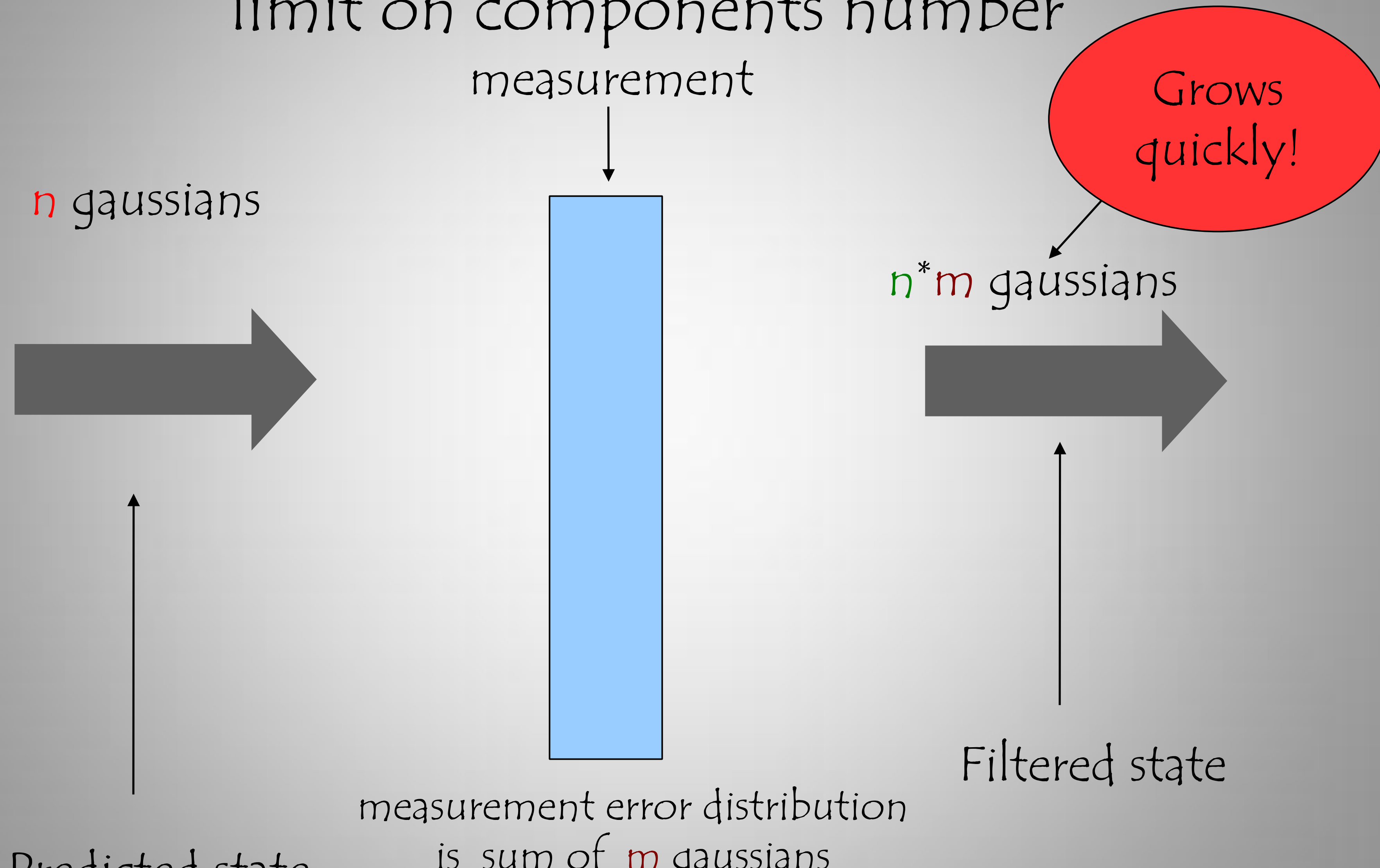
$$\sum_{i=1}^{5} p_i = 1$$

Distribution of measurement errors was obtained from Monte-Carlo data as distribution of measured hit arrival time and theoretical one ($m_k - T_{theor}$)

The GSF resembles a set of Kalman filters running in parallel, each Kalman filter corresponding to one of the components of the state vector mixture.

Each of the filters or components has a weight attached.

Gaussian Sum Filter: limit on components number



Gaussian Sum Filter
limit on number of components

A strict application of the GSF algorithm quickly leads to a prohibitively large number of components due to combinatorics involved each time a hit is added

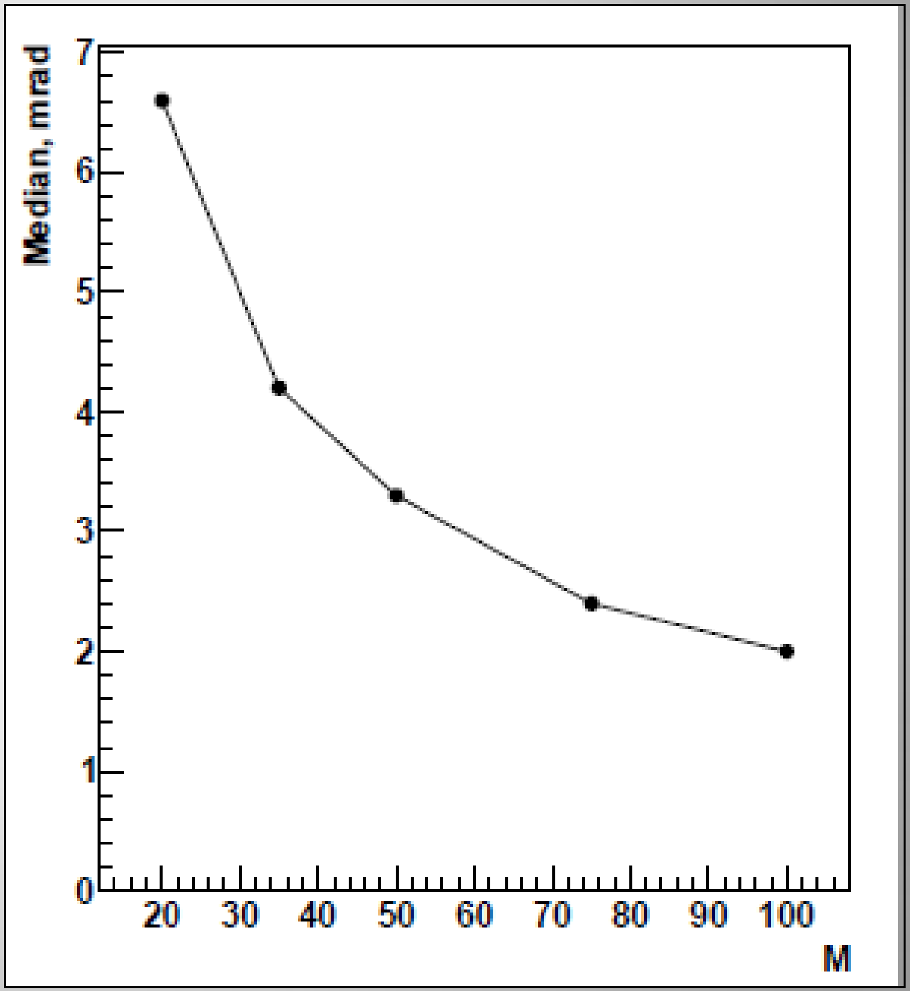
In each filtering step, we obtain 5 times more state vector components than the number of predicted ones: after filtering step for the $k-th$ hit the number of components is 5^k.

Possible strategies:

- select M components with maximal weights,
- clusterization (on base of distance between components) ,
- resampling — random choice of M components with correspondance to them weights.

Gaussian Sum Filter
limit on number of components

By denoting with M the maximum number of components allowed in the filtered step, we studied the dependence of the algorithm track angle error on M:
a value about of at least 50 components is quite good from point of view of the algorithm performances.

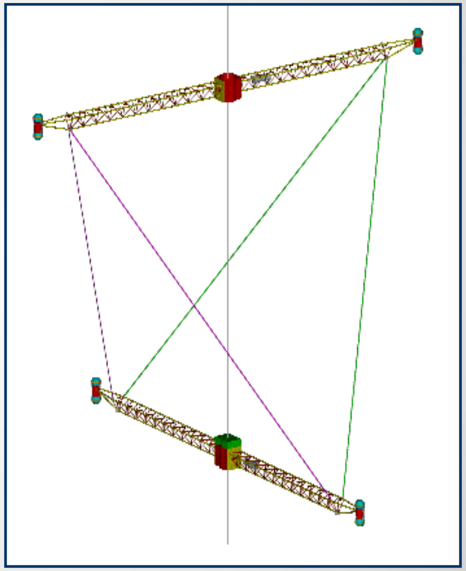


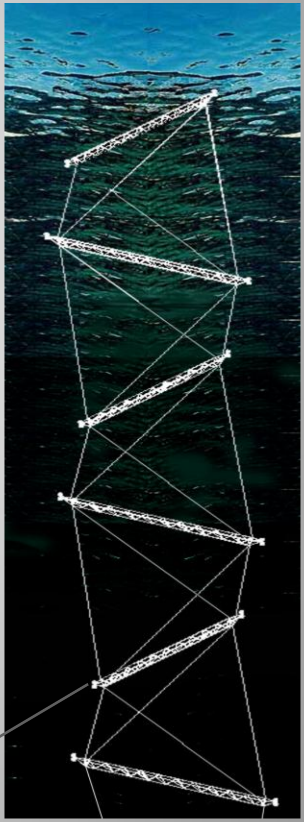
Results

We will refer, for this study, to the one proposed by the NEMO Collaboration as reference detector

A square array of structures (towers) composed of a sequence of "storeys" hosting the PMTs. Each storey will be rotated by 90°, with respect to the upper and lower adjacent ones, around the vertical axis of the tower:

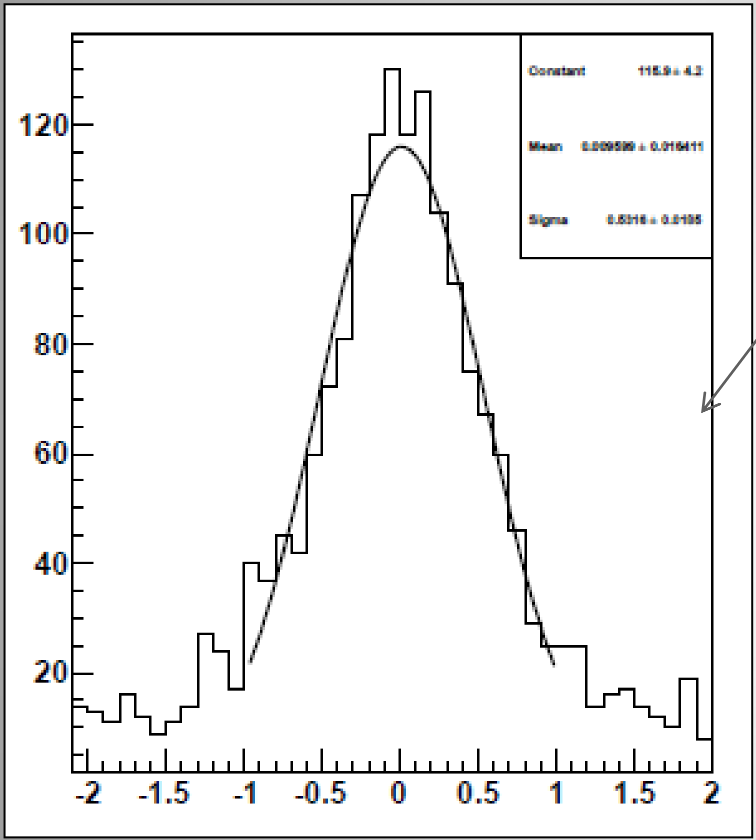
-9×9 towers spaced 140 m, with 72 PMTs for each tower; 5832 PMTs in total
- detector volume ~ 0.9 km^3
- 18 storey per tower
- storey 20 m long with two optical modules (one downlooking and one looking horizontally) at each end (4 OMs per storey)
- Distance between storeys is 40 m



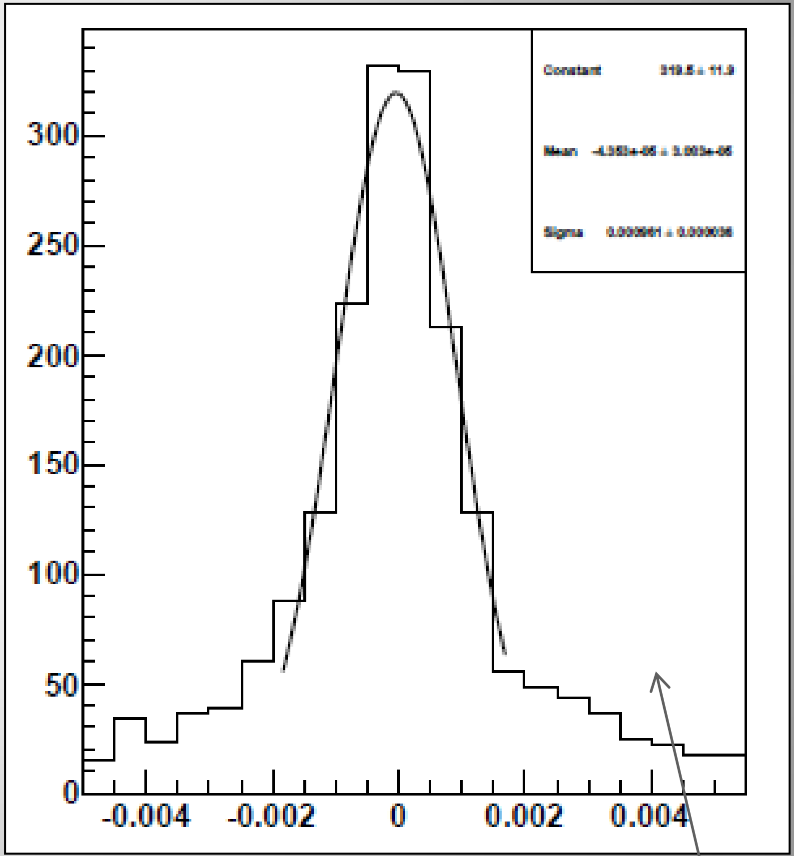


The muon flux is distributed as E^{-1} whitin a range 100 GeV-10^4 TeV. Muons are going upward and distributed uniformly in the hemisphere. Start points of the tracks are always placed outside of the detector.

Results



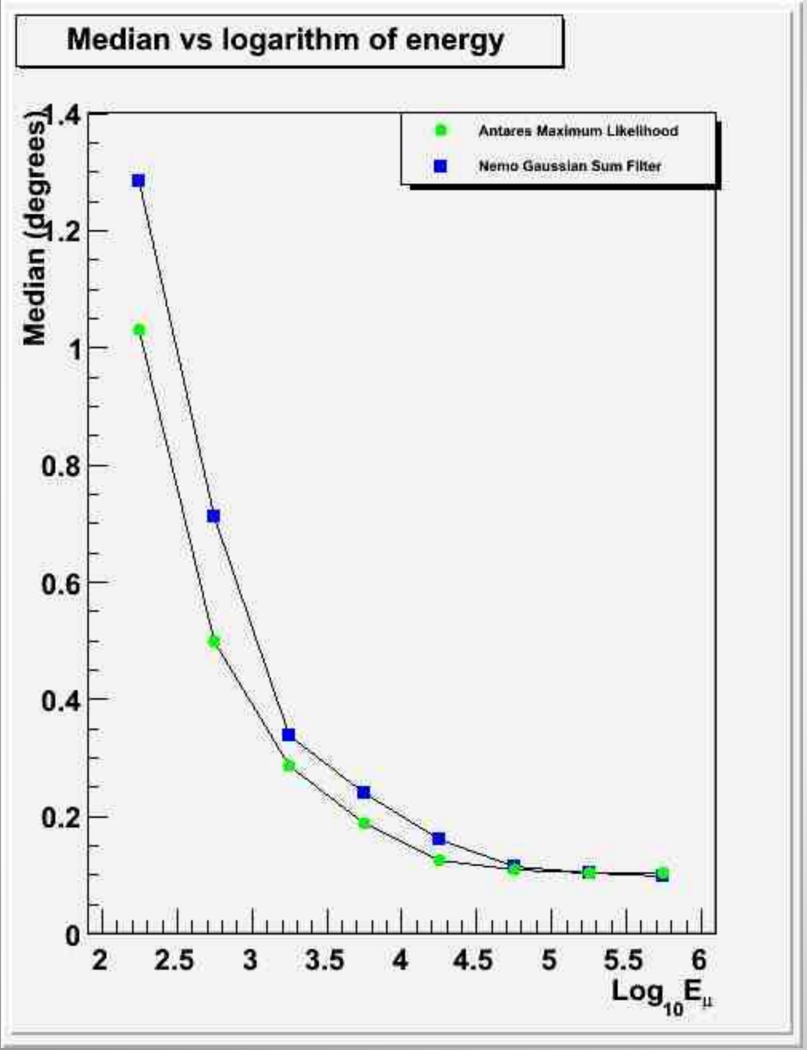
distribution of the difference between reconstructed and generated X-coordinate parameter



distribution of the difference between reconstructed and generated zenith angle parameter

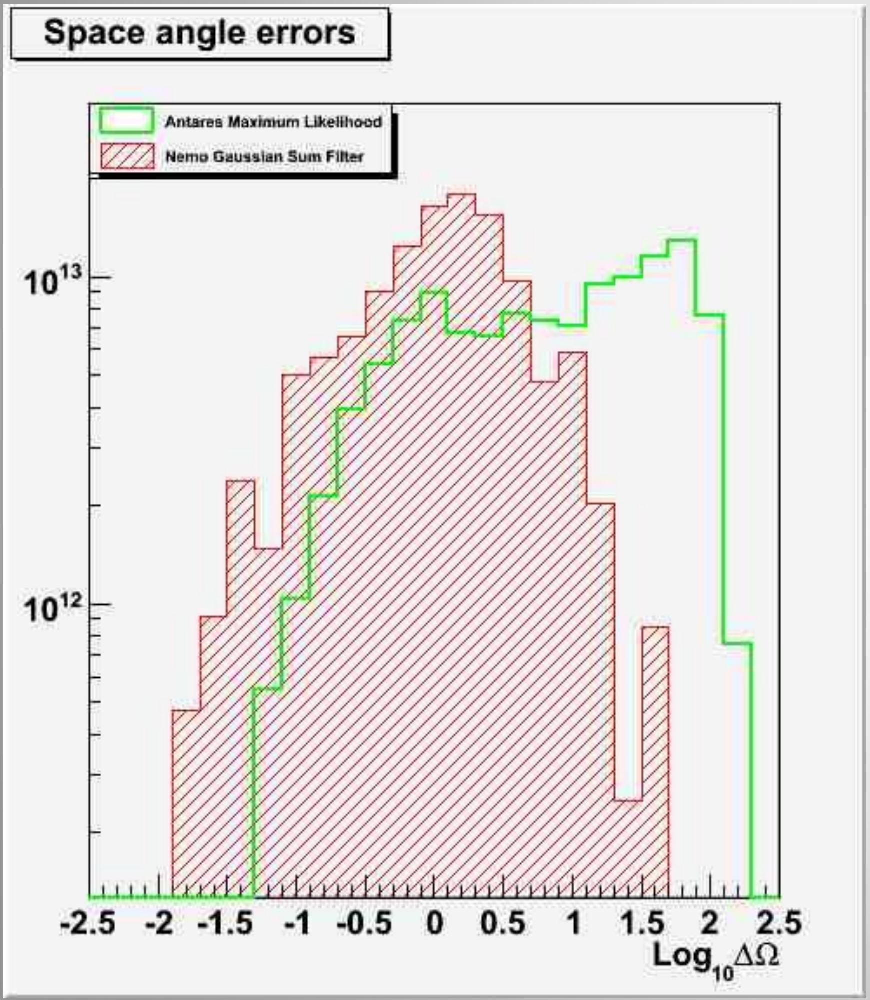
Method	m_θ	σ_θ	σ_X
ANTARES	1.95	0.93	0.66
GSF	2.00	0.96	0.53

Results



angular resolution of the reconstructed muons

KM3NeT design option with MultiPMT optical modules



Preliminary studies on the KM3NeT design option with MultiPMT optical modules

- Prefit track: track prefit algorithm from ANTARES Prefit + «Directional» prefit*
- GSF Algorithm not yet optimized for MultiPMT detector structure

Space angle between incoming neutrino and reconstructed muon

Preliminary Results

* See A. Trovato talk

Results and Outlook

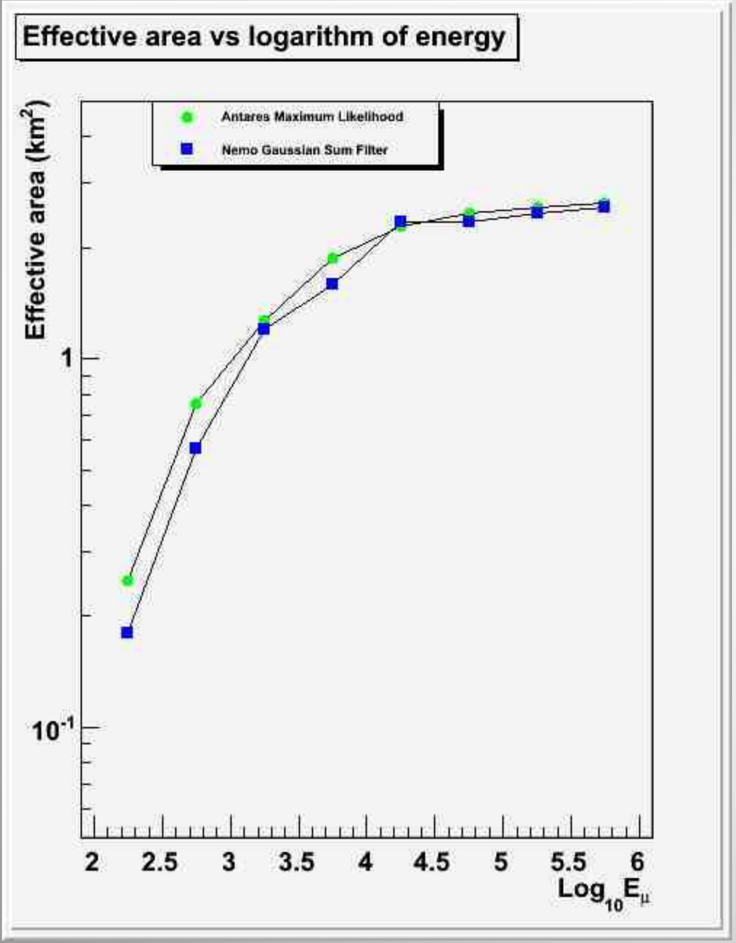
- GSF al works and gives parameters accuracies in agreement with the maximal likelihood ones,
- it is faster then maximal likelihood*,
- improvements are possible: optimization studies for KM3NeT design option with MultiPMT optical modules
- To study quality cut to be applied
- with this approach, also **energy loss** can be taken into account:

  with the GSF reconstruction of muon tracks is possible to obtain a good Gaussian-mixture approximation of the bremsstrahlung energy loss

*The mean CPU speed per event (on Intel Xeon 2.66 Ghz processors) for the GSF algorithms is about 4 time faster than the maximum likelihood method.

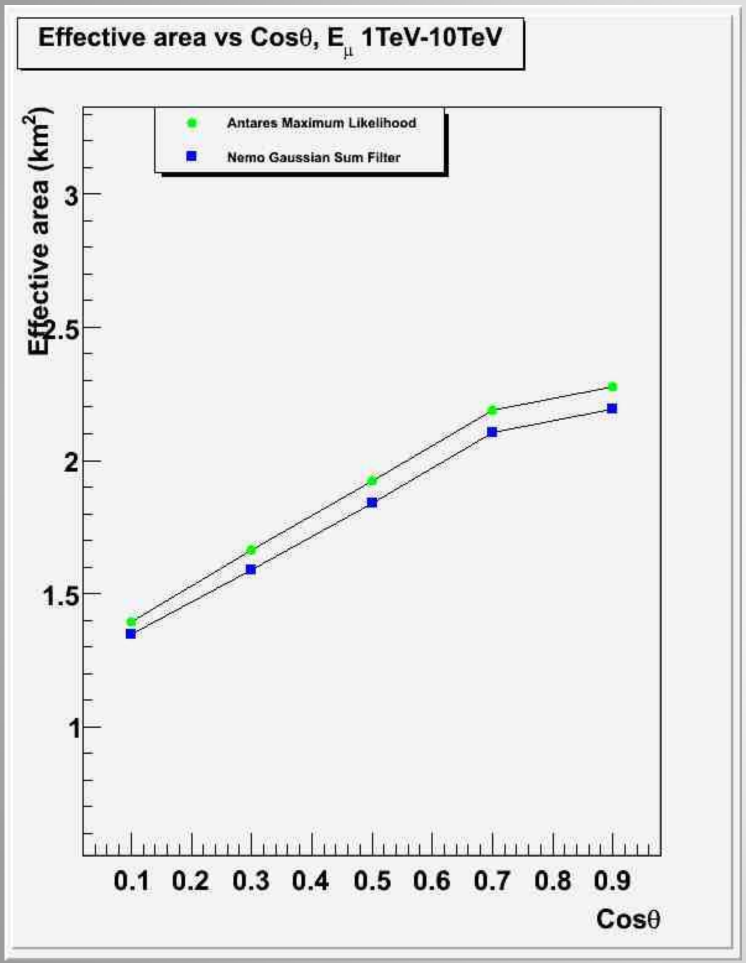
more than…..

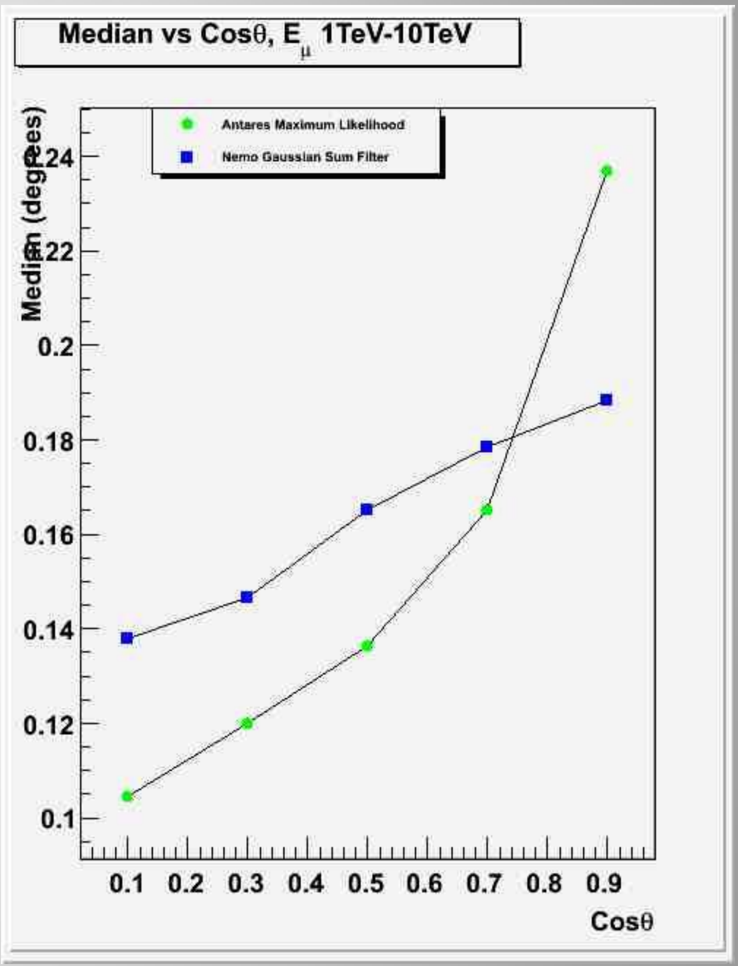
Results



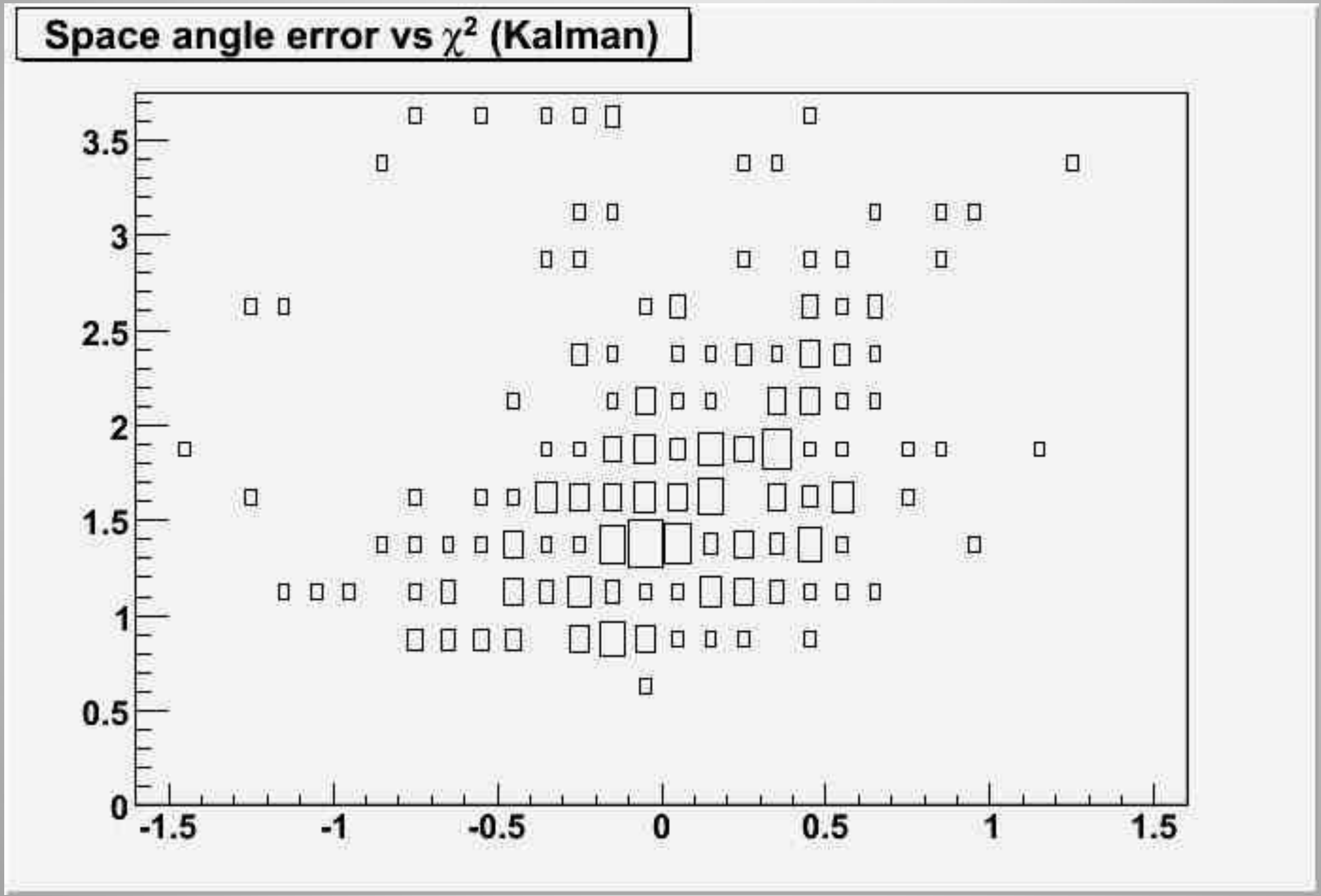
Muon Effective Area

Results


Effective area vs Cosθ, E_μ 1TeV-10TeV
Antares Maximum Likelihood
Nemo Gaussian Sum Filter
Effective area (km^2)
3
2.5
2
1.5
1
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Cosθ


Median vs Cosθ, E_μ 1TeV-10TeV
Antares Maximum Likelihood
Nemo Gaussian Sum Filter
Median (degrees)
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Cosθ

KM3NeT design option with MultiPMT optical modules



The track model

The evolution of the state vector depend on the track parameters via the track model f_k.
In the case under study, we set:

$$\boxed{f_k(\bar{x}_{k-1}) = \bar{x}_{k-1} + w_k}$$

Notice that, for energetic muons, multiple scattering has a negligible effect; the energy loss is not taken into account at this step, thus the process noise w_k **is neglected**

the track is considered as a straight line and is the same for all hits, not depending on hit position

The track model

The measurement function h_k can be parametrized as:

$$h_k(\bar{x}_k) = T_{theor}$$

using the relation

$$(t_j - t_0) = \frac{1}{c}\left(l_j - \frac{d_j}{\tan\theta_C}\right) + \frac{1}{v_{ph}}\frac{d_j}{\sin\theta_C}$$

for the evaluation of T_{theor}