Chiral Anomaly and the BaBar Measurements of the $\gamma\gamma^* \to \pi^0$ Transition Form Factor

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1 Introduction

• The $\gamma^*\gamma \to \pi^0$ transition form factor $F(Q^2)$ at large virtual photon momentum transfer Q^2 is one of the simplest quantities to compute in QCD.

• Relatively easy to measure, as in the $e^+e^- \to e^+e^-\pi^0$ single-tagged experiment at Babar.

• At $Q^2 = 0$, $F(Q^2)$ is given by the two-photon π^0 decay governed by the Adler-Bell-Jackiw(ABJ) triangle chiral anomaly which gives correctly the decay rate.

• At large Q^2, short-distance operator expansion(OPE) [Frishman] or perturbative QCD [Brodsky-Lepage,Kroll] predicts $F(Q^2) \sim 2\, f_\pi/Q^2$ ($f_\pi = 93\,\mathrm{MeV}$).

- The $\gamma^*\gamma \to \pi^0$ transition form factor in pQCD(Brodsky-Lepage et al)

$$F_{\pi\gamma}(Q^2) = 2\sqrt{N_c}(e_{\rm u}^2 - e_{\rm d}^2)\int_0^1 dx_1 dx_2 \int_0^\infty \frac{d^2k_\perp}{16\pi^2}\psi(x_i, k_\perp) \left[\frac{(q_\perp x_2 + k_\perp)\times\epsilon_\perp}{(q_1\times\epsilon_\perp)(q_\perp x_2 + k_\perp)^2} + (x_1 \leftrightarrow x_2)\right] \quad (1)$$

- Wave function peaked at low $k_\perp$
- At large Q^2, $k_\perp \ll q_\perp$, by neglecting $k_\perp$, one has:

$$F_{\pi\gamma}(Q^2) = \frac{2\sqrt{N_c}(e_{\rm u}^2 - e_{\rm d}^2)}{Q^2}\int_0^1 \frac{dx_1 dx_2}{x_1 x_2}\int_0^{Q^2}\frac{d^2k_\perp}{16\pi^2}\psi(x_i, k_\perp) \quad (2)$$

- The quark distribution amplitude(DA) $\phi(x_i, Q)$: amplitude for finding the constituent quark with longitudinal momenta x_i :

$$\phi(x_i, Q) = (\ln\frac{Q^2}{\Lambda^2})^{-\gamma_F/\beta}\int_0^{Q^2}\frac{d^2k_\perp}{16\pi^2}\psi(x_i, k_\perp) \quad (3)$$

- From evolution equation, Brodsky et al obtain solution for $\phi(x_i, Q)$,

$$\phi(x_i, Q) = x_1 x_2 \sum_{n=0}^{\infty} a_n C_n^{3/2} (\ln \frac{Q^2}{\Lambda^2})^{-\gamma_n} \tag{4}$$

- $Q^2 \to \infty$, only a_0 survives, one has the asymptoptic limit:

$$F_{\pi\gamma}(Q^2) = \frac{2(f_\pi)}{Q^2} \tag{5}$$

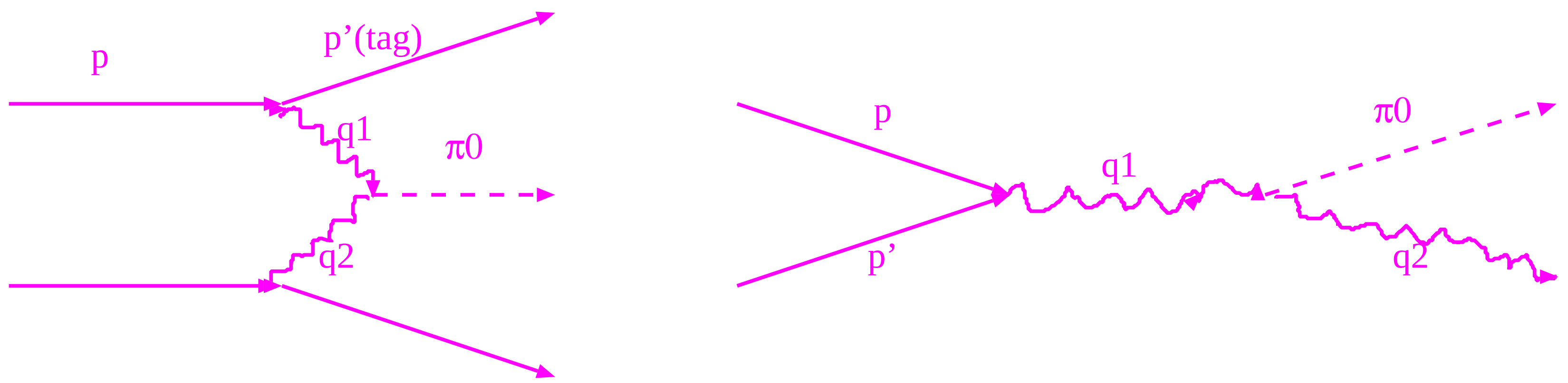

Figure 1: Diagrams for $e^+e^- \to e^+e^-\pi^0$ and $e^+e^- \to \pi^0\gamma$

• The earlier CLEO data for $F(Q^2)$ up to $Q^2 = 8\,\mathrm{GeV}^2$ somewhat below the perturbative QCD(pQCD) prediction, though, with a possible rise for $Q^2\,F(Q^2)$ above $2.5\,\mathrm{GeV}^2$.

• The BaBar Collaboration has produced measurements for Q^2 from 4 to 34 GeV^2 [BaBar] which show spectacular deviation from the perturbative QCD prediction as seen from the data for $Q^2\,F(Q^2)$.

• $Q^2\,F(Q^2)$ of BaBar rises steadily with Q^2 in contrast with the rather flat behavior predicted by pQCD and is more than 50% above the QCD prediction at $34\,\mathrm{GeV}^2$ [Brodsky-Lepage]

• The new Belle results, though somewhat below the BaBar data, indicates some rise with Q^2 for $Q^2\,F(Q^2)$ at large Q^2.

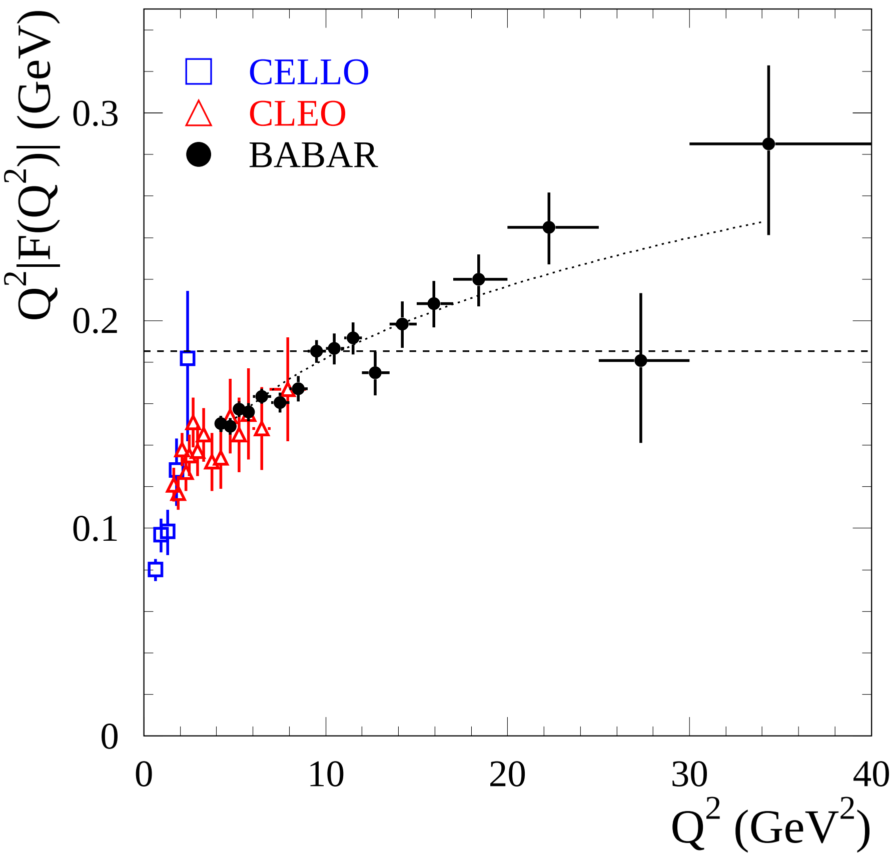

Figure 2: BaBar data for the transition form factor multiplied by Q^2 taken from BaBar published PRD 80,052002 (2009)

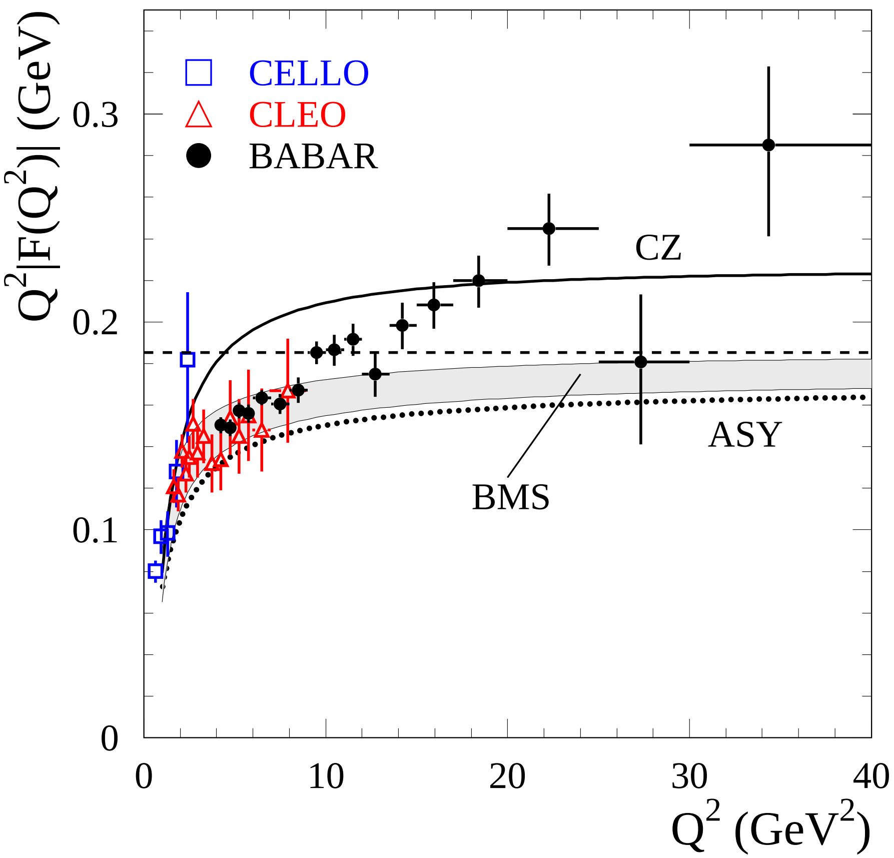

Figure 3: BaBar data for $Q^2\,F(Q^2)$ compared with current theoretical predictions taken from BaBar PRD paper

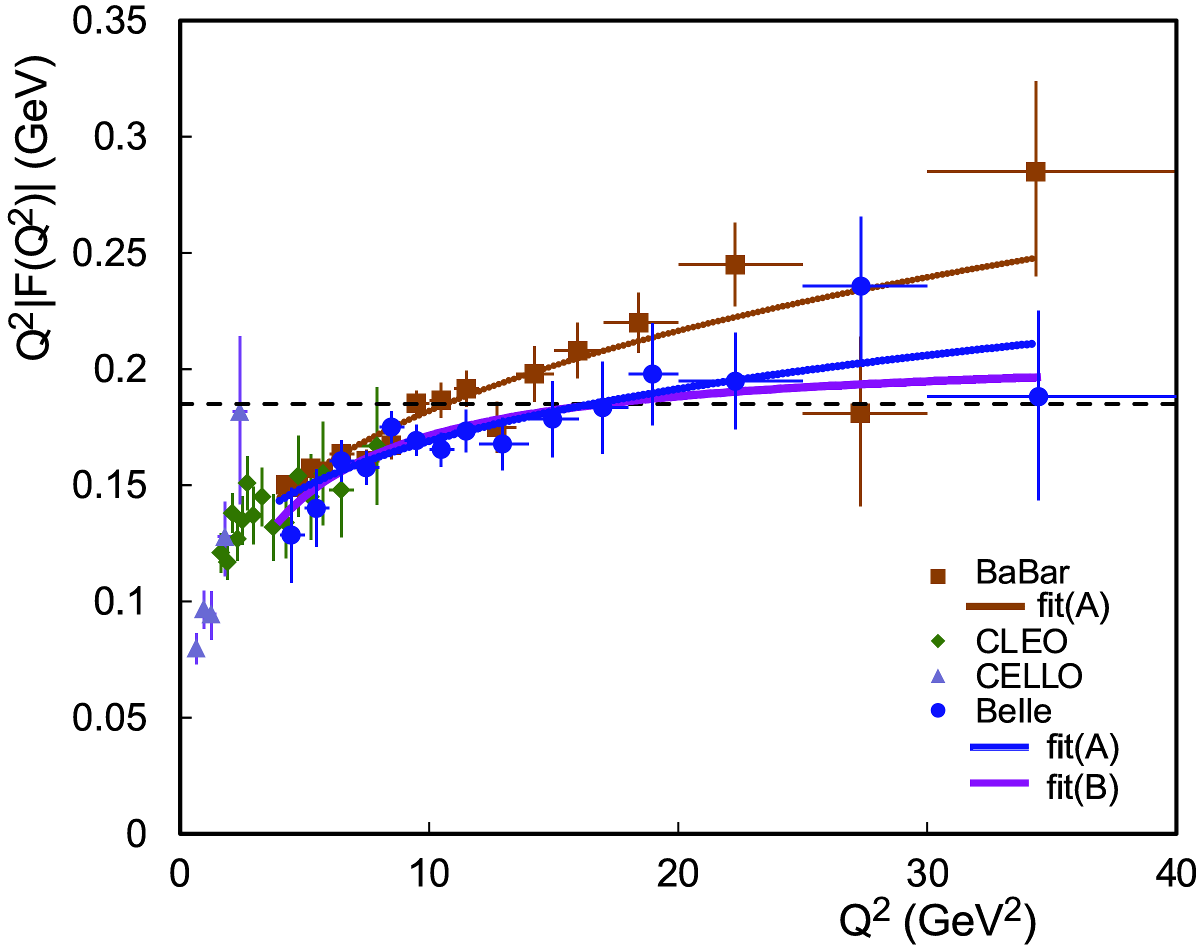

Figure 4: The Belle results for $Q^2\,F(Q^2)$ compared with the BaBar measurements taken from Belle paper

- Recent calculations using the light-cone sum rules method at next-to-leading order with various forms for the pion distribution amplitude, seem to obtain values for the transition form factor higher than the asymptotic limit, but with very different Q^2 behavior than the BaBar data for $Q^2 < 15\,\mathrm{GeV}^2$ and are below the BaBar data for Q^2 in the range from 20 to 40 GeV^2

- As pointed out by BaBar, existing calculations with Chernyak-Zhinitsky(CZ) DA , asymptotic DA(ASY), Bakulev-Mikhailov-Stefanis DA (BMS) for pion, produce only a flat Q^2 dependence for $Q^2 > 10\,\mathrm{GeV}^2$

- The rise with Q^2 for $Q^2\,F(Q^2)$ indicates the presence of hard component in the pion distribution amplitude(DA).

- More recent papers with broad DA distribution, (Huang and Wu (2009), Radyushkin (2009)) and the latest model with flat DA(Agnev, Braun, Offen and Porkett (2011)) could produce a $\log(Q^2)$ rise of $Q^2\,F(Q^2)$ for $Q^2 > 15\,\mathrm{GeV}^2$, though still somewhat below the BaBar data, but could explain the new Belle results at large Q^2 above $30\,\mathrm{GeV}^2$

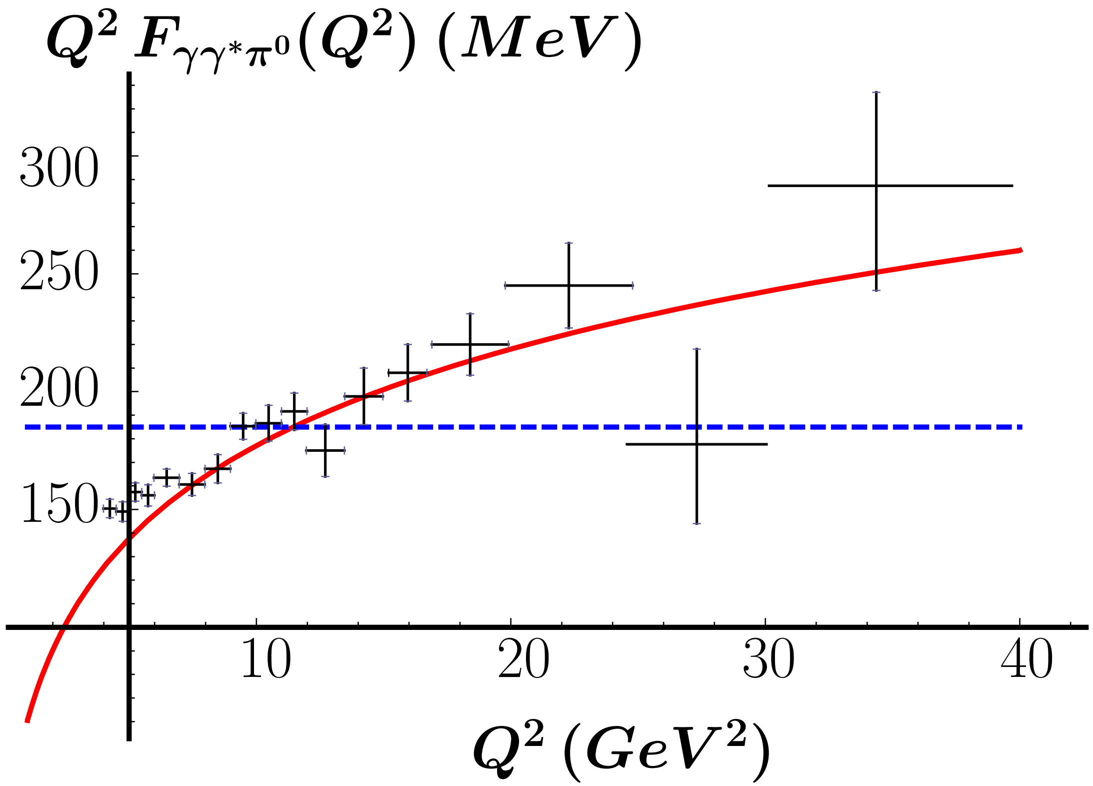

Figure 5: prediction of Radyushkin

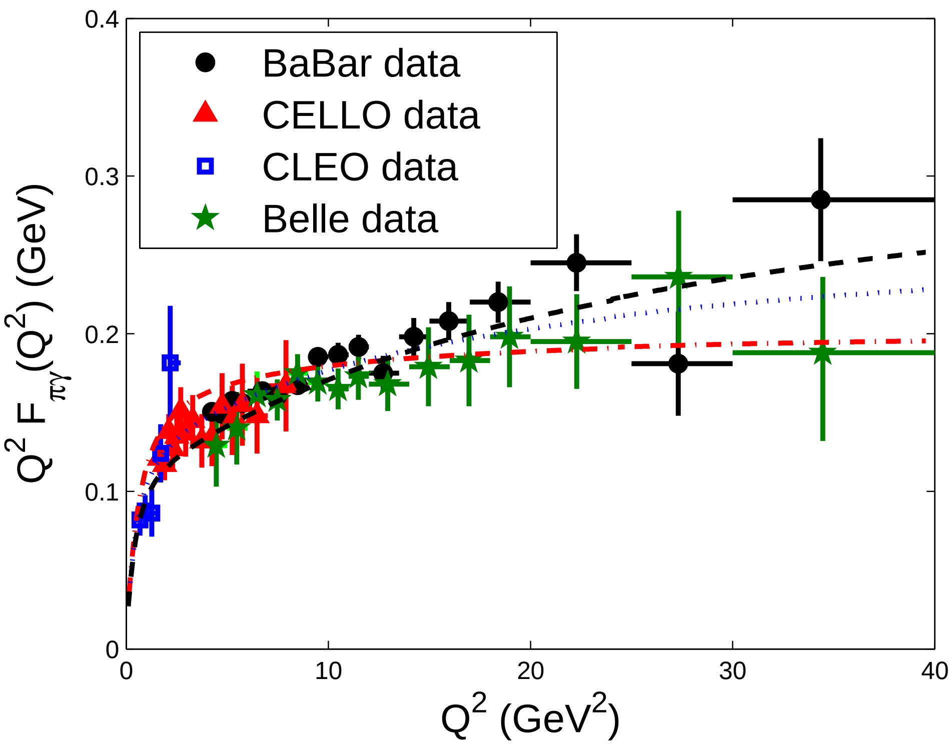

Figure 6: prediction of Wu et al, 2012

• The flat DA distribution model with Sudakov suppression and k_{T} factorization, (the modified perturbation approach(MPA) of Li and Mishima(2009), Kroll(2011)) seems to explain the BaBar data large Q^2

but with extra Q^2 dependence in the Sudakov suppression factor in the quantity $c(Q)$.

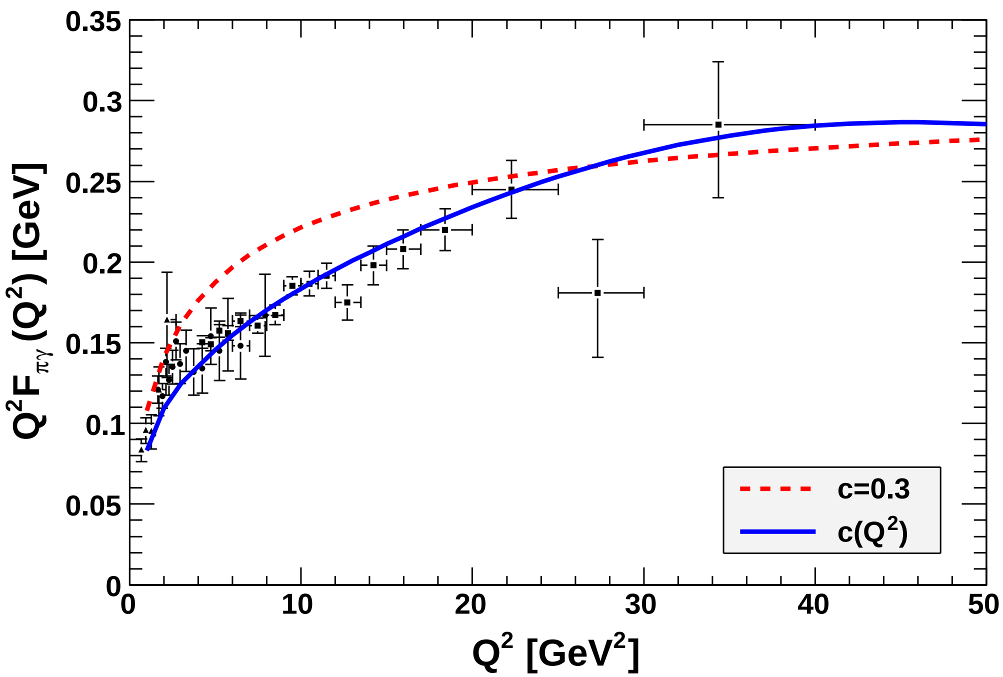

Figure 7: prediction of Li and Mishima with flat DA in $k_{\rm T}$ factorization

• Similarly, in the very recent paper, Agnev *et al* would also need a flat DA or a large contribution from the large invariant mass in the dispersion representation of the transition form factor to explain the BaBar data.

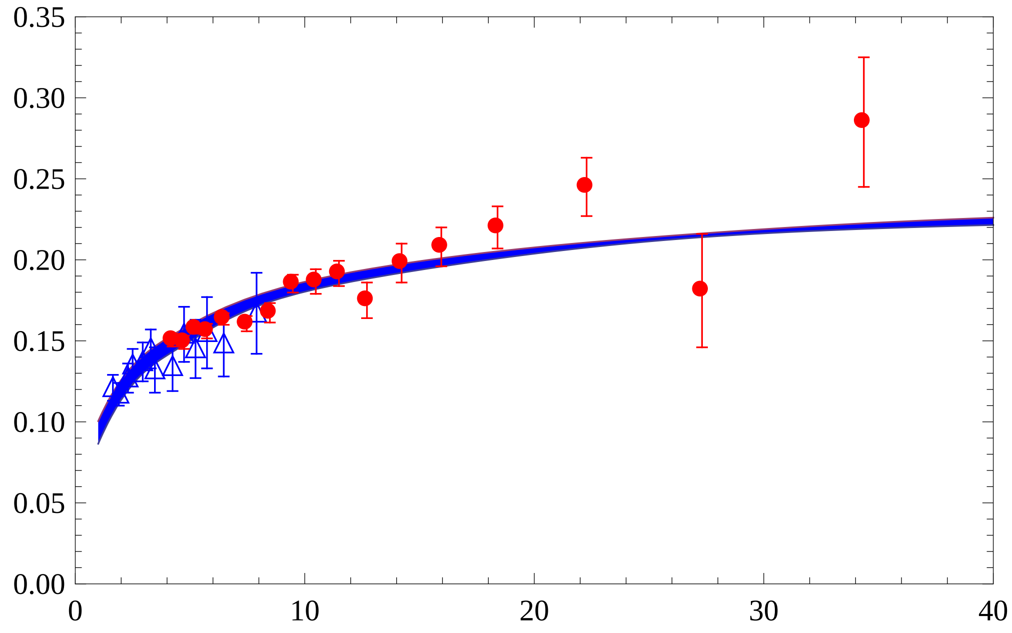

Figure 8: $Q^2F(Q^2)$ obtained by Agnev *et al* for a flat pion DA with reduced second Gegenbauer coefficient $a_2^{\text{flat}} = 0.130$ (model 1)

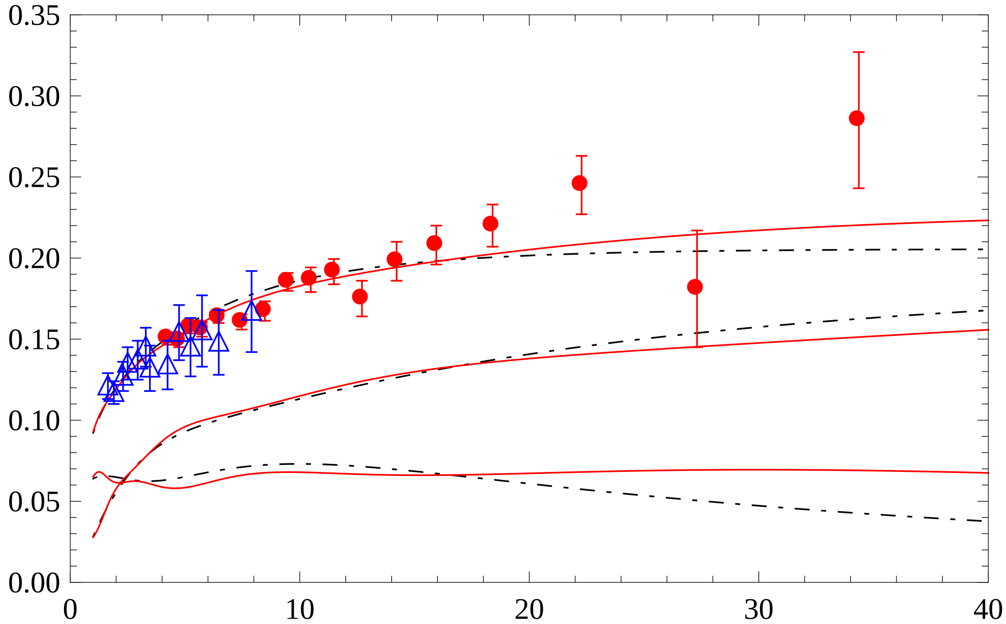

Figure 9: $Q^2F(Q^2)$ from LCSR obtained by Agnev *et al* for 3 models with flat pion DA .with $a_2^{\rm flat} = 0.130$. The curves showing the contribution from large(hard)(middle curve) and small(soft)(lower curve) invariant mass in the dispersion representation and the total hard + soft contribution(the upper curve)

• As pointed out by Dorokhov, a flat DA for the pion corresponds to a point-like coupling of pion to quark. This produces a $\log(Q^2)$ increase with large Q^2 in $Q^2F(Q^2)$ which is however still somewhat below the BaBar data for $Q^2 > 15\,\mathrm{GeV}^2$.

• In a previous work, we have shown(the PLB 1990 paper) that the transition form factor $\gamma^*\gamma \to \pi^0$ with one virtual photon with space-like or time-like Q^2 computed using PCAC and the Adler-Bell-Jackiw chiral anomaly, for large Q^2, behaves as $(\log(Q^2)^2/Q^2$, faster than the simple $\log(Q^2)$ in recent pQCD calculations.

• This work: apply our previous result to to the $\gamma^*\gamma \to \pi^0$ transition form factor.

2 Chiral anomaly effects for the $\gamma^*\gamma \to \pi^0$ transition form factor

• PCAC and Adler-Bell-Jackiw chiral anomaly

The success of the Goldberger-Treiman relation for the pion-nucleon coupling constant obtained from the PCAC hypothesis shows that $SU(2) \times SU(2)$ is a good symmetry for strong interactions.

• Pion is an almost Nambu-Goldstone boson generated by the spontaneous breakdown of chiral $SU(2) \times SU(2)$. The success of the chiral anomaly prediction for $\pi^0 \to \gamma\gamma$ decay is a confirmation of the existence of Adler-Bell-Jackiw chiral anomaly in a theory with quarks and gluons.

Some predictions	Data
$M_\pi^2/M_p^2 = 0$	0.03
$2\,M g_A/(f_\pi g_{pn\pi}) + 1 = 0$	0.06 ± 0.01
$M_\pi^2 a^{\frac{1}{2}} = 0.16$	0.17 ± 0.005
$M_\pi^2 a^{\frac{3}{2}} = -0.078$	-0.088 ± 0.004
$\lambda_{K_{e3}} = 0.021 \pm 0.003$	0.019 ± 0.004
$\Gamma(\pi^0 \to \gamma\gamma) = 7.87\,\mathrm{eV}$	7.95 ± 0.55

Table 1: Chiral symmetry and PCAC predictions(Taken from C. H. Llewllyn Smith, *Proc. of the 1989 Scottish Universities Summer School Physics of the Early Universe*)

• One can derive the Goldberger-Treiman(GT) relation by going to the exact chiral symmetry limit: $m_{u,d} = 0$, $\partial_\mu A_\mu = 0$ and obtain

$$2\,m_N g_A(q^2) + q^2 f_P(q^2) = 0 \tag{6}$$

for the matrix element of the isovector axial vector current between nucleon states.

• Since $M_n \neq 0$, $f_P(q^2)$ must have a pole at $q^2 = 0$. This corresponds to a massless pion since it couples to the nucleon through the pion-nucleon coupling constant:

$$f_P(q^2) = 2g_{\pi N} f_\pi / (-q^2) \tag{7}$$

• Like the $\pi^0 \to \gamma\gamma$ decay, PCAC is modified by the Adler-Bell-Jackiw triangle anomaly.

• Historically, Jacob and Wu apply the modifed PCAC to high energy processes like $Z^0 \to \pi^0\gamma$ and $W^\pm \to \pi^\pm\gamma$ decays

• Suppression of the process $Z^0 \to \pi^0\gamma$ due to cancellation of the anomaly by the axial current matrix element in the triangle graph.

• There remains a $(\log(Q^2))^2/Q^2$ term for the $\gamma^*\gamma \to \pi^0$ transition form factor. Bando and Harada(1994) ; Hayakawa and Kinoshita(1998) also obtained $(\log(Q^2))^2/Q^2$ for the $\gamma^*\gamma \to \pi^0$ transisiton transition form factor from the chiral anomaly.

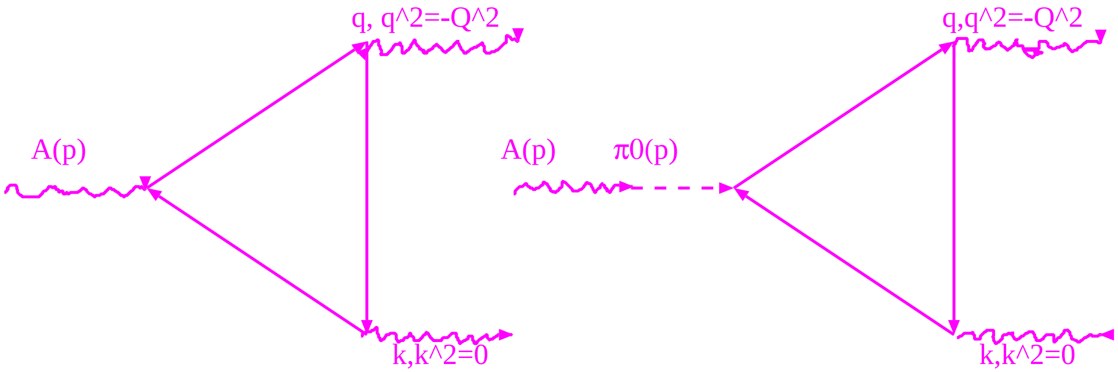

Figure 10: direct and pion pole terms in the triangle graph for $\gamma^*\gamma \to \pi^0$ transisiton transition form factor

- This work: apply our result to the $\gamma^*\gamma \to \pi^0$ transition form factor.
- Modified PCAC by ABJ anomaly

$$\partial_\mu A^\mu = f_\pi m_\pi^2 \phi + S\frac{e^2}{16\pi^2}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta} \tag{8}$$

with $S = 1/2$ in the SM.

- The transition form factor $F(q,k)$ is defined as

$$N^{\mu\nu}(q,k) = e^2 F(q,k) Y^{\mu\nu}, \quad Y^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} q_\alpha k_\beta. \tag{9}$$

- The matrix element $< 0|A_\mu|\gamma^*\gamma >$ is the sum of the direct and the pion pole term.
- The pion term is then:

$$N^{\mu\nu} = \frac{1}{f_\pi}\left(p_\tau \tilde{R}^{\mu\nu\tau}(q,k) - S\frac{e^2}{2\pi^2}Y^{\mu\nu}\right) \tag{10}$$

- The direct term is

$$p_\tau \tilde{R}^{\mu\nu\tau}(q,k) = e^2 S\left(2mP(q,k) + \frac{1}{2\pi^2}\right)Y^{\mu\nu} \tag{11}$$

with

$$P(q,k) = \frac{m}{2\pi^2}\int_0^1 dx \int_0^{1-x}\frac{dy}{D}$$

$$D = k^2 y(1-y) + q^2 x(1-x) - 2q\cdot kxy - m^2 \tag{12}$$

- Anomaly cancellation in the expression for $N^{\mu\nu}$ in Eq. (10) giving

$$N^{\mu\nu} = \frac{1}{f_\pi}\frac{e^2}{2\pi^2} S\left(\frac{m^2}{Q^2} K(m^2, Q^2)\right) Y^{\mu\nu} \tag{13}$$

- The transition form factor is then

$$F(q,k) = \frac{1}{f_\pi}\frac{1}{4\pi^2}\frac{m^2}{s} K(m^2, s) \tag{14}$$

with

$$K(m^2, s) = \left(\ln\frac{1+\rho}{1-\rho} - i\pi\right)^2, \quad \rho = \sqrt{1 - 4m^2/s}, \quad s > 4m^2 \tag{15}$$

For space-like q, with $q^2 = -Q^2$ ($s = -Q^2$), with $s < 0$,

$$K(m^2, Q^2) = \left(\ln\frac{\rho+1}{\rho-1}\right)^2, \quad \rho = \sqrt{1 + 4m^2/Q^2} \tag{16}$$

- At large $Q^2 \gg m^2$, $F(Q^2)$ is given by:

$$F(Q^2) = \frac{1}{f_\pi}\frac{1}{4\pi^2}\frac{m^2}{Q^2}\left(\ln\frac{Q^2}{m^2}\right)^2 \qquad (17)$$

to be compared with the transition form factor for real photon

$$F(q^2 = 0, k^2 = 0, p^2 = 0) = -\left(\frac{1}{4\pi^2 f_\pi}\right) \qquad (18)$$

- As shown below, the behavior of $Q^2F(Q^2)$ for $m = 135\,\mathrm{MeV}$ fits very well the CLEO and BaBar data .

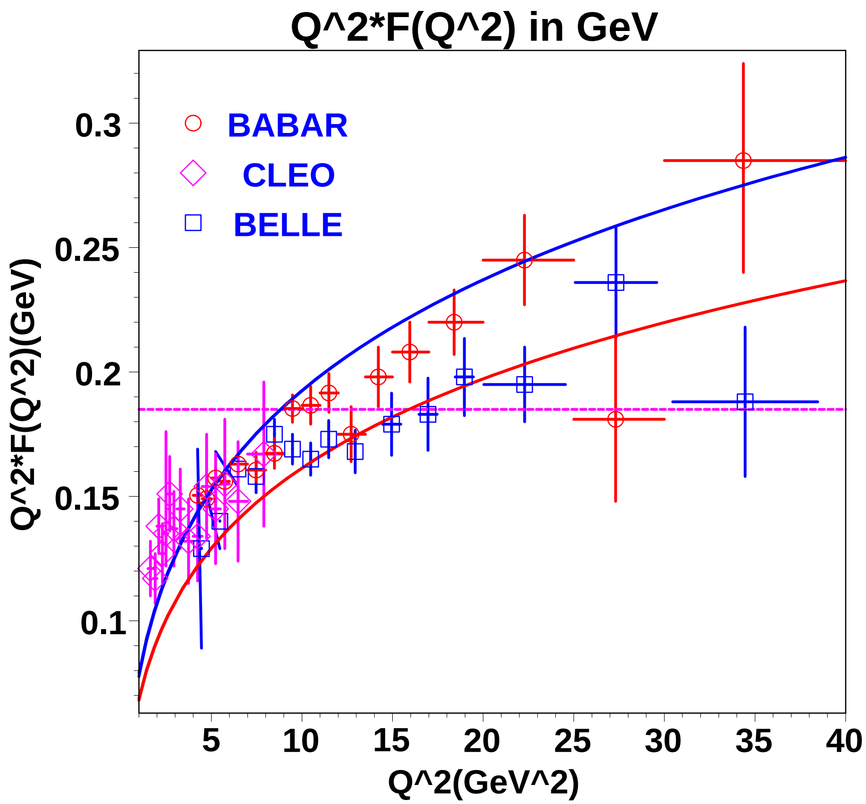

Figure 11: Chiral anomaly prediction(solid line) for $Q^2F(Q^2)$ compared with the BaBar(red) and CLEO(cyan) and the new Belle measured values(blue). The blue solid line for $m = 135\,\mathrm{MeV}$ and the red curve for $m = 120\,\mathrm{MeV}$, pQCD prediction (horizontal line(cyan)) of Brodsly-lepage

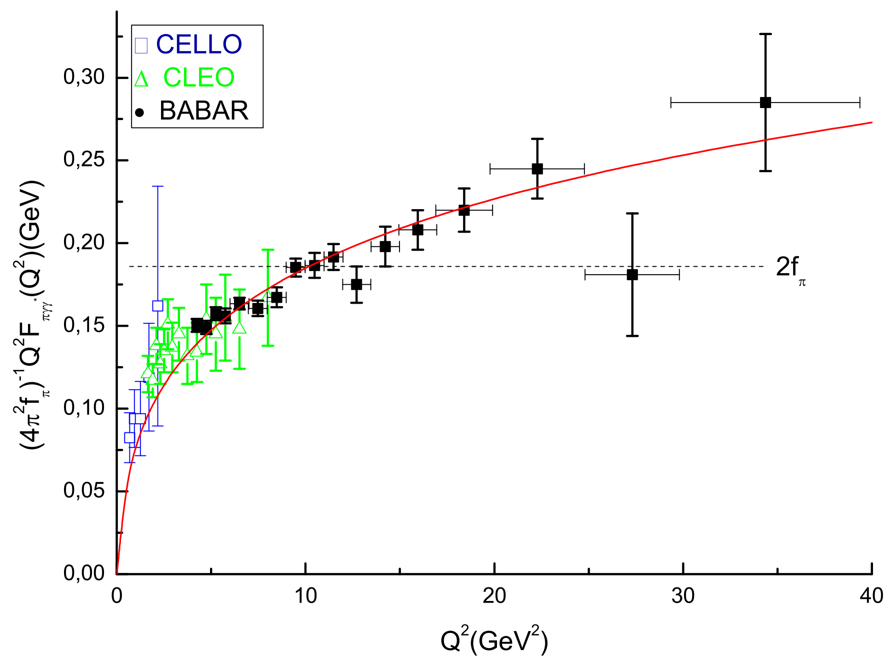

Figure 12: Dorokhov(2010) similar prediction for $Q^2 F(Q^2)$ (solid curve) compared with the BaBar and CLEO measured values and the large Q^2 pQCD prediction (horizontal dash line) of Brodsky-Lepage

3 Conclusion

• Chiral anomaly effects produce, with $m = 135\,\mathrm{MeV}$ the $(m^2/Q^2)(\ln(Q^2/m^2))^2$ behavior for the $\gamma\gamma^* \to \pi^0$ transition form factor at $Q^2 \gg m^2$ in contrast with the $2f_\pi/Q^2$ behavior given by perturbative QCD and in striking agreement with the BaBar data at large Q^2 and also with the CLEO data at lower Q^2.

• The new Belle results are somewhat below the BaBar values but qualitatively are not very different from the BaBar data and could be fitted by lowering the quark mass parameter in the triangle graph from $135\,\mathrm{MeV}$ to $120\,\mathrm{MeV}$.

4 Acknowledgments

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