# Spontaneous electromagnetic superconductivity of QCD×QED vacuum in (very) strong magnetic field

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**Based on:**

**M.Ch., Phys. Rev. D 82, 085011 (2010) [arXiv:1008.1055]**

**M.Ch., Phys. Rev. Lett. 106, 142003 (2011) [arXiv:1101.0117]**

**J. Van Doorsselaere, H. Verschelde, M.Ch., Phys. Rev. D 85, 045002 (2012) [arXiv:1111.4401]**

**+ arXiv:1104.3767 + arXiv:1104.4404 + ...**

# What is «very strong» field? Typical values:

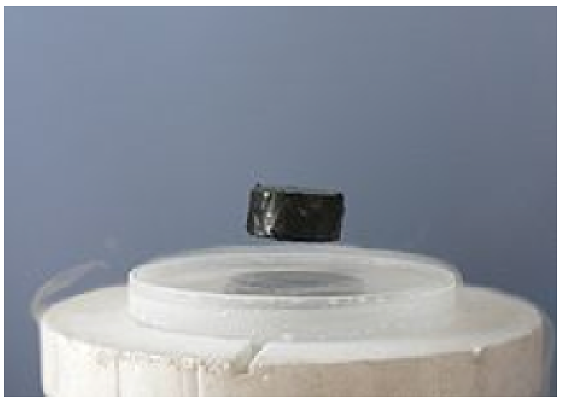
- Thinking — human brain: $10^{-12}$ Tesla
- Earth's magnetic field: $10^{-5}$ Tesla
- Refrigerator magnet: $10^{-3}$ Tesla
- Loudspeaker magnet: 1 Tesla
- Levitating frogs: 10 Tesla
- Strongest field in Lab: $10^{3}$ Tesla
- Typical neutron star: $10^{6}$ Tesla
- Magnetar: $10^{7\ldots9}$ Tesla
- Heavy-ion collisions: $10^{15}$ Tesla (and higher)
- Early Universe: even (much) higher

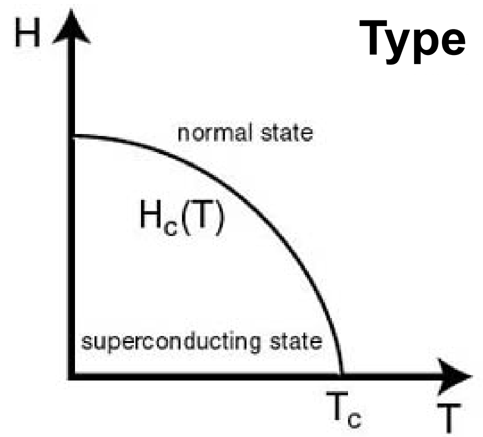
IgNobel 2000 by A.Geim (got Nobel 2010 for graphene)

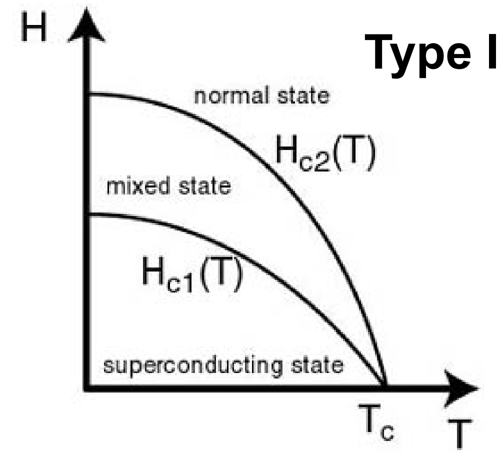
Destructive explosion

# Superconductivity

Discovered by Kamerlingh Onnes at the Leiden University
100 years ago, at 4:00 p.m. April 8, 1911 (Saturday).






I. Any superconductor has zero electrical DC resistance

II. Any superconductor is an enemy of the magnetic field:

1) weak magnetic fields are expelled by all superconductors (the Meissner effect)

2) strong enough magnetic field always kills superconductivity

# Our claim:

**In a background of strong enough magnetic field the vacuum becomes a superconductor.**

**The superconductivity emerges in empty space. Literally, "nothing becomes a superconductor".**

The claim seemingly contradicts textbooks which state that:

1. Superconductor is a material (= a form of matter, not an empty space)
2. Weak magnetic fields are suppressed by superconductivity
3. Strong magnetic fields destroy superconductivity

# General features

Some features of the superconducting state of vacuum:

1. spontaneously emerges <u>above</u> the critical magnetic field

$$B_c \simeq 10^{16} \text{ Tesla} = 10^{20} \text{ Gauss}$$

or

$$eB_c \simeq m_\rho^2 \simeq 31\, m_\pi^2 \simeq 0.6 \text{ GeV}^2$$

**twice stronger field ($eB \simeq$ 1.2 GeV$^2$) can be reached for short time in ultraperipheral Pb+Pb collisions at LHC at $\sqrt{s} = 2.76\text{TeV}$**

W. T. Deng, X. G. Huang, Phys. Rev. C (2012)

2. usual Meissner effect does not exist

3. perfect conductor (zero DC resistance) in one spatial dimension (along the magnetic field axis)

4. insulator in perpendicular directions

# 1+4 approaches to the problem:

0. General arguments; (this talk)

1. Effective bosonic model for electrodynamics of ρ mesons based on vector meson dominance
   [M.Ch., PRD 2010; arXiv:1008.1055] (this talk)

2. Effective fermionic model (the Nambu-Jona-Lasinio model)
   [M.Ch., PRL 2011; arXiv:1101.0117] ~~(this talk)~~

3. Nonperturbative effective models based on gauge/gravity duality (utilizing AdS/CFT duality)
   [Callebaut, Dudal, Verschelde (Gent U., Belgium), arXiv:1105.2217];
   [Erdmenger, Kerner, Strydom (Munich, Germany), arXiv:1106.4551]
   ~~(this talk)~~

5. First-principle numerical simulation of vacuum
   [ITEP Lattice Group, Moscow, Russia, arXiv:1104.3767] (this talk)

# Key players: $\rho$ mesons and vacuum

- $\rho$ mesons:

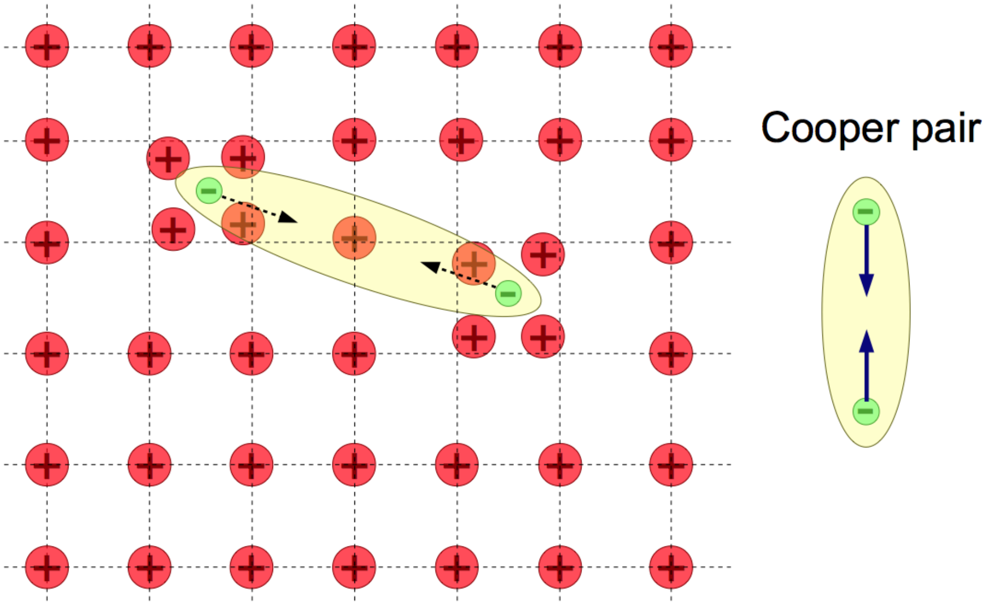
  - electrically charged ($q= \pm e$) and neutral ($q=0$) particles
  - spin: $s$=1, vector particles
  - quark contents: $\rho^+ = u\bar{d}$, $\rho^- = d\bar{u}$, $\rho^{\mathbf{0}} = (u\bar{u} - d\bar{d})/2^{1/2}$
  - mass: $m_\rho$=775.5 MeV (approximately 1550 electron masses)
  - lifetime: $\tau_\rho$=1.35 fm/$c$ (very short: size of the $\rho$ meson is 0.5 fm)

- vacuum: QED+QCD, zero tempertature and density

# Conventional BCS superconductivity

1) The Cooper pair is the relevant degree of freedom!

2) The electrons are bounded into the Cooper pairs by the (attractive) phonon exchange.




Three basic ingredients:

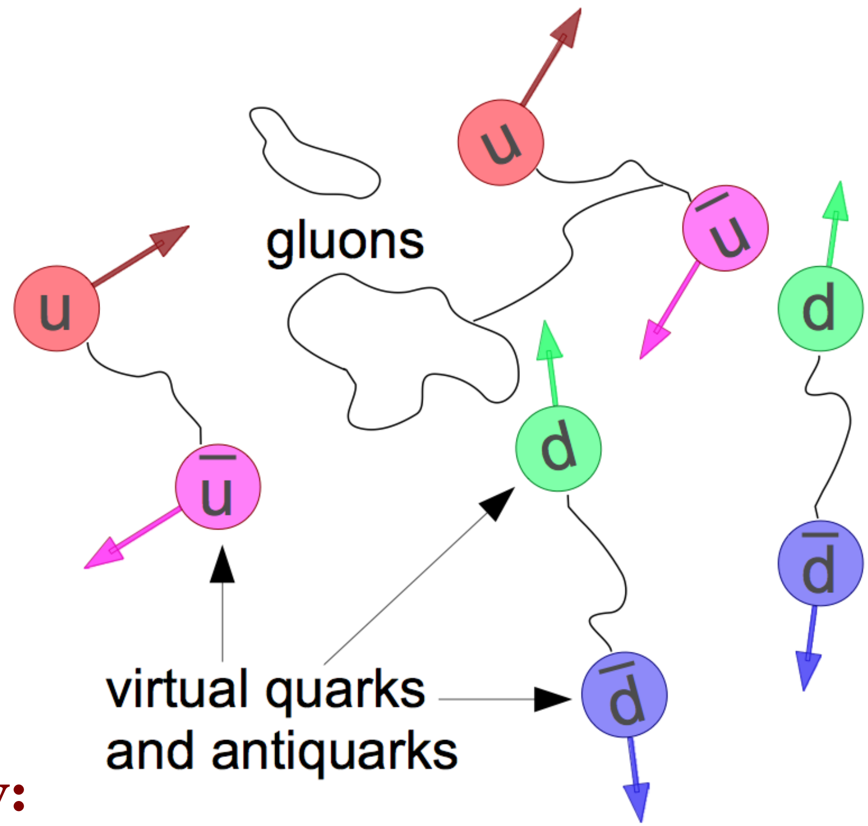
A) the presence of carriers of electric charge (of electric current);

B) the reduction of physics from (3+1) to (1+1) dimensions;

C) the attractive interaction between the like-charged particles.

# Real vacuum, no magnetic field

**1) Boiling soup of everything.**
Virtual particles and antiparticles (electrons, positrons, photons, gluons, quarks, antiquarks ...) are created and annihilated every moment.




**2) Net electric charge is zero.**
An insulator, obviously.

**3) We are interested in "strongly interacting" sector of the theory:**

a) quarks and antiquarks,

i) *u* quark has electric charge $q_u$=+2 $e/3$

*ii)* *d* quark has electric charge $q_d$=- $e/3$

b) gluons (an analogue of photons, no electric charge) "glue" quarks into bounds states, "hadrons" (neutrons, protons, etc).

# The vacuum in strong magnetic field

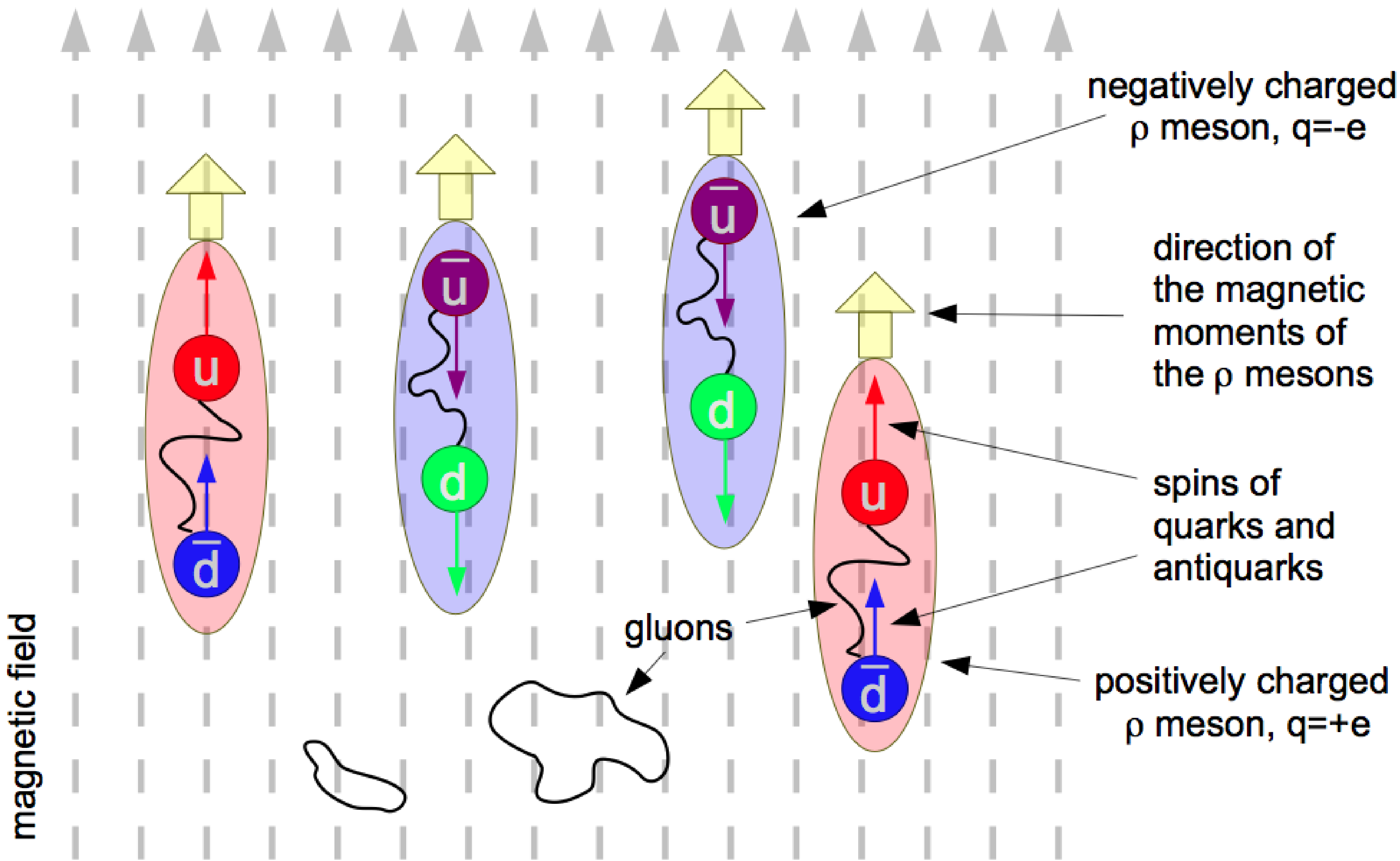
Ingredients needed for possible superconductivity:

A. **Presence of electric charges?**
**Yes,** we have them: there are virtual particles which may potentially become "real" (= pop up from the vacuum) and make the vacuum (super)conducting.

B. **Reduction to 1+1 dimensions?**
**Yes,** we have this phenomenon: in a very strong magnetic field the dynamics of electrically charged particles (quarks, in our case) becomes effectively one-dimensional, because the particles tend to move along the magnetic field only.

C. **Attractive interaction between the like-charged particles?**
**Yes,** we have it: the gluons provide attractive interaction between the quarks and antiquarks ($q_u=+2\,e/3$ and $q_{\bar{d}}=+e/3$)

# Strong magnetic field, picture



# Naïve qualitative argument: charged relativistic particle in magnetic field

- Energy of a relativistic particle in the external magnetic field $B_{\mathrm{ext}}$:

$$\varepsilon^2_{n,s_z}(p_z) = p_z^2 + (2n - 2s_z + 1)eB_{\mathrm{ext}} + m^2$$

momentum along the magnetic field axis

nonnegative integer number

projection of spin on the magnetic field axis

(the external magnetic field is directed along the *z*-axis)

- Masses of $\rho$ mesons and pions in the external magnetic field

$$m^2_{\pi^\pm}(B_{\mathrm{ext}}) = m^2_{\pi^\pm} + eB_{\mathrm{ext}}$$ becomes heavier

$$m^2_{\rho^\pm}(B_{\mathrm{ext}}) = m^2_{\rho^\pm} - eB_{\mathrm{ext}}$$ becomes lighter

$$\rho^\pm \to \pi^\pm \pi^0$$

- Masses of $\rho$ mesons and pions:

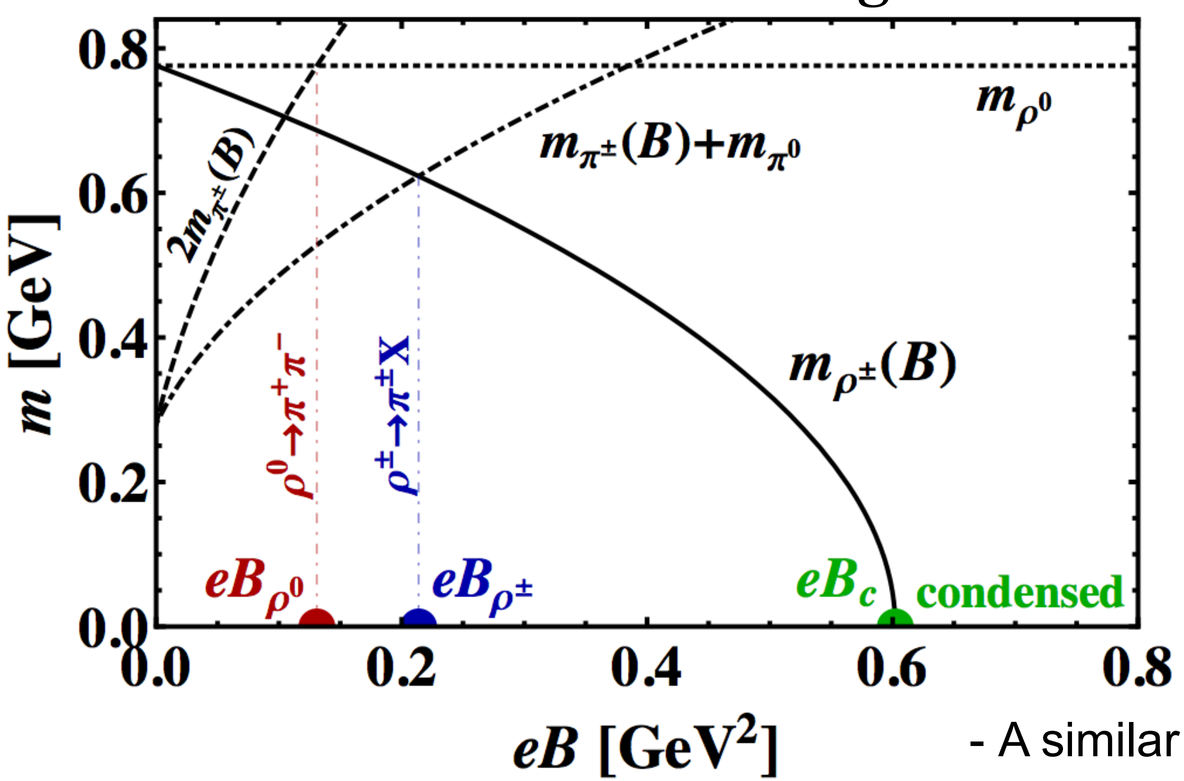
$$m_\pi = 139.6\,\mathrm{MeV}\,, \qquad m_\rho = 775.5\,\mathrm{MeV}$$

# Condensation of $\rho$ mesons

## The $\rho^{\pm}$ mesons become massless and condense at the critical value of the external magnetic field

$$B_c = \frac{m_\rho^2}{e} \approx 10^{16}\,\text{Tesla}$$

masses in the external magnetic field




### Kinematical impossibility of dominant decay modes

The pion becomes heavier while the rho meson becomes lighter

- The decay $\rho^{\pm} \to \pi^{\pm}\pi^{0}$ stops at certain value of the magnetic field

$$m_{\rho^\pm}(B_{\rho^\pm}) = m_{\pi^\pm}(B_{\rho^\pm}) + m_{\pi^0}$$

- A similar statement is true for $\rho^0 \to \pi^+\pi^-$

# Quantitative approaches to the problem: bosonic or fermionic models

Superconductivity: conventional vs. vacuum (simplest models)

| simplest models of superconductivity | | |
|---|---|---|
| | conventional | vacuum |
| bosonic | Ginzburg–Landau model | $\rho$-meson electrodynamics<br>vector dominance model |
| fermionic | Bardeen–Cooper–<br>Schrieffer model | Nambu–Jona-Lasinio model<br>with vector interactions |

1. Effective bosonic model for electrodynamics of ρ mesons based on vector meson dominance
   [M.Ch., PRD 2010; arXiv:1008.1055]

2. Effective fermionic model (the Nambu-Jona-Lasinio model)
   [M.Ch., PRL 2011; arXiv:1101.0117]

# Electrodynamics of $\rho$ mesons

## - Lagrangian (based on vector dominance models):

$$\mathcal{L} = -\frac{1}{4}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\, \rho^\dagger_{\mu\nu}\rho^{\mu\nu} + m_\rho^2\, \rho^\dagger_\mu \rho^\mu$$
$$-\frac{1}{4}\, \rho^{(0)}_{\mu\nu}\rho^{(0)\mu\nu} + \frac{m_\rho^2}{2}\, \rho^{(0)}_\mu \rho^{(0)\mu} + \frac{e}{2g_s}\, F^{\mu\nu}\rho^{(0)}_{\mu\nu}$$

**Nonminimal coupling leads to *g=2***

- Tensor quantities

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,,$$
$$f^{(0)}_{\mu\nu} = \partial_\mu \rho^{(0)}_\nu - \partial_\nu \rho^{(0)}_\mu\,,$$
$$\rho^{(0)}_{\mu\nu} = f^{(0)}_{\mu\nu} - ig_s(\rho^\dagger_\mu \rho_\nu - \rho_\mu \rho^\dagger_\nu)$$
$$\rho_{\mu\nu} = D_\mu \rho_\nu - D_\nu \rho_\mu\,,$$

- Gauge invariance

$$U(1):\quad \begin{cases} \rho^{(0)}_\mu(x) & \to & \rho^{(0)}_\mu(x)\,, \\ \rho_\mu(x) & \to & e^{i\omega(x)}\rho_\mu(x)\,, \\ A_\mu(x) & \to & A_\mu(x) + \partial_\mu \omega(x) \end{cases}$$

- Covariant derivative

$$D_\mu = \partial_\mu + ig_s \rho^{(0)}_\mu - ieA_\mu$$

- Kawarabayashi-Suzuki-Riadzuddin-Fayyazuddin relation

$$g_s \equiv g_{\rho\pi\pi} = \frac{m_\rho}{\sqrt{2} f_\pi} = 5.88$$
$$g_s \gg e \equiv \sqrt{4\pi\alpha_{\text{e.m.}}} \approx 0.303$$

[D. Djukanovic, M. R. Schindler, J. Gegelia, S. Scherer, PRL (2005)]

# Homogeneous approximation

- Energy density:

$$\epsilon \equiv T_{00} = \frac{1}{2}F_{0i}^2 + \frac{1}{4}F_{ij}^2 + \frac{1}{2}\left(\rho_{0i}^{(0)}\right)^2 + \frac{1}{4}\left(\rho_{ij}^{(0)}\right)^2$$
$$+\frac{m_\rho^2}{2}\left[\left(\rho_0^{(0)}\right)^2 + \left(\rho_i^{(0)}\right)^2\right] + \rho_{0i}^\dagger\rho_{0i} + \frac{1}{2}\rho_{ij}^\dagger\rho_{ij}$$
$$+m_\rho^2(\rho_0^\dagger\rho_0 + \rho_i^\dagger\rho_i) - \frac{e}{g_s}F_{0i}\rho_{0i}^{(0)} - \frac{e}{2g_s}F_{ij}\rho_{ij}^{(0)}$$

- Disregard kinetic terms (for a moment) and apply $B_{\text{ext}}$:

$$\epsilon_0^{(2)}(\rho_\mu) = ieB_{\text{ext}}(\rho_1^\dagger\rho_2 - \rho_2^\dagger\rho_1) + m_\rho^2\rho_\mu^\dagger\rho_\mu$$
$$= \sum_{a,b=1}^{2}\rho_a^\dagger\mathcal{M}_{ab}\rho_b + m_\rho^2(\rho_0^\dagger\rho_0 + \rho_3^\dagger\rho_3)$$

$$\mathcal{M} = \begin{pmatrix} m_\rho^2 & ieB_{\text{ext}} \\ -ieB_{\text{ext}} & m_\rho^2 \end{pmatrix}$$

mass matrix

$$\vec{B} = (0,0,B)$$

- Eigenvalues and eigenvectors of the mass matrix:

$$\mu_\pm^2 = m_\rho^2 \pm eB_{\text{ext}}\,, \qquad \rho_\pm = \frac{1}{\sqrt{2}}(\rho_1 \pm i\rho_2)$$

At the critical value of the magnetic field: imaginary mass (=condensation)!

# Homogeneous approximation (II)

- The condensate of the rho mesons: $\rho_1 = -i\rho_2 = \rho$

- The energy of the condensed state:

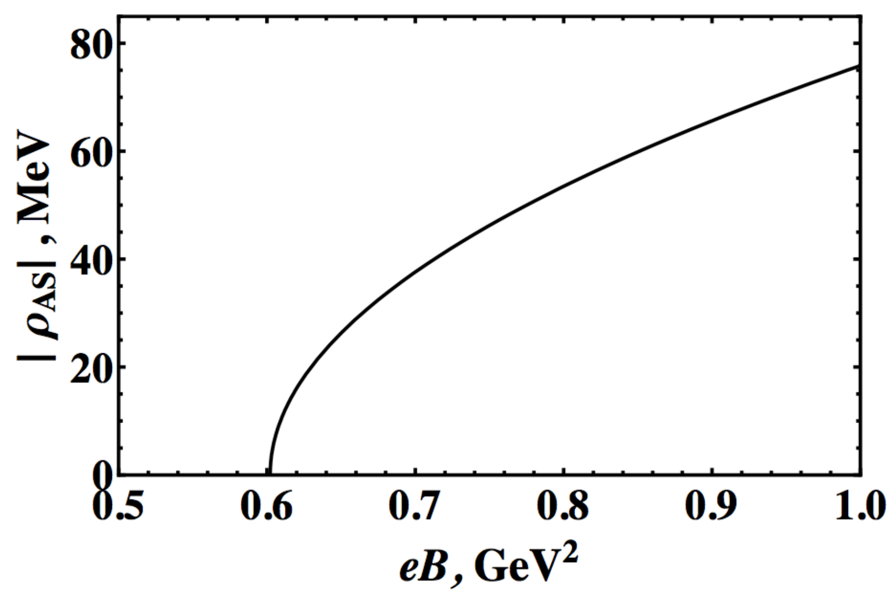
$$\epsilon_0(\rho) = \frac{1}{2}B_{\rm ext}^2 + 2(m_\rho^2 - eB_{\rm ext})\,|\rho|^2 + 2g_s^2\,|\rho|^4$$

(similar to a temperature-dependent Ginzburg-Landau potential for superconductivity!)

(qualitatively similar picture in the Nambu-Jona-Lasinio model)

- The absolute value of the condensate:

$$|\rho|_0 = \begin{cases} \sqrt{\frac{e(B_{\rm ext}-B_c)}{2g_s^2}}, & B_{\rm ext} \geqslant B_c \\ 0, & B_{\rm ext} < B_c \end{cases}$$




Second order (quantum) phase transition, critical exponent = 1/2

# Structure of the condensates

In terms of quarks, the state $\rho_1 = -i\rho_2 = \rho$ implies

$$\langle \bar{u}\gamma_1 d\rangle = \rho(x_\perp)\,, \qquad \langle \bar{u}\gamma_2 d\rangle = i\rho(x_\perp)$$

Depend on transverse coordinates only

(the same structure of the condensates
in the Nambu-Jona-Lasinio model)

$$\vec{B} = (0, 0, B)$$

$U(1)_{\mathrm{e.m.}}:$ $\quad \rho(x) \to e^{i\omega(x)}\rho(x)$ Abelian gauge symmetry

$O(2)_{\mathrm{rot}}:$ $\quad \rho(x) \to e^{i\varphi}\rho(x)$ Rotations around B-axis

- The condensate "locks" rotations around field axis and gauge transformations:

$$U(1)_{\mathrm{e.m.}} \times O(2)_{\mathrm{rot}} \to U(1)_{\mathrm{locked}}$$

(similar to "color-flavor locking" in color superconductors at high quark density)

# Basic features of $\rho$ meson condensation

- The condensate of the ρ mesons appears in a form of an inhomogeneous state, analogous to the Abrikosov lattice in the mixed state of type-II superconductors.

A similar state, the vortex state of W bosons, may appear in Electroweak model in the strong external magnetic field [Ambjorn, Olesen (1989)]

- The condensate forms a lattice, which is made of the new type of topological defects, the "ρ vortices".

- The emergence of the condensate of the charged $\rho^{\pm}$ mesons induces spontaneous condensation of the neutral $\rho^{0}$ mesons.

- The condensate of charged ρ mesons implies <u>superconductivity</u>.
- The condensate of neutral ρ mesons implies <u>superfluidity</u>.

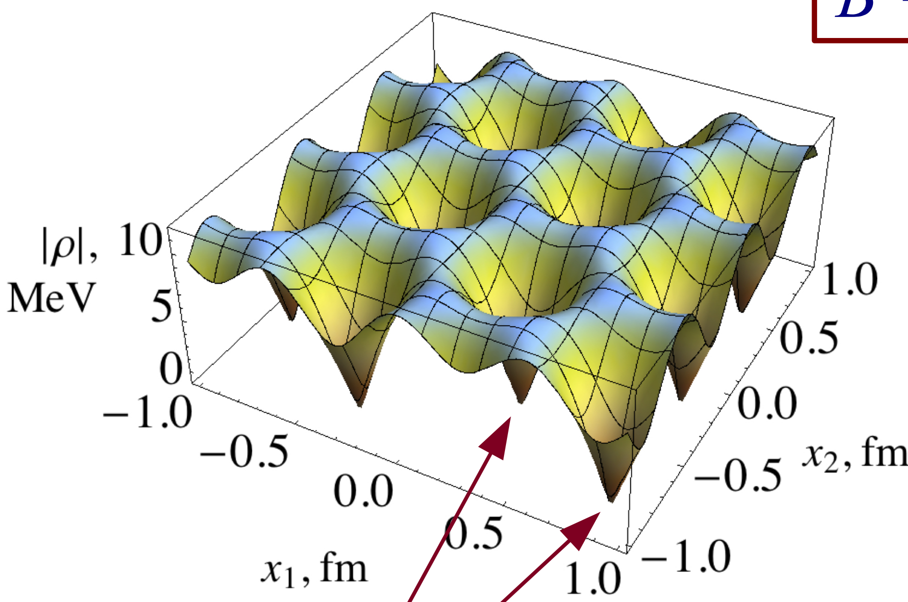
- Unusual optical properties of the superconducting state: It is a metamaterial ("perfect lens" optical phenomenon) with negative electrical permittivity (ε), negative magnetic permeability (μ), negative index of refraction ($n$) [Smolyaninov, PRL 107, 253903 (2011)].

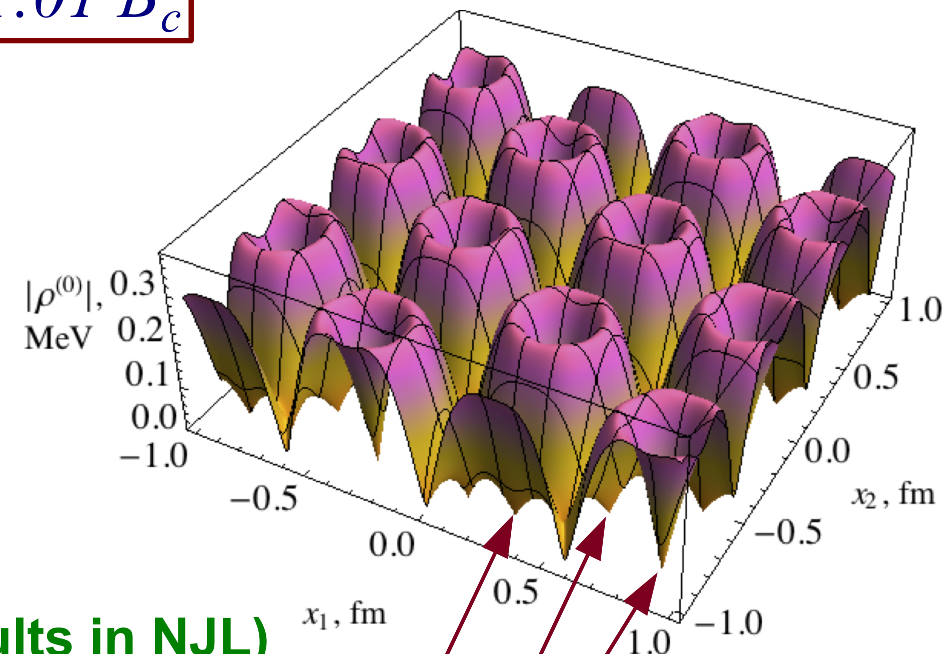
# Condensates of $\rho$ mesons, solutions

## Superconducting condensate
(charged rho mesons)

$\boxed{B = 1.01\, B_c}$

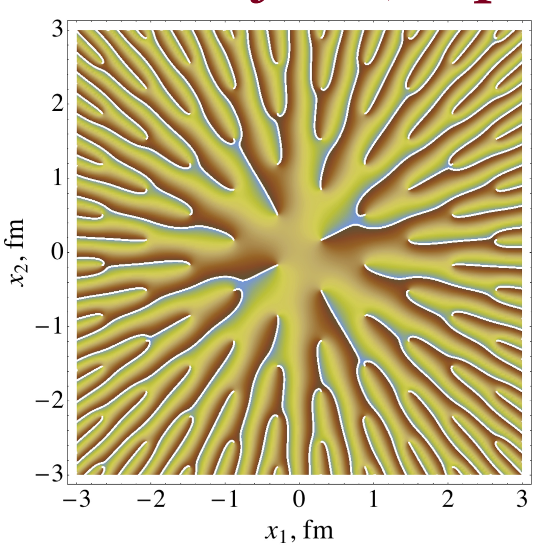
## Superfluid condensate
(neutral rho mesons)



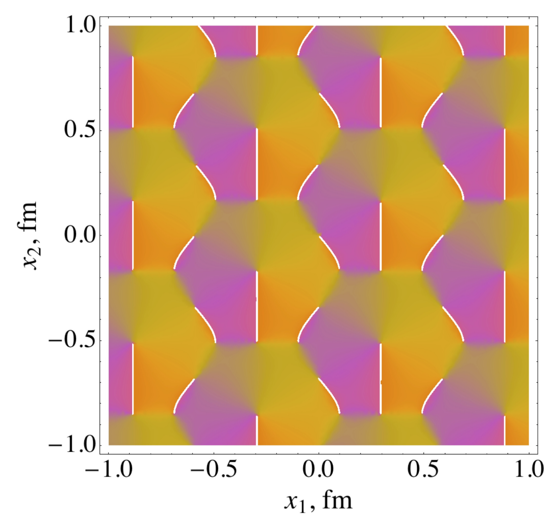




**(similar results in NJL)**

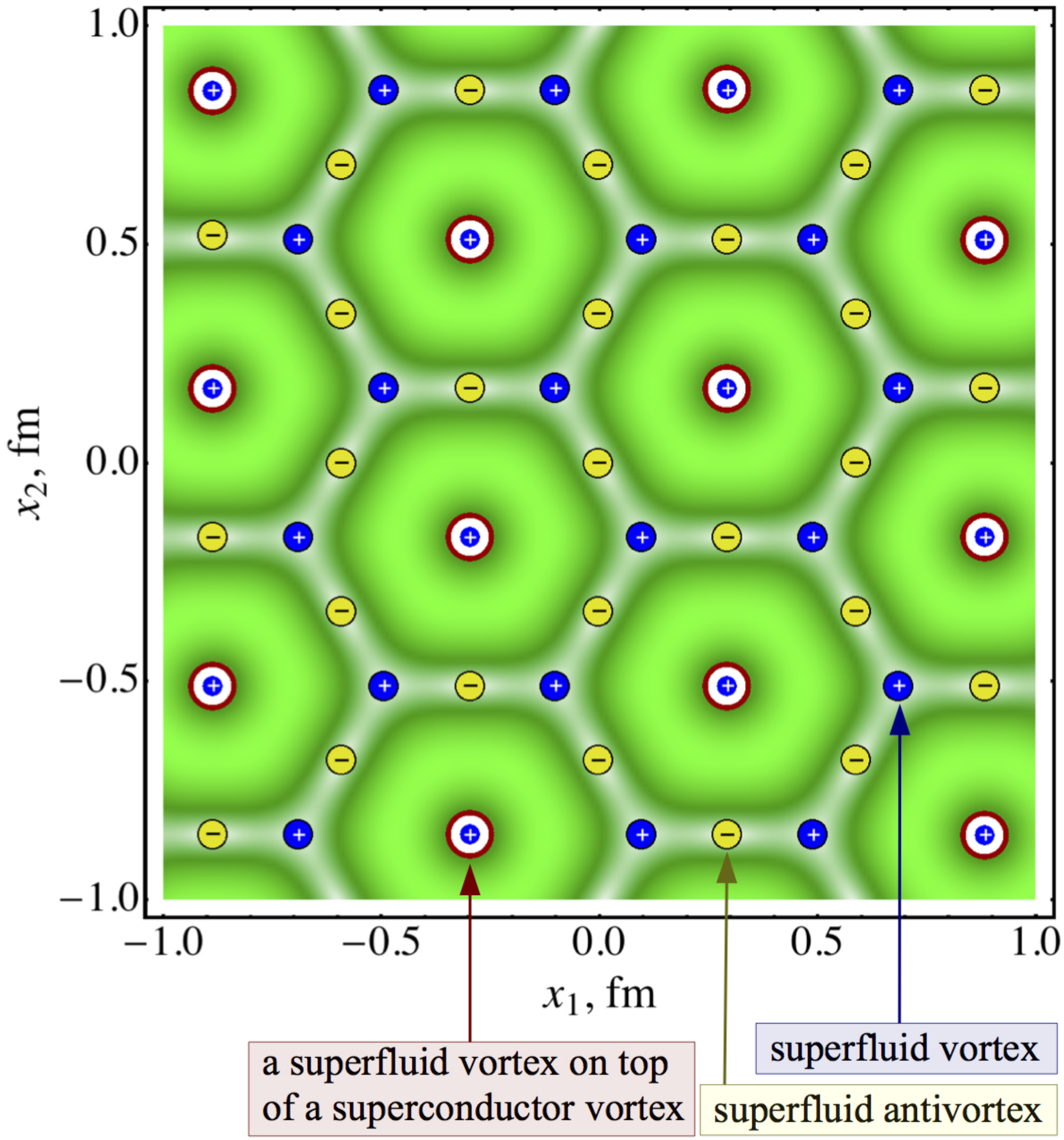
New objects, topological vortices, made of the rho-condensates




The phases of the rho-meson fields wind around vortex centers, at which the condensates vanish.



# Topological structure of the $\rho$ mesons condensates



# Anisotropic superconductivity
## (via an analogue of the London equations)

- Apply a weak electric field $\boldsymbol{E}$ to an ordinary superconductor
- Then one gets accelerating electric current along the electric field:

$$\frac{\partial \vec{J}_{\mathrm{GL}}}{\partial t} = m_A^2 \vec{E}$$ [London equation]

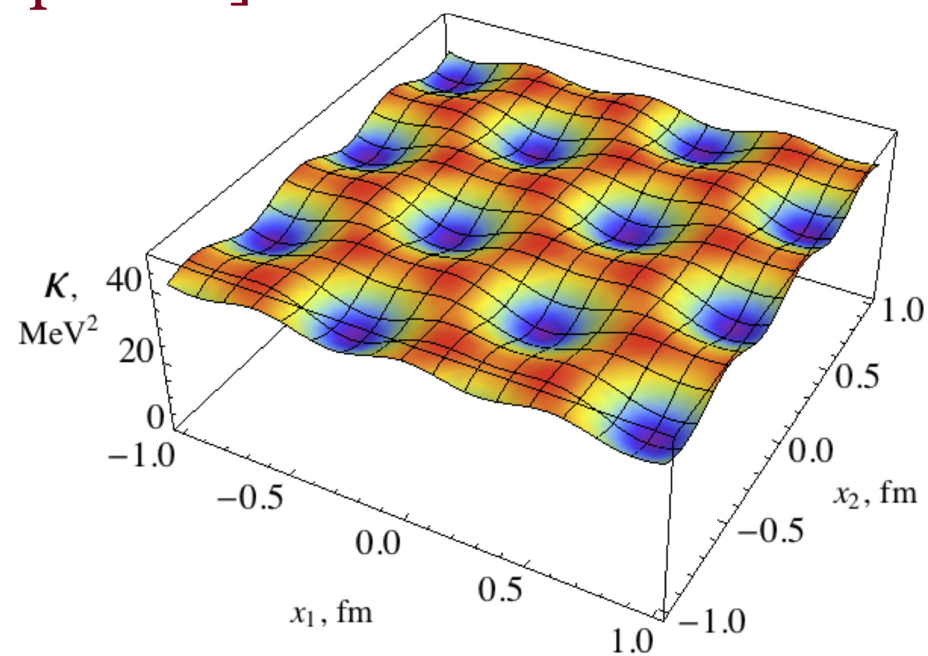
- In the QCDxQED vacuum, we get an accelerating electric current along the magnetic field $\boldsymbol{B}$:

$$\frac{\partial}{\partial t}\langle J_3 \rangle = -\frac{2e^3}{g_s^2}(B_{\mathrm{ext}} - B_c)E_3$$

$$\frac{\partial}{\partial t}\langle J_1 \rangle = \frac{\partial}{\partial t}\langle J_2 \rangle = 0$$

$(\text{for } B \geqslant B_c)$

Written for an electric current averaged over one elementary (unit) rho-vortex cell




**(similar results in NJL)**

# Anisotropic superconductivity

## (Lorentz-covariant form of the London equations)

We are working in the vacuum, thus the transport equations may be rewritten in a Lorentz-covariant form:

Electric current averaged over one elementary rho-vortex cell

$$\partial_{[\mu,} j_{\nu]} = \kappa \frac{(F \cdot \widetilde{F})}{(F \cdot F)} \widetilde{F}_{\mu\nu}$$

A scalar function of Lorentz invariants.
In this particular model:

$\kappa = (e^3/g_s^2)(\sqrt{(F \cdot F)/2} - B_c)$

(slightly different form of κ function in NJL)

Lorentz invariants:

$$(F \cdot \widetilde{F}) = F^{\mu\nu} \widetilde{F}_{\mu\nu} \equiv 4(\vec{B} \cdot \vec{E})$$

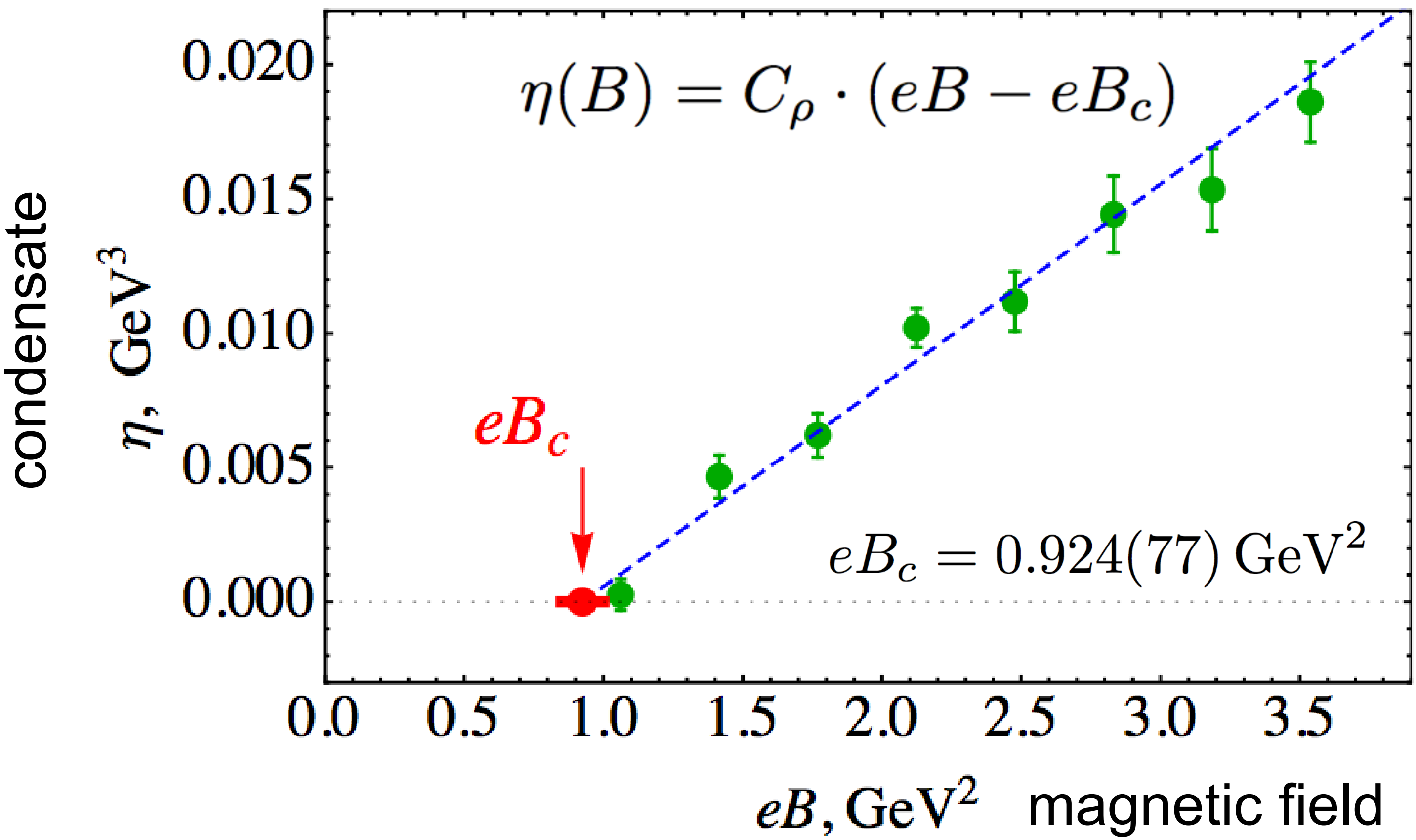
$$(F \cdot F) = F^{\mu\nu} F_{\mu\nu} \equiv 2(\vec{B}^2 - \vec{E}^2)$$

$$\widetilde{F}_{\mu\nu} = \tfrac{1}{2} \varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$$

# Numerical simulations of vacuum in the magnetic field background

V.Braguta, P. Buividovich, M. Polikarpov, M.Ch., arXiv:1104.3767

Numerical simulation: $B_c = (1.56 \pm 0.13) \cdot 10^{16}\,\mathrm{Tesla}$




Theory:

$$B_c = \frac{m_\rho^2}{e} \approx 10^{16}\,\mathrm{Tesla}$$

$$\eta \sim \sqrt{B - B_c}$$

for $B \geqslant B_c$

[qualitatively realistic vacuum, quantitative results may receive corrections (20%-50% typically)]

# Too strong magnetic field?

$$B_c = \frac{m_\rho^2}{e} \approx 10^{16}\,\text{Tesla}$$

Over-critical magnetic fields (of the strength $B \sim 2\,B_c$) may be generated in ultraperiferal heavy-ion collisions (duration is short, however – clarifications are needed)

W. T. Deng and X. G. Huang, Phys.Rev. C85 (2012) 044907 [arXiv:1201.5108]

**A bit of dreams (in deep verification stage):**

**Signatures of the superconducting state of the vacuum could possibly be found in ultra-peripheral heavy-ion collisions at LHC.**
***[ultra-periferal: cold vacuum is exposed to strong magnetic field]***

**Early Universe?**

# Conclusions

- In a sufficiently strong magnetic field condensates with $\rho^{\pm}$ meson quantum numbers are formed spontaneously via a second order phase transition with the critical exponent 1/2.

- The vacuum (= no matter present, = empty space, = nothing) becomes electromagnetically superconducting.

- The superfluidity of the neutral $\rho^0$ mesons emerges as well.

- The superconductivity is anisotropic: the vacuum behaves as a perfect conductor only along the axis of the magnetic field.

- New type of tological defects,"$\rho$ vortices", emerge.

- The $\rho$ vortices form Abrikosov-type lattice in transverse directions.

- The Meissner effect is absent.