Iso-vector Form Factors of the Delta and Nucleon in QCD Sum Rules

A. Özpineci

Physics Department
Middle East Technical University,
Ankara, Turkey

June 20, 2012
QCD@Work

Outline

- Motivation
- Form Factors in Light Cone Sum Rules
- Form Factors Presented:
  - Axial vector form factors of the nucleon,
  - Tensor form factors of the nucleon
  - Axial vector Δ $\rightarrow$Nucleon transition form factors
  - Axial vector Δ baryon form factors
- Sources of uncertainty

Outline

- Motivation
- Form Factors in Light Cone Sum Rules
- Form Factors Presented:
  - Axial vector form factors of the nucleon,

    G. Erkol, A.O, Phys.Rev. D83 (2011) 114022 for all octet, also by Aliev *et al.* (2007)
  - Tensor form factors of the nucleon

    G. Erkol, A.O.,Phys.Lett. B704 (2011) 551-558
  - Axial vector $\Delta \to$ Nucleon transition form factors

    also by T. Aliev, K. Azizi, A.O. (2008)
  - Axial vector Δ baryon form factors
- Sources of uncertainty

Preliminary

Preliminary

Motivation

A peaceful one

Motivation

- Δ and N baryons are the lowest lying baryons that have spin-3/2 and 1/2 respectively.
- Form factors are important quantities describing hadrons (e.g. their shape and size)
- Axial vector form factors can be probed by e.g. ν's or π
- Tensor form factors can be related to spin-dependent generalized parton distributions.
- They need to be calculated using non-perturbative methods
- In Light Cone QCD sum rules, the form factors can be expressed in terms of the distribution amplitudes of the corresponding hadron.

Form Factors in Light Cone QCD Sum Rules

- Consider the correlation function

$$\Pi_\Gamma^{BB'}(p,q) = i\int d^4x e^{iqx}\langle 0|T[\eta_{B'}(0)A_\Gamma(x)]|B(p)\rangle,$$

- Inserting a complete set of hadronic states, it becomes

$$\Pi_\Gamma^{BB'}(p,q) = \sum_h \frac{\langle 0|\eta_{B'}|h(p+q)\rangle}{(p+q)^2-m_h^2}\langle h(p+q)|A_\Gamma|B(q)\rangle$$

- To calculate form factors describing the matrix element $\langle B'(p+q)|A_\Gamma|B(p)\rangle$, choose a current $\eta_{B'}$ such that $\langle 0|\eta_{B'}|B'(p+q)\rangle \neq 0$ (the larger this matrix element, the better it is)

- In the case of axial-vector nucleon form factors

$$\begin{aligned} A_\mu &= \bar{q}\tau^3\gamma_\mu\gamma_5 q \\ \eta_N &= 2\epsilon^{abc}\sum_{\ell=1}^{2}(u^{aT}(x)CJ_1^\ell d^b(x))J_2^\ell u^c(x) \\ \langle 0|\eta_N|N(p')\rangle &= \lambda_N u(p') \\ \langle N(p')|A_\mu|N(p)\rangle &= \bar{u}(p')\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu}{2m_N}\gamma_5 G_P(q^2)\right]u(p), \end{aligned}$$

  where $J_1^1 = I$, $J_1^2 = J_2^1 = \gamma_5$ and $J_2^2 = \beta$ is an arbitrary parameter.
- $\beta = -1$ corresponds to Ioffe current

- In terms of the form factors, the correlation function becomes:

$$\Pi = \lambda_N \frac{G_A}{m_N^2 - p'^2} \not{q}\gamma_\mu\gamma_5 u(p) + \lambda_N \frac{G_P}{m_N^2 - p'^2} q^\mu \not{q}\gamma_5 u(p) + \cdots$$

- The coefficients of the structures $\not{q}\gamma_\mu\gamma_5$ and $q^\mu \not{q}\gamma_5$ give us the form factors G_A and G_P respectively.
- The correlation function can also be calculated using holography (see talk by F. Bigazzi), lattice, or OPE.

- In terms of the QCD parameter, in the $p^2, p'^2 \to -\infty$ limit, the correlation function can be calculated using OPE:

$$\Pi_\mu^B = \frac{1}{2}\int d^4x e^{iqx} \sum_{\ell=1}^{2}$$
$$\Big\{ c_1 (C J_1^\ell)_{\alpha\gamma} \left[J_2^\ell S(-x)\gamma_\mu\gamma_5\right]_{\rho\beta} 4\epsilon^{abc} \langle 0| q_{1\alpha}^a(0) q_{2\beta}^b(x) q_{3\gamma}^c(0)|B\rangle$$
$$+ c_2 (J_2^\ell)_{\rho\alpha} \left[(C J_1^\ell)^T S(-x)\gamma_\mu\gamma_5\right]_{\gamma\beta} 4\epsilon^{abc} \langle 0| q_{1\alpha}^a(x) q_{2\beta}^b(0) q_{3\gamma}^c(0)|B\rangle$$
$$+ c_3 (J_2^\ell)_{\rho\beta} \left[C J_1^\ell S(-x)\gamma_\mu\gamma_5\right]_{\alpha\gamma} 4\epsilon^{abc} \langle 0| q_{1\alpha}^a(0) q_{2\beta}^b(0) q_{3\gamma}^c(x)|B\rangle \Big\},$$

where

$$G_N : \{c_1 = c_2 = 1,\ c_3 = -1,\ q_1 \to u,\ q_2 \to u,\ q_3 \to d\},$$
$$G_\Sigma : \{c_1 = c_2 = 1,\ c_3 = 0,\ q_1 \to u,\ q_2 \to u,\ q_3 \to s\},$$
$$G_\Xi : \{c_1 = c_2 = 0,\ c_3 = 1,\ q_1 \to s,\ q_2 \to s,\ q_3 \to d\},$$

- The matrix elements $4\epsilon^{abc}\langle 0|u_\alpha^a(a_1 x)u_\beta^b(a_2 x)d_\gamma^c(a_3 x)|N\rangle$ are calculated by V. Braun, *et al.*. They can be written as:

$$= \mathcal{S}_1 m_N C_{\alpha\beta}(\gamma_5 N)_\gamma + \mathcal{S}_2 m_N^2 C_{\alpha\beta}(\not{x}\gamma_5 N)_\gamma + \left(\mathcal{V}_1 + \frac{x^2 m_N^2}{4}\mathcal{V}_1^M\right)(\not{p}C)_{\alpha\beta}(\gamma_5 N)_\gamma + \cdots$$

  where $\mathcal{S}_1 = \int \mathcal{D}x_i e^{-i\sum_i a_i x_i p\cdot x}\mathcal{S}_1(x_i)$, etc.
- $\mathcal{S}_1(x_i)$, etc. describe how the quarks are distributed in the nucleon

- Contributions of higher states and continuum are subtracted using quark hadron duality
- To eliminate unknown polynomials in the spectral representation and to suppress contributions of higher states and continuum, Borel transformation is applied

$$\frac{1}{m^2 - p'^2} \to e^{-m^2/M^2}$$

- Sum rules for the form factors can be obtained from the integral

$$\lambda_N f(Q^2) e^{-\frac{m_N^2}{M^2}} = \int_0^{s_0} e^{-\frac{s}{M^2}} \rho^{QCD}(s; Q^2)$$

where ρ^{QCD} can be expressed in terms of the QCD parameters only

Results

$\beta = \tan\theta$

Dependence of Form Factors on $\cos\theta$

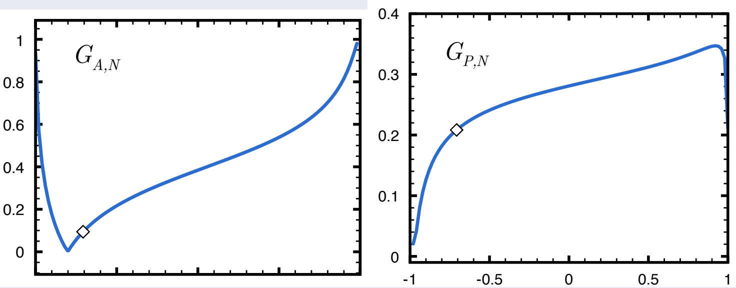
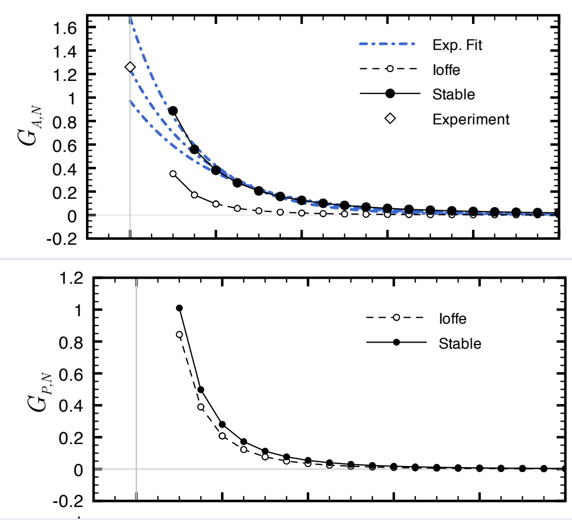


Figure : $M^2 = 2\ GeV^2$ and $s_0 = 2.25\ GeV^2$

Dependence of Form Factors on Q^2



Extrapolation

Extrapolation function used to extrapolate the predictions on g_A out of the validity region of sum rules:

$$G_{A,B} = g_{A,B} e^{-Q^2/m_{A,B}^2}$$

Neither dipole nor exponential fit function describes predictions well for g_P.

Fit parameters

Baryon	Fit Region (GeV^2)	$g_{A,B}$	$m_{A,B}$ (GeV)	$g_{A,B}$(Exp)	$g_{A,B}$(Lattice)
	[1.0-10]	1.68	1.20		
N	[1.5-10]	1.24	1.33	1.2694(28)	1.280(15)
	[2.0-10]	0.97	1.42		
	[1.0-10]	1.11	1.32		
Σ	[1.5-10]	0.92	1.40		0.998(14)
	[2.0-10]	0.77	1.48		
	[1.0-10]	0.46	1.25		
Ξ	[1.5-10]	0.41	1.29		0.282(6)
	[2.0-10]	0.35	1.35		

Nucleon Tensor Form Factors

- The tensor current is defined as:

$$T_{\mu\nu} = \bar{u} i \sigma_{\mu\nu} u - \bar{d} i \sigma_{\mu\nu} d$$

- The tensor form factors are:

$$\langle N(p')|T_{\mu\nu}|N(p)\rangle = \bar{u}(p')\Big[i\sigma_{\mu\nu} H_T(q^2) + \frac{\gamma_\mu q_\nu - \gamma_\nu q_\mu}{2m_N} E_T(q^2) + \frac{P_\mu q_\nu - P_\nu q_\mu}{2m_N^2} \tilde{H}_T(q^2)\Big] u(p),$$

where $P = p' + p$ and $q = p' - p$

Tensor Form Factors

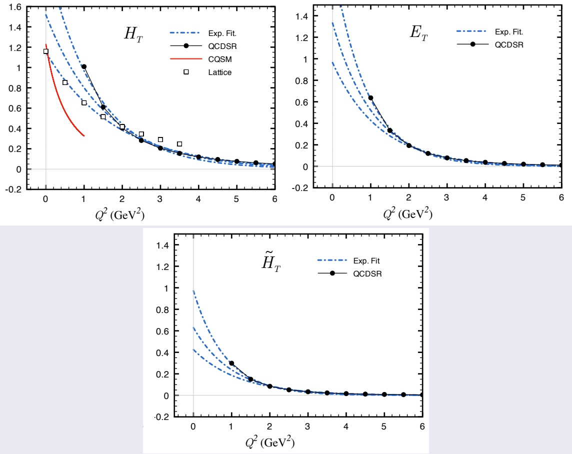


Figure : $\cos\theta = 0$, $M^2 = 2\ GeV^2$, and $s_0 = 2.25\ GeV^2$

Exponential Fit Parameters

Form Factor	Fit Region (GeV^2)	$F_T(0)$	m_T (GeV)
H_T	[2.0-10]	1.15	1.35
	[1.5-10]	1.52	1.25
	[1.0-10]	2.11	1.13
E_T	[2.0-10]	0.96	1.11
	[1.5-10]	1.33	1.03
	[1.0-10]	1.92	0.94
$\tilde{H}_T$	[2.0-10]	0.43	1.10
	[1.5-10]	0.63	1.01
	[1.0-10]	0.97	0.91

$\Delta \to N$ Axial Vector Form Factors

- $j^{\mu}_{\Delta} = \frac{1}{\sqrt{3}} \epsilon^{abc} [2(u^{aT} C \gamma_\mu d^b) u^c(x) + (u^{aT} C \gamma_\mu u^b) d^c]$
- The form factors are defined as:

$$\langle \Delta(p', s') | A_\nu(x) | N(p, s) \rangle = i \overline{\upsilon}^{\lambda}(p', s') \left[\left\{ \frac{C_3^A(q^2)}{M_N} \gamma_\mu + \frac{C_4^A(q^2)}{M_N^2} p'_\mu \right\} (g_{\lambda\nu} g_{\rho\mu} - g_{\lambda\rho} g_{\mu\nu}) q^\rho + C_5^A(q^2) g_{\lambda\nu} + \frac{C_6^A(q^2)}{M_N^2} q_\lambda q_\nu \right] u(p)$$

- BUT.....

- $\langle 0|j_\Delta|s=\frac{1}{2}(p')\rangle \neq 0$, i.e. (lighter) spin-1/2 particles also contribute to the correlation function
- In general

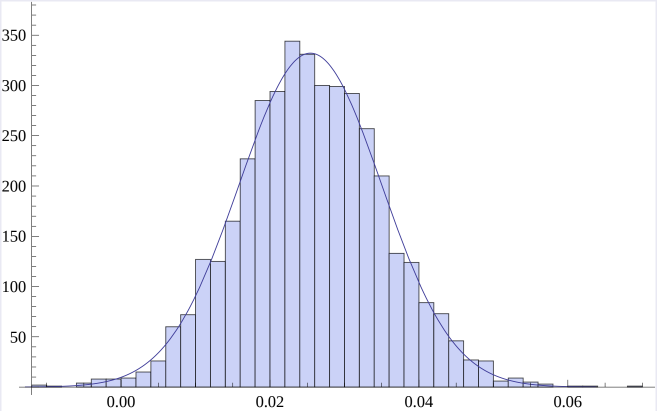
$$\langle 0|j_\Delta^\mu|s=1/2(p')\rangle = (Ap'^\mu + B\gamma^\mu)u(p')$$

  hence all the contribution from spin-1/2 baryons are either proportional to p'_μ or have γ^μ at the far left
- Other Dirac structures do not receive contributions from spin-1/2 baryons.

Monte Carlo Analysis of Uncertainties Due to Input Parameters

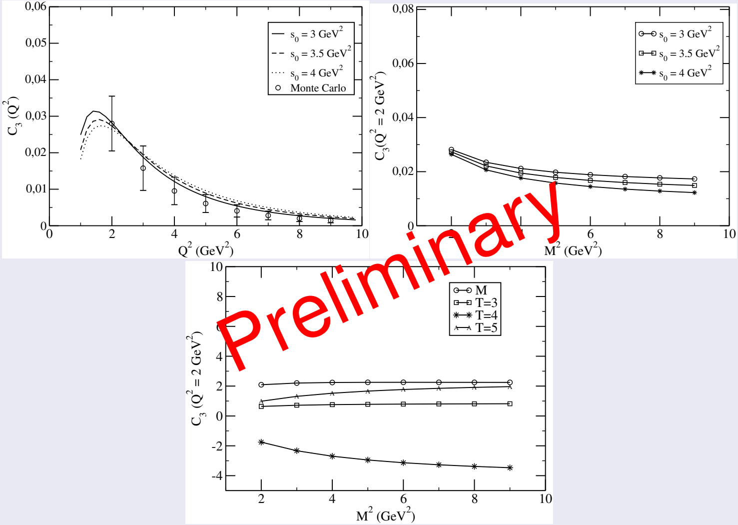
- Proposed by D. Leinweber 1995 (for mass sum rules)
- $\lambda_N f(Q^2) e^{-\frac{m_N^2}{M^2}} = \int_0^{s_0} e^{-\frac{s}{M^2}} \rho^{QCD}(s; Q^2)$
- For each value of Q^2, choose a random value for the input parameters (normally distributed) within their uncertainties and obtain $f(Q^2)$
- Fit the distribution of $f(Q^2)$ to a normal distribution to obtain its mean and variation.
- M^2 and s_0 are randomly chosen (in the working region) with a flat distribution

Histogram For $C_3(2\ GeV^2)$

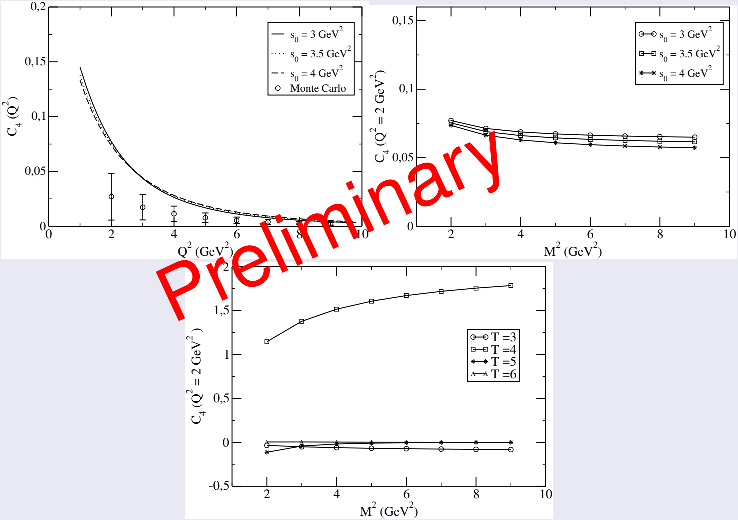


$$\overline{C_3(2\ GeV^2)} = 0.025$$
$$\sigma = 0.0096$$

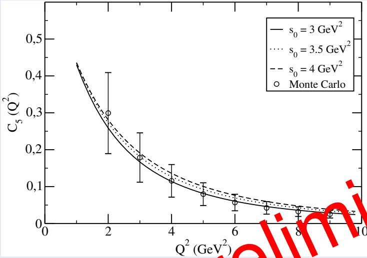
$C_3(Q^2)$

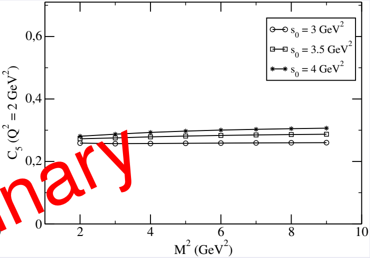

C3 (Q2)
Q2 (GeV2)
s0 = 3 GeV2
s0 = 3.5 GeV2
s0 = 4 GeV2
Monte Carlo
C3(Q2 = 2 GeV2)
M2 (GeV2)
C3 (Q2 = 2 GeV2)
M
T=3
T=4
T=5
Preliminary

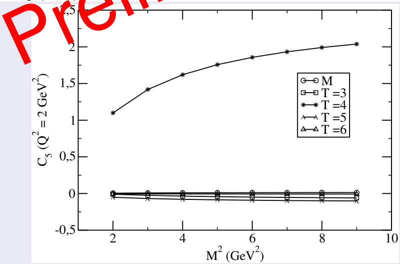
$C_4(Q^2)$


Preliminary
$C_4(Q^2)$
Q^2 (GeV^2)
$s_0 = 3$ GeV^2
$s_0 = 3.5$ GeV^2
$s_0 = 4$ GeV^2
Monte Carlo
$C_4(Q^2 = 2$ GeV$^2)$
M^2 (GeV^2)
$s_0 = 3$ GeV^2
$s_0 = 3.5$ GeV^2
$s_0 = 4$ GeV^2
$C_4(Q^2 = 2$ GeV$^2)$
M^2 (GeV^2)
T =3
T =4
T =5
T =6

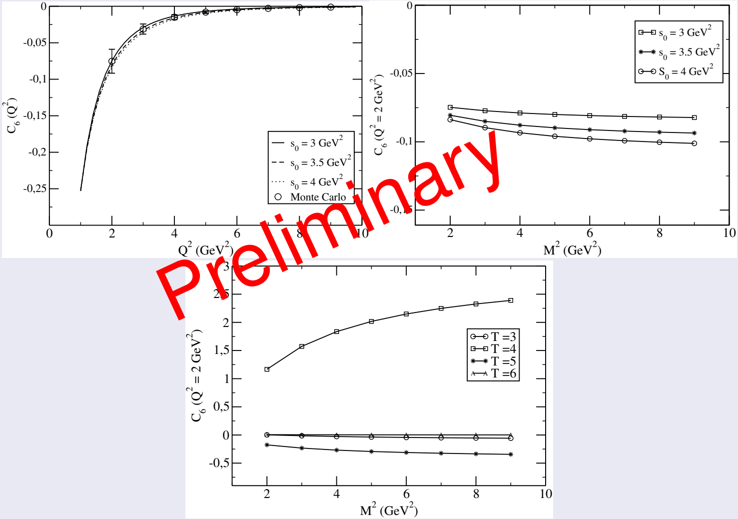
$C_5(Q^2)$


s0 = 3 GeV2
s0 = 3.5 GeV2
s0 = 4 GeV2
Monte Carlo
C5 (Q2)
Q2 (GeV2)
0,5
0,4
0,3
0,2
0,1
0
0
2
4
6
8
10
Preliminary


s0 = 3 GeV2
s0 = 3.5 GeV2
s0 = 4 GeV2
C5 (Q2 = 2 GeV2)
M2 (GeV2)
0,6
0,4
0,2
0
2
4
6
8
10


M
T =3
T =4
T =5
T =6
C5 (Q2 = 2 GeV2)
M2 (GeV2)
2
1,5
1
0,5
0
-0,5
2
4
6
8
10

$C_6(Q^2)$


$C_6(Q^2)$
Q^2 (GeV^2)
$s_0 = 3$ GeV^2
$s_0 = 3.5$ GeV^2
$s_0 = 4$ GeV^2
Monte Carlo
$C_6(Q^2 = 2$ GeV$^2)$
M^2 (GeV^2)
$s_0 = 3$ GeV^2
$s_0 = 3.5$ GeV^2
$S_0 = 4$ GeV^2
Preliminary
$C_6(Q^2 = 2$ GeV$^2)$
M^2 (GeV^2)
T =3
T =4
T =5
T =6

Fit Function

$$f(Q^2) = \frac{f(0)}{(1+Q^2/m^2)^2}$$

Fit Parameters

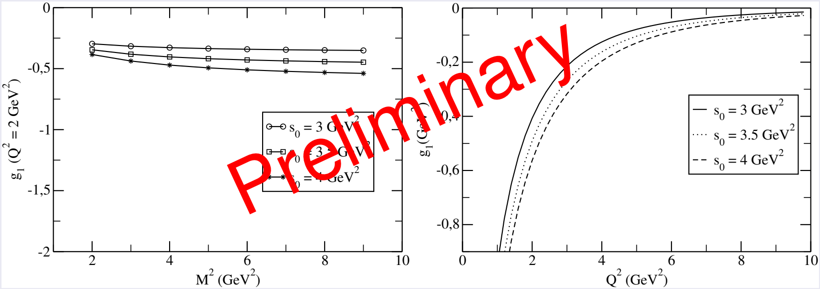
	$f(0)$	$m(GeV)$
C_3	0.049	2.10
C_4	0.448	1.24
C_5	1.11	1.52
C_6	-1.66	1.40

- The Axial Vector Δ baryon vertex is defined as:

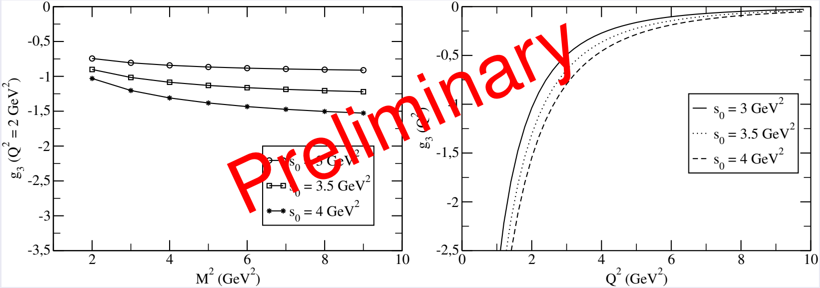
$$\langle \Delta(p', s') | A_\nu(x) | \Delta(p, s) \rangle = \frac{-i}{2} \overline{\upsilon}^\alpha(p', s') \left[g_{\alpha\beta} \left(g_1(q^2) \gamma_\nu \gamma_5 + g_3(q^2) \frac{q_\nu \gamma_5}{2M_\Delta} \right) + \frac{q^\alpha q^\beta}{4M_\Delta^2} \left(h_1(q^2) \gamma_\nu \gamma_5 + h_3(q^2) \frac{q_\nu \gamma_5}{2M_\Delta} \right) \right] \upsilon^\beta(p, s)$$

- Only the leading twist distribution amplitudes of the Δ baryon are calculated (C.E. Carlson and J. L. Poor, 1988)
- Leading twist is not enough to calculate h_1 and h_3 form factors due to additional factors of q in their coefficients.
- Calculation of $\Delta \to N$ form factors using Δ distribution amplitudes can give an idea on how well the leading twist DA describes the Δ baryon.

$g_1(Q^2)$


Preliminary
$g_1\,(Q^2 = 2\text{ GeV}^2)$
$M^2\,(\text{GeV}^2)$
$s_0 = 3\text{ GeV}^2$
$s_0 = 3.5\text{ GeV}^2$
$s_0 = 4\text{ GeV}^2$
$Q^2\,(\text{GeV}^2)$

$g_3(Q^2)$


Preliminary
g_3 (Q^2 = 2 GeV^2)
M^2 (GeV^2)
s_0 = 3 GeV^2
s_0 = 3.5 GeV^2
s_0 = 4 GeV^2
g_3 (Q^2)
Q^2 (GeV^2)

Fit Function:

$$f(Q^2) = \frac{f(0)}{\left(1+\frac{Q^2}{m^2}\right)^2}$$

Fit Parameters

	$f(0)$	$m\ (GeV)$
g_1	-5.45	0.85
g_3	-24.21	0.81

Conclusions

- Nucleon and Delta isovector form factors are presented
- Baryon mass corrections are important.
- Need more information of Δ baryon DAs.
- Uncertainties due to the input parameters, fit region are analyzed
- Using M^2 dependent s_0 might reduce the uncertainty? (to be done, see next talk)