Sigma meson and lowest possible glubeall candidate in an extended linear σ model

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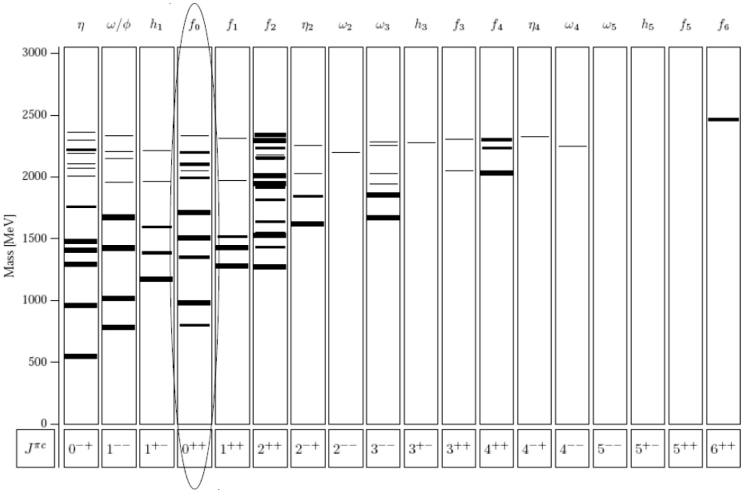
Reference: arXiv:1203.5717 [hep-ph]

Experimental Light Flavoured Iso-Scalar Meson Spectrum:



Unusual Spectroscopy

Vector Mesons:

I $= 1$:	m$[\rho(776)] \approx 776 MeV$	$n\bar{n}$
I $= 0$:	m$[\omega(783)] \approx 783 MeV$	$n\bar{n}$
I $= \frac{1}{2}$:	m$[K^\star(892)] \approx 892 MeV$	$n\bar{s}$
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Scalar Mesons:

- I = 0: m[$f_0(600)$] $\approx 500 MeV$ $\quad \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$
- I = $\frac{1}{2}$: m[κ] $\approx 800 MeV$ $\quad \bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d$
- I = 0: m[$f_0(980)$] $\approx 980 MeV$ $\quad \bar{s}s$
- I = 1: m[$f_0(980)$] $\approx 980 MeV$ $\quad \bar{u}d, \bar{d}u, \sqrt{\frac{1}{2}}(\bar{u}u - \bar{d}d)$

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Light Scalars are tetraquark state: Jaffe (Phys. Rev. D 15 (1977))
The States above consecutively can be represented as:
$nn\bar{n}\bar{n}, nn\bar{n}\bar{s}, ns\bar{n}\bar{s}, ns\bar{n}\bar{s}$

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- Basic Lagrangian:

$$\mathcal{L} = \text{Tr}\ (\partial_\mu \Phi\ \partial^\mu \Phi^\dagger) + \text{Tr}\ (\partial_\mu \Phi'\ \partial^\mu \Phi^{\dagger\prime}) + \partial_\mu Y \partial^\mu Y^\star - V_0 - V_{SB}$$

Some Remarks

- Tetraquark field:
  a) molecular type :

$$M^b{}_a = (q_{bA})^\dagger \gamma_4 \frac{1+\gamma_5}{2} q_{aA}; \Phi^b{}_a = \epsilon_{acd}\epsilon^{bef}(M^\dagger)^c{}_e(M^\dagger)^d{}_f$$

  b) scalar di-quark + anti-diquark :

$$\phi_i = \sqrt{\frac{1}{2}}\epsilon_{ijk} q^\dagger{}_j C\gamma^5 q_k; \Phi_{ij} = \phi^\dagger{}_i \phi_j$$

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- Glueball field:
  We interpret the spurion field as effective glueball field. To accommodate realistic glueball field it is widely used practice to introduce a flavor singlet complex field to the linear/non-linear sigma model.[Phys Rev. D 21, 3393 (1980), Nucl. Phys. B175, 477 (1980), Prog. Theor. Phys. 66, 1789 (1981), Phys. Rev. D 80, 014014 (2009)].

Lagrangian

$$\mathcal{L}_{\mathcal{S}} = Tr(\partial_\mu \Phi \partial^\mu \Phi^\dagger) + Tr(\partial_\mu \Phi' \partial^\mu \Phi^{\dagger\prime}) + \partial_\mu Y \partial^\mu Y^\star - {m_\Phi}^2 Tr(\Phi^\dagger \Phi)$$
$$- {m_{\Phi'}}^2 Tr(\Phi^{\dagger\prime}\Phi') - {m_Y}^2 YY^\star - \lambda_1 Tr(\Phi^\dagger\Phi\Phi^\dagger\Phi) - {\lambda_1}' Tr(\Phi^{\dagger\prime}\Phi'\Phi^{\dagger\prime}\Phi')$$
$$- \lambda_2 Tr(\Phi^\dagger\Phi\Phi^{\dagger\prime}\Phi') - \lambda_Y (YY^\star)^2 - [\lambda_3 \epsilon_{abc}\epsilon^{def} {\Phi_d}^a {\Phi_e}^b {\Phi_f}'^c + h.c.]$$
$$+ [kYDet(\Phi) + h.c.] \qquad (1)$$

$$\mathcal{L}_{\mathcal{SB}} = [Tr(B.\Phi) + h.c.] + [Tr(B'.\Phi') + h.c.] + (D.Y + h.c.)$$
$$- [\lambda_m Tr(\Phi\Phi^{\dagger\prime}) + h.c.] \qquad (2)$$

Mixing and Parameter Fixing:

Isospin	$I = 1$	$I = \frac{1}{2}$	$I = 0$
PseudoScalars(P=-1)	$\{\pi, \pi'\}$	$\{K, K'\}, \{K^*, K^{*\prime}\}$	$\{\eta_1, \eta_2, \eta_3, \eta_4, \eta_5\}$
Scalars(P=1)	$\{a, a'\}$,	$\{\kappa, \kappa'\}, \{\kappa^*, \kappa^{*\prime}\}$,	$\{f_1, f_2, f_3, f_4, f_5\}$

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- For I = 0 scalar and pseudoscalar states two, four quarks as well as glueball states mixed with each other.

- Input Parameters:
  Mixing angles for π and K within the range $\{-\frac{\pi}{4}, \frac{\pi}{4}\}$ along with their decay constants.
  Two condensates (below 2 GeV)
- Symmetry Breaking Parameters:
  $\frac{B_s}{B_{u,d}} = \frac{m_s}{m_{u,d}} = \frac{{B_s}'}{{B_{u,d}}'}$

Parameter Fixing contd..

- Vacuum Stability Conditions: $\frac{\partial V}{\partial < v_i >} = 0$.
- Physical Input Mass: $(R^{-1})M^2{}_{bare}(R) = M^2{}_{phys}$

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- Parameters related to Glueball sector:

$$Tr[M_\eta{}^2]_{Model} = Tr[M_\eta{}^2]_{Exp} \ , \tag{3}$$

$$Det[M_\eta{}^2]_{Model} = Det[M_\eta{}^2]_{Exp} \ . \tag{4}$$

π' Mass (GeV)	Field	Our Value (GeV)	quarkonia (%)	tetraquark (%)	Experimental Value (GeV)
1.2	a	1.055	38.14	61.8 6	0.98
	a'	1.417	61.86	38.14	1.47
	κ	1.13	62.14	37.86	0.80
	κ'	1.186	37.86	62.14	1.43

Table: Mass spectra and components for the triplet and doublet sector based on our fit are demonstrated where the best value of $m_{\pi'}$ is found to be $m_{\pi'} = 1.2$ GeV.

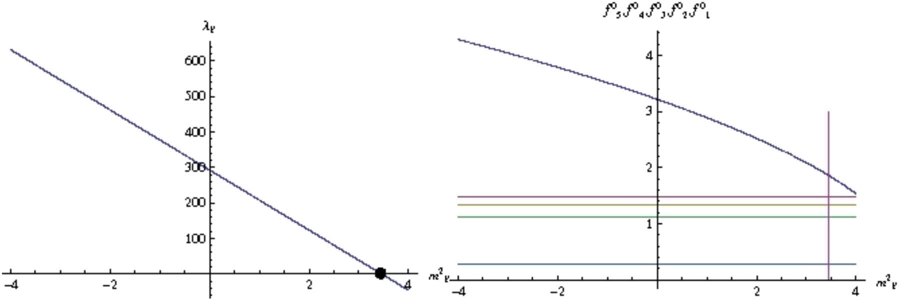
π' Mass (GeV)	$J^{PC} = 0^{-+}$	Our Value (GeV)	quarkonia (%)	tetraquark (%)	glueball (%)	Experimental Value (GeV)
1.2	η_5	1.858	0.037	0.001	99.962	1.756 ± 0.009
	η_4	1.380	75.803	24.167	0.03	1.476 ± 0.004
	η_3	1.291	26.700	73.294	0.006	1.294 ± 0.004
	η_2	0.907	15.852	84.145	0.003	0.95766 ± 0.00024
	η_1	0.595	81.607	18.393	0.0	0.547853 ± 0.000024

Table: Mass spectra and components for the pseudo-scalar mesons based on our fit are shown where the best value of $m_{\pi'}$ is found to be $m_{\pi'} = 1.2$ GeV.

π' Mass (GeV)	$J^{PC} = 0^{++}$	Our Value (GeV)	quarkonia (%)	tetraquark (%)	glueball (%)	Experimental Value (GeV)
1.2	${f_5}^0$	2.09	0.01	0.0	99.99	-
	${f_4}^0$	1.487	77.469	22.53	0.001	1.505 ± 0.006
	${f_3}^0$	1.347	22.177	77.82	0.003	1.2-1.5
	${f_2}^0$	1.124	21.561	78.439	0.0	0.980 ± 0.010
	${f_1}^0$	0.274	78.784	21.211	0.005	0.4-1.2

Table: Mass spectra and components for the scalar mesons based on our fit are shown where the best value of $m_{\pi'}$ is found to be $m_{\pi'} = 1.2$ GeV.

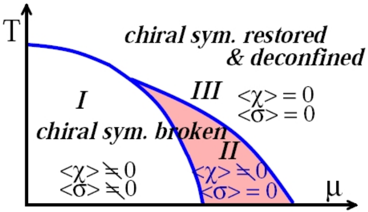
Bounded potential constraint $\lambda_Y > 0$



Dependence of λ_Y and scalar meson masses on the scanning parameter ${m_Y}^2$

Summary and Outlook

- Without the mixing between tetraquark and glueball the heaviest scalar is glueball dominated and the lowest scalar is quarkonia dominated with a seizable porition of tetraquakr percentage.
- It would be interesting to check if the mixing between quarkonia and tetraquark changes the scenario.
- Understanding of the vacuum phenomenology $\rightarrow$ medium behaviours.
- Implication for chiral symmetry restoration. $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V \times (Z_{N_f})_A \rightarrow SU(N_f)_V$. M. Harada et al. arXiv:0908.1361



THANK YOU!