Mutual friction
in
color-flavor locked quark matter

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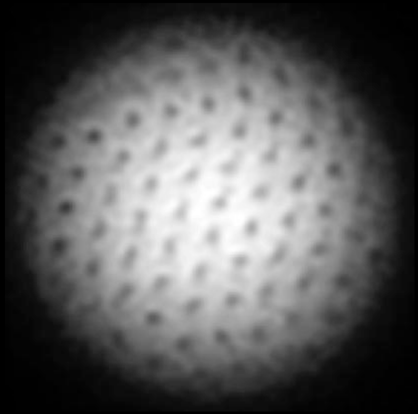
A message in this talk is...

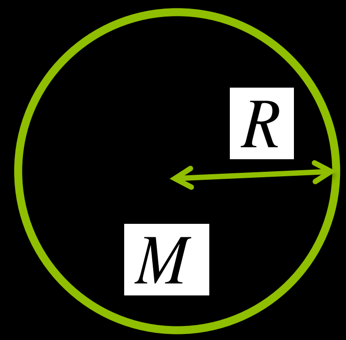
Vortex structure appears everywhere



How nuclear physics meets astrophysics

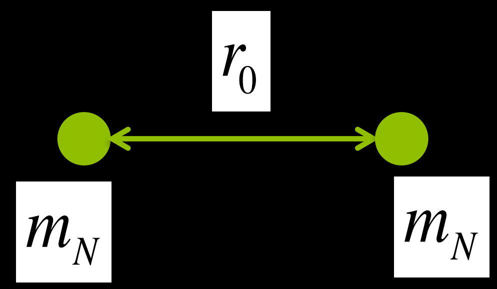
General relativity

$$R = 2GM$$



Nuclear physics

$$R = r_0 A^{1/3}, \quad M = m_N A$$



$$R \sim 7km \quad M \sim 2.3 M_{SUN}$$

Not bad!

Major questions

1. What is the ground state of dense QCD matter as a function of temperature/chemical potential(s)?

2. At which density does the phase transition among nuclear and quark matter happen if it does?

3. What kind of form is made after the phase transition?

4. How do the above staffs affect compact star physics?

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R-mode instability of compact stars
Mutual friction as a possible mechanism

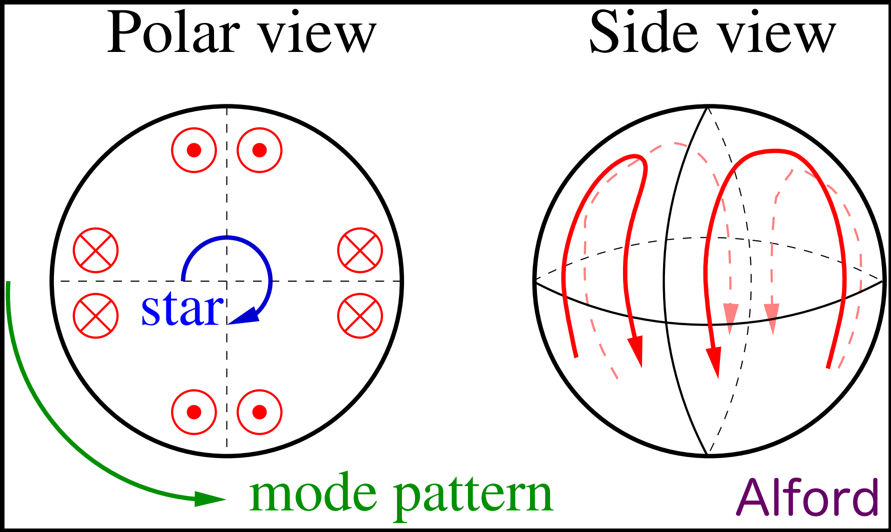
Mutual friction in CFL quark matter

Work in progress w/ M. Mannarelli & M. Ruggieri

The r(otational) mode instability of NS

(eg. N. Andersson)

A quadrupole flow that emits gravitational wave radiation.



If dissipative phenomena are not strong enough, the oscillations will grow exponentially, and the star will keep slowing down until some dissipation mechanism can damp the r-modes.

Useful in constraining the stellar structure.

Neutron star as a gigantic nuclei

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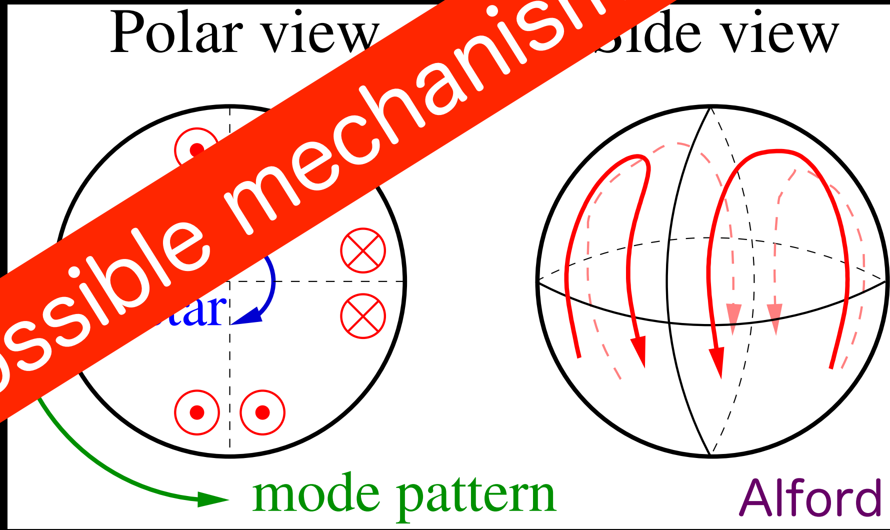

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Mutual friction as a possible mechanism?

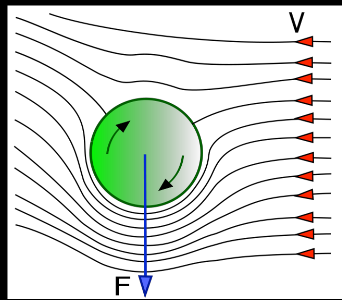
Neutron star as a gigantic nuclei

Mutual friction in superfluid vortex system

A force btw normal & superfluid components via vortices

- Magnus force btw superfluid comp. and vortex

$$\vec{F}_M = \kappa\rho_s(\vec{v}_s - \vec{v}_L)\times\vec{\hat{z}}$$



- Force produced by normal excitations

$$\vec{F}_N = -D(\vec{v}_n - \vec{v}_L) - D'\vec{\hat{z}}\times(\vec{v}_n - \vec{v}_L)$$

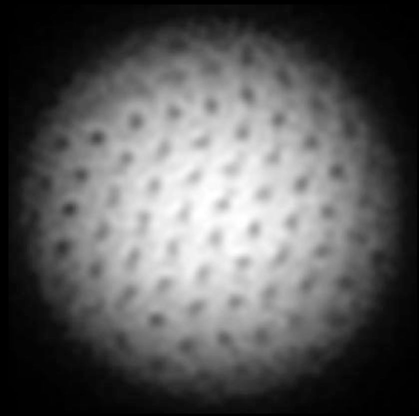
“transport coefficients”

- Force balance condition for a vortex

$$\vec{F}_M + \vec{F}_N = 0$$

The vortex velocity

$$\vec{v}_L = \vec{v}_s + \alpha'(\vec{v}_n - \vec{v}_s) + \alpha\vec{\hat{z}} \times (\vec{v}_n - \vec{v}_s)$$



where,

$$\alpha = \frac{d_{\parallel}}{d_{\parallel}^2 + (1-d_{\perp})^2}, \quad \alpha' = \frac{1-d_{\perp}}{d_{\parallel}^2 + (1-d_{\perp})^2}$$

$$d_{\parallel} = D/\kappa\rho_s, \quad d_{\perp} = D'/\kappa\rho_s$$

depend on micro -scopic physics and related to scattering cross sections

"Hall-Vinen parameters" (1956)

$$D = \sigma_{\parallel} c_s \rho_n$$

$$D' = \sigma_{\perp} c_s \rho_n$$

Mutual friction

$$\vec{F}_{MF} = -\vec{F}_M$$

If a perturbation of the superfluid velocity $\boxed{\delta\vec{v}_L}$ is introduced, there is no guarantee that two forces are balanced

$$\boxed{\delta\vec{F}_v \equiv \delta\vec{F}_N + \delta\vec{F}_M = m_v \frac{d\delta\vec{v}_L}{dt}}$$

"hydro"

Energy dissip: $\boxed{\left(\frac{dE}{dt}\right)_{MF} = \delta\vec{F}_v \cdot \delta\vec{v}_s}$ $\Rightarrow$ $\boxed{\frac{1}{t_{MF}} = -\frac{1}{2E}\left(\frac{dE}{dt}\right)_{MF}}$

<u>Application to color-flavor locked quark matter</u>

In CFL phase, baryon number symmetry $U(1)_B$ is broken. So CFL quark matter is a superfluid. If such a state exists in NS, then it will be worth to consider the mutual friction.

Analog of gravity

Unruh '81

A relation btw hydrodynamics and field theory in curved space

Action of phonon field moving in superfluid background = Action of a boson field propagating in a curved space-time

Analog of gravity

Unruh '81

A relation btw hydrodynamics and field theory in curved space

"Acoustic metric" and "Effective action"

velocity field

$$ds^2 = G_{\mu\nu} dx^\mu dx^\nu, \qquad G_{\mu\nu} = \eta_{\mu\nu} + \left(c_s^2 - 1\right) v_\mu v_\nu$$

$$S[\phi] = \frac{1}{2} \int d^4 x \sqrt{-G} G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

acoustic metric

※frame independent

"phonon" field

$$v_\mu \xrightarrow[r \to \infty]{} \left(1, 0, \frac{\kappa}{2\pi r}, 0\right)$$

$$\kappa = \frac{2\pi}{\mu}$$

Our strategy

We consider color-flavor locked (CFL) quark matter,
where global U(1) symmetry is spontaneously broken.

Comments

- Neglecting the Caroli—de-Gennes—Matricon mode

$$E_g \sim \mathrm{O}(1) \times \frac{\Delta^2}{\varepsilon_F}, \quad \delta E = \frac{\Delta^2}{\varepsilon_F}$$

(mini-gap)

- Just taking into account Abelian vortices

  (※) non-Abelian vortices in color superconductor
  (see for instance, Eto-Nitta, PRD80(2009))

- Considering amplitudon mode as well as phason

Ref. Mannarelli-Manuel, PRD77(2008)

Mannarelli-Manuel-Sa'd (2008) PRL101

- CFL quark matter with $U(1)_B$ NG boson
- Analog of gravity & elastic scattering

$$\frac{1}{t_{MF}} = 2 \times 18.1 \left(\frac{T}{\mu}\right)^2 \Omega \quad > \quad \frac{1}{t_{GR}} = \frac{1}{3.26}\left(\frac{\Omega^2}{\pi G_N \rho}\right)^3$$

Mutual friction — GW radiation

$$\Rightarrow \quad \nu \leq \nu_c \cong 1[Hz] @ T = 0.01 eV, \quad \mu = 400 MeV$$

Effective field theory for CFL mater

-Higgs and NG boson-

Anglani-Mannarelli-Ruggieri, New J. Phys 13 (2011)

$$\langle qq \rangle \sim \left(\Delta_{CFL} + \rho \right) e^{i\phi}$$

Higgs
(amplitudon)

NG boson
(phason)

Why Higgs?

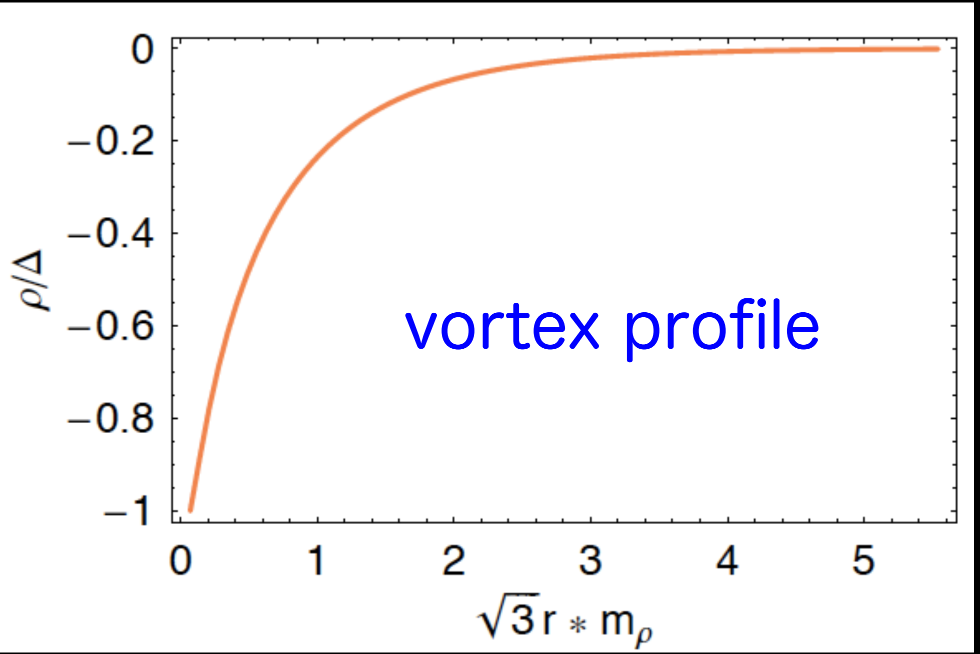
$$m_\rho = 4\Delta_{CFL}$$

In the vicinity of vortex, Δ is small,
which means ρ becomes light d.o.f.
So Higgs could come into the game.

Yet we are on the way. . .

$$\mathcal{L}(\rho) = \frac{1}{2}\left[(\partial_0\rho)^2 - \frac{1}{3}(\boldsymbol{\nabla}\rho)^2\right] - \frac{6}{\pi^2}\left(\frac{4\pi^2\Delta^2}{3\mu^2}\right)\left[(\mu-\partial_0\phi)^2 - \frac{1}{3}(\boldsymbol{\nabla}\phi)^2\right]\rho^2$$
$$-\frac{2\mu^2}{\pi^2\Delta}\left(\frac{4\pi^2\Delta^2}{3\mu^2}\right)^{3/2}\rho^3 + \frac{4\pi^2\Delta^2}{9\mu^2}\rho^4 \,.$$

effective Lagrangian



asymptotic solution

$$\rho(r) \approx C\left(\frac{\pi}{2\sqrt{3}m_\rho r}\right)^{1/2} e^{-\sqrt{3}m_\rho r}\left[1 + O\left(\frac{1}{m_\rho r}\right)\right]$$

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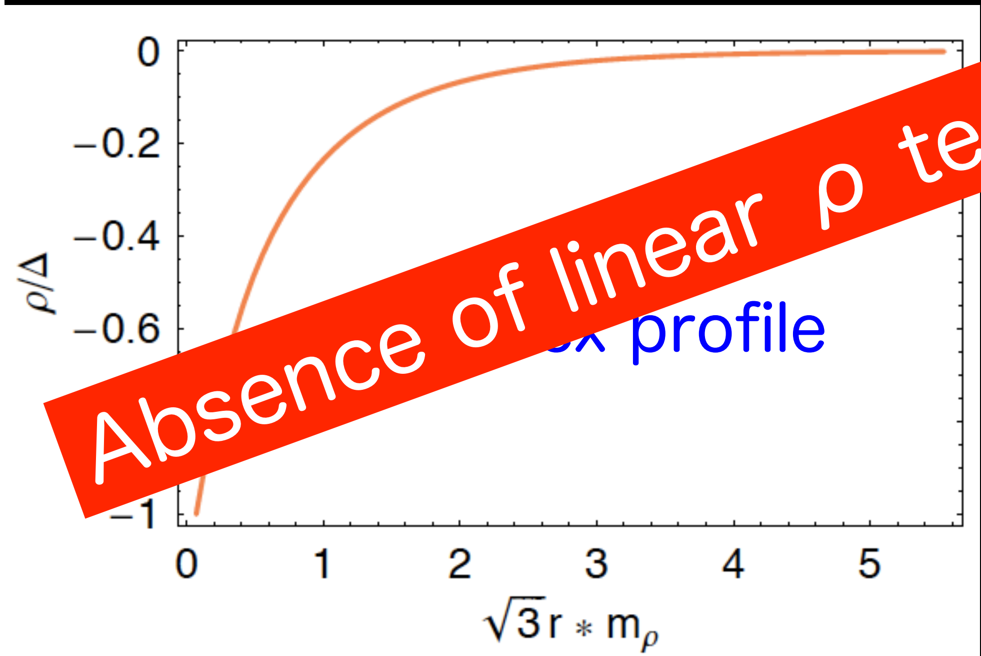
effective Lagr



Absence of linear ρ term is problematic . .

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Astrophysical applications of dense matter

1. R-mode instability of compact stars
2. Mutual friction in CFL quark matter

Perspectives

1. CFL effective action w/ Higgs and NG boson (derived based on Hong's high density theory)
2. Estimating the time scales τ_{MF} and τ_{GR}
3. Non-Abelian vortices