Meson – photon transition form factors at spacelike momentum transfers

Dmitri Melikhov

HEPHY, Vienna, Austria & SINP, Moscow State University, Russia

We present the analysis of the $F_{P\gamma}(Q^2)$, $P=\pi,\eta,\eta',\eta_c$ form factors making use of the local-duality (LD) version of QCD sum rules.

I.Balakireva, W.Lucha, DM, Phys.Rev. **D85** *(2012) 036006;*

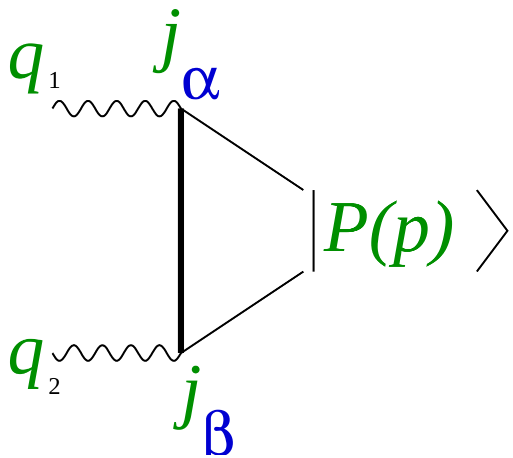
DM, B.Stech, Phys.Rev. **D85** *(2012) 051901;*

W. Lucha, DM, J.Phys. **G39**, *045003 (2012); Phys.Rev.* **D85** *(2012) N9.*

Introduction

The amplitude of $\gamma^*\gamma^* \to P$, $(P = \pi^0, \eta, \eta', \eta_c,)$ **contains only one form factor:**

$$\langle \gamma^*(q_1)\gamma^*(q_2)|P(p)\rangle = \mathrm{i}\epsilon_{\varepsilon_1\varepsilon_2 q_1 q_2} F_{P\gamma\gamma}(q_1^2, q_2^2).$$



QCD factorization theorem predicts at asymptotically large spacelike momentum transfers $q_1^2 = -Q_1^2 < 0$**,** $q_2^2 = -Q_2^2 < 0$**:**

$$F_{P\gamma\gamma}(Q_1^2, Q_2^2) \to 2e_c^2 \int\limits_0^1 \frac{\mathrm{d}\xi \phi_P^{\mathrm{ass}}(\xi)}{Q_1^2\xi + Q_2^2(1-\xi)}, \qquad \phi_P^{\mathrm{ass}}(\xi) = 6 f_P \xi(1-\xi),$$

Introduce $Q^2 \equiv Q_2^2$, $0 \le \beta \equiv Q_1^2/Q_2^2 \le 1$ (Q_2^2 is the larger virtuality):

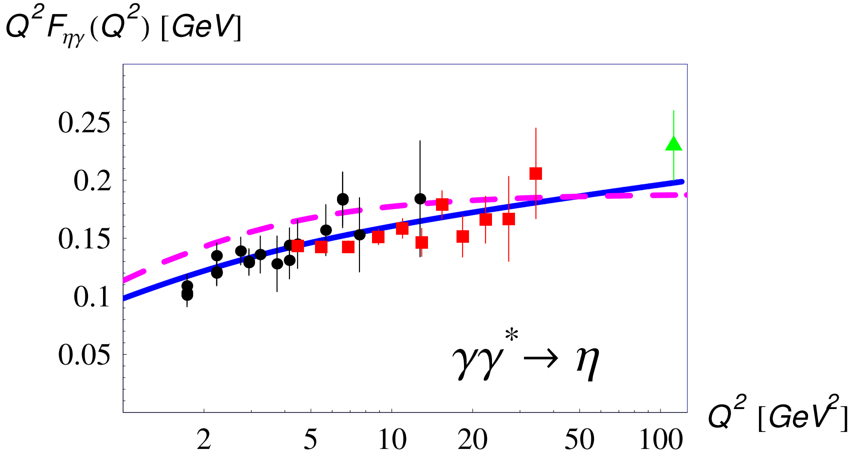
$$F_{P\gamma\gamma}(Q_1^2, Q_2^2) = \frac{6e_c^2 f_P}{Q^2} I(\beta), \qquad I(\beta) = \frac{1 + 2\beta \log\beta - \beta^2}{(1-\beta)^3}, \qquad I(0) = 1, \qquad I(1) = 1/3.$$

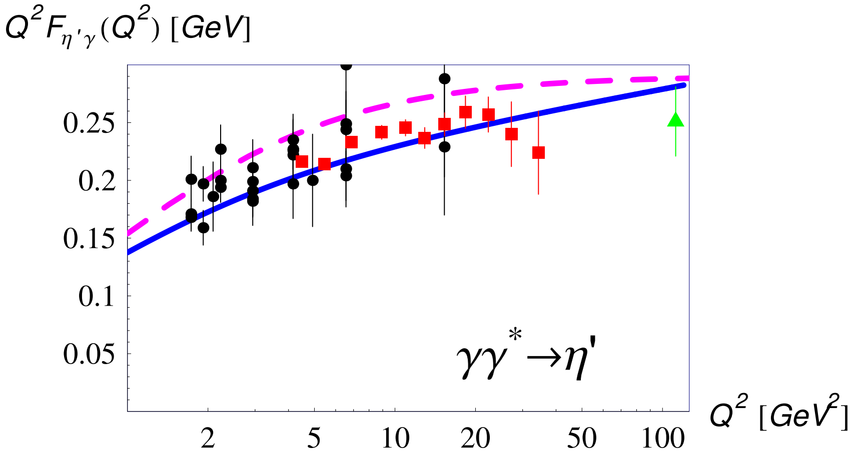
Experimentally relevant kinematics is $Q_1^2 \simeq 0$ and $Q_2^2 = Q^2$ large.

For the pion

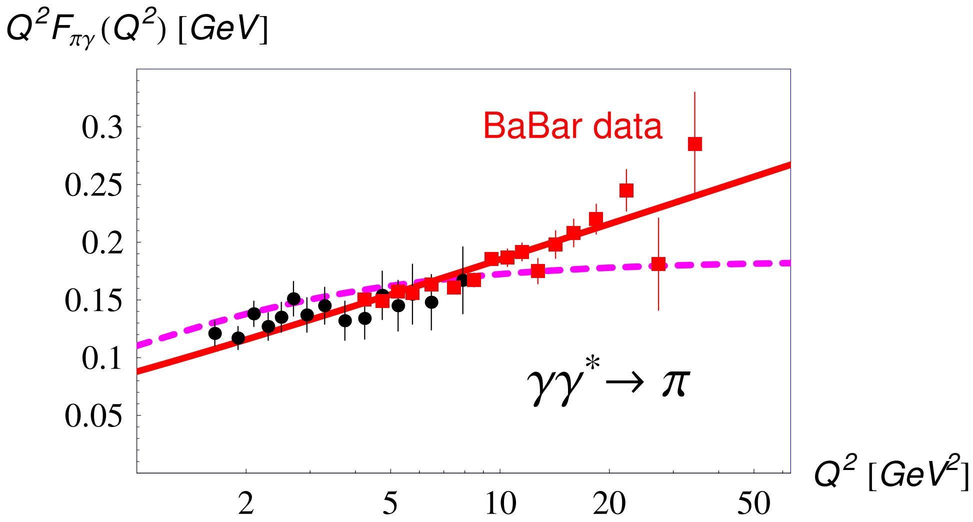
$$Q^2 F_{\pi\gamma}(Q^2) \to \sqrt{2} f_\pi \qquad f_\pi = 0.130 \ \mathbf{GeV}.$$

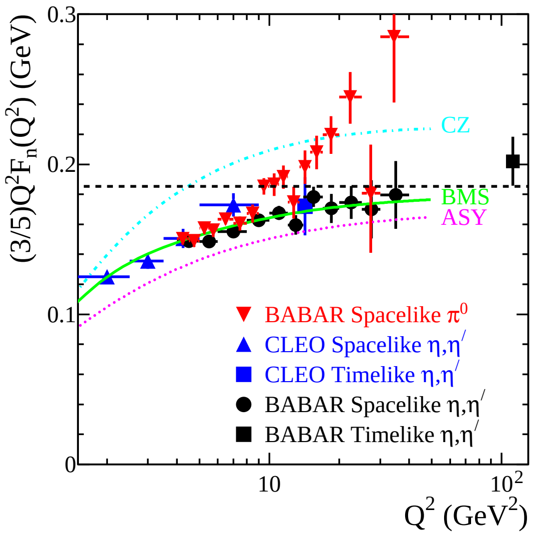
Similar scaling relations emerge for η and η' after taking into account the mixing effects.





The η and η' data is not in contradiction with saturation $Q^2 F(Q^2) \sim$ const



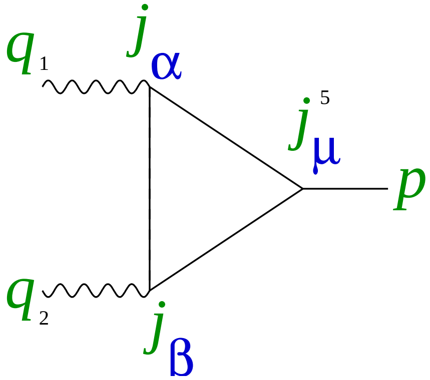


The BaBar pion form factor seems more compatible with $Q^2 F_{\pi\gamma}(Q^2) \sim \log(Q^2)$.

QCD sum rule in LD limit

The basic object is the 3-point function $\langle AVV\rangle = \langle 0|T(j^5_\mu j_\alpha j_\beta)|0\rangle$**,**

$j^5_\mu = \frac{1}{\sqrt{2}}\left(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d\right)$**;** $j_\alpha = \frac{2}{3}\bar{u}\gamma_\alpha u - \frac{1}{3}\bar{d}\gamma_\alpha d$**;** $j^5 = \frac{1}{\sqrt{2}}\left(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d\right)$**;**



The amplitude has the general decomposition $(p = q_1 + q_2)$**:**

$$T_{\mu\alpha\beta}(p|q_1, q_2) = p_\mu \epsilon_{\alpha\beta q_1 q_2} iF - (q_1^2 \epsilon_{\mu\alpha\beta q_2} - q_{1\alpha}\epsilon_{\mu q_1 \beta q_2}) iF_1 - (q_2^2 \epsilon_{\mu\beta\alpha q_1} - q_{2\beta}\epsilon_{\mu q_2 \alpha q_1}) iF_2.$$

In the language of hadron intermediate states, the pseudoscalar contributes to the structure $\sim p_\mu$**:**

$$T_{\mu\alpha\beta}(p|q_1, q_2) \sim \frac{p_\mu}{m_P^2 - p^2} i\epsilon_{\alpha\beta q_1 q_2} f_P F_{P\gamma\gamma}(q_1^2, q_2^2) + \dots$$

Thus, the form factor $F(p^2, q_1^2, q_2^2)$ **contains the pseudoscalar contribution.**

Consider also the amplitude induced by j_5:

$$\langle\gamma(q_1)\gamma(q_2)|j_5|0\rangle = -\epsilon_{\alpha\beta q_1 q_2}\varepsilon_1^\alpha\varepsilon_2^\beta F_5(q_1^2, q_2^2, p^2).$$

The two-photon amplitude of the divergence of the axial current

$$\langle\gamma(q_1)\gamma(q_2)|\partial^\mu j_\mu^5|0\rangle = -\epsilon_{\alpha\beta q_1 q_2}\varepsilon_1^\alpha\varepsilon_2^\beta(p^2 F - q_1^2 F_1 - q_2^2 F_2).$$

For the form factors F_i one can write spectral representation in p^2:

$$F_i(p^2, q_1^2, q_2^2) = \frac{1}{\pi}\int\limits_{4m^2}^{\infty}\frac{ds}{s-p^2}\,\Delta_i(s, q_1^2, q_2^2).$$

The spectral densities $\Delta_i(s, q_1^2, q_2^2)$ obey the classical equation of motion

$$s\,\Delta(s, q_1^2, q_2^2) - q_1^2\,\Delta_1(s, q_1^2, q_2^2) - q_2^2\,\Delta_2(s, q_1^2, q_2^2) = 2m\,\Delta_5(s, q_1^2, q_2^2).$$

The form factors then satisfy

$$p^2 F(p^2, q_1^2, q_2^2) - q_1^2 F_1(p^2, q_1^2, q_2^2) - q_2^2 F_2(p^2, q_1^2, q_2^2) = 2m\,F_5(p^2, q_1^2, q_2^2) - \frac{1}{\pi}\int\limits_{4m^2}^{\infty} ds\,\Delta(s, q_1^2, q_2^2|m).$$

In pQCD, one obtains $\Delta(s, q_1^2, q_2^2|m)$ **as an expansion**

$$\Delta(s, q_1^2, q_2^2|m) = \Delta^{(0)}_{\rm QCD}(s, q_1^2, q_2^2|m) + \frac{\alpha_s}{\pi}\Delta^{(1)}_{\rm QCD}(s, q_1^2, q_2^2|m) + O(\alpha_s^2).$$

The integral

$$\int\limits_{4m^2}^{\infty} ds\, \Delta^{(0)}_{\rm QCD}(s, q_1^2, q_2^2|m) = \frac{1}{2\pi},$$

independently of the values of m **and** $q_{1,2}^2$ **and represents the axial anomaly.**

The *exact* **relation (no radiative corrections on the r.h.s., Adler-Bardeen theorem)**

$$\boxed{\int\limits_{4m^2}^{\infty} ds\, \Delta(s, q_1^2, q_2^2|m) = \tfrac{1}{2\pi}.}$$

In the hadron language,

$$\Delta(s, q_1^2, q_2^2|m) = \pi f_\pi F_{\pi\gamma\gamma}(q_1^2, q_2^2)\delta(s - m_P^2) + \textbf{hadron continuum}$$

A remarkable relation emerges for both real photons and massless fermion:

$$p^2 F(p^2, q_1^2, q_2^2) - q_1^2 F_1(p^2, q_1^2, q_2^2) - q_2^2 F_2(p^2, q_1^2, q_2^2) = 2m\, F_5(p^2, q_1^2, q_2^2) - \frac{1}{2\pi^2}$$

$$F(p^2, 0, 0) \;=\; -\frac{1}{2\pi^2 p^2} \quad \textbf{(exactly!)}$$

$$F(p^2, 0, 0) \;=\; -\frac{f_\pi F_{\pi\gamma\gamma}(0,0)}{p^2} + \textbf{excited states}$$

In the chiral limit and for both real photons remarkable quark – hadron duality relation :

one (triangle) diagram → one hadron state

The anomaly should be reproduced by confined bound states of the theory leading to the *exact* anomaly sum rule

$$\boxed{\pi f_P F_{P\gamma\gamma}(q_1^2, q_2^2) + \int\limits_{\rm cont}^{\infty} ds\, \Delta_{\rm hadr}(s, q_1^2, q_2^2|m) = \frac{1}{2\pi}.}$$

The anomaly sum rule

$$\pi f_P F_{P\gamma\gamma}(q_1^2, q_2^2) + \int\limits_{\text{cont}}^{\infty} ds\, \Delta_{\text{hadr}}(s, q_1^2, q_2^2|m) = \int\limits_{4m^2}^{\infty} ds\, \Delta_{\text{QCD}}(s, q_1^2, q_2^2|m) = \frac{1}{2\pi}.$$

Duality implemented in a standard way as a low-energy cut on the spectral representation gives

$$\pi f_P F_{P\gamma\gamma}(Q_1^2, Q_2^2) = \int\limits_{4m^2}^{s_{\text{eff}}(Q_1^2, Q_2^2)} ds\, \Delta_{\text{QCD}}(s, Q_1^2, Q_2^2|m).$$

The effective threshold should depend on external kinematical variables, $s_{\text{eff}}\,(Q_1^2, Q_2^2)$

E.g., at large $Q_2^2 \equiv Q^2 \to \infty$ and fixed ratio $\beta = Q_1^2/Q_2^2$, the effective threshold $s_{\text{eff}}(Q_1^2, Q_2^2)$ may be determined by matching to the asymptotic pQCD factorization formula.

One finds that $s_{\text{eff}}(Q^2 \to \infty, \beta)$ in the general case $m \neq 0$ indeed depends on β.

The only exception, in the massless fermion $m = 0$: in this case the asymptotic factorization formula is reproduced for any β if one sets

$$s_{\rm eff}(Q^2 \to \infty, \beta) = 4\pi^2 f_\pi^2$$

For $Q_1^2 = 0$ and $m = 0$, the LD expression for the form factor for the one-flavour case reads:

$$F_{P\gamma}(Q^2) = \frac{1}{2\pi^2 f_P} \frac{s_{\rm eff}(Q^2)}{s_{\rm eff}(Q^2) + Q^2}.$$

Independently of the behaviour of $s_{\rm eff}(Q^2)$ at $Q^2 \to 0$, $F_{P\gamma}(Q^2 = 0)$ is related to axial anomaly.

The LD *model* for the transition form factor emerges when one *assumes* that at finite values of Q^2, $s_{\rm eff}(Q^2, \beta)$ may be well approximated by its value at $Q^2 \to \infty$

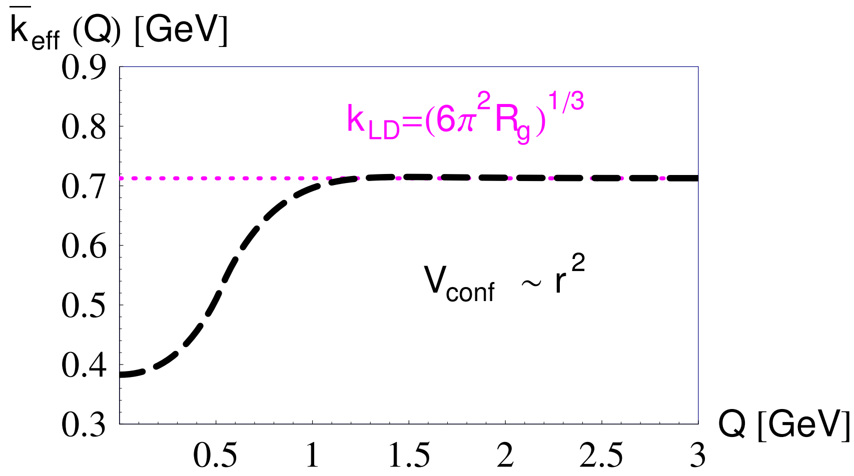
$$s_{\rm eff}(Q^2, \beta) = s_{\rm eff}(Q^2 \to \infty, \beta).$$

P $\to \gamma\gamma^*$ transition form factor in quantum mechanics

$$F_{\pi\gamma}(Q^2) = \frac{1}{f_\pi} \int\limits_0^{s_{\rm eff}(Q^2)} \Delta_{\rm pert}(s, Q^2)\, ds, \qquad s_{\rm eff}(Q^2 \to \infty) \to 4\pi^2 f_\pi^2.$$

Quantum mechanics:

Here is the exact effective threshold obtained for a quantum-mechanical model with HO potential. The parameters are chosen such that the ground state has a typical hadron size 1 Fm.

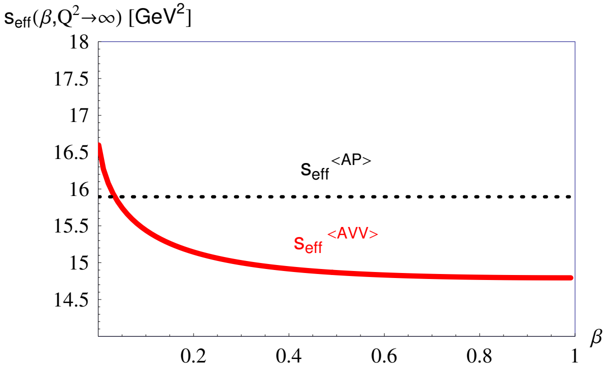


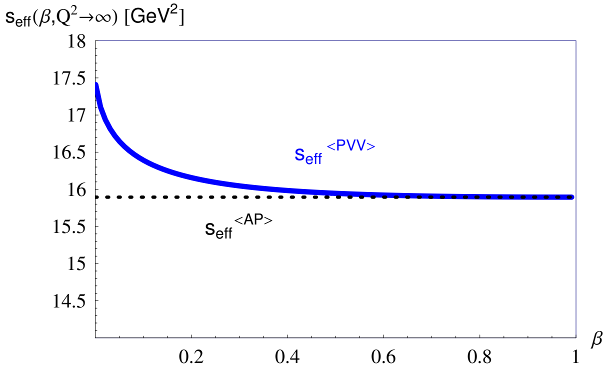
For "light" quarks, the LD threshold gives a very good approximation to the exact threshold at $Q > 1.5$ GeV. For "charm" quarks, works at $Q > 3 - 4$ GeV.

The accuracy of the LD approximation further increases with Q in this region.

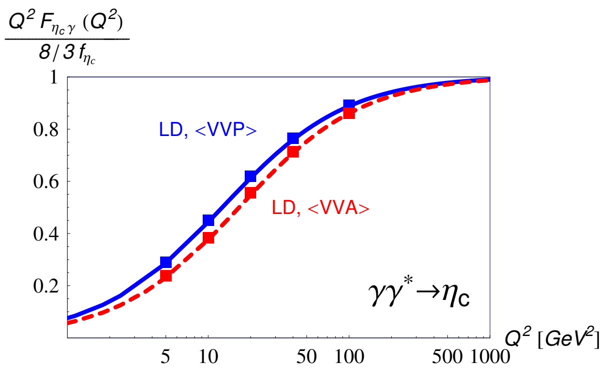
$\eta_c \to \gamma\gamma^*$ transition form factor

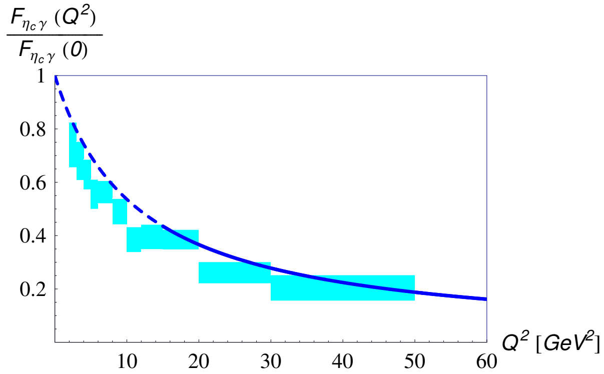
One can consider $\langle AVV\rangle$ and from $\langle PVV\rangle$. LD model for each case may be constructed. From matching to pQCD factorization formula, we find $s_{\rm eff}(Q^2 \to \infty, \beta)$:





Assuming that $s_{\rm eff}(Q^2, \beta) = s_{\rm eff}(Q^2 = \infty, \beta)$:





Try to go to $Q^2 = 0$? Exp: $F_{\eta_c\gamma}(Q^2 = 0) = 0.08 \pm 0.01$ GeV^{-1}.

$\langle AVV\rangle$ yields $F_{\eta_c\gamma}(0) = 0.067$ GeV^{-1}, $\langle PVV\rangle$ yields $F_{\eta_c\gamma}(0) = 0.086$ GeV^{-1}.

Optimistically: LD model for $\langle PVV\rangle$ gives reliable form factor for all Q^2.

Notice: Asymptotics is reached “relatively” fast!

$\eta, \eta' \to \gamma\gamma^*$ transition form factor

$\eta - \eta'$-mixing scheme:

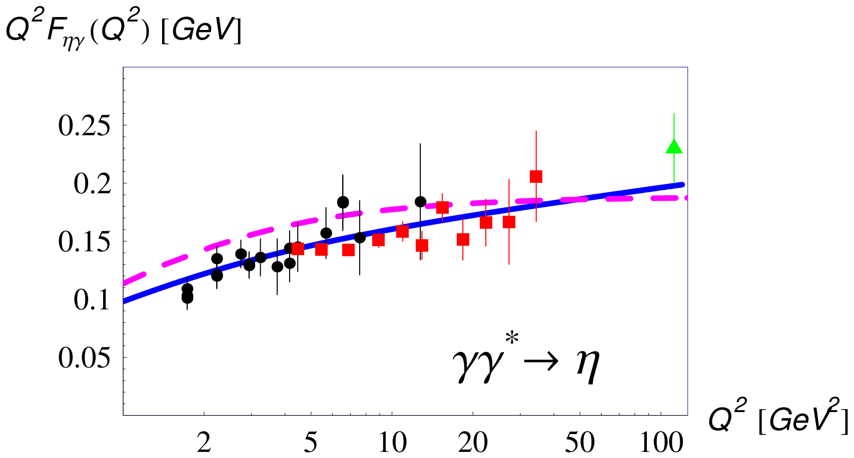
$$F_{\eta\gamma} = \cos(\phi)F_{n\gamma} - \sin(\phi)F_{s\gamma}, \quad F_{\eta'\gamma} = \sin(\phi)F_{n\gamma} + \cos(\phi)F_{s\gamma}, \quad \phi \simeq 38^0$$

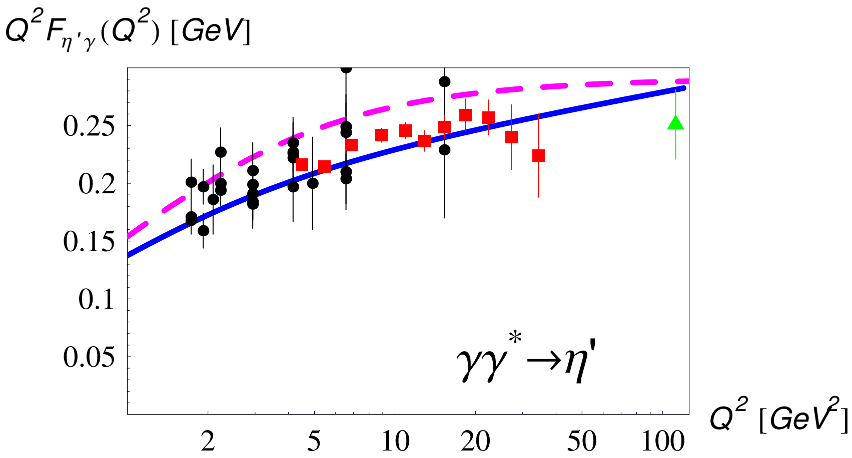
with $n \to \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$ **and** $s \to \bar{s}s$.

Two LD expressions for these form factors:

$$F_{n\gamma}(Q^2) = \frac{1}{f_n} \int\limits_0^{s_{\rm eff}^{(n)}(Q^2)} \Delta_n(s, Q^2)\, ds, \qquad F_{s\gamma}(Q^2) = \frac{1}{f_s} \int\limits_0^{s_{\rm eff}^{(s)}(Q^2)} \Delta_s(s, Q^2)\, ds,$$

Two separate effective thresholds: $s_{\rm eff}^{(n)} = 4\pi^2 f_n^2, \quad s_{\rm eff}^{(s)} = 4\pi^2 f_s^2, \quad f_n \simeq 1.07 f_\pi, \quad f_s \simeq 1.36 f_\pi$.

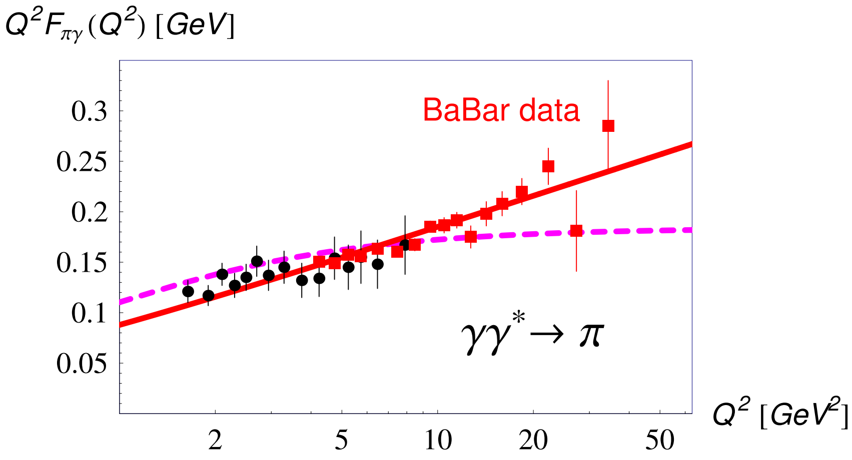


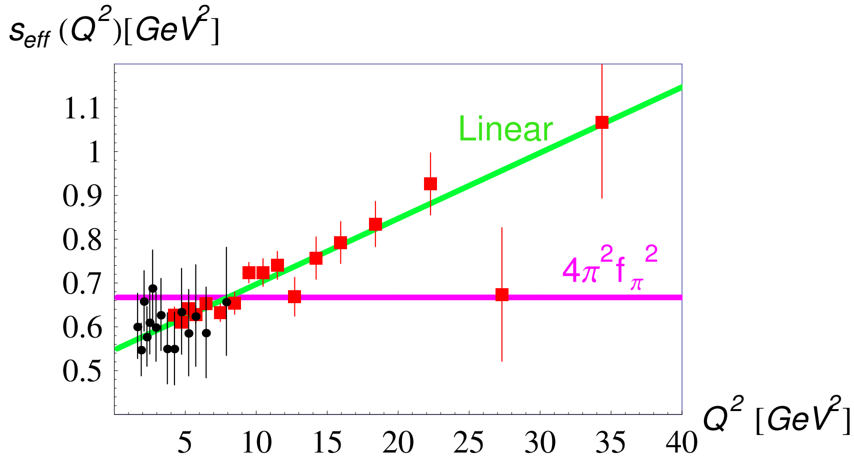


No disagreement between the LD model and the data.

$\pi^0 \to \gamma\gamma^*$ transition form factor

For the pion transition form factor one observes a clear disagreement of the LD model with the BaBar data.





Left: CLEO+CELLO (black), BaBar (red)data vs LD prediction for F_π.

Right: equivalent threshold for the BaBar data. It may be well approximated by a linear rising function.

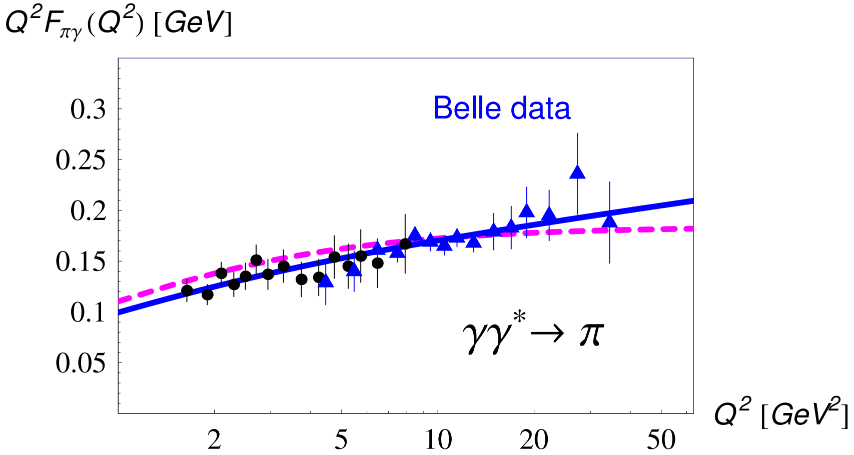
This means that - opposite to

(i) the η and η' cases and

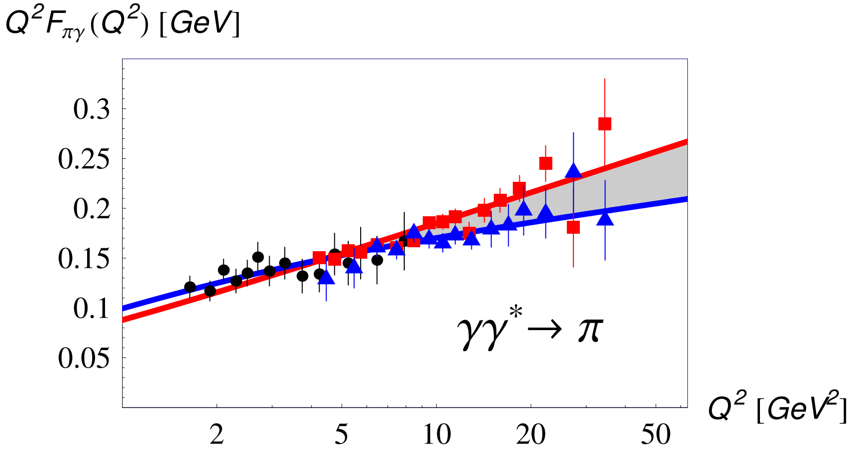
(ii) the lessons from quantum mechanics,

the violations of LD rise with Q even in the region $Q^2 \simeq 40$ GeV^2!

Puzzle: why nonstrange components in η, η' and π^0 should behave so much differently?



$Q^2F_{\pi\gamma}(Q^2)$ [GeV]
0.3
0.25
0.2
0.15
0.1
0.05
Belle data
$\gamma\gamma^* \to \pi$
Q^2 [GeV^2]
2
5
10
20
50



$Q^2F_{\pi\gamma}(Q^2)$ [GeV]
0.3
0.25
0.2
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0.1
0.05
$\gamma\gamma^* \to \pi$
Q^2 [GeV^2]
2
5
10
20
50

Can ~log (Q^2) rise of $Q^2 F(Q^2)$ be understood?

Anomaly sum rule for one real and one virtual photon:

$$\int_0^\infty ds\, \Delta(s, Q^2) = \int_{4m^2}^\infty ds\, \Delta_{\rm QCD}(s, Q^2) = \frac{1}{2\pi}.$$

The absorptive part of $F(p^2, Q^2)$ has the form

$$\Delta(s, Q^2) = \pi\delta(s - m_\pi^2)\ \sqrt{2} f_\pi\ F_{\pi\gamma}(Q^2) + \theta(s - s_{\rm th})\ \Delta^{I=1}_{\rm cont}(s, Q^2).$$

$F_{\pi\gamma}(Q^2)$ **then takes the form**

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\sqrt{2}\,\pi^2 f_\pi}\left[1 - 2\pi \int_{s_{\rm th}}^\infty ds\ \Delta^{I=1}_{\rm cont}(s, Q^2)\right].$$

$$F_{\bar{q}q}(Q^2) = \frac{1}{2\sqrt{2}\,\pi^2 f_q}\left[1 - 2\pi \int_{s_{\rm th}}^\infty ds\, \Delta^{I=0}_{\rm cont}(s, Q^2)\right],$$

$$F_{\bar{s}s}(Q^2) = \frac{1}{2\sqrt{2}\,\pi^2 f_s}\left[1 - 2\pi \int_{s_{\rm th}}^\infty ds\, \Delta^{\bar{s}s}_{\rm cont}(s, Q^2)\right].$$

The calculation of the $P\gamma$ form factors requires an Ansatz for the continuum spectral densities $\Delta_{\rm cont}(s, Q^2)$ for all three cases.

Duality concept: for $s \to \infty$, $\Delta_{\rm cont}(s,Q^2) \to \Delta_{\rm QCD}(s,Q^2)$.

A simple Ansatz for $\Delta_{\rm cont}(s,Q^2)$:

$$\Delta_{\rm cont}(s,Q^2) = \theta(s - s_{\rm th}) R(s) \Delta^{(0)}_{\rm QCD}(s,Q^2), \ \textbf{with}\ R(s) = \left(1 - \frac{r}{s}\right).$$

One readily calculates the form factor

$$Q^2 F(Q^2) \sim \frac{Q^2}{Q^2 + s_{th}}(s_{th} - r) + r \log\left(\frac{Q^2 + s_{th}}{s_{th}}\right).$$

• **Details of the $R(s)$ at small s are irrelavant for large-Q^2 behavior of the form factor; the presence of higher-order terms $O(1/s^2)$ is irrelevant too: they do not modify the scaling behaviour of the form factor $Q^2F(Q^2) \sim$ const.**

• **The log rise of $Q^2F(Q^2)$ requires $1/s$ terms in the relation between $\Delta_{\rm cont}(s,Q^2)$ and $\Delta_{\rm QCD}(s,Q^2)$. This correction however then leads to violation of pQCD factorization theorems.**

The "best" fit to BaBar data on η, η' and Belle data on π suggest $r = 0.05$ GeV^2.

The "best" fit to BaBar data on π requires $r = 0.17$ GeV^2.

Summary and conclusions

We investigated the π^0, η, η', and η_c tranistion form factors by means of a LD version of QCD sum rules. The key parameter—the effective continuum threshold—was determined by matching the LD form factors to QCD factorization formulas.

Our main conclusions are as follows:

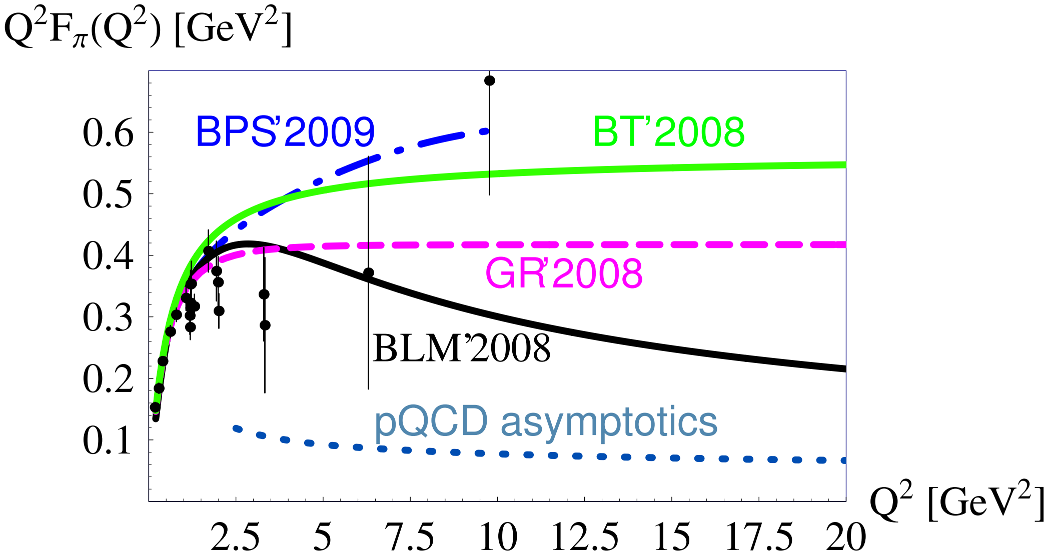
• For the $P\gamma\gamma^*$ form factors, the LD model should work well in the region $Q^2 \geq$ a few GeV^2. LD model works reasonably well for $\eta_c \to \gamma\gamma^*$, $\eta \to \gamma\gamma^*$ and $\eta' \to \gamma\gamma^*$ form factors. For $\pi \to \gamma\gamma^*$, the BaBar data indicate extreme violation of local duality prompting a linearly rising (instead of a constant) effective threshold. On the contrary, the Belle data indicate an agreement with the predictions of the LD model.

• Nevertheless, a better fit to the full set of the meson-photon form factor data seem to prefer a small logarithmic rise of $Q^2F(Q^2)$. If established experimentally, this rise would require the presence of $1/s$ duality-violating term in the ratio of the hadron and the QCD spectral densities.

• A good accuracy of the LD model has implications for the pion *elastic* form factor: one can show that the accuracy of the LD model for the *elastic* form factor increases with Q^2 in the region $Q^2 \approx 4-8$ GeV^2. The accurate data on the pion form factor suggest that the LD limit for the effective threshold $s_{\rm eff}(\infty) = 4\pi^2 f_\pi^2$ may be reached already at $Q^2 = 5-6$ GeV^2. Should be tesable with JLab upgrade.

Elastic form factor

Some recent results on the pion elastic form factor are shown on the plot:



No conclusive results have been obtained and we still have a strong discrepancy between the results from various theoretical approaches.

The basic object: $\langle 0|T\, j^5_\alpha j_\mu j^5_\beta|0\rangle$.

j^5_α, j^5_β **- are the pion interpolating axial currents.** j_μ **is the electromagnetic current.**

In QCD this correlator may be calculated by applying OPE. Duality assumption says that the contribution of the excited states is dual to the high-energy region of the perturbative diagrams.

Using this assumption, the sum rule takes the form

$$f_\pi^2 F_\pi(Q^2) \;=\; \int\limits_0^{s_{\rm eff}(\tau,Q^2)} ds_1 \int\limits_0^{s_{\rm eff}(\tau,Q^2)} ds_2 e^{-\frac{(s_1+s_2)\tau}{2}} \Delta_{\rm pert}(s_1, s_2, Q^2) + \frac{\langle\alpha_s G^2\rangle}{24\pi}\tau + \frac{4\pi\alpha_s\langle\bar{q}q\rangle^2}{81}(13 + Q^2\tau)\tau^2 + \cdots$$

$\Delta_{\rm pert}$ **are double spectral densities of 3-point diagrams of perturbation theory.**

We want to study the form factor at large Q. The form factor of a bound state should decrease with Q; however, power corrections on the r.h.s. are polynomials in Q and thus rise with Q. So, this expression cannot be directly used at large Q. To apply sum rule at large Q one of the few possibilities is just set $\tau = 0$.

The Local – duality (LD) limit $\tau \to 0$: then "bad" power corrections vanish

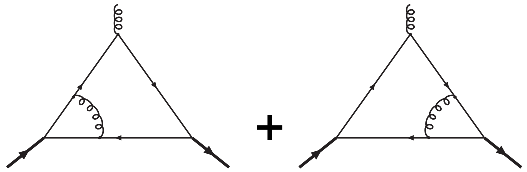
$$F_\pi(Q^2) = \frac{1}{f_\pi^2} \int\limits_0^{s_{\rm eff}(Q^2)} ds_1 \int\limits_0^{s_{\rm eff}(Q^2)} ds_2\; \Delta_{\rm pert}^{(VAV)}(s_1, s_2, Q^2).$$

For any given prediction for the form factor $F_\pi(Q^2)$, one can calculate the equivalent $s_{\rm eff}(Q^2)$. The problem is now how to determine the "true" $s_{\rm eff}(Q^2)$.

Properties of the spectral functions

- **Vector Ward identity at $Q^2 = 0$ relates 3-point and 2-point functions.**

- **Factorization at $Q^2 \to \infty$: the leading $1/Q^2$ behavior of the spectral function is given by**



If we set

$$s_{\rm eff}(Q^2 = 0) = \frac{4\pi^2 f_\pi^2}{1 + \alpha_s/\pi} \qquad s_{\rm eff}(Q^2 \to \infty) = 4\pi^2 f_\pi^2,$$

then the form factor obtained from the LD sum rule satisfies the correct normalization at $Q^2 = 0$ and reproduces the asymptotic behavior according to the factorization theorem for the form factor at $Q^2 \to \infty$.

The two values are not far from each other, construct an interpolation function $s_{\rm eff}(Q^2)$ for all Q^2.

The local – duality model for hadron elastic form factors :

a. Based on a dispersive three-point sum rule at $\tau = 0$ (i.e. infinitely large Borel mass parameter). In this case dangerous power corrections $\sim (\tau Q^2)^n$ vanish and the details of the non-perturbative dynamics are hidden in one quantity — the effective threshold $s_{\rm eff}(Q^2)$.

b. Makes use of a model for $s_{\rm eff}(Q^2)$ based on a smooth interpolation between its values at $Q^2 = 0$ determined by the Ward identity and at $Q^2 \to \infty$ determined by factorization. Since these values are not far from each other, the details of the interpolation are not essential.

Obviously, the LD model for the effective continuum is a model which does not take into account the details of the confinement dynamics. The only property of theory relevant for this model is factorization of hard form factors.

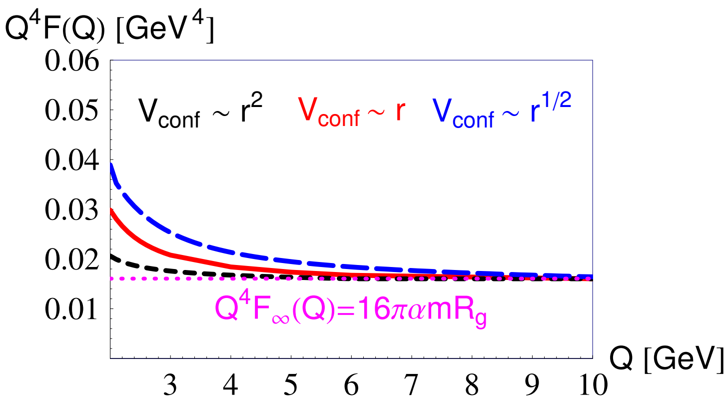
The model may be tested in quantum mechanics for the case of the potential containing the Coulomb and Confining interactions.

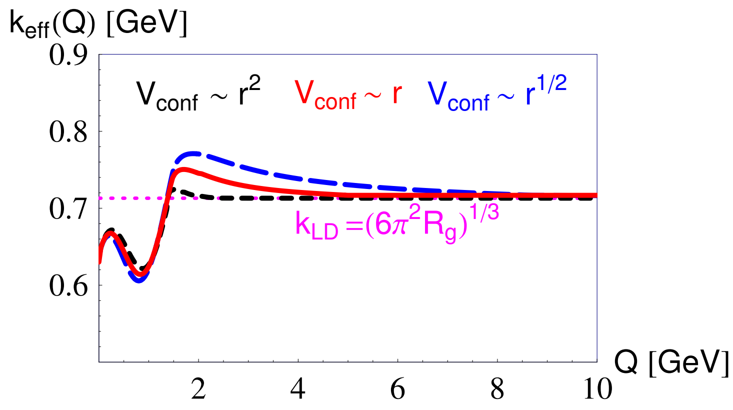
• The form factor satisfies factorization theorem similar to QCD. LD sum rules are very similar to QCD; the spectral densities are calculated from diagrams of NR field theory.

• The exact form factor may be calculated and confronted with LD model, probing its accuracy.

Elastic form factor

Results for elastic form factor in quantum-mechanical potential model





The plots show the results for the elastic form factor in potential model. The potential containing a Coulomb interaction and a confining part for several different confining parts: HO potential, linear potential, and $r^{1/2}$ potential.

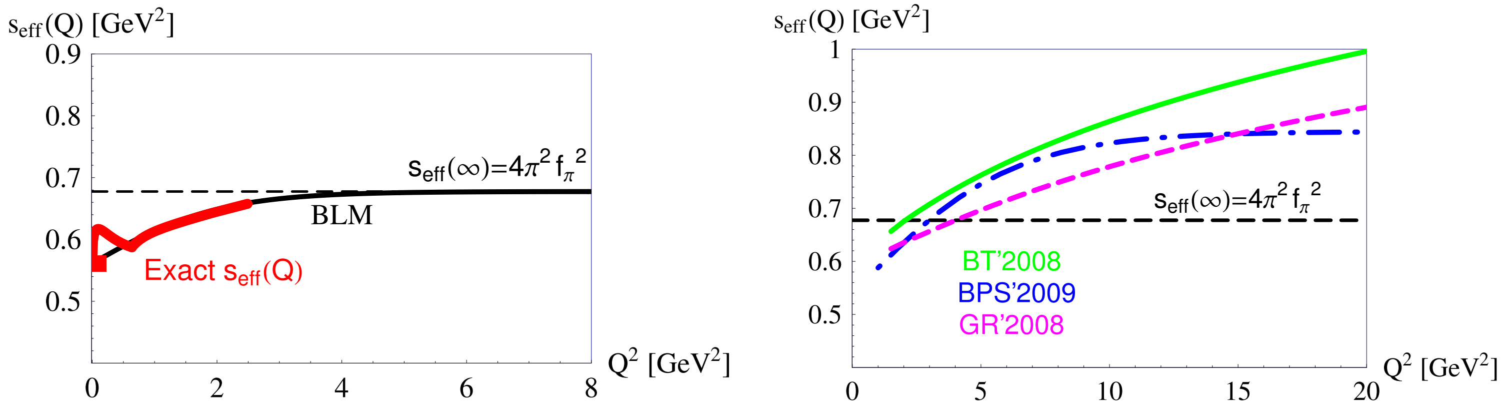
Left: the exact form factors for these potentials

Right: the corresponding equivalent effective thresholds.

An important conclusion from these plots:

Independently of the form of the confining part, the accuracy of the LD model increases with Q already starting with relatively low values $Q \simeq 2-3$ GeV.

Results for elastic pion form factor in QCD:



Left plot: the equivalent threshold extracted from the experimental data and the LD model. So far one can see no disagreement between the two.

Right: the equivalent thresholds for other theoretical predictions are given on the right plot. Obviously, these results imply that the accuracy of the LD model decreases with Q^2 even at Q^2 as large as $Q^2 = 20$ GeV^2. Let us notice that this is in conflict with our experience from quantum mechanics.

The future accurate data expected from JLab in the range up to $Q^2 = 8$ GeV^2 should decide.