A nonperturbative light-front coupled-cluster method

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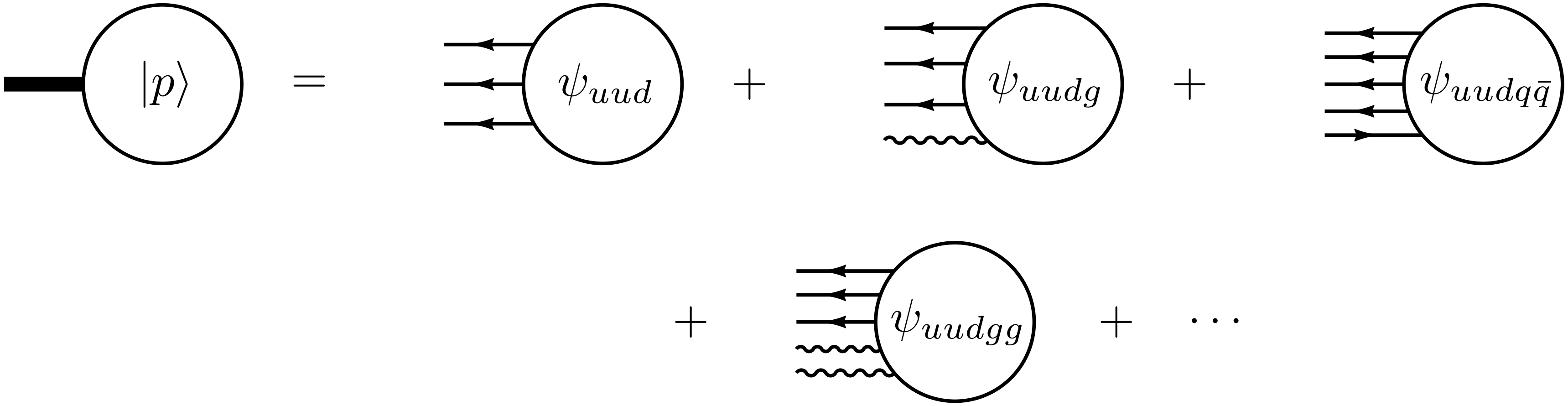
Outline

- introduction
- bound-state eigenvalue problem
- light-front coupled-cluster method
  - Chabysheva and jrh, PLB **711**, 417 (2012)
  - also next talk by Chabysheva
- sample application
- form-factor calculation
- summary

Introduction

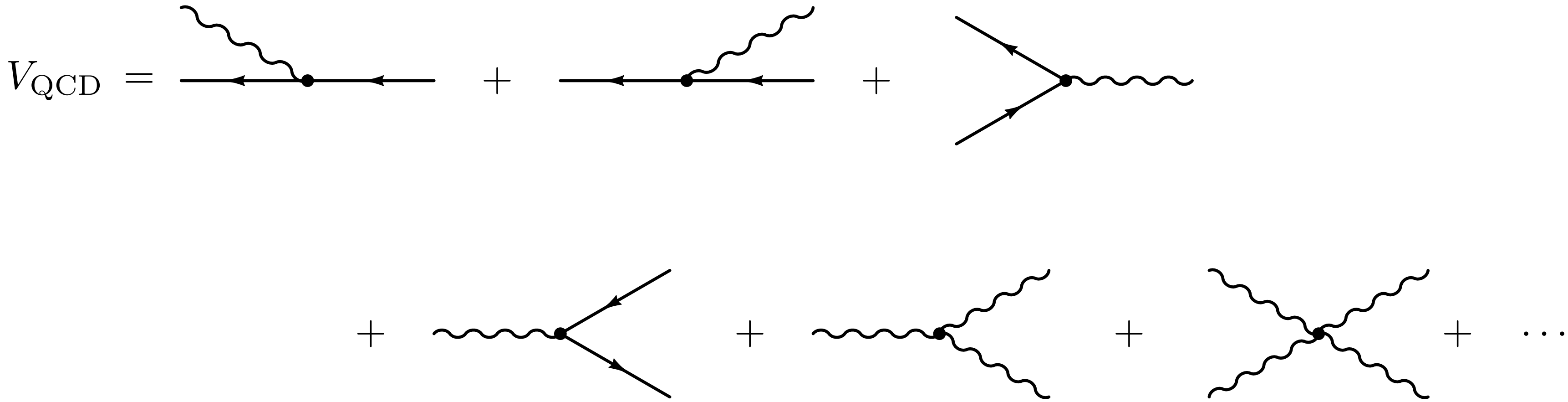
- wish to compute hadron structure in terms of wave functions



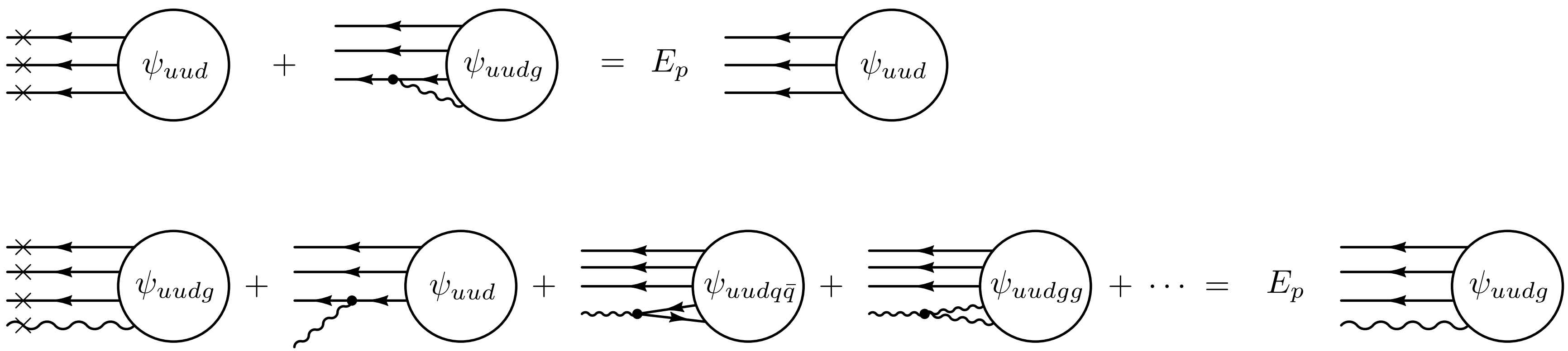
- must truncate in some fashion
- the LFCC method avoids Fock-space truncation

Equations for wave functions

Solve $\left(\text{K.E.} + V_{\rm QCD}\right)|p\rangle = E_p|p\rangle$ with $E_p = \sqrt{m_p^2 + p^2}$ and



Equivalent to coupled integral equations



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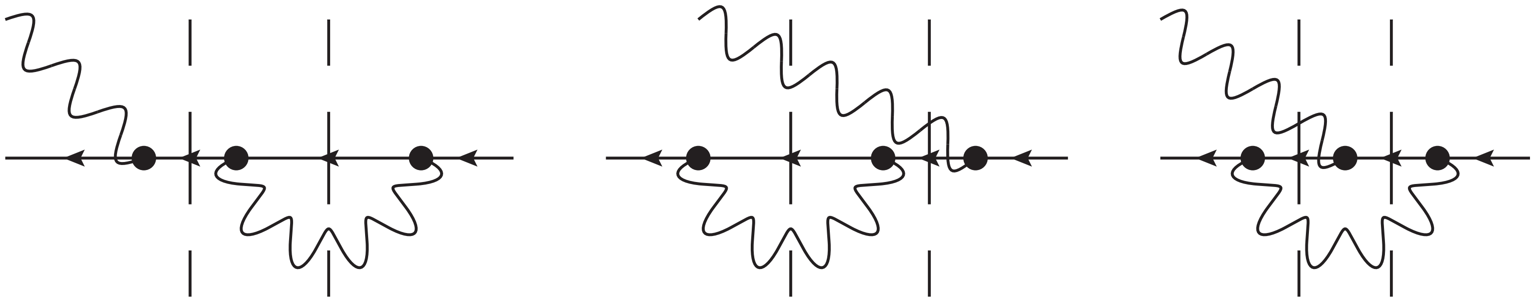
Uncanceled divergences

- for example, the Ward identity of gauge theories is destroyed by truncation

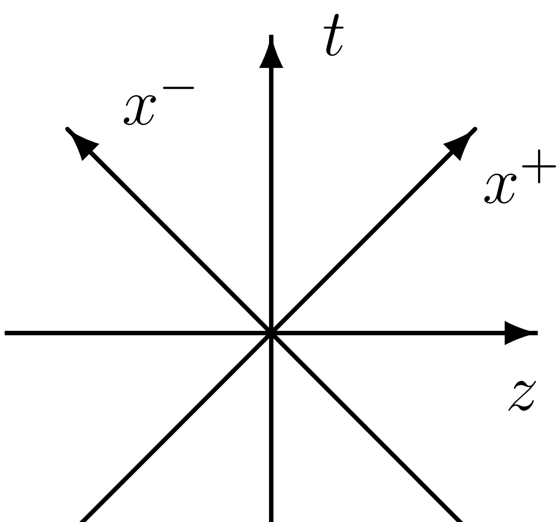


- analog in Feynman perturbation theory
  - separate diagrams into time-ordered diagrams
  - discard time orderings that include intermediate states with more particles than some finite limit
  - destroys covariance, disrupts regularization, and induces spectator dependence for subdiagrams
- in the nonperturbative case, this happens not just to some finite order in the coupling but to all orders

Light-cone coordinates

Dirac, RMP **21**, 392 (1949);
Brodsky, Pauli, and Pinsky, Phys. Rep. **301**, 299 (1997).

- time: $x^+ = t + z$
- space: $\underline{x} = (x^-, \vec{x}_\perp), \quad x^- \equiv t - z, \quad \vec{x}_\perp = (x, y)$
- energy: $p^- = E - p_z$
- momentum: $\underline{p} = (p^+, \vec{p}_\perp), \quad p^+ \equiv E + p_z, \quad \vec{p}_\perp = (p_x, p_y)$
- mass-shell condition: $p^2 = m^2 \quad \Rightarrow \quad p^- = \frac{m^2 + p_\perp^2}{p^+}$



Mass eigenvalue problem

- Pauli and Brodsky, PRD **32**, 1993 (1985); 2001 (1985)

$$\mathcal{P}^-|\underline{P}\rangle = \frac{M^2 + P_\perp^2}{P^+}|\underline{P}\rangle, \quad \underline{\mathcal{P}}|\underline{P}\rangle = \underline{P}|\underline{P}\rangle.$$

- no spurious vacuum contributions to eigenstates
  - $p^+ > 0$ for all particles
  - cannot produce particles from vacuum and still conserve p^+
  - (but difficult to analyze structure of physical vacuum)
- boost-invariant separation of internal and external momenta
  - longitudinal momentum fractions $x_i \equiv p_i^+/P^+$
  - relative transverse momenta $\vec{k}_{i\perp} \equiv \vec{p}_{i\perp} - x_i\vec{P}_\perp$

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Coupled-cluster (CC) method

- originated with Coester, Nucl. Phys. **7**, 421 (1958) and Coester and Kümmel, Nucl. Phys. **17**, 477 (1960), with applications to the many-body Schrödinger equation in nuclear physics.
- applied to many-electron problem in molecules by Čižek, J. Chem. Phys. **45**, 4256 (1966).
- form eigenstate as $e^T|\phi\rangle$ where
  - $|\phi\rangle$ is product of single-particle states
  - terms in T annihilate states in $|\phi\rangle$ and create excited states, to build in correlations
  - truncate T at some number of excitations
- review: RJ Bartlett and M Musial, RMP **79**, 291 (2007).

Light-front CC (LFCC) method

- wish to solve $\mathcal{P}^-|\psi\rangle = \frac{M^2+P_\perp^2}{P^+}|\psi\rangle$.
- write eigenstate as $|\psi\rangle = \sqrt{Z}e^T|\phi\rangle$
  - Z controls normalization: $\langle\psi'|\psi\rangle = \delta(\underline{P}' - \underline{P})$.
  - $|\phi\rangle$ is the valence state, with $\langle\phi'|\phi\rangle = \delta(\underline{P}' - \underline{P})$.
  - T contains terms that only increase particle number.
  - T conserves J_z, light-front momentum $\underline{P}$, charge, $\ldots$
  - $p^+ > 0 \Rightarrow T$ must include annihilation and powers of T include contractions.
- construct $\overline{\mathcal{P}^-} = e^{-T}\mathcal{P}^- e^T$ and let P_v project onto the valence Fock sector. Then, have coupled system:

$$P_v\overline{\mathcal{P}^-}|\phi\rangle = \frac{M^2+P_\perp^2}{P^+}|\phi\rangle \text{ and } (1-P_v)\overline{\mathcal{P}^-}|\phi\rangle = 0.$$

A soluble model

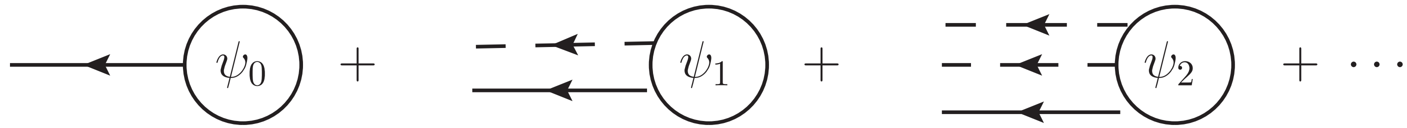
- light-front analog of the Greenberg–Schweber model
  - static fermionic source that emits and absorbs bosons without changing its spin

Chabysheva and jrh, PLB **711**, 417 (2012);
Brodsky, jrh, and McCartor, PRD **58**, 025005 (1998);
Greenberg and Schweber, N Cim **8**, 378 (1958).

- not fully covariant
  - hides some of the power of the LFCC method, but is sufficient to show how the method can be applied.
  - states are all limited to having a fixed total transverse momentum $\vec{P}_\perp$, which we take to be zero.

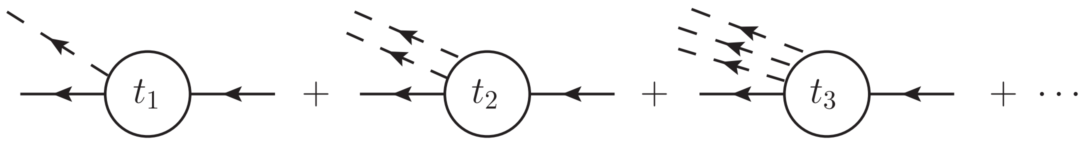
Fock-state expansions

$|\psi\rangle =$

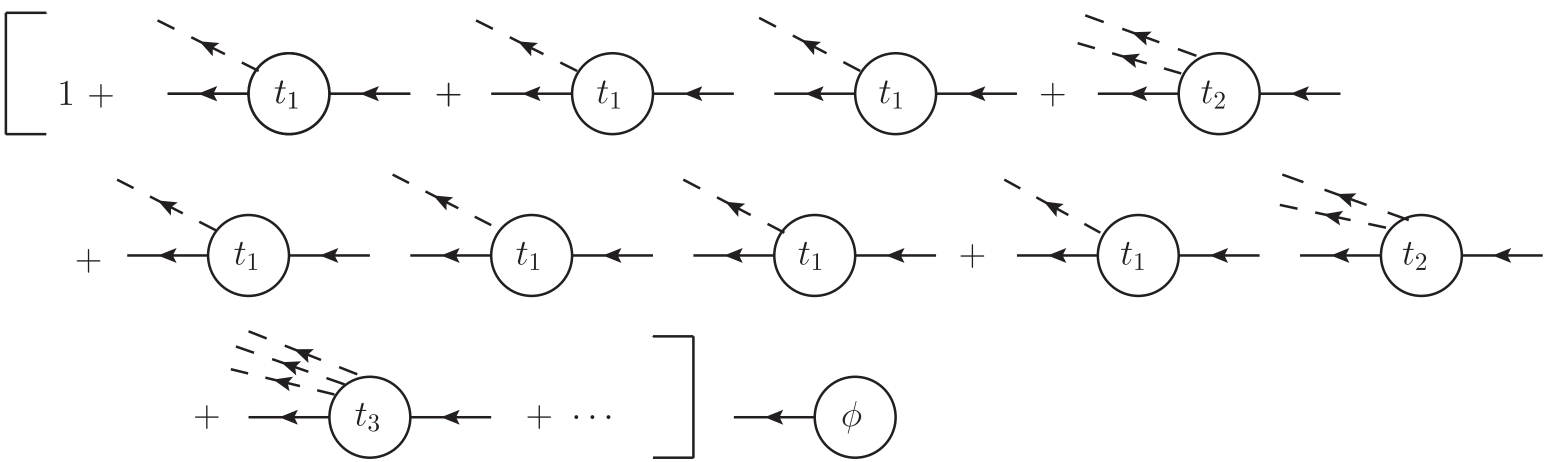


Instead, in LFCC:

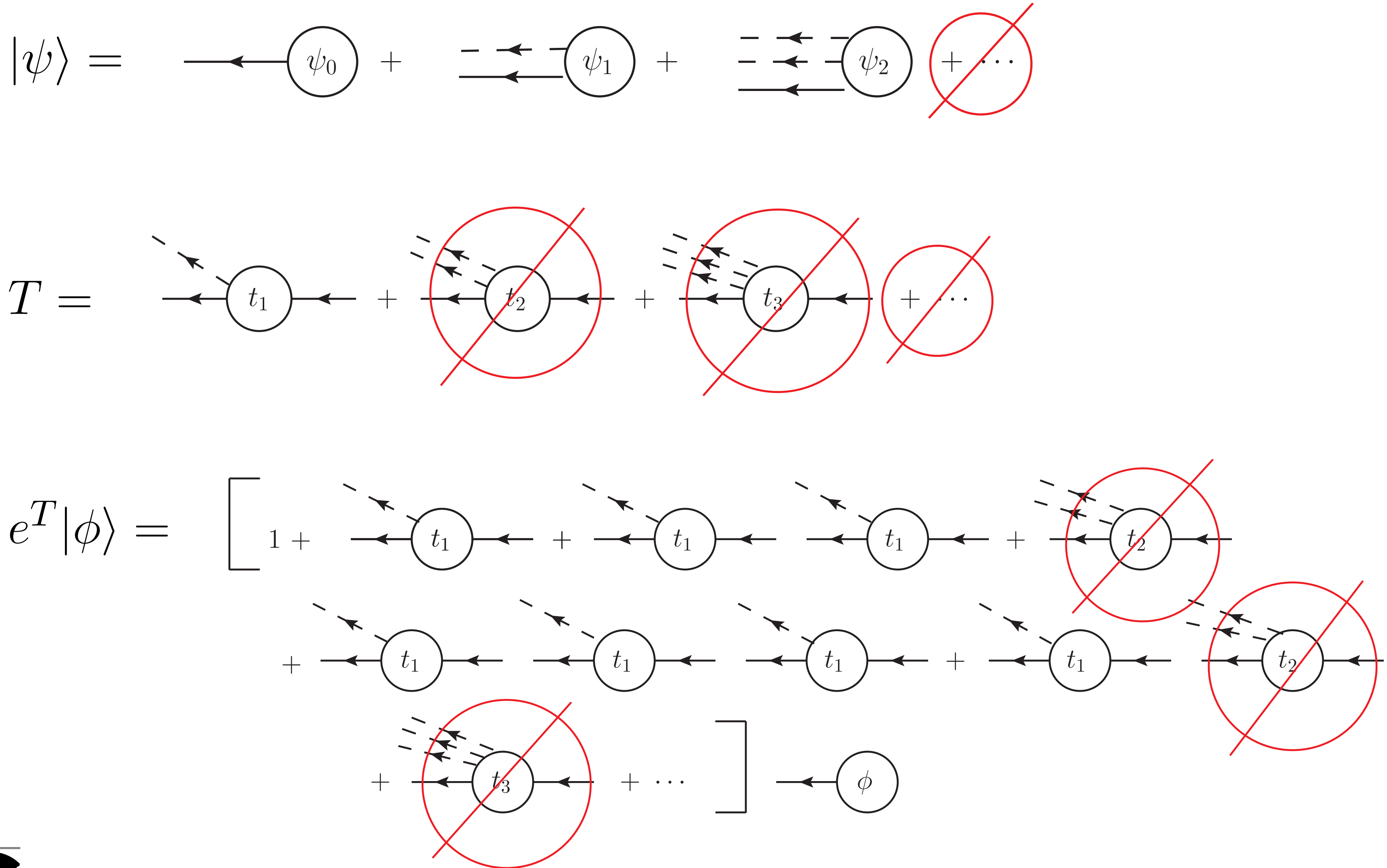
$T =$



$e^T|\phi\rangle =$



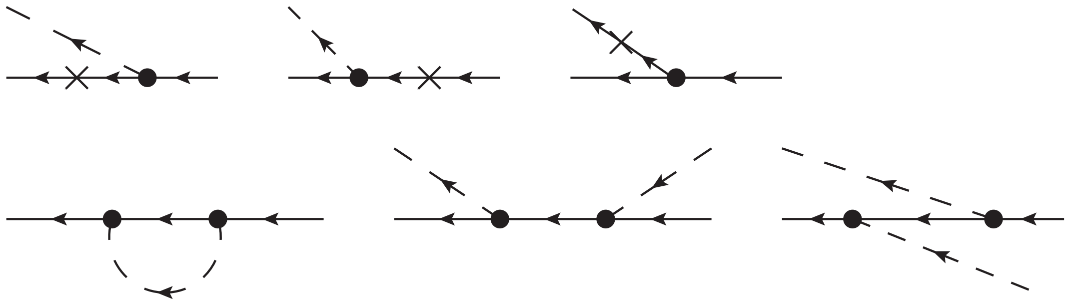
Truncation



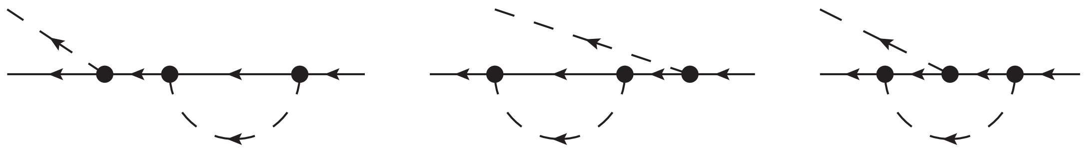
Effective Hamiltonian

- constructed from Baker–Hausdorff expansion.

$[\mathcal{P}^-, T] \to$

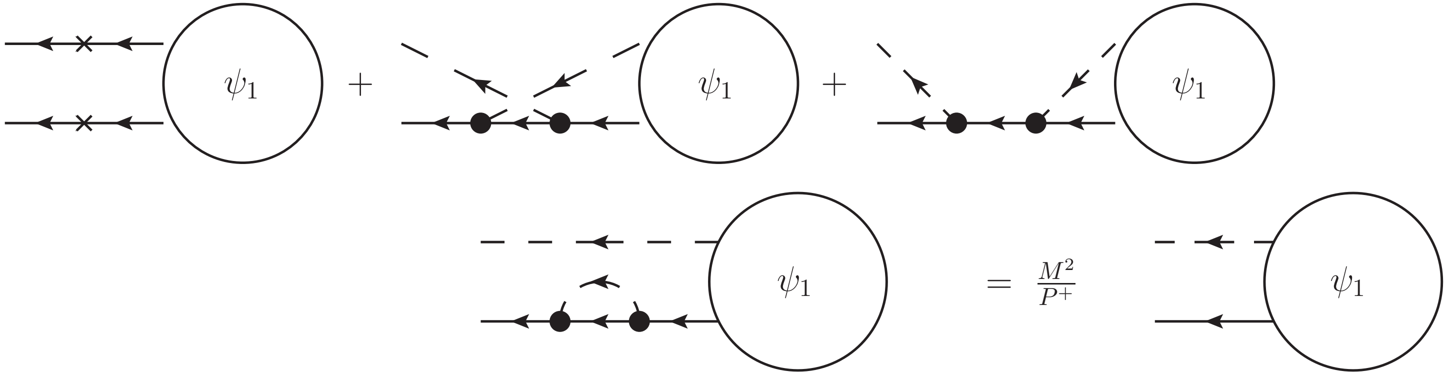


$[[\mathcal{P}^-, T], T] \to$

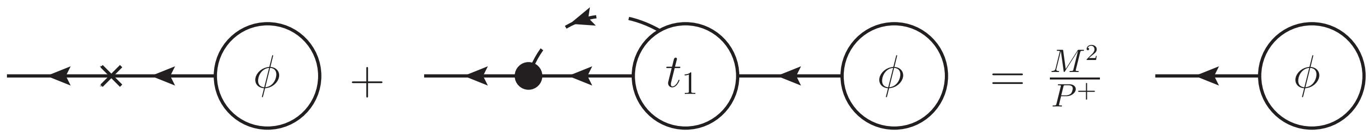


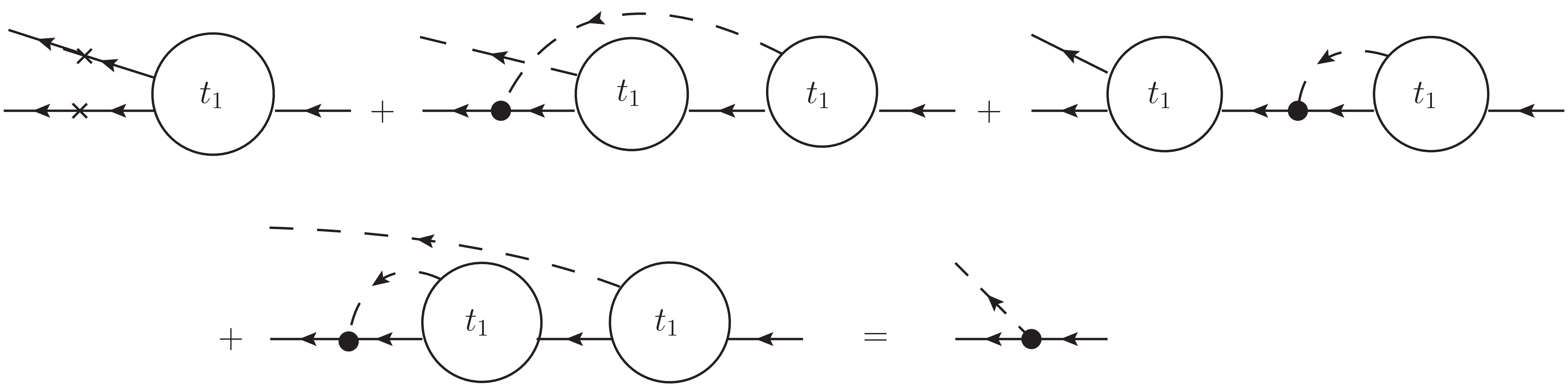
- list only terms that connect the lowest Fock sectors.
- the self-energy contribution is the same in all Fock sectors
- contains all three of the diagrams analogous to those for the Ward identity in QED

Eigenvalue problems



In LFCC:





Exact solution

- in this special case, the exponential operator e^T generates the exact solution, with
$t_{ls}(\underline{q},\underline{p}) = \frac{-g}{\sqrt{16\pi^3 q^+}}\left(\frac{p^+}{p^+ + q^+}\right)^\gamma \frac{q^+/P^+}{\mu_l^2 + q_\perp^2}$.
- the fact that the self-energy loop is the same in the valence sector and the one-fermion/one-boson sector plays a critical role. It contributes

$$-\frac{g}{P^+}\int\frac{d\underline{q}}{\sqrt{16\pi^3 q^+}}\theta(P^+ - q^+)\left(\frac{P^+ - q^+}{P^+}\right)^\gamma \sum_l (-1)^l t_{l\pm}(\underline{q},\underline{P}-\underline{q}),$$

regulated by the PV ($l=1$) term.
- the expression for the loop obtained in the valence sector is exactly what is needed to obtain the necessary cancellations.

Dirac form factor

- compute the Dirac form factor for the dressed fermion from a matrix element of the current $J^+ = \overline{\psi}\gamma^+\psi$
- the current couples to a photon of momentum q
- the matrix element is generally

$$\langle\psi^\sigma(\underline{P}+\underline{q})|16\pi^3 J^+(0)|\psi^\pm(\underline{P})\rangle = 2\delta_{\sigma\pm}F_1(q^2) \pm \frac{q^1 \pm iq^2}{M}\delta_{\sigma\mp}F_2(q^2)$$

  with F_1 and F_2 the Dirac and Pauli form factors.
- in the present model, the fermion cannot flip its spin; therefore, F_2 is zero, and we investigate only F_1

Expectation values

- expectation value for op $\hat{O}$: $\langle\hat{O}\rangle = \frac{\langle\phi|e^{T^\dagger}\hat{O}e^T|\phi\rangle}{\langle\phi|e^{T^\dagger}e^T|\phi\rangle}$
- direct computation requires infinite sum.
- define $\overline{O} = e^{-T}\hat{O}e^T$ and $\langle\tilde{\psi}| = \langle\phi|\frac{e^{T^\dagger}e^T}{\langle\phi|e^{T^\dagger}e^T|\phi\rangle}$
- then $\langle\hat{O}\rangle = \langle\tilde{\psi}|\overline{O}|\phi\rangle$ and
  $\langle\tilde{\psi}'|\phi\rangle = \langle\phi'|\frac{e^{T^\dagger}e^T}{\langle\phi|e^{T^\dagger}e^T|\phi\rangle}|\phi\rangle = \delta(\underline{P}' - \underline{P})$
- $\overline{O}$ computed from Baker–Hausdorff expansion:
  $\overline{O} = \hat{O} + [\hat{O}, T] + \frac{1}{2}[[\hat{O}, T], T] + \cdots$
- $\langle\tilde{\psi}|$ is a left eigenvector of $\overline{\mathcal{P}^-}$:
  $\langle\tilde{\psi}|\overline{\mathcal{P}^-} = \langle\phi|\frac{e^{T^\dagger}\mathcal{P}^-e^T}{\langle\phi|e^{T^\dagger}e^T|\phi\rangle} = \langle\phi|\overline{\mathcal{P}^-}^\dagger\frac{e^{T^\dagger}e^T}{\langle\phi|e^{T^\dagger}e^T|\phi\rangle} = \frac{M^2+P_\perp^2}{P^+}\langle\tilde{\psi}|$

LFCC approximation

- the form factor is approximated by the matrix element

$$F_1(q^2) = 8\pi^3 \langle \widetilde{\psi}^{\pm}(\underline{P}+\underline{q}) | \overline{J^+(0)} | \phi^{\pm}(\underline{P}) \rangle,$$

  with $\overline{J^+(0)} = J^+(0) + [J^+(0), T] + \cdots$

- for this model, there are no contributions from fermion-antifermion pairs, so that

$$J^+(0) = 2\sum_s \int \frac{d\underline{p}'}{\sqrt{16\pi^3}} \int \frac{d\underline{p}}{\sqrt{16\pi^3}} b_s^\dagger(\underline{p}') b_s(\underline{p}),$$

- only the first two terms of the Baker–Hausdorff expansion contribute to the matrix element
- the first term contributes $1/8\pi^3$

Evaluation of the form factor

- the left-hand wave function takes the form
$$l^\sigma_{ls}(\underline{q},\underline{P}) = \delta_{\sigma s}\frac{-g}{\sqrt{16\pi^3 q^+}}\left(\frac{P^+-q^+}{P^+}\right)^\gamma \frac{q^+/P^+}{\mu_l^2+q_\perp^2}\tilde{l}(q^+/P^+)$$
with $\tilde{l}$ the solution of a 1D integral equation
- if $\tilde{l}$ is computed in quadrature, the integrals remaining in F_1 can be computed from the same quadrature rule for any chosen value of q^2
- if $\tilde{l}$ is instead constructed as an expansion in g^2, F_1 can also be constructed as an expansion
- in any case, in the limit of $q^2 \to 0$, we have $F_1(0) = 1$, consistent with the unit charge in the current $J^+ = \bar{\psi}\gamma^+\psi$

Summary

- advantages of LFCC approach:
  - no Fock-space truncation.
  - no sector dependence or spectator dependence.
  - systematically improvable.
- future work:
  - QED (see 1203.0250 and talk by Chabysheva):
    - dressed-electron-state
    - dressed-photon state.
    - extend dressed-electron state to include e^+e^-.
    - muonium, positronium.
  - symmetry breaking in scalar theories.
  - QCD
    - holographic & full